Faster than Fourier -- Ultra-Efficient Time-to-Frequency Domain Conversions for FDTD

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Abstract

Two highly efficient methods of computing magnitude and phase from time-domain data are demonstrated. These methods, based on solution of linear equations, are found to be equally accurate and more efficient than Fourier transform methods (DFT and FFT) for limited numbers of frequencies. These methods provide a significant savings in computation time and storage requirements for FDTD simulations which require a large number of time-to-frequency domain conversions. Although some FDTD researchers may have applied the method for single frequency simulations, it is not widely known or used. The multiple frequency extension of this method has not been used previously, to my knowledge.

1. Introduction

Many applications of the finite-difference time-domain (FDTD) method require conversion of time-domain field data to magnitude and phase (frequency-domain data) over large regions of the model. Applications include bioelectromagnetic dosimetry of cellular telephones [1] and power lines [2], radar cross section calculations [3], and calculations of S-parameters [4]. This has traditionally been accomplished with either the Fast Fourier transform (FFT) [5] or the Discrete Fourier transform (DFT) [6]. Fourier transform calculations provide an accurate time-to-frequency domain conversion, however their computation and storage requirements are very high, often as large as or larger than the FDTD simulation itself.

This paper presents two methods that are extremely efficient, accurate, and easy to program for converting time to frequency domain data. These methods can obtain magnitude and phase information from the low kHz range to the high GHz range with the added advantage that for many applications, they can be applied with virtually no memory or computational requirement (beyond the FDTD requirements themselves). Although the single-frequency method may have been used by some FDTD reserachers, it is not widely-known, and the multiple frequency extension has not been previously applied, to my knowledge.

2. Two Equations - Two Unknowns Method

A simple, direct method for solving for the magnitude and phase of a sine wave in the time-domain is based on writing two equations in two unknowns (magnitude and phase) for the time-domain fields, and then solving them directly for the magnitude and phase. At a given location in space the fields as a function of time can be written:

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$$\begin{aligned} A \sin(\omega t_1 + \theta) &= q_1 \\ A \sin(\omega t_2 + \theta) &= q_2 \end{aligned} \qquad (2)$$

where A is the magnitude, θ is the phase angle, and ω (= 2 • F) is the angular frequency. At two times, t_1 and t_2, the values q_1 and q_2 are obtained from the FDTD simulation. Therefore, these equations can be solved for the unknowns, A and θ, to give direct relationships for these values.

This method provides dramatic savings in computation time and memory over Fourier transform methods as shown in Figure1. These savings are obtained because the Fourier transform methods require calculations to be made over the last half cycle of the simulation, and the two-equation method requires only a single calculation. Values for the FFT are not plotted, because it has already been shown to be less efficient than the DFT. [6]

The choice of t_1 and t_2 depends on the simulation. For most FDTD simulations, the spatial resolution Δx, is on the order of $\lambda / 10$ to $\lambda / 100$. For these simulations, t_1 and t_2 can be the last two time steps of the simulation. For higher-resolution simulations, the time resolution is also high ($\Delta t = \Delta x/2c$), so q_1 and q_2 are nearly equal if t_1 and t_2 are so close. This results in errors due to numerical round-off when calculating A and θ. For these simulations, it is better to choose t_1 to be a few time steps before the end of the simulation, and t_2 to be the final time step. Optimized sample spacings can be found using the method described in Section 3 below. In this case, the number of frequencies is 1, and the condition number of the matrix as a function of sample spacing is indicative of the resultant accuracy of the solution.

The equations for A and θ can be programmed one of two ways, depending on t_1 and t_2. The first is to store (or output to disk) the value of q_1 at time step t_1, and then when the final time step, t_2, is reached, the values of A and θ can be calculated. This is necessary if t_1 and t_2 are not subsequent time steps. An alternate method of eliminating the memory requirement can be used when t_1 and t_2 are taken to be the last two time steps. For the final time step, q_1 is stored in a single location (not an array), then q_2 is calculated from the FDTD algorithm, then A and θ are calculated and output to disk. This is then repeated for each location.

This method provides accurate magnitude and phase calculations for simulations with clean, sine wave output. Noise and dc offsets will cause errors. Ramped sine excitations known not to cause a dc offset should be used or a pulse with a very smooth turn-on. Ramped sine excitations have also been observed to reduce or eliminate numerical noise in FDTD simulations.

3. N-Equations N-Unknowns

The two-equations two-unknowns method can be extended to multiple frequencies. In this case, N-equations are solved for N-unknowns (amplitude and phase at N/2 frequencies of interest). This requires samples at N time steps and results in the following equations for two frequencies:

$$\begin{aligned} A_1 \sin(\omega_1 t_1 + \theta_1) + A_2 \sin(\omega_2 t_1 + \theta_2) &= q_1 \\ A_1 \sin(\omega_1 t_2 + \theta_1) + A_2 \sin(\omega_2 t_2 + \theta_2) &= q_2 \\ A_1 \sin(\omega_1 t_3 + \theta_1) + A_2 \sin(\omega_2 t_3 + \theta_2) &= q_3 \\ A_1 \sin(\omega_1 t_4 + \theta_1) + A_2 \sin(\omega_2 t_4 + \theta_2) &= q_4 \end{aligned} \qquad (4)$$

This can be extended to multiple frequencies:

$$q_i = \sum_{n=1}^{N} A_n \sin(\omega_n t_i + \theta_n) \quad i = 1,2,3,\ldots,2N \qquad (5)$$

Using trigonometric identities on the sine function, this can be broken into a matrix equation which can be solved for functions ($A_n\cos(\theta_n)$)) of A_n and θ_n .

$$\begin{bmatrix} \sin(\omega_1 t_1) & \cos(\omega_1 t_1) & \sin(\omega_2 t_1) & \cos(\omega_2 t_1) & \ldots & \sin(\omega_N t_1) & \cos(\omega_N t_1) \\ \sin(\omega_1 t_2) & \cos(\omega_1 t_2) & \sin(\omega_2 t_2) & \cos(\omega_2 t_2) & \ldots & \sin(\omega_N t_2) & \cos(\omega_N t_2) \\ \vdots & \vdots & \vdots & \vdots & & \ldots & \ldots \\ \sin(\omega_1 t_N) & \cos(\omega_1 t_N) & \sin(\omega_2 t_N) & \cos(\omega_2 t_N) & \ldots & \sin(\omega_N t_N) & \cos(\omega_N t_N) \end{bmatrix} \begin{bmatrix} A_1\cos(\theta_1) \\ A_1\sin(\theta_1) \\ A_2\cos(\theta_2) \\ A_2\sin(\theta_2) \\ \vdots \\ A_N\cos(\theta_N) \\ A_N\sin(\theta_N) \end{bmatrix} =$$

A standard matrix solution method such as Gaussian elimination via Linpack is used to obtain the vector of functions, and the unknowns A_n and θ_n are found from these functions.

This specific solution method requires a multi-frequency source in the form of (5), which does not utilize methods such as sine wave ramping to prevent high frequency transients or dc offsets. These specialized ramps could be included in the source type, and the same solution method could be followed. This would change the specifics of the matrix above. In practice, this was not found to be necessary for human dosimetry models, although it may be in other types of applications.

In theory, the method above provides an exact conversion from time to frequency domain. In practice, however, the matrix can be ill-conditioned. This happens when the cosine and sine samples become very close together so that they are numerically indistinguishable. In particular, this occurs when the time samples (t_1, t_2, t_3, etc.) are too close together, when a very large number of frequencies are involved, or when the frequencies are too close together. There is a also a problem when their relative magnitudes are several orders of magnitude different. This paper discusses each of these problems and the efficiency tradeoffs for solving them. In practice, this method, which we will call "N-equations N-unknowns" or (NENU) is the most efficient method of converting from time to frequency domain data for up to about 30 frequencies, which covers most of the applications of FDTD to-date. Singular Value Decomposition is found to significantly increase the range of application of this method.

3. Conclusions and Summary

Figure 1 shows the relative cost of the Fourier transform, 2E2U and NENU methods. For up to 30 frequencies, the NENU method is most efficient. The cost of taking the pseudo-inverse of the matrix for this method eventually outstrips the cost of the additional FDTD simulations required in the 2E2U method, and after 30 frequencies the 2E2U method is more efficient. Both of these methods are more efficient than the DFT method.

Memory and storage are tradeoffs for these methods. The DFT must store a complex value (equal to 2 real values) for every location and parameter of interest. The 2E2U can completely eliminate this requirement if the sampling resolution is sufficient to allow computation of magnitude and phase from the final two time steps of the simulation. The NENU method must store the matrix which is $(2\ NF)^2$ where NF is the number of frequencies for each location and parameter of interest. These values would generally be written to disk, with final solution being done as a post-processing step, but this may be prohibitively expensive for some applications.

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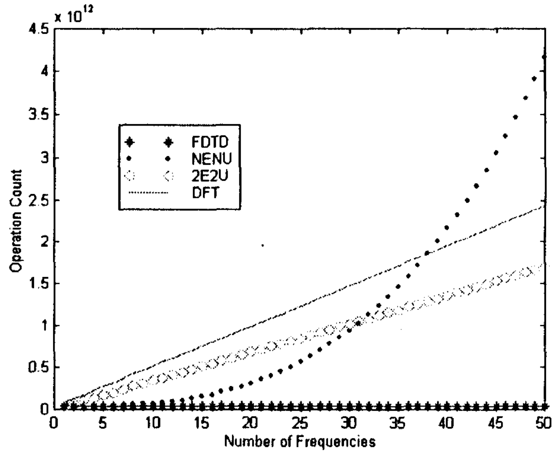

Figure 1: Computational requirements of the FDTD algorithm and associated time-to-frequency domain conversions for the parameter values :
# of FDTD time steps = 2000, # of parameters of interest = 6 (all **E** and all **H**)
# of FDTD cells =# of FDTD cells of interest = 100 x 100 x 100