# Apparent Symmetries in Range Data 

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#### Abstract

A procedure for extracting symmetrical features from the output of a range scanner is described which is insensitive to sensor noise and robust with respect to object surface complexity. The acquisition of symmetry descriptors for rigid bodies from a range image was in this case motivated by the need to direct pre-grasp configurations in dextrous manipulation systems. However, object symmetries are powerful features for object identification/matching and correspond explicitly to useful geometric object models such as generalized cylinder representations ${ }^{1}$.


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## 1. Introduction

Work has been done previously concerning the segmentation of range imagery based on sparse data that relies on surface hypothesis testing [1, 4]. These techniques use sparse sets of data to construct hypotheses which are either supported or denied by other such "local" hypotheses. This approach could conceivably be generalized to identify that class of objects which can be represented by generalized cylinders. The resulting procedure is efficient and may be implemented as a VLSI range image preprocessor. The algorithm described by Perez et al. [4] requires that hypotheses are constructed from a model of the object's surface. The resulting number of competing "interpretations" of the object derived from such sparse surface measurements, in noisy data, over a large set of object models could prove to be prohibitive. It is in this case that global information concerning the symmetries apparent in the range image can be used to effectively prune the tree of competing interpretations.

Symmetries are a powerful feature in object identification and are also quite useful in applications which require no further surface information. Consider, for example, the problem of selecting initial orientations and approach vectors for a manipulator which has functional symmetries. Knowledge of the principal symmetries of an object and a manipulator are sufficient to prune the set of initial hand/object interaction configurations to an optimal subset. It is in this context that the approach described here was developed.

The procedure has proven useful for identifying symmetrical features within range images. It will, however, produce symmetry parameters (eigenvalues and eigenvectors) for any object and is robust with respect to object orientations. The procedure begins by assigning a normal to each point in the range image from which we identify approximately planar, contiguous segments. This step produces essentially a least squares planar approximation to the object's surface and therefore removes random noise in the sensed data. The planar patch approximation of the object consists of a set of patches each described by centroid, area and normal (directed outward). The centroids are projected backwards along their respective normals to identify positions in space where several such projections converge. To these points of convergence we attach a weight which represents the combined area of the contributing surface patches. These weighted positons might be regarded as a group of distributed point masses for a homogeneous object from which we may compute a real, symmetric $3 \times 3$ matrix discribing the object centered mass distribution. The eigenvectors of this distribution of "mass-nodes" are the principal axes of the object and the corresponding eigenvalues are the principal moments of inertia.

The following text discusses the implementation of this approach and describes some of the results produced so far. We conclude with some remarks which extend our results to the general case of object identification and localization.

## 2. Identifying Local Mass Nodes

Mass nodes are identified by means of an analysis of planar patches recovered from the range data. First, a surface normal is computed at each range data point using standard techniques over some minimum set of neighboring points [2]. Next, the normals are grouped together using blob coloring techniques where "color" is represented by the normal. This set of patches is then made available to the symmetry finder described below.

Once planar patches have been identified, a blob volume description is defined which includes the
volume of the object as suggested by the pointwise data. The volume can be represented in multiple resolutions, but it was found in the experiments conducted that a rather coarse regular decomposition of the volume is sufficient. The centroid of each planar element of the surface is then projected backwards along its normal a distance proportional to the maximum range in the pointwise data. As the back projection passes through elements of this blob volume, accumulators in each element or bin increment the number of such intersections and add the surface element's area into a total contributing "mass" for the bin. The accumulator value determines the relative significance of the resulting set of bin volumes, and can be used to threshold the bins so that only significant results are saved. The accumulator values can be interpreted physically as the number of surface patches which appear to be symmetrical with respect to the mass node position. Locations with large accumulator values are relatively reliable symmetry focii. The center of mass of the resulting spatial distribution of "masses" is a trivial computation. An illustration of a simple case is presented in Figure 2-1.


Figure 2-1: The Distribution of Apparent Mass Nodes in an Undersampled Object

In our experiments, we noted that cases similar to that illustrated in Figure 2-1 produce relatively few mass nodes. This stems from the fact that the computation which produces the planar patch approximation segments this surface into the largest possible contiguous planar faces. If this object had been a perfect cylinder, it may well have produced only one mass node, from which symmetries could not be inferred. The solution to this problem is the definition of a maximum area for contiguous surface elements which creates a new patch if areas grow too large.

The examples presented in this paper were computed without using a maximum area in the planar patches segmentation. To improve the results in those cases where few mass nodes were produced, we
lowered the accumulator significance threshold. This permits more data, but unfortunately, data which is less significant. We note in Figure 2-1 that this undersampled case accurately identifies the approximate center of mass, but in fact suggests planar and not axial symmetry in the range image of a soda can.

## 3. Computing the Principal Axes

After having identified an object's center of mass and the apparent distribution of mass, a real valued, symmetric inertia matrix, M, may be computed:

$$
M=\sum_{i=1}^{n} \omega_{i} \vec{x}_{\mathrm{i}}^{\top} \vec{x}_{\mathrm{i}}
$$

where:

$$
\begin{aligned}
& M=\text { a } 3 X 3 \text { inertia matrix for the object, } \\
& \omega_{i}=\text { weight of mass node } i \text {, and } \\
& \vec{x}_{i}=\text { world frame position of mass node } i .
\end{aligned}
$$

The principal axes of the apparent mass distribution are just the eigenvectors of the inertia matrix, M. In our experiments we found that a simple iterative method (Jacobi's method) suited our objectives. The magnitude of eigenvalues associated with each of these eigenvectors is proportional to the spread of mass along that particular eigenvector. That is, large magnitude eigenvalues indicate a wide range in the mass distribution along the corresponding axis. If $\lambda_{\mathrm{i}}, \lambda_{\mathrm{j}}$, and $\lambda_{\mathrm{k}}$ represent the three eigenvalues:

- $\lambda_{\mathrm{i}}, \lambda_{\mathrm{j}} \gg \lambda_{\mathrm{k}} \Rightarrow$ plane whose normal is in direction defined by the $\mathrm{k}^{\text {th }}$ eigenvector,
- $\lambda_{i}, \lambda_{j} \ll \lambda_{k} \Rightarrow$ axis whose direction is defined by the $k^{\text {th }}$ eigenvector, or
- $\lambda_{i}-\lambda_{\mathrm{j}}-\lambda_{\mathrm{k}} \Rightarrow$ spherical swarm of mass, or two planes of symmetry.


## 4. Results : Some Repesentative Examples

Some of the results produced so far by this procedure are presented in the following figures. These demonstrations were all made using the UTAH Range Database [3]. All figures depict the original pointwise data, the mass nodes which satisfy the threshold (radius is proportional to "mass") and the resulting principal axes. The relative magnitude of the eigenvectors associated with these axes are the principal moments of inertia of the object. It is possible, therefore, to identify the dominant type of symmetry: axial, planar or spherical. In the figures, the relative length of the principal axes expresses the apparent symmetry class of the object.

These preliminary results illustrate the utility of this method and motivate a number of logical improvements. Figure 4-1 illustrates a case where the accumulator threshold indicating the significance of the apparent mass nodes was lowered to include more of them. The lower portion of this surface was represented by tall thin planar patches, whose centroids are prone to error due to the shape of the plane produced in the segmentation. It was not probable that the backcast centroids would intersect. As was mentioned before, the correct solution is to segment the surface into planar patches which do not exceed


Figure 4-1: The Apparent Symmetries in UTAH Range Database: Bottle
an area limit. The result is a roughly uniform segmentation of these large, but somewhat featureless surface regions. The examples presented in Figures 4-2 and 4-3 did not suffer from a lack of apparent mass nodes and the results are relatively good. Figure 4-3 demonstrates by the lack of mass nodes near the threaded portion of the light bulb, that the area constraint on the planar patches will undoubtedly improve the results.

The weight of the mass nodes shown here are simply the total area of all the patches which contributed to them. Since the accumulator is used to judge the confidence that a node reflects a symmetry in the object, it stands to reason that areas that contribute to a node be roughly equivalent. A node that reflects the intersection of a large and a small surface area may have a significant total area, but may not reflect the correct symmetrical nature of the object. Future versions of this procedure will investigate other methods of assigning a weight to a mass node. A simple approach is to replace the total area with the average area of the patches contributing to a node, or to multiply the total area by a coefficient which is inversely proportional to the variance of the patch areas that contribute to the node.

Finally, we make the distinction between symmetry and the computations performed by this procedure. It is actually more correct to describe the results of this analysis as the apparent principal axes of the object which correspond to the axes of symmetry in symmetrical objects. The program also finds these axes for entirely non-symmetrical, irregular objects and effectiveness of the procedure to disambiguate these objects is equivalent to its effectiveness for symmetrical objects.


Figure 4-2: The Apparent Symmetries in UTAH Range Database : Bottle_1


Figure 4-3: The Apparent Symmetries in UTAH Range Database : Bulb

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