A New Approach to Tolerance Analysis

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Abstract

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The common question is whether parts with uncertain shape fulfill certain functional specification. The question is expressed as geometrical relationship between toleranced objects. Unfortunately, tolerance based relations are often inconsistent, unlike relations between exactly represented objects.

In this paper we survey current tolerance representation and analysis methods. We then derive our method of intuitionistic tolerance handling from a method developed for robust solid modeling. A new representational framework is proposed, which serves as the basis for robust geometric modeling and tolerance analysis. We illustrate the framework with examples of assembly design.

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1 Introduction: Robust Tolerance-based Geometric Computation in Computer Integrated Design and Manufacturing

The concurrent design and manufacturing of industrial parts requires a multi-layered set of software support systems. The components of such a system exchange data concerning shape and other information about parts. The geometric shape is an important part of the data and is used, in part, to convey aesthetic, functional and manufacturing aspects of the design. Shape consists of symbolic and numeric information. The symbolic information is for instance the type of shape, the degree of a polynomial, or relations between components (adjacency, inside, etc.). The numeric part determines the values for the parameters specified in the symbolic part. The numeric values are often computed by some geometric algorithm, come from measured data, or are determined by design parameters, etc. Theoretically, geometric shapes are defined in Euclidean space, meaning that the numeric values in the representation are defined as real numbers; however, in practice such ideal objects rarely exist. On a computer objects usually need to be approximated, e.g., by floating point numbers. Any geometric operation implemented with floating point representation and floating point arithmetic is therefore accurate only to a limited degree. When manufacturing a designed part this can be done with a precision that is limited by the machine tools. Likewise, when measuring data (e.g., with a coordinate measuring machine) the accuracy is limited by the precision of the sensors.

When representing data with uncertainty, we usually distinguish between *nominal geometry* and the *deviation from the nominal shape* (the tolerance). It is clear that we cannot determine certain geometric relations for uncertain objects in a straightforward way (e.g., that two planes are coincident, parallel or that a line is incident with a surface, etc.). The tolerance effects every such computation. The amount of tolerance is determined by the accuracy of the data. In mechanical engineering design tolerances are specified as part of the functionality of the product. Unfortunately, no matter how small the error is, tolerance based decisions are a matter of arbitrary interpretation, and therefore ambiguous. Tolerance-based relations do not obey the same rules as the exact relations would. For instance, the coincidence relation for objects in Euclidean space is transitive, but not for approximated objects. This leads to many problems, such as contradictory decisions and inconsistent object representations. Typical consequences of this problem in solid modeling systems are, for instance, dangling edges, missing surfaces, or even crashing algorithms.

To make these systems more reliable they need to be based on a strong theory. We are therefore studiing the application of intuitionistic geometry to a tolerance based robustness method which can provide consistent underpinnings for a variety of design and manufacturing processes, including:

- geometric tolerances in mechanical design,
- robust solid modeling algorithms,
- manufacturing process planning,
- assembly planning.

It is crucial that the tolerances used during geometric computations are compatible with the tolerances established during the design and the manufacturing phase, as well as when the part is inspected and serviced. Just as the same physical units, and the same floating point arithmetic, should be used throughout the system, so must consistent geometric analysis and relations be determined.

In the following sections, we describe the proposed tolerance based robustness method, as well as how we intend to apply it to robust solid modeling, computer aided design, manufacturing, and process planning. We believe that such a capability is obligatory for any consistent CIM system.

2 Related Work in Mechanical Tolerancing and Geometric Modeling

2.1 Robust Geometric Modeling

Geometric algorithms, in particular, solid modeling algorithms regularly yield inconsistent representations in their output, due to numerical inacurracy and the arbitrary but ambiguous decisions based on approximated, tolerance based relations. These inconsistencies manifest themselves, for instance, in dangling edges, missing faces, etc., and sometimes cause the system to crash entirely. The problems occur mostly in situations where surfaces or edges are coincident which is quite common in mechanical engineering design, for instance, when using Boolean set operations. A number of publications have addressed the robustness problem in geometric modeling[4, 8, 15, 26, 27, 35, 45, 49]. A survey can be found in[26]. Approaches to solve the robustness problem in geometric algorithms and solid modeling are using perturbation of the numerical data, to avoid the difficulties with coincidences, also exact rational numbers are used, or geometric reasoning[8, 27, 49, 35], rather than treating numbers as elements of a continuum. Approaches that deal with uncertainty in geometric algorithms are described, for instance in[15, 45]. However, there was no provably robust solution to geometric modeling with curved surfaces using floating point computation.

We have been working on a solution to the geometric robustness problem for over three years and we developed and implemented two tolerance-based robustness methods for geometric computation, in general, and geometric modeling algorithms, see[4, 5, 9, 10, 62]. We believe that a tolerance based method is necessary to support the various data representations and applications in CAD/CAM in a uniform way.

2.2 Tolerances in CAD/CAM

The concept of tolerance is used in mechanical engineering to represent permissible errors in manufacturing parts. Traditionally, these tolerances have been represented as annotations to mechanical drawings. Geometric dimensioning and tolerancing is used to define the range of acceptable geometry, including the description of essential functional relationships[43]. In the ANSI Y14.5 standard on dimensioning and tolerancing[1], a dimension is defined as numerical value defining the size or the characteristics of parts or part features, while a tolerance is defined as the total amount by which a specific dimension is permitted to vary. There are many problems with the traditional definition of tolerances, as they lead to ambiguities, and don't lend themselves to the use in computer aided geometric design systems.

Currently, several types of representations for tolerancing are proposed for CAD/CAM systems. In [38], tolerance is represented as form, size and position deviations. In [38, 12], tolerance is defined by tolerance zones and datum. A tolerance zone is a region of space in which a portion of a surface of a real part must lie. A datum is a point, an infinite line or a plane. In[39] an experimental tolerance-based implementation of the solid modeling system PADL-2 is described. Woo has summarized the research that has been done in the dimensioning and tolerancing representation and processing in his survey article[43]. The latter two papers provide the theory and experiments that have been done in the tolerancing area. In [59], tolerance classification is addressed with respect to functionality. In [43], tolerances are classified as conventional or geometrical tolerances. Conventional tolerances specify upper and lower limits for dimensions, while geometric tolerances are used to control the form, profile, attitude, orientation, location and runout of the product.

In recent years [40, 41, 42], work has been done towards refining their approach so as to correspond more closely to current practices, and also on algorithms for tolerance analysis to automatically analyze issues of fit and clearance, however the results are not available yet. Turner and Wozny [58, 57] have formulated an approach based on a notion of *feasibility spaces*. This approach is derived from variational modeling and geometric tolerances, and is the basis of the Geos tolerancing system at RPI, [53, 55, 56] which was integrated with a boundary-based modeler, IBM's CATIA. One potential caveat of their approach is that the dimension of the feasibility space will grow with the complexity of the user's model. They intended to explore the use of pruning strategies to eliminate irrelevant model variables and tolerances from the problem definition. Some progress has been made by Martino [33, 34] to carry out prior examination of the model to eliminate non-contributing parameters. They are about to address the tolerance synthesis problem, which seeks to find a set of tolerance limits given a desired variation in the design function. The objective is to minimize the manufacturing cost based on the Taguchi method.

Many geometric modeling systems currently on the market provide some tolerancing facilities. Mostly these are two dimensional tolerance analysis and tolerance synthesis modules, that are separate units, and therefore can neither fully support automatic manufacturing planning nor perform some of the spatial reasoning required for assembly planning. Requicha in his paper [38] proposed a theory for integrating tolerance into geometric modeling systems.

Research is currently also done in the following areas: Constructing geometric primitives or features from measured data[61, 46, 17], different ways of representing geometric tolerance [2, 3, 38, 25], synthesis and analysis of tolerance[47, 59, 16, 12, 28, 14]. In [59] functional tolerance and assembly tolerance concepts have been proposed. In [39], a tolerance-based representation for surface features is studied to build the automation of assembly. Also constraint relations between toleranced geometric object have been studied, e.g. in [54]. [19] proposes a new idea of soft constraints solving strategy, that could lead the way in integrating constraints and tolerances.

Other than in geometric modeling, tolerances have also been widely addressed in vision, and robotics. In [32], model based vision programs have been developed, and uncertainty models are used. In [60], geometric tolerance for position and orientation computations are described.

3 Research in Tolerance Representation and Analysis

We started to extend our previous research in intuitionistic geometry (briefly described below), to extend the foundations, and to build a tolerance based geometric library, and a solid modeling kernel and apply the algorithms and data structures to robust manufacturing planning, CAD-based inspection, and reverse engineering

3.1 Intuitionistic Geometry for Robust Spatial Data Handling

Geometric algorithms are designed for objects defined in a real number space which, on a computer, can only be approximated using floating point numbers. Due to these arithmetic errors, but also because of geometric approximation errors, or measuring errors (in the case of measured data input), geometric algorithms are rarely robust without special robustness handling. The usual naive use of tolerances on geometric objects does not guarantee robustness, especially for geometric problems with complicated and degenerate geometric relations, occurring in applications such as 3D Boolean operations[26], or when interpreting measured data, as in CAD-based inspection, and when simulating manufacturing operations.

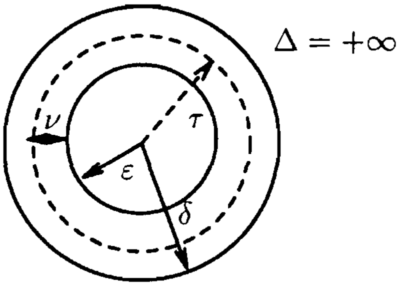


Figure 1: Initial tolerance definition of a point

Since tolerance based decisions are always arbitrary, it is not generally possible to guarantee freedom of inconsistencies. The main idea behind intuitionistic geometry is to dynamically update the tolerance regions of geometric objects, as a consequence of the relations determined between them. This way we either detect possible ambiguities which gives us an opportunity to fix them, or we are guaranteed the correctness. It is not possible that an inconsitency goes undetected.

In this paragraph the intuitionistic robustness method is introduced for geometric relations between points, and finite curves and surfaces. We first define tolerance regions of geometric shapes and operations on them, independent of representation of the shape:

An *r-region* of an object O is the set of points with a Euclidean distance of r or less from O. In the approach presented here the tolerance of an object consists of three such regions: the ε-, the δ- and the Δ-region. We assume an initial tolerance τ which represents the allowable deviation from the nominal value that is still accepted equivalent to the nominal value (the value of τ is somewhat arbitrary and determined by the application), and the error bound ν (representing the numerical error associated with the data or the computation of geometric relations such as coincidence, incidence, etc. with other data). The radii of the ε, δ and Δ regions are initialized as $\varepsilon = \tau - \nu$, $\delta = \tau + \nu$ and $\Delta = +\infty$. Figure 1 shows the tolerance definition of a point object.

Objects inside the ε-region satisfying a certain set of constraints are called models for the representation. According to Hoffmann's notion of *representation* and *model*[26], a model must satisfy all the constraints defined for the object, while the existence of a model of the resulting object ensures the robust implementation of the algorithm. The δ region is used to separate apart objects (minimal feature separation).

Requiring a model to strictly satisfy all the constraints defined for the object is somewhat too restrictive as discussed in [9]. It can make the algorithm extremely inefficient and the tolerances to be so big that the original problem's meaning is changed (e.g., two obviously apart objects are considered coincident). Therefore, in the 'approximated-model' version of intuitionistic geometry models are not required to have the same mathematical form as the original object they represent. For instance, a model of a line can be any curve within the line's ε-region passing through all the points that were detected to be on the line (incidence constraints).

We define the relation between two objects to be either apart, coincident, incident, intersecting or ambiguous with the following tolerance based relation definitions and tolerance updating rules. With the approximated model definition, a model always exists if the ε-region of the represented

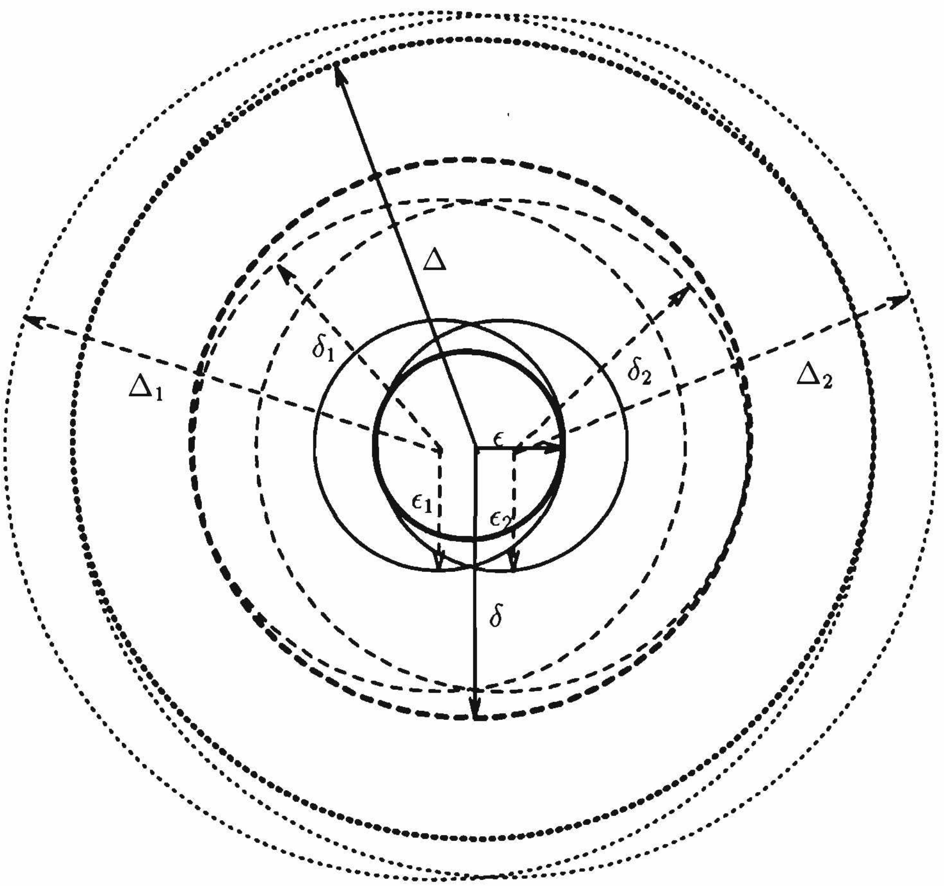
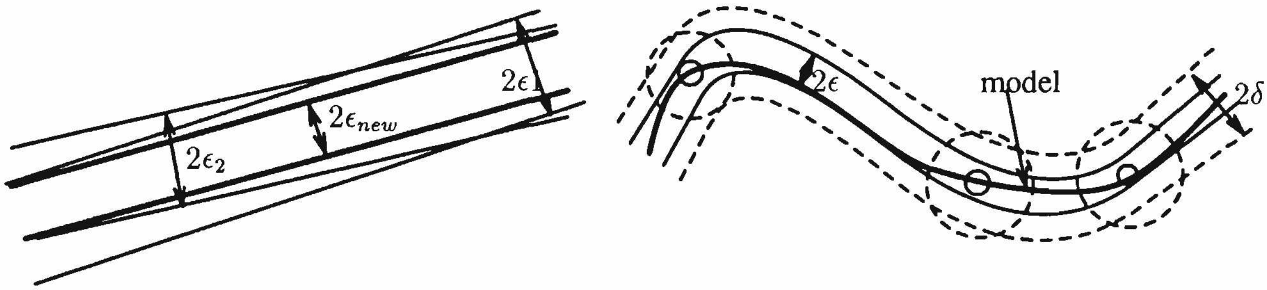


Figure 2: Tolerance updating for two coincident points

object is not empty (objects are always finite). The tolerance update rules for the approximated model method (defined below) are less restrictive than for the analytic model method (there are fewer occurrences of ambiguous situations).

- If the δ-regions of two objects do not intersect, the objects are apart. The radii of their Δ-regions will be updated to be half of the smallest distance between these two objects, if Δ is currently larger than this distance.
- If O_1 and O_2 are of the same dimension and if there exists a common model for the two objects, the two objects are said to be coincident and merged into one single object. The new ε and Δ-regions are the maximal possible regions for the new object inside the intersections of the previous ε and Δ-regions, respectively (Figure 2 shows the updated ε, δ and Δ regions of two coincident points). (Figure 3(a) shows the updated ε region of two coincident lines.) The new δ-region is the minimal region of the new object enclosing the union of the previous δ-regions of the two objects.
- If object O_1 is a lower dimensional object than O_2 and there exists a model of O_1 inside O_2's



(a) (b)

Figure 3: Tolerance updating with coincidence and incidence relations

ε-region, then O_1 is incident on O_2. The new ε and Δ-regions of O_1 are the maximal regions of O_1 inside the intersections of the ε and Δ-regions of the old O_1 and O_2 (see figure 3(b)).

- If two objects have neither coincidence nor incidence relations, and their ε-regions intersect, then the two objects are intersecting. The tolerances of the intersecting objects are updated in such a way that there exists an intersection object that is incident on both of the original objects.

- When the tolerance of an object O is updated, all the tolerances of those objects that have been detected to be incident on O also need to be updated so that the incidence relations still hold, i. e. the ε and Δ regions of these objects must be shrunk (if necessary) so that they are inside the ε and Δ regions of O, respectively.

- If the ε-region becomes empty or the δ-region is no longer contained in the Δ-region for some object, then we detect an ambiguity. Certain steps (usually increasing the tolerances) have to be taken to solve the ambiguity before the algorithm continues.

In [9] we showed how important properties are preserved automatically for tolerance-based geometric relations just by observing the tolerance updating rules. These properties include most of the basic properties preserved by the corresponding relations in Euclidean space (e.g., the transitivity of the coincidence relation; two lines only intersect in one point, etc.). When implementing a geometric algorithm with above definitions, tolerance operations and properties, we either detect an ambiguity or guarantee the existence of a model that has the properties expected from the representation. In a recent paper [10] we showed that this method is a vast improvement over the analytic model method [4], in that it reduces the likelihood of ambiguities by several orders of magnitude.

3.2 A Robust Solid Modeling Kernel

We have begun implementing a 3D Boolean operation algorithm based on our robustness method and a hybrid solid representation method in an experimental solid modeler. In this modeler, a solid is bounded by planes and natural quadric surfaces. These surfaces are the most commonly used in engineering because of their useful geometric properties (symmetry, etc.) and simplicity in terms of geometric computations such as surface intersections.

For Boolean operations on solids, we must determine the relation between two surfaces using our robustness method, and find intersections of the two surfaces if they are detected intersecting. Algebraic solutions are available for relation detections of planes and natural quadric surfaces[13].

In the future we need to extend the implementation by general intersections between quadrics (not yielding conic sections). Levin's parametric approach[31], which uses piecewise linear segments to approximate the intersection curves, may be used to find other general intersections of two quadric surfaces. However, a piecewise linear approximation requires much larger tolerances to be used in the algorithm which in turn may yield more ambiguous relations between objects. Using algebraic methods, as we do for the special cases, is more efficient and more robust for the reasons mentioned.

In most modern sculptured surface modelers surfaces are represented by trimmed parametric surfaces (B-splines, NURBS). Intersections between surfaces will be computed as piecewise linear polylines by recursive subdivision techniques. We plan to implement the necessary operations for these surfaces, using our robustness method. We argue that for general intersections degenerate (coincidences) relations are less likely, therefore these will not be a big problem (it is very unlikely that two general 3D curves are coincident, unless they have a common history: for instance, one curve is a duplicate of the other, or some of the intersecting surfaces are identical, or a curve is incident with a surface, if it is an intersection of this surface with another surface, etc.). We will annotate all the geometric objects with this dependency information, and use this information to derive special relations between general curves and surfaces, rather than relying on the numerical representation.

Nevertheless, many applications (e.g., CAD-based inspection) still require a way of representing tolerances even for sculptured (parametric) surfaces, and to do tolerance based computation (e.g., finding the intersection curve of two surfaces and its tolerance, as a function of the tolerance of the surfaces. A survey on surface-to-surface intersection algorithms can be found in[37]. We plan to use an interval spline approach similar to the one described in [44]. We plan to implement it as an extension to the public domain spline package which is part of the IRIT solid modeling system, developed at the Technion, Israel. To this end we started a collaboration with the Technion and the HP Research Laboratory in Haifa.

Recently we showed that shapes computed by set operations that result in a 2D manifold topology can be computed without redundancies, reducing the necessity for tolerance updates to a minimum[62]. We will take advantage of these results to increases the reliability of the algorithm also for non-manifold objects.

3.3 Robust Design, Manufacturing and Process Planning

With the data structures and operations proposed in the previous two sections we will be able to also simulate manufacturing tolerances, and thus simulate the validity of the design under these tolerances. For instance, we want to find out whether a functional feature can be manufactured, and whether it has certain relationships with other features (within tolerance), and whether the relationships are logically consistent. The geometric operations that need to be carried out can be very similar to the solid modeling operations done in the design stage. We geometrically construct the object by machinable features (e.g., drilling a hole corresponds to a Boolean subtraction of a cylinder, etc.), however, this time we associate tolerances that correspond to the tool precision to the geometric elements, rather than the floating point, or design tolerances, used previously. This is done by simply setting the parameters τ and ν used in the definition of the tolerance regions to the appropriate value. The tolerance ν, instead of the numeric precision, now represents the precision of the physically manufactured object (the actual tolerance in a mechnical engineering sense). The parameter τ may represent the compliance, clearance, or 'play'. Generally, it indicates how much a the position or orientation of a part can be varied, such that it still fulfills its function.

After the geometric construction we will query relationships between geometric elements and features to test the validity of the functional features (e.g., we want to find out whether the two holes manufactured by two independent drilling operations line up within the tolerance of the design specifications. The adaptive tolerance method of the intuitionistic geometry approach (see above) will facilitate tolerance analysis and synthesis in that it can be used to determine whether certain relationships can be achieved unambiguously under the current tolerances, and it provides us with the necessary feedback (in form of a smaller resulting ε-region), indicating that the precision ν of the individual objects in the relation need to be tightened in some cases, indicating that the features need to be manufactured with a more precise tool, or an additional finishing stage may become necessary. In other cases the analysis may tell us that the tolerances can be relaxed, or that the clearance τ needs to be increased. Examples are given below.

Many geometric operations used in the different processes are similar in principle, but different values for the tolerances need to be assigned for different tasks (e.g., a larger tolerance to simulate the rough cut and a smaller tolerance for the finishing stage). We therefore need a software library that is very flexible, and allows us to parameterize the tolerances, and also use different tolerance updating rules as they are appropriate for the various applications. The geometric queries that are necessary, in addition to incidence and apartness are, for instance, distance, parallelism, right angles, etc. The tolerance-based implementation of these queries can be derived from the definitions of incidence and apartness. In some sense the tolerance regions in this approach are similar to the tolerance zones, proposed in[39] with the additional advantage of providing a built-in mechanism for consistency.

3.4 Relation Between Shape and Transformation Uncertainties

In assembly design, the shape uncertainty of an object's surface directly affects the uncertainty of the relative rigid body transformation between two objects attached to each other. Figure 4 illustrates the effect of an object mounted on an uncertain surface. The object can be rotated and translated within the range determined by the tolerance zone.

Figure 5 illustrates (in one dimension) how the rotation uncertainty $\delta\alpha$ follows from the shape tolerance δx, for a contact surface of diameter r. This rotation uncertainty causes a position uncertainty Δx in parts of the assembly that are translated by a distance R.

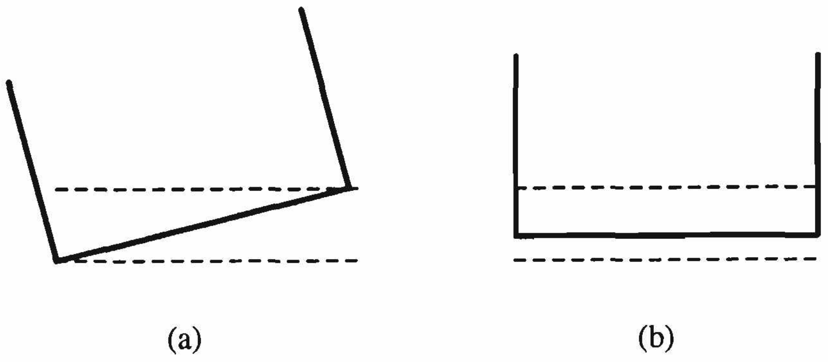


Figure 4: Shape uncertainty results in transformation uncertainties.

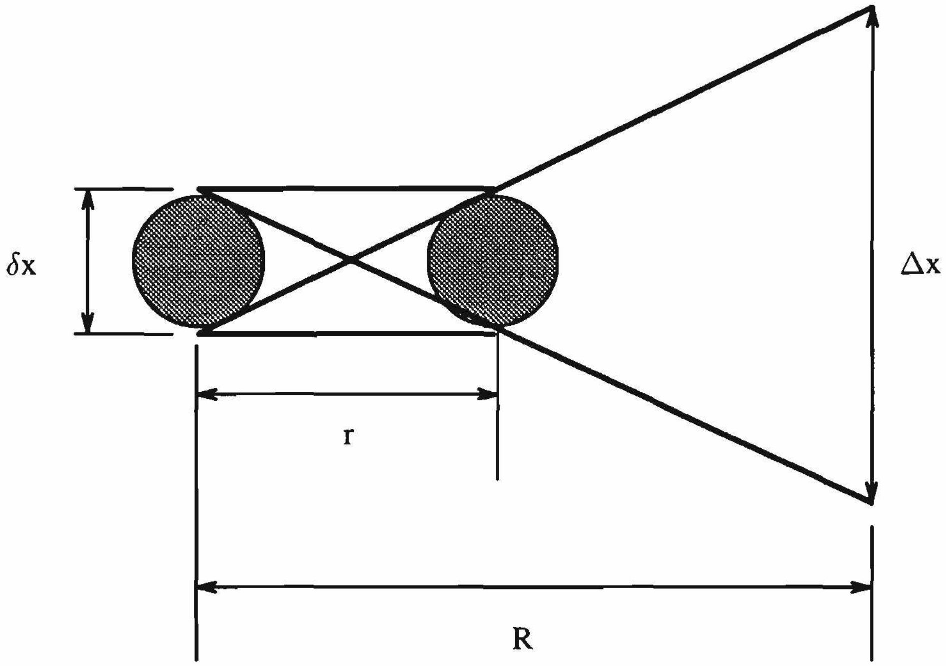


Figure 5: The transformation uncertainty caused by shape uncertainty.

	peg	hole
diameter	9.5	10.0
τ	0.5	-0.5
ν	0.1	0.1
ε	0.4	-0.4
δ	0.6	-0.6

Table 1: Tolerance environments for the peg and the hole.

Assume that δx is small, we can approximate the angle $\delta\alpha$ with the following formula:

$$\delta\alpha = \frac{2\delta x}{r}$$

Therefore, Δx can be calculated as:

$$\Delta x = (R - \frac{r}{2}) \times \delta\alpha = \delta x(\frac{2R}{r} - 1)$$

If several objects are assembled in a chain, the tolerance stackup for the wole assembly can be calculated by repeated application of this principle.

The geometric principle applies equally to the tolerance zone ν (the precision) as well as to the feasibility region represented by τ. We can calculate these zones independently and then check whether the resulting ε and δ regions are unambiguous.

3.5 Examples: Assembly of Mechanical Parts

In this section, we will show several examples of how the previous approach is applied to tolerance analysis.

Consider the assembly of a peg and a hole as shown in figure 6 with a clearance τ and a tolerance ν. The tolerance environments are summarized in table 1. To verify whether the peg can fit inside the hole, we merge their tolerance environments using the tolerance updating rule for the coincident objects. The new tolerance environment has an ε equal to 0.3 and a δ equal to 0.7. The ε region is not empty, therefore, the above tolerance specification describes a valid assembly condition. Figure 7 shows a picture of the final tolerance environment for the example just described.

In the example of figure 8 the peg has a larger diameter which yields an ambiguous assembly condition in form of and empty (negative) ε region (because the tolerance is larger than the clearance).

Also, we can calculate the transformation uncertainty of the peg resulting from the shape uncertainty of both the hole and the peg (shown in figure 9). If we want to fit the same peg through another hole on the other end, the feasible region of the peg will be reduced. This can be derived by doing the same analysis of the assembly of the peg and the second hole independently, as shown in figure 10. The resulting transformation uncertainty of the assembly is then obtained by merging ("intersecting")the two individual ε regions (as shown on the right hand side). If the resulting

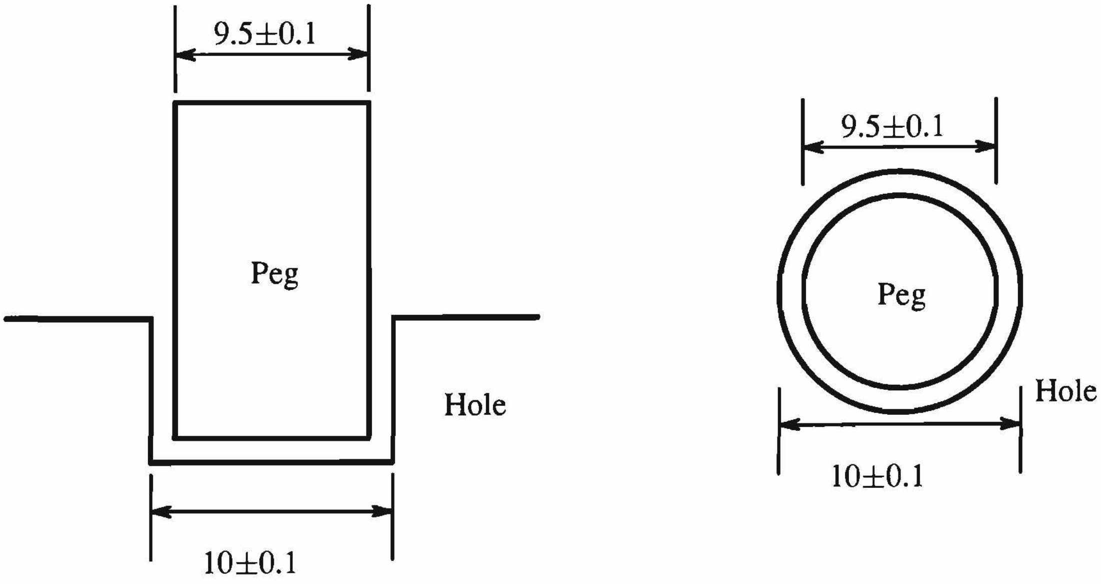


Figure 6: A peg and a hole.

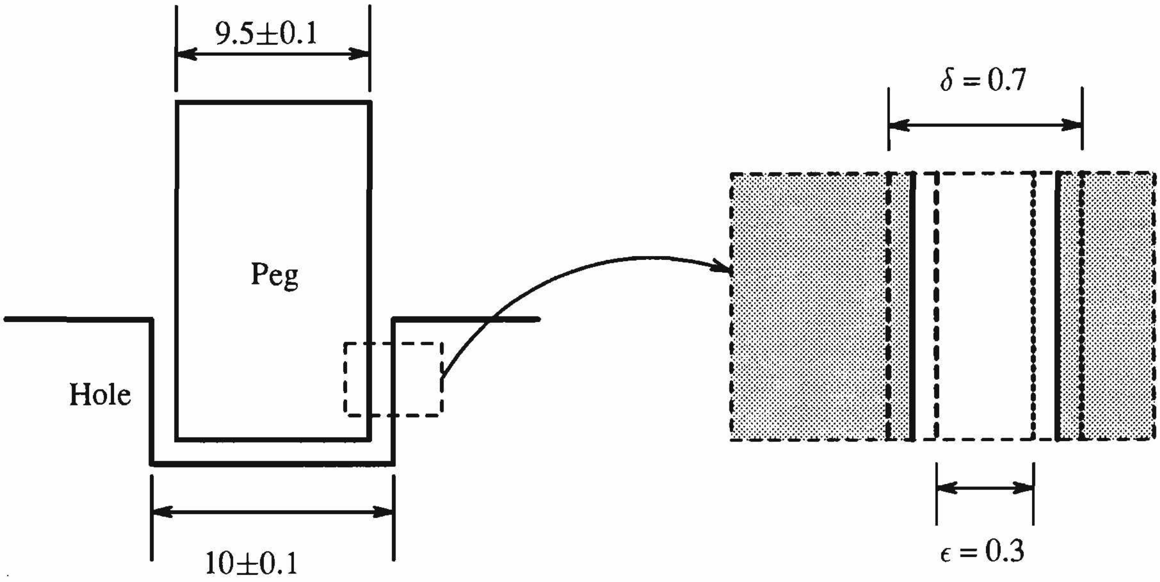


Figure 7: The tolerance zone of the hole and the peg.

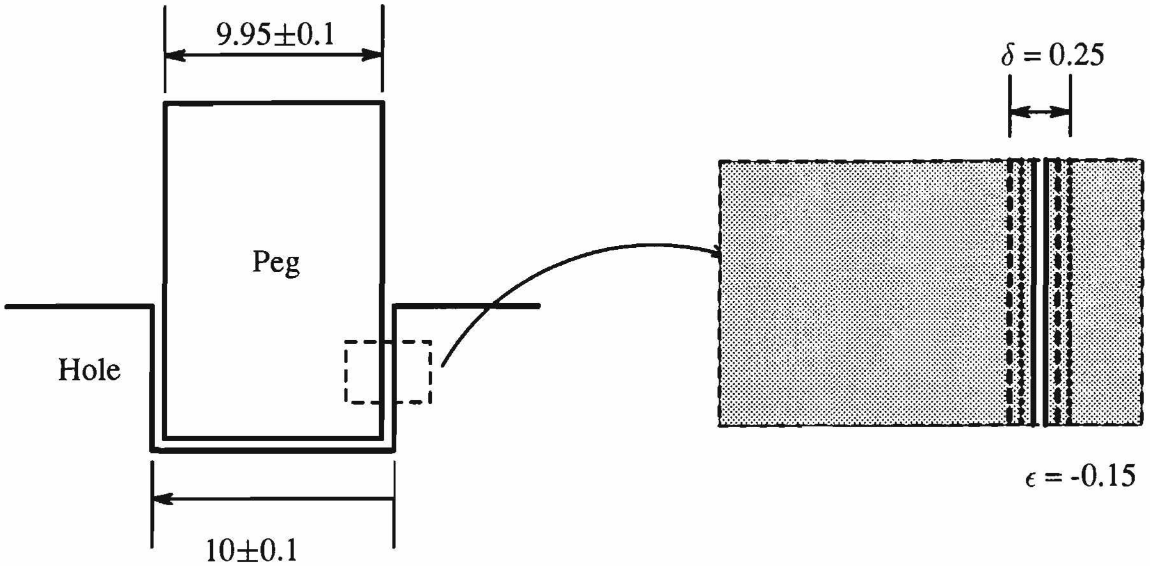


Figure 8: The tolerance zone of the hole and the enlarged peg.

feasibility space becomes empty the assembly is no longer guaranteed to work. We may have to increase the clearance or achieve higher manufacturing precision for the two holes.

Next we show how the tolerance analysis ideas can be used in assembly planning. As in previous examples we represent the clearance as τ-regions, and the manufacturing tolerance as ν-regions. The fixture in figure 11 has holes of different size on both sides, resulting in different transformation uncertainties in an assembly situation.

The larger holes will allow larger clearance for the bolt. If we tighten the bolts with the larger clearance first, we may not have enough play to fit the bolts with the smaller holes, because the necessary degrees of freedoms are elimintated, leading to ambiguous relations, later on. When as-

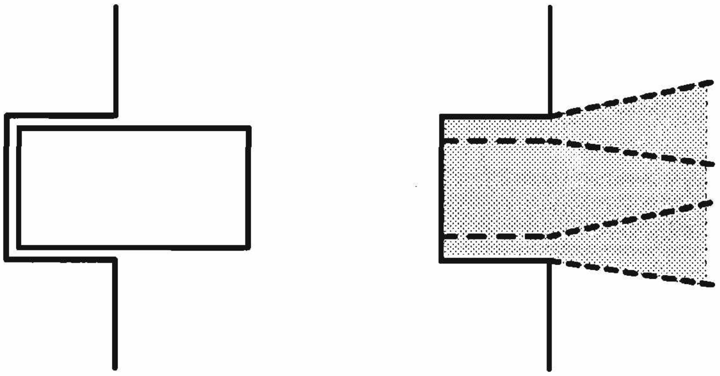


Figure 9: The transformation uncertainty of the peg

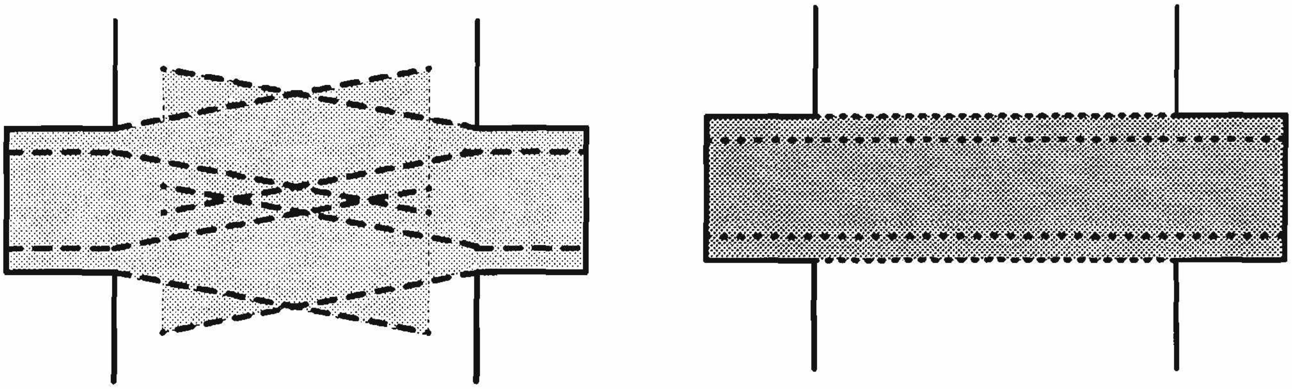


Figure 10: Reduced uncertainties.

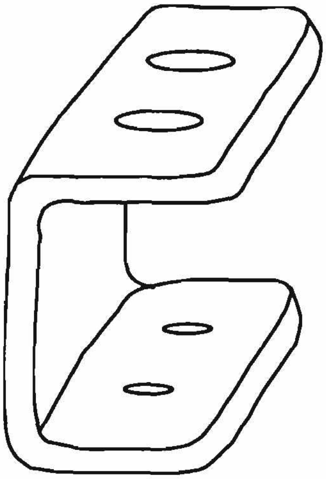


Figure 11: Assembly planning.

sembling the fixture it is therefore better to first tighten the bolts with the smaller clearance.

4 Conclusion

With the approach proposed here we found a unified approach for handling data with uncertainty. The same geometric relations and operations can be applied to the tolerance regions, whether they represent numerical inaccuracy, measuring uncertainty stemming from sensing data, or manufacturing tolerances. Naturally, different numerical values have to be used, depending on the origin and meaning of the uncertainty information. We therfore propose to make the tolerance regions part of every geometric model.

5 Acknowledgements

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