Computationally Efficient Multiuser Detection for Coded CDMA

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Abstract: Computationally efficient multiuser detection for coded asynchronous CDMA systems is investigated. The particular receiver studied is a near-far resistant multi-user detector known as the projection receiver (PR) [originally developed in [1, 2]]. The PR performs multiple access interference resolution for CDMA with error control coding. The output of the front-end of the projection receiver yields a metric for decoding of the coded sequences. This metric allows the use of a standard sequence decoder (e.g., Viterbi algorithm, M-algorithm) for the error control code. The metric computation can be performed adaptively by an extension of the familiar recursive least-squares (RLS) algorithm. The adaptive PR operates on a single sample per chip. In this paper it is shown that for random spreading codes this algorithm simplifies and can be executed an order of magnitude faster by exploiting the average cross-correlation of the spreading sequences. The performance of both algorithms are studied for CDMA with random spreading sequences and compared to theoretical performance bounds.

1. INTRODUCTION

In Code-Division Multiple Accessing (CDMA) systems, multiple users transmit simultaneously and independently over a common channel using preassigned spreading waveforms or signature sequences that uniquely identify the users. In practical applications these signature sequences cannot be made orthogonal, and the conventional correlation receivers suffer from the *near-far* problem which requires strict power control for satisfactory operation. Furthermore, the multi-user interference degrades performance of the conventional detector significantly when the number of active users exceeds about 10% of the processing gain.

Multi-user receivers whose performance is largely unaffected by power variations of other users are called *near-far resistant*. Optimal detection of asynchronous CDMA is theoretically possible [4], but it's practical realization is not feasible due to the large complexity of such detectors. The addition of forward error control (FEC) coding to CDMA complicates the receiver structure, and often FEC systems are simply concatenated with the multiuser detector using heuristic arguments. In this paper we study combined multi-user detection and FEC decoding with a proper transfer of appropriate 'soft' information between receiver stages.

2. THE PROJECTION RECEIVER (PR)

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2.1 Review of Results

The PR [2, 3] is employed as the first receiver stage for multiuser interference cancellation in a structured manner, followed by decoding of the FEC code. The resultant class of receivers is ideally near-far resistant (a property it shares with the decorrelator) and can be tailored to achieve complexity-performance tradeoffs between the optimal detector at the high end and the standard decorrelator (zero-forcing) detector at the low end. The PR cancels the interference statistically, i.e., without explicitly detecting the transmitted data of the interfering users. The PR utilizes a linear preprocessor which projects the received signal onto the signal subspace orthogonal to the signal space "contaminated" by the interference. The projection operation results in an "effective energy loss" $\mathcal{L}$ (in dB) with respect to an interference-free reference system, i.e., in the presence of multi-access interference, an amount $\mathcal{L}$ of extra energy needs to be expended to achieve the performance of the interference-free reference system.

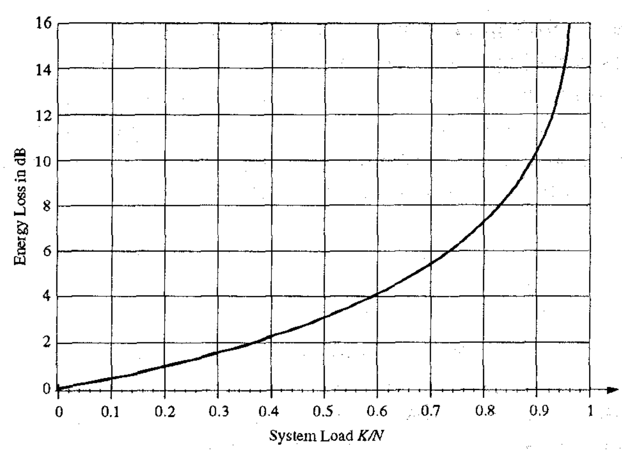

Figure 1: Loss factor of the projection receiver (PR).

This energy loss is given by

$$\mathcal{L} \approx 10\log_{10}\left(\frac{N}{N-K+1}\right)[\text{dB}], \qquad (1)$$

where N is the processing gain and K is the number of simultaneous asynchronous CDMA users. This loss factor is plotted in Figure 1 as a function of K/N. It is remarkable that $\mathcal{L} \leq 3\text{dB}$ for $K \leq N/2$, i.e., even with a 50% system load only 3dB excess energy is required.

Equation (1) can be proven as a lower bound for random spreading codes [3], and simulations demonstrate its achievability for both random spreading sequences and fixed spreading sequences (using Gold codes).

2.2 Detection Algorithm Derivation

The projection receiver is derived from the formulation of the maximum-likelihood receiver, i.e., from

$$\underline{d}^{(\mathrm{ML})} = \arg\min_{\underline{d}\in\mathcal{D}} |\underline{e} - \mathcal{A}\mathcal{W}\underline{d}|^2 , \quad (2)$$

where $\mathcal{A}$ is the matrix whose columns are the asynchronous, and possibly time-varying, spreading sequences, $\mathcal{W}$ is a diagonal matrix of the square root powers of the different users, $\underline{e}$ is the vector of received chip samples of dimension LN, where L is the frame length of the transmission, and $\underline{d}$ is the KL vector of encoded symbols of the different users. The set $\mathcal{D}$ denotes the sequence candidates $\underline{d} \in \{-1, +1\}$ which conform to the FEC codes. The projection receiver reduces decoder complexity by partitioning $\underline{d}$ into two sets,

$$\underline{d} \to \{\underline{d}_u, \underline{d}_c\}, \quad (3)$$

where $\underline{d}_u$ is estimated over the real (unconstrained) domain, and only $\underline{d}_c$ is estimated over the constrained domain, now denoted by $\mathcal{D}_c$. This leads to the partitioned minimization

$$\underline{d}_{(\mathrm{PR})} = \arg\min_{\underline{d}_c\in\mathcal{D}_c}\min_{\underline{d}_u} |\underline{e} - \mathcal{A}\mathcal{W}\underline{d}|^2 . \quad (4)$$

The receiver in (4) is maximum-likelihood if the powers $\mathcal{W}_u$ of the interfering users are unknown. Let $\mathcal{A}_u$ be $\mathcal{A}$ with the columns corresponding to the unprojected user(s) removed, and let $\mathcal{A}_c$ be the matrix whose columns are the spreading sequences of the unprojected users. The minimization of $\underline{d}_u$ can now be done in closed form and this leads to a sequence metric for $\underline{d}_c$, given by

$$\Lambda(\underline{d}_c) = |\mathcal{M}\left(\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c\right)|^2 . \quad (5)$$

The matrix

$$\mathcal{M} = \mathcal{I} - \mathcal{A}_u\left(\mathcal{A}_u^T\mathcal{A}_u\right)^{-1}\mathcal{A}_u^T \quad (6)$$

is the projection matrix onto the nullspace of $\mathcal{A}_u^T$, i.e., onto the signal sub-space *not* covered by the interfering users spreading sequences which are contained in $\mathcal{A}_u$. From this derives the name "projection receiver".

The FEC detector will now choose as hypothesis the sequence $\underline{d}_c$ with the smallest metric $\Lambda(\underline{d}_c)$, and we show how this metric can be calculated in a recursive way, leading to a sequence detector for the unconstrained user(s), which can be accomplished with a sequence detector like the Viterbi algorithm, or the M-algorithm.

3. ADAPTIVE METRIC GENERATION

We will concentrate on the case where all but one of the users are projected, and let that be user k. The metric calculation according to (5) requires the calculation of the matrix $\mathcal{M}$. For a time-varying CDMA system, i.e., a system where different spreading sequences are used in the different symbol intervals, this poses a significant computational burden in the order of $O(NK^3)$ per symbol. In this section we discuss a recursive procedure which avoids the matrix inversions required for the calculation of (6).

From the sequence metric (5) and (6) we can write

$$\begin{aligned}\Lambda(\underline{d}_c) &= |\mathcal{M}\left(\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c\right)|^2 \\ &= \left|\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c - \mathcal{A}_u\hat{\underline{d}}_u\right|^2 \\ &= |\tilde{\underline{e}} - \mathcal{A}_u\hat{\underline{d}}_u|^2 ,\end{aligned} \quad (7)$$

where $\tilde{\underline{e}} = \underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c$. The estimate $\hat{\underline{d}}_u$ for the unconstrained symbols is obtained as the least-squares (LS) solution to

$$\min_{\underline{d}_u} |\tilde{\underline{e}} - \mathcal{A}_u\underline{d}_u|^2 = \min_{\underline{d}_u} |\tilde{\underline{e}} - \hat{\underline{e}}|^2 \quad (8)$$

The exact LS solution to (8) is given by [3]

$$\hat{\underline{d}}_u = \left(\mathcal{A}_u^T\mathcal{A}_u\right)\mathcal{A}_u^T\left(\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c\right) \quad (9)$$

However, attempting to avoid its algebraic evaluation, we propose to use an adaptive generation of $\mathcal{A}_u\hat{\underline{d}}_u$ to be used in (7). It is based on the recursive least-squares (RLS) algorithm [5, 6], and recursively generates

$$\hat{\underline{e}}(i) = \sum_{j=1}^{L}\sum_{\substack{l=1\\(l\neq k)}}^{K} v_j^{(l)}(i-1)\underline{a}_j^{(l)}(i), \quad (10)$$

where $v_j^{(l)}(i-1)$ is the current estimate of $d_j^{(l)}$, and the index i is the chip index running from $1 \to LN$. $\hat{\underline{e}}(i)$ and $\underline{a}_j^{(l)}(i)$ denote the i-th component of the vectors $\hat{\underline{e}}$ and $\underline{a}_j^{(l)}$, respectively.

The standard LS solution to (8) at chip index n is given by ([5], Page 479),

$$\boldsymbol{\Phi}(n)\boldsymbol{v}(n) = \boldsymbol{\theta}(n), \quad (11)$$

where

$$\boldsymbol{v}(n) = \left[v_1^T(n), \cdots, v_L^T(n)\right] \quad (12)$$

with

$$v_j^T(n) = \left[v_j^{(1)}(n), \ldots, v_j^{(k-1)}(n), v_j^{(k+1)}(n), v_j^{(K)}(n)\right] \quad (13)$$

is the LS estimate of all *projected* users at chip time n, and k is the unprojected user. Furthermore, denoting the i-th row of $\mathcal{A}_u$ by $\boldsymbol{a}(i)$,

$$\boldsymbol{\Phi}(n) = \sum_{i=1}^{n}\boldsymbol{a}(i)\boldsymbol{a}^T(i) \quad (14)$$

is the $L(K-1) \times L(K-1)$ correlation matrix of the spreading sequences, and

$$\boldsymbol{\theta}(n) = \sum_{i=1}^{n}\tilde{\underline{e}}(i)\boldsymbol{a}(i) \quad (15)$$

is the $L(K-1) \times 1$ cross-correlation vector between the spreading sequences and the received signal hypothesis.

The standard RLS algorithm for (11) is now easily summarized by the following steps [5], page 485:

Step 1: Initalize the $L(K-1) \times L(K-1)$-matrix $\mathcal{P}(0) = \delta^{-1}\mathcal{I}$, where δ is a small positive constant, and $\boldsymbol{v}(0) = 0$.

Step 2: $\boldsymbol{\pi}^T(n) = \boldsymbol{a}^T(n)\mathcal{P}(n-1)$, where $\boldsymbol{\pi}^T(n)$ is a $1 \times L(K-1)$ vector.

Step 3: $\boldsymbol{k}(n) = \frac{\mathcal{P}(n-1)\boldsymbol{a}(n)}{1+\boldsymbol{\pi}(n)\boldsymbol{a}(n)}$, where $\boldsymbol{k}(n)$ is an $L(K-1) \times 1$ vector, called the *Kalman gain vector*.

Step 4: $\underline{\alpha}(n) = \tilde{\underline{\epsilon}}(n) - \boldsymbol{v}^T(n-1)\boldsymbol{a}(n)$, where $\underline{\alpha}$ is a vector of size NL, and its n-th entry is the apriori error at chip time n.

Step 5: $\boldsymbol{v}(n) = \boldsymbol{v}(n-1) + \boldsymbol{k}(n)\,\underline{\alpha}(n)$.

Step 6: $\underline{\beta}(n) = \tilde{\underline{\epsilon}}(n) - \boldsymbol{v}^T(n)\boldsymbol{a}(n)$, where $\underline{\beta}(n)$ is the aposteriori error at chip time n.

Step 7: $\mathcal{P}(n) = \mathcal{P}(n-1) - \boldsymbol{k}(n)\boldsymbol{\pi}(n)$.

Step 8: $\underline{\epsilon}(n) = \underline{\epsilon}(n-1) + \underline{\alpha}(n)\,\underline{\beta}(n)$, which is the updated scalar error at chip time n.

Steps 2 – 8 are executed recursively from $n = 1$ to NL. Clearly, for large L, this turns into a computationally infeasible task. Furthermore, the RLS algorithm has to be executed for each hypothesis $\underline{d}_c$. Moreover, it can be shown that the above algorithm can be executed by considering a data window of width only K [3]. This then allows us to compute the metric in (8) recursively.

4. COMPUTATIONAL SIMPLIFICATION OF THE PR ALGORITHM

One of the greatest difficulties associated with the PR algorithm is its computational complexity. Each adaptive filter in the system requires $O(NK^2)$ arithmetical operations per encoded symbol, where K represents the number of coefficients in the adaptive filter. However, the structure of the input signals to the adaptive filters make significant computational simplifications possible. To see this, consider the coefficient update equation for the adaptive filters from Step 5 of the algorithm in the previous section,

$$\boldsymbol{v}(n) = \boldsymbol{v}(n-1) + \boldsymbol{k}(n)\,\underline{\alpha}(n) \tag{16}$$

where $\boldsymbol{k}(n)$ is the Kalman gain vector defined as

$$\boldsymbol{k}(n) = \boldsymbol{R}^{-1}(n)\boldsymbol{a}(n). \tag{17}$$

In the above expression, $\boldsymbol{a}(n)$ represents the input vector at time n, and $\boldsymbol{R}(n)$ represents the autocorrelation matrix of the input vector, i.e., the expectation of (14). The input signal is formed by appropriate samples of the spreading code. We assume that the spreading code is a pseudo-random sequence. In this case, the autocorrelation matrix is given by the identity matrix. Using this result, the adaptive filter is updated as

$$\boldsymbol{v}(n) = \boldsymbol{v}(n-1) + \mu\boldsymbol{a}(n)\,\underline{\alpha}(n), \tag{18}$$

where we have added a scaling factor μ to the update equation. This update equation is extremely simple to implement. Furthermore, it is identical to the coefficient update equation of the LMS adaptive filter [5]. This result implies that the PR algorithm employing the LMS adaptive filter with a scaling factor $\mu = 1$ has essentially the same performance as the algorithm that employs the recursive least-squares adaptive filter when a pseudo-random spreading code is employed. This is an important result since it reduces the complexity of the PR from $O(NK^2)$ to $O(NK)$ per encoded symbol, i.e., it now has the same order of complexity as the standard correlation receiver.

The simplified algorithm is now given by the following steps:

Step 1: Initalize $\boldsymbol{v}(0) = 0$.

Step 2: $\underline{\alpha}(n) = \tilde{\underline{\epsilon}}(n) - \boldsymbol{v}^T(n-1)\boldsymbol{a}(n)$, where $\underline{\alpha}$ is a vector of size NL, and its n-th entry is the apriori error at chip time n.

Step 3: $\boldsymbol{v}(n) = \boldsymbol{v}(n-1) + \mu\boldsymbol{a}(n)\,\underline{\alpha}(n)$.

Step 4: $\underline{\beta}(n) = \tilde{\underline{\epsilon}}(n) - \boldsymbol{v}^T(n)\boldsymbol{a}(n)$, where $\underline{\beta}(n)$ is the aposteriori error at chip time n.

Step 5: $\underline{\epsilon}(n) = \underline{\epsilon}(n-1) + \underline{\alpha}(n)\,\underline{\beta}(n)$, which is the updated scalar error at chip time n.

Again, the algorithm can operate with a window size of $K-1$, the number of "canceled" interferers.

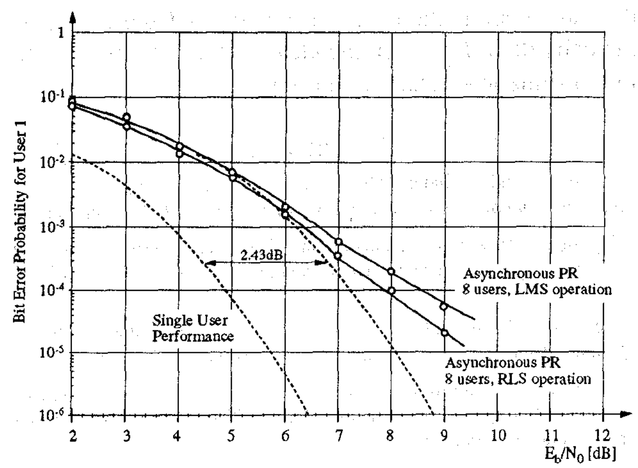

Figure 2: Simulated performance of the projection receiver for random codes of length $N = 15$ with $K = 8$ users with delays $\tau^{(1)} = 0, \tau^{(2)} = 3, \tau^{(3)} = 5, \tau^{(4)} = 6, \tau^{(5)} = 7, \tau^{(6)} = 9, \tau^{(7)} = 11, \tau^{(8)} = 12$. The bound (1) is also shown. The FEC system used is a maximum free distance convolutional code with 4 states.

Simulation results demonstrate that this adaptive receiver performs very close to the theoretical lower bounds as shown in Figure 2. The actual delays of the users have virtually no influence on performance. Furthermore, the low-complexity implementation of the receiver suffers little degradation with respect to the RLS implementation, which in turn achieves the theoretical lower bounds. The theoretical loss $\mathcal{L} = 2.73$dB, and the error control code used in the simulations is a 4-state, maximum free distance convolutional code.

In many applications (e.g., [7, 8]) a deterministic code is used as spreading sequence. Simplification of the PR algorithm is still possible, and the key property is that such codes are periodic. One of the consequences of this periodicity is that the Kalman gain vector is periodic with the same period as the spreading sequences. Therefore, we can precompute the Kalman gain sequence and use it to update the coefficients of the adaptive filter, making implementations relatively easy. It is important to note that computationally efficient realizations of recursive least-squares adaptive filters often have poor numerical error propagation properties. The simplifications described above result in numerically stable realizations of adaptive filters that require only $O(KN)$ arithmetical operations per encoded symbol, and are hence of the same complexity order as matched filtering. We will present numerical results for this situation.

5. CONCLUSIONS

We have proposed a low-complexity implementation of a multiuser receiver, which virtually achieves the theoretical lower bound for that receiver. The low-complexity implementation is based on an adaptive LMS-type algorithm for random spreading codes, and exploits the code periodicity for fixed codes, with a complexity of $O(KN)$ per encoded symbol, i.e., in the same order of magnitude as matched filtering.

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