Statistical Comparison of Capacity Predictions for Realistic MIMO Channels

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Introduction

The impact of antenna polarization on channel capacity is explored in multiple-input, multiple-output (MIMO) systems. An idealized polarization model involving branch power rations (BPR's) and channel cross-coupling is incorporated into channel-specific capacity calculations. Results are compared for several measured channels including line-of-sight (LOS) and non-line-of-sight (NLOS) both indoors and outdoors yielding valuable sensitivity analyses for channel capacity. Virtually all channels achieve per-channel peak capacities of 50% above that of single-input, single-output (SISO) channels and are well suited to opportunistic scheduling. However, systems exclusively dependent on polarization diversity will often exhibit outage capacities of just 10% above SISO capacity and will perform worse than those dependent on additional degrees of freedom.

Representative Model Parameters

For a narrowband channel, transmit voltages, x, and receive voltages, y, are related as $y = Hx$. As discussed in [1], the channel transfer matrix, H, expresses both spatial and polarization effects. The two effects are independent and can be treated separately. This paper assumes H is 2 x 2, and consists exclusively of polarization-dependent effects, thus excluding spatial correlation effects. Given the branch power ratio, $\beta^2 = |y_v|^2/|x_v|^2$, cross polarization, $X_{hv} = |h_{vv}|/|h_{hv}|^2$, and a pair of phase terms, φ and ε, one may express realizations of the channel matrix as [1]:

$$\boldsymbol{H} \approx \begin{bmatrix} 1 & \dfrac{e^{j\varphi+\varepsilon}}{\beta\sqrt{X_{vh}}} \\ \dfrac{e^{j\varphi}}{\sqrt{X_{hv}}} & \dfrac{e^{j\varepsilon}}{\beta} \end{bmatrix}. \qquad (1)$$

Antennas are assumed to be omni-directional in the horizontal plane, uncoupled and have perfect cross-polarization discrimination. X_{vh}, X_{hv} and β are characteristics of the environment and each should be measured separately for a specific channel. Phase terms are modeled statistically with φ uniformly distributed over $[0, 2\pi)$ and ε following a zero-mean Gaussian distribution with standard deviation of 0.3 [1].

Table 1 reports idealized and realistic parameters for channels taken from the literature. Various capacities are predicted for each model based on both a modified Saleh-Valenzuela angular (SVA) model and a purely statistical method. Results from each method generally agree to within 5% of the single-channel capacity, so only the results of the modified SVA approach are given in Table 1. A discussion of each method follows.

Table 1 Representative Channel Parameters and Capacities. A dash (-) indicates a value not reported in the reference.

	β^2 (dB)	χ_{HV} (dB)	χ_{VH} (dB)	Source	$C_{0.1}$ % above C_{SISO}	$E\{C\}$ % above C_{SISO}	$C_{0.95}$ % above C_{SISO}
Const ideal	0	∞	∞	X, β^2 const	54	54	54
Ideal	0	∞	∞	β^2 lognorm.	25	44	50
Indoor LOS	-	15	15	Kyritsi [2]	22	41	53
(hallway)	-	6.8	6.8	Wallace [3]	16	37	51
Indoor	-	0	0	Kyritsi [2]	12	34	50
NLOS	-	2.5	2.5	Cox [5]	12	33	50
Outdoor LOS	6-7	13	13	Oestges [6]	9	32	51
Outdoors	±3 >	6	6	Lee [7]	13	34	50
NLOS	-	3.5	3.5	Cox [5]	12	34	54

Capacity Predictions

Wallace uses a form of the Saleh-Valenzuela channel model as a tool for modeling the time-variant channel capacity in [3]. Dozens of statistically-generated rays are summed to predict H, from which the water-filling solution may be used to compute capacity [3]. After replacing β in [3] with b to avoid confusion, this model can be simplified for a collocated 2 x 2 dual-polar system and extended to include the BPR as follows [4]:

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{h_{vv}} = \sum_{k=1}^{K_{vv}} b_{vv,k} \boldsymbol{e}^{j\varphi_{vv,k}} & \boldsymbol{h_{vh}} = \frac{1}{\beta X_{vh}} \sum_{k=1}^{K_{vh}} b_{vh,k} \boldsymbol{e}^{j\varphi_{vh,k}} \\ \boldsymbol{h_{hv}} = \frac{1}{X_{hv}} \sum_{k=1}^{K_{hv}} b_{hv,k} \boldsymbol{e}^{j\varphi_{hv,k}} & \boldsymbol{h_{hh}} = \frac{1}{\beta} \sum_{k=1}^{K_{vv}} b_{hh,k} \boldsymbol{e}^{j\varphi_{hh,k}} \end{bmatrix}. \quad (2)$$

To generate the final columns of Table 1, 5000 realizations of H are generated for each pair (β, X). In addition, X_{hv} and X_{vh} are set equal, as is commonly the case. Two common summary statistics are used—mean (or ergodic) capacity and capacity outage, $C_{0.1}$ [8]—and a third statistic, peak capacity, is introduced. Analogous to outage capacity, peak capacity, $C_{0.95}$, is achieved 5% of the time and may be a practical predictor of performance in opportunistic scheduling systems, such as CMDA 2000 [9].

A purely statistical approach further establishes the results given in Table 1. Figure 2, summarizes the results of 130,000 Monte-Carlo channel matrix simulations. Both Oestges and Wallace, report that X and β may be modeled as lognormal random variables with the average values reported in Table 1. Each pair (β_i, X_i) plotted in this figure, represents the mean values of the respective random variables with fixed variance used for 5 x 10^4 realizations of each of the random variables, φ, ε, β and X. The capacity variations are summarized as $C_{0.95}$, $E\{C\}$ and $C_{0.1}$ and plotted for each mean-value pairing (β_i, X_i). Channel parameters for real channels reported in Table 1 are identified as interpolated values on the curves of Figure 2.

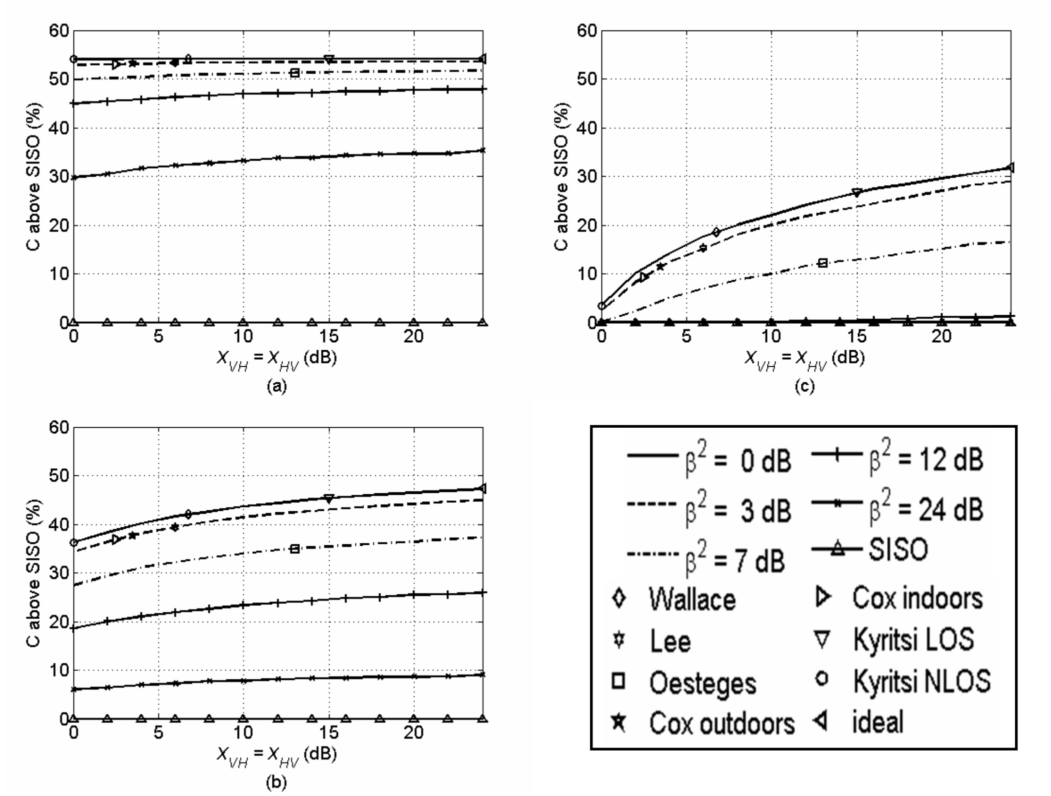

Figure 1 Summary statistics, (a) $C_{0.95}$, (b) mean C, and (c) $C_{0.1}$, for capacity simulations, reported as the fractional amount by which capacity exceeds that of a SISO channel.

Results of this statistical approach agree to within 5% of SISO capacity for both $C_{0.95}$ and $E\{C\}$ and to within 9% for $C_{0.1}$. This level of agreement between the approach used by Oestges and the approach taken by Wallace helps confirm the validity of each approach and helps validate the results given in Table 1. To generate a lognormal variable with mean, μ, one may start from a normal variable with mean $\ln(\mu)-\sigma^2/2$ and variance σ^2, adjusting the variance for an appropriate fit to measured data. Using a fixed value of $\sigma^2 =$ 0.5 matches Oestges's reported value and offers a good visual fit to probability distribution function data reported in [3], Figure 8.25a. However, adjusting this variance parameter to $\sigma^2 = 1.5$ better matches the Wallace capacity complementary cumulative density functions reported by Wallace. Agreement between the tabulated values of Table 1 generated with the modified SVA model and the statistically derived points labeled in Figure 2 is very good. For example, consider Wallace's channel, marked by the diamond on the uppermost curve of each subplot of Figure 2 at $X_{vh} = X_{hv} = 6.8$ dB. For this channel, $C_{0.95} = 53\%$, $E\{C\} = 41\%$ and $C_{0.1} = 19\%$ above SISO capacity, all of which agree to within 4% of the SISO capacity values computed using the modified SVA.

$C_{0.95}$ is only mildly sensitive to X and is essentially constant for $\beta^2 < 7$ dB. Peak capacity does eventually deteriorate as β^2 increases, causing H to look increasingly rank deficient. This suggests that opportunistic scheduling schemes should perform fairly consistently over a wide range of physical channels. In contrast, $C_{0.1}$ is strongly tied to both β^2 and X. As X approaches 0 dB, e.g. Cox's channels in Fig 1, $C_{0.1}$ drops to that of a SISO channel. This will degrade the performance of single-user links without the inclusion of further diversity techniques such as time, space or frequency.

Conclusions

Channel characterizations in the literature are compared and extended to offer predictions of channel capacity in terms of its mean, outage, and peak levels. Each is estimated from Monte-Carlo simulations via a modified Saleh-Valenzuela model and directly from parameter probability distributions. Both approaches yield consistent results, which suggest that channels will typically support at least 50% more peak per-channel capacity and 30-45% more average per-channel capacity than single-polarized elements offer, by using independently-fed, idealized dual-polarized, collocated elements.

Multi-user or delay tolerant links can exploit peak capacity making them least sensitive to branch polarization and cross-polarization discrimination. Such channels therefore require the least careful characterization. Single-user channels with less delay tolerance require more careful characterization, but virtually all channels are expected to profit significantly from the use of polarization diversity, as a means of boosting channel capacity. The impact of these parameters is quantifiable. As time-windowed estimates of average branch polarization vary from 0 dB to 7 dB, estimates of the average capacity vary by roughly 10% of the SISO capacity for the channel. When link quality must be continuously high, outage capacity is of primary interest and one must estimate channel parameters much more carefully. The $C_{0.1}$ contours exhibit steeper slopes than the other capacity measures, requiring much tighter estimates of channel parameters to adequately predict $C_{0.1}$. Such dependencies can be justified either through arguments about the rank of the channel matrix or by considering the independent and distinguishable character of either polarization channel.

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