EQUALIZATION OF EXCURSION AND CURRENT-DEPENDENT NONLINEARITIES IN LOUDSPEAKERS

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ABSTRACT

This paper presents a novel equalizer for nonlinear distortions in direct-radiator loudspeakers in a closed cabinet by constructing an exact inverse of an electro-mechanical model of the loudspeaker. This exact inverse compensates for distortions introduced by excursion and current-dependent nonlinearities. The equalizer compensates for the nonlinearities in the force factor, voice coil inductance, eddy currents and the stiffness of the loudspeaker. Simulation results demonstrating substantial reduction in the harmonic distortions at the output of the loudspeaker are included in this paper.

***Index Terms*—** equalizers, nonlinear distortion, nonlinear systems, loudspeakers.

1. INTRODUCTION

Direct-radiator loudspeakers are used in many applications such as entertainment devices, mobile phones, tablet computers, laptop computers and desktop speakers. They are inexpensive and usually have a good linear response over a wide band of frequencies. However, when they are driven with large amplitude inputs, nonlinear distortions degrade the audio quality. Pre-processing of the input signal using a digital corrector for the distortions can mitigate these nonlinear effects and reproduce high-fidelity audio. This paper presents a pre-equalizer for direct-radiator loudspeakers that compensates for distortions introduced into audio signals by the nonlinearities in the force factor, the voice coil inductance, the stiffness of the loudspeaker diaphragm and eddy currents.

These nonlinear effects of the loudspeaker depend on its geometric construction and the materials used in the voice coil, the diaphragm and the enclosure [1]. Figure 1 displays a schematic diagram of the loudspeaker, explicitly showing how the electrical and mechanical components are coupled together. The figure also shows the primary components in the loudspeaker that exhibit nonlinear behavior.

A lumped parameter electro-mechanical model of the loudspeaker is shown in Figure 2. The suspension stiffness

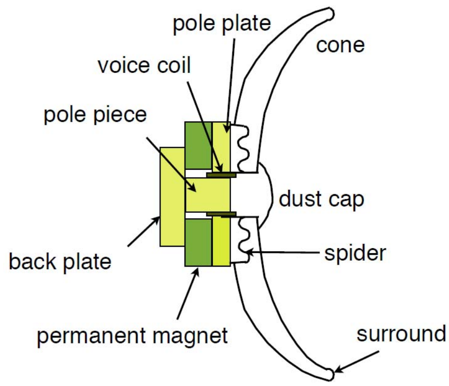

Fig. 1. Diagrammatic representation of a direct-radiator loudspeaker (adapted from [1, 2]).

$K_{ms}(x)$ arises from the material properties in the surround and the spider of the loudspeaker. The force factor nonlinearity, $Bl(x)$, describes the frequency-independent electromechanical coupling in the loudspeaker. This is modeled as a gyrator. The inductance of the voice coil, $L_0(x, i)$, is a function of its displacement $x(t)$ and the current in the voice coil, $i(t)$. To model eddy currents in the pole pieces and the plates, a parallel combination of an inductance L_2 and a resistance R_2 is included [1, 3, 4]. These inductances L_0 and L_2 also induce a reluctance force in the moving components of the speaker. The electrical voice coil windings have a resistance R_{vc}. The electrical components are modeled via Kirchoff's voltage law (KVL) [1, 5]. A force equation is written for the mechanical components to include the mechanical load resistance (like friction) R_{ms} that affects the diaphragm and M_{ms}, the moving mass of the diaphragm. In this paper, we construct an equalizer for the direct-radiator loudspeaker using the model in Figure 2. Pre-equalization of the input audio using this equalizer provides substantial reduction in harmonic distortion at the output of the loudspeaker.

The rest of this paper is organized as follows. Section 2 provides an overview of the previous work on nonlinear equalization in loudspeakers. Section 3 describes a simplified loudspeaker model and a formal derivation of an inverse to this loudspeaker model. Evaluation results are discussed in Section 4. Conclusions and future work follow in Section 5.

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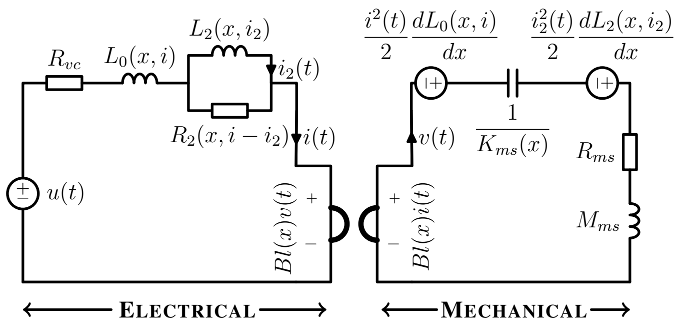


Fig. 2. Lumped-parameter representation of a loudspeaker.

2. PRIOR WORK

Nonlinear equalizers were based on truncated Volterra filters in [6, 7] and on Volterra-Wiener-Hammerstein models in [8][1]. Gao and Snelgrove [9] introduced an adaptive truncated Volterra system equalizer for loudspeakers. The mirror filter [10] was obtained by "reflecting" the order of nonlinearities in a lumped parameter model of the speaker.

We model the loudspeaker using the electrical and mechanical components of the loudspeaker, and then implement a pre-inverse of this model that pre-distorts the input to the speaker model. Our equalizer is obtained by inverting all operations in the loudspeaker model. Unlike prior work [10, 11, 12, 13, 14], our model and the equalizer includes effects of current-dependent nonlinearities as well as the distortions resulting from eddy currents. Unlike the approaches using truncated Volterra filters, our method is an exact inverse. Also, this equalizer can be implemented using one differentiator. This is an advantage over the mirror filter in real-time applications.

3. LOUDSPEAKER EQUALIZER

3.1. Loudspeaker model

Consider the equivalent representation in Figure 2. Let the voltage $u(t)$ denote the input stimulus to the system. Then, the voice coil current $i(t)$ is related to the parameters of the system through the KVL as,

$$u(t) - Bl(x)v(t) = i(t)R_{vc} + \frac{d}{dt}\left(L_0(x,i)i(t)\right) + \frac{d}{dt}\left(L_2(x,i_2)i_2(t)\right) \quad (1)$$

where, $i_2(t)$ is the current in the para-inductance, $Bl(x)v(t)$ represents the back-emf induced by the force factor and $v(t)$ is the velocity of the speaker diaphragm. Using KVL in the L_2-R_2 loop, we get

$$\frac{d}{dt}\left(L_2(x,i_2)i_2(t)\right) = \left(i(t) - i_2(t)\right)R_2(x, i-i_2) \quad (2)$$

Similarly, for the mechanical components in the lumped parameter circuit, the equation for the force on moving diaphragm is given by,

$$M_{ms}\frac{dv(t)}{dt} = \frac{i^2(t)}{2}\frac{dL_0(x,i)}{dx} - K_{ms}(x)\int_0^t v(\tau)d\tau + Bl(x)i(t) - R_{ms}v(t) + \frac{i_2^2(t)}{2}\frac{dL_2(x,i_2)}{dx} \quad (3)$$

An equivalent block diagram for the loudspeaker can be derived as in Figure 3. by combining (1), (2) and (3). The force signal input $f_{in}(t)$ to the suspension block is given by,

$$f_{in}(t) = Bl(x)i(t) + \frac{i^2(t)}{2}\frac{dL_0(x,i)}{dx} + \frac{i_2^2(t)}{2}\frac{dL_2(x,i_2)}{dx} \quad (4)$$

This force moves the diaphragm modulating the air pressure in the region around the speaker. The resulting pressure signal $p(t)$ is modeled as,

$$p(t) = P_{const}\frac{dv(t)}{dt} \quad (5)$$

where, $P_{const} = \frac{\rho\pi r_{spkr}^2}{2\pi d}$, d is the distance at which the pressure is measured, πr_{spkr}^2 represents the effective area of the diaphragm, r_{spkr} represents the radius of the diaphragm and ρ corresponds to the air density.

Figure 3 can be further simplified as shown in Figure 4 by combining a cascade of the voice coil, the force factor nonlinearity and the suspension system into an equivalent system **H**. The operations in **H** require the signals $x(t)$ and $v(t)$.

3.2. Exact pre-equalization

We assume that the input signal $p_{in}(t)$ to the equalizer is the pressure waveform that we wish to reproduce at some distance from the loudspeaker. The diaphragm velocity waveform $v_p(t)$ can be estimated using (5), and the displacement

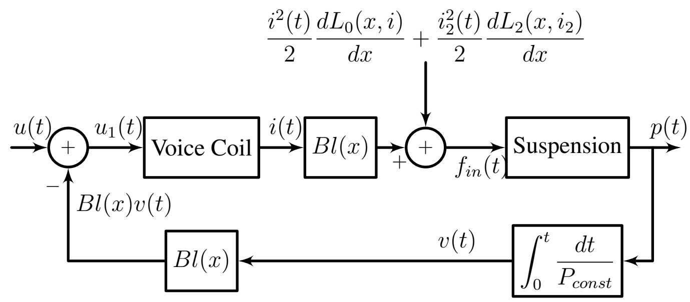


Fig. 3. Block diagram for the loudspeaker model.

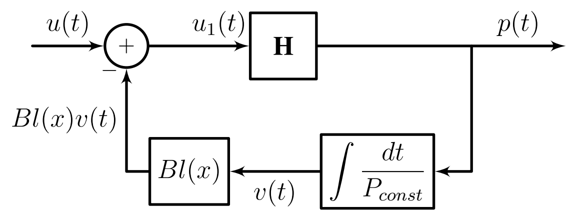


Fig. 4. Simplified model of a loudspeaker.

[1]The authors thank the anonymous reviewers for this reference.

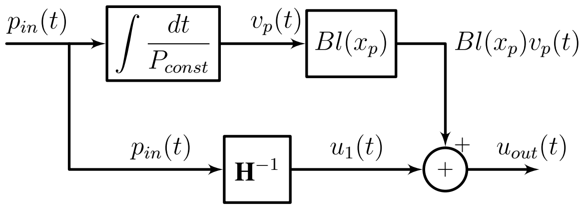


Fig. 5. Loudspeaker pre-inverse.

signal $x_p(t)$ can be obtained from the velocity waveform. Figure 5 shows a block diagram of the pre-inverse of the model in Figure 4. To see this, we start with an expression for $u_{out}(t)$ in Figure 5.

$$u_{out}(t) = \mathbf{H}^{-1}[p_{in}(t)] + Bl(x_p)v_p(t) \tag{6}$$

This output voltage $u_{out}(t)$ from the pre-equalizer is applied to the loudspeaker model to generate an output $y(t)$. Using Figure 4, this is formulated as,

$$y(t) = \mathbf{H}\left[u_{out}(t) - Bl(x_y)v_y(t)\right] \tag{7}$$

$x_y(t), v_y(t)$ were derived from $y(t)$. Substituting for $u_{out}(t)$ using (6) in (7),

$$y(t) = \mathbf{H}\left[\mathbf{H}^{-1}[p_{in}(t)] + Bl(x_p)v_p(t) - Bl(x_y)v_y(t)\right] \tag{8}$$

Thus, $y(t) = p_{in}(t)$ for all t can be proved by mathematical induction and causality of the system.

For system $\mathbf{H}^{-1}$, a pressure input signal $p_{in}(t)$ is converted to a voltage output $u_1(t)$. System $\mathbf{H}^{-1}$ is implemented by reversing the order of operations in **H**, namely suspension followed by nonlinear scaling, followed by the voice coil, i.e., (3), (2) and (1). Using the estimated velocity waveform, the displacement signal and the acceleration waveform, the current in the voice coil $i_p(t)$ is estimated as,

$$\begin{aligned} i_p(t) \quad = \quad & \frac{1}{Bl(x_p)}\left(K_{ms}(x_p)x_p(t) - \frac{i_p^2(t)}{2}\frac{dL_0(x_p, i_p)}{dx}\right. \\ & \left.\frac{M_{ms}dv_p(t)}{dt} + R_{ms}v_p(t) - \frac{i_{2_p}^2(t)}{2}\frac{dL_2(x_p, i_{2_p})}{dx}\right) \end{aligned} \tag{9}$$

where $i_{2_p}(t)$ is the estimated para-inductance current. The nonlinearities in $\mathbf{H}^{-1}$ are obtained using signals $i_p(t)$ and $x_p(t)$. Using (9), we may estimate the output voltage signal of system $\mathbf{H}^{-1}$, $u_1(t)$, as

$$\begin{aligned} u_1(t) \quad = \quad & \frac{d}{dt}\left(L_0(x_p, i_p)i_p(t)\right) + \frac{d}{dt}\left(L_2(x_p, i_{2_p})i_{2_p}(t)\right) \\ & i_p(t)R_{vc} \end{aligned} \tag{10}$$

where the voltage across the para-inductance L_2 is computed recursively as,

$$\frac{d}{dt}\left(L_2(x_p, i_{2_p})i_{2_p}(t)\right) \quad = \quad \left(i_p(t) - i_{2_p}(t)\right)R_2(x_p, i_p - i_{2_p}) \tag{11}$$

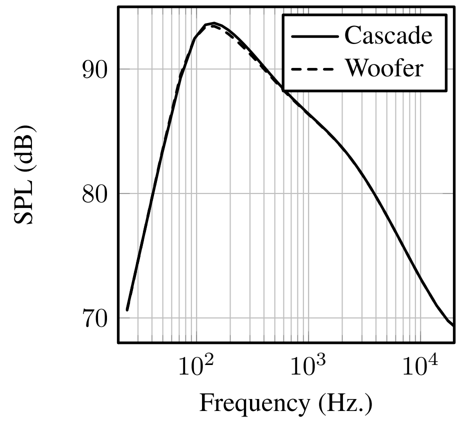


Fig. 6. Comparison of linear responses.

4. PERFORMANCE EVALUATION

An explicit Runge-Kutta (RK) algorithm [15] of order 4 was used to perform integration operations in the model. An approximate differentiator was constructed as the inverse of this RK-integrator. All signals involved were sampled at 48 kHz. The parameters of the loudspeaker model were obtained through actual characterization of a 3-inch woofer using the Klippel Analyzer at JBL Pro, Northridge, CA. The model assumed that the nonlinearities $L_0(x, i)$, $L_2(x, i_2)$ and $R_2(x, i - i_2)$ were separable, i.e.,

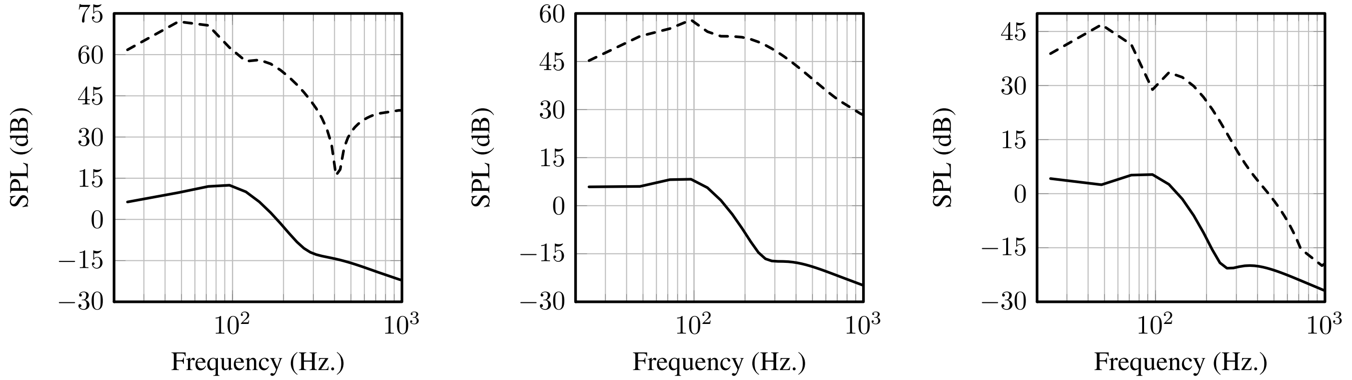
$$\begin{aligned} L_0(x, i) &= L_0(x)F_0(i), & (12) \\ L_2(x, i_2) &= L_2(x)F_2(i_2), & (13) \\ R_2(x, i - i_2) &= R_2(x)G_2(i - i_2) & (14) \end{aligned}$$

The coefficients for $F_0(i)$, $F_2(i_2)$ and $G_2(i - i_2)$ were assumed to be equal [16]. The excursion-dependent terms $L_0(x)$, $L_2(x)$ and $R_2(x)$ were related as in [1, 16].

Figure 6 compares the linear responses of the woofer and a cascade comprising of this shaping filter, the equalizer and woofer. The shaping filter used had a linear response that was similar to that of the loudspeaker. Minor differences in these responses were observed in Figure 6.

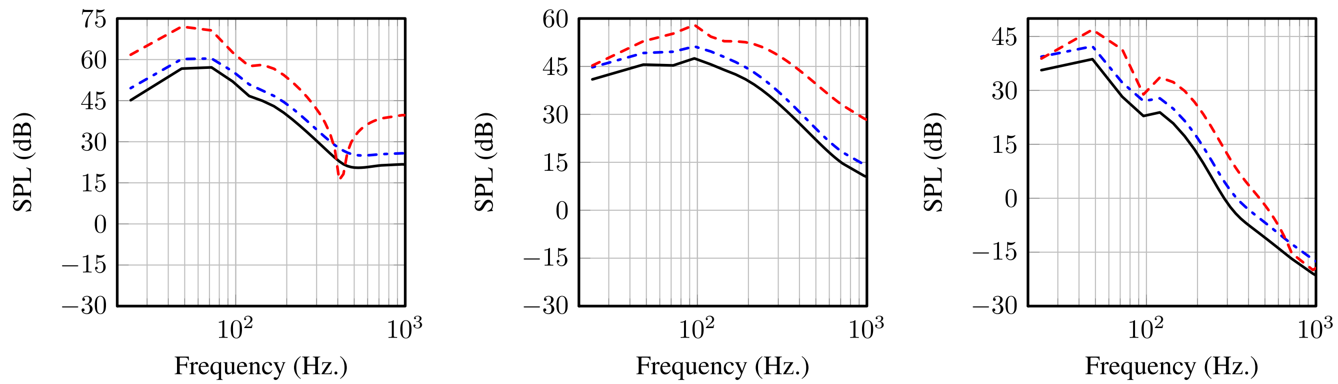
Second, third and fourth order harmonic distortions were measured for the forward loudspeaker model and the cascade of the shaping filter, equalizer and the loudspeaker model using sinusoidal waveforms of frequencies in the range 24 Hz. and 20 kHz and amplitude 8V. The sound pressure level (SPL) for the n-th harmonic of a sine waveform with frequency ω_0, was computed as the ratio of the power at frequency $n\omega_0$ to the reference sound pressure level, 20 μPa.

Figure 7 displays the measured harmonic distortions for the woofer and the equalized woofer. A substantial reduction in the harmonic distortion was observed because of equalization using our pre-equalizer. Ideally, the distortions associated with the equalized woofer should be zero. These non-ideal results in the figure were attributed to computation and measurement errors.



(a) 2nd order harmonic distortion (b) 3rd order harmonic distortion (c) 4th order harmonic distortion

Fig. 7. Harmonic distortion (HD) responses for the cascaded system (bold lines) and for the nonlinear woofer (dashed lines).



(a) 2nd order harmonic distortion (b) 3rd order harmonic distortion (c) 4th order harmonic distortion

Fig. 8. Harmonic distortion (HD) for the cascaded system when the nonlinear parameters in the woofer are perturbed using a random error with $\alpha = 0.1$. Black bold lines : mean HD response over 100 iterations. Blue dash-dot lines : mean HD response + σ (standard deviation) of the HD responses. Red dashed lines : Response of the nonlinear woofer.

In order to test the system for robustness against inaccuracies in the loudspeaker model, we introduced a mismatch between equalizer and loudspeaker model parameters. This could also be used to simulate aging in the loudspeaker. Specifically, the parameters of the loudspeaker model were perturbed by multiplicative noise in the form

$$w_p = w(1 + \kappa) \tag{15}$$

where w is any coefficient of the original model and κ is a uniformly distributed random variable in the range [-α, α]. The equalizer used the original parameters w while the woofer used the perturbed parameters w_p.

Figure 8 displays the harmonic distortions for orders two, three and four for this case of model mismatch with $\alpha = 0.1$. We can see that even with this level of mismatch, the system on average is able to reduce the harmonic distortion in the output of the equalizer by 10-15 dB at most frequencies. Accurate modeling of the loudspeaker is key to additional, substantial mitigation of the distortions.

5. CONCLUDING REMARKS

A novel pre-equalizer structure for nonlinear equalization of loudspeakers was presented in this paper. This structure was able to compensate for nonlinear effects due to excursion and current-dependent nonlinearities and those due to eddy currents. Performance evaluation involving perturbed coefficients of the model indicates that our approach is reasonably robust to inaccuracies in the loudspeaker model. The stability of this inverse can be demonstrated but was not included in this paper because of space limitations. Additional work on parameter estimation for loudspeaker models, real-time implementation and adaptive equalization of time-varying nonlinearities is underway at this time.

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