Mattis and Pan Reply: After several independent calculations failed to confirm our published[1] numbers on the ground-state energy of the $s=\frac{1}{2}$ antiferromagnet in two dimensions, we checked our computer programs and found some deplorable errors introduced in proceeding from one dimension to two. The new numbers for $s=\frac{1}{2}$ are the following: $E_0(3)=-4.7493273$, $\lambda_1^2=(0.4582)^2=0.2100$, and $\lambda_2^2=(-0.2946)^2=0.0868$ resulting in $e_0=0.5592$, a much less satisfactory ground-state energy per spin than before, and $\Lambda^2=0.51$. (These numbers correct the first line of Table I in our paper which now agrees with the calculation of Yedidia as presented in his preceding Comment,[2] and with the identical results of von der Linden.[3]) This value of $\Lambda^2<1$ fails indeed to prove long-range order (LRO) for $s=\frac{1}{2}$, which becomes once again an open question.

Our calculation for $s=1$ was not subject to this unfortunate programming error, and so the numbers for $s=1$ quoted in the table stand intact. Nevertheless, we missed an obvious feature which Lin[4] kindly pointed out to us (too late, unfortunately, to include in the published manuscript), and which the readers will wish to note, viz., for $s=1$ the total renormalization-group energy per site, $e_0=-1.907$, is not as good as that of the far simpler Ising-Néel state, $e_0=-2.0$, a state which, incidentally, exhibits perfect LRO! We recall that real-space renormalization group was unable to prove this, although the calculated $\Lambda^2=0.9136$ came to within 10% of doing so.

Thus, far from yielding fortuitously excellent results as we believed, the real-space renormalization-group method which we used in our paper seems to converge as slowly in two dimensions as it did in one, and the conclusions which can be derived from 3×3 clusters are totally inadequate. This is unfortunate, as the calculation of Λ for clusters of the next allowed size, 5×5, is just at the threshold of possibility, and for 7×7, outside the realm of possibility with present-day computing technology.[5] However, it remains interesting, and important, to determine whether LRO exists.

Following the discovery and correction of our programming error, we extended the $s=\frac{1}{2}$ calculation to the anisotropic Heisenberg antiferromagnet. If $J_x=J_y=1$ and $J_z\geq1$, then evidently $\Lambda_z\geq\Lambda_x=\Lambda_y$. If J_z is sufficiently large, the transverse interactions are minor perturbations on an Ising-Néel antiferromagnet, and ground-state LRO necessarily sets in [as manifested by $\Lambda_z^2(J_z)\geq1$ at large J_z]. Also, $\Lambda_z^2(J_z)<1$ in the "x-y" phase, $J_z<1$. *Thus, if there is no LRO at the isotropy point $J_z=1$, $\Lambda_z^2(J_z)$ must be discontinuous there* [i.e., $\Lambda_z^2(1+\delta)\neq\Lambda_z^2(1-\delta)$ for $\delta\rightarrow0$]. Conversely, if $\Lambda_z^2(J_z)=1$ and $d[\Lambda_z^2(J_z)]/dJ_z<\infty$ at $J_z=1$, we can safely infer the existence of LRO at the isotropy point.

Our new calculations for 3×3 clusters show $\Lambda_z^2(J_z)\geq1$ for $J_z\geq1.23$. We have extended this calculation to 5×5 clusters using a variational approach. The resulting threshold for LRO drops, and it is now $J_z\geq1.13$. *The curves show no hint of discontinuity* over a wide range of J_z. Although these preliminary results are inconclusive, we are hopeful that the question of LRO in the isotropic antiferromagnet can ultimately be settled in this manner, and are therefore proceeding with a rigorous calculation for the 5×5, and ultimately, when data storage problems are resolved, for the 7×7 clusters.

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[1]D. C. Mattis and C. Y. Pan, Phys. Rev. Lett. **61**, 463 (1988).

[2]J. S. Yedidia, preceding Comment [Phys. Rev. Lett. **61**, 2278 (1988)].

[3]Wolfgang von der Linden, private communication.

[4]H.-Q. Lin, private communication.

[5]It should be noted that calculation of Λ requires more knowledge of the ground-state correlations than is obtained from conventional Monte Carlo calculations.