TRANSFER FUNCTION MAPPING FOR A GRID CONNECTED PV SYSTEM USING REVERSE SYNTHESIS TECHNIQUE

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Abstract— **Mathematical modeling of power electronic converters is a historical problem. Numerous efforts have been documented in literature to model different converter topologies and their control schemes since the 1940s. Traditional modeling approaches avoid transfer function derivation due to high degree of nonlinearity involved with the power converter's switched operation. In this paper, a simple transfer functions for a grid connected PV system is derived. First an ideal transfer function is derived from the steady state input and output relationship of ideal converters. Then, it is compared with the non-ideal model of a practical converter. Using this technique, it is possible to find generalized modules to represent a complex power converter, and it is possible to deduce new converter topologies using the reverse mapping.**

Keywords – Modeling, power converter, grid, PV, mapping, synthesis.

I. INTRODUCTION

Since the evolution of the power electronic converters, different mathematical modeling and analysis techniques of those power converter and associate control circuits have been proposed [1]-[6]. Small-signal analysis based on state-space averaging or circuit averaging for the pulse-width modulated (PWM) switches is most widely used [2] [3], and these approaches utilize the circuit structure of a converter to mathematically model the power converter. These methods are suitable for slow-scale dynamics, and these fail to predict the fast scale dynamics. Another widely practiced technique of system identification of power converter is analyzing the input and output responses of a power converter, and this identification methods can be divided in two wide categories: non-parametric and parametric system identification [8][11].

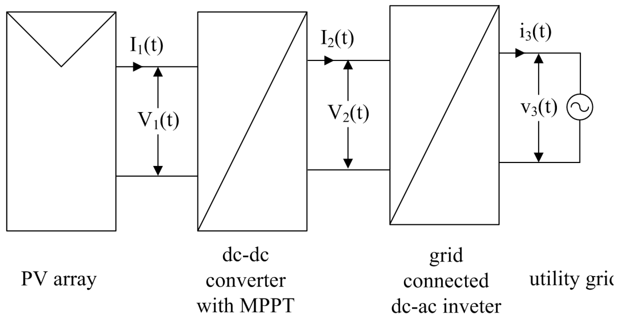

Fig. 1. Two stage grid connected PV inverter

An initial assumption of the transfer function is made in parametric identification, and frequency domain or autocorrelation analysis is generally performed in non-parametric identification without any prior assumption on the converter's model [4]-[14]. However, the transfer function derivation of power converters in Laplace or s-domain using Laplace transform to model the system are usually limited due to the non-linear nature of the converters. Most of the non-linearity is introduced by power switches used in different power converters [7]. Moreover, the power converters are in general multiple input multiple output (MIMO) system and most of the articles model the system as single input single output system (SISO).

In this paper, a simple transfer function for a grid connected PV system has been derived considering a two stage conversion. The first stage is a dc-dc converter followed by a second dc-ac conversion stage. The transfer function of the overall system is derived for an ideal system consists of ideal power converters, and the transfer function is then modified to introduce other non-linear factors considering the practical implementation of those converters. The aim of this work is to derive new converter topologies in future in the form of smaller generalized modules originated from the overall transfer function of the system.

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II. TWO STAGE GRID CONNECTED PV SYSTEM

A two stage grid connected PV system has been analyzed in this section, and a schematic of the system is shown in Fig. 1. A PV array is generally consists of parallel connected strings, and each string is formed by connecting PV cells in series. An equivalent circuit of a PV array is shown in Fig. 2, and *I-V* characteristic of a PV array can be written as shown in eqn. (1) [15].

$$I_1 = I_{PV} - \mathrm{I}_0 \; exp(\frac{V_1 + R_s I_1}{V_t a} - 1) - \frac{V_1 + R_s I_1}{R_P} \tag{1}$$

Here,

I_1 = Output current of the PV array

I_{PV}= Array current generated by the incident sunlight

$= N_P \times I_{PV,cell}$

N_P=Number of parallel strings

$I_{PV,cell}$= Photocurrent generated by each PV cells

I_0= Reverse saturation current of the PV array

V_1= Output voltage of the PV array

$V_t = N_s kT/q$= Thermal voltage of the array

N_s= Number of series connected cells in each string

k= Boltzmann constant

T=Temperature

q= Electron charge (1.602×10^{-19} C)

a= Diode ideality constant

R_s= Equivalent series resistance of the array

R_p= Equivalent parallel resistance of the array

$I_d = \mathrm{I}_0 \; exp(\frac{V_1 + R_s I_1}{V_t a} - 1)$ = Diode current

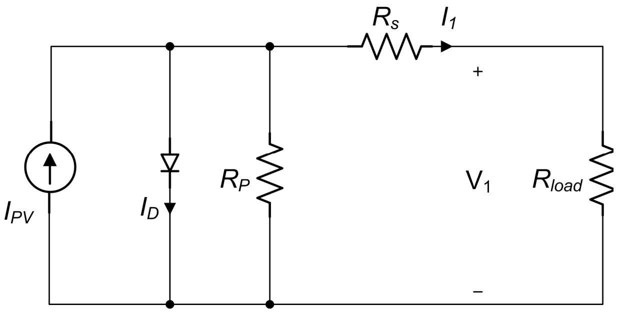

Fig. 2. Electrical equivalent circuit model of PV array

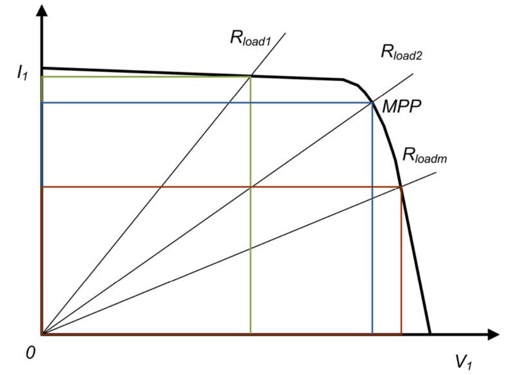

Fig. 3. *I-V* curve of a PV array with illustrating the concept of MPP.

A typical *I-V* curve of a PV array is shown in Fig.3, and the operating point of the PV array is defined by the equivalent output resistance seen by the PV array at its output port (R_{load}). Area of the rectangle with diagonal connecting the origin and the operating point on the *I-V* curve defines the amount of power delivered by the PV array, and maximum amount of power is delivered by the PV array when the slope generated by R_{load} coincides with diagonal of the largest possible rectangle under the *I-V* curve. Adjusting R_{load} to shift the operating point where the PV array generates the maximum power it termed as maximum power point tracking (MPPT), and the operating point on the *I-V* curve is referred as maximum power point (MPP).

In general, a PV array/string is connected to a boost dc-dc converter to step up the voltage of the PV array/string to a voltage level which is defined by the dc-ac inverter, and this dc-dc converter performs the maximum power point tracking operation as well. The dc-dc converter is followed by a dc-ac inverter, and the output of the inverter is connected to the grid.

Throughout this paper it will be assumed that:

1) The PV array is working at its maximum power point (MPP).
2) The ambient temperature, irradiance, insolation etc do not vary with time, and thereby, the MPP of the PV array is fixed as well.
3) The voltage and current output of the inverter are in phase. Therefore, the system is working at unity power factor and no reactive power is supplied to the grid from the PV system.

As shown in Fig. 1, $V_1(t)$, $V_2(t)$, $v_3(t)$ are the voltage outputs from the PV array, dc-dc boost converter and inverter respectively. Similarly, $I_1(t)$, $I_2(t)$, $i_3(t)$ are the current outputs from the PV array, dc-dc boost converter and inverter respectively. V_m, I_m, T_0 and ω are the amplitude of the grid

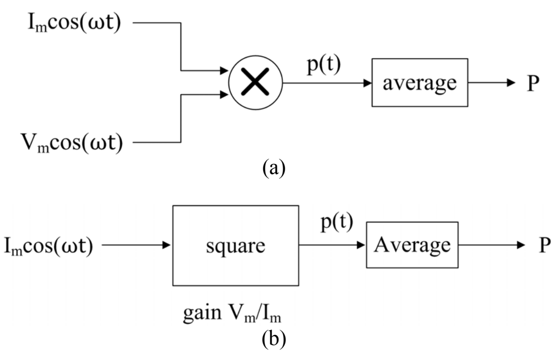

Fig. 4. Diagram of instantaneous and average power calculation is shown in (a). Single input single output relation of input current and power of the inverter is shown in (b).

voltage, amplitude of the grid current, time period and angular frequency of the grid voltage respectively. M_1 is the voltage gain of the dc-dc converter, and M_2 is the dc voltage to ac voltage peak gain of the inverter. P and $p(t)$ represent the average power and instantaneous power respectively, and these quantities will remain unchanged since a lossless system is considered here.

III. TRANSFER FUNCTION DERIVATION

Transfer function of an ideal dc-dc boost converter with gain M_1 can be written as eqn. (2).

$$G_1 = M_1 = \frac{V_2}{V_1} = \frac{I_1}{I_2} \tag{2}$$

Transfer functions for current and voltage conversion of the inverter are shown in eqn. (3)-(4):

$$\frac{v_3(s)}{V_2(s)} = \frac{M_2 s^2}{s^2 + \omega^2} \tag{3}$$

$$\frac{i_3(s)}{I_2(s)} = \frac{2s^2}{M_2(s^2 + \omega^2)} \tag{4}$$

Where,

$$M_2 = \frac{V_m}{V_2} \tag{5}$$

$$v_3(t) = V_m \cos(\omega t) \tag{6}$$

$$i_3(t) = i_m \cos(\omega t) \tag{7}$$

$$p(t) = v_3(t) i_3(t) \tag{8}$$

$$P = V_1 I_1 = V_2 I_2 = \int_{T_s} p(t)dt = \int_{T_s} v_3(t) i_3(t)dt \tag{9}$$

Since the grid RMS voltage amplitude is constant, the overall voltage transfer function may be written as eqn. (10).

$$\frac{v_3(s)}{V_1(s)} = \frac{M_1 M_2 s^2}{s^2 + \omega^2} \tag{10}$$

Similarly, the overall current transfer function may be written as given in eqn. (11).

$$\frac{i_3(s)}{I_1(s)} = \frac{2s^2}{M_1 M_2 (s^2 + \omega^2)} \tag{11}$$

In order to calculate the average power output from the inverter, we need to average the instantaneous output power of the inverter from the inverter's instantaneous output voltage and current. One way to calculate the average power is to multiply the instantaneous voltage and the instantaneous current and then averaging the product over a certain period of time T_s.

In order to derive a transfer function of instantaneous power to input current injected to the grid, the system can be converted to a single input single output system by as shown in Fig. 4. The transfer function is given in eqn. (12).

$$\frac{p(s)}{i_3(s)} = \frac{V_m (s^2 + \omega^2)(s^2 + 2\omega^2)}{s^2(s^2 + 4\omega^2)} \tag{12}$$

The transfer function of the PV array current to the instantaneous power is given in eqn. (13).

$$\begin{aligned} \frac{p(s)}{I_1(s)} &= \frac{p(s)}{i_3(s)} \frac{i_3(s)}{I_1(s)} = \frac{p(s)}{i_3(s)} \frac{i_3(s)}{I_2(s)} \frac{I_2(s)}{I_1(s)} \\ &= \frac{2s^2}{M_1 M_2 (s^2 + \omega^2)} \frac{V_m (s^2 + \omega^2)(s^2 + 2\omega^2)}{s^2(s^2 + 4\omega^2)} \end{aligned} \tag{13}$$

The transfer function from the solar panel's input current to the output of the averaging module, P can be written as shown in eqn. (14).

$$\frac{P}{I_1} = \frac{s^2}{s^2 + \omega^2} \frac{V(s^2 + \omega^2)(s^2 + 2\omega^2)}{ks^2(s^2 + 4\omega^2)} \frac{(1 - e^{-sT_s})}{sT_s} \tag{14}$$

However, transfer function of a boost converter in real time implementation can be written more precisely as shown in eqn. (15) [4]-[6].

$$G'_1(s)=\frac{M_1}{1+s\dfrac{L}{R}+s^2LC}=\frac{M_1/LC}{s^2+\dfrac{s}{RC}+\dfrac{1}{LC}} \tag{15}$$

Here, R is the output resistance seen by the dc-dc converter and can be written as shown in eqn. (16).

$$R=\frac{V_2^2}{P}=\frac{P}{I_2^2} \tag{16}$$

The transfer function of a rectangular PWM inverter followed by a first order low pass filter is given in eqn. (17).

$$\frac{1-e^{-sT_o/2}}{1+e^{-sT_o/2}}\frac{M_2}{1+s\tau} \tag{17}$$

Therefore a more practical voltage transfer function for the overall system is given in eqn. (18).

$$\frac{M_1/LC}{s^2+\dfrac{s}{RC}+\dfrac{1}{LC}}\frac{1-e^{-sT_o/2}}{1+e^{-sT_o/2}}\frac{M_2}{1+s\tau} \tag{18}$$

IV. DESIGN EXAMPLE

The PV system shown in Fig. 1 has been simulated in Simulink, and the PV array has an operating voltage of 38V with 400 watt output power at MPP. The simulation is performed with M_1=10, M_2=0.4466, V_m=120√2 V and ω=377 rad/sec, and the Simulink model provided in Fig. 5. The current and instantaneous power output from the inverter is shown in Fig. 6 and Fig. 7 respectively.

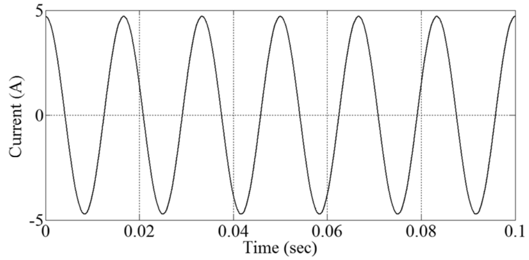

Fig. 6. Waveshape of $i_3(t)$ from the model.

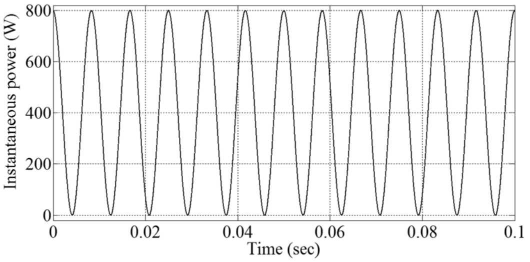

Fig. 7. Waveshape of $p(t)$ from the model.

V. CONCLUSIONS AND FUTURE WORK

A complete Laplace transfer function of a grid connected PV system is derived in this paper. The transfer function is derived for an ideal system that is linear and simpler to model. Then a more practical model is derived. The practical circuit implementation of power electronic converters introduces different types of non-linearity and requires more rigorous mathematical analysis. Accomplished tasks up to this point could be considered as the first part of this project. Authors believe that it is possible to synthesize the overall transfer function as a product of multiple functional blocks, and each block could represent a smaller module. This reverse synthesis thus results in achieving new modulation schemes and converter topologies. The discussed inverter system has been modeled in Simulink at MATLAB environment, and the results completely agree with the expected results of these

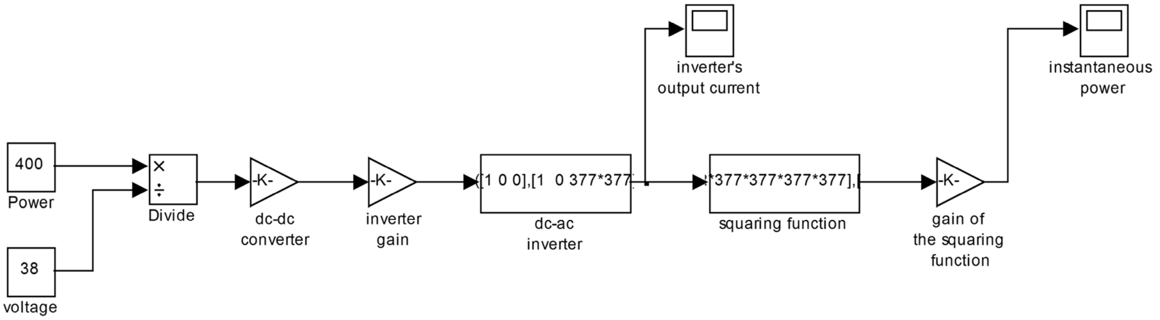

Fig. 5. Simulink model of the system

well known systems.

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