OPTIMAL CONTROL OF A PROCESS WITH DISCRETE AND CONTINUOUS DECISION VARIABLES

by

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FOREWARD

Automation of the control of an industrial process generally entails three basic tasks:

i) design of a computer system (the computer configuration, the sensors, the transistors, and the control software);

ii) design of a control strategy or control law

iii) representation, i.e., modeling of the process

There is normally a natural ordering of these tasks; that is, one does not design the computer system without knowledge of the control strategy, and one does not design a control strategy without a model of the process.

Often the first task is less process-dependent than the others, and common computer technology is applicable. The second task is straightforward for certain simple models; for example, linear systems, systems with restricted size, and systems with only discrete or only continuous control (decision) variables.

For many complex industrial processes, it is the second and third tasks that limit realization of automated control. This is true for the copper converter aisle described in this report. The converter aisle process is large, nonlinear, and has mixed continuous and discrete control and decision variables. For this type of system work is being done on development of representative models and improved sensors, but work on design of control strategies has lagged. In part this is because, in concept at least, optimal control design can be achieved using standard techniques such as Bellman's dynamic programming. Unfortunately, for systems such as the converter aisle, standard control design techniques lead to prohibitively large computational tasks.

This paper reports a new and simple approach for computation of the control for a process as typified by the copper converter aisle. The resultant control is open loop and locally optimal. The models used to represent the process and to illustrate results are unrealistically simple, but they serve to illustrate application of the technique.

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ABSTRACT

The task of dynamic optimization consists of manipulating the inputs to a dynamic system (i.e., one in which the state varies with time) so that the system performs in an advantageous manner.

This paper presents a systematic technique for solving the problem of optimally controlling a converter aisle in a copper smelter. The converter aisle is distinguished from the usual dynamic system in that some of the control variables occur as discrete decisions while others may vary continuously with time. In this sense, the converter aisle typifies many industrial processes. The aisle is viewed as a total system with the objective of optimizing overall performance as evaluated using a mathematical performance criterion. Typical criteria reflect total processing time and operating costs.

An essential step towards optimization is the development of a mathematical model to predict the state of the system. A simple mathematical model of the converter aisle is developed; and using this model, two optimization approaches are examined: direct optimization of the total system and partitioning the system into interacting subproblems. The partitioned approach is pursued in detail with techniques for solving the optimization subproblems presented and illustrated by numerical examples.

INTRODUCTION

The converter aisle in a copper smelter is composed of a number of converters (or furnaces), a supply of input material, a receiver of molten copper, and a crane (or cranes) which serves the converters. The converters oxidize the input material to produce first an intermediate material, and later molten copper. The crane delivers material to the converters and removes waste material and molten copper. Viewed as a "system" to be controlled in some advantageous manner, the aisle has an important characteristic in that some of the "controls" are discrete decisions (for example, the times that the crane visits each converter) and others are continuous variables (for example, oxygen flow rate to each converter).

The application of optimal control techniques to industrial processes has commonly been limited to techniques for static optimization, such as allocation and scheduling problems involving discrete decisions and "set-point" type control of continuous variables. Because of its complexities, there has been little attention given to the copper aisle. Recently some papers have reported on this process, but these have dealt only with parts of the total optimal control problem.

This paper presents a technique for solving the problem of optimal control of the converter aisle viewed as a total system. We first establish a perspective of the problem from a general systems viewpoint and then view optimal control of a converter aisle as a general systems optimization problem. We point out major steps and approaches to aisle optimization. Two optimization approaches are presented. The first is a direct approach which results in a problem that is computationally large--so large, in fact, that a major investment in time and effort is required to produce any results. Consequently, the direct optimization approach is not used but rather, an alternative method is developed whereby the problem is partitioned into smaller interacting subproblems which may be solved with a reasonable effort in a relatively short period of time. The contribution of this paper is the systematic method of partitioning a large complex problem, such as the converter aisle, into a class of more easily solved subproblems and then composing the solutions to the subproblems to form a total optimal solution.

CHAPTER I

SYSTEMS CONCEPTS

Modern control theory offers a richly developed body of techniques for optimally controlling a dynamic system. These techniques will be increasingly applied to control of industrial processes. This section establishes basic systems terminology and perspective relating to process control. We note the distinction between the physical process and the system model and then discuss the role of the model in design of the optimal control.

Associated with a physical process, we identify certain (time varying) quantities as *controlled variables*, *control variables* (or *decision variables*), and *internal variables*. Controlled variables are those which we wish to observe, control, or regulate in some prescribed manner, or which are used in evaluation of a performance criterion which measures the process performance. Control variables are variables to be manipulated or decisions to be made so as to achieve desired process performance. Internal variables are those additional variables which are physically related (directly or indirectly) to either the controlled or control variables. As a conceptual abstraction, the physical process is viewed as a *system* represented as the block diagram in Figure 1, where the arrows represent flow of information. The system input is a vector of the process control variables and the system output is a vector of the controlled variables of the process. The *state vector* is derived from the internal

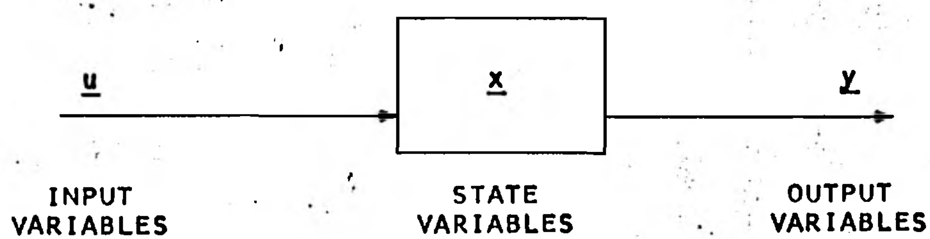

Figure 1 SYSTEM REPRESENTATION
(Underlined variables denote vectors.)

and controlled variables of the process; the state variables are the minimum set of variables necessary to specify at any time the value of the output given the value of the input at the same time. The state vector is not necessarily unique, as it depends upon the choice of output variables and upon the degree of approximation in specifying the value of the output vector.

In design of the system control, it is necessary to evaluate the output in response to choices of the input. Generally it is unfeasible or prohibitively expensive to experiment with the actual process; consequently, a *model* of the system is developed. The model consists of two sets of rules which specify the output and the state. The *output equation* is generally algebraic and it specifies the output as a function of the state and the input. Commonly, the output will be merely a subset of the state variables. The *state equation* depends upon the type of model, there being two primary types of models, *static models* in which the state does not vary with time and *dynamic models* in which the state does vary with time. In addition, a dynamic model is *stationary* if the equations do not vary with time. For a static model, the state equations are generally algebraic. Dynamic model state equations take a variety of forms: for an analytic *continuous model*, i.e., with continuous variables, the state equations are differential equations which specify the time rate of change of the state variables. If the state variables are naturally discrete or are quantized in time, the *discrete model* may be a difference equation or state transition equation which specifies the next state as a function of the current state and input. Alternately, a state transition equation may be given in a tabular or graphic form.

Computer simulation models incorporate analytic, tabular, and partially verbal representationsof the model. Whereas the analytic and tabular models will be directly useful in design of optimal controls, simulation models are commonly used for parametric experimentation on the system.

The system represented in Figure 1 is shown as a single block, although systems are often modeled as a set of interacting subsystems, for example, as illustrated in Figure 2. System structure may be naturally implied by identification of subsystems associated with the physical process, or the system structure may be imposed to achieve computational advantages. Decomposition of a system into subsystems has the advantage that the individual subsystems are more easily analyzed, but on the other hand, the interaction of the subsystems may be difficult to handle. (It should be kept in mind that subsystem input variables are not necessarily physical quantities, but may represent decisions, schedules, etc.)

Generation of a system model is a complex task (so much so that there is danger of temporarily ignoring the optimization objective). The modeling task has two basic steps: determination of the model form, i.e., structure and type, and data acquisition and numerical construction of the model. The form of the model is usually motivated by physical knowledge of or familiarity with the process. Numerical design of the model often reduces to statistical estimation of a set of parameters, which in turn involves subtasks of filtering data and estimating state variables which are not directly measurable.

The role of *optimal control* theory is to provide a method for determining dynamic inputs which cause the system to behave optimally in the

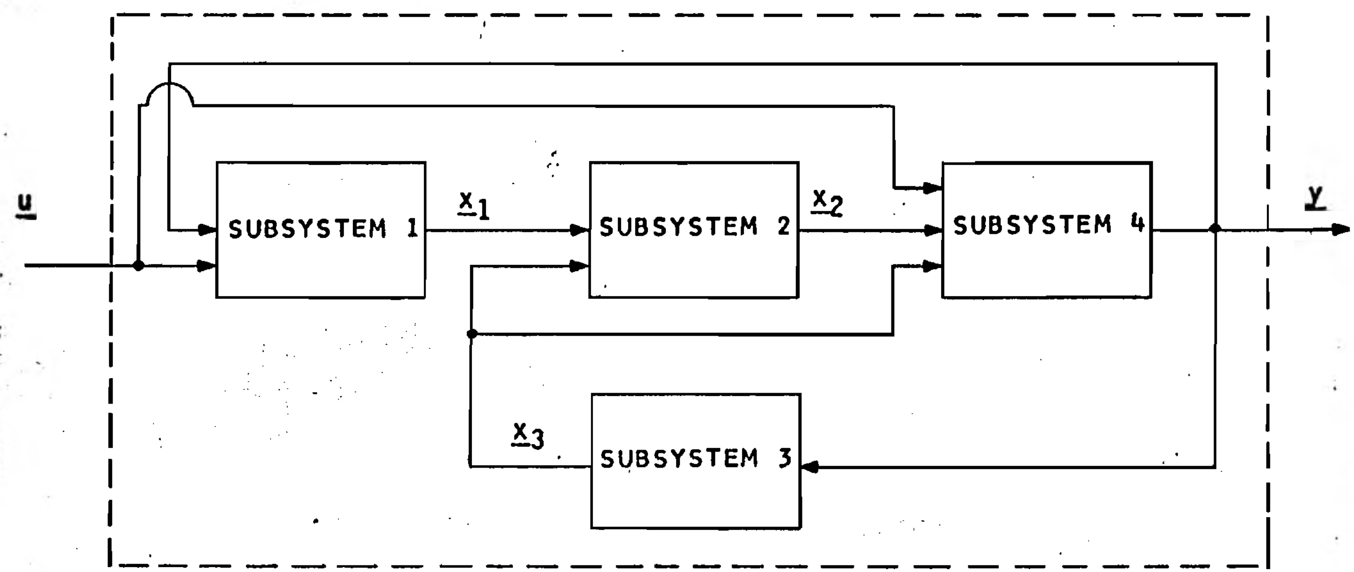

Figure 2 DECOMPOSITION OF A SYSTEM INTO SUBSYSTEMS

that a performance criterion J, which measures the influence of the inputs upon the system performance, will be minimized (or maximized) subject to system constraints. For application of optimization techniques the *performance criterion* is commonly of the form

$$J = \theta\left[\underline{x}(t_f),\ t_f\right] + \int_{t_o}^{t_f} \phi(\underline{x},\ \underline{u},\ \tau)\,d\tau \qquad (1.1)$$

for a continuous dynamic model or

$$J = \sum_{t=0}^{N} \phi[\underline{x},\ \underline{u},\ t] \qquad (1.2)$$

for a discrete model, where θ and ϕ are functionals (i.e., they take on scalar values) and t_o to t_f is the time interval of interest or t = 0,..., N are the discrete times of interest, although the final time need not be fixed. The performance criteria Equations 1.1 and 1.2 are functions of the system *trajectory* $\{\underline{x}(t),\ \underline{u}(t)\}$, that is, the time history of the state and associated control. The system *constraints* reflect the physical or economic constraints on the process. Given the system model, performance criterion, and constraints, the optimization problem may be represented by the diagram in Figure 3, which should be distinguished from the system block diagram. (The output of the problem diagram is the optimal solution $\underline{u}°$ which is the input to the system.)

Depending upon the model, the criterion, and the constraints, there are a number of techniques available for determining $\underline{u}°$. Commonly, $\underline{u}°$ is found as a *control policy* $\underline{u}° = \underline{u}°(t)$ so that the control is "open

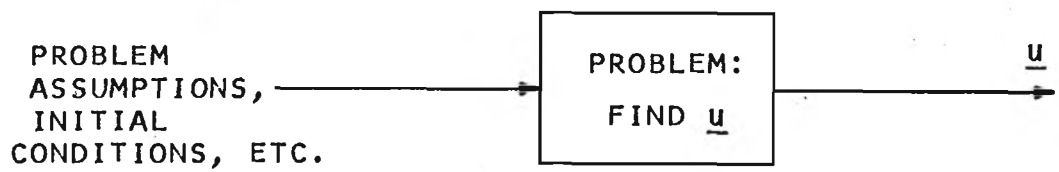

Figure 3 DIAGRAM OF A SINGLE STAGE PROBLEM

loop." In contrast, for practical implementation, it is desired that the control be a feedback law $\underline{u}^\circ = \underline{u}^\circ\ [x,t]$, a function of the measured (or estimated) state, as illustrated in Figure 4. In some cases, the feedback law may be directly determined, but more often a *neighbor optimal* controller is used. In this case, the open loop $\underline{u}^\circ(t)$ and corresponding $\underline{x}^\circ(t)$ are calculated and a simple controller is designed to keep the system "near" the desired state in spite of random disturbances which cause deviation from desired performance. We now apply these systems concepts to the copper aisle process.

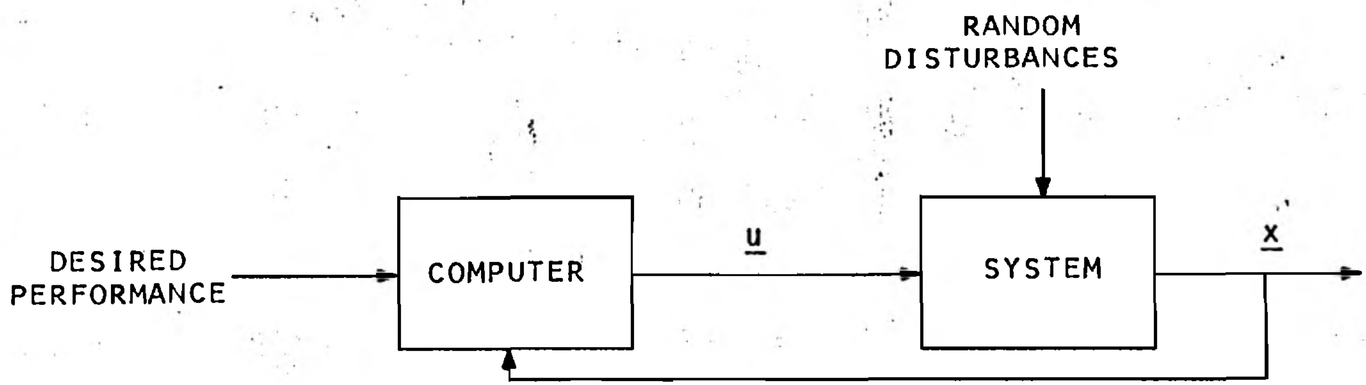

Figure 4 CLOSED LOOP CONTROL SYSTEM

CHAPTER II

CONVERTER AISLE SYSTEM

We will now formulate the converter aisle as a general system. In order to identify the input variables, state variables and output variables, a sequence of events in a converter aisle is briefly described. For the purposes of illustration, let an example converter aisle consist of one reverberatory furnace, three converter furnaces and a holding furnace as shown in Figure 5.

Liquid *matte* (FeS and Cu_2S) is poured into the mouth of a converter from a ladle which is manipulated by an overhead crane. Often *cold material*, including matte shells, scrap copper, etc., is added with the initial *charge* of matte. The converter is then rolled into the *blowing* position, submerging the air inlet openings (tuyeres) below the surface of the molten *bath* and air is blown into the bath, thus removing by oxidation, or *slagging*, the undesirable elements (Fe,S) which are in the bath. *Flux* is added through an overhead hopper or by means of a conveyor belt which dumps the flux into the hood of the converter. Blowing is continued long enough to use up the flux and to form slag which is periodically skimmed (poured) from the converter by turning it out of the blowing position. More matte, flux and cold material are added and then the converter is turned back to the blowing position and blowing is resumed. These partial blows continue until all the iron sulfide has been oxidized and the converter contains essentially pure cuprous sulfide (Cu_2S). Several of these matte charges must be slagged

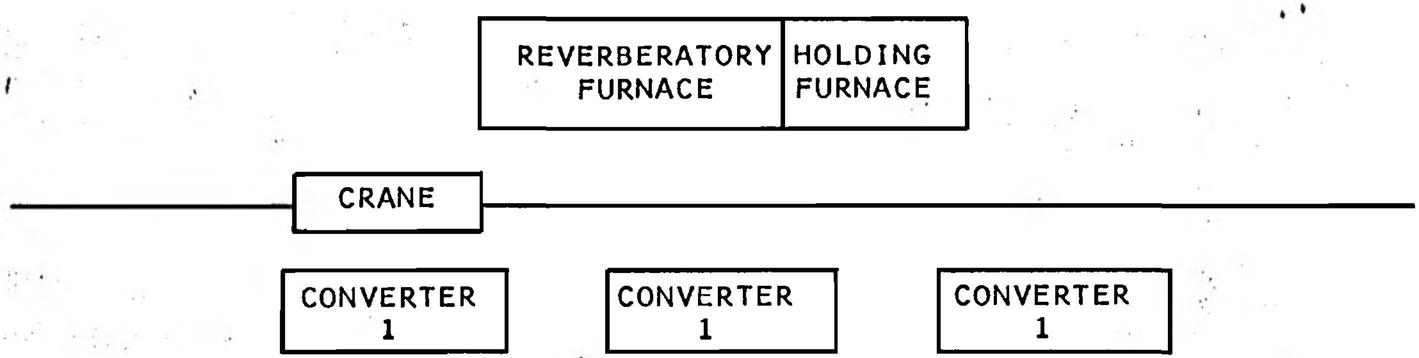

Figure 5 CONVERTER AISLE MODEL

before there is enough Cu_2S accumulated to proceed with the *finish* or copper *blow*. As soon as all of the sulfur in the Cu_2S has been oxidized, the metal, *blister copper*, is poured out and the converter is available to begin another charge sequence.

As the converters process material, the crane moves along the aisle track to deliver and remove material in response to needs of the converters. The crane obtains matte and cold material at the reverberatory furnace site and delivers slag to the reverberatory furnace as it delivers blister copper to the holding furnace, which is assumed to be located near the reverberatory furnace. Generally, each service to a converter requires several trips by the crane. Also, when a converter is being serviced, it does not blow between visits by the crane.

The above descriptions suggest two activities that characterize the converter aisle: (1) material handling by the cranes and (2) charge processing by the converter furnaces. The behavior of the crane is characterized by its location and the full or empty status of the ladle which we will establish together as a two-dimensional state vector. The input variable which affects the state of the crane is a decision which specifies the position to which the crane will move. The result, or output, of the crane activity is the transfer of material into or out of the converters. From this it is apparent that discrete material inputs to the converters are directly associated with the activity of the crane.

The second activity which characterizes the behavior of the converter aisle is the processing of matte by the converter furnaces. The input quantities that affect the state of the converter can be measured in a straightforward way. Air flow into and exhaust gases from each converter can be measured continuously and the material input and output of the converter can be weighed and the composition analyzed with some degree of confidence. For the slag phase, essential inputs indicated by the previously described events are:

u_1 = Matte weight

u_2 = Flux weight

u_3 = Cold material weight

u_4 = Oxygen input rate

In contrast to the input variables, the activities directly associated with the progress of each converter are difficult to measure directly. The state variables chosen to characterize the converting process are somewhat arbitrary and certainly do not describe every activity of the process exhaustively. The state variables are chosen in accordance with the information needed to adequately and realistically describe the observable and measurable activities of the process. Since the function of the converter is to remove undesirable elements from the copper bearing matte, the relative amount of FeS contained in the bath at any time is of interest during the slag phase. Similarly, the amount of Cu_2S provides information concerning the progress of the finish phase. The amount of slag produced would have an effect on the requirement for skimming. The copper accumulated is of fundamental importance since that is the objective of the converter furnace. The bath temperature would surely have an effect on the reaction rates within the process. Summarizing, a typical set of state variables required to describe the process activities during the slag phase are:

x_1 = FeS concentration

x_2 = Cu_2S weight

x_3 = Flux weight

x_4 = Slag weight

x_5 = Temperature

x_6 = Magnetic (Fe_3O_4) weight

The finish phase is similar in concept to the slag phase, except that during the finish phase, the bath consists essentially of Cu_2S which is being converted to blister copper.

The discrete decision variables of the system are:

1. Crane movement decision
2. Converter schedule
3. Batch material inputs

Of course, these are not independent decisions. The continuous control variables are the continuous inputs such as oxygen blow rate and possibly flux addition rate. The rules for determining the time history of the state variables are discussed in detail in the succeeding chapter on modeling. The copper production rate is specified as an output variable as a matter of convenience in the development of the optimization objective.

One objective of optimization of the converter aisle system is to specify the control values which will influence the system to achieve the greatest possible value for the output, where the output performance criterion is expressed by:

$$J = \sum_{i=1}^{3} \frac{Cu_i}{T_i} \qquad (2.1)$$

where

Cu_i = Blister copper produced per charge cycle in converter i

T_i = Duration of a charge cycle for converter i

The charge cycle time is taken to be the time to complete a charge cycle on a converter as shown in Figure 6. Crane movement decisions influence the charge cycle time particularly when two or more converters are in conflict for service. The converter furnace schedule affects the charge cycle time by adjusting the time staggering between converters. The batch material inputs influence the charge cycle times and the blister copper production, while continuous material inputs affect the blowing times. Thus, we observe that the performance criterion of Equation 2.1 is an implicit function of the state and control variables.

We next turn to establishing a system model.

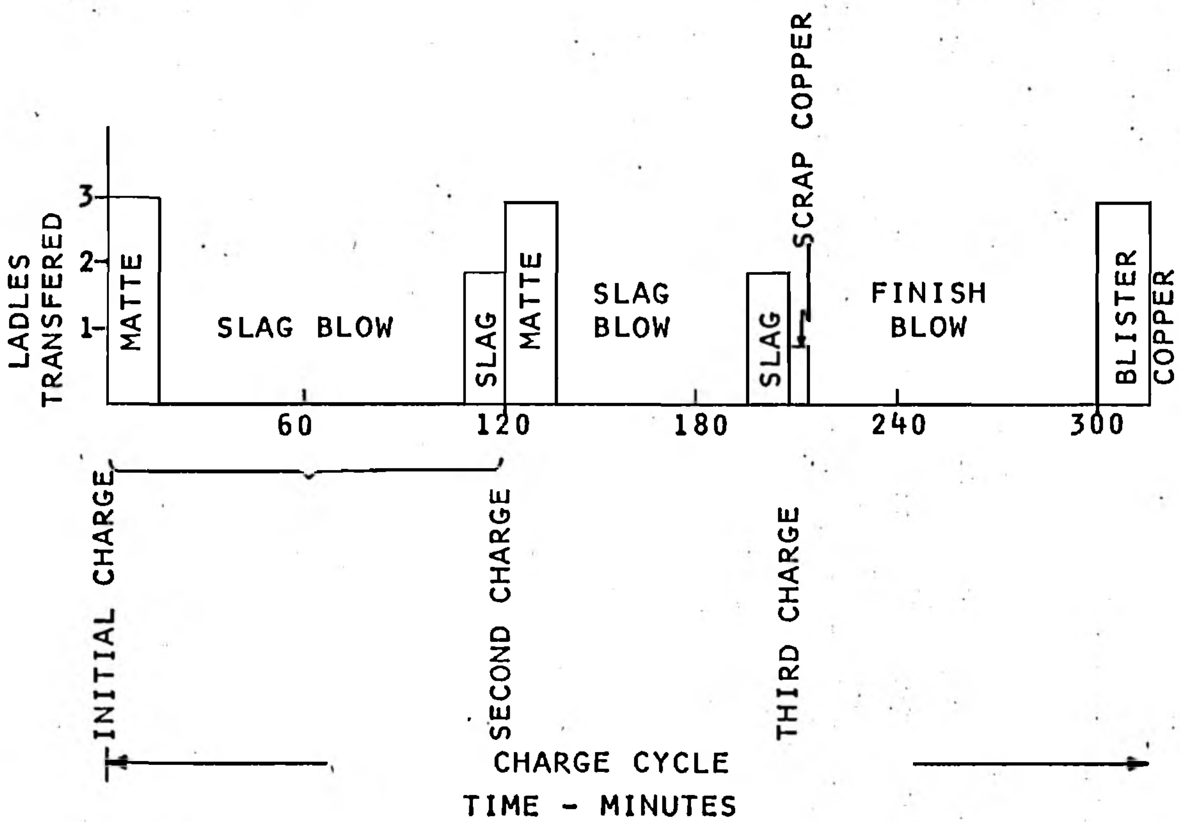

Figure 6 EXAMPLE CHARGE CYCLE

CHAPTER III

CONVERTER AISLE MODELS

Motivated by the ideas presented in the previous chapter, we will develop a model of each subsystem of Figure 7. The emphasis of this section is on modeling concepts rather than the exhaustive derivation of a specific model. Thus, the models that are developed are purposely simple with the assumptions used clearly stated.

It is not possible to specify general rules by which system models are formulated, but a number of guiding principles can be stated:

1. The system will be organized in subsystems so as to simplify the specification of interactions within the system. The subsystems will be characterized by one type of variable (i.e., discrete or continuous).

2. The model will include only those variables which are relevant to the optimization objective.

3. The accuracy of the model will be limited by the data used to develop the model.

4. The model must be capable of reproducing the behavior of the system within an acceptable accuracy.

Crane System Model

The physical structure of the converter aisle suggests a crane routing model. The crane state will be a two-vector $\underline{P}_j$ which represents the state after the jth crane move:

$$\underline{P}_j = \begin{bmatrix} P_{1_j} \\ P_{2_j} \end{bmatrix} \qquad (3.1)$$

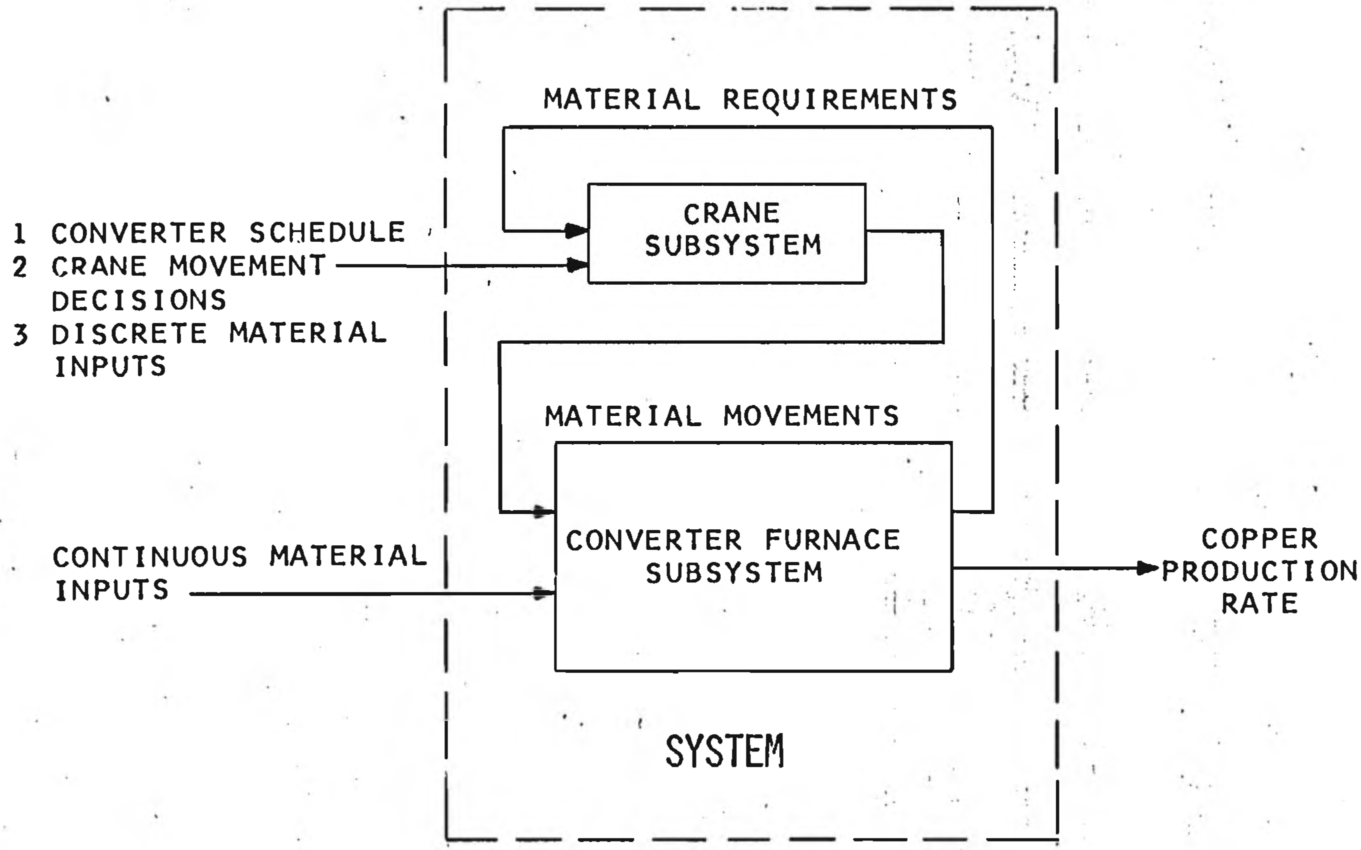

Figure 7 CONVERTER AISLE SYSTEM

The first component P_1 represents the crane location, which can have four possible values

P_1 = 0 crane at reverberatory furnace or holding furnace

P_1 = 1 crane at converter 1

P_1 = 2 crane at converter 2

P_1 = 3 crane at converter 3

The second component P_2 represents the ladle condition during the move, where

P_2 = 0 ladle empty

P_2 = 1 ladle loaded

The crane is modeled by a state transition equation represented as

$$\underline{P}_{j+1} = \underline{h}\left(\underline{p}_j, d_j\right) \tag{3.2}$$

where $\underline{P}_j$ is the current state, d is the crane move decision, and $\underline{P}_{j+1}$ is the next state. $\underline{P}_{j+1}$ is determined by d_j subject to the following assumed constraints:

1. Only one ladle of material will be input or output from a converter at each visit.

2. Only an empty ladle will be moved to a converter that needs to be skimmed or emptied.

3. Only a loaded ladle will be moved from a converter that has been skimmed or (partially) emptied.

4. Only a loaded ladle will be moved to a converter which requires input material, provided it does need output service.

5. Only an empty ladle will be moved from a converter after delivery of input material.

With these assumptions, it is seen that choice of the value of the decision d_j requires knowledge of the converter states; and given that knowledge, the decision d_j is a scaler value, i.e., it is necessary only to choose $(P_1)_{j+1}$ subject to the constraints and then $(P_2)_{j+1}$ is uniquely determined.

For a given crane decision, the additional information required to model the crane is the time required to move from the current location to the next location. This transition time information will be given in the form of Tables 1 and 2, where it is assumed that the transition time includes the loading time at the reverberatory furnace and the loading or unloading time at the converter to which the crane moves. The blanks in the tables indicate illegal moves.

As noted above, crane movements are made in response to material input/output needs of the converters, and the moves in turn effect the states of the converters. The constraints of converter requirements on the crane decision can be described in terms of the converter state variables, and the effect of a crane move on the converters can be modeled by including the crane variables in the converter models. (This is done in Chapter V in the direct optimization approach.) However, even if the converters are not explicitly modeled, it may still be desirable to deal with the crane subsystem. For this case, to model the crane interaction with the converters, we model requirement variables as an input queue and an output queue. The input queue is a vector

$$\underline{q}_{in} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

TABLE 1

STATE TRANSITION TIMES, $(p_2)_{j+1} = 1$

State $(p_1)_{j+1}$ / $(p_1)_j$	0	1	2	3
0	-	t_{01}	t_{02}	t_{03}
1	t_{10}	-	-	-
2	t_{20}	-	-	-
3	t_{30}	-	-	-

TABLE 2

STATE TRANSITION TIMES, $(p_2)_{j+1} = 0$

State $(p_1)_{j+1}$ / $(p_1)_j$	0	1	2	3
0	-	t'_{01}	t'_{02}	t'_{03}
1	t'_{10}	-	t'_{12}	t'_{13}
2	t'_{20}	t'_{21}	-	t'_{23}
3	t'_{30}	t'_{31}	t'_{32}	-

where m_i is the number of ladles to be input to the ith converter. The output queue is expressed by

$$\underline{q}_{out} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

where s_i is the number of ladles of material to be removed from the ith converter. Clearly, the queue variables can have only non-negative integer values. In terms of the queues, the crane moves only in response to nonzero queue values or to return to the reverberatory or holding furnace. Each time the crane moves to a converter, the appropriate queue variable for that converter decreases by one. Each time the crane moves from a converter, P_2 must change, i.e., $(P_2)_{j+1} \neq (P_2)_j$. A crane move to the reverberatory furnace or holding furnace does not change the queues.

In addition to the changes in queue variables, the state of the converter must change since material is being removed or added. The change in converter state due to crane service is given by the following relationship

$$x_1(k+1) = x_1(k) + e_1 m_i$$

$$x_2(k+1) = x_2(k) + e_2 m_i$$

$$x_3(k+1) = x_3(k) + e_3 e_2 m_i$$

$$x_4(k+1) = 0$$

$$x_5(k+1) = [x_5(k) x_2(k) + (e_1 m_i + e_2 m_i + e_3 e_1 m_i)\ e_4] /$$
$$[x_1(k+1) + x_2(k+1) + x_3(k+1)]$$

$$x_6(k+1) = 0$$

where the subscripts (k+1) denote the time following crane service and (k) the time preceeding crane service. The parameters e_1, e_2, e_3, e_4 are defined as

e_1 = weight of FeS per ladle of input

e_2 = weight of Cu_2S per ladle of input

e_3 = number of tons of flux required per ton of FeS

e_4 = average temperature of input material

The model of temperature change is a simple linear extrapolation and assumes there is no change in bath temperature due to radiation losses during crane service.

The above crane service state transition equations for the ith converter may be summarized by

$$\underline{x}(k+1) = \underline{x}(k) + \Delta\, \underline{x}(\underline{e}, m_i) \qquad (3.3)$$

where $\underline{e}$ is a vector of model parameters defined above and m_i is the number of ladles of input material.

The number of ladles of material to be removed is a function of the amount of material to be removed and the ladle capacity. For the ith converter this may be expressed by

$$n_i = \left[\frac{x_4(k)}{W}\right]^+$$

for the slag and by

$$n_i = \left[\frac{(-.8)\,x_2(k)}{w} \right]^+$$

for the finish phase, where:

x_4 (k) = amount of slag at end of slag blow

x_2 (k) = amount of Cu_2S at the start of the finish blow

w = weight capacity per ladle (assume slag and copper are of equal density)

and where $[\,\cdot\,]^+$ denotes the nearest equal or greater integer.

The crane service time between any two consecutive blows is, of course, a function of the routing used in performing service. If a converter is serviced exclusively (i.e., the crane is not shared with another converter) then the service time is given by

$$TS(i,j) = (t_{io} + t'_{oi})\ n_i + (t_{oi} + t'_{io})\ m_i \qquad (3.4)$$

where TS(i,j) is the time to service the j^{th} blow on the i^{th} converter.

This modeling of the crane-converter interaction completes a model of the crane subsystem.

Converter Furnace Model

We recognize from Chapter II that the activities which characterize the behavior of the converter furnace involve two major reactions. The slag phase is characterized by the elimination of iron sulfide from the matte charge as governed by the following chemical equations:

$$2FeS + 3O_2 \rightarrow 2FeO + 2SO_2 \qquad \Delta H = -105{,}000 \text{ Cal/mole}$$

$$2FeO + S_iO_2 \rightarrow 2FeO \cdot S_iO_2 \qquad \Delta H = -306 \text{ Cal/mole} \qquad (3.5)$$

$$6FeO + O_2 \rightarrow 2Fe_3O_4 \qquad \Delta H = -133{,}000 \text{ Cal/mole}$$

$$3Fe_3O_4 + FeS \rightarrow 10FeO + SO_2 \qquad \Delta H = +110{,}000 \text{ Cal/mole}$$

where ΔH is the heat of reaction at molten temperatures. The second phase is described by the reaction

$$Cu_2S + O_2 \rightarrow 2Cu + SO_2 \qquad \Delta H = -56{,}000 \text{ Cal/mole} \tag{3.6}$$

In practice, these equations have been used for many years to establish material balance and heat balance relationships. These equations do not, however, offer any information regarding the behavior of the process in the course of time. It is an interest in controlling the processing time of the converter furnace that motivates us to develop a dynamic model which will predict the value of the state variables at any time. Defining the slag phase state variables as in Chapter II, we wish to formulate a slag phase model using oxygen input rate as the only continuous control.

During each blow, the process involves only continuous variables, so that it may be modeled as a vector differential equation of the form

$$\frac{d\underline{x}}{dt} = \dot{\underline{x}} = \underline{f}(\underline{x}, u) \tag{3.7}$$

where $\underline{x}$ is the state vector defined in Chapter II and u is the oxygen input rate. The function $\underline{f}$ is in general a nonlinear function of the arguments $\underline{x}$, u and it is assumed to be stationary. Because of limited theoretical knowledge about the dynamics of copper converter furnaces, we will approximate the function $\underline{f}$ with an empirical, stationary, linear function. We offer no experimental justification for such a simple model but use it to illustrate a modeling technique and to serve as a simple model in optimization examples. While the assumption of linearity may seem unjustified, it does have practical merit. A non-linear system such as Equation 3.7, operating "near" a known trajectory $\{x_o(t), u_o(t)\}$ may be linearized by the expansion.

$$\underline{x} \approx \underline{f}(\underline{x}_o, u_o) + \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{\underline{x}_o, u_o} (\underline{x} - \underline{x}_o) + \left.\frac{\partial \underline{f}}{\partial u}\right|_{\underline{x}_o, u_o} (u - u_o)$$

Evaluating the partial derivatives as indicated and assuming equality yields an equation of the form

$$\dot{\underline{x}} = \underline{\underline{A}}(t)\underline{x} + \underline{B}(t)\underline{u} + \underline{C}(t)$$

This equation represents a nonstationary linear system. A further approximation to this system is made by assuming the system is stationary over an interval of time. For the slag phase model, we assume the system is stationary over all of the blows, thus yielding an approximate model with form

$$\dot{\underline{x}} = \underline{\underline{A}}\underline{x} + \underline{B}u + \underline{C} \qquad (3.8)$$

where A, B, and C are constant matrices which will be evaluated.

To further simplify the model, we assume that x_2 (Cu_2S weight) is constant during slag blows and that all other bath constituents vary in proportion to the FeS oxidation rate. (These assumptions are supported by observation of converter data.) With these assumptions, the slag blow state equations become

$$
\begin{aligned}
\dot{x}_1 &= A_{11}x_{.} + A_{12}x_2 + \ldots + A_{16}x_6 + B_1u + C_1 \\
\dot{x}_2 &= 0 \\
\dot{x}_3 &= K_3\dot{x}_1 \\
\dot{x}_4 &= K_4\dot{x}_1 \\
\dot{x}_5 &= A_{51}x_1 + A_{52}x_1 + \ldots + A_{56}x_6 + B_5u + C_5 \\
\dot{x}_6 &= K_6\dot{x}_1
\end{aligned}
\tag{3.9}
$$

The unknown constants in Equation 3.9 will be determined empiracally based upon observation and estimation of converter state trajectories. Direct measurements of many of the state variables associated with the converter furnace are difficult to obtain. Consequently, some of the data is obtained indirectly, such as through the use of material balance relationships. For example, smoothed trajectories of the state variables are shown in Figure 8 for a typical converter furnace with fixed oxygen input rate during each blow. The original data was obtained by direct measurement of SO_2 discharge rate, oxygen input rate, temperature, and initial material inputs and by computation of the trajectories of the concentration of FeS and weights of flux, slag, and Fe_3O_4 using the reaction Equations 3.5. Denote the data points in Figure 8 by $x_k(t_{ij})$ and the slope at each data point as $\dot{x}_k(t_{ij})$, where k denotes the kth state variable and t_{ij} the ith discrete time in the jth slag blow.

To evaluate the constants, in particular the B_k coefficients, we require a collection of data for different values of u. However, using only the data in Figure 8, a very simple initial model can be obtained by arbitrarily choosing

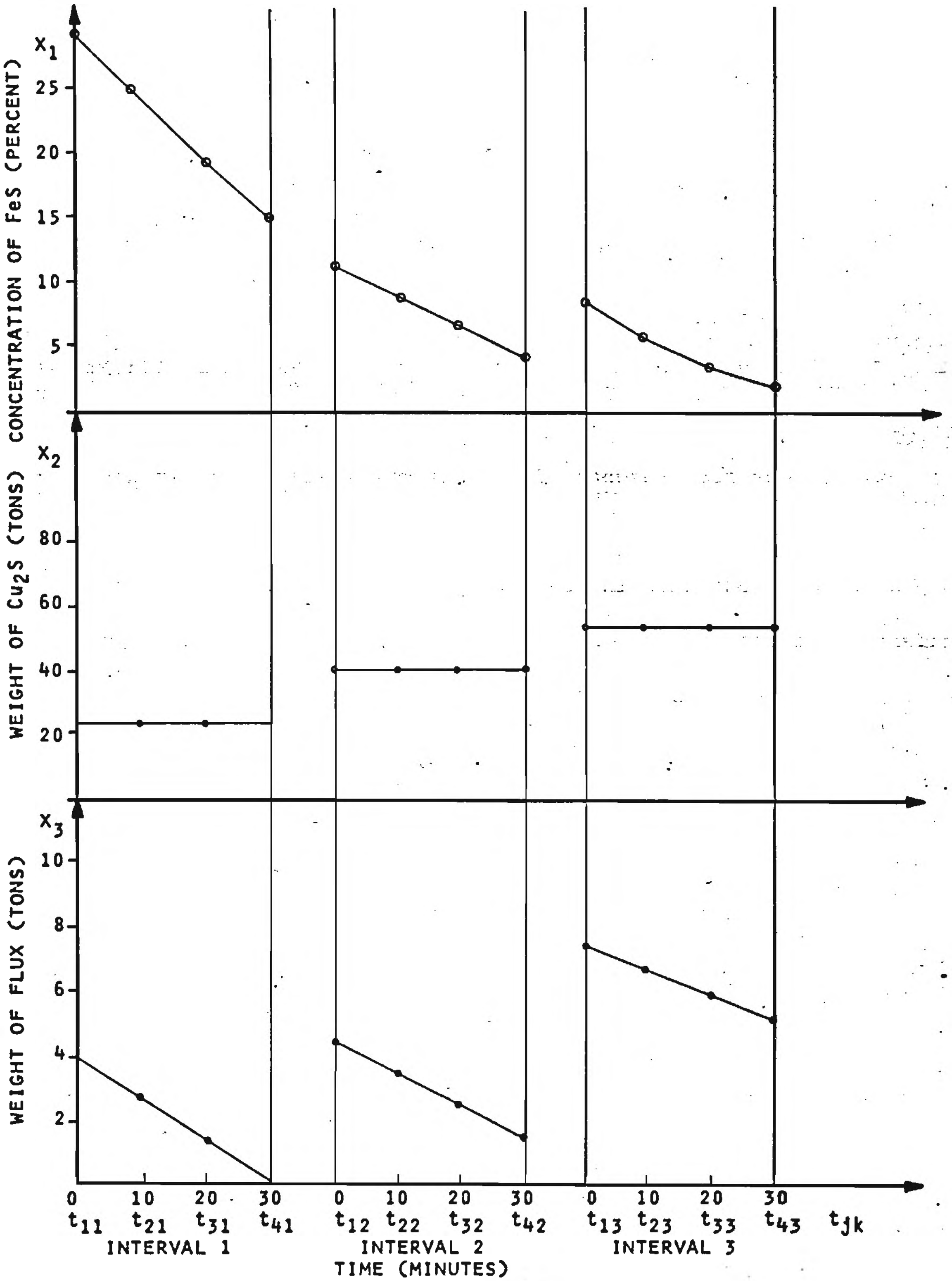

Figure 8 EXAMPLE STATE VARIABLES (SLAG PHASE)

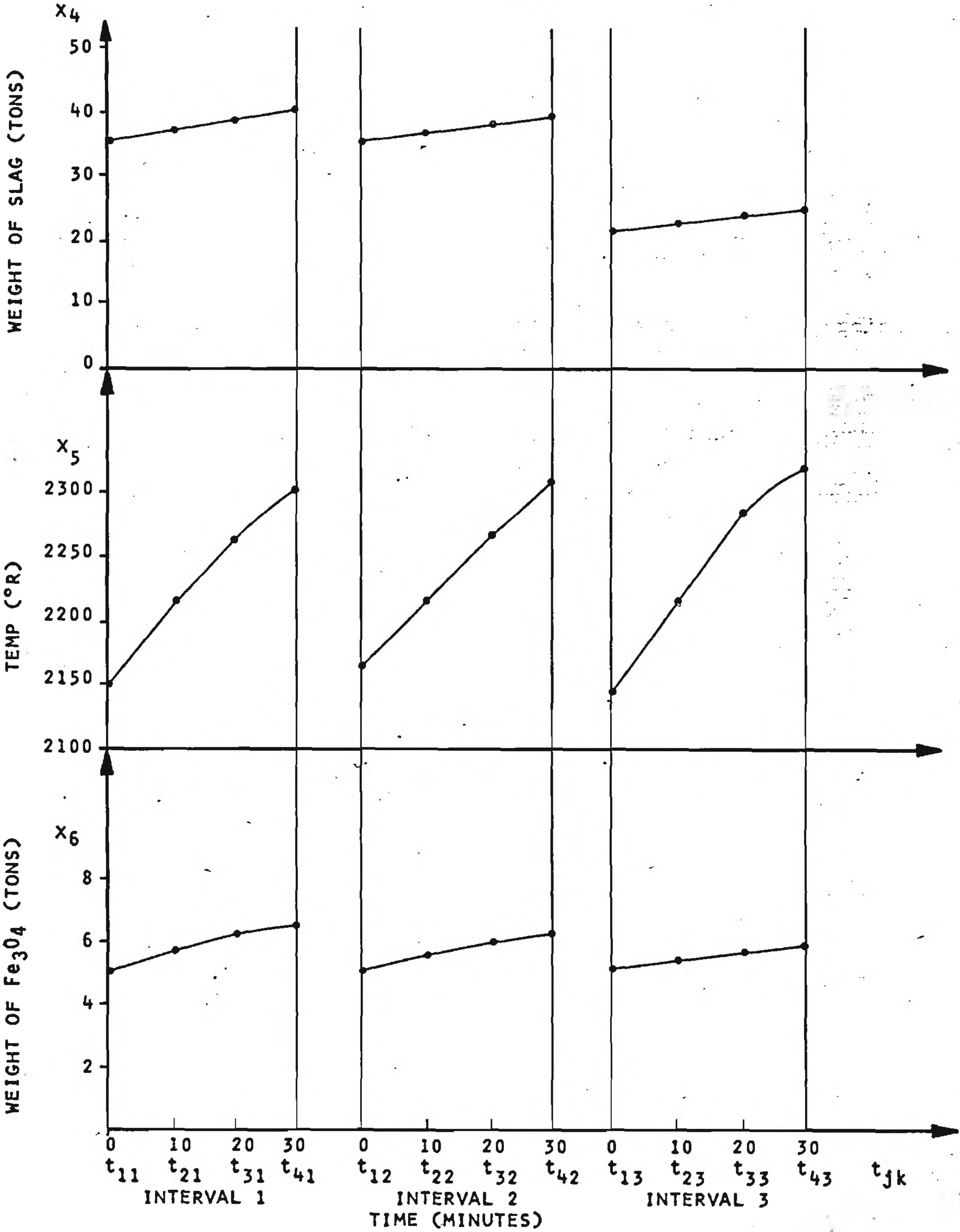

Figure 8 (cont) EXAMPLE STATE VARIABLES (SLAG PHASE)

$$B_k = \frac{\overline{\dot{x}_k(t_{ij})}}{\overline{u(t_{ij})}}, \; k = 1, 5 \qquad (3.10)$$

where the superscript bar denotes the average over all sample times t_{ij}. Equation 3.10 has some physical justification in that oxygen input rate is dominant in determining FeS oxidation rate. To evaluate the remaining constants, arrange the data in the matrix forms

$$\underline{\underline{X}} = \begin{bmatrix} x_1(t_{11}) & x_2(t_{11}) & \cdots & x_6(t_{11}) & 1 \\ x_1(t_{21}) & x_2(t_{21}) & \cdots & x_6(t_{21}) & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ x_1(t_{43}) & x_2(t_{43}) & \cdots & x_6(t_{43}) & 1 \end{bmatrix}$$

and

$$\underline{Y}_k = \begin{bmatrix} x_k(t_{11}) - B_k u(t_{11}) \\ x_k(t_{21}) - B_k u(t_{21}) \\ \vdots \\ x_k(t_{43}) - B_k u(t_{43}) \end{bmatrix}, \; k = 1, 5$$

For a least sum-square-error fit of the data,

$$\underline{Y}_k = \begin{bmatrix} A_{k1} \\ \cdot \\ \cdot \\ \cdot \\ A_{k6} \\ \\ C_{1k} \end{bmatrix} = \left[\underline{\underline{X}}^t \underline{\underline{X}}\right]^{-1} \underline{\underline{X}}^t \underline{Y}_k \ , \ k = 1,5 \tag{3.11}$$

where superscript t denotes transpose. The constants K_3, K_4, and K_6 in Equation 3.9 may be estimated as

$$K_k = \overline{\frac{\dot{x}_k(t_{ij})}{\dot{x}_1(t_{ij})}} \ , \ k = 3, 4, 6 \tag{3.12}$$

To complete the model, the system constraints are simply

$$\begin{aligned} &0 \leq u(t) \leq u_{max} \\ &0 \leq x_k(t) \leq X_{k\ max} \ , \ k = 1,\ldots,6 \end{aligned} \tag{3.13}$$

where U_{max} and $X_{k\ max}$ are fixed constants.

Together, Equations 3.9-3.13 constitute a simple initial slag blow model. The model has validity only during the blows and only for state and control values in the range of data used to derive the model coefficients. Between slag blows, oxidation stops and the converter state changes in accordance with equation 2.3 as previously discussed. Thus, between blows, the crane actions determine the change in the converter state. When a sufficient amount of Cu_2S has been accumulated and when the FeS has been removed, the finish phase is started.

A finish phase model can be developed in manner similar to that used for the slag blow model, but with appropriate changes in the state variables. A typical state trajectory for the finish phase is shown in Figure 9. The temperature during the finish phase remains relatively constant for a given oxygen input rate. As expressed by Equation 3.6, there is only one principle reaction involved, thus the state vector will be approximated by a scalar as

$$x_2 = Cu_2S \text{ weight} \tag{3.14}$$

As in the slag phase model, the finish phase model may be expressed as a differential equation of the form of Equation 3.7.

$$\frac{dx_2}{dt} = f(x_2,u) \tag{3.15}$$

where f is to be determined. The curve of Figure 9 suggests a non-linear exponential form for the function f

$$f(x_2,u) = 2\ \alpha\ ute^{-\alpha\ ut^2} \tag{3.16}$$

where α is a constant to be determined and t is the independent variable time. The choice of an exponential function is based simply on the fact that data best fit this form of curve. Clearly, the choice of f is not unique but represents a judgment based on observation and experience. Combining Equations 3.15 and 3.16, we have the finish phase model given by

$$\dot{x}_2 = -2\ \alpha\ ute^{-\alpha ut^2} \tag{3.17}$$

the constant α can be determined by solving equation 3.15 and fitting the solution to the data in Figure 9 using a standard curve fitting tech-

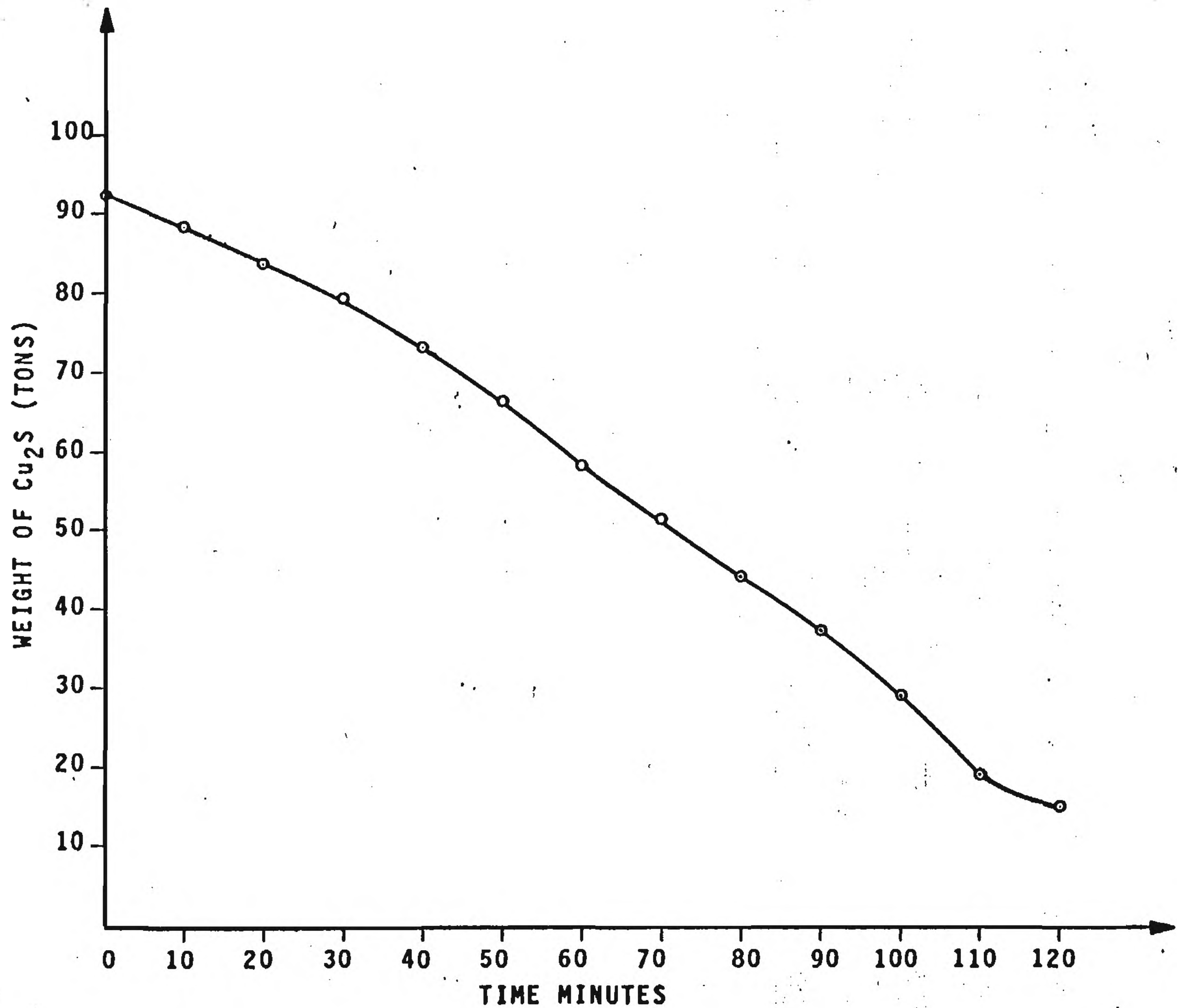

Figure 9 EXAMPLE STATE VARIABLES (FINISH PHASE)

nique, similar to the least squares method used in modeling the slag phase.

The model illustrated in this section is, of course, quite simple, and much remains to be done in modeling of copper conversion dynamics. Towards this objective, more complex models have been considered, for example, Nenonen and Paqurek [8] and Niemi and Koskinen [9]. However, this simple model will provide a basis for the optimization concepts which follow.

CHAPTER IV

SIMULATION STUDIES

In this chapter a simulation of the simple, single crane, three converter aisle shown in Figure 5 is described. The purpose of the simulation is twofold; first, to validate the model, and second, to provide the motivation for studying the converter aisle as an optimization problem. In the simulation study opportunities for optimal control are identified and a measure of possible improvement in performance is established. The simulation program is briefly described, some results are presented, and a summary of promising optimization opportunities is given.

Simulation Program

This simulation study is conducted with the use of an IBM 1800 Process Control Digital Computer and a FORTRAN based simulation program called GASP II (General Activity Simulation Program) [15]. Two principle activities of the converter aisle system are identified as crane service and processing of material by the converters (blowing). The crane service activity is performed by an entity of the system, namely the crane, while the blowing activity is performed by another entity, the converter furnace. Each of these entities have certain attributes which characterize their behavior as shown in Table 3.

Clearly a mixture of continuous and discrete event models is found at the boundary between the crane subsystem and the converter furnace subsystem. Two types of events are specified. A type 1 event is

TABLE 3

ENTITIES AND ATTRIBUTES OF THE CONVERTER AISLE SYSTEM

ENTITY	ATTRIBUTE
CRANE	Transit time, load and unload time: Position Ladle Status
CONVERTER	Material Capacity Processing Rate. Minimum blow duration Current state

defined at the endpoint of a trajectory which satisfies the differential equations of the converter furnace model (i.e., event type 1 occurs at the end of a blow). This event represents a discrete decision point. The type 2 event is defined at the completion of crane service. This event represents a continuous decision point since it is necessary at this event time to decide oxygen input rate for the blow that is to follow. Figure 10 illustrates the event model used in the simulation.

Since there are three converters operating simultaneously, the program maintains an event file as shown in Table 4. The time a particular event will occur is stored in the event time. On each iteration in the program, the smallest event time is determined and the event code, converter number and blow number are removed from the event file. At this point, another file is accessed to determine the duration of the activity associated with the event code. The file containing the activity duration is the activity file as illustrated in Table 5. The program (EVNTS) which stores and retrieves information from these files is shown in Appendix 1.

The event code determines which one of two model subprograms is to be executed. If the event code is 1, then the event marks an end of blow, and crane service is called for. The crane service subprogram (CRNSV; appendix 1) calculates the change in state due to the action of the crane and also the duration of the crane service activity. It is assumed that once crane service has been started on a converter, it will continue until the needs of that converter have been satisfied (i.e. time-sharing of the crane is not allowed). Thus the crane service mod-

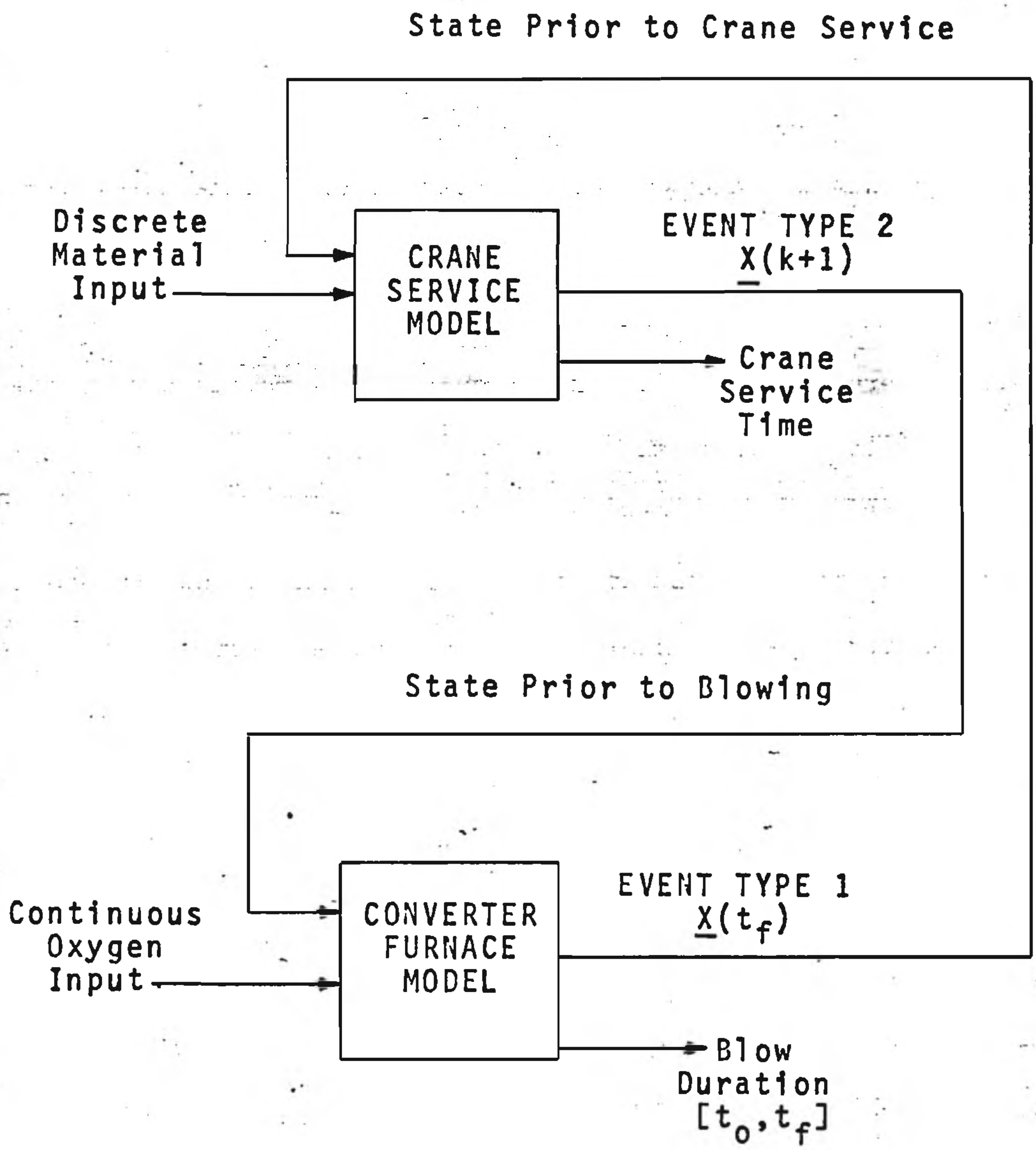

Figure 10 SYSTEM EVENT MODEL

TABLE 4

TYPICAL EVENT FILE

EVENT TIME	EVENT CODE	CONVERTER NUMBER	BLOW NUMBER
0.0	1	1	3
50.0	2	2	4
75.0	1	3	1

TABLE 5

TYPICAL ACTIVITY FILE

ACTIVITY DURATION	CONVERTER NUMBER
70.0	1
23.0	2
50.1	3

el is given by Equations 3.3 and 3.4. The time for the next event on the particular converter is calculated by adding the activity duration to the current time. The event code is changed to an end of crane service and a blow is initiated. The converter furnace subprogram (CONVT; appendix 1) calculates the change in state due to the activity of the converting process. The converter furnace model as given by Equations 3.9 for the slag activity duration is the time required for the converter to transfer the state from the initial condition at the end of crane service to a specified terminal state constrained by

$$x_i(t_f) \geq 0 \text{ for } i = 1, 2, 3, 4, 6$$

$$x_5(t_f) \leq 2250°F$$

Violation of any of the above terminal constraints will terminate the blow simulation. In addition, the last slag blow and finish blow have special terminal conditions. At the end of the final slag blow $x_1(t_f) = 0$ and at the end of the finish blow $x_2(t_f)/x_2(0)$ is specified.

The model parameters and assumed oxygen input rates are found in Appendix 2.

Simulation Results

Figure 11 shows some sample state trajectories obtained from a solution of the slag phase model and a comparison of interval 1 trajectories from the model with interval 1 trajectories from observed data. Figure 12 illustrates a comparison of the state trajectory obtained from the solution of the finish phase model and the state trajectory from actual observation. The simulation results and actual results are not directly comparable because some of the parameters associated with the

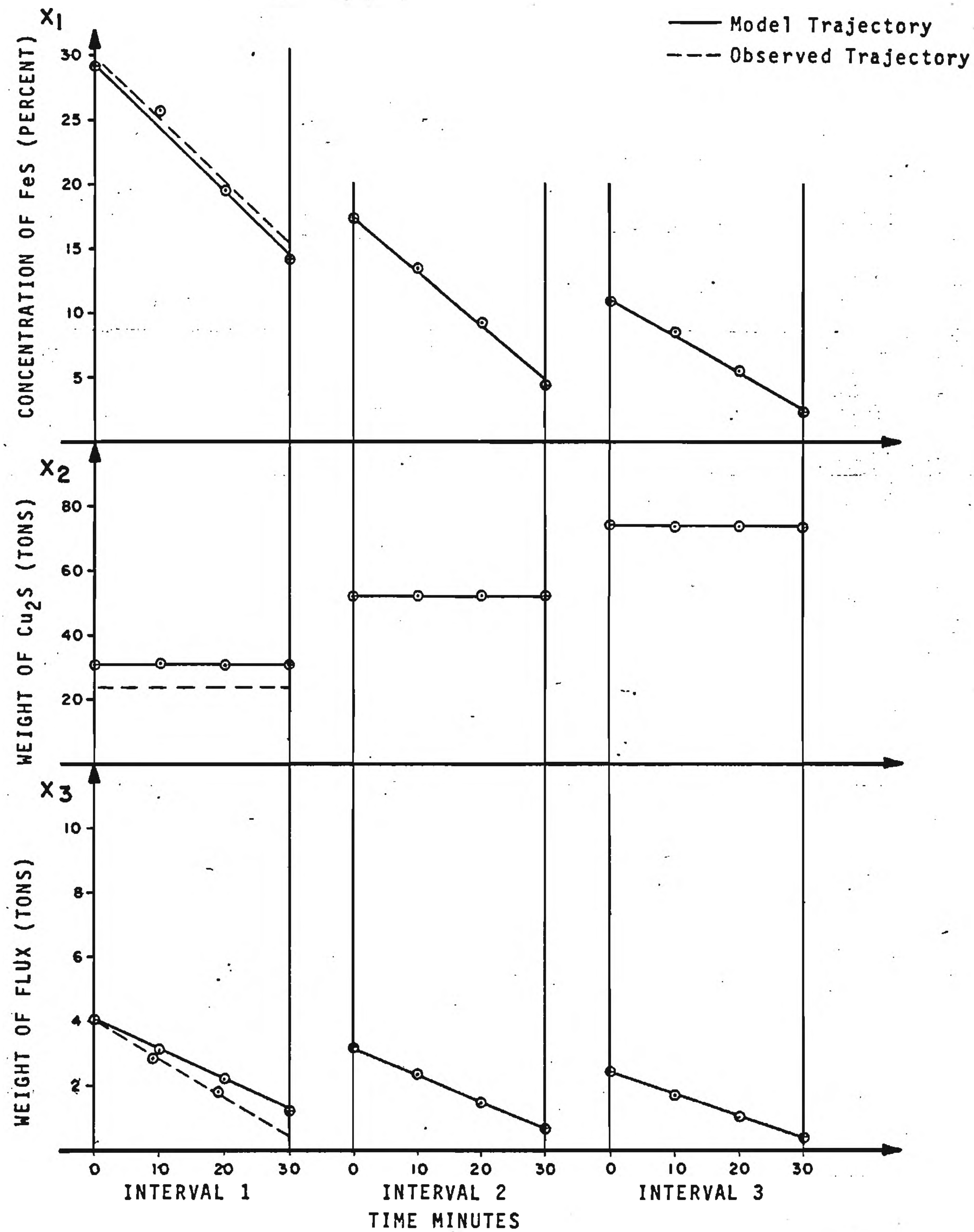

Figure 11 STATE VARIABLES (SLAG PHASE) COMPUTED FROM MODEL

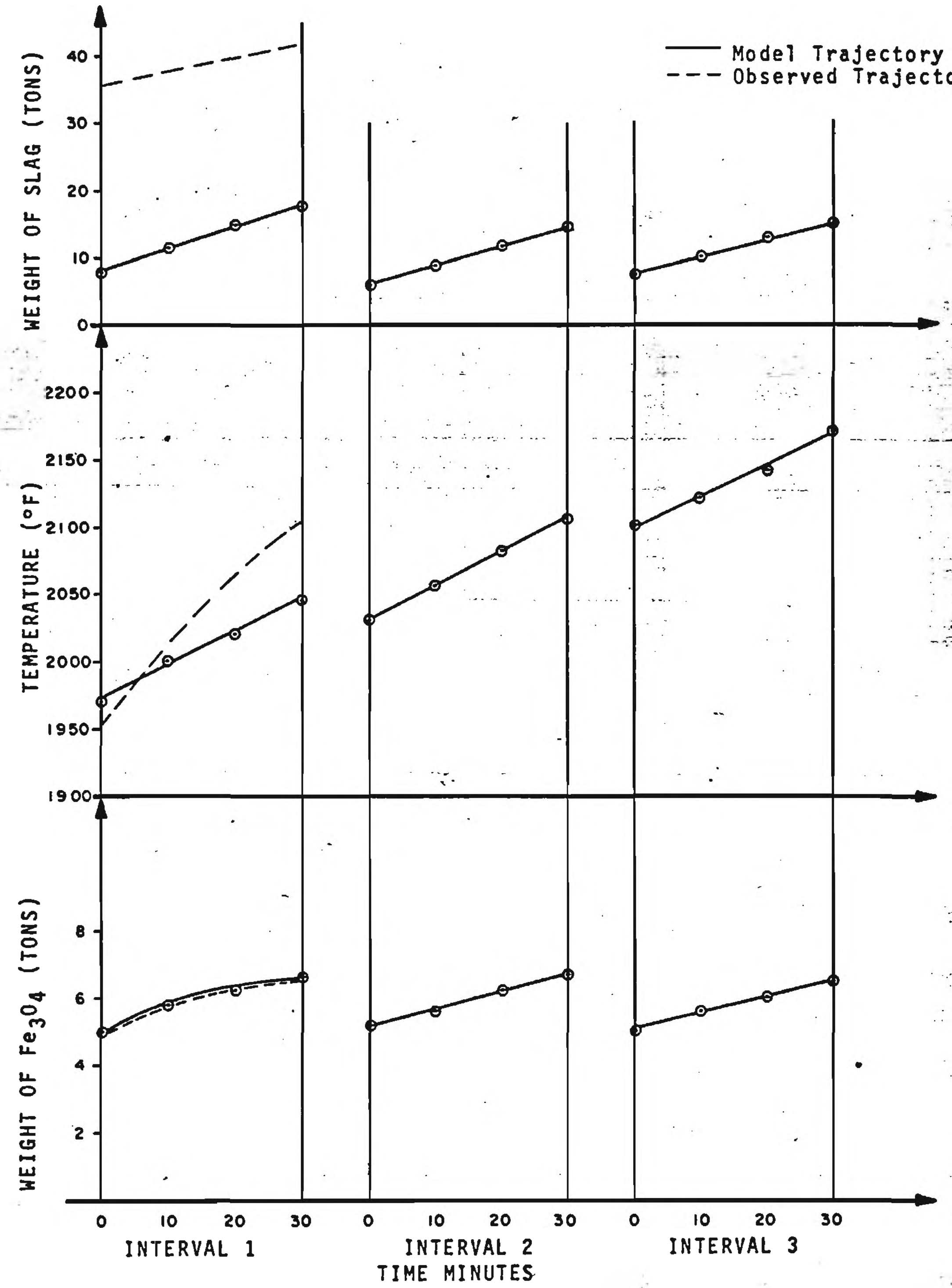

Figure 11 (cont) STATE VARIABLES (SLAG PHASE) COMPUTED FROM MODEL

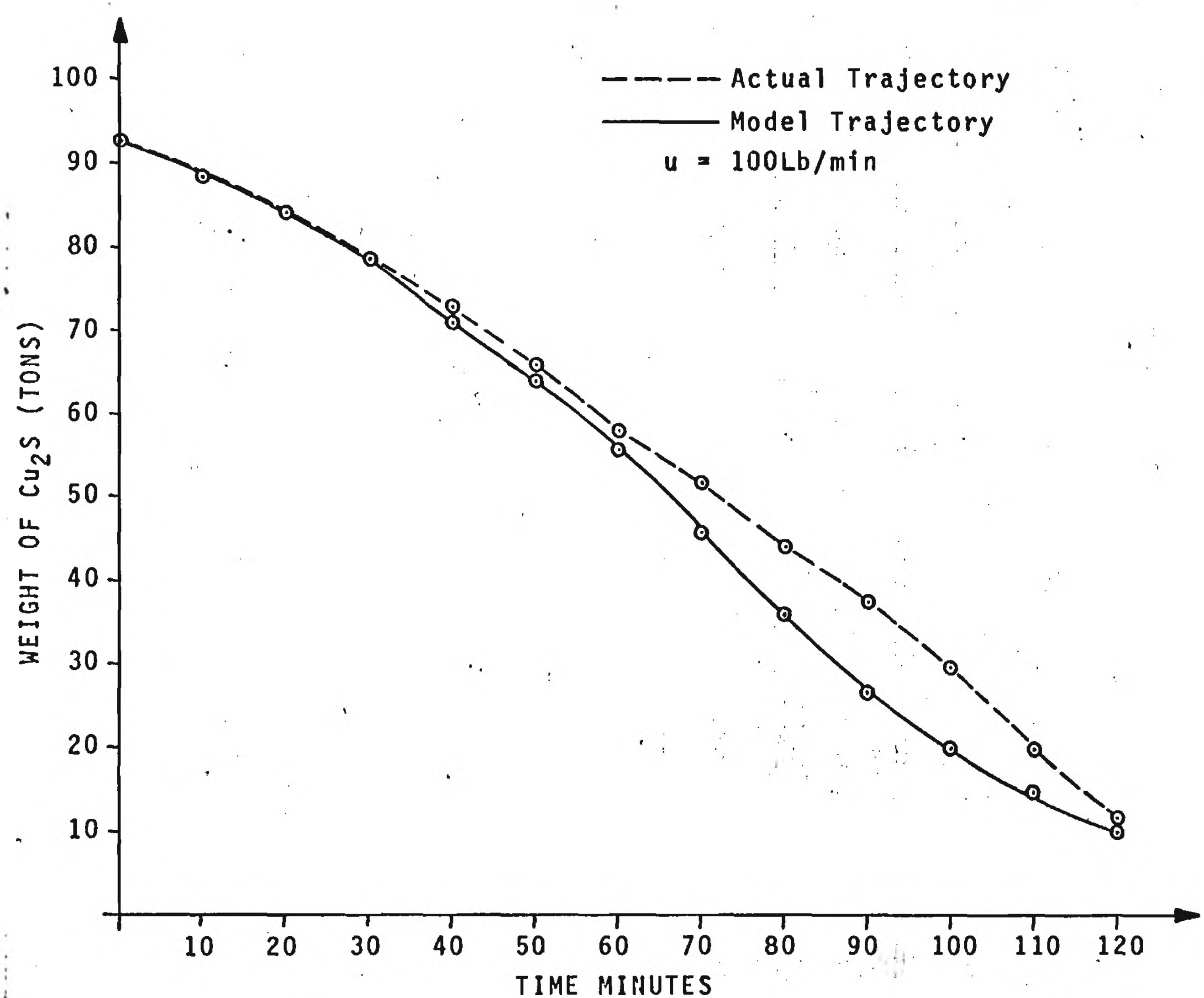

Figure 12 STATE VARIABLES (FINISH PHASE) CALCULATED FROM MODEL

actual data are unknown and thus are assumed for the model. These assumptions lead to some differences in initial conditions between the actual data and model results as observed in Figure 11. However, comparisons made between actual observation and model results showed considerable similarity; and, therefore, this model will serve as a nominal starting point for further optimization studies.

The results of six simulation runs, each with a different assumption or initial condition show a distinct difference in process performance. The model parameter values for each run are shown in Appendix 2. The first two simulation runs are designed to illustrate a difference in process performance due to two different material input decisions as given in Table 6. The results of this experiment are shown in Table 7. This variation in material input decision shows an improvement in copper production rate of 10.4 percent. This is primarily a result of the smaller number of conflicts for crane service in run 2.

Runs 3 and 4 illustrate the difference in performance for two different initial starting conditions as given in Table 8. No significant difference in performance was detected for these particular initial conditions, however, the number of conflicts for crane service on each converter varied considerably.

Run 6 illustrates the performance assuming infinite crane service flexibility (i.e. the crane is always available when a converter requires service). The initial conditions are as in run 3. Table 9 shows a comparison between run 3 and run 6.

A production rate improvement of 13.1 percent is obtained by assuming infinite crane flexibility. An improvement this large may not be

TABLE 6

MATERIAL INPUT DECISIONS

BLOW	RUN 1 LADLES OF MATTE	RUN 2 LADLES OF MATTE
1	5	4
2	4	3
3	3	3
4	2	3
5	1	2
6	1	1

TABLE 7

RESULTS OF VARYING MATERIAL INPUT DECISIONS

Run	Average Production Rate Tons/Hour	Average Crane Service Time Minutes	Average Blow Duration Minutes	Average Duration of Wait Minutes/Hour	Average No. of Conflicts per Charge Cycle
1	5.78	35.4	113.8	1.0	8
2	6.4	36.6	109.8	0.71	6

TABLE 8

INITIAL STARTING CONDITIONS

	Converter	Event Time	Event Code	Blow Number	Activity Duration (Min.)
	1	0.0	2	2	50
Run 3	2	40	2	6	60
	3	45	1	3	58
	1	0.0	2	5	50
Run 4	2	40.0	2	6	60
	3	45.0	1	3	58

TABLE 9

RESULTS OF INFINITE CRANE SERVICE

Run	Average Production Rate Tons/Hour	Average Crane Service Time Minutes	Average Blow Duration Minutes	Average Duration of Wait Minutes/Hour	Average No. of Conflicts per Charge Cycle
3	6.5	35.2	115.8	6.46	21
6	7.35	34.8	94.3	0.0	0

feasible in practice, however, because a no-conflict crane schedule will require increasing blow durations in some cases to avoid conflicts.

Optimization Opportunities

Summarizing the above results, we observe that at least four possible optimization problems may be identified. These include:

1. Optimum crane routing so as to minimize the time a converter must wait for crane service.

2. Optimum scheduling of blow durations on each converter so as to maximize the copper production rate on that converter.

3. Optimum scheduling of activities so as to minimize the number of conflicts for crane service.

4. Control of the converter furnaces in accordance with blow duration specified by the crane schedule.

In the next chapter, these problems are clearly defined and several optimization techniques which may be applied are presented together with some simple illustrative examples.

CHAPTER V

OPTIMIZATION

If it were not for the mixed continuous control variables and discrete decisions with unspecified times of occurrence, optimization of the aisle system would be relatively straight forward. As is, the problem requires modification of the model or modified application of standard optimization techniques. In the following sections, we focus on two general approaches to aisle optimization. First, for direct total system optimization, continuous variables are quantized in time and combined with discrete decision variables to form one large discrete time model which is theoretically amenable to solution; for the second approach, the primary optimization problem is partitioned into a number of smaller, less complex optimization subtasks and these are related to total system optimization.

Because the problem for the direct approach is so large and complex that it is unwieldy, it is not solved. Instead the partitioned method is developed to the point where a solution can be achieved.

Direct Optimization Approach

A number of optimization techniques apply to large discrete time (or discrete state) models. The aisle model given in Chapter III can be converted to a single combined model with this form. Let Δ be a time increment such that time duration of any blow or any crane movement can be represented as an integral multiple of Δ. A state transition equation for the ith furnace takes the general form:

$$\underline{x}^i_{j+1} = \underline{g}_i(\underline{x}^i_u, \underline{p}_j, u^i_j) \tag{5.1}$$

where j = 0, 1,... corresponds to t = 0, Δ, 2Δ,...; $\underline{x}^i_j$ and u^i_j are the ith furnace state and control at time j, and p_j is the crane state at time j (which defines the crane service at time j); and u_j is assumed to be constant for $j\Delta \leq t \leq (j+1)\Delta$.

The functions $\underline{g}_i$ are obtained by integrating Equation 3.7, and adjusting the state to account for possible crane service. Of course, $\underline{p}_j$ only affects $\underline{x}^i_{j+1}$ if the crane services furnace i at time j. Note that by including $\underline{p}_j$ as an argument in $\underline{g}_i$, Equation 5.1 models the furnace for the complete cycle including the times of the crane visit.

Similarly, the crane transition equation can be written as

$$\underline{p}_{j+1} = \underline{h}(\underline{p}_j, d_j) \tag{5.2}$$

where $\underline{p}_j$ and d_j are the crane state and transition decision as defined in Chapter III, except for the altered choice of discrete times. Since the discrete times are not restricted to the times of the crane visits, $(p_1)_j$ assumes the discrete states 0, 1, 2, 3 described in Chapter III, and an additional value corresponding to "crane in transit." Of course, the crane can change state only at times when it has completed a previously initiated move.

Combining Equations 5.1 and 5.2, the total aisle system state transition equation takes the form:

$$\underline{z}_{j+1} = \underline{g}(\underline{z}_j, \underline{v}_j) \tag{5.3}$$

where $\underline{z}_j$ is the combined state,

$$\underline{z}_j = \begin{bmatrix} \underline{x}_j^1 \\ \underline{x}_j^2 \\ \underline{x}_j^3 \\ \underline{p}_j \end{bmatrix}$$

$\underline{v}_j$ is the combined control,

$$\underline{v}_j = \begin{bmatrix} u_j^1 \\ u_j^2 \\ u_j^3 \\ d_j \end{bmatrix}$$

and $\underline{g}$ is given as

$$\underline{g}(\underline{z}_j, \underline{v}_j) = \begin{bmatrix} \underline{g}_1(x_j^1, \underline{p}_j, u_j^1) \\ g_2(\underline{x}_j^2, \underline{p}_j, u_j^2) \\ g_3(\underline{x}_j^3, \underline{p}_j, u_j^3) \\ \underline{h}(p_j, d_j) \end{bmatrix}$$

The system constraints and the performance criterion may be readily expressed in terms of $\underline{z}_j$ and $\underline{u}_j$. Let us now examine the magnitude of the optimization task.

The model, Equation 5.3, still has mixed control variables with continuous and discrete values. This can be treated, but for simplicity assume that each control u_j^i (oxygen input rate) is quantized into γ values. The control d_j has five possible values; but these are not permissible at each time j; hence, assume that there are effectively ν crane decisions possible at each time. Assume that the total production cycle lasts N time intervals. If an initial state were specified, there would be $(3\gamma + \nu)^N$ possible controls -- a number which easily becomes unwieldly. To compound matters, the initial state is not known. Specification of the initial state is equivalent to specification of the staggering of charge cycles, which need not be known.

In spite of the enormity of the problem, direct optimization may be feasible. We comment on three techniques.†

<u>Dynamic programming</u>.-- This is a highly efficient technique for solving many N-stage decision processes [2]. The technique is illustrated later using a much smaller process model. The computational saving afforded by dynamic programming is that the number of controls to be evaluated is proportional to $(3\gamma + \nu)N$ rather than $(3\gamma + \nu)^N$, again assuming an initial state is specified. Larson [6] illustrates a further computational savings over direct application of dynamic programming, for the case in which a nominal starting state can be specified.

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†Because of the size of the total aisle model, no examples are presented in this section, but optimization techniques are illustrated in the partitioning approach.

Rather than determining the optimal control for each possible starting state, only initial states "near" the nominal starting state are examined.

Gradient search.-- Gradient search in the space of control sequences $\{\underline{v}_1,\ldots,\underline{v}_N\}$ is normally a very simple technique for control calculation [7, 10], but it requires that the system performance J vary continuously with each control variable $(v_k)_j$ and that each control vector $\underline{v}_j$ be independent of the value $\underline{v}_i$ of the control at any other time. Unfortunately, for the formulation of the model in this section, the control does not meet these requirements. Thus, the gradient technique is not directly applicable for determination of the optimal aisle control. On the other hand, the gradient technique may be applicable for searching on values of the initial state, for use in conjunction with some other technique for determining an optimal control with an initial state specified.

Random search.-- It has been shown, for example [4,14], that as the number of decision variables in a problem becomes large, a random search may become more efficient than a gradient search. Intuitively, the reason for this is the gradient must be computed for each decision variable whereas in the same time, the random search is likely to have found an improved value of the control. Further, control constraints do not seriously hamper a random search because non-allowed controls are merely ignored. Thus, random search may be a candidate for direct aisle optimization. Lest the random approach seem unjustified, we note that it is the basis for EVOP [3], an "evolutionary optimization" scheme which has been applied to control of chemical processes.

Optimization by Partitioning

Some of the difficulties encountered in the direct optimization approach may be solved by use of the principle of invariant embedding [12], according to which a very difficult or unsolvable problem is embedded into a class of simpler, solvable problems. For aisle optimization, we consider partitioning the problem into a number of more simple subproblems which relate to the overall objective. The structure of the aisle problem suggests a possible hierarchy of control subproblems as listed below:

1. Given material requirements of the converters, find an optimum crane routing which will minimize the service time for each converter service.

2. Given material input decisions for a converter, find the minimum converter blow durations to minimize the total charge cycle time.

3. Given fixed crane service times and minimum blow durations for each converter, find an optimum converter schedule which will minimize the charge cycle time for each converter.

4. Given a schedule of converter services, find optimum continuous inputs (e.g., oxygen input rate) to each converter furnace which will achieve that schedule and satisfy the physical constraints of the converting process.

A systematic structuring of these four problems is shown in Figure 13. Of course, this structure is not unique, but it does have the following advantages:

1. The complexity of each subproblem is less than that of the total problem.

2. The control variables in each subproblem are of one type (i.e., discrete or continuous).

3. The subproblems can be implemented separately, thus allowing opportunity to evaluate the results of one problem before proceeding to the next.

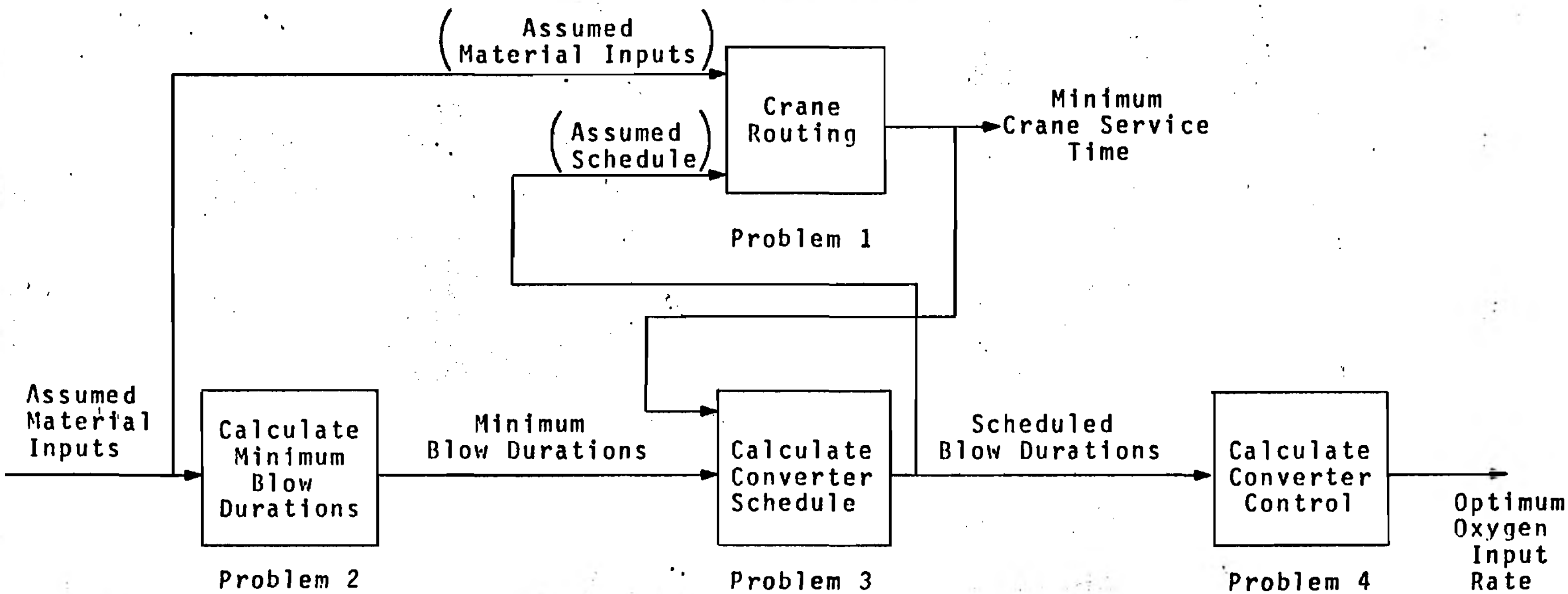

Figure 13 PARTITIONED OPTIMIZATION PROBLEM

The primary disadvantage to this approach comes with the problem of composing a total system solution from the solutions to the subproblems. In the following subsections a method for solving each of the above subproblems is presented. Then, in Chapter VI, numerical examples are given.

Crane routing.-- Let a converter aisle material handling process be characterized by the network† shown in Figure 14. The nodes in the network represent crane locations p_1 and the numbers on the branches represent the ladle state during transit and the transit times as given in Tables 10 and 11.

The crane routing problem is: given requirements for converter services, determine the sequence of crane moves to meet the requirements in minimum time for each converter. Clearly, if only one converter needs service, the crane routing is trivial and the service time is immediately determined. If several converters need service simultaneously, it may be more efficient to service them jointly rather than sequentually. This problem arises if either (i) several converters have been scheduled for simultaneous service, or (ii) in actual operation of an aisle, random fluctuations cause several converters to need services at the same time.

The performance criterion of the total system was given by Equation 2.1 as

$$J = \sum_{i=1}^{3} \frac{Cu_i}{T_i}$$

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†Figure 14 is not a state-transition diagram because the states are partially associated with the branches.

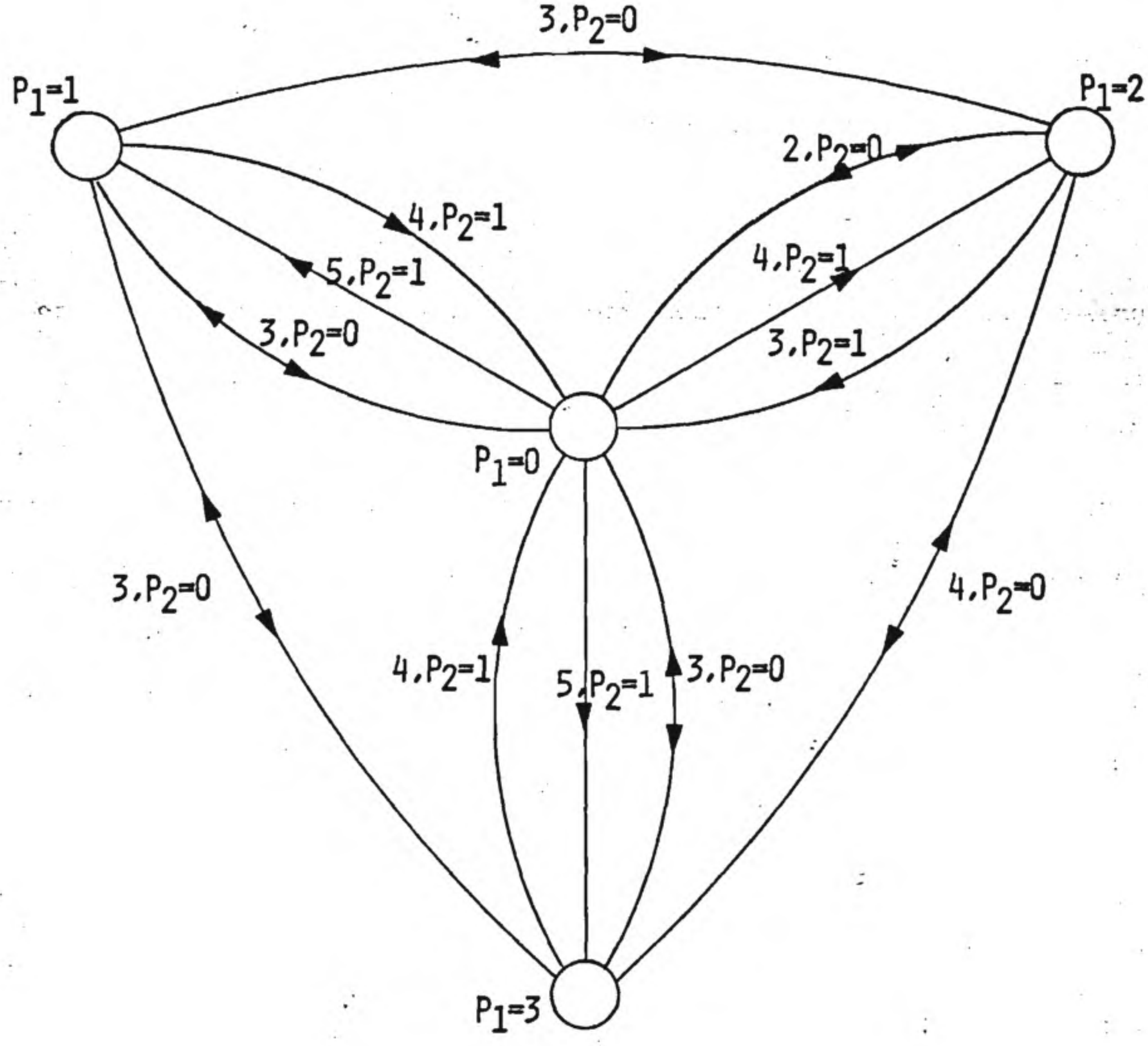

Figure 14 CRANE ROUTE NETWORK

TABLE 10

TRANSITION TIMES FOR $(p_2)_j$ = 1, LADLE LOADED

State $(p_1)_{j+1}$ / $(p_1)_j$	0	1	2	3
0	-	5	4	5
1	4	-	-	-
2	3	-	-	-
3	4	-	-	-

TABLE 11

TRANSITION TIMES FOR $(p_2)_j$ = 0, LADLE EMPTY

State $(p_1)_{j+1}$ / $(p_1)_j$	0	1	2	3
0	-	5	4	5
1	3	-	3	3
2	2	3	-	4
3	3	3	4	-

where the maximum of J is sought over all decisions and controls. Holding all decisions and controls fixed except the crane routing decisions d_k, $k = 0,\ldots, K$ required to meet a specified service requirement, the charge cycle time T_i may be decomposed as $T_i = T_i^* + T_i^S$ where T_i^* is fixed and T_i^S is the time required for the service to be completed. Because T_i^S is small compared with T_i^*, the performance objective can be approximated as

$$\max J \approx \underset{d_o,\ldots,d_K}{\text{maximum}} \sum_{i=1}^{3} Cu_i (T_i^* - T_i^S)$$

which is equivalent to

$$\max J \approx \underset{d_o,\ldots,d_K}{\text{minimum}} \sum_{i=1}^{3} Cu_i \, T_i^S + \text{constant}$$

This determines as a crane routing criterion to be minimized

$$J_K = \sum_{k=0}^{K} \sum_{i=1}^{3} Cu_i \, t_k \, \delta_{ik} \tag{5.4}$$

where t_k is the crane transition time for the kth crane move and $\delta_{ik} = 1$ if the ith converter requires service during the kth crane move and $\delta_{ik} = 0$ otherwise.

Formalizing the crane routing problem now: given an initial crane state $\underline{p}_o$ and initial service requirements $\underline{q}in_o$, $\underline{q}out_p$, determine crane routing decisions $d_o,\ldots,d_K$ such that $\underline{q}in_K = \underline{0}$, $\underline{q}out_K = \underline{0}$, $\underline{p}_K = \underline{0}$ and J_K is minimum.

The method of dynamic programming offers a systematic approach to problems of this type. The essential feature of this method is given by Bellman's "Principle of Optimality" [1,2]. A mathematical statement of the principle of optimality is the recursion formula

$$J_k^o = \min_{d_k} \; t_k + J_{k-1}^o \qquad (5.5)$$

where J_k^o is the optimal value of J_k, expressed as a function of the combined state ($\underline{q}in_k$, $\underline{q}out_k$, $\underline{p}_k$). In practice, the problem is solved using a technique of successive approximation. It is assumed that the optimal J_{k-1}^o is tabulated for each combined state under consideration at the (k-1)th step. For each state at the kth step, J_k^o is evaluated by searching over all tabulated states at the (k-1)th step and all values of the decision d_k. The procedure eliminates all routes to each state that have J_k greater than minimum for that state. Incrementing k, the optimal route will be determined if the final state is achieved for all possible routes or if all other partial routes have J_K^o greater than J_K^o for a route which achieves the final state.

To illustrate this technique, let initial vectors be $\underline{q}in_o = [2\ 2\ 0]^t$, $\underline{q}out_o = [0\ 2\ 0]^t$, $\underline{p}_o = [0\ 0]^t$. Since converter three is not involved, the third variable will be dropped from the queue vectors. The dynamic programming tabulation of J_k^o as a function of the combined state is shown in Table 12, where the eliminated rows were nonoptimal routes. For ease in reading the table, the states for each k value are labeled with lower case letters. The table is generated up to k = 10, at which stage the optimal route is determined. Tracing backwards through the table, the optimal sequence of states is marked with asterisks. The optimal sequence is

TABLE 12

DYNAMIC PROGRAMMING CRANE ROUTING EXAMPLE

k	State	p_k^t	$q_{in_k}^t$	$q_{out_k}^t$	j_k^0	$(p_1)_{k-1}$
0	a	[0 0]	[2 2]	[0 2]	0	
1	a	[1 1]	[1 2]	[0 2]	10	a *
	b	[2 0]	[2 2]	[0 1]	8	a
2	a	[0 0]	[1 2]	[0 2]	16	a
	b	[2 0]	[1 2]	[0 1]	16	a *
	c	[0 1]	[2 2]	[0 1]	14	b
3	a	[1 1]	[0 2]	[0 2]	26	a
	b	[2 0]	[1 2]	[0 1]	24	a worse than 2b
	c	[0 1]	[1 2]	[0 1]	22	b *
	d	[1 1]	[1 2]	[0 1]	24	c
	e	[2 0]	[2 2]	[0 0]	22	c
4	a	[2 0]	[0 2]	[0 1]	29	a
	b	[1 1]	[0 2]	[0 1]	32	c *
	c	[2 0]	[1 2]	[0 0]	30	c
	d	[0 0]	[1 2]	[0 1]	30	d worse than 3c
	e	[2 0]	[1 2]	[0 0]	30	d same as 4c
	f	[0 1]	[2 2]	[0 0]	28	e
5	a	[0 1]	[0 2]	[0 1]	32	a
	b	[2 0]	[0 2]	[0 0]	35	b *
	c	[0 1]	[1 2]	[0 0]	36	c
	d	[1 1]	[1 2]	[0 0]	38	f
	e	[2 1]	[2 1]	[0 0]	36	f

TABLE 12 continued

k	State	p_k^t	$q_{in_k}^t$	$q_{out_k}^t$	j_k^0	$(p_1)_{k-1}$
6	a	[2 0]	[0 2]	[0 0]	36	a worse than 5b
	b	[0 1]	[0 2]	[0 0]	38	b *
	c	[1 1]	[0 2]	[0 0]	46	c
	d	[2 1]	[1 1]	[0 0]	44	c
	e	[0 0]	[1 2]	[0 0]	44	d worse than 5c
	f	[0 0]	[2 1]	[0 0]	40	e
7	a	[2 1]	[0 1]	[0 0]	42	b *
	b	[0 0]	[0 2]	[0 0]	49	c worse than 6b
	c	[0 0]	[1 1]	[0 0]	48	d
	d	[1 1]	[1 1]	[0 0]	50	e
	e	[2 1]	[2 0]	[0 0]	48	e
8	a	[0 0]	[0 1]	[0 0]	44	a *
	b	[1 1]	[1 0]	[0 0]	58	c worse than 9e
	c	[2 1]	[0 1]	[0 0]	56	c worse than 7a
	d	[0 0]	[1 1]	[0 0]	56	d worse than 7c
	e	[0 0]	[2 0]	[0 0]	50	e
9	a	[2 1]	[0 0]	[0 0]	48	a *
	b	[0 0]	[1 0]	[0 0]	61	b from 8b
	c	[1 1]	[1 0]	[0 0]	55	e
10	a	[0 0]	[0 0]	[0 0]	48	a optimal *
	b	[0 0]	[1 0]	[0 0]	58	c

$$\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix} .$$

While the dynamic programming example may seem complex, the algorithm can be implemented simply and efficiently. The computational savings over direct enumeration of all possible routings is evidenced by the early termination of nine partial routes in Table 12. By comparison, only one route which reached the desired state was evaluated. In spite of these arguments, the illustration example was contrived and it could reasonably have been solved by enumeration of routes. On the other hand, for a more complex aisle with additional cranes, converters, and reverberatory and holding furnaces, routing problems can become less trivial. Also, dynamic programming can be extended to total system optimization as was discussed previously.

Minimum Blow Durations

Given the material input decision for each blow on the jth converter, the problem is to find the oxygen input rate, u, and the blow durations, TB(n) which maximize $\frac{Cu_j}{T_j}$. Since Cu_j is fixed by the material input decisions, then maximizing the copper production rate is equivalent to minimizing

$$T_j = \sum_{i=1}^{n} TB_i(n) \qquad (5.6)$$

where TB(n) is the duration of the nth blow. If u is to be held constant at the maximum rate during the finish blow, then the duration of the finish phase blow is determined by the relationship

$$TB_i \text{ (finish)} = \frac{1}{u} \ln \frac{x_2(t_f)}{x_2(o)} \tag{5.7}$$

where the ratio $x_2(t_f)/x_2(o)$ is the concentration of Cu_2S at the end of the finish blow. The minimum blow duration of the finish blow then is specified by the maximum u available. Thus, only the duration of slag blows is considered in the optimization problem.

This problem, like the routing problem, yields to solution by using the dynamic programming technique. Expressing the state equations for the slag phase in terms of the two independent state variables FeS and temperature, we obtain

$$\begin{aligned} \dot{x}_1 &= a'_{11}x_1 + a_{15}x_5 + b_1u + C'_1 \\ \dot{x}_5 &+ a'_{51}x_1 + a_{55}x_5 + b_5u + C'_5 \end{aligned} \tag{5.8}$$

where

$$a_{11} = a_{11} + k_3a_{13} + k_4a_{14} + k_6a_{16}$$

$$a_{51} = a_{51} + k_3a_{53} + k_4a_{54} + k_6a_{56}$$

$$C_1 = C_1 + a_{12}X_2$$

$$C_5 = C_5 + a_{52}X_2$$

where unprimed constants and variables are defined in Chapter III. The state variables are quantized into q levels for X_1 and p levels for X_2 where X_1 and X_2 vary over the range

$$\begin{aligned} 0 \le X \le X_1 &\text{ max tons} \\ 1900 \le X_5 \le 2250°&\text{F} \end{aligned} \tag{5.9}$$

The blow time decision variable TB may be quantized into γ levels in the interval given by

$$TB\ min \leq TB \leq TB\ max$$

The oxygen input rate u may be a constant during the slag blow or a continuous function of time. If u is a continuous function the search space becomes infinite dimensional and thus adds a higher level of complexity to the problem. For simplicity, assume u to be constant during each slag blow, thus it may be quantized into m levels in the interval

$$u_{min} \leq u \leq u_{max}$$

Since there are n slag blows, the problem is formulated as an n stage decision process as illustrated in Figure 15. The state transition rule given by

$$\underline{X}(t_n) = \underline{\underline{\phi}}\ \underline{X}(n-1) + \int_o^{t_{fn}} \underline{\underline{\phi}}\ \underline{b}\ u \ \ d\ t \qquad (5.10)$$

where $\underline{X}(t_n)$ is the state vector at the end of the nth blowing period, $\underline{x}(n)$ is the state vector at the end of crane service following the nth blow, and $\underline{\underline{\phi}}$ is the fundamental solution matrix to the homogeneous form of Equation 5.8. The state vector at the end of crane service following the nth blow is given by

$$\underline{X}(n) = \underline{X}\ (t_{fn}) + \Delta X(e,m) \qquad (5.11)$$

where $\Delta\underline{X}$ is the change in state due to crane service in Equation 3.3. At each stage, all possible combinations of TB and u are considered for each of the states attained from two separate states at stage (n-1)

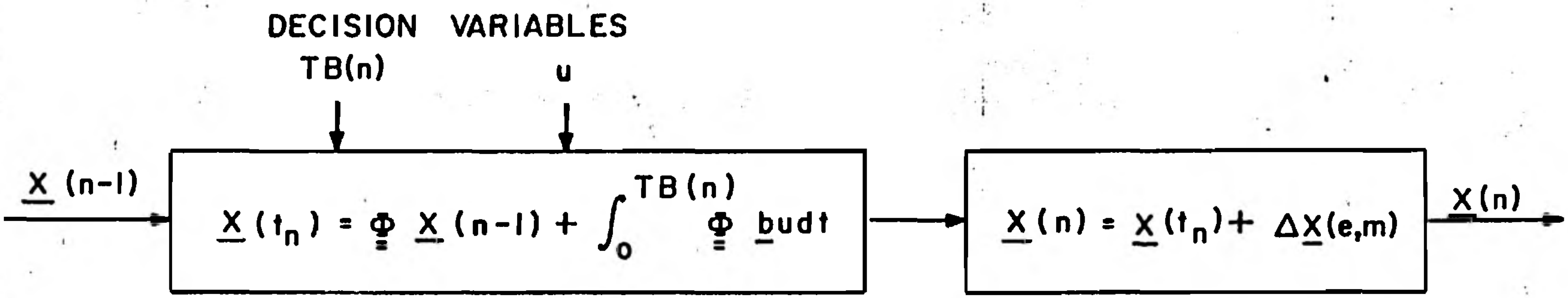

Figure 15 STAGE n OF AN n STAGE DECISION PROCESS

as illustrated in Figure 16. By the principle of optimality, we choose the decisions (TB, u) which correspond to the minimum of

$$s_i = \min \left[S_{i-1} + TB(n) \right] \qquad (5.12)$$

thus trajectory AC is the optimum path to state C at stage n. Clearly each of the states at stage n-1 must be considered for all decisions (TB, u).

Converter schedule.-- The third subproblem is that of several converters systematically receiving all services via a limited facility, i.e., a single crane. This leads to a scheduling problem wherein the objective is to systematically arrange in time the services to each converter so that the total production rate of all converters is maximized. Solution of the scheduling problem results in a schedule for the crane operator, which specifies the times crane service is required for each converter, and a set of schedules for the converter operators, which specify the blow durations. To illustrate these ideas, we present a simple example based upon the following assumptions:

1. The number of blows per charge cycle is given. This determines a fixed number of services, one between each pair of consecutive blows.

2. The number of ladles of output and input to complete each service is given.

3. For simplicity, it is assumed that once a service is initiated, it will be completed, i.e., all output and input requirements will be met, before service to another converter is begun. In other words, we require a schedule with no conflicts in demands for crane service. As previously noted service times are known.

assumption 3 may be unjustified. The crane routing example showed that it can be more efficient to service two converters simultaneously ra-

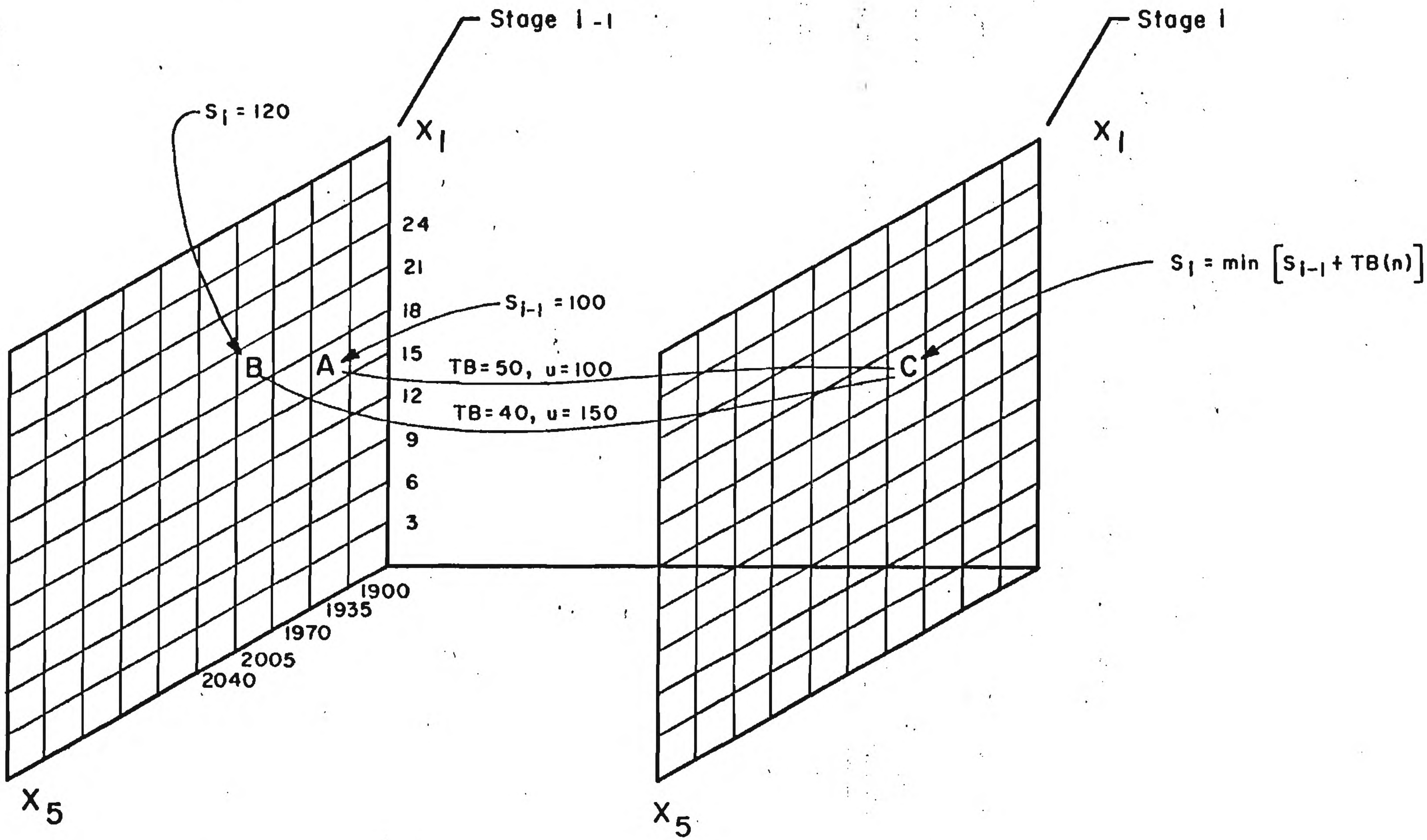

Figure 16 MINIMUM BLOW DURATION DYNAMIC PROGRAMMING

ther than causing one converter to wait until the other completes service. However, the routing example assumed that all other decisions were fixed, whereas purposefully scheduling two converters for simultaneous service effects all other times for service. This requires further investigation.

With the above assumptions, generation of a schedule is a small enough problem that it may be solved by direct enumeration of all possible schedules which meet all the system constraints. A simple example will illustrate the scheduling ideas. Typical service times and minimum blow durations for a three converter, three-blow-per-charge cycle design are shown in Table 13. Service times are denoted as TS(i, j) and minimum blow durations by TB(i, j), where i denotes the ith converter and j denotes the jth blow. Define TO(i, j) as the scheduled time to service the ith converter prior to the jth blow. The system constraints require that

$$TO(i, j) \geq TO(i, j - 1) + TS(i, j - 1) + TB(i, j - 1) \quad (5.13)$$

and

$$TO(i, j) \neq TO(n, m) + \alpha\, TS(n, m)$$

for $n \neq i$, $m = 1, 2, 3$, and $0 \leq \alpha \leq 1$. The objective of scheduling is to maximize J, Equation 2.1, and hence, with the assumed fixed material input and output, to minimize the charge cycle times T_i, given as

$$T_i = \sum_{j=1}^{3} [TB(i, j) + TS(i, j)] \quad (5.14)$$

where TB(i, j) is the scheduled blow time, which is determined as

$$TB(i, j) = TO(i, j + 1) - TO(i, j) - TS(i, j) \quad (5.15)$$

TABLE 13

SERVICE TIMES AND MINIMUM BLOW DURATIONS

		Converter 1		2		3	
		Service Time TS(1,j)	Minimum Blow Duration TB (1,j)	TS(2,j)	TB (2,j)	TS(3,j)	TB (3,j)
	1	40	80	37	80	39	80
Blow j	2	30	50	25	50	29	50
	3	28	90	22	90	25	90

TABLE 14

CRANE SERVICE SCHEDULE, TO(i, j)

		Converter 1	2	3
	1	0	120	200
Blow j	2	40	150	228
	3	65	172	267

For the sample data in Table 13, a schedule was determined as shown in Table 14, where we assume TO(1, 1) = 0.

In order to achieve the blow durations determined by scheduling, we turn attention to control of a converter furnace.

Converter furnace control.-- We are motivated to study converter furnace optimization by the desire to control the blow durations in accordance with a specified schedule. However, there are several other variables that might be of interest in addition to blow duration. These include temperature, sulfur dioxide (SO_2) production rate and magnetite formation. Temperature is important because high bath temperatures damage the refractory lining of the converter and at low temperatures, magnetite is formed, which is generally undesirable. SO_2 production rate is important since efficient operation of the facilities which process SO_2 often depend on a constant output from the converter aisle. We now survey a number of optimal control solution techniques.

Perhaps the simplest control is to assume that the oxygen input rate is constant over the entire blowing period. The behavior of the converter furnace is determined for each constant value of oxygen input by solving the model equations, for example, Equations 3.9 and 3.13. For a linear model state equation of the form

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\underline{x}(t) + \underline{B}u + \underline{C} \tag{5.16}$$

where u is the scalar control variable (oxygen input rate), the solution takes the form

$$\underline{x}(t) = \underline{\underline{\Phi}}(t, t_o)\ \underline{x}(t_o) + \int_{t_o}^{t} \underline{\underline{\Phi}}(t, \tau)\ [\underline{B}u + \underline{C}]\ d\tau \tag{5.17}$$

where $\underline{\underline{\Phi}}$ is the fundamental solution matrix corresponding to the homogeneous system [12]. Since we have assumed u to be constant, it can be removed from under the integral sign, thus yielding an equation of the form

$$\underline{x}(t) = \underline{y}_1(t) + \underline{y}_2(t)\, u \tag{5.18}$$

where $\underline{y}_1(t)$ and $\underline{y}_2(t)$ are known. Substituting various values of u into the above equation, we obtain state trajectories which are to be evaluated. By choosing from these the one that represents the most desirable behavior, we have in a very simple way solved the problem.

If a constant control is not adequate, or it does not satisfy the constraints (Equation 3.13), then we define a performance criterion J and determine, in general, a time varying control function which will influence the converter to minimize $\hat{J}$. To illustrate, we pose the following problem: using the slag blow model developed in Chapter III and assuming SO_2 production rate to be linearly proportional to FeS oxidation rate, we express a performance criterion as

$$\hat{J} = \min_{u} \int_{t_0}^{t_f} (K_1 \dot{x}_1 - R)^2 \, dt \tag{5.19}$$

where

t_f = scheduled blow duration

$K_1 \dot{x}_1$ = SO_2 discharge rate

R = desired SO_2 discharge rate

u = oxygen input rate

We offer as a justification for this performance criterion, the physical argument that it is desirable to maintain a constant SO_2 discharge during each blow duration. The rate of oxidation of FeS was expressed by the model Equation 3.9 as

$$\dot{x}_1 = \underline{a}_1\underline{x} + b_1u + c_1 \qquad (5.20)$$

where $\underline{a}_1 = [a_{11} \ldots a_{16}]$. Substituting Equation 5.20 into Equation 5.19 and combining constants yields $\hat{J}$ of the form

$$\hat{J} = \min \int_0^{t_f} (\underline{\alpha}_1\underline{x} + \beta_1 u + d_1)^2 \, dt \qquad (5.21)$$

Note that $\hat{J}$ is a quadratic function of u and thus u will be directly determined by minimizing J.

The system constraints are rewritten as

$$0 \le x_k\,(t_f) \le x_{k\ max},\ k = 1,\ 5 \qquad (5.22)$$

$$0 \le x_k(t_f),\ k = 2,\ 3,\ 4,\ 6$$

$$0 \le u \le u_{max} \qquad (5.23)$$

We constrain the terminal value of the state variables via Equation 5.22 rather than constraining the state trajectories, because of the monotonic nature of the state variables and because the terminal constraint problem is generally easier to solve.

With the problem now expressed mathematically, we point out two dynamic optimization techniques: (i) solution via a set of necessary conditions determined by the maximum principle [7, 10], and (ii) solution via a gradient search in the space of control functions [7, 10]. Both techniques employ a function H called the Hamiltonian, which is based upon calculus of variations.

For the approach via the maximum principle, the constraint Equations 5.22, 5.23 are absorbed in two ways, and then the set of necessary equations are written which specify the optimal $u^o(t)$. Equation 5.22 is absorbed by attaching a penalty function [7, 10] to Equation 5.21.

$$J_1 = \hat{J} + w_1 x_1(t_f) + w_1 x_5(t_f) \tag{5.24}$$

where w_1 and w_5 are penalty weights which must be determined, perhaps on a trial basis, so that Equation 5.22 will be satisfied. Equation 5.23 is absorbed by defining new variables γ_1 and γ_2, where $[\gamma_1(t)]^2 = u(t)$, and using the new constraint

$$\gamma_1^2 + \gamma_2^2 = u_{max} \tag{5.25}$$

The Hamiltonian for this problem is

$$H = (\underline{\alpha}_1 \underline{x} + \beta_1 \gamma_1^2 + d_1)^2 + \underline{\lambda}^t(\underline{\underline{A}}\underline{x} + \underline{B}\gamma_1^2 + \underline{C}) + \Gamma(\gamma_1^2 + \gamma_2^2 - u_{max}) \tag{5.26}$$

where Γ is a scalar Lagrange multiplier and $\underline{\lambda}(t)$ is a vector Lagrange multiplier. The necessary equations are

$$\frac{\partial H}{\partial \gamma_i} = 0, \quad \frac{\partial H}{\partial \Gamma} = 0$$

$$\frac{\partial H}{\partial \underline{\lambda}} = \dot{\underline{x}} \tag{5.27}$$

$$\frac{\partial H}{\partial \underline{x}} = -\dot{\underline{\lambda}}$$

subject to the boundary conditions

$$\underline{x}(t_o) = \underline{x}_o$$

$$\lambda_k(t_f) = w_k \ , \ k = 1, 5; \ \lambda_k = 0, \ k = 2, 3, 4, 5$$

where $\underline{x}_o$ is assumed given and w_1 and w_5 must be determined. Equations 5.27 together with the boundary conditions, constitute a two-point boundary value problem [7, 10] which yields the optimal control $u^o(t)$. In general, such a system is difficult to solve, but for this particular case, Equation 5.27 reduces to a linear differential system which may be readily solved.

In some cases, control solution via the above approach may be too difficult. For example, if the performance criteria are not a quadratic function of the control u, the control may be singular [8]. In these cases, the control may be determined by an iterative gradient search, which is simpler but less direct. At each iteration of the gradient search, an approximate u(t) is known and

$$\frac{\partial H}{\partial \underline{x}} = -\underline{\lambda}^o$$

and

$$\frac{\partial H}{\partial \underline{\lambda}} = \dot{\underline{x}}$$

are solved subject to known boundary conditions. Then, $\frac{\partial H}{\partial u}$ is computed and used to update the approximate u(t), and the next iteration is begun. Nononen and Paquurek [8] illustrate a gradient search technique using flux addition rate as the continuous control variable and temperature as a controlled variable.

Thus far we have assumed the blow duration to be fixed by a schedule, based upon an estimate of the minimum blow times. If the scheduled blow times were not realistic, however, it may not be possible to find a satisfactory solution, or any solution for that matter, with the blow terminal time fixed. This is motivation to consider a variable end time problem. Two basic methods of dealing with this problem are: (i) to imbed time into the original problem as a new variable and optimize using one of the above techniques, or (ii) simply to search on the end-time by solving the problem for various values of ending time. Solution of the variable end-time problem may result in blow durations which are inconsistent with the schedule. To resolve this difficulty, we turn to the final task of coordinating the subproblems that have been presented so as to achieve a total system optimization.

CHAPTER VI

OPTIMIZATION RESULTS

In the previous chapter, several optimization problems are presented along with a survey of techniques for solving them. We now focus attention on the optimization of the overall converter aisle performance. To achieve the goal of system optimization, the problem is partitioned into three subproblems as shown in Figure 17. The crane routing subproblem is not included in this study since it applies primarily to converter aisles having a large number of converters, say eight or nine, while this is an example of a three-converter aisle. To eliminate this problem, we schedule to avoid crane conflicts. Assuming material input decisions, the minimum blow durations and corresponding terminal states are determined. Using these minimum blow durations and material input decisions, an optimum no-conflict schedule is calculated. As a result of scheduling, the blow duration may be longer than the minimum, thus the converter furnaces are regulated in accordance with these durations by manipulating the oxygen input rates. The important advantage of the partitioned approach is that the problem has been reduced to a search problem on a small set of decision variables, namely, the material input decisions. The subproblems may be formulated as standard optimization problems and yield to solution by well-developed techniques.

For simplicity in solving each of the subproblems of Figure 17, the following assumptions are made held constant at any level within a given range.

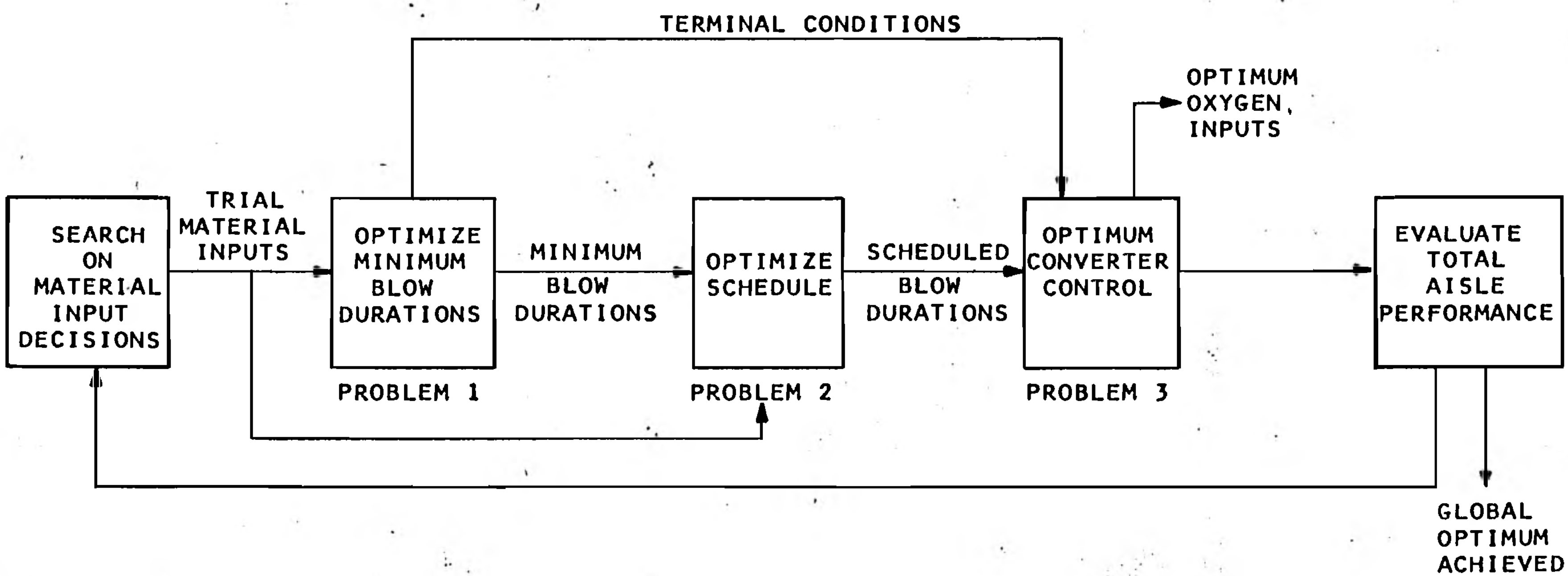

Figure 17 TOTAL SYSTEM OPTIMIZATION

1. The oxygen input rate is constant during each blow.

2. Material inputs are identical for all converters.

3. Five slag blow are assumed.

4. The system is not subject to random perterbations.

Using these assumptions, numerical examples of the three subproblems of Figure 17 are presented. Finally, the converter aisle is optimized for three choices of material inputs and the results are compared, but a total search of the material input space is not done.

Minimum Blow Durations

The objective is, given material input decisions, to find the oxygen input rate, u, and minimum blow durations TB(n) which minimize the objective function of Equation 5.6.

Since it is necessary to numerically integrate Equation 5.11 for each investigation, the computer time required to solve the problem (assuming one second per integration) could be several hours. To establish the quantization size for the states and controls, it is necessary to try several combinations of each. To shorten the time required to obtain a solution, the state model is approximated by the linear relationship

$$\begin{aligned} x_1(t_n) &= x_1(n-1) + .00177\ u\ TB(n) \\ x_5(t_n) &= x_5(n-1) + .0156\ u\ TB(n) \end{aligned} \qquad (6.1)$$

This approximation is reasonable for this problem since the model solutions shown in Figure 11 are linear for x_1 and x_5. An analysis of the solutions to Equation 5.10 for various values of u shows x_1 and x_5 to be linear in u. To complete the state transition model for a given state Equation, 5.11 is used. The amount of Cu_2S (i.e. x_2) does not change during a slag blow due to blowing but accumulates with each material addition. Thus the state transition equations of 6.1 do not reflect changes in x_2 but Equation 5.11 does. The solution begins at the initial state and solves forward by considering all combinations of TB and u for all states $\underline{x}$ at each state. The solution has the constraint that all FeS must be oxidized at the end of the fifth blow. The principle of optimality given by Equation 5.12 specifies the decision rule for selecting the optimum decisions on TB and u. The dynamic programming procedure is implemented on a digital computer and a listing of the program is shown in Appendix 3. The results of a dynamic programming solution using the material inputs of Table 6 Run 2, are given in Table 15. For this solution, the states are quantized into 5 ton increments for $0 \leq x_1 \leq 30$ tons and 50°F increments for $1900°F \leq x_2 \leq 2250°F$. The controls are quantized into 10 minute increments for 30 min $\leq$ TB $\leq$ 100 min and 50 lb/minute increments for $100 \leq u \leq 250$.

Scheduling

Given the material inputs of Table 6, Run 2; the crane service times TS(i,j) as given in Table 16; and the minimum blow durations as given in Table 15; find a schedule of converter aisle activities which will maximize the copper production rate given by Equation 2.1.

$$J = \sum_{i=1}^{3} \frac{Cu_i}{T_i}$$

TABLE 15

MINIMUM BLOW TIMES

Blow	$X_1(TB)$ tons	$X_5(TB)^{o}F$	TB	U
1	0	2075	50	250
2	0	2125	50	200
3	0	2175	30	250
4	0	2225	40	250
5	0	2250	30	200

TABLE 16

CRANE SERVICE TIMES

Blow j	Converter 1 TS(i,j) Minutes Run 1	Run 2	Converter 2 TS(i,j) Run 1	Run 2	Converter 3 TS(i,j) Run 1	Run 2
1	89	81	65	59	89	81
2	46	38	34	28	46	38
3	38	38	28	28	38	38
4	23	38	17	28	23	38
5	15	23	11	17	15	23
6	8	8	6	6	8	8

Cu_i is a constant determined from the material input vector by the relationship

$$Cu_i = \sum_{j=1}^{6} m_j GW \qquad (6.2)$$

where G = matte grade (percent Cu)

W = weight of matte per ladle

m = number of ladles of input material

Since Cu_i is a constant, the maximization of Equation 2.1 is achieved by minimizing the charge cycle time T_i which is given by equation 5.14

$$T_i = \sum_{j-1}^{6} \left[TB(i,j) + TS(i,j) \right]$$

The solution is a schedule of times to start the ith blow on the jth converter and a Table of converter blow durations. The solution is subject to the constraints of Equation 5.18. A computer program which performs an exhaustive search on all possible schedules is shown in Appendix 4. On each iteration of the search, the value of J is computed and compared against the previous value. The schedule associated with the optimum choice is retained and another iteration is begun. The starting time TO(1,1) is assumed to be zero in every iteration. The results of this procedure using the material inputs of Run 2 from Table 6, the minimum blow durations from Table 15 and the crane service times for Run 2, Table 16 are shown in Table 17 and and 18.

TABLE 17

CRANE SERVICE SCHEDULE (TO(i,j))

Converter i Blow j	1	2	3
1	0	642	701
2	147	81	109
3	251	185	213
4	355	289	317
5	433	393	410
6	486	456	463

TABLE 18

CONVERTER BLOW DURATIONS ((TB(i,j)) MINUTES

Converter i Blow j	1	2	3
1	66	162	109
2	66	76	66
3	66	76	66
4	40	76	55
5	30	46	30
6	288	180	230

Having specified the blow durations as given in Table 18, we turn to designing the oxygen input rates which will control the oxidation rates of the converters in accordance with these durations.

Converter Furnace Control

Regulation of the converter furnace activity to achieve a fixed blow duration is a terminal control problem. In general, the objective for the slag blows is to find a continuous control $u^o(t)$ which transfers the system from an initial state $\underline{x}(t_o) = \underline{x}_o$ to a terminal state given by

$$0 \leq \underline{x}_k(t_f) \leq \underline{x}_k max \quad k = 1,5 \tag{6.3}$$

in a time interval $[t_f - t_o] = TB(i,j)$ for $j = 1,\ldots,5$. For the finish blow, the objective is to transfer the initial state $x_2(t_o) = X_{20}$ to the terminal state $x_2(t_f) \approx 0$ in the interval $[t_f - t_o] = TB(i,6)$.

The type of control functions u(t) which is to be obtained depends on the method of its implementation. If the control adjustments are to be made manually by an operator, they must be much less frequent than the case where an automatic controller is used. Thus, the optimization method for finding the control is chosen in accordance with the desired form of the control. For manual adjustment u must be a discrete form where the period between adjustments is sufficient to allow operator response. For automatic control, u may be a continuous function of the form $u = u(t)$ for all t in $[t_o, t_f]$.

In Chapter V, two techniques for determining the optimum continuous oxygen input rate are presented. The Maximum Principle of Pontryagin [10] specified the necessary conditions for an optimum solution. The gradient search technique was considered as an alternative method for a problem where the control is singular (i.e., the control cannot be expressed explicitly as a function of the state variables and Lagrange multipliers).

For a discrete solution, the simplest control is to assume the oxygen input rate is constant over the entire blowing period. As a practical solution, the constant oxygen input problem is considered.

The problem is to find the constant oxygen input rate which will transfer the state from a given initial value to a given terminal value in a given period of time TB(n), subject to the constraints of Equation 5.9. The oxygen input rate is implicitly constrained by the solution obtained in the minimum blow time problem. An iterative search technique is used to find u. This is accomplished by solving Equation 5.10 for various values of u and testing to determine if the terminal constraints have been achieved. A computer program which performs an iterative search for the optimum, u, is shown in Appendix 5.

The control for the finish phase may be obtained directly by solving the finish phase state equation and inverting thus

$$u = -\frac{1}{t_f^{\,2}} \ln \frac{X_2(t_f)}{X_2(o)} \qquad (6.4)$$

where $X_2(t_f)/X_2(o)$ is the terminal concentration of Cu_2S.

A numerical example for a solution to this problem is shown in Table 19. The blow durations TB(n) are obtained from Table 18 and the initial and terminal states are obtained from Table 15.

Comparative Results

Three optimization studies were conducted, corresponding to material input decision of Run 1 and Run 2 in Table 6 and Run 3 of Table 20.

Applying the optimization technique for finding minimum blow durations to the material inputs of Run 1 yield a solution which does not satisfy the constraints of the minimum blow time optimization problem, namely that all FeS must be oxidized at the end of Blow 5. Therefore, Run 1 is eliminated from future consideration as a material input choice. In practice, when the situation arises where FeS remains in the bath at maximum temperature, cold copper scrap is added to cool the bath and blowing is resumed until all FeS has been oxidized.

The results of optimizing for the material inputs of Runs 2 and 3 are compared in Table 21. Comparing the optimized Run 2 in Table 21 with the non-optimized Run 2 in Table 7 shows an improvement of 20 percent in favor of the optimally controlled aisle. Only a slight difference is noted between the results of Runs 2 and 3 for the optimally controlled aisle.

We now investigate the size of the material input search space. The range of material input choices for this example is constrained by the physical size of the converters and the assumption that only an integer number of ladles may be transferred during crane service.

TABLE 20
MATERIAL INPUT DECISIONS FOR
RUN 3 OF THE OPTIMIZATION STUDIES

Blow	Number of Ladles of Input
1	4
2	4
3	3
4	2
5	2
6	1

TABLE 21

RESULTS OF OPTIMUM OPERATING STRATEGY
WITH FIXED MATERIAL INPUTS

	Average Production Rate Tons/Hour	Average Crane Service Time Minutes	Average Blow Duration Minutes	Average Duration of Wait Minutes/Hour	Average No. of Conflicts per Charge Cycle
Run 2	7.68	34.2	96	0	0
Run 3	7.8	33.2	94.8	0	0

Assuming the maximum capacity of the furnaces to be M_c tons, the objective is to determine the number of ladles of material and the number of blows which minimize

$$M_c - \sum_{j=1}^{i} M_j W G_1 \geq 0 \qquad (6.7)$$

where W = weight per ladle of matte

G_1 = weight fraction of Cu_2S per ton of matte

i = number of slag blows

M_j = number of ladles on the jth blow

For this example the range of i is 3 to 13 and the range of M is 1 to 9.

Because of the accumulation of Cu_2S on successive blows and the capacity constraints, the search space is approximately 200 points. With this size of space, direct enumeration is possible. However, a search may be more efficient.

CONCLUSIONS

A complex industrial optimization problem exemplified by the converter aisle in a copper smelter has been identified. Two approaches to solving this optimization problem have been presented. The direct approach offered well developed methods for solution, but the problem was too large from a practical point of view. Thus, an alternative method of optimization by partitioning the problem into managable subproblems was developed. Then the solutions to the subproblems together compose a solution to the total problem. The results of applying the partitioned optimization technique showed that an improvement of at least 20% in the copper production rate was achieved. This was accomplished by minimizing the blowing times, scheduling the crane service activities, and regulating the converter furnaces in accordance with an optimum schedule. Some comments are in order regarding the interpretation of the findings in the numerical example.

The model used throughout this work is hypothetical. Its purpose is to characterize the dynamics of a converter aisle with sufficient realism for posing a practical optimization problem. The important contribution of this research is the development of a technique for solving this type of optimization problem, which otherwise had not yet been solved. To view in perspective the meaningful results of the partitioning approach, two important sensitivity considerations should be recognized. First, consider the sensitivity of the model parameters to actual process operating data. The form of the model, for example, may significantly influence the sensitivity of the model to the data used in its formulation. One may also ask, "How are the results obtained from the model changed, by varying the model parameters (e.g., the coefficients in Equation 3.9)?" This question has not been investigated in this work but clearly is a prerequisite for accepting the results of any specific study where this optimization method is applied. A second consideration is the sensitivity of the optimization results to the model of the process being optimized. Future work on this problem should include an investigation of results obtained by optimizing with several models.

Some considerations which may influence the results obtained from the optimization approach are:

1. The numerical integration technique used to solve the model equations.
2. The step size of the numerical integration.
3. The quantization levels used for the state and control variables in the minimum blow duration problem.

Although these considerations were not investigated in this paper, it should be recognized that they may have an important influence on the results obtained from an actual application of the optimization technique.

Practical application of this technique requires initially an abundance of data from which to build and verify a model. For many processes such as the converter aisle, the type of data needed to accomplish this is very difficult to obtain, principally due to the lack of instrumentation for measuring the state variables. It should be recognized that the physical environment is very harsh and special instrumentation is required for measuring many of the variables. Once a model has been developed and verified, application of the optimization techniques presented herein will provide an optimal operating strategy. However, a strategy such as the one given in Table 22 does not account for perturbations in the process such as changes in matte grade, oxygen input rates, or equipment failures. To compensate for this, we seek closed loop (i.e. feedback) control which continuously monitors the activities of the process and adjusts the control variables in accordance with the optimum operating strategy. This eventually requires a control computer which performs the functions of monitoring the system variables, filtering measurements, estimating the system state, and computing the optimal control values. The necessity for feedback control may be a factor in rejecting the direct optimization approach as a method of solving this problem. In the case where the process disturbances are such that it is necessary to compute new optimal control values, the direct approach would take a prohibitive length of time to compute an updated operating strategy while the partitioned approach produces a solution in a much shorter time. The direct approach does have an advantage, however, in that a true optimal solution is guaranteed. For the partitioned approach that claim cannot be made since the composition of the subproblems into a total solution may result in a suboptimal control which is a local optimum.

TABLE 22

OPTIMUM CONVERTER AISLE OPERATING STRATEGY

Blow	CONVERTER 1			CONVERTER 2			CONVERTER 3			Material Input Decision
	Time to Service	Blow Duration	Oxygen Input Rate	Time to Service	Blow Duration	Oxygen Input Rate	Time to Service	Blow Duration	Oxygen Input Rate	
1	0	66	145	642	162	25	701	109	70	4 ladles
2	147	66	150	81	76	130	109	66	150	3 ladles
3	251	66	155	185	76	135	213	66	155	3 ladles
4	355	40	250	289	76	125	317	55	205	3 ladles
5	433	30	200	393	46	125	410	30	200	2 ladles
6	486	288	185	456	180	475	463	230	290	1 ladle

Further research might include continued study of the direct approach to seek methods of reducing size and solution time. Further research should definitely be done on the partitioned approach to consider improved models and the significance of the simplifying assumptions.

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APPENDICES

1. Computer Program of the Simulation Model
    a. Subroutine (EVNTS)
    b. Subroutine (CRNSV)
    c. Subroutine (CONVT)

2. Model Parameters and Assumed Oxygen Input Rates
    a. Coefficients of the slag phase model
    b. Coefficients of the finish phase model

3. Computer Algorithm for the Minimum Blow Duration Problem

4. Computer Algorithm for the Optimum Scheduling Problem

5. Computer Algorithm for the Converter Furnace Control Problem

Appendix 1

Computer Program of the Simulation Model

Subroutine (EVNTS)

```
      SUBROUTINE EVNTS(IX,NSET)
      DIMENSION NSET(6,1)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNO(4)
      COMMON ATRIB(4),ENQ(4),INN(4),JCELS(5,22),KRANK(4),JCLR,MAXNQ(4),
     1MFE(4),MLC(4),MLE(4),NCELS(5),NQ(4),PARAM(20,4),QTIME(4),SSUMA(10,
     25),SUMA(10,5),NAME(6),NPROJ,MON,NDAY,NYR
      COMMON XMATL(3,6),X(3,6),XN(14),XD(6),CRNB,IC,IB,NB,TCU(3),TS(3,6)
     1,DELT,CU(3)
      IB=ATRIB(4)
      IC=ATRIB(3)
      VIC=IC
      CALL FIND(VIC,5,2,2,KCOL,NSET)
      CALL RMOVE(KCOL,2,NSET)
      VAL=ATRIB(1)
      GO TO(1,2),IX
    1 IF(CRNB-1.)3,4,4
    3 CALL CFNSV(TSV)
      CALL COLCT(VAL,IC,NSET)
      ATRIB(1)=TSV+TNOW
      IF(IB-NE)5,6,6
    5 IB=IB+1
      GO TO 7
    6 IB=1
      TCU(IC)=TCU(IC)+CU(IC)
    7 ATRIB(4)=IB
      ATRIB(2)=2.
      ATRIB(3)=IC
      CRNB=1.
      SAV=ATRIB(1)-TNOW
      CALL FILEM(1,NSET)
      ATRIB(1)=SAV
      ATRIB(2)=IC
      CALL FILEM(2,NSET)
      RETURN
    4 CALL FIND(TNOW,2,1,1,KCOL,NSET)
      CALL RMOVE(KCOL,1,NSET)
      WTM=ATRIB(1)-TNOW+.2
      ICW=IC+3
      CALL COLCT(WTM,ICW,NSET)
      CALL TMST(CRNB,TNOW,1,NSET)
      WAIT=ATRIB(1)+.2
      CALL FILEM(1,NSET)
      ATRIB(1)=WAIT
      ATRIB(4)=IB
      ATRIB(3)=IC
      ATRIB(2)=1.
      CALL FILEM(1,NSET)
      ATRIB(1)=VAL+WTM
      ATRIB(2)=IC
      CALL FILEM(2,NSET)
      RETURN
```

Appendix 1 (Cont'd.)

Subroutine (EVNTS) Cont'd.

```
2   ATRIB(4)=IB
    ICP=IC+6
    CALL COLCT(VAL,ICP,NSET)
    CALL CONVT(TMC)
    ATRIB(1)=TMC
    PAV=TNOW+ATRIB(1)
    CALL FILEM(2,NSET)
    ATRIB(1)=PAV
    ATRIB(2)=1.
    ATRIB(3)=IC
    ATRIB(4)=IB
    CRNB=0.
    CALL FILEM(1,NSET)
    RETURN
    END
```

Appendix 1 (Cont'd.)

Subroutine (CRNSV)

```
*ONE WORD INTEGERS
      SUBROUTINE CRNSV(TSV)
      DIMENSION TPL(3)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(4)
      COMMON ATRIB(4),ENQ(4),INN(4),JCELS(5,22),KRANK(4),JCLR,MAXNQ(4),
     1MFE(4),MLC(4),MLE(4),NCELS(5),NQ(4),PARAM(20,4),QTIME(4),SSUMA(10,
     25),SUMA(10,5),NAME(6),NPROJ,MON,NDAY,NYR
      COMMON XMATL(3,6),X(3,6),XN(14),XD(6),CRNB,IC,IB,NB,TCU(3),TS(3,6)
     1,DELT,CU(3),OXY(6)
      TPL(1)=.7
      TPL(2)=.5
      TPL(3)=.7
      IF(IB-6)4,5,4
    5 XLDL=CU(IC)/15.+1.
      YLDL=0.
      GO TO 7
    4 XLDL=X(IC,4)/15.+1.
      YLDL=XLDL-1.
    7 LDL=XLDL
      XLDL=LDL
      TSL=XLDL*TPL(IC)
      IF(IB-5)1,2,1
C COMPUTE CONVERTER STATE AT THE END OF CRANE SERVICE FOR SLAG BLOW
    1 X(IC,1)=XMATL(1,IB)+X(IC,1)
      X(IC,2)=XMATL(2,IB)+X(IC,2)
      X(IC,3)=X(IC,2)*.274+X(IC,3)
      X(IC,4)=X(IC,4)-YLDL*15.
      X(IC,5)=X(IC,5)*.01
      X(IC,6)=(X(IC,6)*(X(IC,1)-XMATL(1,IB))+(XMATL(1,IB)+XMATL(2,IB)*1.
     1274)*1900.)/(X(IC,1)+XMATL(2,IB)*1.274)
      TSV=(TS(IC,IB)+TSL)*RNORM(4)
      RETURN
    2 DO 3 I=2,5
    3 X(IC,I)=0.0
      X(IC,1)=X(IC,1)+XMATL(1,IB)*RNORM(1)
      X(IC,6)=(X(IC,6)*(X(IC,1)-XMATL(1,IB))+(XMATL(1,IB)+XMATL(2,IB)*1.
     1274)*1900.)/(X(IC,1)+XMATL(2,IB)*1.274)
      TSV=(TS(IC,IB)+TSL)*RNORM(4)
      RETURN
      END
```

Appendix 1 (Cont'd.)

Subroutine (CONVT)

```
*ONE WORD INTEGERS
      SUBROUTINE CONVT(TMC)
      DIMENSION XM(14)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(4)
      COMMON ATRIB(4),ENQ(4),INN(4),JCELS(5,22),KRANK(4),JCLR,MAXNQ(4),
     1MFE(4),MLC(4),MLE(4),NCELS(5),NQ(4),PARAM(20,4),QTIME(4),SSUMA(10,
     25),SUMA(10,5),NAME(6),NPROJ,MON,NDAY,NYR
      COMMON XMATL(3,6),X(3,6),XN(14),XD(6),CRNB,IC,IB,NB,TCU(3),TS(3,6)
     1,DELT,CU(3),OXY(6)
      WRITE(NPRNT,101)IC,IB
      SCL=10.
  101 FORMAT(5X,2I6)
      T=0.
      IF(IB-6)1,2,2
    1 IF(X(IC,6)-2250.)3,4,4
    3 IF(X(IC,2))4,20,20
   20 IF(X(IC,3))4,21,21
   21 JL=1
      WRITE(NPRNT,100)T,(X(IC,L),L=1,6)
      JN=0
      DO 6 I=1,6
    6 XD(I)=0.
      DO 15 I=1,14
   15 XM(I)=XN(I)
      K=2
      DO 8 I=1,2
      XM(JL+1)=XM(JL+1)/(X(IC,1)+X(IC,2)+X(IC,3))
      DO 7 J=1,6
      JL=JL+1
      JN=JN+1
    7 XD(K)=XM(JN)*X(IC,J)+XD(K)
      JL=JL+1
      JN=JN+1
      XD(K)=XD(K)+XM(JN)
    8 K=K+4
      U=OXY(IB)
      XD(1)=0.
      XD(2)=XD(2)-.0018*U
      XD(3)=XD(2)*.274
      XD(4)=XD(2)*(-.866)
      XD(5)=XD(2)*(-.19)
      XD(6)=XD(6)+.016*U
      DO 9 I=1,6
    9 X(IC,I)=X(IC,I)+XD(I)*DELT
      T=T+DELT
      GO TO 1
    4 TMC=T/SCL
      WRITE(NPRNT,100)T,(X(IC,L),L=1,6)
      RETURN
    2 T=0.
      CU(IC)=X(IC,1)*.8
```

Subroutine (CONVT) Cont'd.

```
 10 IF(X(IC,1)-.1)12,11,11
 11 U=OXY(IB)
    WRITE(NPRNT,100)T,(X(IC,L),L=1,6)
    ARG=.150E-06*U*T**2.
    X(IC,1)=X(IC,1)*EXP(-ARG)
    T=T+DELT
    GO TO 10
 12 TMC=T/SCL
    WRITE(NPRNT,100)T,(X(IC,L),L=1,6)
100 FORMAT(2X,F8.3,6(2X,E14.4))
    RETURN
    ENC
```

Appendix 2

Model Parameters and Assumed Oxygen Input Rates

a. Coefficients of the Slag Phase Model as expressed by Equation 3.9

$A_{11} = -.29$	$A_{51} = .085$
$A_{12} = .0005265$	$A_{52} = .000106$
$A_{13} = .008117$	$A_{53} = -.007658$
$A_{14} = .001004$	$A_{54} = -.0004252$
$A_{15} = -.00006333$	$A_{55} = -.00003307$
$A_{16} = .00769$	$A_{56} = -.005419$
$C_1 = .06282$	$C_5 = .2167$
$K_3 = .274$	$b_5 = .016$
$K_4 = -.93$	
$K_6 = -.19$	
$b_1 = -.0018$	

Assumed $u = 150$ lb/minute

b. Coefficients of the Finish Phase Model as expressed by Equation 3.17

$\alpha = .15 \times 10^{-6}$

Assumed $u = 100$ lb/minute

Appendix 3

Computer Algorithm for the Minimum Blow Duration Problem

```
AFTER DATA CARD INSERT II = 1 (FOR 360 COMPUTER) OR I1 = 2 (FOR 1800 COMPUTER)
DATA CARDS MUST BE PUNCHED AS FOLLOWS AND STACKED IN THIS ORDER:
  1ST CARD - CU2S (AMOUNT ADDED FOR EACH OF 5 BLOWS) FORMAT 5F10.0
  2ND CARD - FES (AMOUNT ADDED FOR EACH OF  5 BLOWS) FORMAT 5F10.0
  3RD CARD - FLUX (AMOUNT ADDED FOR EACH OF 5 BLOWS) FORMAT 5F10.0
  4TH CARD - TBI (INITIAL VALUE OF BLOW DURATION), DELTB (INCREMENT FOR BLOW
             DURATION), M3 (NUMBER OF TIMES TO INCREMENT BLOW DURATION) FORMAT
             2F10.C,I10.
  5TH CARD - VI (INITIAL VALUE OF OXYGEN RATE), DELTV (INCREMENT FOR OXYGEN
             RATE),M4 (NUMBER OF TIMES OXYGEN RATE IS INCREMENTED) FORMAT
             2F10.0,I10.
      DIMENSION T(10,10,6), U(10,10,6),X(6),CU2S(5),FES(5),FLUX(5),XS(6)
      DATA T,U,X/1206*0.0/
      I1 = 1
      READ(I1,1)(CU2S(I),I=1,5)
      READ(I1,1)(FES(I),I=1,5)
      READ(I1,1)(FLUX(I),I=1,5)
    1 FORMAT(5F10.0)
      READ(I1,2)TBI,DELTB,M4
      READ(I1,2) VI,DELTV,M3
    2 FORMAT(2F10.0,I10)
      X(4)=0.0
      X(5) = 1900.00
      X(6)=0.0
      X(1) = FES(1)
      DO 35 N=1,5
      X(2) = CU2S(N) + X(2)
      X(3) = FLUX(N)
      N1=1
      N2=1
      IF(N-1)20,20,4
    4 IT = T(N1,N2,N)
      IF(IT-0)31,31,5
    5 X(1) = 5 * N1
      X(1) = X(1) - 2.5
      X(1) = FES(N) + X(1)
      X(5) = 50.*N2
      X(5) = X(5) - 25.0 + 1900.00
       A = FES(N) + CU2S(N) + FLUX(N)
      B= X(1) + X(2) -FES(N)-CU2S(N)
      C = X(1) + X(2) + X(3)
      X(5) = (X(5) * B + 1900.0 * A)/C
   20 V = VI
      DO 30 M1=1,M3
      TB = TBI
      DO 29 M2=1,M4
      TBT = TB + T(N1,N2,N)
      CALL DONOT (TB,V,X,XS)
      IF(N-5)1000,1001,1001
 1001 IF(XS(1)-5.)21,21,29
 1000 IF(XS(1)-0.0)29,29,21
   21 IF(XS(5)-2250.0)22,22,29
   22 DO 23 I=1,6
      R = 5*I
      IF(XS(1)-R)24,24,23
   23 CONTINUE
   24 S=1950.
      DO 25 J=1,7
      IF(XS(5)-S)26,26,25
   25 S = S + 50.0
```

Appendix 3 (Cont'd.)

```
26 NS=N+1
   IT = T(I,J,NS)
   IF(IT-0)28,28,27
27 IF(T(I,J,NS)-TBT)29,29,28
28 T(I,J,NS) = TBT
   U(I,J,NS) =V
   WRITE(3,70)TBT,V,I,J,N1,N2
70 FORMAT(5X,2E12.5,4I8)
29 TB = TB + DELTB
30 V = V + DELTV
   IF(N-1)35,35,31
31 N2 = N2 + 1
   IF(N2-7 )4,4,32
32 N2 = 1
   N1 = N1 + 1
   IF(N1-6 )4,4,35
35 CONTINUE
   WRITE(3,44)
44 FORMAT('1',' BLOW   TOTAL BLOW TIME   OXYGEN RATE   I(FES)   J(TEM
  1P)')
   DO 50 K=2,6
   DO 50 I=1,6
   DO 50 J=1,7
   L=K-1
   IT=T(I,J,K)
   IF(IT-0)50,50,45
45 WRITE(3,46)L,T(I,J,K),U(I,J,K),I,J
46 FORMAT(' ',2X,I1,5X,F10.2,6X,F10.2,8X,I2,7X,I2)
50 CONTINUE
   CALL EXIT
   END
   SUBROUTINE DONOT(TEND,U,X,XS)
   DIMENSION X(6),XS(6)
   XS(1)=X(1)-.00177*U*TEND
   XS(5)=X(5)+.01565*U*TEND
   RETURN
   END
```

Appendix 4

Computer Algorithm for the Optimum Scheduling Problem

```
      DIMENSION TS(3,6),TSI(3,6),TB(3,6),TBI(3,6),TSO(3,6),TBO(3,6),TD(3
     1,6),T(3,6),TO(3,6),AT(3,6),LB(3,6),TDO(3,6)
      DATA TDO/18*0.0/
C   READ AND WRITE INPUT VALUES OF SERVICE TIME AND MINIMUM BLOW DURATION
      DO 110 I=1,3
      READ(1,1)(TSI(I,K),K=1,6)
      READ(1,1)(TBI(I,K),K=1,6)
    1 FORMAT(6F10.0)
  110 CONTINUE
      WRITE(3,39)
   39 FORMAT(' ','INPUT: CONVERTER BLOW  SERVICE TIME  MIN. BLOW DURATIO
     1N')
      DO 30 M=1,3
      DO 30 N=1,6
      WRITE(3,40)M,N,TSI(M,N),TBI(M,N)
   40 FORMAT(' ',11X,I1,6X,I1,5X,F10.2,5X,F10.2)
   30 CONTINUE
C   ROTATES INPUT VALUES TO TRY EVERY COMBINATION
      DTT=0
      DO 13 I=1,6
      DO 13 IA=1,6
      DO 13 IB=1,6
C   GENERATES NEW ARRAYS OF INPUT VALUES FOR EACH COMBINATION OF INPUT DATA
      DO 62 L=1,6
      LC=L+I-I
      GO TO (47,47,47,47,47,47,41,42,43,44,45,46),LC
   41 LC=1
      GO TO 47
   42 LC=2
      GO TO 47
   43 LC=3
      GO TO 47
   44 LC=4
      GO TO 47
   45 LC=5
      GO TO 47
   46 LC=6
   47 TS(1,L)=TSI(1,LC)
      TB(1,L)=TBI(1,LC)
      DO 62 LA=1,6
      LD=LA + IA - 1
      GO TO (54,54,54,54,54,54,48,49,50,51,52,53),LD
   48 LD=1
      GO TO 54
   49 LD=2
      GO TO 54
   50 LD=3
      GO TO 54
   51 LD=4
      GO TO 54
   52 LD=5
      GO TO 54
   53 LD=6
   54 TS(2,LA)=TSI(2,LD)
      TB(2,LA)=TBI(2,LD)
      DO 62 LL=1,6
      LE=LL+IB-1
      GO TO (61,61,61,61,61,61,55,56,57,58,59,60),LE
```

Appendix 4 (Cont'd.)

```
   55 LE=1
      GO TO 61
   56 LE=2
      GO TO 61
   57 LE=3
      GO TO 61
   58 LE=4
      GO TO 61
   59 LE=5
      GO TO 61
   60 LE=6
   61 TS(3,LL)=TSI(3,LE)
   62 TB(3,LL)=TBI(3,LE)
C    COMPUTES TOTAL TIME FOR A COMPLETE CYCLE (6 BLOWS AT ALL 3 CONVERTERS).
      DO 28 IM=1,3
      DO 28 IN=1,6
   28 TO(IM, IN) = 0.0
      DO 11 KAN=1,3
      DO 11 J=1,6
      J1=J-1
      IF(J1-0)3,2,3
    2 J1=6
    3 J2=J+1
      IF(J2-7)5,4,5
    4 J2=1
    5 T(1,J)=TS(2,J)+TS(3,J)+TD(2,J1)+TD(3,J1)
      TD(1,J)=TB(1,J)-T(1,J)
      IF(TD(1,J))6,6,7
    6 TD(1,J)=0
    7 T(1,J)=T(1,J)+TD(1,J)
      T(2,J)=TS(1,J2)+TS(3,J)+TD(1,J)+TD(3,J1)
      TD(2,J)=TB(2,J)-T(2,J)
      IF(TD(2,J))8,8,9
    8 TD(2,J)=0
    9 T(2,J)=T(2,J)+TD(2,J)
      T(3,J)=TS(1,J2)+TS(2,J2)+TD(1,J)+TD(2,J)
      TD(3,J)=TB(3,J)-T(3,J)
      IF(TD(3,J))10,10,11
   10 TD(3,J)=0
   11 T(3,J)=T(3,J)+TD(3,J)
      TT=0
      DO 22 K=1,3
      DO 22 L=1,6
   22 TT=TD(K,L) + TS(K,L) + TT
C    SELECTS MINIMUM CYCLE TIME AND STORES ARRAYS OF SERVICE TIME, MINIMUM BLOW
C    DURATION, AND OPTIMUM BLOW DURATION
      TTI=1./TT
      IF(TTI-OTT)13,13,12
   12 OTT=TTI
      DO 21 M=1,3
      DO 21 N=1,6
      TSO(M,N)=TS(M,N)
      TBO(M,N)=TB(M,N)
      TDO(M,N)=TD(M,N)
   21 TO(M,N)=T(M,N)
      IO=I
      IAO=IA
      IBO=IB
      TTO=1./OTT
   13 CONTINUE
```

Appendix 4 (Cont'd.)

```
C     OUTPUTS RESULTS OF OPTIMIZATION.  STATES OPTIMUM STARTING CONDITIONS FOR
C     EACH CONVERTER AND OPTINUM TOTAL CYCLE TIME.
      WRITE(3,14)IO,IAO,IBO,TTO
   14 FORMAT('0','AT T=0: CONV. 1 IS ON BLOW ',I1,', CONV. 2 IS ON BLOW
     1',I1,', CONV. 3 IS ON BLOW ',I1,'.'/ ' THIS GIVES OPTIMUM TOTAL TI
     2ME OF ',F10.2,' MINUTES.')
C     GENERATES AN INDEX FOR EACH CONVERTER, TO INDICATE CORRECT BLOW NUMBERS
      DO 71 L=I,6
      LB(1,L)=L +IO -1
      LB(2,L) = L + IAO - 1
   71 LB(3,L) = L + IBO - 1
      DO 77 M=I,3
      DO 77 N=1,6
      L = LB(M,N)
      GO TO (77,77,77,77,77,77,72,73,74,75,76),L
   72 LB(M,N)=1
      GO TO 77
   73 LB(M,N) = 2
      GO TO 77
   74 LB(M,N) = 3
      GO TO 77
   75 LB(M,N) = 4
      GO TO 77
   76 LB(M,N) = 5
   77 CONTINUE
C     OUTPUTS AN ARRAY OF OPTIMUM BLOW DURATIONS
      WRITE(3,100)
  100 FORMAT('1',16X,'OPTIMUM BLOW DURATIONS'//' ',15X,'BLOW 1   BLOW 2
     1  BLOW 3   BLOW 4   BLOW 5   BLOW 6')
      DO 103 I=1,3
      WRITE(3,101)I,(TO(I,K),K=1,6)
  101 FORMAT('0',' CONVERTER ',I1,6(2X,F7.2))
  103 CONTINUE
C     GENERATES A TIME SCHEDULE FOR OPTIMAL CRANE SERVICE FOR CONVERTERS.
      DO 85 I=1,3
      DO 85 J=1,6
   85 AT(I,J) = 0.0
      DO 82 KEN=1,3
      IF(KEN-1)96,96,98
   96 AT(1,1)=0.0
      AT(2,1) =TSO(1,1)
      AT(3,1)=TSO(1,1) + TSO(2,1)
      DO 97 I=1,3
      DO 97 J=2,6
   97 AT(I,J) = AT(I,J-1) + TO(I,J-1) + TSO(I,J-1)
      GO TO 99
   98 CONTINUE
      DO 70 K=1,3
      DO 70 L=1,6
      L3=L-1
      IF(L3-0)70,95,70
   95 L3=6
   70 AT(K,L) = AT(K,L3) + TO(K,L3) + TSO(K,L3)
   99 CONTINUE
C     OUTPUTS CRANE TIME SCHEDULE.
      WRITE(3,80)
   80 FORMAT('0',3X,'TIME',4X,'CONVERTER',4X,'BLOW',4X,'OPTIMUM BLOW DUR
     1ATION  SERVICE TIME')
      DO 82 J=1,6
      DO 82 I=1,3
      WRITE (3,81)AT(I,J),I,LB(I,J),TO(I,J),TSO(I,J)
   81 FORMAT('0',F8.2,7X,I1,9X,I1,11X,F10.2,9X,F10.2)
   82 CONTINUE
      CALL EXIT
      END
```

Appendix 5

Computer Algorithm for the Converter Furnace Control Problem

```
*ONE WORD INTEGERS
*LIST SOURCE PROGRAM
      SUBROUTINE TONOD(TEND,U,XX,XTEST,T)
      DIMENSION XM(14),A(14),XX(6),XS(6),XD(6),DUMYX(6)
      DATA            XM /-0.0029E 02,0.5265E-03,0.8117E-02,0.1004E-02,
     1 -0.6333E-04,0.7690E-02,0.6232E-01,0.0850E 00,0.1060E-03,-0.7653E-
     2 02,-0.4252E-03,-0.3307E-04,-0.5419E-02,0.2167E 00/
C.....THIS PROGRAM IS GIVEN THE STATE VARIABLES AND BLOW TIME FOR THE
C.....CONVERTERS AND FINDS THE OPTIMUM U .
C.....XM(I) IS THE A(I,J) COEFFICIENTS
      DELT=5.0
      U=25.
C.....DUMMY ARRAY TO PRESERVE A11 FOR CALCULATING A11 PRIME
      DO 125 NQ=1,4
      A(NQ)=XM(NQ)
  125 CONTINUE
   11 T=0.
      DO 124 NP=1,6
      XS(NP)=XX(NP)
  124 CONTINUE
   13 JL=0
      DO 19 IL=1,6
      XD(IL)=0.
   19 CONTINUE
      T=T+DELT
      DO 69 IY=1,6
      DUMYX(IY)=XS(IY)
   69 CONTINUE
      K=1
      DO 16 I=1,2
      XM(JL+1)=A(JL+1)/(XS(1)+XS(2)+XS(3))
      DO 15 J=1,6
      JL=JL+1
      XD(K)=XM(JL)*XS(J)+XD(K)
   15 CONTINUE
      JL=JL+1
      XD(K)=XD(K)+XM(JL)
      K=K+4
   16 CONTINUE
      XD(1)=XD(1)-.0018*U
      XD(3)=XD(1)*.274
      XD(4)=XD(1)*(-.866)
      XD(5)=XD(5)+.016*U
      XD(6)=XD(1)*(-.19)
      DO 25 I=1,6
      XS(I)=XS(I)+XD(I)*DELT
   25 CONTINUE
      IF(XS(1)-XTEST)77,77,100
  100 IF(T-TEND)13,79,79
   79 U=U+5.
      GO TO 11
   77 IF(T-(TEND-5.))44,101,101
  101 IF(T-(TEND+5.))102,102,78
```

Appendix 5 (Cont'd.)

```
C.....LINEAR INTERPOLATION
   79 DO 69 IZ=1,6
      XS(IZ)=(XS(IZ)+DUMYX(IZ))/2.
   69 CONTINUE
      T=T-(DELT/2.)
  102 WRITE(3,2)T,U,(XS(I),I=1,6)
    2 FORMAT(8E11.4)
      RETURN
   44 WRITE(3,45)
   45 FORMAT('   WHAT HAPPENED?   MISSED RESULT.')
      RETURN
      END
```

VITA continued

Papers and Publications

Templeton, F. E., Hankley, W. J., "Dynamic Optimal Control of a Process with Discrete and Continuous Decision Variables," in Techniques for Decision Making in the Mineral Industry (1970 International Symposium), Canadian Institute of Mining and Metallurgy, Montreal, Quebec, Canada, 1971.