Task Defined Grasp Force Solutions

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Abstract

Force Control for dextrous manipulation has been approached algebraically with a great deal of success, however, the computational burden created when such approaches are applied to grasps consisting of many contacts is prohibitive. This paper describes a procedure which restricts the complexity of the algebraic system of equations, and makes use of mathematical programming techniques to select a solution which is optimal with respect to an objective function. The solution is constrained by contact surface friction properties and the kinematic limitations of the hand. The application of the procedure to the selection of minimal internal grasp forces which allow the application of task defined external forces is described. Examples of the procedure are presented.

1 Introduction

Salisbury has demonstrated that the solution for contact wrench intensities which produce a desired external wrench on the object could be found by inverting the Grip Jacobian[4]. The procedure involves first, augmenting the W matrix presented in section 1.1 row wise with the command vectors which describe its null space. The result is the Grip transform, a square N X N matrix, where N is the number of contact wrenches in the contact system. The desired external wrench and the magnitude of the internal wrench systems are inputs to the resulting system of equations. The computation involved to build the Grip Transform and to invert it for large N is prohibitive. We propose a method for defining minimal contact interaction forces with which to accomplish the external wrench command. The selection of forces is constrained by the coefficient of friction for tangential forces, and the geometry of the robot hand which produces the contact. The latter is accomplished by the so called *Principally Conditioned Axes* for the manipulator, and is discussed in detail in [2]. The result is a reduction in rank for the system of equations used, and the use of an objective function to produce ***optimal*** solutions. The procedure discussed here is similar to a method described by Kerr and Roth[3]. The algorithm presented here is somewhat more effecient, however, since it employs the singular value decomposition of the augmented wrench matrix rather than separate homogeneous and particular solutions.

1.1 The Singular Value Decomposition

The *Singular Value Decomposition* is a very powerful method for describing the character of a linear transform. For a very complete and detailed description of its properties, see Golub *et al.* [1]. In this section, we will present our nomenclature and the role of the svd in the selection of grasp forces.

DEFINITIONS:

$\vec{\omega}_i$ a particular wrench space vector resulting from the i^{th} interaction force.

W_i the set of non orthogonal wrenches produced at the i^{th} contact position.

$\overline{W}_i$ an orthogonal basis for $R(W_i)$ with associated magnitude limits.

W $[\ \overline{W}_1\ \overline{W}_2\ \dots\ \overline{W}_n\ ]$, for 4 fingertip grasp modeled as point contacts with friction, W is 6X12.

CONSIDER:

$$\mathbf{W} \in \mathcal{R}^{\mathrm{mxn}}$$

$$R(\mathbf{W}) = \{\mathrm{y} \in \mathcal{R}^{\mathrm{m}} \mid \mathrm{y} = \mathbf{W}\,\mathrm{x} \text{ for } \mathrm{x} \in \mathcal{R}^{\mathrm{n}}\}$$

$$N(\mathbf{W}) = \{\mathrm{x} \in \mathcal{R}^{\mathrm{n}} \mid \mathbf{W}\,\mathrm{x} = \vec{0}\}$$

$\exists$ orthogonal matrices:

$$U = [u_1, \dots, u_m] \in \mathcal{R}^{mxm} \; and \; V = [v_1, \dots, v_n] \in \mathcal{R}^{nxn}$$

such that:

$$U^T_{mxm}\, \mathrm{W_{mxn}}\, \mathrm{V_{nxn}} = \Sigma = \mathrm{diag}[\sigma_1, \dots, \sigma_{\mathrm{p}}]$$

where, $p = \min(m, n)$.

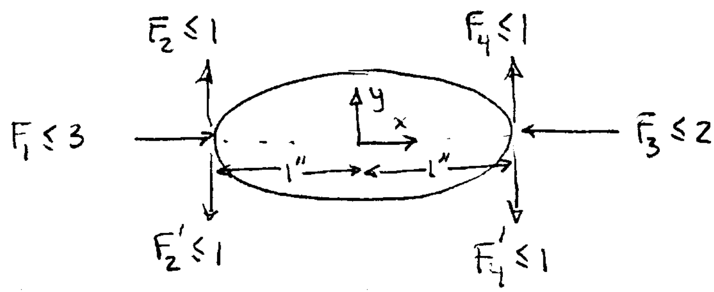

Figure 1: Example Grasp Geometry and Force Magnitude Limits

INTERPRETING THE SVD:

First of all, the hyperellipsoid...

$$E \;=\; \{\, y \mid y \;=\; \mathbf{W}\mathrm{x},\; \|\, \mathrm{x} \,\|_2 \;=\; 1 \,\}$$

Then if we define r to be the number of non-zero singular values, that is...

$$\sigma_1 \geq \;\ldots\; \geq \sigma_r \;>\; \sigma_{r+1} \;=\; \ldots \;=\; 0$$

we may make the following observations about the vector space described by matrix W.

1. rank(W) = r
2. N(W) = span [$v_{r+1}, \ldots, v_n$] = null space of W
3. R(W) = span [$u_1, \ldots, u_r$] = space spanned by W

EXAMPLE:

A two dimensional grasp geometry consisting of two fingers and a spherical object, with associated force magnitude limits is illustrated in Figure 1. This system of forces produces a cooresponding system of wrenches at contact positions 0 and 1 as follows:

$$\begin{array}{llcccrcccrl}
\overline{W}_0: & (\hat{w}_1) & 3 & 0 & [& 1 & 0 & 0\;0\;0 & 0 &] \\
 & (\hat{w}_2) & 1.414 & 1.414 & [& 0 & 0.707 & 0\;0\;0 & -0.707 &] \\
\overline{W}_1: & (\hat{w}_3) & 2 & 0 & [& -1 & 0 & 0\;0\;0 & 0 &] \\
 & (\hat{w}_4) & 1.414 & 1.414 & [& 0 & 0.707 & 0\;0\;0 & 0.707 &]
\end{array}$$

The positive and negative sense magnitude limits, respectively, preceed a normalized wrench representing the effect of a particular interaction force.

If we wish to identify the null space of this contact system, we perform the singular value decomposition of the composite W matrix.

$$\mathbf{W} \;=\; [\, \overline{\mathrm{W}}_0^{\mathrm{T}} \mid \overline{\mathrm{W}}_1^{\mathrm{T}} \,] \;=\; \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0.707 & 0 & 0.707 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -0.707 & 0 & 0.707 \end{bmatrix}$$

$$SVD\,[\,\mathbf{W}\,] \Rightarrow \Sigma = \text{diag}[1.414\ \ 1.0\ \ 1.0\ \ 1.0\ \ 0.0]$$

$$U = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.707 & 0 & 0 & 0 & -0.707 \\ 0 & 0 & 0.707 & 0.707 & 0 \\ 0.707 & 0 & 0 & 0 & -0.707 \\ 0 & 0 & 0.707 & -0.707 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

There are four non-zero singular values which defines the rank of the contact system to be four. Cooorespondingly, the first four column vectors in U define the space spanned by the contact system, and the fifth column vector in V defines the null space of the system. The magnitude of a wrench applied within the null space of the contact system may be increased within the magnitude limits without producing a net external wrench on the object. Therefore, we define the family of such solutions as:

$$\lambda\,[\,-0.707\hat{w}_1 \ - \ 0.707\hat{w}_3\,].$$

2 Internal Wrench Commands for External Tasks

The W matrix defined in section 1.1 is augmented by the external wrench desired as follows...

$$\mathbf{W}_{\mathbf{aug}} = [\,-\vec{w}_{\text{ext}} \mid \mathbf{W}\,]$$

A reaction force equal in magnitude and opposite in direction is assumed to act on the object, and is, therefore, added to the wrench system created by finger/object interactions. The null space of the resulting augmented vector space defines the particular solution of the original system for this external wrench, and when constrained by the magnitude limits at each contact, describes the homogeneous solution space of the original system. A quadratic programming search through this solution space is used to identify the solution which minimizes the magnitude of the internal wrenches required for the task.

The procedure is given a geometry for the grasp which defines the contact positions and the geometry of the hand. This information is used to determine limits on the magnitude of the forces that may be generated by the hand and/or transmitted by the contact surface. These constraints are expressed in the local coordinate frame, that is, along vector directions normal to and tangential to the contact surface.

The contacts are then grouped into disjoint sets for which internal wrenches are not desired. This is a modification of the vitual fingers approach of Arbib *et al.* in that we wish to define sets of contacts that do not produce useful internal wrench systems for particular tasks. The wrench system produced over the set can then be described by an orthogonal basis which spans the same vector subspace as the constituent wrench vectors. We may then condense the W matrix by using (learning) appropriate sets of virtual contacts for particular classes of tasks. Commands submitted to a virtual contact must be decomposed into commands for each independently expressed interaction force.

2.1 Posing the Problem

We augment the W matrix representing the contact system by including the negative external wrench command in the first column. If we require the example presented earlier to be capable of producing a unit force in the x direction and a unit moment about the z axis, The W matrix becomes...

$$W = [\ \vec{w}_0\ \hat{w}_1\ \hat{w}_2\ \hat{w}_3\ \hat{w}_4\ ],\ or$$

$$W = \begin{bmatrix} -1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0.707 & 0 & 0.707 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -0.707 & 0 & 0.707 \end{bmatrix}$$

The family of solutions which occupy the null space of this augmented contact system is then:

$$\begin{array}{lllllll} \lambda_1\ [& (0.6324)\vec{w}_0 & +(0.3162)\hat{w}_1 & -(0.4473)\hat{w}_2 & -(0.3612)\hat{w}_3 & +(0.4473)\hat{w}_4 &] \\ \lambda_2\ [& & -(0.707)\hat{w}_1 & & -(0.707)\hat{w}_3 & &] \end{array}$$

The system of equations representing the null space can be generalized as follows.

$$\begin{bmatrix} \lambda_1\alpha_{10} & \lambda_1\alpha_{11} & & \lambda_1\alpha_{1n} \\ \lambda_2\alpha_{20} & \lambda_2\alpha_{21} & & \lambda_2\alpha_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \lambda_m\alpha_{m0} & \lambda_m\alpha_{m1} & & \lambda_m\alpha_{mn} \end{bmatrix} \begin{bmatrix} \vec{w}_0 \\ \hat{w}_1 \\ \vdots \\ \hat{w}_n \end{bmatrix} = \begin{bmatrix} \vec{0} \\ \vec{0} \\ \vdots \\ \vec{0} \end{bmatrix}$$

2.2 Constraining the Solution Space

The system of equations representing the null space of the augmented W matrix is now subject to constraints which are enumerated according to rules that fire based on the type of force being considered.

External Constraint : for $\vec{w}_0$

$$\sum_{i=1}^{m} \lambda_i\alpha_{i0} = 1 \tag{1}$$

Constraints for $\hat{w}_j(\vec{f}_j)$ where $\vec{f}_j$ is a normal force

$$\sum_{i=1}^{m} \lambda_i\alpha_{ij} \geq -negative\ magnitude\ limit\ for\ \hat{w}_j,\ and \tag{2}$$

$$\sum_{i=1}^{m} \lambda_i\alpha_{ij} \leq positive\ magnitude\ limit\ for\ \hat{w}_j, or$$

$$-\sum_{i=1}^{m} \lambda_i\alpha_{ij} \geq -positive\ magnitude\ limit\ for\ \hat{w}_j. \tag{3}$$

In order to express the effects of friction on the solution space, we first define a geometric magnification factor which relates the magnitude of $\hat{w}_j$ to the magnitude of $\vec{f}_j$, that is,

$$\kappa_j = \frac{|\vec{f}_j|}{|\vec{w}_j|} .$$

With such a relationship, we may convert wrench commands into force commands, and thus, express frictional force constraints.

Constraints for $\hat{w}_j(\vec{f}_j)$, where $\vec{f}_j$ is a tangential force with associated normal force, $\vec{f}_k$

$$|\vec{f}_j| \leq \mu|\vec{f}_k|, \textit{ so that,}$$

$$\kappa_j \sum_{i=1}^{m} \lambda_i\alpha_{ij} \leq \mu\kappa_k \sum_{i=1}^{m} \lambda_i\alpha_{ik}, \textit{ or,}$$

$$\kappa_j \sum_{i=1}^{m} \lambda_i\alpha_{ij} - \mu\kappa_k \sum_{i=1}^{m} \lambda_i\alpha_{ik} \leq 0, \textit{ or,}$$

$$\sum_{i=1}^{m} \lambda_i[-\kappa_j\alpha_{ij} + \mu\kappa_k\alpha_{ik}] \geq 0. \tag{4}$$

$$\sum_{i=1}^{m} \lambda_i\alpha_{ij} \leq \textit{positive magnitude limit for } \hat{w}_j, \textit{ or,}$$

$$-\sum_{i=1}^{m} \lambda_i\alpha_{ij} \geq -\textit{positive magnitude limit for } \hat{w}_j. \tag{5}$$

$$|\vec{f}_j| \geq -\mu|\vec{f}_k|, \textit{ so that,}$$

$$\kappa_j \sum_{i=1}^{m} \lambda_i\alpha_{ij} \geq -\mu\kappa_k \sum_{i=1}^{m} \lambda_i\alpha_{ik}, \textit{ or,}$$

$$\sum_{i=1}^{m} \lambda_i[\kappa_j\alpha_{ij} + \mu\kappa_k\alpha_{ik}] \geq 0. \tag{6}$$

$$\sum_{i=1}^{m} \lambda_i\alpha_{ij} \geq -\textit{negative magnitude limit for } \hat{w}_j. \tag{7}$$

Equations 1 through 7 delimit the allowable volume of the solution space. The procedure used to select a solution from this space is a straight forward quadratic programming technique. The specific procedure used is part of the IMSL mathematical library. Given objective functions of the form,

$$g^T x + \frac{1}{2} x^T H x,$$

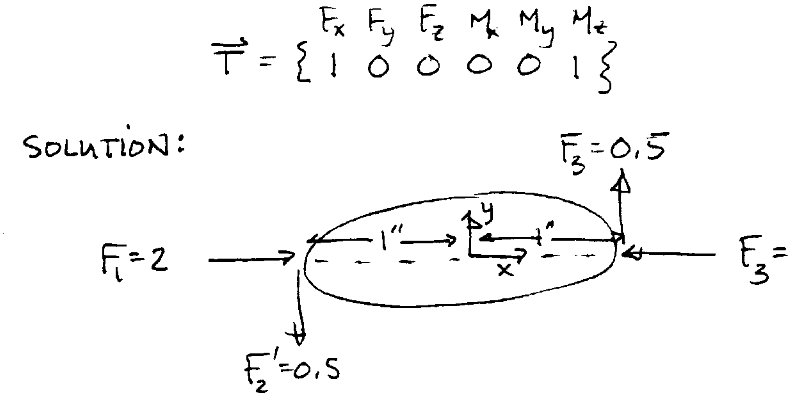

Figure 2: Minimal Grasp Forces required to produce External Wrench

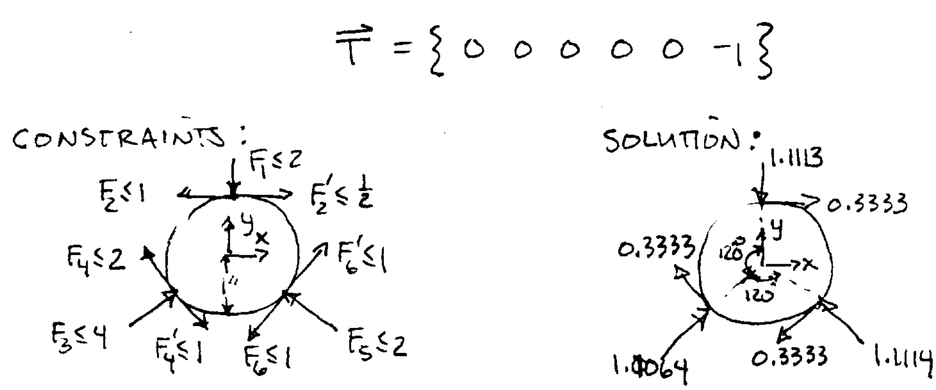

Figure 3: Minimal Grasp Forces required to produce External Wrench

subject to constraints of the form,

$$A_1 x = b_1, \; and$$
$$A_2 x \geq b_2,$$

the procedure moves along the solution space surface toward the position which minimizes the object function. The euclidean length of the solution vector can easily be expressed in the form of this objective function.

When the system described earlier is solved in this manner, the result is a solution for the minimal $\vec{\lambda}$ force system required to accomplish the task. This force system is illustrated in Figure 2.

Another example is presented in Figure 3. Illustrated is a two dimensional object interacting with three contacts. The magnitude limits for each contact are the result of a kinematic analysis of the hand geometry which achieves the contacts. When a task consisting of a moment about the $-\hat{z}$ axis is submitted to the procedure, it yields the minimal force interaction presented in the Figure.

References

[1] G.H. Golub and C.F. Van Loan. *Matrix Computations.* The Johns Hopkins University Press, Baltimore Maryland, 1983.

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[3] J. Kerr and B. Roth. Analysis of Multifingered Hands. *Journal of Robotics Research*, 4(4):3–17, Winter 1986.

[4] J.K. Salisbury. *Kinematic and Force Analysis of Articulated Hands.* PhD thesis, Stanford University, May 1982.