Optimizing Inverse Electrocardiographic Problem: Hybrid and High-Order Finite Element Method

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Motivation and Goals

One type of inverse problems in electrocardiography (ECG) is to non-invasively reconstruct epicardial electric potentials from body-surface measurements. We study how to design the finite element discretization of such problem, so as to optimize the conditioning and stability of the resulting numerical system.

The inverse ECG problem is ill-posed, requiring different discretization strategies from its corresponding forward problem (see Fig.1). We developed two new techniques: 1) a unified finite element framework that accepts tetrahedral, hexahedral and prismatic elements, and 2) a high-order finite element method with a flexible hierarchical structure.

We plan to integrate these techniques into the ECG toolkit within SCIRun. The toolkit will facilitate realistic simulation of clinical applications such as ischemia and arrhythmia.

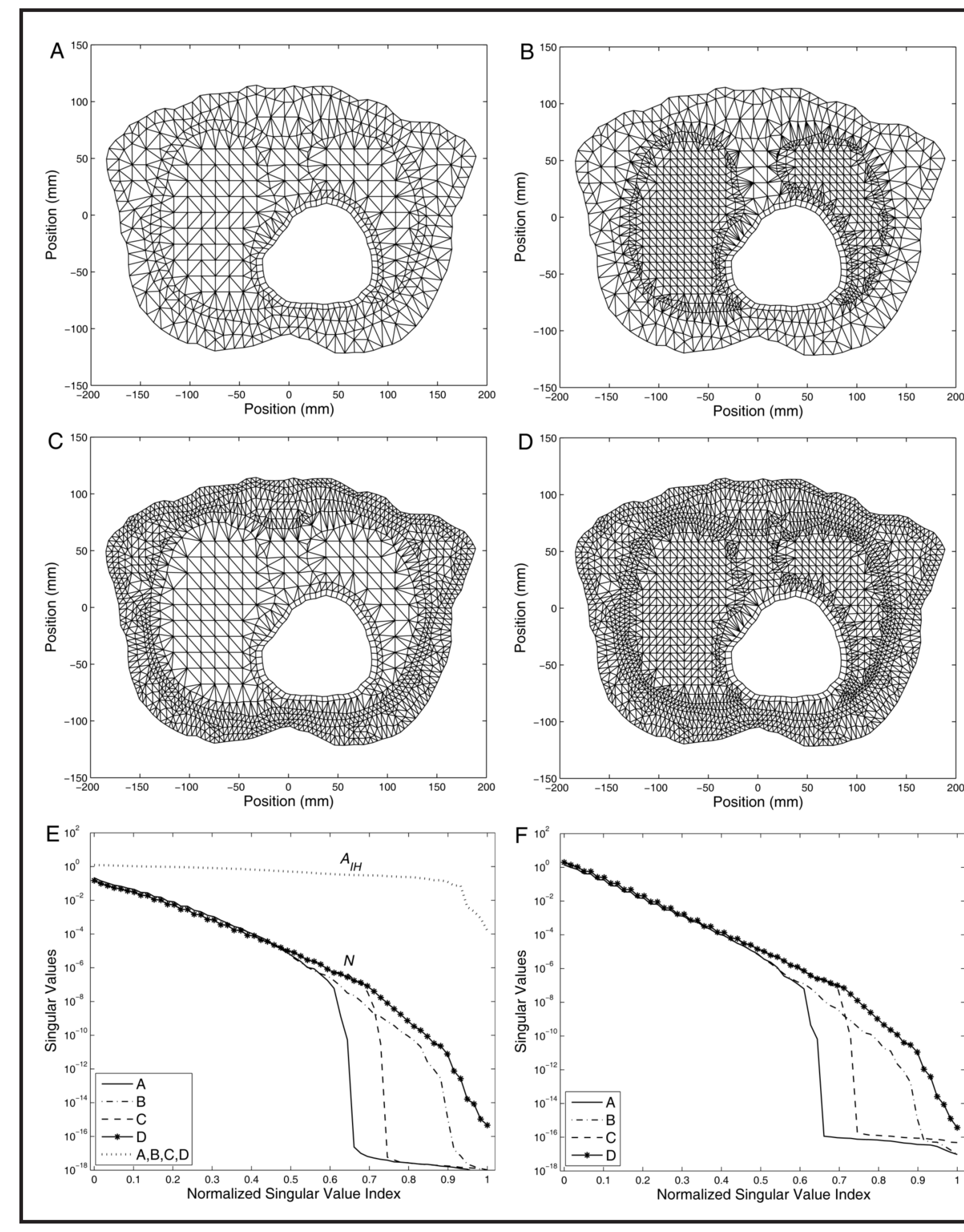


Fig 1. Simulation based on a 2D torso mesh shows the effect of different refinements on the conditioning of the numerical system to be inverted.

Hybrid Elements

Introducing prismatic elements into conventional tetrahedral mesh brings extra benefits:

- Control the resolution on epicardium, which determines the ill-conditioning of the resulting numerical system.
- Capture the high-potential-gradient field around the heart.
- Avoid ill-shaped elements with bad aspect-ratio problems.

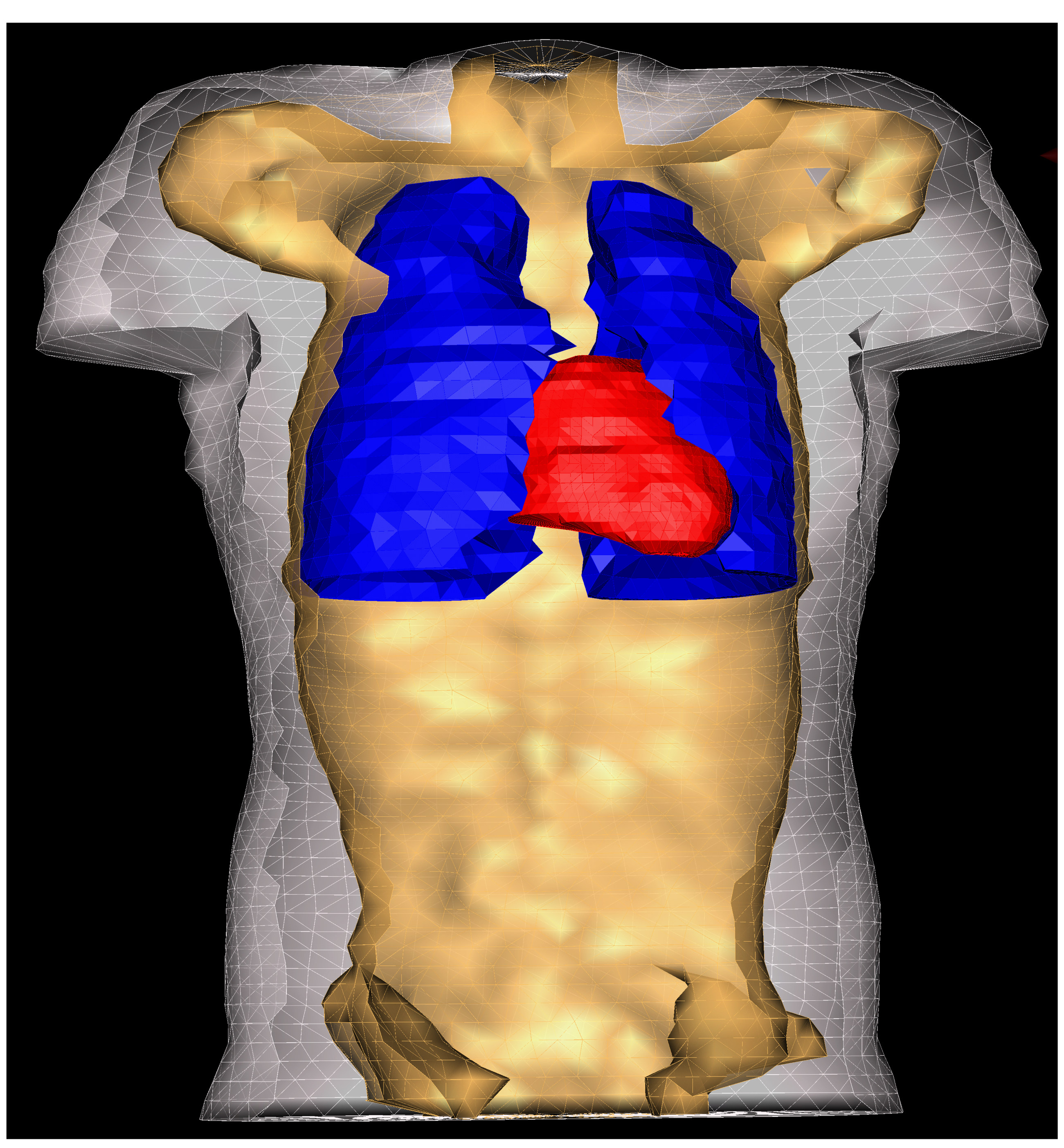


Fig 2. The Utah Torso model contains well-segmented tissues with realistic conductivity values assigned. The model consists of 168 thousand nodes and 1 million elements.

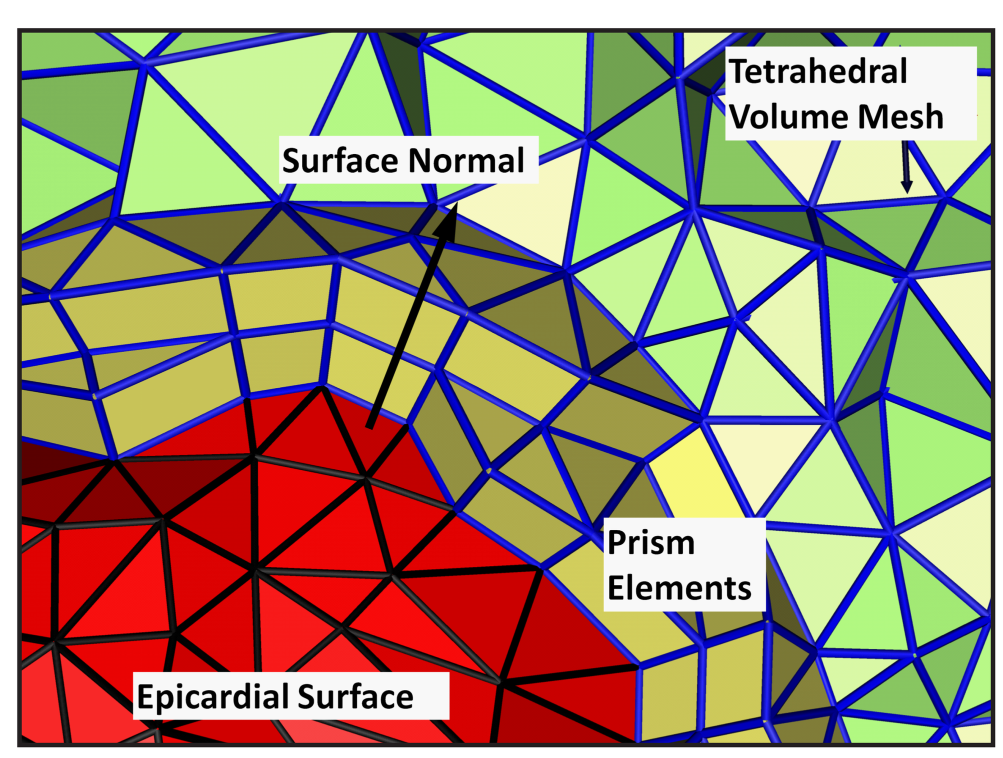


Fig 3. Hybrid mesh elements (prisms and tetrahedra) around the heart surface.

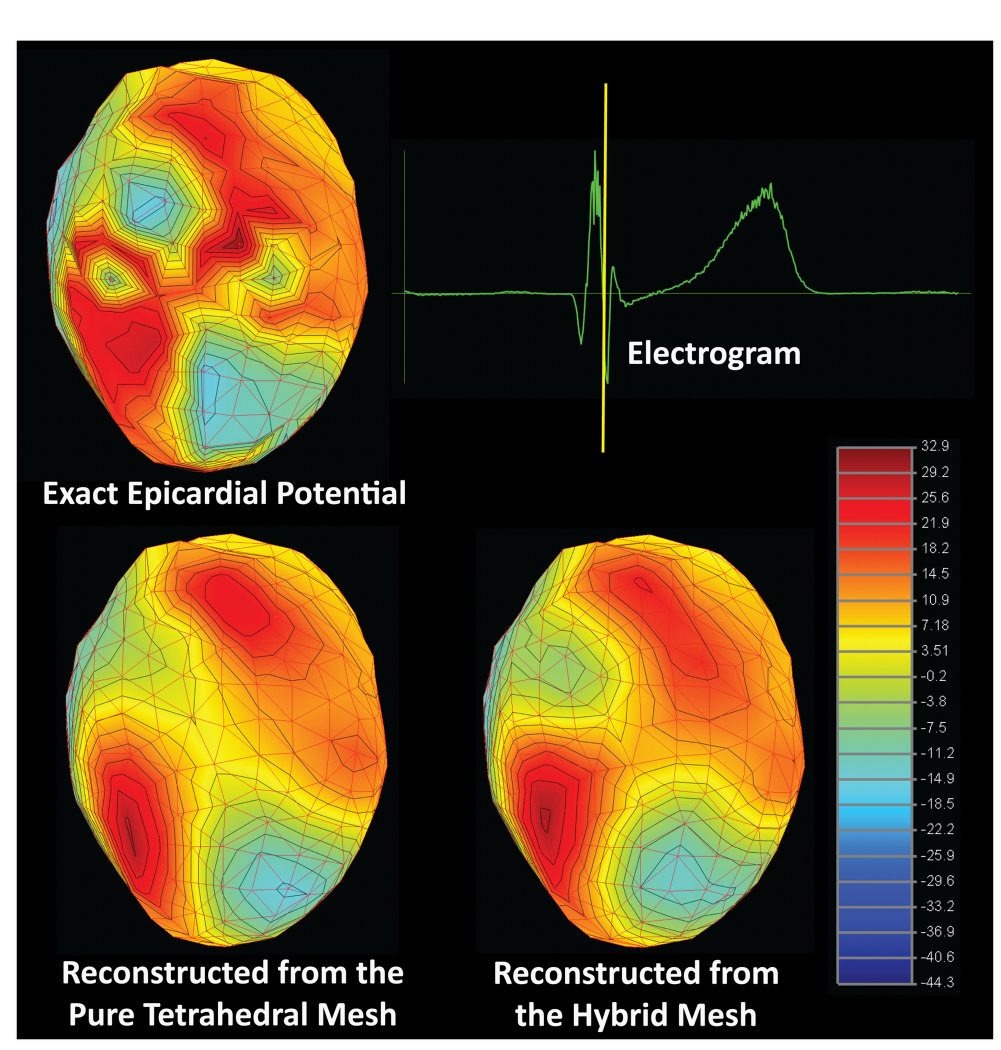


Fig 4. Epicardial potentials measured during the QRS-interval, compared to potentials inversely calculated under 0.1% input noises.

High-Order Finite Elements with Hierarchical Truncation

We devised high-order finite elements with a hierarchical structure that provides a seamless approach of refining the volume mesh, limiting ill-conditioning, and avoiding aspect-ratio problems.

What the conventional linear finite element method gives:

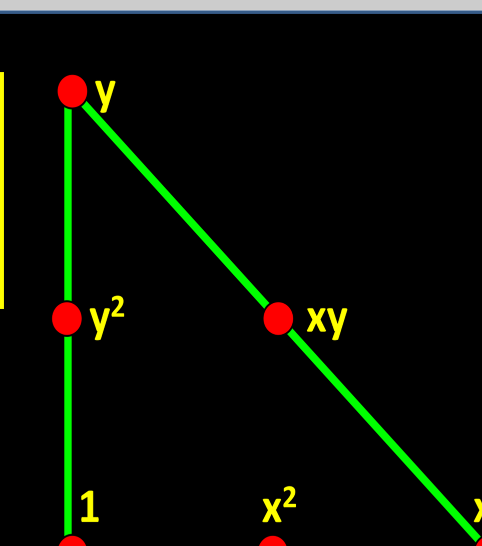
$$u_T = K_{T,H}\, u_H, \quad K_{T,H} = A_{T,V} A_{V,V}^{-1} A_{T,V}$$

$K_{T,H}$ maps the heart potential uH to the torso potential uT.
$K_{T,H}$ is composed of linear torso surface, ***linear volume***, and linear heart surface.

Applying second-order finite elements and tailoring the resulting numerical system for inverse calculation:

$$\begin{pmatrix} u_T^1 \\ u_T^2 \end{pmatrix} = \begin{pmatrix} K_{T,H}^{1,1} & K_{T,H}^{1,2} \\ K_{T,H}^{2,1} & K_{T,H}^{2,1} \end{pmatrix} \begin{pmatrix} u_H^1 \\ u_H^2 \end{pmatrix}$$

$$u_T^1 = K_{T,H}^{1,1} u_H^1$$



$K_{T,H}^{1,1}$ is composed of linear torso surface, ***quadratic volume***, and linear heart surface.

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