# Halo independent comparison of direct dark matter detection data

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We extend the halo-independent method of Fox, Liu, and Weiner to include energy resolution and efficiency with arbitrary energy dependence, making it more suitable for experiments to use in presenting their results. Then we compare measurements and upper limits on the direct detection of low mass (∼10 GeV) weakly interacting massive particles (WIMPs) with spin-independent interactions, including the preliminary upper limit on the annual modulation amplitude from the CDMS collaboration. We find that isospin-symmetric couplings are severely constrained, but isospin-violating couplings are still possible if for example the local Galactic escape speed is small, as found in recent surveys.

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The nature of dark matter is one of the fundamental problems of physics and cosmology. Weakly interacting massive particles (WIMPs), i.e. particles with weakly interacting cross sections and masses in the GeV–10 TeV range, are among the best motivated candidates for dark matter. Of particular interest is a low mass region, $\sim 10$ GeV, suggested by data from three direct dark matter experiments: DAMA [1], CoGeNT [2, 3] and CRESST-II [4]. DAMA and CoGeNT report annual modulations with the expected characteristics of a WIMP signal [5]. CRESST-II observes an excess of events above their known background, excess which may be interpreted as due to dark matter WIMPs. Stringent upper limits have been placed on dark matter WIMPs by other direct detection experiments. The most stringent limits in the region of $\sim 10$ GeV WIMPs come from the XENON10 [6], XENON100 [7], SIMPLE [8], and CDMS experiments [9]. All but one of these limits result from an upper bound on the total unmodulated event rate. The exception is a very recent preliminary result by the CDMS collaboration [10], which has searched for an annual modulation in their data and, not finding it, has placed a stringent upper limit on its amplitude.

In this paper, we compare the above measurements and upper limits in a halo-model independent fashion. We concentrate on light WIMPs with spin independent (SI) interactions, and extend the halo-independent method of Fox, Liu, and Weiner [11], later extensively employed in [12], by including the energy-dependent energy resolution, efficiency, and form factors. In our form, the method can be used by any experiment to present their own results in a way that would allow for an immediate comparison between experiments in a halo-independent manner.

The differential recoil rate per unit detector mass, typically in units of counts/kg/day/keV, for the scattering of WIMPs of mass $m$ off nuclei of mass number $A$, atomic number $Z$, and mass $m_{A,Z}$ is

$$\frac{dR_{A,Z}}{dE} = \frac{\sigma_{A,Z}(E)}{2m\mu_{A,Z}^2}\,\rho\,\eta(v_{\min},t), \qquad (1)$$

where $E$ is the nucleus recoil energy, $\rho$ is the local WIMP density, $\mu_{A,Z} = mm_{A,Z}/(m+m_{A,Z})$ is the WIMP-nucleus reduced mass, $\sigma_{A,Z}(E)$ is (a multiple of) the WIMP-nucleus differential cross-section $d\sigma_{A,Z}/dE$, and

$$\eta(v_{\min},t) = \int_{|\mathbf{v}|>v_{\min}} \frac{f(\mathbf{v},t)}{v} d^3v \qquad (2)$$

is a velocity integral carrying the only dependence on the (time-dependent) distribution $f(\mathbf{v},t)$ of WIMP velocities $\mathbf{v}$ relative to the detector. Here

$$v_{\min} = \sqrt{\frac{m_{A,Z}E}{2\mu_{A,Z}^2}} \qquad (3)$$

is the minimum WIMP speed that can result in a recoil energy $E$ in an elastic scattering with the $A,Z$ nucleus. Due to the revolution of the Earth around the Sun, the $\eta$ function has an annual modulation generically well approximated by the first terms of a harmonic series

$$\eta(v_{\min},t) = \eta_0(v_{\min}) + \eta_1(v_{\min})\cos\omega(t-t_0), \qquad (4)$$

where $\omega = 2\pi/\text{yr}$ and $t_0$ is the time of maximum signal.

For spin-independent interactions (SI), the WIMP-nucleus cross-section can be written in terms of the effective WIMP-neutron and WIMP-proton coupling constants $f_n$ and $f_p$ as

$$\sigma_{A,Z}^{SI}(E) = \sigma_p \frac{\mu_{A,Z}^2}{\mu_p^2}[Z+(A-Z)(f_n/f_p)]^2\ F_{A,Z}^2(E)\,, \qquad (5)$$

where $\sigma_p$ is the WIMP-proton cross-section and $F_{A,Z}^2(E)$ is a nuclear form factor, which we take to be a Helm form factor normalized to $F_{A,Z}(0) = 1$. In most models the couplings are isospin conserving, $f_n = f_p$. Isospin-violating couplings $f_n \neq f_p$ have been considered as a possibility to weaken the upper bounds obtained with heavier target elements, which being richer in neutrons than lighter elements, have their couplings to WIMPs suppressed for $f_n/f_p \simeq -0.7$ [13].

Fox, Liu, and Weiner [11] observed that the factor $\tilde{\eta}(v_{\min}) = \sigma_p(\rho/m)\eta(v_{\min})$ in Eq. (1) with SI interactions is common to all experiments, and compared direct detection experiments without any assumption about the dark halo of our galaxy by expressing the data in terms of $v_{\min}$ and $\tilde{\eta}(v_{\min})$. This was done extensively in [12], separately for $\tilde{\eta}_0(v_{\min})$ and $\tilde{\eta}_1(v_{\min})$. Since the $E$-$v_{\min}$ relation depends explicitly on the WIMP mass $m$, this procedure can be carried out only by fixing $m$ (except when $m$ is much smaller than the masses of all nuclei involved, in which case the combination $mv_{\min}$ becomes independent of $m$).

However most experiments do not measure the recoil energy $E$ directly, but rather a detected energy $E'$ subject to measurement uncertainties and fluctuations. These are expressed in an energy response function $G_{A,Z}(E, E')$ that incorporates the energy resolution $\sigma_E(E')$ and the mean value $\langle E' \rangle = E\, Q_{A,Z}(E)$, where $Q_{A,Z}(E)$ is the quenching factor. In this context, recoil energies are often quoted in keVnr, while detected energies are quoted in keVee (keV electron-equivalent) or directly in photoelectrons. Moreover, experiments have an overall counting efficiency or cut acceptance $\epsilon(E')$ that depends on $E'$. A compound detector with mass fraction $C_{A,Z}$ in nuclide $A, Z$ has an expected event rate equal to

$$\frac{dR}{dE'} = \epsilon(E') \int_0^\infty dE \sum_{A,Z} C_{A,Z}\, G_{A,Z}(E, E')\, \frac{dR_{A,Z}}{dE}. \quad (6)$$

We observe that the factor $\tilde{\eta}(v_{\min})$ is common to all experiments also when the rates are expressed in terms of the detected energies $E'$ as in Eq. (6). This observation allows us to extend Fox et al.'s method to the more realistic case of finite energy resolutions and $E'$-dependent efficiencies, without restrictions on how rapidly these quantities change with energy.

For this purpose, using $dE = (4\mu_{A,Z}^2/m_{A,Z}) v_{\min}\, dv_{\min}$, we write the average of Eq. (6) over a detected energy interval $[E'_1, E'_2]$ as

$$R_{[E'_1,E'_2]} = \int_0^\infty dv_{\min}\; \mathcal{R}^{SI}_{[E'_1,E'_2]}(v_{\min})\, \tilde{\eta}(v_{\min}). \quad (7)$$

Here we have defined the response function for SI WIMP interactions, with $E_{A,Z} = 2\mu_{A,Z}^2 v_{\min}^2/m_{A,Z}$,

$$\mathcal{R}^{SI}_{[E'_1,E'_2]}(v_{\min}) = \sum_{A,Z} \frac{2\, v_{\min}\, C_{A,Z}\, \sigma^{SI}_{A,Z}(E_{A,Z})}{m_{A,Z}\, \sigma_p\, (E'_2 - E'_1)} \times \int_{E'_1}^{E'_2} dE'\, G_{A,Z}(E_{A,Z}, E')\, \epsilon(E'). \quad (8)$$

When several energy bins are present, like when binning the data in energy or computing the maximum gap upper limit, we label each energy interval with an index $i$ and write $R_i(t)$, $\mathcal{R}^{SI}_i(v_{\min})$, etc, for quantities belonging to the $i$-th energy interval. For example, binning the harmonic series in Eq. (4) in energy gives

$$R_i(t) = R_{0i} + R_{1i} \cos[\omega(t - t_0)]\,. \quad (9)$$

Our task is to gain knowledge on the functions $\eta_0(v_{\min})$ and $\eta_1(v_{\min})$ from measurements $\hat{R}_{0i} \pm \Delta R_{0i}$ and $\hat{R}_{1i} \pm \Delta R_{1i}$ of $R_{0i}$ and $R_{1i}$, respectively. This is possible when a range of detected energies $[E'_1, E'_2]$ corresponds to only one range of $v_{\min}$ values $[v_{\min,1}, v_{\min,2}]$, for example when the measured rate is due to interactions with one nuclide only. In this case, $[v_{\min,1}, v_{\min,2}]$ is the $v_{\min}$ interval where the response function $\mathcal{R}^{SI}_{[E'_1,E'_2]}(v_{\min})$ is significantly different from zero. Ref. [12] approximated this interval with $v_{\min,1} = v_{\min}(E'_1 - \sigma_E(E'_1))$ and $v_{\min,2} = v_{\min}(E'_2 + \sigma_E(E'_2))$. When isotopes of the same element are present, like for Xe or Ge, the $v_{\min}$ intervals of the different isotopes almost completely overlap, and $v_{\min,1}$, $v_{\min,2}$ could be the $C_{A,Z}$-weighted averages over the isotopes of the element. When there are nuclides belonging to very different elements, like Ca and O in CRESST-II, a more complicated procedure should be followed (see below).

Once the $[E'_1, E'_2]$ range has been mapped to a $[v_{\min,1}, v_{\min,2}]$ range, we can estimate the $v_{\min}$-weighted averages

$$\overline{\tilde{\eta}_{[E'_1,E'_2]}} = \frac{\int_{v_{\min,1}}^{v_{\min,2}} \mathcal{R}^{SI}_{[E'_1,E'_2]}(v_{\min})\, \tilde{\eta}(v_{\min})\, dv_{\min}}{\int_{v_{\min,1}}^{v_{\min,2}} \mathcal{R}^{SI}_{[E'_1,E'_2]}(v_{\min})\, dv_{\min}} \quad (10)$$

as

$$\overline{\tilde{\eta}_{[E'_1,E'_2]}} = \frac{\hat{R}_{[E'_1,E'_2]}}{\mathcal{A}^{SI}_{[E'_1,E'_2]}}, \quad (11)$$

where

$$\mathcal{A}^{SI}_{[E'_1,E'_2]} = \int_{v_{\min,1}}^{v_{\min,2}} \mathcal{R}^{SI}_{[E'_1,E'_2]}(v_{\min})\, dv_{\min}. \quad (12)$$

In the case of binned data, these equations read $\overline{\tilde{\eta}_{0i}} = \hat{R}_{0i}/\mathcal{A}^{SI}_i$, $\overline{\tilde{\eta}_{1i}} = \hat{R}_{1i}/\mathcal{A}^{SI}_i$, with errors $\Delta\overline{\tilde{\eta}_{0i}} = \Delta R_{0i}/\mathcal{A}^{SI}_i$ and $\Delta\overline{\tilde{\eta}_{1i}} = \Delta R_{1i}/\mathcal{A}^{SI}_i$.

Upper limits on binned data can be set by replacing $\hat{R}_{[E'_1,E'_2]}$ above with the measured upper limit. Upper limits on unbinned data can be set using the method of Fox et al. [11]. The smallest decreasing function $\eta(v_{\min})$ passing through a point $(v_s, \eta_s)$ is the downward step function $\eta(v_{\min}) = \eta_s$ for $v_{\min} \le v_s$ and zero otherwise. The minimal event rate is correspondingly

$$R^{\min}_{[E'_1,E'_2]} = \eta_s \int_0^{v_s} dv_{\min}\; \mathcal{R}^{SI}_{[E'_1,E'_2]}(v_{\min}). \quad (13)$$

We use this equation in the maximum gap method [14] to bound the value of $\eta_s$ as a function of $v_s$ for CDMS, XENON10, XENON100, and SIMPLE unbinned data. For compound detectors like SIMPLE, Eq. (13) is equivalent but more transparent than the method in Appendix A.1 of [12].

The data and detector properties we use are as follows. (We acknowledge criticism of some experimental analyses [15], and try to be conservative.)

*CoGeNT.* We use the list of events, quenching factor, efficiency, exposure times and cosmogenic background given in the 2011 CoGeNT data release [16]. We separate the modulated and unmodulated parts with a chi-square fit after binning in energy and in 30-day time intervals (we fix the modulation phase to DAMA's best fit value of 152.5 days from January 1). We correct the unmodulated part by surface-event correction factors $C(E) = 1 - e^{-E^2/E_C^2}$, which are similar to those in [17] for $E_C = 1.04$ keVee ("CoGeNT high"), 0.92 keVee ("CoGeNT med."), and 0.8 keVee ("CoGeNT low"). We leave it to the reader to subtract a possible constant background contribution.

*CDMS.* For the upper limit on the total event rate we use only the T1Z5 detector [9], which gives the most stringent limits at low WIMP masses. The energy resolution is $[0.293^2 + (0.056E)^2)]^{1/2}$, and the range for the maximum gap method is 2 keV–20 keV. For the modulation amplitude we use the 95% upper bound of 0.045 events/kg-day-keV for a modulation phase equal to DAMA's in the energy range 5 keV–11.9 keV [10].

*DAMA.* We read the modulation amplitudes from [1]. We consider scattering off Na only, since the I component is under threshold for low mass WIMPs and reasonable local Galactic escape velocity. We show results for two values of the Na quenching factor: 0.3 and 0.45 (the latter suggested in [18]). No channeling is included, as per [19].

*XENON100.* The exposure is 48 kg $\times$ 100.9 days. We convert the energies of the three candidate events in Ref. [7] into S1 values, and use the Poisson fluctuation formula Eq. (15) in [20] to compute the energy fluctuations. We use the light efficiency function $\mathcal{L}_{\rm eff}$ in Fig. 1 of [7]. We obtain the cut acceptance by multiplying two factors: the overall cut acceptance, which we set to a conservative value of 0.6 since it is unclear why in Fig. 2 of [7] it would depend on the WIMP mass when expressed as a function of S1, and the S1/S2 discrimination acceptance, taken from the just mentioned Fig. 2. We use a maximum gap method over the interval $4 \leq S1 \leq 30$ photoelectrons.

*XENON10.* We follow Ref. [6] and use only S2 without S1/S2 discrimination. The exposure is 1.2 kg $\times$12.5 days. We consider the 32 events within the 1.4 keV-10 keV acceptance box in the Phys. Rev. Lett. article (not the arxiv preprint, which had an S2 window cut). We take a conservative acceptance of 0.94. For the energy resolution, we are more conservative than [6]: we convert the quoted energies into number of electrons $n_e = E\mathcal{Q}_y(E)$, with $\mathcal{Q}_y(E)$ as in Eq. 1 of [6] with $k = 0.11$, and use the Poisson fluctuation formula in [21].

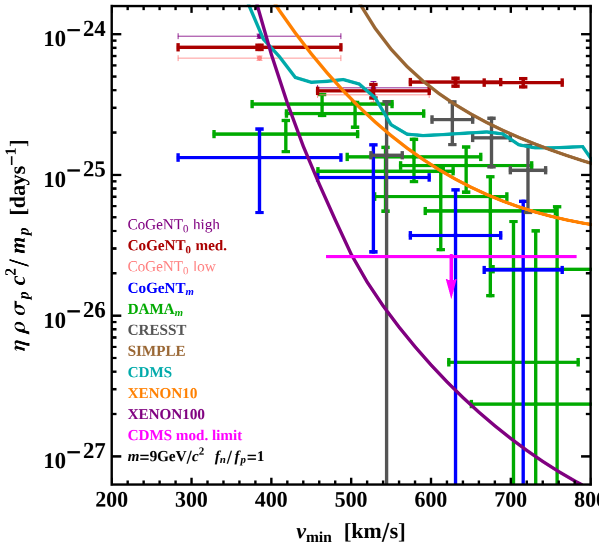


FIG. 1: Measurements and upper bounds on the unmodulated and modulated part of the velocity integral $\eta(v_{\rm min})$ as a function of $v_{\rm min}$. For this case of spin-independent isospin-symmetric couplings and WIMP mass of 9 GeV, the XENON100 and CDMS bounds exclude all but the lowest energy CoGeNT and DAMA bins.

*SIMPLE.* We consider only Stage 2, with an exposure of 6.71 kg days and no observed candidate event. We take an efficiency $\eta'(E) = 1 - \exp\{-\Gamma[(E_{dep}/E_{thr}) - 1]\}$ with $\Gamma = 4.2\pm0.3$. With no events observed, the Poisson and maximum gap upper limits coincide.

*CRESST-II.* We take the histogram of events in Fig. 11 of Ref. [4]. The acceptance is obtained by adding each module at its lower energy acceptance limit in their Table 1. The electromagnetic background is modeled as one $e/\gamma$ event in the first energy bin of each module. The exposure is 730 kg days. We assume a maximum WIMP velocity in the Galaxy such that W recoils can be neglected. To take into account the Ca and O components, we follow the same philosophy as Method 2 in Appendix A.2 of [12], but, without having to assume a constant efficiency in each energy bin, we are able to cover the CRESST-II energy range without gaps with the following binning: three high-energy bins ($i = 4, 5, 6$) with scatterings off O only (assuming a maximum $v_{\rm min}$ of $\sim 750$ km/s): $[17, 20]$, $[20, 23]$, and $[23, 26]$ keV; and three corresponding low-energy bins ($i = 1, 2, 3$) with the same $v_{\rm min}$ range and scatterings off O and Ca: $[11, 13]$, $[13, 15]$, and $[15, 17]$ keV. To avoid complications with the overlap of the tails of the weight functions $\mathcal{R}_i^{SI}(v_{\rm min})$, we cut them outside the $v_{min}$ interval $[v_{\rm min}(E'_{1i}), v_{\rm min}(E'_{2i})]$, i.e. we do not enlarge the $v_{\rm min}$ interval using the energy resolution. Having determined $\overline{\eta_{0i}} = \hat{R}_{0i}/\mathcal{A}_{{\rm O},i}^{SI}$ for $i = 4, 5, 6$ using O only in $\mathcal{A}_{{\rm O},i}^{SI}$, we estimate the Ca contribution

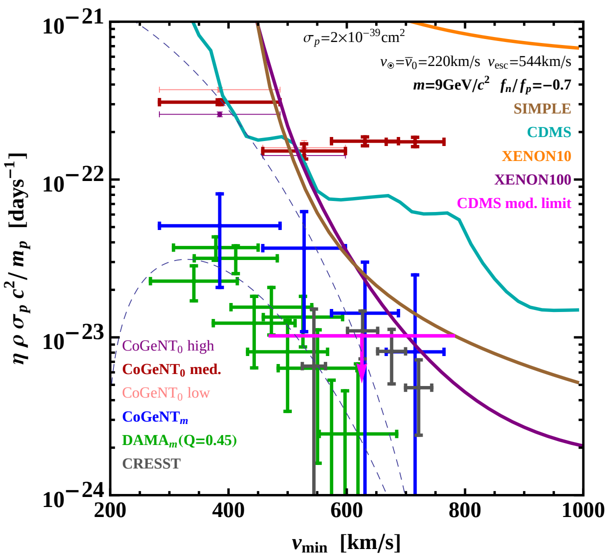


FIG. 2: Same as Fig. 1 but for spin-independent isospin-violating couplings $f_n/f_p = -0.7$ and DAMA quenching factor $Q_{\rm Na} = 0.45$. The first CoGeNT energy bin and the first five DAMA energy bins are compatible with all existing bounds. The dashed lines show the expected $\eta_0(v_{\rm min})$ (upper line) and $\eta_1(v_{\rm min})$ (lower line) for a WIMP-proton cross section of $2 \times 10^{-39}$ cm$^2$ and a Maxwellian halo with Solar System speed relative to the WIMPs of 220 km/s, 1d velocity dispersion $220/\sqrt{2}$ km/s, and Galactic escape speed 544 km/s.

to bins $j = 1, 2, 3$ as $R^{SI}_{{\rm Ca},j} = \mathcal{A}^{SI}_{{\rm Ca},j}\overline{\tilde{\eta}_{0,j+3}}$, where $\mathcal{A}^{SI}_{{\rm Ca},j}$ contains only Ca. Then to reduce the effect of the propagation of errors in subtracting the Ca contribution, we combine the three low-energy bins into one, obtaining for it $\overline{\tilde{\eta}_0} = \sum_{j=1}^{3}(\hat{R}_{0j} - R^{SI}_{{\rm Ca},j})/\sum_{j=1}^{3}\mathcal{A}^{SI}_{{\rm O},j}$.

Fig. 1 shows the measurements and upper bounds on $\tilde{\eta}_0$ and $\tilde{\eta}_1$ for a 9 GeV WIMP with spin-independent isospin-symmetric couplings. Here we took $Q_{\rm Na} = 0.3$. The XENON100 and CDMS bounds exclude all but the lowest energy CoGeNT and DAMA bins. Although a larger value of $Q_{\rm Na}$ would shift the DAMA points upward and to the left by a factor $Q_{\rm Na}/0.3$, the tension between CoGeNT and DAMA on one side and XENON100 and CDMS on the other is strong. Varying the WIMP mass from 6 to 12 GeV does not improve the situation.

The tension is alleviated for isospin-violating couplings $f_n/f_p = -0.7$, especially if the DAMA Na quenching factor is taken as $Q_{\rm Na} = 0.45$ (see Fig. 2). In this case, the first CoGeNT energy bin and the first five DAMA energy bins are compatible with all existing bounds. Since CDMS and CoGeNT both use Ge, the tension of the higher energy CoGeNT bins with the CDMS modulation constraint remains. If the unmodulated CoGeNT rate at high recoil energies is subtracted throughout the energy range, the relative modulation amplitude would have to be high, $\sim 30\%$.

As an intriguing possibility, the dashed lines show the expected $\eta_0(v_{\rm min})$ (upper line) and $\eta_1(v_{\rm min})$ (lower line) for a WIMP-proton cross section of $2 \times 10^{-39}$ cm$^2$ and a Maxwellian halo with Solar System speed relative to the WIMPs of 220 km/s, 1d velocity dispersion $220/\sqrt{2}$ km/s, and WIMP Galactic escape speed 544 km/s. These theoretical lines, representing the canonical model of the dark halo with a low escape speed in accord with the most recent determinations [22], pass tantalizingly close to the data points. Given our very limited knowledge of the velocity distribution of WIMPs in the Galactic halo, it is not inconceivable that a distribution function with a sharp cut-off at large velocities may account for the current data. It is therefore of the utmost interest that CDMS extend their modulation analysis to lower energies, so as to confirm or exclude the spin-independent interpretation of the CoGeNT and DAMA annual modulations over the full $v_{\rm min}$ range.

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