Transport equations for a two-dimensional electron gas with spin-orbit interaction

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The transport equations for a two-dimensional electron gas with spin-orbit interaction are presented. The distribution function is a 2×2-matrix in the spin space. Particle and energy conservation laws determine the expressions for the electric current and the energy flow. The derived transport equations are applied to the spin-splitting of a wave packet and to the calculation of the structure factor and the dynamic conductivity.

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Spin injection and coherent control of spins in various nanostructures represent two principal challenges for the field of spintronics. Recently, the amount of spintronics research has grown up extensively with the ultimate goal of applications to the quantum computing and information processing.[1] A number of spin-based devices have been designed and studied.[2–5] Spin manipulation in such devices can be achieved by optical[6] or electric[7–10] methods or by ferromagnetic gating.[11] A controlled coupling between spin and orbital degrees of freedom is considered to be a particularly promising tool of efficient spin manipulation dating back to the seminal proposal by Datta and Das.[2]

Spin-orbit interaction in two-dimensional electron gas (2DEG) confined at GaAs/AlGaAs, GaN/AlGaN or similar heterojunctions arises because of the quantum well asymmetry in the perpendicular $[z]$ direction. The resulting perpendicular electric field leads to the coupling of spin to the electron momentum.[12] The strength of this coupling can be experimentally tuned by a gate voltage.[13,14]

Experimental advances in spin manipulation present a certain challenge to develop a proper theoretical description for various phenomena related to the spin-orbit interaction. In particular, modification of universal conductance fluctuations and weak localization has been studied in quantum dots.[15–17] The phenomenon of weak localization has been considered in 2DEG as well.[14,18] The Friedel oscillations in the presence of spin-orbit interaction[19] and the ac conductivity and the plasmon attenuation[20] are calculated.

The principal goal of the present paper is to derive general transport equations for the spin-dependent distribution function of 2DEG including the effects of spin-orbital coupling. We assume that the spin-orbit interaction in a two-dimensional electron gas has the form,

$$H_{\rm so}=\alpha(\hat\sigma_x p_y-\hat\sigma_y p_x)+\beta(\hat\sigma_x p_x-\hat\sigma_y p_y), \tag{1}$$

where the first term is the Bychkov–Rashba term[12] and the second term is the linear Dresselhaus (or anisotropy) term present in semiconductors with no bulk inversion symmetry.[21] The expression of Eq. (1) corresponds to the confinement along the (001) growth direction.[22] Hereinafter we neglect cubic Dresselhaus terms. The free particle Hamiltonian can therefore be written in compact notations as

$$H=\left[\frac{\mathbf{p}^2}{2m}-\mu\right]\hat\sigma_0+\alpha_{ik}\hat\sigma_i p_k, \quad \alpha_{ik}=\begin{pmatrix}\beta & \alpha\\ -\alpha & -\beta\end{pmatrix}. \tag{2}$$

Here, as usual, $\hat\sigma_0,\hat\sigma_x,\hat\sigma_y,\hat\sigma_z$ constitute the set of Pauli matrices. We use italic subscripts for spatial coordinates and reserve greek subscripts for the spin indexes.

Spectral properties: Before deriving the transport equation we describe briefly the spectral properties of the Hamiltonian (1). Its diagonalization is straightforward and reveals the existence of two spin-split subbands in the electron spectrum,

$$\epsilon_{1\mathbf{p}}=\xi_p+\Delta_{\mathbf{p}}, \quad \epsilon_{2\mathbf{p}}=\xi_p-\Delta_{\mathbf{p}}, \tag{3}$$

where

$$\xi_p=\frac{p^2}{2m}-\mu, \quad \Delta_{\mathbf{p}}=\sqrt{p^2(\alpha^2+\beta^2)+4\alpha\beta p_x p_y}, \tag{4}$$

and $p^2=p_x^2+p_y^2$ is the total electron momentum. The eigenfunctions corresponding to the eigenstates (3) are

$$\psi_{1,2}(\mathbf{x})=\frac{1}{\sqrt{2}}\begin{pmatrix}e^{i\chi_{\mathbf{p}}/2}\\ \pm e^{-i\chi_{\mathbf{p}}/2}\end{pmatrix}e^{i\mathbf{px}}, \tag{5}$$

with the upper (lower) sign corresponding to the ψ_1 (ψ_2) state. The phase factor $\chi_{\mathbf{p}}$ depends on the direction of the electron momentum,

$$\tan\chi_{\mathbf{p}}=\frac{\alpha p_x+\beta p_y}{\alpha p_y+\beta p_x}. \tag{6}$$

For the isotropic 2DEG ($\beta=0$) the phase $\chi_{\mathbf{p}}$ coincides with the angle between the electron momentum and the y axis.

Further and more convenient description of the spectral properties can be obtained by considering the spin-dependent retarded Green function, defined as usual by

$$iG^R_{\alpha\beta}(x,x')=\theta(t-t')\langle\langle\psi_\alpha(x)\psi^\dagger_\beta(x')+\psi^\dagger_\beta(x')\psi_\alpha(x)\rangle\rangle. \tag{7}$$

Here we have used the shorthand notation for the space and time variables, $x=(\mathbf{x},t)$. In a homogeneous system the correlation functions depend on the relative coordinates $x-x'$ only. Using the above expressions (3)–(5) we can write the expression for the retarded Green function of free electrons in the momentum representation, $\hat G^R(\epsilon,\mathbf{p})=\int dt\,d\mathbf{x}\,\hat G^R(x)e^{i\epsilon t-i\mathbf{px}}$, which after simple transformations takes the form

$$\hat{G}^R(\epsilon,\mathbf{p}) = \sum_{\mu=1,2} \hat{G}^R_\mu(\epsilon,\mathbf{p}) = \frac{1}{2}\frac{\hat{\sigma}_0 + \cos\chi_\mathbf{p}\hat{\sigma}_x - \sin\chi_\mathbf{p}\hat{\sigma}_y}{\epsilon - \epsilon_{1\mathbf{p}} + i\eta} + \frac{1}{2}\frac{\hat{\sigma}_0 - \cos\chi_\mathbf{p}\hat{\sigma}_x + \sin\chi_\mathbf{p}\hat{\sigma}_y}{\epsilon - \epsilon_{2\mathbf{p}} + i\eta}, \tag{8}$$

with the indexes $\mu=1,2$ corresponding to the first and second terms, respectively, in the last expression of Eq. (8).

The central quantity in the transport theory is the density matrix,

$$f_{\alpha\beta}(x,x') = \langle\langle \psi^\dagger_\beta(x')\psi_\alpha(x)\rangle\rangle. \tag{9}$$

Its value in the thermal equilibrium is related to the imaginary part of the retarded Green function via the fluctuation-dissipation theorem,

$$\hat{f}(\epsilon,\mathbf{p}) = \sum_{\mu=1,2} n_{\mu\mathbf{p}}[\hat{G}^{R\dagger}_\mu(\epsilon,\mathbf{p}) - G^R_\mu(\epsilon,\mathbf{p})], \tag{10}$$

here $n_{\mu\mathbf{p}}$ is the Fermi–Dirac distribution function for the μth state (3). It is convenient to expand the density matrix over the complete set of the Pauli matrices. In particular, for the equal-time density matrix,

$$\hat{f}_\mathbf{p} = \int \frac{d\epsilon}{2\pi}\hat{f}(\epsilon,\mathbf{p}) = \frac{1}{2}f_\mathbf{p}\hat{\sigma}_0 + \frac{1}{2}\mathbf{g}_\mathbf{p}\cdot\hat{\boldsymbol{\sigma}}, \tag{11}$$

we observe according to Eq. (10) that in the thermal equilibrium,

$$f_\mathbf{p} = (n_{1\mathbf{p}} + n_{2\mathbf{p}}), \quad g_{\mathbf{p}x} = \cos\chi_\mathbf{p}(n_{1\mathbf{p}} - n_{2\mathbf{p}}), \quad g_{\mathbf{p}z} = 0, \quad g_{\mathbf{p}y} = -\sin\chi_\mathbf{p}(n_{1\mathbf{p}} - n_{2\mathbf{p}}), \tag{12}$$

Transport equations: In a generic nonequilibrium state, the density matrix (9) obeys a set of conjugated equations that can be obtained from the equations of motion for the electron operators $\psi(x)$ and $\psi^\dagger(x)$ determined by the Hamiltonian (2),

$$\left[i\partial_t + \frac{\nabla^2}{2m} + \mu - e\phi_x\right]\hat{f}(x,x') + i\alpha_{ik}\hat{\sigma}_i\nabla_k\hat{f}(x,x') = 0,$$

$$\left[i\partial_{t'} - \frac{\nabla'^2}{2m} - \mu + e\phi_{x'}\right]\hat{f}(x,x') + i\alpha_{ik}\nabla'_k\hat{f}(x,x')\hat{\sigma}_i = 0. \tag{13}$$

The equations (13) neglect impurity scattering. This is justified for ballistic systems when the mean free path exceeds the characteristic system size, e.g., in high-mobility 2DEG in semiconductor heterostructures (we discuss the impurity scattering at the end of the paper). In Eq. (13) we allowed for the scalar external field $\phi_x = \phi(\mathbf{x},t)$. In the absence of electron-impurity or electron-electron collisions no self-energy terms appear in the equations (13) which makes it sufficient to consider the equal-time ($t=t'$) functions only.

Following the known route of deriving kinetic equations[23] we utilize the Wigner transformation for the density matrix,

$$\hat{f}_\mathbf{p}(\mathbf{x},t) = \int d\mathbf{r}\ e^{-i\mathbf{p}\mathbf{r}}\hat{f}\left(\mathbf{x} + \frac{\mathbf{r}}{2}, \mathbf{x} - \frac{\mathbf{r}}{2}, t\right). \tag{14}$$

By taking the sum of Eqs. (13) in the Wigner representation we obtain

$$[\partial_t + \mathbf{v}\cdot\nabla]\hat{f}_\mathbf{p} + ie\int d\mathbf{q}\ \phi_\mathbf{q}(\hat{f}_{\mathbf{p}-(\mathbf{q}/2)} - \hat{f}_{\mathbf{p}+(\mathbf{q}/2)})e^{i\mathbf{q}\mathbf{x}} + i\alpha_{ik}p_k[\hat{\sigma}_i, \hat{f}_\mathbf{p}] + \frac{1}{2}\alpha_{ik}\nabla_k\{\hat{\sigma}_i, \hat{f}_\mathbf{p}\} = 0, \tag{15}$$

where $\mathbf{v} = \mathbf{p}/m$. Here we introduced the spatial Fourier transform for the scalar potential $\phi_\mathbf{q} = \int d\mathbf{x}\,\phi(\mathbf{x},t)e^{i\mathbf{q}\mathbf{x}}$, with the shorthand notation for the momentum integration $d\mathbf{q} = d^2q/(2\pi)^2$.

Finally, to present Eq. (15) in a more transparent way we turn to the Pauli matrix representation (11) to write,

$$[\partial_t + \mathbf{v}\cdot\nabla]f_\mathbf{p} + ie\int d\mathbf{q}\ \phi_\mathbf{q}(f_{\mathbf{p}-(\mathbf{q}/2)} - f_{\mathbf{p}+(\mathbf{q}/2)})e^{i\mathbf{q}\mathbf{x}} + \alpha_{ik}\nabla_k g_{\mathbf{p}i} = 0, \tag{16}$$

$$[\partial_t + \mathbf{v}\cdot\nabla]g_{\mathbf{p}i} + ie\int d\mathbf{q}\ \phi_\mathbf{q}(g_{\mathbf{p}-(\mathbf{q}/2)i} - g_{\mathbf{p}+(\mathbf{q}/2)i})e^{i\mathbf{q}\mathbf{x}} - [\mathbf{b}_\mathbf{p}\times\mathbf{g}_\mathbf{p}]_i + \alpha_{ik}\nabla_k f_\mathbf{p} = 0, \tag{17}$$

with the following notation for the precession frequency, $b_{\mathbf{p}i} = 2\alpha_{ik}p_k = 2\Delta_\mathbf{p}(\cos\chi_\mathbf{p}, -\sin\chi_\mathbf{p}, 0)$.

Conservation laws: The transport equations (16) and (17) are of the Boltzmann type and therefore fulfill certain particle and energy conservation conditions which will now be obtained. By integrating Eq. (16) with respect to the momentum we find the continuity equation for the particle flow,

$$\partial_t\rho + \frac{1}{e}\nabla\cdot\mathbf{j} = 0, \tag{18}$$

where the electron density and the electric current are given, respectively, by

$$\rho = \int d\mathbf{p} f_\mathbf{p}, \quad j_k = e\int d\mathbf{p}[v_k f_\mathbf{p} + \alpha_{ik}g_{\mathbf{p}i}]. \tag{19}$$

The terms containing the external potential $\phi_\mathbf{q}$ cancel as is readily seen by the change of integration variables. To obtain the energy continuity condition we multiply Eq. (16) by ξ_p and Eq. (17) by $\mathbf{b}_\mathbf{p}$ and add them together. After simple transformations the conservation of energy can be written in the conventional form,

$$\partial_t\rho^\epsilon + \nabla\cdot\mathbf{j}^\epsilon = \mathbf{j}\cdot\mathbf{E}, \tag{20}$$

where the energy density and energy current are,

$$\rho^\epsilon = \int d\mathbf{p}[\xi_p f_\mathbf{p} + b_{\mathbf{p}i}g_{\mathbf{p}i}],$$

$$j^\epsilon_k = \int d\mathbf{p} v_k[\xi_p f_\mathbf{p} + b_{\mathbf{p}i}g_{\mathbf{p}i}] + \alpha_{ik}\int d\mathbf{p}[\xi_p g_{\mathbf{p}i} + b_{\mathbf{p}i}f_\mathbf{p}].$$

The equation (20) means that the local energy change is due to the energy flow to the neighboring points in space as well as a result of the local Joule heating (right-hand side).

Wave packet splitting: To give a specific application of the derived equations let us now use them to describe the propagation of a wave packet in 2DEG with a spin-orbit coupling. We neglect a spin-orbit anisotropy $\beta=0$ for simplicity. The wave packet propagates along the y direction and is uniform along the x axis. The transport equations (16) and (17) are then one-dimensional and (with no external field applied) yield

$$[\partial_t + v\partial_y]f = -\alpha\partial_y g_x,$$

$$[\partial_t + v\partial_y]g_x = -\alpha\partial_y f,$$

$$[\partial_t + v\partial_y]g_y = -2\Delta_p g_z,$$

$$[\partial_t + v\partial_y]g_z = 2\Delta_p g_y. \tag{21}$$

First, we consider a spin-unpolarized Gaussian wave packet injected at the point $y=0$ at the time $t=0$ and moving with the average momentum $\bar{p}$,

$$\hat{f}_{\mathbf{p}}(\mathbf{x},t=0) = \hat{\sigma}_0 F(y), \quad F(y) = e^{-y^2\delta p^2 - [(p_y-\bar{p})^2/\delta p^2]}. \tag{22}$$

In this geometry the phase factor $\chi_{\mathbf{p}}=0$, which means that the precession vector $\mathbf{b}$ is directed along the x axis. We also observe that $g_y = g_z = 0$. The remaining two of the equations (21) are easily solved by Fourier transforming them into a set of linear algebraic equations. The general solution of Eqs. (21) takes the form

$$f(y,t) = A(y - v_+ t) + B(y - v_- t),$$

$$g_x(y,t) = A(y - v_+ t) - B(y - v_- t),$$

where we have introduced subband velocities $v_\pm = v \pm \alpha$. So far, $A(x)$ and $B(x)$ are two arbitrary functions which have to be determined from the initial condition (21) yielding, $A(y)=B(y)=F(y)$. We find that the incident wave packet (22) is decomposed into two independent constituents oppositely polarized along x-direction and moving with different velocities. The spatial distribution of the electron density is given by the integral over all momenta, i.e.,

$$\rho_\pm(y,t) = \frac{1}{2\pi^{1/2}\delta x(t)} \exp\left[-\frac{(x-\bar{v}_\pm t)^2}{\delta x^2(t)}\right], \tag{23}$$

with the average velocities $\bar{v}_\pm = \bar{p}/m \pm \alpha$, and the Gaussian width at finite times, $\delta x^2(t) = \delta p^{-2} + (t^2\delta p^2/m^2)$. To observe the spin-orbit induced splitting of a wave packet the following conditions should be satisfied:

$$\frac{\delta p}{m} \ll \alpha \ll \frac{t}{\delta p}.$$

The first of the two conditions ensures that the splitting dominates over the wave packet broadening, while the second condition means that enough time has to elapse before the splitting becomes larger than the intrinsic packet width.

Now let us consider an injection of a packet initially polarized along the y direction.

$$\hat{f}_{\mathbf{p}}(\mathbf{x}) = (\hat{\sigma}_0 + \hat{\sigma}_y)F(y), \tag{24}$$

The equations for the f and g_x components of the density matrix remain unchanged with the above analysis still valid. The second pair of Eqs. (21) is independent of the first pair and have a solution

$$g_y(y,t) = F(y - vt)\cos(2\Delta_p t),$$

$$g_z(y,t) = -F(y - vt)\sin(2\Delta_p t). \tag{25}$$

According to the expressions (25) the initial spin polarization precesses with a frequency $2\Delta_p$ around the axis perpendicular to the propagation direction. Note that the precessing spin propagates with the center-of-mass velocity $\bar{v}$ rather than with the subband velocities $\bar{v}_\pm$.

The above analysis assumes that a wave packet is injected with a given momentum $\bar{p}$. Such an injection into 2DEG with a spin-orbit coupling is not easy to achieve. For example, injection through an interface with a ''normal'' (with no spin-orbit interaction) 2DEG[24–26] would not result in a spatial splitting of a wave packet. This is due to the fact that the injection happens with a conservation of energy rather than momentum. As seen from Eqs. (3) and (4) the two states with the same energy propagate with the same velocity[26,27] within the approximations of this paper. However, if we take into account the cubic Dresselhaus terms, which have been omitted in our discussion, there can be a splitting of velocities at the same energy. In order to achieve splitting without the cubic terms we need to consider a more complicated setup. As a demonstration of principle, we consider the following example. Let us inject a wave packet propagating along the y direction with the spin polarized along the interface (x axis), e.g., by injection from a ferromagnetic contact. The states forming the wave packet belong to the subband 1, with a spin polarization $(1/\sqrt{2})(1,1)$. Let us now switch on ac magnetic field along the y axis rotating the spin direction until it is aligned with the z axis (1,0), and then switch the magnetic field off. The resulting state will be an equal mixture of both eigenstates $(1/\sqrt{2})(1,1)$ and $(1/\sqrt{2})(1,-1)$ without any change of momentum (the energy is no longer conserved). The velocities of these states are different and the packet will split.

The above picture holds not only for the injection of initially polarized packet. If the incident packet is unpolarized and has a given energy, upon entering the interface it will become a mixture of two states: $(1/\sqrt{2})(1,1)$ with the momentum $p_0 - m\alpha$, and $(1/\sqrt{2})(1,-1)$ with the momentum $p_0 + m\alpha$. Both velocities remain equal to $v_0 = p_0/m$. After switching on the ac magnetic field with the frequency $\omega \approx 2\alpha p_0$ (which is a resonant frequency for the transition between the two subbands), the first state will evolve into the mixture of the states: $(1/\sqrt{2})(1,1)$ and $(1/\sqrt{2})(1,-1)$, both with the momentum $p_0 + m\alpha$, meaning two different velocities v_0 and $v_0 - 2\alpha$. The same reasoning shows that the other

initial state will develop two velocities v_0 and $v_0+2\alpha$. Therefore, the initially unpolarized packet will split into three parts.

Ballistic spin injection: We envisage a spin injection from ferromagnetic contacts into ballistic 2DEG among the applications for the equations derived above. In this case the injection occurs with conservation of energy, and can be described by the time-independent solution of the equations (16) and (17) with the appropriate boundary conditions, which require a conservation of the normal components of the electric current, Eq. (19), at the interfaces. A corresponding theory would generalize the existing approach for the ballistic spin-injection based on the ordinary Boltzmann equation.[28] In the latter case the Boltzmann equation method is more convenient for the calculation of spin polarization of current and magnetoresistance than the direct solution of the single-particle Schrödinger equation.

Structure factor: The electron density fluctuations are described by the structure factor[29] defined as the retarded correlation function,

$$\chi(x,x')=-i\theta(t-t')\langle\langle\rho(x)\rho(x')-\rho(x')\rho(x)\rangle\rangle, \tag{26}$$

of the electron density operators $\rho(x)=\psi_\alpha^\dagger(x)\psi_\alpha(x)$. At equilibrium the structure factor (26) depends on the relative coordinates $x-x'$ only. The imaginary part of the Fourier transform $\chi(\omega,\mathbf{q})$ measures the energy dissipation of the external field at a given frequency ω and a wave vector $\mathbf{q}$. In the isotropic system the structure factor is related to the ac conductivity by the relation

$$\sigma(\omega)=\lim_{q\to 0}\frac{ie^2\omega}{q^2}\chi(\omega,\mathbf{q}). \tag{27}$$

The formula (27) is readily checked using the Kubo formula for the conductivity and the continuity equation (18).

According to the fluctuation-dissipation theorem, the structure factor can be determined from calculations of the linear response to an external scalar field. The field-induced modulation of electron density is related to the magnitude of the external perturbation through the structure factor according to,[29]

$$\delta\rho(\omega,\mathbf{q})=e\chi(\omega,\mathbf{q})\phi(\omega,\mathbf{q}). \tag{28}$$

The electron density modulation is given by the deviation of the function $f_\mathbf{p}(t,\mathbf{x})$ from its equilibrium value, $\delta\rho(\omega,\mathbf{q})=\int d\mathbf{p}\,\delta f_\mathbf{p}(\omega,\mathbf{q})$, and can be found from the linearized equations (16) and (17). In the linear approximation by the external field $\phi(\omega,\mathbf{q})$, the distribution function is a small deviation

$$f_\mathbf{p}=f_\mathbf{p}^0+\delta f_\mathbf{p},\quad \mathbf{g_p}=\mathbf{g}_\mathbf{p}^0+\delta\mathbf{g_p}, \tag{29}$$

from its equilibrium value (12). The linearized transport equations (16) and (17) take the form

$$(\omega-\mathbf{qv})\delta f_\mathbf{p}-\alpha_{ik}q_i\delta g_{\mathbf{p}k}=e\phi(\omega,\mathbf{q})(f^0_{\mathbf{p}+(\mathbf{q}/2)}+f^0_{\mathbf{p}-(\mathbf{q}/2)}),$$

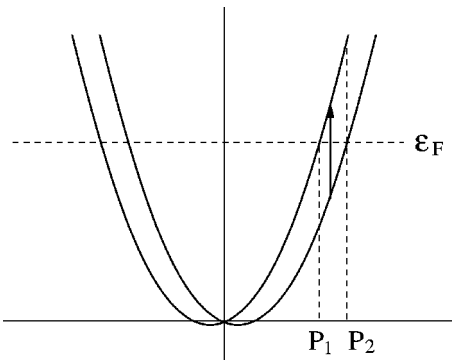

FIG. 1. Spin-orbit induced subbands of an isotropic two-dimensional electron gas, $\epsilon_p=p^2/2m\pm\alpha|p|$. At $T=0$ all states below the Fermi energy ϵ_F are filled. The Fermi momenta for the two subbands are $p_{1,2}=p_0\mp m\alpha$. The direct transitions, $q=0$ (shown by the arrow) are possible for the states between the dashed lines, $p_1<p<p_2$.

$$(\omega-\mathbf{qv})\delta g_{\mathbf{p}i}-i[\mathbf{b_p}\times\delta\mathbf{g_p}]_i-\alpha_{ik}q_k\delta f_\mathbf{p}$$
$$=-e\phi(\omega,\mathbf{q})(g^0_{\mathbf{p}+(\mathbf{q}/2)i}+\mathbf{g}^0_{\mathbf{p}-(\mathbf{q}/2)i}). \tag{30}$$

Solving these equations for the variation of the electron density (19) we obtain the structure factor with the help of the relation (28),

$$\chi(\omega,\mathbf{q})=\frac{1}{2}\sum_{\mu\mu'}\int d\mathbf{p}[1+(-1)^{\mu\mu'}\cos(\chi_\mathbf{p}-\chi_{\mathbf{p}'})]$$
$$\times\frac{n_{\mu\mathbf{p}_-}-n_{\mu'\mathbf{p}_+}}{\omega-\epsilon_{\mu'\mathbf{p}_+}+\epsilon_{\mu\mathbf{p}_-}}, \tag{31}$$

where $\mathbf{p}_\pm=\mathbf{p}\pm\mathbf{q}/2$. The expression (31) with $\omega=0$ corresponds to the previously derived result for the static dielectric function.[19] To simplify further the subsequent discussion we will disregard the anisotropy, $\beta=0$, and consider the zero-temperature limit $T=0$. The two spin-orbit subbands are axially symmetric, shown on Fig. 1. The subbands are filled up to the same Fermi energy level ϵ_F but have two different Fermi momenta, p_1 and p_2, determined from the equations $\epsilon_i(p_i)=\epsilon_F$, where $\epsilon_i(p)$ are given by Eqs. (3) and (4) with $\beta=0$. This leads to the values

$$p_1=p_0-m\alpha+O(m^2\alpha^2/p_0^2),$$
$$p_2=p_0+m\alpha+O(m^2\alpha^2/p_0^2), \tag{32}$$

where p_0 is determined by $\epsilon_F=p_0^2/2m$, namely p_0 is the Fermi momentum in the absence of spin-orbit interaction. Note that the Fermi velocities for the two subbands,

$$v_i=\frac{\partial\epsilon_i(p)}{\partial p}\Big|_{p=p_i}=\frac{p_0}{m}+O(m^2\alpha^2/p_0^2), \tag{33}$$

are the same and (up to higher-order terms) equal to the Fermi velocity in 2DEG with no spin-orbit coupling $\alpha=0$. The imaginary part of the structure factor $\chi(\omega,\mathbf{q})$ determines the absorption, or Landau damping, of the external field at given frequency and wave vector. The points in the electron

momentum space that contribute to the Landau damping correspond to the zeros of the denominators. There are total four determined by the equations

$$\omega = \mathbf{q}\mathbf{v} \pm \alpha p_+ \pm \alpha p_-, \tag{34}$$

with the opposite signs of the last two terms corresponding to the (gapless) transitions within the same subbands [Eq. (3)] and equal signs describing the transitions between different subbands.

The terms with $\mu = \mu'$ in Eq. (31) represent the effect of intrasubband transitions. Only indirect ($q \neq 0$) transitions contribute to the imaginary part of the structure factor. For small transferred momenta $q \ll p_0$ one can disregard the deviation of the cosine factor from unity and also approximate $n_{i\mathbf{p}_-} - n_{i\mathbf{p}_+} \simeq -\mathbf{q}\partial n_i/\partial \mathbf{p}$. Taking the momentum integral we obtain for the contribution of the ith subband,

$$\Im\chi_i(\omega,q) = -\nu_i \frac{\omega}{\sqrt{q^2 v_0^2 - \omega^2}} \theta(q^2 v_0^2 - \omega^2), \tag{35}$$

where $v_0 = p_0/m$ and ν_i stands for the density of states of the ith subband at its Fermi surface $p = p_i$:

$$\nu_1 = \frac{m}{2\pi}\left(1 - \frac{m\alpha}{p_0}\right), \quad \nu_2 = \frac{m}{2\pi}\left(1 + \frac{m\alpha}{p_0}\right).$$

Note that the sum of the two contributions (35) is *independent* of the spin-orbit interaction (up to higher-order terms), a consequence of the fact that the two subbands have the same value of the Fermi velocity. Spin-orbit interaction results only in a redistribution of the spectral weight between the subbands controlled by the changes in the densities of states.

The terms with $\mu \neq \mu'$ in Eq. (31) correspond to the intersubband transitions. Their contribution to the structure factor for $m\alpha \ll q \ll p_0$ is negligible compared to the above considered intrasubband transitions by the factor $\sim q^2/p_0^2$ (due to the small $\sin^2$ prefactor). However, the presence of the two subbands is important as it makes the direct, $q = 0$, transitions possible. The factor $n_{1p} - n_{2p}$ then defines the momentum space available for the direct transitions, $p_1 < p < p_2$ (see Fig. 1), which corresponds to the frequency domain $2\Delta_0 - 2m\alpha^2 < \omega < 2\Delta_0 + 2m\alpha^2$, where $\Delta_0 = \Delta_{p_0}$,

$$\chi(\omega, q \to 0) = \frac{\alpha q^2}{4\pi} \int_{p_1}^{p_2} \frac{dp}{(\omega + i0)^2 - 4\Delta_p^2}. \tag{36}$$

The imaginary part of this expression is

$$\Im\chi(\omega, q \to 0) = -\frac{q^2 \operatorname{sgn}\omega}{32\Delta_0} \theta[4m^2\alpha^4 - (\omega - 2\Delta_0)^2]. \tag{37}$$

The equation (36) corresponds to the previously obtained result[20] for the optical conductivity $\sigma(\omega)$. The expression (36) goes to zero with the wave vector, which is easily understood by noting that the matrix elements for the transitions between $\psi_1(\mathbf{p}_-)$ and $\psi_2(\mathbf{p}_+)$ states are suppressed at small transferred momenta since they are orthogonal at $q = 0$. However, their contribution to the conductivity [according to Eq. (27)] remains finite, which is clear since the operator of electron velocity has nonzero matrix elements for the intersubband transitions even at $q = 0$.

The experimental observation of the direct transitions (37) is feasible in the measurements of the resonant microwave absorption in high-mobility semiconductor heterostructures.

Screened electron-electron interaction and plasmon excitations: So far our analysis has been restricted to the noninteracting electron gas. To incorporate the effects of the electron-electron interaction in the random phase approximation one has to account for the self-consistent electric field induced by the variations of the electron density. The potential for this field $\phi_{\rm sc}$ obeys the Poisson equation. In two dimensions the Fourier transform of the Poisson equation has the form

$$e\phi_{\rm sc}(\omega, \mathbf{q}) = V_q \rho(\omega, \mathbf{q}), \tag{38}$$

where $V_q = 2\pi e^2/q$ is the bare Coulomb propagator. The random phase approximation (RPA) is then equivalent to the substitution $\phi(\omega, \mathbf{q}) \to \phi_{\rm sc}(\omega, \mathbf{q}) + \phi(\omega, \mathbf{q})$ on the right-hand side of Eq. (30). It is straightforward to see that the structure factor takes the familiar RPA form,

$$\chi_{\rm RPA}(\omega, \mathbf{q}) = \frac{\chi(\omega, \mathbf{q})}{1 - V_q \chi(\omega, \mathbf{q})}. \tag{39}$$

The pole of this expression determines the plasmon spectrum $\omega = \omega_q + i\gamma_q$, where $\omega_q^2 = v^2 \kappa q/2$, with $\kappa = 2\pi e^2 \nu$ standing for the static screening radius. The plasmon linewidth is given by the imaginary part of the bare structure factor at $\omega = \omega_q$,

$$\gamma_q = \tfrac{1}{2} V_q \omega_q |\Im\chi(\omega_q, q)|. \tag{40}$$

For the plasmon to be an undamped excitation its frequency should lie above the electron-hole continuum, $\omega_q > qv$, which requires $\kappa > 2q$. As was already pointed out in Ref. 20 the plasmon acquires damping when $\omega_q \sim 2\Delta$. Since $q \ll (q\kappa)^{1/2} \sim m\alpha$ at this range, the direct transitions (37) make the principal contribution to Eq. (40).

Impurity scattering: The equations presented in this paper assume ballistic electron motion. The absence of impurities allows one to write kinetic equation as a closed set of equations, Eq. (15), for the density matrix integrated over the energy variable ϵ [see Eq. (11)], i.e., at coinciding times. In the presence of disorder the self-energy due to impurity scattering should be added to the right-hand side of Eq. (13). In general, since plain waves are no longer eigenstates of the system with impurities, the equations for the distribution function depending on the momentum $\mathbf{p}$ (and not on the energy ϵ) become not very convenient. More natural (though more complicated) equations would result from integration over ξ_p, similar to the usual spin-degenerate case.[31] Such equations are beyond the scope of the present paper.

Special case $\alpha = \pm\beta$: Recently, Schliemann *et al.*[30] proposed a spin field-effect transistor based on a particular tuning of the spin-orbit coupling constants such that $\alpha = \beta$ (or $\alpha = -\beta$). This special system is expected to preserve spin coherence even in the presence of disorder. This is due to the fact that the spin eigenstates (5) are independent of the elec-

tron momenta, $\chi_{\mathbf{p}}=$ const, therefore a scalar impurity potential does not result in the intersubband transitions. The same observation holds for the structure factor. Since the matrix elements of the density are identically zero for the transitions between different subbands the second line of Eq. (31) is absent in this case and the structure factor is intact by the presence of spin-orbit interaction (up to higher order corrections).

Conclusions: To summarize, we have derived transport equations for the distribution function of a two-dimensional electron gas with spin-orbit interaction of both the Bychkov-Rashba and the Dresselhaus mechanisms. The distribution function is a 2×2-matrix in the spin space. General expressions for the particle and energy currents and densities are available in terms of the density $f_{\mathbf{p}}$ and spin $\mathbf{g}_{\mathbf{p}}$ distribution functions. The obtained equations are applied to the wave-packet propagation in a ballistic 2DEG and to the calculation of the density-density correlation function $\chi(\omega,q)$. We observe that for $q>m\alpha$ the structure factor $\chi(\omega,q)$ is almost not affected by the spin-orbit interaction, but it reveals new features when $q\ll m\alpha$ due to the direct transitions between different spin-orbit subbands.

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