Analysis of Electromagnetic Field Variability in Magnetized Ionosphere Plasma using the Stochastic FDTD Method

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***Abstract*—A stochastic finite-difference time-domain (S-FDTD) algorithm is presented for electromagnetic wave propagation in anisotropic magnetized plasma. This new algorithm efficiently calculates in a single simulation not only the mean electromagnetic field values, but also their variance as caused by the variability or uncertainty of the content of the ionosphere. By accounting for fully three-dimensional, high resolution (even cm-scale) structures and uncertainty in the ionosphere, this algorithm represents a paradigm shift in our ability to analyze realistic, complex wave propagation in the ionosphere.**

I. Introduction

Communications, surveillance, and navigation capabilities rely heavily on accurate knowledge of electromagnetic (EM) signal propagation characteristics through and reflected by the Earth's ionosphere. The variability of the ionosphere renders many propagation problems too complex to be solved using a deterministic formulation, however. The structure of the ionosphere can depend not only on the altitude, time of day, and season, but also on the latitude, longitude, sun spot cycle, and occurrence of space weather events. A useful approach to such a highly complex problem is to consider it as a random medium problem. The Monte Carlo method is a widely-used brute force technique for evaluating random medium problems via multiple realizations. However, it is significantly more efficient to formulate the problem in such a way that its ensemble averages may be run in a single realization scheme. In this paper, we develop the first stochastic FDTD (S-FDTD) [1] algorithm for electromagnetic wave propagation in three-dimensional (3-D) anisotropic magnetized plasma.

II. Methodology

The magnetized (anisotropic) cold plasma governing equations are cast in terms of Maxwell's equations coupled to current equations derived from the Lorentz equation of motion [2]. The resulting whole governing equation set is given by:

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{H} = -\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}_e \tag{2}$$

$$\frac{\partial \mathbf{J}_e}{\partial t} + v_e \mathbf{J}_e = \epsilon_0 \omega_{Pe}^2 \mathbf{E} + \boldsymbol{\omega}_{Ce} \times \mathbf{J}_e \tag{3}$$

Here v_e, J_e and ω_{Pe} are the collision frequency, the current density and the plasma frequency of electrons, respectively. The plasma frequency is a function of the electron density n_e given by,

$$\omega_{Pe} = \sqrt{\frac{q_e^2 n_e}{\epsilon_0 m_e}} \tag{4}$$

Ionosphere electron densities vary in a complex manner as a function of location and time. Thus, we consider the electron density as a random variable with its own statistical variation. This variability in the electron density causes variability in the EM fields, which will also be treated as random variables. $\boldsymbol{\omega}_{Ce}$ is the cyclotron frequency of the electrons given by $\boldsymbol{\omega}_{Ce} = q_e \mathbf{B}/m_e$. Here, $\mathbf{B}$ is the applied magnetic field.

For the S-FDTD derivation, there are initially three stochastic equations (1), (2) and (3) that for Cartesian coordinates contain ten random variables for the 3-D case: E_x, E_y, E_z, H_x, H_y, H_z, J_{ex}, J_{ey}, J_{ez} and ω_{Pe}. By using the delta method [3], Smith and Furse demonstrated that the average (or expected) fields can be found by solving the field equations using the mean or averages of the variables [1]. Thus, the mean EM field and current density values are found by using the mean plasma frequency of ω_{Pe}, or equivalently, the mean of electron density n_e.

In order to derive the standard deviation (or variance) equations, we must take the variance of (1), (2) and (3). This step results in two cases as described below:

1) Case 1: If a function is formed by the sum of multiple variables (equations (1) and (2)), its variance is:

$$\sigma^2 \left\{ \sum_{i=1}^{n} a_i X_i \right\} \tag{5}$$
$$= \sum_{i=1}^{n} a_i^2 \sigma^2 \{X_i\} + 2 \sum_{1 \le i < j \le n}\sum a_i a_j \rho_{X_i,X_j} \sigma\{X_i\} \sigma\{X_j\}$$

Here, ρ_{X_i,X_j} is the correlation coefficient $(-1 \le \rho_{X_i,X_j} \le 1)$. The closer this coefficient is to zero, the more independent the terms are from each other. If the correlation coefficients

$\rho_{X_i,X_j}(1 \leq i < j \leq n) = 1$, we obtain:

$$\sigma\left\{\sum_{i=1}^{n} a_i X_i\right\} = \sum_{i=1}^{n} a_i \sigma\{X_i\} \quad (6)$$

2) Case 2: If a function is formed by the product of multiple variables (equation (3)), its variance is solved by using the delta method [3]:

$$\sigma^2\{f(X_1, X_2, \ldots, X_m)\, g(X_{m+1}, X_{m+2}, \ldots, X_{m+2})\} = \sum_{i=1}^{m+n}\sum_{j=1}^{m+n} \frac{\partial(fg)}{\partial X_i}\frac{\partial(fg)}{\partial X_j}\Bigg|_{\mu_{X1},\ldots,\mu_{X\,m+n}} Cov(X_i, X_j) \quad (7)$$

Equations (6) and (7) are used in the derivation of the standard deviation equations. When the standard deviation equations are derived, covariances are needed of the E, H fields and current density J_e in both time and space. These equations also relate the electric field to the plasma frequency of the ionosphere, resulting in additional covariance terms between the electric field and the plasma frequency. As for the Maxwell's equations S-FDTD methodology of [1], for the 3-D S-FDTD magnetized cold plasma algorithm, the magnetic fields, electric fields and current densities are highly correlated to each other. As such, the correlation coefficients of the E, H fields and current density J_e may be approximated as 1. However, it is challenging to decide which method should be used to evaluate the remaining cross correlation coefficients between the electric field and the plasma frequency. There are many factors in choosing the best $\rho_{\omega,E}$ values, such as the field component orientation, the cell's location relative to the source, the type of source wave, and the direction of the background magnetic field. The approximation of these correlation coefficients will control the accuracy of the algorithm.

III. Validation of the Algorithm

The performance of the fully 3-D S-FDTD cold plasma model of Section II is evaluated by running a similar validation test as for the FDTD plasma model of [2]. An x-polarized, z-directed Gaussian-pulsed plane wave source is implemented. The lattice space increments in each Cartesian direction of the grid are $\Delta x = \Delta y = \Delta z = 1mm$, the time step $\Delta t = \Delta x/(c \times 0.55)$. An magnetic field $B = 0.06T$ is applied to the plasma (a large value so that we may observe an effect of the plasma over a short distance for validation purposes). An example of the standard deviation of the E_x field as recorded 10-cells away from the source in the z-direction is shown in Fig.1.

For validation, 100 Monte Carlo simulations are used to predict the exact standard deviation of the fields. The input electron densities n_e for each simulation are generated in a random manner with a normal distribution given by mean $\mu_{n_e} = 1.0 \times 10^{18} m^{-3}$ and standard deviation $\sigma\{n_e\} = 2.0 \times 10^{16} m^{-3}$. All of the simulation responses are collected and analyzed to obtain their statistical properties (standard deviation and variance values). Then, using S-FDTD, three separate simulation cases are run using approximations for the correlation coefficients between the plasma frequency and the electric fields of 1, 0.5 and 0.05, respectively.

Fig.1 shows that a higher correlation coefficient leads to a higher standard deviation of the electric field. As expected, the approximations for the cross correlation of the plasma frequency and the electric fields have a direct impact on the accuracy of the S-FDTD method. The correlation coefficient of 1.0 yields a maximum (upper bound) of the standard deviation. In this data set, a cross correlation value of 0.05 provides the best agreement with the Monte Carlo simulations. As part of future research, systematic studies will be performed to evaluate the best methodology for determining the appropriate correlation coefficients for different plasma modeling scenarios.

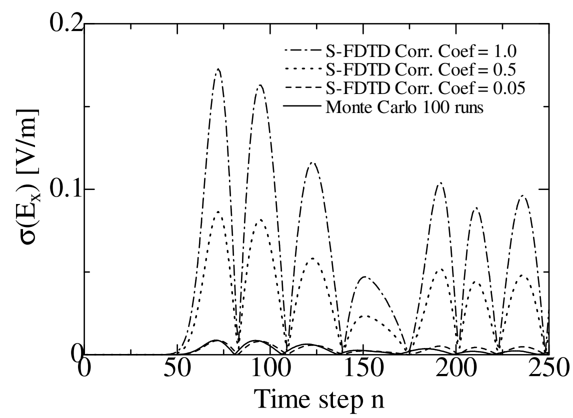

Fig. 1. Standard deviation of E_x (observed at a point 10 cells away from the source).

IV. Conclusion

A 3-D S-FDTD model of EM wave propagation in anisotropic magnetized cold plasma was introduced. The plasma S-FDTD model of this paper is derived from Maxwell's equations coupled to the current equations derived from the Lorentz equation of motion. It uses as input not only average electron densities, but also their variance due to uncertainties or variances due to factors such as space weather events.

S-FDTD offers an exceptional improvement in simulation time compared to the brute-force Monte Carlo method. S-FDTD may therefore serve as an important tool for EM ionospheric propagation studies, especially for large 3-D plasma scenarios wherein Monte Carlo simulations would be impractical to run.

References

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