# Automatic Sculptured Five-Axis <br> Milling with Check Surfaces ${ }^{1}$ 

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#### Abstract

An approach to 5 -axis milling of B -spline surfaces is presented. Within its domain, it provides better check surface handling than APT. The scheme for tool position generation is based on a B-spline curve refinement method and a set of criteria for tolerance control which allows the tool positions to be automatically determined by a set of user specified tolerances. The approach has been used to generate the tool paths for a relatively complex sculptured model, and shows promising results.


## 1 Introduction

Traditionally, a surface is machined by directing the tool to follow a number of paths which run along an approximation to the surface. Critical to this process is deciding how many tool paths on a surface should be generated, where they should be, how many points one must specify along a particular tool path, and where these points should be placed. Except for a few methods [6,2], tool paths for a surface are either chosen to be parallel lines in either the parametric or geometric domain. That is, isoparametric curves are specified or the intersection curves of a family of parallel planes and the surface are used [5,4,1].

In this paper a set of isoparametric curves is used to generate the tool paths for a sculptured surface. Currently, the curves are selected by the user. Then, an automatic tool position generation scheme is applied to each of the curves to produce the tool positions along the curve. The scheme is based on a B-spline curve refinement estimation method and is very robust, especially for the common occurrence of surfaces with non-uniform parametrization.

Gouging is a major problem in NC machine tool path generation. Traditionally, the prevention of gouging is handled by imposing check surfaces. The check surfaces in most of

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Figure 1: The Wedge Between the Tool and the Surfaces.
the current NC systems have the same semantics as those specified in APT. In APT a check surface is used to truncate the tool path, i.e., the tool will cease its motion along the current path whenever the check surface is encountered. In this strategy, some stock material between the check surface and part surface may remain uncut. One such situation is shown in Figure 1. In the figure the tool moves on the part surface from the left to the right. It stops when it encounters the check surface, but the wedge shape area below the tool and above the part and check surfaces remains, even though it is possible to remove the remaining material in the area with 5 -axis milling.

In this paper, a different semantic is given to a check surface so that the wedge shape area in Figure 1 can be properly removed. At the same time, the new semantic allows the region around the intersection of the check surface and the part surface to be machined more accurately.

If a ball end tool is used, the only major advantage in multiaxis milling over 3-axis milling is that the number of setups required in multiaxis milling may be fewer. But for a fixed tool size, flat end tools can provide higher efficiency and can produce a better finish when the surface can be cleaned by face milling with falt end tools. When face milling with a flat end tool, the tool axis must be aligned with the surface normal at every tool position. In general, multiaxis milling is required to perform such alignment. In this paper we investigate the use of flat end tools with multiaxis face milling on parts with check surfaces.

The rest of this paper is organized as follows. In Section 2, the domain of the new method is defined. Supporting arguments for the domain selection are provided. Section 3
describes our tool path generation method. An example with reasonable complexity is given in Section 4. This example shows the process of utilizing the new strategies in tool path generation and also demonstrates the capability of the new scheme. Finally, comments, remaining issues and suggested future improvements are discussed in Section 5.

## 2 Domain Selection

As mentioned earlier, flat end tools are capable of producing a better finish than ball end tools for certain surfaces. In general, convex surfaces are suitable to 5 -axis milling with flat end tools, while surfaces with concave regions can not be produced exactly by flat end tools in face milling mode.. However, confining the domain of possible surfaces to convex surfaces will result in unnecessarily restricting the range of the application of the resulting methods. To take full advantage of multiaxis milling, we restrict ourselves to flat end tools and surfaces which are "nearly convex", as that term is defined below, which includes "feature lines" in the part surfaces and allows check surfaces in the domain.

The "feature lines" are strips of concave, high curvature regions on the part surface and must be narrow compared with the tool size. It is assumed that the feature lines are oriented so that they are crossed by the tool paths on the part surface. For example, the connecting region between the lower edge of the windshield and the body of the helicopter in Figure 5 is a feature line if the tool paths on the body are chosen to run across the connecting region. Note that feature lines are frequently not isoparametric lines.

Using feature line curves as tool paths, or even choosing a tool position on a feature line in face milling will frequently cause the tool to gouge into the part surface. Also, the production of the exact geometry of the part surface near the feature lines is impossible with flat end cutter face milling since feature lines are concave regions of the surface. We handle the feature lines by approximation, the details of which are explained in Section 3.4. Hereafter, the term "nearly convex surfaces" will be used to denote convex surfaces with feature lines.

The term "check surface" is defined slightly differently than than in APT. Full details on the semantics of our check surfaces will be given in Section 3. In our domain, only one check surface is allowed for each tool path, and at most one contact point between the tool and the check surface at any tool position is allowed. Both the check and the part surfaces are assumed to be represented as nonuniform rational B-spline (NURBS) surfaces. Since so many sculptured surfaces, including common surfaces such as spheres, cylinders, and cones, can be represented as NURBS surfaces, this is not a severe restriction.

We assume the sides of the tool will not gouge the surfaces, i.e., if the face of the tool does not gouge into the surfaces, neither will the periphery of the tool. In face milling, the assumption will be violated if the tool is required to undercut at a point after its axis has been oriented to the normal at the point. But such an undercut is eliminated and the assumption is automatically satisfied in 5 -axis face milling if both the check and the part surfaces are convex and if the angle at the intersection of these surfaces is greater than or equal to $90^{\circ}$. With that assumption for tool path calculation, the representation of the flat end tool can be a circle at the end of the tool.

Since differential geometry and numerical methods are used in our approach, we also assume that the surfaces are differentiable to the order which makes the numerical methods used in the calculations work properly and that the local surface properties such as normals and curvatures exist at the required points on the surfaces.

## 3 The Approach

In this approach, we use isoparametric curves to define the tool path and compute the tool positions for each tool path on the part surface automatically. The current version still requires manual selection of the collection of isoparametric curves used. To generate the tool path for a part, the following steps are used:

1. Break the surfaces of the part into pieces which satisfy the constraints on the domain of the method.
2. Select the isoparametric curves on each of the surface pieces to generate tool paths.
3. Generate the tool positions on each of the curves with the method.
4. Combine the tool positions on one surface piece into one continuous tool path.

In the rest of this section, we give the details on how the method presented here generates the tool positions from an isoparametric curve. The application of the above steps for a part will be shown by an example in Section 4.

### 3.1 Generation of Potential Tool Position

We first use the refinement estimator in Alpha_1 [7] to compute a piecewise linear approximation to the curve which accurately portrays the shape of the isoparametric curve. In general, tools paths require fewer points than are generated through the refinement method for a specific resolution. So the points generated from the refinement estimator are used as input to the tool generation function which produces tool positions from this oversized set. Roughly, the role of the tool position generator is to decide which points of the set are unimportant and discard them. It also searches for certain critical points, creating them if they are not already included. Before we describe the criteria and the process to decide the importance of a point, an approach to solving the check surface uncut region problem, pointed out in Section 1, is described.

### 3.2 Tool Offset

We handle the check surface problem by offsetting tool positions on a point-by-point scheme and by imposing a different semantic on check surfaces. In APT, when a check surface is encountered by the tool, the tool stops; this results in the uncut wedge shown in Figure 1. In our method, the tool does not stop when it encounters the check surface, instead we start to offset the tool. This offset operation is performed for at every tool position along the tool path


Figure 2: Illustration for the Closest Point Finding.
where the tool gouges the check surface. In the following discussion, we detail the tool offset operation first, and then the way that we handle a tool path with a check surface is described.

Both the tool offset operation and the detection of the gouging between the tool and the check surface are based on a closest point finding operation, that is, we want to find the closest point on the check surface to $P$, the potential tool position. The state is shown in Figure 2, with $P$ and its normal $N_{p}$. The tangent plane $P_{t}$ is the plane passing through $P$ with normal $N_{p}$. Denote the check surface as $C_{s}$ and the part surface as $P_{s}$. We find the closest point to $P$ which is on the check surface and also on the tangent plane. Since we are using a flat end tool, this is appropriate. Let $I_{c}$ be the intersection curve of the tangent plane and the check surface, i.e., $I_{c}=\left\{x: x \in P_{t} \cap C_{s}\right\}$. The closest point $Q$ to $P$ is $Q=\left\{x: \min d(x, P), \forall x \in I_{c}\right\}$.

Once $Q$ is found, it is easy to detect the gouging between the tool and the check surface. We can simply compare the tool radius with the distance between $Q$ and $P$. If gouging between the check surface and the tool occurs at $P$, we calculate an offset tool position $P_{o}$ to replace the point $P$. The offset position is calculated by first computing the normal $N_{q}$ of the check surface $C_{s}$ at point $Q$. Then $N_{q}$ is projected onto the plane $P_{t}$. The new tool position $P_{0}$ is obtained by offsetting point $Q$ by the amount of the tool radius along the projection of $N_{q}$. $N_{p}$, the normal at $P$, is used for the normal at $P_{o}$. The new tool position usually is not on the original tool path.

Newton's method is used to compute the closest point $Q$, to provide fast convergence. Detailed formulation of the closest point finding can be found in the appendix. Unfortunately, Newton's method suffers from the need for initial guess values for the parameters at $Q$. We
use geometric coherence to automatically estimate an initial value and to alleviate the need for the user to specify an initial guess. Use of the coherence allows us to specify only one set of initial values along each tool path and then to use the final parameter values at one point as the initial value for the next tool position parameters along the path.

Geometric coherence among the sequence of tool paths on a surface is also used. If the corresponding tool position on a neighboring path has been recorded to have its closest point found, the parametric values of that closest point will also be used to locate the closest point to the current tool position, if necessary.

### 3.3 Tool Position Selection

For a given sequence of points generated from the automatic refinement of a curve, we select and create tool positions from the points by using criteria specifying which points are unncessary. Hence, a point in the sequence is eliminated if the criteria are not violated by so doing. If two adjacent points in the sequence fail any of the criteria, then additional points between them may be necessary to create a good tool path. Hence, new points are created between them and then are tested against the criteria. The criteria we have developed are:

1. To reduce the chord height error on the part surface, the difference of the normals between two consecutive tool positions should be less than a tolerance. This insures that the surface cannot twist too much between tool positions.
2. For points which need the offset operation, that is, when there is a check surface detected, the difference between the normals $N_{q}$ of the closest points of two consecutive tool positions should be less than a tolerance. Otherwise the tool may gouge into the check surface heavily when it moves from one of these tool position to the other, since in this case the tool makes a linear move between the closest points on the check surface and large difference between the normals means large chord height on the check surface.
3. When the tool is moved along an isoline of which only part encounters a check surface and hence, only part of whose tool path will be offset, the tool transitions from no offset to a first position with an offset. For such a curve, we compute the point along the curve at which the tool is tangent to the check surface. If such a tangent point is not included in the original sequence of points, we numerically compute it.
4. The direction of the tool must not change dramatically if a point along the path is discarded. If this criterion is violated, the selected tod positions may not follow the shape of the given curve. One extreme case is that the original sequence of points is obtained from a curve with a "C" shape on a flat plane. Since the points from the curve are on a plane, we probably can remove all the points except the last and the first ones and still satisfy the first three criteria. The resulting tool path would be a linear move connecting the two ends of the " C ".

The chord height error on the part surface is controlled by the first criterion, while that on the check surface is limited by the second. The third criterion assures that the tool will not gouge into the check surface when it comes in contact with the check surface and


Figure 3: Tool Positions From the New Check Surface Handling.
when it disengages the check surface. These three criteria guarantee smooth cutting between consecutive tool positions in the resulting tool path. Along with the last criterion, these criteria also result in a tool path which closely follows the position and shape in the surface of the given curve.

In Figure 3 the tool positions generated in a tool path by the above scheme are shown. Comparing with Figure 1, we find the wedge left in Figure 1 will be properly removed by the tool.

### 3.4 Feature Line Handling

We have defined, a feature line as a narrow strip of concave region on the part surface that is crossed by the set of tool paths on the surface. We handle a feature line by approximating the surface in the concave region by a sequence of intersecting planes. Two planes are on every tool path, and each of those is the tangent plane of a tool position on one side of the feature line. The two planes on a tool path intersect each other and form a corner at the feature line. The narrowness of the feature lines allows us to use the approximation without introducing too much error.

To determine the planes forming the corner on a tool path for a feature line, we have to detect the line first. By checking curvature values at the points in the automatic refinement sequence, we can detect a feature line and, hence, points on both sides of the feature lines.


Figure 4: One Tool Path.

The points on the feature line are thrown away and so are not included in the tool path. After the points on both sides of the feature line are found, the point closest to the feature line on each of the sides is marked. The tangent planes through the marked points are taken as the intersecting planes approximating the region around the feature line.

Gouges into the part surface by the tool can be caused by having tool positions on the feature line and also if the tool positions are less than a tool radius from the feature line. For example, a tool position just above the lower edge of the windshield in Figure 5 will cause the tool to gouge across the feature line and into the surface forming the front of the fuselage.

To avoid such gouging, the points on each side of the feature line are offset with the method described in Section 3.2, taking the part surface as both the check and the part surfaces. The parametric values of the marked point on the other side of the feature line are taken as the initial guess values for the closest point finding. By doing so, the region around the feature line is approximated by the tangent planes through the marked points. Figure 4 shows the tool positions generated across a feature line. The crosses in the figure represent the tool points before the offset operation. The circles represent the face of the tool after the offset.

## 4 An Example

The body part of the helicopter model (see Figure 5) was chosen as an example to demonstrate the power of the above methods. The model was constructed in 1983 [3].

Figure 5: The Helicopter Model (Courtesy of Elaine Cohen).


### 4.1 Model Selection

There are several reasons why the helicopter model was appropriate. First, there are only two surfaces in the body of the helicopter. The first surface is for the cowling which starts right behind the windshield and extends all the way to the tail. The cowling is above another surface, the main body, which makes up the front nose, windshield, side vent windows, passenger cabin and the main boom. This surface extends through the full length of the helicopter. The twosurface setting will result in at most one check surface when we move the tool on the other surface. This conforms to the one check surface limitation in our tool path generation scheme.

In addition, most parts of the surfaces are convex. The windshield and the side vent windows are nearly flat pieces and are blended into the body with sharp turns at the transitions. Both of the surfaces bulge in the front half and then contract abruptly to meet a conical boom. Except for several feature lines, the only portion of the model which is not convex is the blend region between the body and the boom, which, unfortunately, is too wide to be considered a feature line. There are several feature lines on the model to which we applied our strategy. The set of feature lines along the transitions where the triangular vents are connected to the passenger cabin is an example of the case to which our method is applied. The feature lines are strips with very narrow width compared with any reasonably sized tools. The angle at the intersection of the two surfaces is also always greater than 90 degrees. The convexity and the property of the angle make it possible to use the hypothesized flat end tool with face milling on most areas of the model and to apply the above tool path generation strategy. The feature lines and the two surfaces make it an interesting problem to attack.

Finally, the model is relatively well constructed. Both surfaces in the model are tangent continuous everywhere except at the tip of the tail. The cowling is a quadratic $\times$ cubic surface while the main body is bicubic. Both surfaces are nonuniform B-spline surfaces.

### 4.2 Tool Path Generation for Finishing

The first step in utilizing the method to generate tool paths is to separate the surfaces into manageable pieces. Several criteria are used in deciding where and how to separate the surfaces. Since the above scheme can only handle convex surfaces with feature lines, but the model contains concave regions which are too wide for feature lines, each of the original surfaces has to be divided into three pieces: two nearly convex surfaces with a concave piece in the middle. The concave pieces resulting from this division must be machined with ball end tools. Another reason to do such a division is that the longitudinal isoparametric curves within each of the pieces from the division have more uniform spacings than those in the original surfaces. This allows fewer tool paths to be used on the surface pieces of the boom.

The model has an elongated shape. One isoparametric direction of the surfaces is also in the lengthwise direction of the model. In order to maximize the possible length between two tool positions and to minimize the number of tool paths, the tool paths are chosen to be along the isoparametric lines of the surfaces in the longitudinal direction. We encounter one difficulty when we do so, i.e., the feature lines on the vents will be parallel to our tool paths. This invalidates the application of the tool path generation scheme described above, since the scheme assumed the feature lines crossed the tool paths. To use the developed scheme, the
areas around the vents, including the feature lines, must be taken as separate pieces, and the tool paths must be crosswise from the top of the vent to the feature line on these pieces.

The next step is to select isoparametric curves on the surface pieces for tool path generation. Given the number of curves, a preliminary set of curves uniformly separated in the parametric space is generated. The final curves are obtained from this preliminary set with some interactive adjustment.

To generate the tool positions from the selected curves, a single function which embodies the above tool position generation scheme is used. The input to the function is the sequence of curves from which the tool paths are generated, the check surface, the part surface and other milling information, e.g., radius of the tool to be used. The output of the function is the sequence of tool positions on each of the tool paths. Then another function is used to connect the tool positions on each of the tool paths for a surface piece to form a zigzag tool path.

One tool path is shown in Figure 4. The tool path runs across the feature line below the windshield and is offset against the cowling surface. From the figure we can see that the tool position offset scheme allows the region around the intersection of the cowling and passenger cabin to be machined smoothly.

### 4.3 Tool Path Generation for Roughing

The finish cut, described above, is performed after the "rough" cut. The rough cut decreases the amount of material that the finish cut tool must remove. This is an important process since the forces on and subsequent deflections of the finish tool (the inaccuracies) are related to the amount of material that must be removed. A requirement for the rough cut is to keep the tool at a constant distance from the part. If the distance is too large or the variation of the distance is too large, the good effects of the rough cut on the finish cut is diminished. We choose to use a ball end tool of a large radius to perform the roughing in 3 -axis machining mode. Tool paths are taken along the isoparametric curves on the offset surfaces of the two surfaces.

### 4.4 Results

Figure 6 shows a photo of the helicopter model produced by the NC machine. The model is made of aluminum, and no hand finishing was done after milling. Two halves of the model were machined and then were put together. Roughing was done by a three-quarter inch diameter ball end mill. Finishing was done by a quarter inch diameter flat end mill on all the convex surface pieces. The two concave pieces were done by quarter and half inch diameter ball ends. The NC code required to cut each half had a size of approximately 360 kbyte (equivalent tape length of approximately 3000 feet).

## 5 Remaining Problems

To increase the domain of the scheme, more contact points between the tool and the check surface should be allowed. A related problem to multiple contact points is to allow multiple

Figure 6: The Aluminum Helicopter Model.


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check surfaces.
In the machining aspect, the tool path generation scheme we used for the rough cut does produces a roughly constant layer of remaining material on the model. But the depth of cuts of the tool path can vary. Unless a sophisticated method is used to calculate the feedrate and speed on a step by step base along the tool path, the rough-cut tool path can produce very high forces.

Since the machine and/or the workpiece rotates when making linear moves in multiaxis milling, the possibility of the interference between the tools and the workpiece is high. Manual pre-determination of such interference is also very difficult. The traditional way of determining such interference by test runs is sometimes dangerous and laborious. Software for automatic interference checking is important in this case.

## 6 Conclusion

A different approach is presented for 5 -axis milling of $B$-spline surfaces. In the special domain, the tool position generation strategy places tool positions automatically. Gouge checking is included in the tool position generation. The amount of human interaction is reduced in this method. The tool path also produces a relatively good finish so only limited hand finishing is required to get the machined part into the final product form.

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## Appendix: Closet Point Finding

The equation of the intersection curve $I_{c}$ of the tangent plane $P_{t}$ and the check surface can be written as,

$$
\begin{equation*}
g \equiv D \cdot N_{p}=0, \text { where } D=P-C_{s}(u, v) \tag{1}
\end{equation*}
$$

The closest point $Q$ to $P$ on $I_{c}$ can be found by minimizing

$$
\begin{equation*}
f \equiv D \cdot D \tag{2}
\end{equation*}
$$

with the restriction of equation 1 . Note that on $I_{c}, u$ and $v$ are related. Taking the derivatives of both equations 1 and 2 with respect to $u$, results in

$$
\begin{equation*}
\frac{\partial f}{\partial u}+\frac{\partial f}{\partial v} \frac{\partial v}{\partial u}=0 \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial g}{\partial u}+\frac{\partial g}{\partial v} \frac{\partial v}{\partial u}=0 \tag{4}
\end{equation*}
$$

Then eliminate the relation between $u$ and $v$ from equation 3 by equation 4, so

$$
\begin{equation*}
\frac{\partial g}{\partial v} \frac{\partial f}{\partial u}-\frac{\partial f}{\partial v} \frac{\partial g}{\partial u}=0 \tag{5}
\end{equation*}
$$

Given an estimated value of ( $u_{0}, v_{0}$ ), Newton's method is used to find the $(\bar{u}, \bar{v})$ which satisfies both equation 1 and equation 5 . Then the closest point is obtained by $Q=C_{s}(\bar{u}, \bar{v})$.


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