Volumetric Parameterization and Trivariate B-spline Fitting using Harmonic Functions

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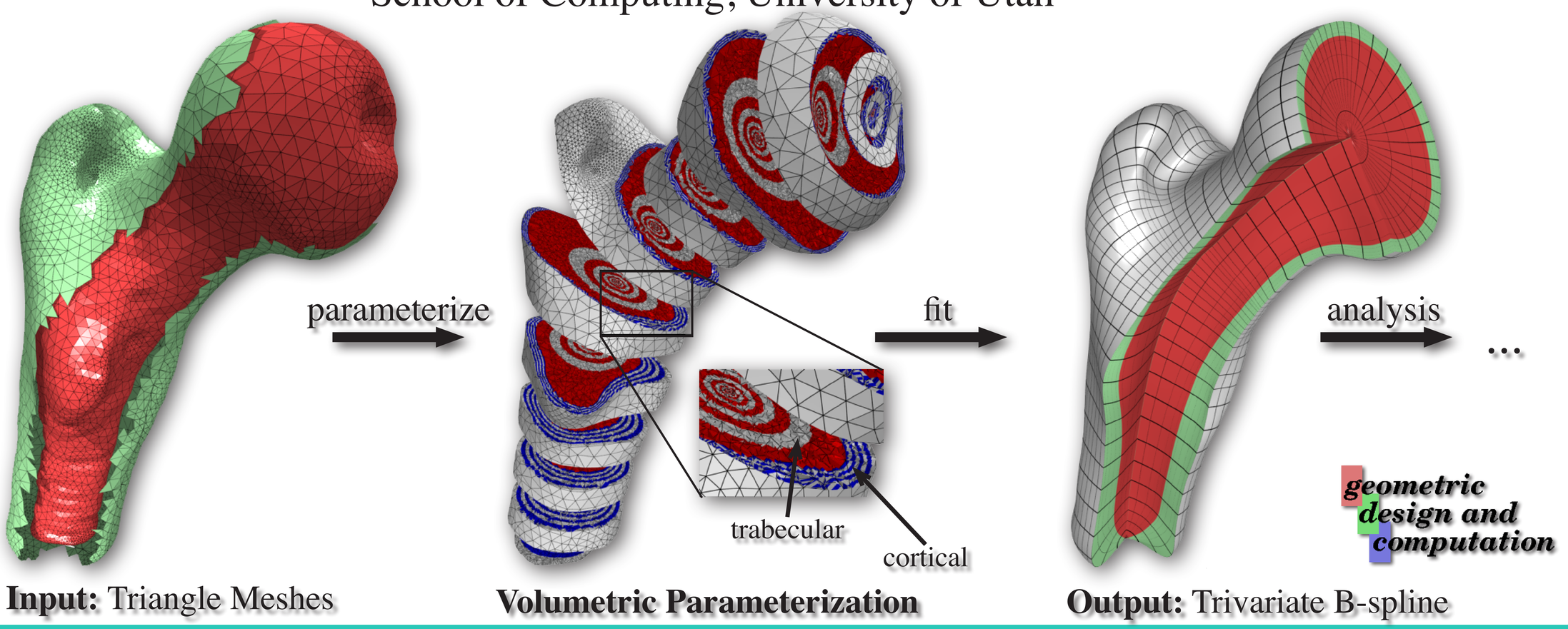
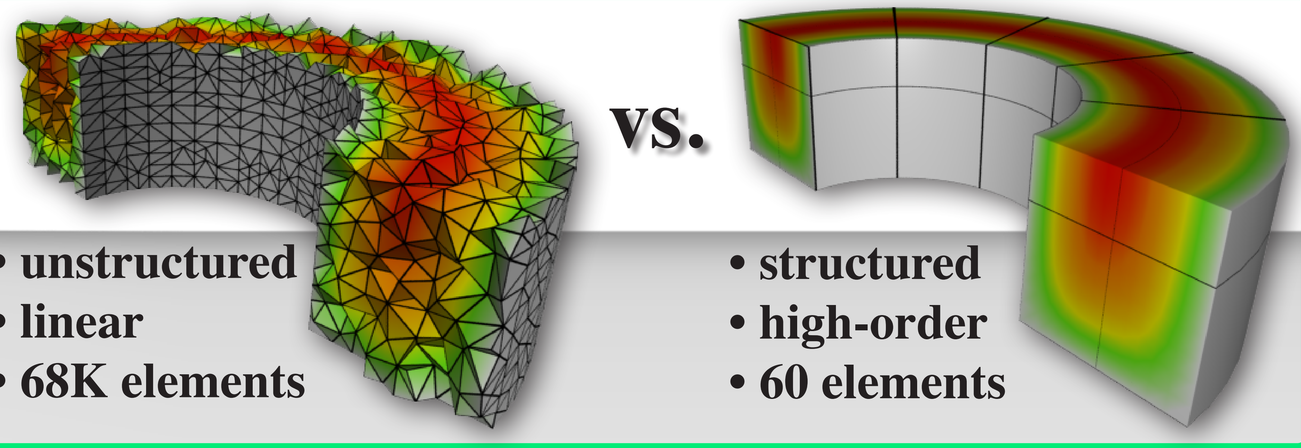


Input: Triangle Meshes **Volumetric Parameterization** **Output:** Trivariate B-spline

Summary:

A methodology to establish a bijective mapping (parameterization) of a volumetric model in a way that it can be used to fit a single volumetric (trivariate) B-spline to data so that simulation attributes can also be modeled and physical simulation can be directly applied to the representation. The methodology is based on functions which do not have local maxima and minima (harmonic functions). Input data consists of both a closed triangle mesh representing the exterior geometric shape of the object and interior triangle meshes representing material attributes (e.g. different bone materials) or other interior features. Trivariate B-spline geometric and attribute representations are generated from the resulting parameterization with guaranteed quality of approximation to the original data.

- **B-splines** are piecewise polynomials with minimal support with respect to degree, smoothness and domain partition.
- **Goal in Geometric Modeling:** Given exterior and interior boundaries of a model, generate a **single trivariate B-spline** representation.



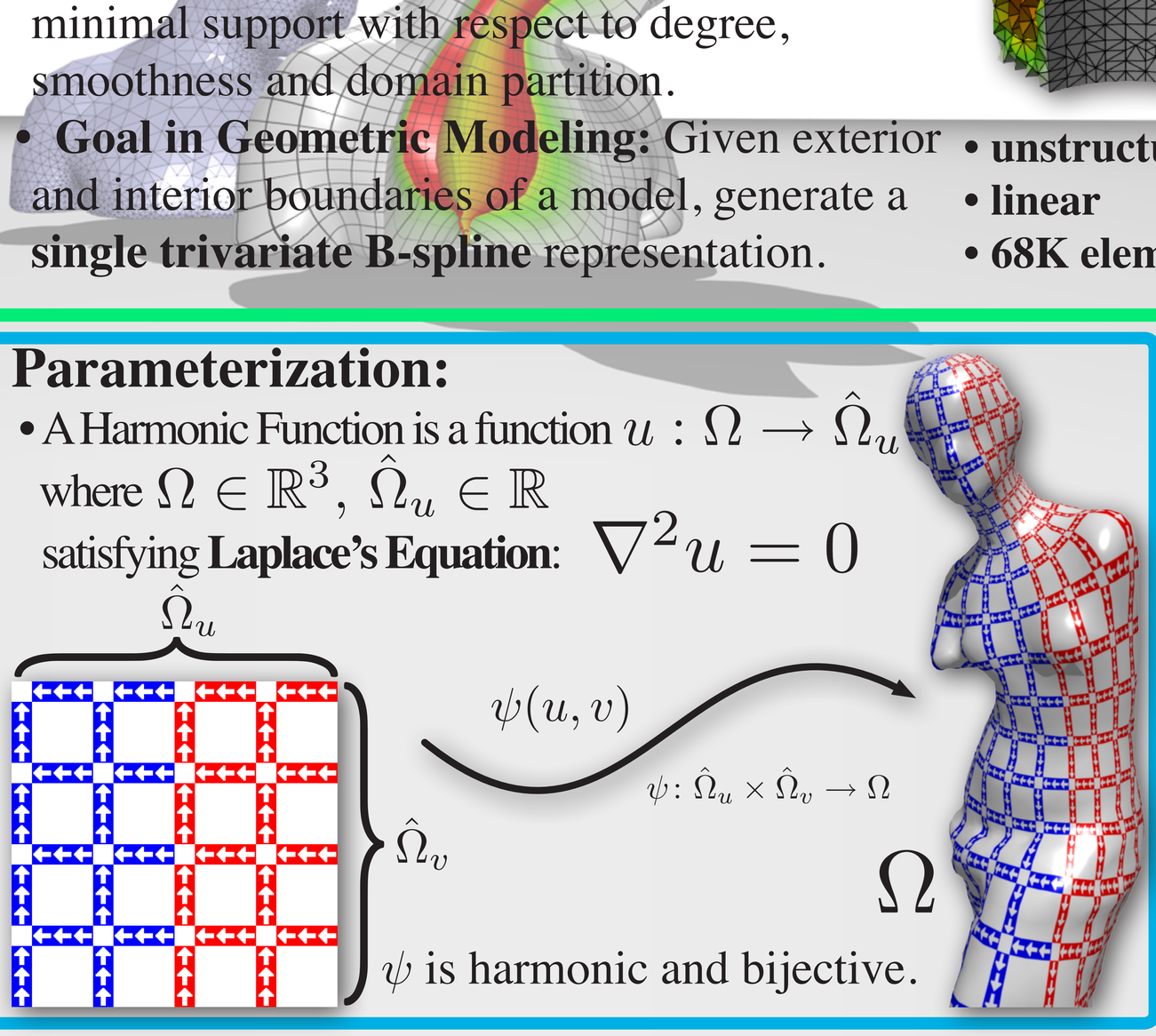
- **unstructured**
- **linear**
- **68K elements**

vs.

- **structured**
- **high-order**
- **60 elements**

Parameterization:

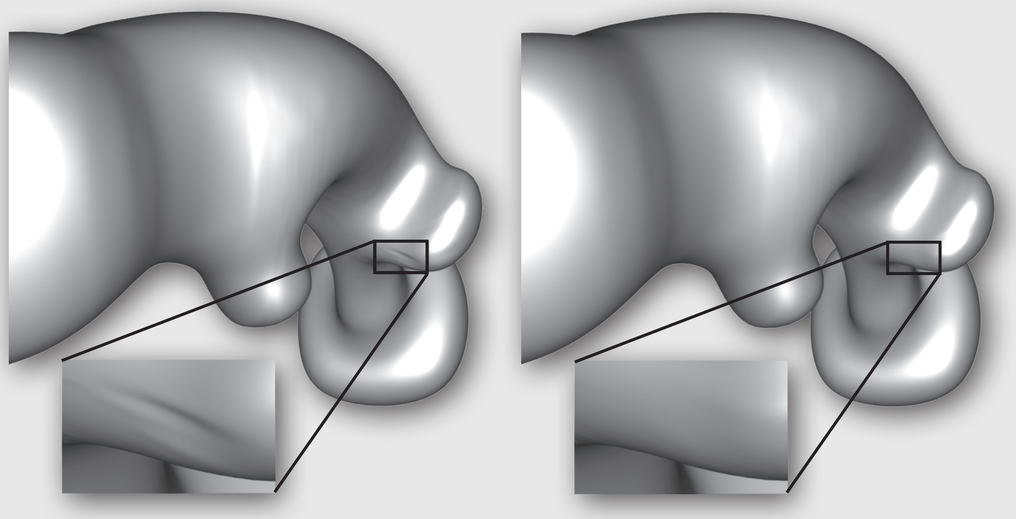
- A Harmonic Function is a function $u : \Omega \rightarrow \hat{\Omega}_u$ where $\Omega \in \mathbb{R}^3$, $\hat{\Omega}_u \in \mathbb{R}$ satisfying **Laplace's Equation**: $\nabla^2 u = 0$



ψ is harmonic and bijective.

Data Fitting:

- Hybrid Approximation/Interpolation Method



interpolation **our method**

References:

- Proceedings of the 2008 ACM symposium on Solid and physical modeling, pages 269–280, NY, USA, 2008. ACM. **Best Paper.**
- Computer Aided Geometric Design, 26(6), pages 648–664, 2009