A KINETIC APPROACH TO BALL MILL SCALE-UP FOR DRY AND WET SYSTEMS

by

Mohammad Siddique

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ABSTRACT

In this investigation detailed experimentation has been carried out to establish a relationship for mill scale-up using a linear population balance model in dry and wet grinding systems. Data were obtained in three different sizes of mill (10, 15 and 30 inches in diameter) for limestone grinding. In each case the selection and breakage parameters of the population balance model were determined. Analysis of these data showed that the selection functions are proportional to the specific power draft of the mill ($S_i = S_i^E(P/H)$) for both wet and dry grinding. In addition the breakage functions were found to be independent of mill size, the same for wet and dry grinding. For dry grinding the specific selection functions (S_i^E) were found to be independent of fineness of grind. While for wet grinding the specific selection functions varied with fineness of grind. These relationships were found to constitute a basis for mill scale-up. By incorporating the specific selection functions and breakage functions into the linear population balance model it was possible to accurately predict dry product size distributions in the larger mills from data obtained in the 10-inch diameter mill. Equally accurate predictions were achieved for wet grinding by employing a linearization procedure termed as the "similar fineness of grind approach".

ACKNOWLEDGEMENTS

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CHAPTER I

INTRODUCTION

Grinding operations are of great importance to the mineral and cement industries in particular and the U.S. economy as a whole. At present, approximately 10^9 tons of material is ground annually in the United States. In the process about 1% of the U.S. steel production is consumed as grinding media and about 1% of the U.S. energy production is consumed in driving the mills. In many regards grinding is the most important unit operation in a mineral processing plant. The objectives in the mineral industry are to liberate valuable minerals from the gangue materials and to reduce the size of the mineral particles so that they are suitable for subsequent mineral processing operations.

Unfortunately grinding circuits involve high capital and operating costs and are notoriously inefficient. Thus there is considerable room for improvement in grinding circuits which would reduce the processing costs. The two areas in which the potential is greatest are: (i) mill scale-up design and (ii) automatic control. This study is concerned with improving procedures for scale-up design. For the last three decades most of the mill scale-up work has been based, directly or indirectly, on the empirical Bond energy-size-reduction equation(1). Although this approach has been used extensively in the mineral industry, occasionally serious design errors are made.

In fact, two industrial surveys suggest that the design risk associated with this traditional scale-up method is on the order of ± 20 percent[2,3]. The principal limitations of the Bond equation appears to be its failure to account for important grinding circuit subprocesses, i.e., breakage kinetics, particle transport through the mill, and size classification in an explicit fashion. The Bond scale-up approach lumps the influence of all of these subprocesses into a single empirical correlation[1].

In recent years significant advances have been made in the development of detailed phenomenological grinding models derived from population balance considerations[1,4,5,6]. In these models the explicit accounting of grinding subprocesses (size reduction kinetics, material transport in the mill and size classification) gives them a significant advantage over the simpler energy-size-reduction equations. This type of model, in its complete form, is capable of describing the size distribution in a tumbling mill grinding device as a function of time and mill position. The breakage process is characterized by two physically interpretable quantities, a <u>selection function</u>, which gives the fractional rate of breakage of particles in each size interval, and a <u>breakage function</u>, which gives the average size distribution of daughter fragments resulting from primary breakage events. These two quantities allow the behavior of each size fraction in the mill to be represented mathematically for grinding conditions of industrial importance[1]. For these reasons, the more detailed models hold considerable promise for reducing the design risk associated with traditional mill scale-up methods.

To date population balance models have been used primarily in the analysis of the performance of laboratory scale grinding mills. In the very recent past there have been a few attempts to evaluate the appropriateness of population balance models for use in the scale-up design of dry ball milling systems[1,6,7]. Efforts in this area have been devoted to finding the required relationship between model parameters and mill design and operating variables. Basically two approaches have been explored. The first involves the correlation of selection and breakage functions with mill diameter[6,7]. The second involves the correlation of model parameters with the specific power draft of the mill[1,6,7]. As discussed by Herbst et. al.[1] the mill diameter correlation is really a special case of the specific power draft correlation which is only valid if complete dynamic similarity of charge motion in mills of different diameters is maintained. Both types of correlation have been used successfully in predicting dry ball milling behavior in large batch mills based on experimental measurements made in small mills.

Virtually nothing has been done in the area of wet ball mill scale-up using population balance models. Although wet grinding is more important industrially than dry grinding, wet processes are much more difficult to treat due to inherent non-linearities in the breakage process[4,8]. Recently Kim[8] has shown that the parameters of a linearized population balance model for wet grinding can be correlated with mill operating variables in a similar manner to the specific power draft correlation used in dry grinding. It has been speculated that this approach may also be successful in accounting for the effect of mill design variables[8].

The objective of this thesis research was to provide additional confirmation of the appropriateness of population balance models as a basis for dry ball mill scale-up and to extend this approach to wet systems. The experimental portion of this study involves the dry and wet batch milling of limestone in two laboratory scale ball mills and in a pilot scale mill. In addition, preliminary test work with a wet continuous open circuit mill has been conducted. The data have been analyzed in the context of the population balance framework using specific power draft as a basis for model parameter scale-up. Predictions of large mill performance from small scale batch mill data suggest that the population balance approach to scale-up is capable of yielding considerably more accurate designs for wet and dry ball mills than is possible with the traditional approach.

CHAPTER II

REVIEW OF RELATED STUDIES

In this chapter the formal basis of population balance models for ball milling is reviewed along with existing experimental evidence supporting these models. Then actual applications of this type of model to the scale-up design problem are discussed.

The character of a mathematical model can be determined by the physical details it can handle and the computational complexity associated with its application. A completely predictive model will have a complex mathematical solution while simple models are usually obtained by ignoring several subprocesses in the system. Bond has put forward a simple empirical equation for specific energy calculations[9,10,11]. Bond's equation contains three parameter: a work index, a feed size parameter and a product size parameter. These parameters are used to compute the specific energy requirement for commercial grinding processes. For the scale-up purpose Bond's equation implicitly assumes that breakage kinetics, transport through the mill and classification subprocesses are characterized by a single parameter, a work index. The basic assumption in Bond's equation is that the breakage behavior of all materials is the same as that of an 'ideal Bond material'[1]. Unfortunately the work index of many materials does not remain constant with the product size[10].

The work index is evaluated by locked cycle grinding tests in a standard grindability mill using perfect classification. But in industrial scale classifiers it is extremely difficult to have a perfect separation. In a Bond test it is also assumed that a full scale mill has a plug flow characteristic and that equilibrium is equivalent to steady state in a continuous plug flow mill operated in closed circuit.

In contrast to Bond's equation, population balance models are derived in a physically meaningful way[1,4,5,8,12]. Such models have been shown to provide very accurate predictions of product size distribution produced in batch, locked-cycle and continuous open and closed circuit mills[1]. Several investigators have presented convincing cases for the use of population balance models as an alternate to the Bond equation for scale-up design[1,4,13,14].

Model Framework

The formulation of the population balance models and subsequent discussions of their characteristics have been presented by several authors[4,8,12,15,16]. Most authors agree that the most useful form of these models is the size-discretized, time continuous description. In this model the breakage process is completely characterized by two sets of physically interpretable quantities, the size discretized selection functions and the size discretized breakage functions. The development of this model is briefly described below.

Consider that the size range of mass H in a batch mill be divided into n intervals with maximum size x_1 and minimum size x_{n+1}. The ith interval, bounded by x_i above and x_{i+1} below, contains a mass fraction of material $m_i(t)$ at time t. In many instances x_i and x_{i+1} are related by $x_i = r\, x_{i+1}$ $(i=1,2,\ldots\ldots\ldots,n-1)$ where r is the geometric sieve ratio. A mass balance for ith size interval yields the kinetic model:

$$\frac{d[H\, m_i(t)]}{dt} = - S_i(t) H\, m_i(t) + \sum_{j=1}^{i-1} b_{ij}\, S_j(t)\, H\, m_j(t) \qquad \text{(II-1)}$$

where $m_i(t)$ is the mass fraction of the material in the interval i at any time t, $S_i(t)$ is the size discretized selection function for the ith size interval, denoting the fractional rate at which material is broken out of the ith size interval and b_{ij} is the size discretized breakage function representing the fraction of primary breakage product of material from the jth size interval which appears in the ith size interval[1,4]. The size discretized selection functions can, in general, be dependent upon the size consist in the mill at any time t, i.e.,

$$S_i(t) = S_i\ (H,\ m_k(t), \qquad k = 1,2,\ldots\ldots\ldots,n)$$

but are not explicitly time dependent[1,4,8]. The kinetic model is said to be linear with constant coefficients when both selection and breakage functions are independent of size consist in the mill.

The following assumptions are associated with the development of equation (II-1)[4,8]:

1. The size interval i is sufficiently narrow to ensure that the behavior of all particles in the interval can be

described by the interval average parameters $S_i(t)$ and b_{ij}.

2. The size discretized breakage functions do not depend on the size consist in the mill, i.e.,

$$b_{ij} \neq b_{ij} \ (H, m_i(t), \qquad i = 1,2,........,n).$$

3. Agglomeration of particles is non-existent and attrition is negligible.

The distinct advantage of equation (II-1) is that the parameters $S_i(t)$ and b_{ij} can be obtained directly by experiments[14,17,18]. In the dry ball milling, the selection and breakage functions have been shown to be independent of the size consist in the mill. Also the breakage functions are independent of mass hold-up, H, however, the selection functions are strongly dependent on mass hold-up[8], i.e.,

$$S_i(t) = S_i \ (H, m_k(t) \ k = 1,2,...,n) = S_i(H)$$

When the selection functions are independent of mill size consist the set of n differential equations represented by equation (II-1) can be expressed as a single matrix equation with constant coefficients[1,4,8].

$$\frac{d\,\underline{m}\,(t)}{dt} = - \left[\underline{\underline{I}} - \underline{\underline{B}}\right] \underline{\underline{S}}(H)\ \underline{m}(t) \ldots\ldots. \qquad \text{(II-2)}$$

where $\underline{m}(t)$ is a nx1 vector denoting mass fractions in n size intervals at any time t ($m_i(t)$, i=1,2,........,n), $\underline{\underline{B}}$ is a nxn lower triangular matrix of breakage functions, $\underline{\underline{S}}(H)$ is the diagonal matrix of selection functions and $\underline{\underline{I}}$ the identity matrix. For batch grinding with an arbitrary initial feed, H $\underline{m}(0)$, the equation (II-2) has an analytical solution of the type:

$$\underline{m}(t) = \exp\left[-(\underline{\underline{I}} - \underline{\underline{B}})\underline{\underline{S}}(H)t\right] \underline{m}(0) \tag{II-3}$$

For the case when no two selection functions are equal, the exponential in equation (II-3) can be simplified by similarity transformation to yield[1,4,8,14,20].

$$\underline{m}(t) = \underline{\underline{T}}\ \underline{\underline{J}}(t)\ \underline{\underline{T}}^{-1}\ \underline{m}(0) \tag{II-4}$$

where the elements of matrices $\underline{\underline{T}}$ and $\underline{\underline{J}}$ are given by:

$$T_{ij} = \begin{cases} 0 & i < j \\ 1 & i = j \\ \sum_{k=1}^{i-1} \dfrac{b_{ik} S_k(H)}{S_i(H) - S_j(H)} T_{kj} & i > j \end{cases}$$

and

$$J_{ij} = \begin{cases} \exp\left[-S_i(H)t\right] & i = j \\ 0 & i \neq j \end{cases}$$

Equation (II-4) is useful for batch mill simulations. However, it can be extended for the continuous grinding by incorporating residence time distribution information in the J matrix and attains the form[19].

$$\underline{m}_{MP} = \underline{\underline{T}}\ \underline{\underline{J}}_c(\tau)\underline{\underline{T}}^{-1}\ \underline{m}_{MF} \qquad \text{(II-4A)}$$

where

$$J_{cij}(\tau) = \begin{cases} \int_0^\infty \exp\,[-(S_i\tau)\theta]E(\theta)d\theta & i = j \\ 0 & i \neq j \end{cases}$$

and $\underline{m}_{MP}$ is the steady state size distribution of mill product, $\underline{m}_{MF}$ is the steady state size distribution of mill feed, θ is dimensionless time variable, τ is the mean residence time and $E(\theta)$ is the dimensionless exit age distribution for material transport through the mill.

For the successful application of the model to the simulation of grinding behavior an efficient method for estimating model parameters is required. This method is discussed in Chapter V.

The above model has successfully been applied to dry ball milling systems but is not strictly applicable to wet systems due to an inherent non-linearity in wet ball milling which apparently arises from the classifying action of the pulp in the mill[4,8]. A detailed non-linear model has been developed to account for this behavior[4], however, it is mathematically and computationally quite complex.

Since it is highly desirable to take advantage of the mathematical simplicity of a linear model for engineering applications, there is considerable impetus to explore the applicability of linear models to non-linear systems. Herbst and Mika[21] have discussed this possibility from a theoretical standpoint demonstrating that for any non-linear system there will be regions of operation (grind times, retention times, feed size distributions, etc.) over which a linear or linearized model will be applicable. Kim[8] found that at low speeds and high ball loads, the cascading action of the media results in minimum turbulence conditions in which the kinetics of breakage are "nearly linear" for short and intermediate grind times. For these "nearly linear" breakage conditions a linear or linearized model will be applicable.

Applications Related to Scale-up

Because the population balance models are phenomenological in nature, there is no apriori method for predicting the dependence of model parameters on mill design and operating variables[4,8]. Instead the required relationships must be obtained from a fundamental analysis of the breakage phenomena and/or correlations. Various attempts have been made to correlate kinetic parameters with mill dimensions, mill speed, media shape, media load and density, ball diameter and hold-up mass[1,6,7,22]. However, considerable experimental effort will have to be devoted to define these relationships accurately. Recently a somewhat different approach to such a correlation was attempted[4,5,6]. In this work it was shown that for a dry ball milling operation, the size-discretized selection functions are proportional to the specific power input to the mill (P/H), i.e.,

$$S_i = S_i^E(P/H) \tag{II-5}$$

where S_i^E termed the 'specific selection function', is essentially independent of mill operating conditions. In addition it was found that the breakage function, b_{ij}, is to a good approximation invariant. With equation (II-5) the solution to equation (II-1) for the top size (i=1) can be shown to be:

$$m_1(t) = m_1(0) \exp [-S_1^E(P/H)t] \tag{II-6}$$

Since the product of the specific power and time is equal to the specific energy input to the mill $\overline{E}$, equation (II-6) can be expressed as:

$$m_1(\overline{E}) = m_1(0) \exp [-S_i^E \overline{E}] \tag{II-7}$$

and equation (II-1) can be rewritten in the normalized form as:

$$\frac{d\, m_i(\overline{E})}{d\overline{E}} = - S_i^E\, m_i(\overline{E}) + \sum_{j=1}^{i-1} b_{ij}\, S_j^E\, m_j(\overline{E}) \tag{II-8}$$

Similarly equations (II-2) and (II-3) can be written in normalized forms by replacing t by $\overline{E}$ and S_i by S_i^E. These normalized grinding equations have also been successfully applied to the prediction of dry grinding behavior in different sizes of batch mill[6,7]. The scale-up procedure used in these studies involved 1) obtaining batch grinding and power data in a small laboratory scale mill 2) estimating reduced selection functions and breakage functions from this data 3) using the parameter estimates obtained in (2) along with specific power draft information for larger mills to predict the size distributions expected for these mills. The range of mill diameter examined (at constant L/D) was 5 inches to 20 inches--a 64 fold range of mill volume. In all instances the predictions were found to be in good agreement with experimentally measured size distributions.

Herbst et. al.[1] have discussed the relationship between specifix power scale-up and direct mill diameter scale-up. They point out that the mill power and material hold-up H can be expressed in terms of mill dimensions and dimensionless operating conditions as follows:

$$P = \phi_1(N^*, M_B^*, M_P^*, q^*)\ L\ D^{(2.5+\delta)} \qquad \text{(II-9)}$$

$$H = \phi_2(M_B^*, M_P^*)\ L\ D^{(2.0)} \qquad \text{(II-10)}$$

where L is the mill length and D is the mill diameter and ϕ_1 and ϕ_2 are functions which depend on the dimensionless mill speed N^*, dimensionless ball load M_B^*, dimensionless particle load M_P^* and dimensionless ball size and lifter geometry variables q^*. The parameter δ is zero by dimensional analysis, but takes a value between zero and 0.1 in practice[1].

Substitution of equations (II-9) and (II-10) into the scale-up expression for selection functions,equation (II-5), yields:

$$S_i = S_i^E \cdot \phi_3(N^*, M_B^*, M_P^*, q_B^*)\ D^{(0.5+\delta)} \qquad \text{(II-11)}$$

where the function ϕ_3 is given by ϕ_1/ϕ_2.

Considering two mills of diameters D_1 and D_2, lengths L_1 and L_2 with complete dynamic similarity i.e., N^*, M_B^*, M_P^*, ball diameter and lifter configuration, the ratio of selection functions for the two mills should, according to equation (II-11), be given by

$$\frac{(S_i)_1}{(S_i)_2} = \left(\frac{D_1}{D_2}\right)^{0.5+\delta} \qquad \text{(II-12)}$$

This type of relationship has been observed in two studies involving mills operated with complete dynamic similarity. The values of δ observed were δ=0.06 (5 inch to 20 inch diameter mills)[5] and δ=0.1 (8 inch to 24 inch diameter mills)[13].

At the outset of this study no direct information was available for wet ball mill scale-up using population balance models. Wet grinding results obtained by Kim[8] do, however, appear to have important implications concerning scale-up. In a series of batch experiments in a 10-inch mill, using various ball loads, mill speeds and pulp percent solids, he found that 1) the selection functions are to a good approximation proportional to the specific power draft (P/H) and inversely proportional to the pulp percent solids 2) the breakage functions for a given percent solids are to a good approximation independent of mill speed, ball load and particle load. If Kim's wet grinding results can be extended to mills of different diameters in the same manner that Herbst and Fuerstenau's[1] original results were extended to dry grinding in different size mills, it may be possible to define an accurate basis for wet ball mill scale-up. Wet grinding systems do, however, have the additional complication of non-linear breakage behavior which must be taken into account in any scale-up strategy which is to be developed.

CHAPTER III

EQUIPMENT AND EXPERIMENTAL PROCEDURES

In this chapter the equipment and the experimental procedures followed are discussed. The experiments were designed to provide a quantitative guide for correlation of three mills of different size. The data were obtained from the three mills for testing the predictive capability of the model described in Chapter II. Dry and wet batch grinding experiments were performed in each of the mills to provide the basis for comparison of the two systems and verification of the model. Mono-size (10 x 14 mesh) and 'natural' (-10 mesh) size distribution feeds were examined.

The material used was a limestone acquired from Utah Calcium, Aragonite, Utah. The X-ray diffraction pattern showed that the material contained limestone and calcite crystals. The specific gravity as measured by picnometer was found to be 2.69. The transparent calcite crystals and opaque limestone particles could be visually differentiated in the feed, but the breakage properties of the two types of particles were similar enough to ensure homogeneous breakage behavior.

Equipment

Three mills of 10-inch, 15-inch and 30-inch diameter were used for dry and wet batch tests. The 10-inch and 15-inch diameter mills were of stainless steel construction (Figure 1), each 11.5 inch long

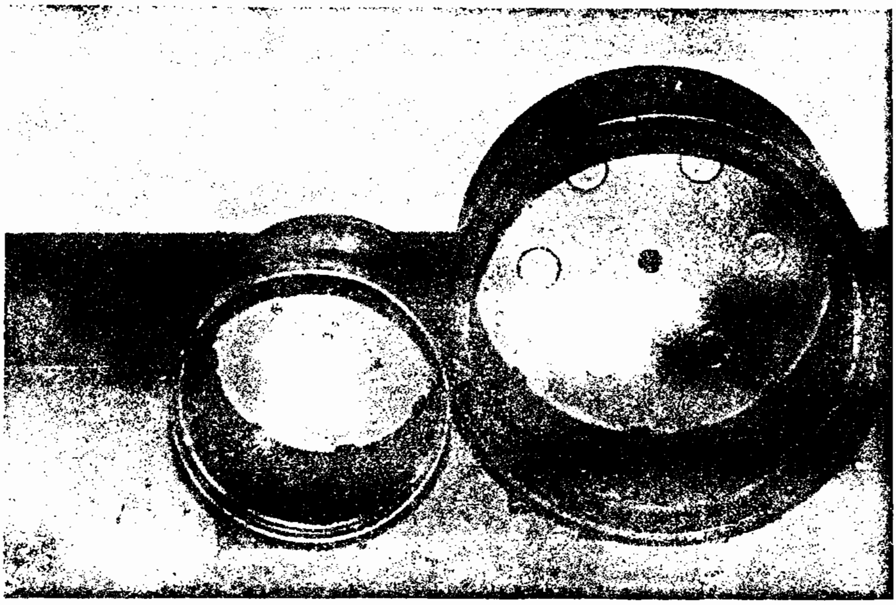

Figure 1. Photograph showing the 10-in. and 15-in. diameter mills with square lifters.

having eight square lifters (Figure 2b). The laboratory scale mills were equipped with a Graham variable speed transmission coupled with a BLH torque sensor and a Sargent recorder to measure power draft (torque) directly from the drive shaft between the transmission and the mill (Figures 3 & 4). The system was identical to the one described by Yang et. al.,[23] and other investigators[4,8,19]. Stainless steel balls were used in the 10-inch mill experiments. Mild steel balls were mixed with stainless steel balls in the 15-inch mill as a grinding media. Experiments performed in the 10-inch mill showed that grinding behavior was identical with media charges consisting of entirely stainless steel balls or mild steel balls or a mixture of the two. The ball sizes used in all experiments ranged from 1/2 inch to 1 1/2 inch in diameter. A ball load of 30.5 kg was used in the 10-inch mill and 68.6 kg in the 15-inch mill, each corresponds to a ball filling of 50% (M_B^*=0.5)† of the struck volume of the mill. The ball size distribution approximated that of an "equilibrium charge distribution"†† frequently used in laboratory tests for mill scale-up design[4,8]. The precise distribution used in each of the mills is given in table on page 22.

The pilot scale mill used in this study is a standard Denver mild steel ball mill having an internal diameter of 30-inch and a

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† $M_B^*=M_B/M_{BC}$, where M_B is the mass of balls, M_{BC} the mass of balls when the mill is completely filled with balls under static conditions.

†† In practice only one size of ball is charged to a commercial mill. With the passage of time the ball size reduces and new balls are added. After a certain time period the ball size distribution attains an equilibrium. This ball size distribution is called the "Equilibrium Charge Distribution."

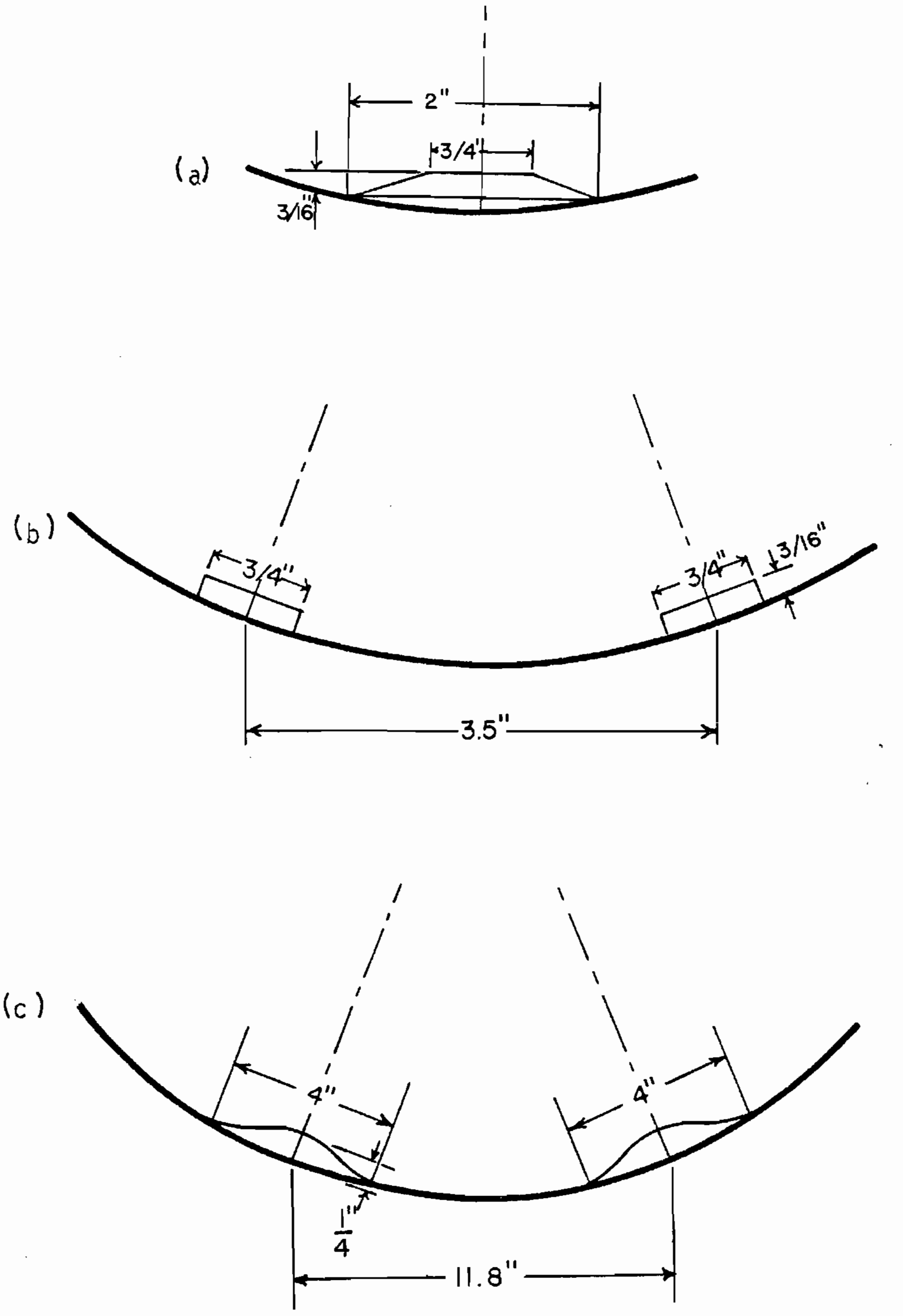

Figure 2. (a) The configuration of ramp lifters used in the 10-in. mill
(b) The configuration of square lifters used in the 10-in. and 15-in. mills
(c) The configuration of rounded lifters used in the 30-in. mill.

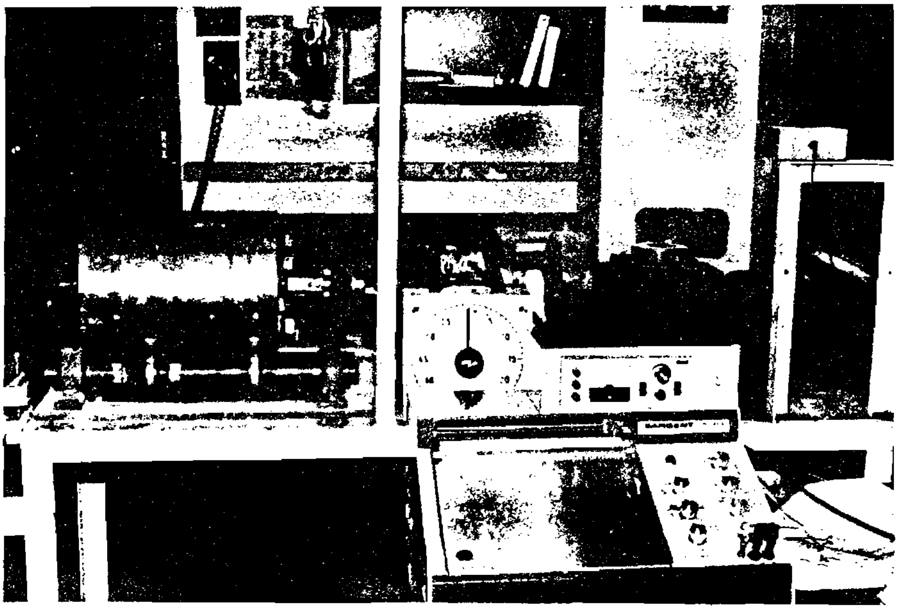

Figure 3. Photograph showing the 10-in. mill with variable speed transmission, the torque sensor, coupling and Sargent recorder.

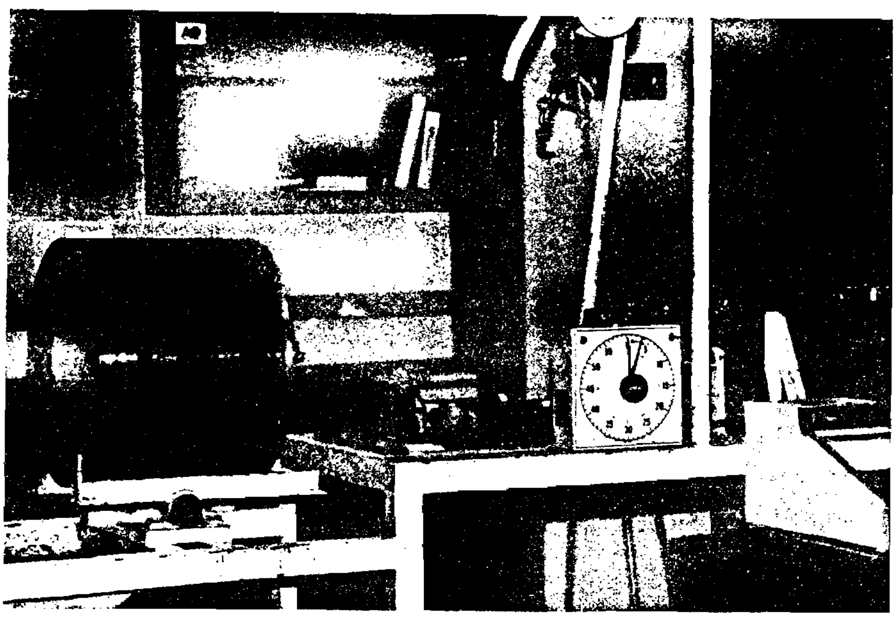

Figure 4. Photograph showing the 15-in. mill on the loading port.

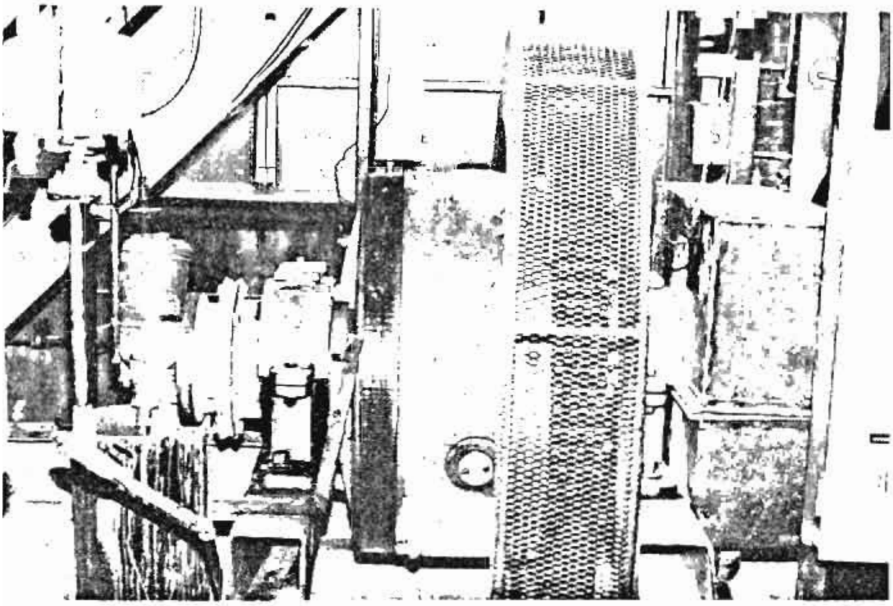

Figure 5. Photograph showing the sideview of the 30-in. mill.

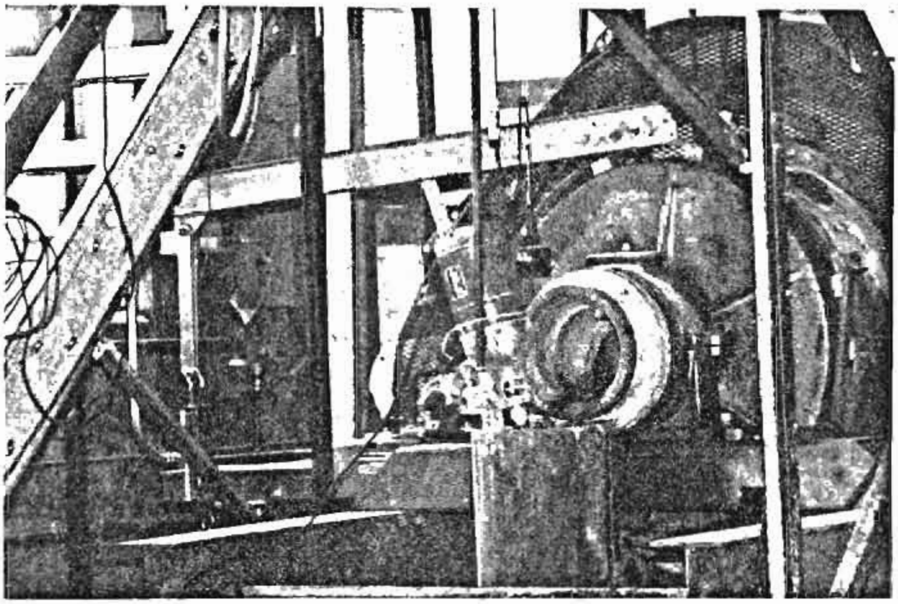

Figure 6. Photograph showing a view of the 30-in. mill with prony brake in position.

The Equilibrium Charge Distribution of Balls

Ball Diameter (inches)	Percent (by wgt)	Mass (Kilograms) 10-in mill 50% filling	15-in mill 50% filling	30-in mill 40% filling	30-in mill 50% filling
1.50	52.79%	16.10	36.23	181.40	226.80
1.00	30.16%	9.20	20.70	103.70	129.60
0.75	11.80	3.60	8.05	40.60	50.70
0.50	5.25%	1.60	3.60	18.10	22.60
	100.00	30.50	68.58	343.80	429.70

length of 18-inch. This mill comes fitted with eight rounded lifters (Figure 2c), similar in configuration to a wavy liner. The mill was designed to be a component of a continuous system. It was mounted on two heavy duty ball bearings and driven in two stages by an electric motor through belts and gears as shown in Figures 6 and 7. Since the mill did not have a central shaft, the BLH torque sensor was coupled to the pinion shaft to record the power input transmitted through the shaft to the mill (Figure 7). The sensor was coupled to a Sargent recorder. The power input to the mill was obtained by multiplying the recorded torque by the gear ratio and applying appropriate corrections for losses. The speed of the pinion shaft could be varied by an adjustable pulley mounted on the motor shaft. For dry grinding experiments a ball load of 429.70 kg, corresponding to 50% ball filling, was used and for wet grinding experiments a ball load of 343.80 kg was used corresponding to 40% ball

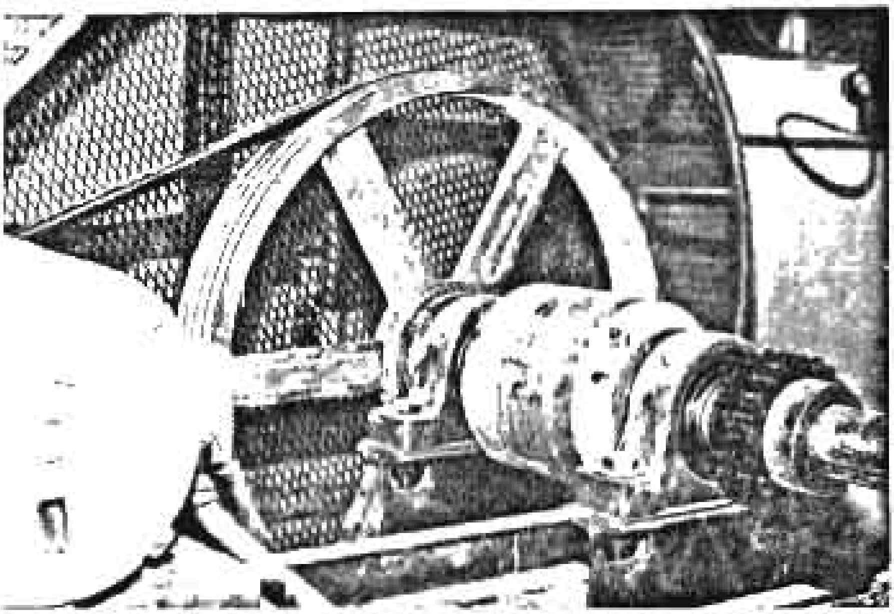

Figure 7. Photograph showing pinion shaft of the 30-in. mill with the belt pulley, the torque sensor and the chain coupling.

filling. The ball size distribution corresponded to the "Equilibrium Charge Distribution" mentioned earlier.

Experimental Procedure

To prepare a 10 x 14 mesh feed for the 10-inch, 15-inch and 30-inch mills, the "as received" limestone was first sieved through a set of Sweco screens, discarding +10 mesh and -14 mesh material. The product was further sieved for thirty minutes through Tyler $\sqrt{2}$-interval screens to obtain a narrow size of 10 x 14 mesh. The material was then washed with water through a 28 mesh screen in order to remove any fine particles which may have been attached to the 10 x 14 mesh material.

The grinding procedures for the 10-inch and 15-inch mills were identical except that the total mass of material ground in the two mills was 3.3 kg and 7.425 kg, respectively. These particle loadings corresponded to 100% filling of the interstices ($M_p^*=1.0$)† of the ball charges in the two mills. The speed of the mills for all experiments was kept at 60% of the critical speed ($N^*=0.6$)†† and in wet grinding the percent solids was also kept constant at 60% by weight (F=0.6).

A detailed description of the procedure used for each batch experiment in the 10-inch and 15-inch mills is given below:

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† $M_p^*=M_p/M_{PC}$, where M_p is the mass hold-up and M_{PC} the mass of particles that completely fills the interstices between the balls (M_{BC}) under static conditions.

†† $N^*=N/N_C$, where N is the mill speed, N_C the critical speed of mill in revolutions per minute at which the first layer of the balls will centrifuge ($N_C=265/\sqrt{D-d}$ rpm, D the mill diameter in inches and d the maximum ball diameter in inches).

1. The mill was charged with balls and feed material in a layer loading manner[4,8] that is, alternating layers of balls and feed, to ensure thorough mixing at the start of each test. The replaceable end plate was locked into place for proper sealing. In the case of wet grinding 2200 grams and 4950 grams of water were added to the 10-inch and 15-inch mills, respectively, through the ports provided in the back plate of each mill (Figure 1). The mill was then transferred to the rollers and coupled to the drive shaft (Figure 3).
2. The mill speed was adjusted with variable speed arrangement provided on the shaft. 60% of critical speed corresponded to 54 rpm and 42 rpm for the 10-inch and 15-inch mills, respectively. The mill was allowed to run for a predetermined time. The mill revolutions were recorded by a counter attached to the shaft and the torque was recorded on a Sargent strip chart recorder.
3. After completion of a particular grind, the mill was uncoupled and transferred back to the loading port (Figure 4). The end plate was removed and the mill was gently discharged over a grizzley. The material was washed with water through the grizzley and was collected in buckets.
4. The ground product was wet screened through a 400 mesh screen mounted on a vibrator. The -400 mesh slurry was dewatered using a pressure filter, the cake and +400 mesh product thus obtained were dried under the infrared heat lamps.
5. The dry +400 mesh fraction of the product was weighed and

split using a rifle splitter to obtain a representative sample of about 500 grams. This sample was then sieved using $\sqrt{2}$-series Tyler screens (from 14 mesh down to 400 mesh) for 30 minutes on a rotap sifter.

6. The weight of each size fraction was determined using a two place Mettler balance. Then each screen was well cleaned to ensure the recovery of all the material sieved.
7. The -400 mesh cake was weighed and mixed with +400 mesh product, and additional -400 mesh material was added to make up for any weight loss (typically less than 0.4%), presuming the losses in handling were of very fine particles. The material was then kept for subsequent grinds.

The experimentation in the 30-inch mill was somewhat cumbersome as the design was not suitable for batch tests. The sequence of procedural steps was essentially the same as outlined for the 10-inch and 15-inch mills. The mill was loaded and unloaded through three 2 1/4 inch diameter ports, spaced at 120^{o} along the periphery of the mill (Figure 5). Extreme care was taken in loading and unloading of the mill to avoid particle breakage. The inlet and discharge ends of the mill were sealed with tin plates and putty. In the case of dry grinding tests the feed material weighed 46.54 kg for a 50% ball filling, while in wet grinding tests the feed weighed 37.23 kg for a 40% ball filling. Since, ultimately this scale-up study will be applied to a continuous system in which a maximum of 40% ball filling is practical, wet tests were carried out at a 40% ball filling. For 60% solids, 24.82 kg of water was added. The mill was rotated for a pre-determined time. The material was unloaded through the bottom port into several buckets. The balls and material were then separated by hand. The product was washed, dried and sieved as in the case of the 10-inch and 15-inch mills.

Batch tests were performed in the three different mills wet and dry with both the 10 x 14 mesh feed material and 'natural' (as received feed with the +10 mesh material removed). In addition an open circuit test was conducted in the 30-inch mill (N^*=.6, M_B^*=.4, F=.6) with 'natural' size distribution feed material. In this continuous test the mill was run with a feedrate of 1040 lbs/hr for 60 minutes to allow all transients in the mill to die out. Once steady state was achieved, samples of feed and product were taken simultaneously. In addition mill contents were discharged for analysis. For the analysis of these samples the same procedure was followed as in the 10-inch and 15-inch mill.

The data obtained from the above mentioned three mills for dry and wet grinding experiments are tabulated in Appendix I. The procedure used to determine net power draft for each of the mills and specific energy inputs for each experiments are given in Appendix II.

CHAPTER IV

ANALYSIS OF BREAKAGE KINETICS

In this chapter a preliminary analysis of data is presented. Breakage kinetics in the dry and wet grinding systems are described. The dependence of breakage and selection functions on the mill operating variables and verification of scale-up relationships is discussed.

Dry grinding kinetics were found to be essentially linear for all grind times in each mill. In contrast the wet grinding behavior exhibited a significant deviation from linearity. Feed size selection functions can be determined from the slope of feed disappearance plots ($\ln(m_i(t)/m_i(0))$ vs. t) according to the rearranged form of equation (II-1) i.e.

$$S_1(t) = - \frac{d}{dt} [\ln(m_i(t)/m_i(0))] \qquad \text{(IV-1)}$$

Figures 8 and 9 show plots of the fraction of feed remaining ($m_i(t)/m_i(0)$) versus time (t) for dry and wet grinding respectively for each mill. It is found that for the dry system the breakage kinetics are linear, that is the feed size selection function (S_1) is independent of the extent of grind time. In the wet system the deviation from linearity becomes significant when the fraction of feed remaining in the top size interval drops below 0.1 in the case of all the three mills (see Figure 9). It is also observed that

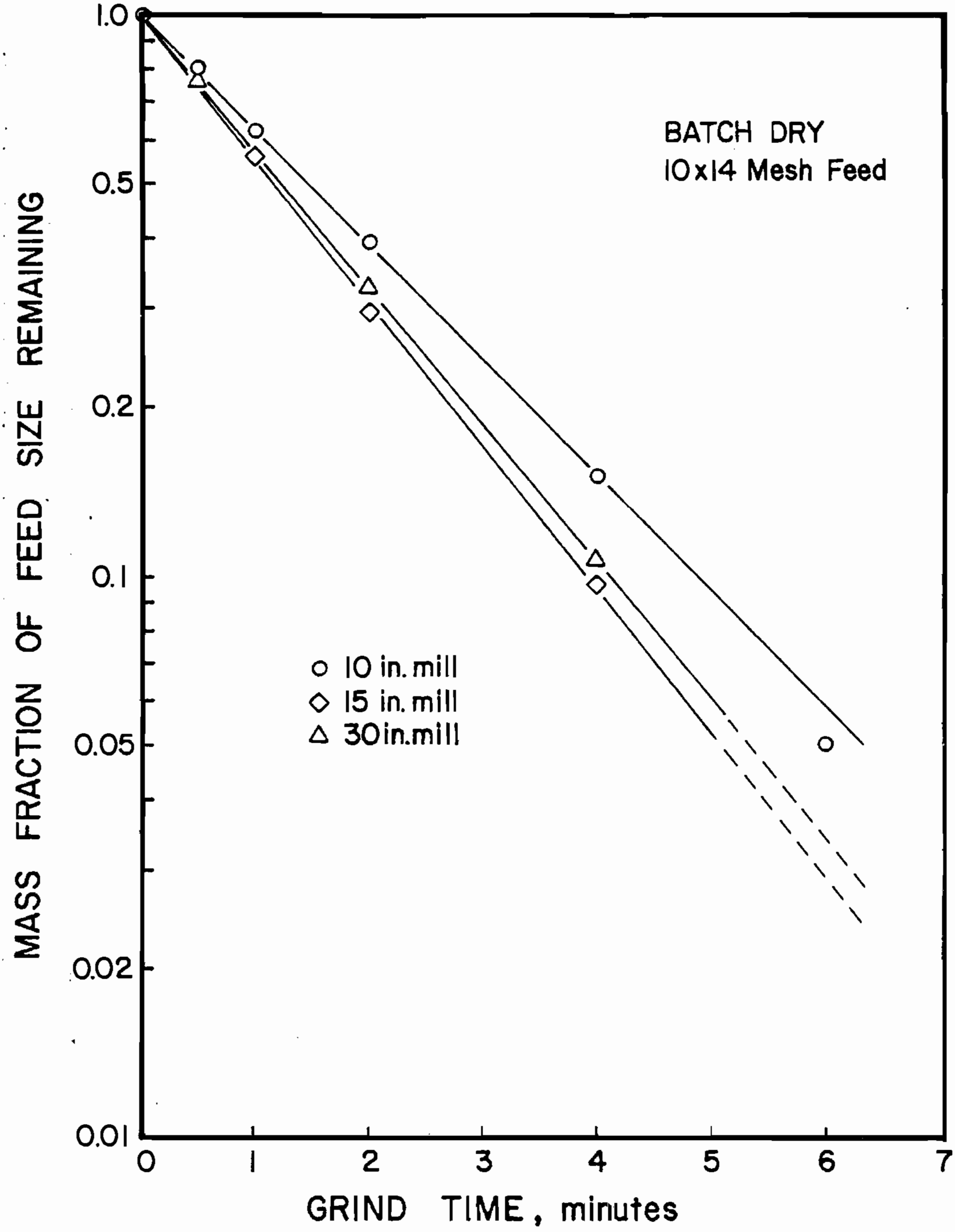


Figure 8. Feed disappearance plot for 10-in., 15-in. and 30-in. diameter mills for dry grinding (N^*=0.6, M_B^*=0.5).

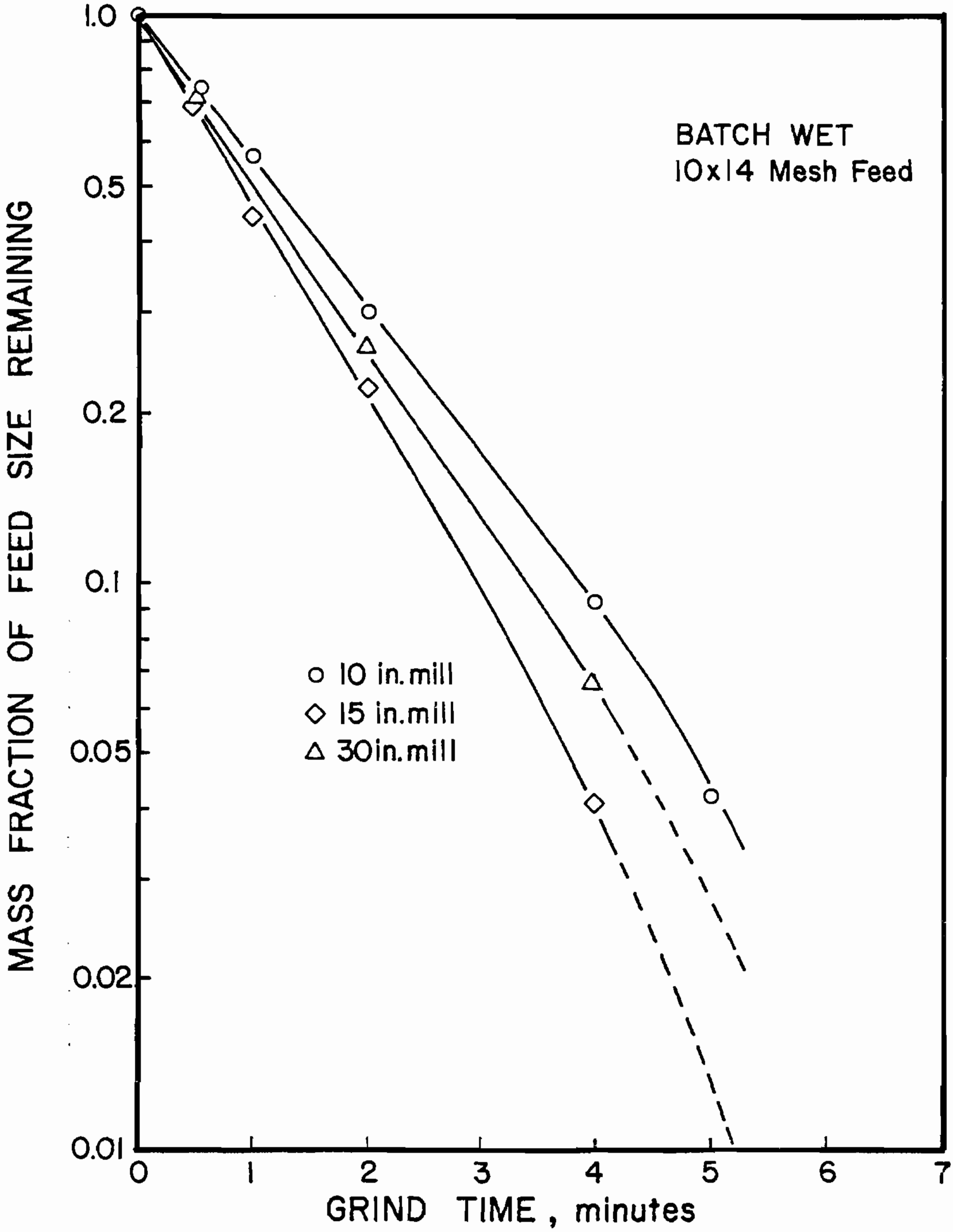


Figure 9. Feed disappearance plot for 10-in., 15-in. and 30-in. diameter mills showing the wet grinding nonlinearity (N^*=0.6, M_B^*=0.5, for 30-in. mill M_B^*=0.4, F=0.6).

the feed size selection functions depend on the mill diameter. This observation is in accordance with the findings of an earlier study by Malghan and Fuerstenau[6]. However, it is noticed from Figures 8 and 9 that in the case of 30-inch mill the feed size selection functions cannot satisfy equation (II-12) in relation to any of the two laboratory scale mills. The reason which could be put forward was that the lifters of the 30-inch mill were rounded (Figure 2c) and did not lift the balls high enough to draw more energy resulting in slower grinding. The effect of lifter configuration of the mills, on the breakage and selection functions is discussed later.

Writing the first-order disappearance kinetic equation for the normalized model (equation (II-8)), the following expression is obtained:

$$m_1(\overline{E}) = m_1(0) \exp [-S_1^E \overline{E}] \qquad \text{(IV-2)}$$

It predicts that the kinetics of breakage for the top size interval can be normalized with respect to the specific energy input to the mill. The appropriateness of this normalized relationship is illustrated in Figures 13 and 14 for the dry and wet systems, respectively. It is shown that all the kinetic data from the three mills reduce to a single line when grinding time is replaced by the specific energy input. For the dry grinding system the results are consistent with Malghan's[6,7] findings. Malghan has demonstrated the same results with various operating variables (mill speed, ball load, ball size and lifter configuration, mill diameter). For the wet system Kim[8] has described the normalizability phenomena with various operating variables (mill speed, ball load, particle load, percent solids) but only in 10-inch diameter batch mill.

Comparison of the specific selection functions for dry and wet grinding under the same operating conditions in the 10-inch mill show a slightly smaller value of S_1^E for dry grinding than for wet grinding. This may be due to the preferential grinding in the wet system in which the probability of breakage of coarse particles is higher resulting in higher rate of breakage (S_1) of material in the top size intervals. The energy drawn by the wet system is slightly higher than the dry system (Appendix II), but is insufficient to drop the numerical value of the specific selection function values of wet system. The value of S_1^E was 1.438 $(KWH/T)^{-1}$ for dry grinding and 1.714 $(KWH/T)^{-1}$ for wet grinding (Table 11-3, Appendix II).

The feed size cumulative breakage functions B_{i1} can be computed from the relationship[17]:

$$B_{i1} = \frac{F_i}{S_1} \qquad \text{(IV-3)}$$

where S_1 is the feed size selection function determined from a feed disappearance plot and F_i is the initial slope of the fines production plot for material finer than size X_i (Figure 10). Figures 11 and 12 show plots of feed size cumulative breakage functions versus particle size in dry and wet systems respectively, for the three different mills. It is found that to a good approximation the breakage functions are the same in the three mills.

The lifters in the 30-inch mill were not geometrically identical to the ones in the 10-inch and 15-inch mills. Therefore, it was

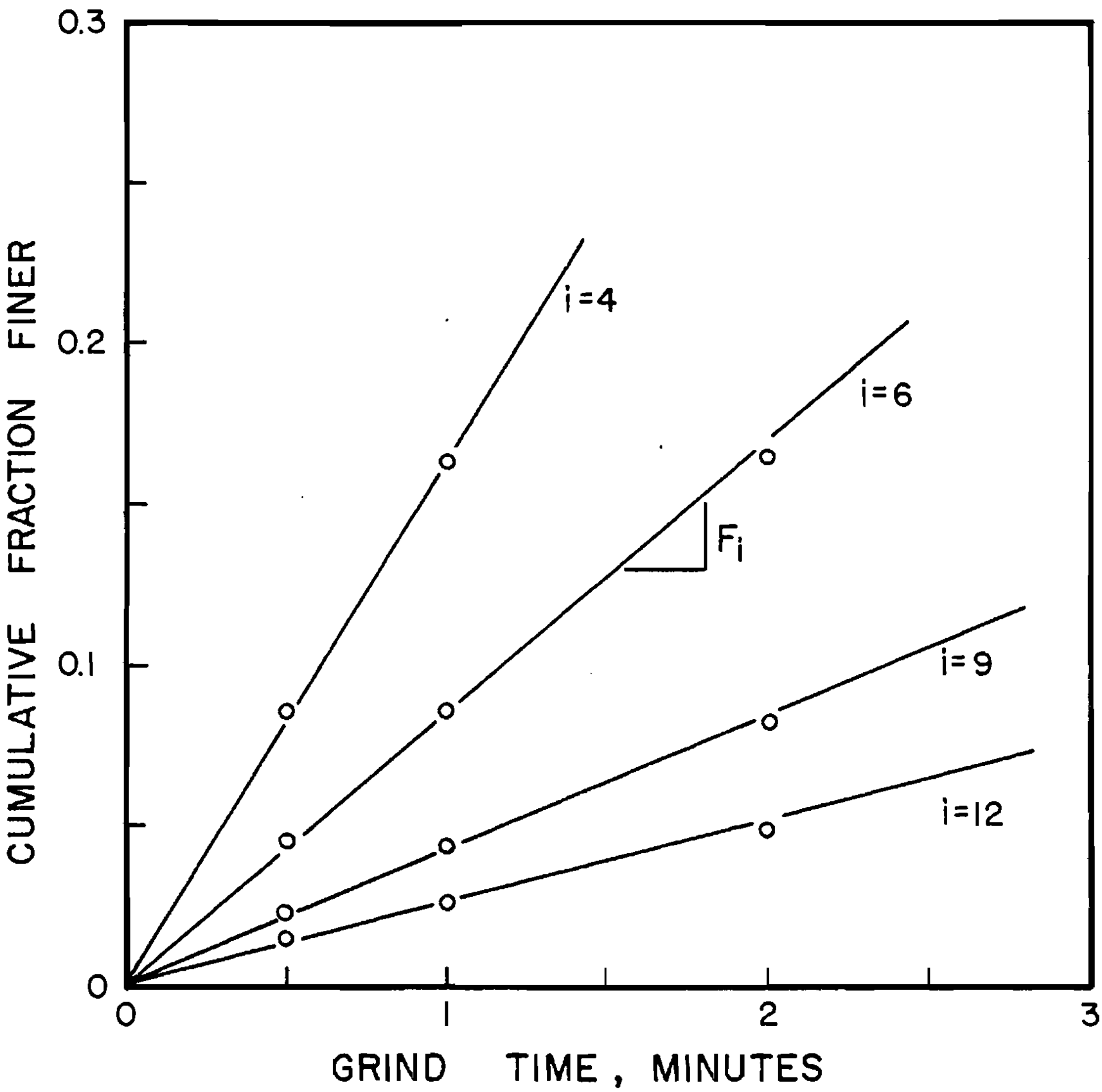


Figure 10. A sample of fines production plots for arbitrary chosen size intervals.

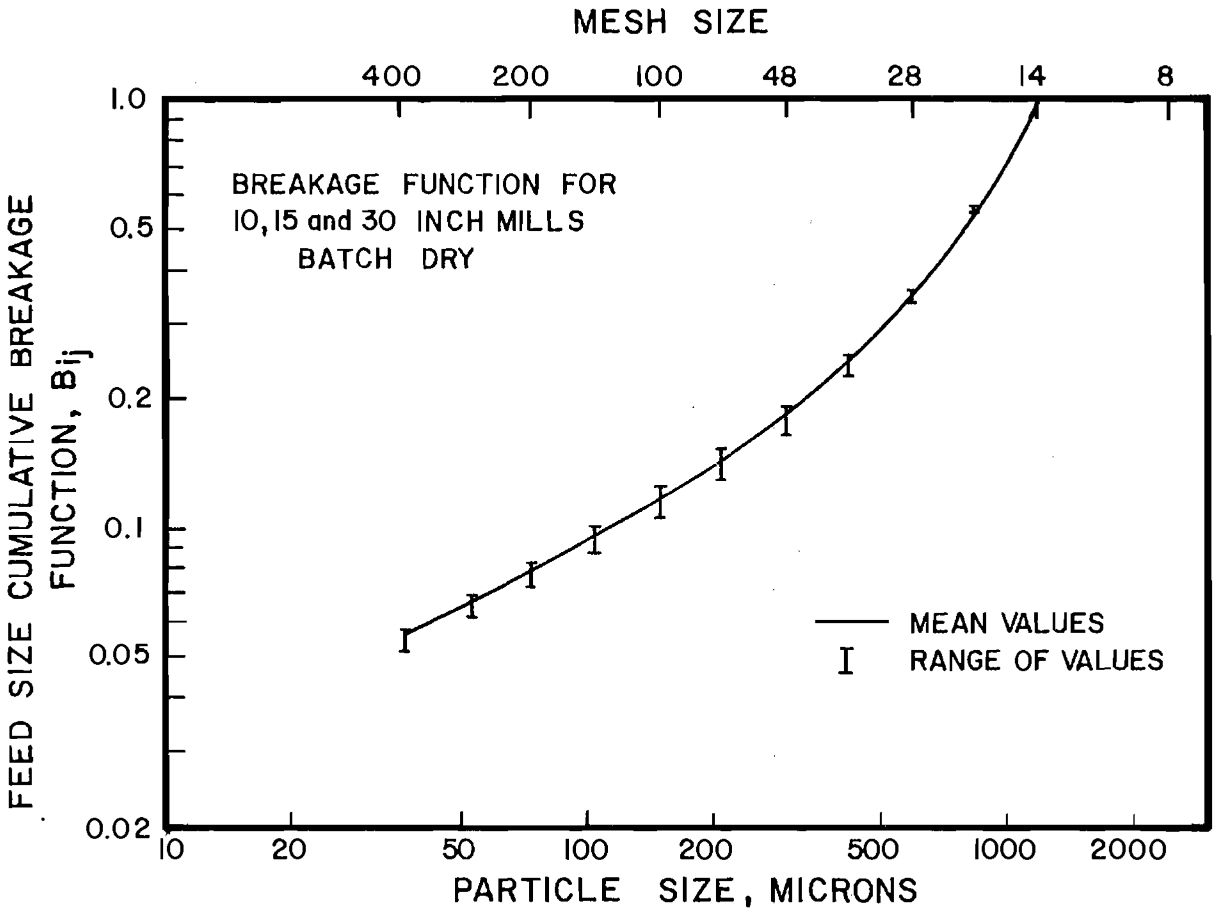


Figure 11. Feed size breakage functions for dry grinding in 10-in., 15-in. and 30-in. diameter mills (N^*=0.6, M_B^*=0.5).

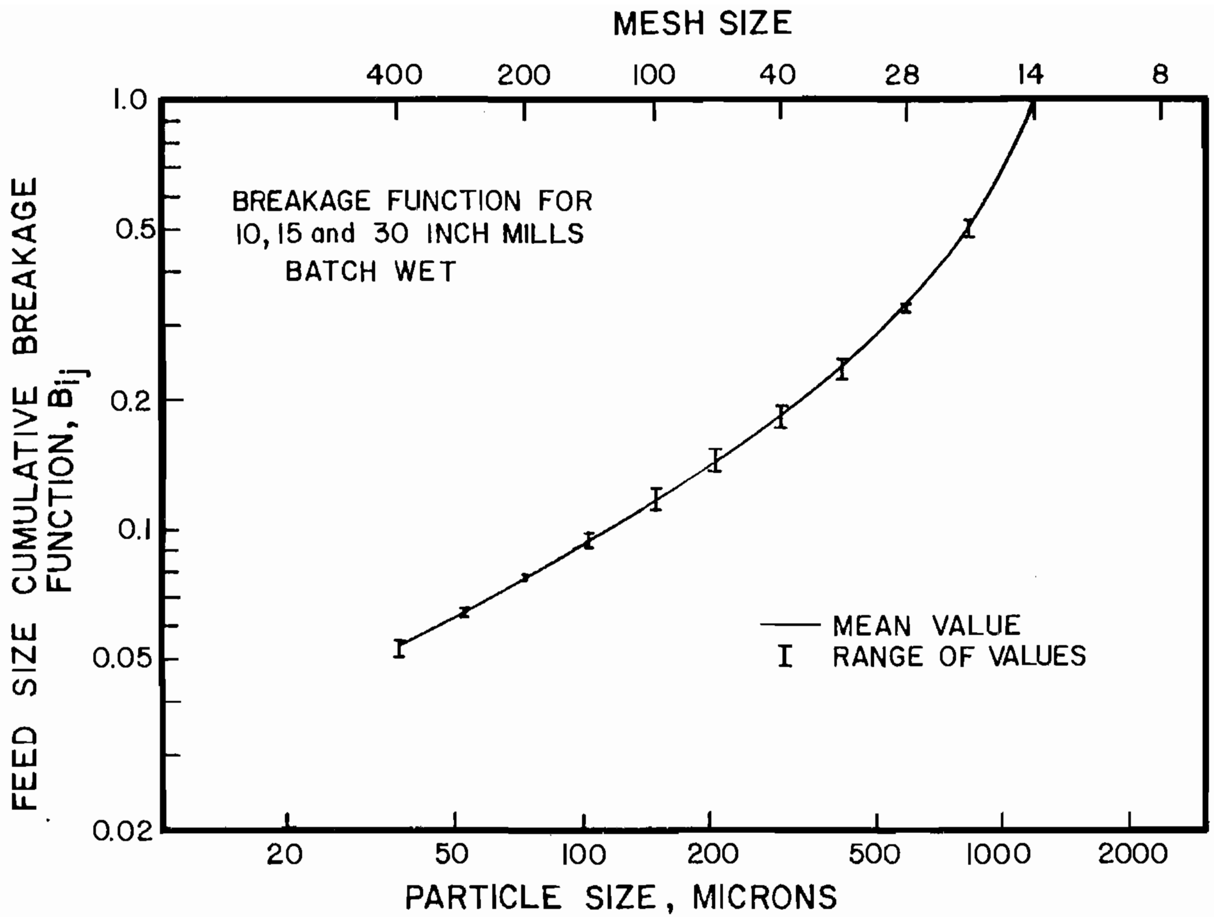


Figure 12. Feed size breakage functions for wet grinding in 10-in., 15-in. and 30-in. diameter mills. (N^*=0.6, M_B^*=0.5, for 30-in. mill M_B^*=0.4, F=0.6).

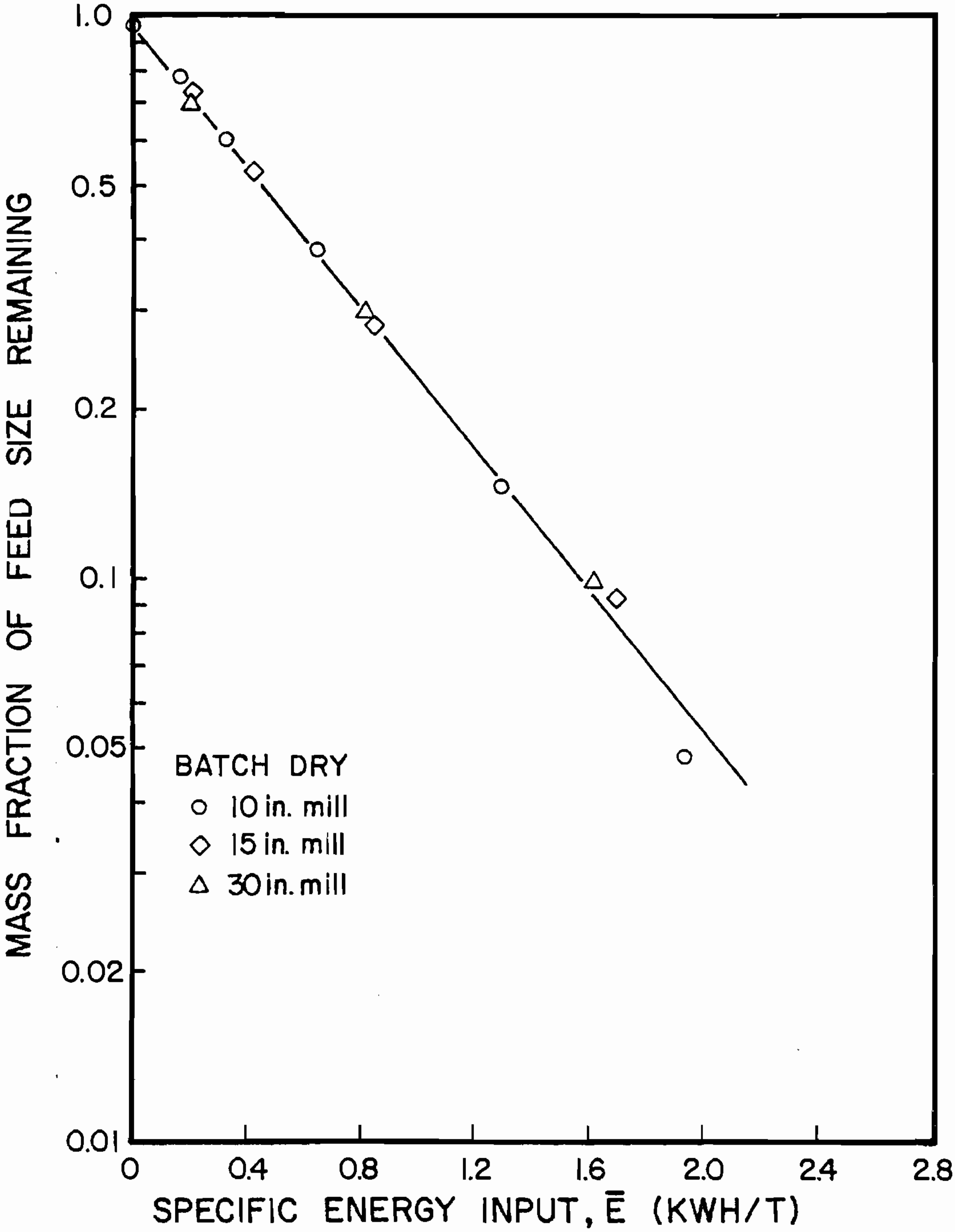


Figure 13. Normalized feed size disappearance plot for dry grinding in 10-in., 15-in. and 30-in. diameter mills (N^*=0.6, M_B^*=0.5).

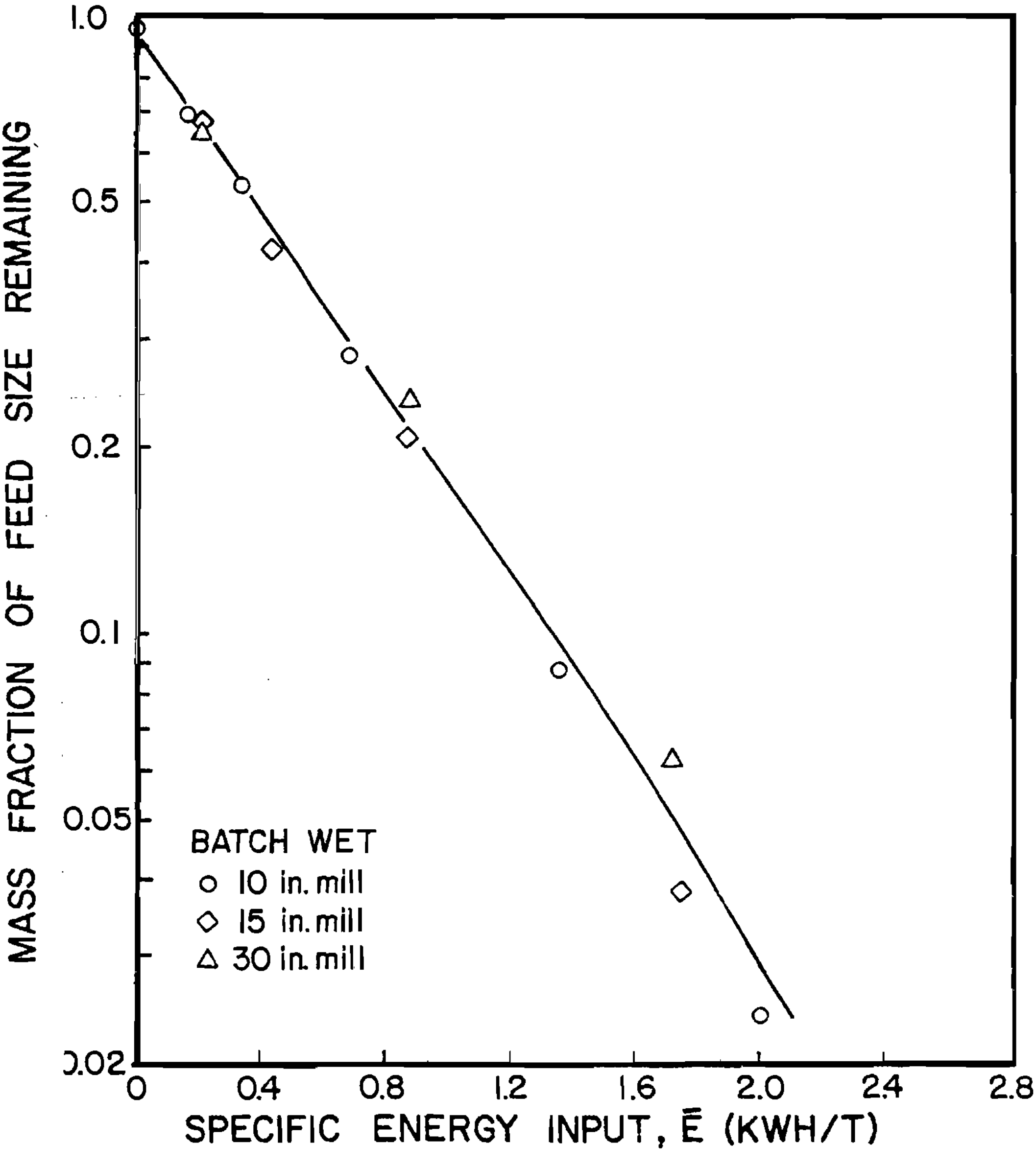


Figure 14. Normalized feed size disappearance plot for wet grinding in 10-in., 15-in. and 30-in. diameter mills, showing nonlinearity (N^*=0.6, M_B^*=0.5 for 30-in. mill M_B^*=0.4, F=0.6).

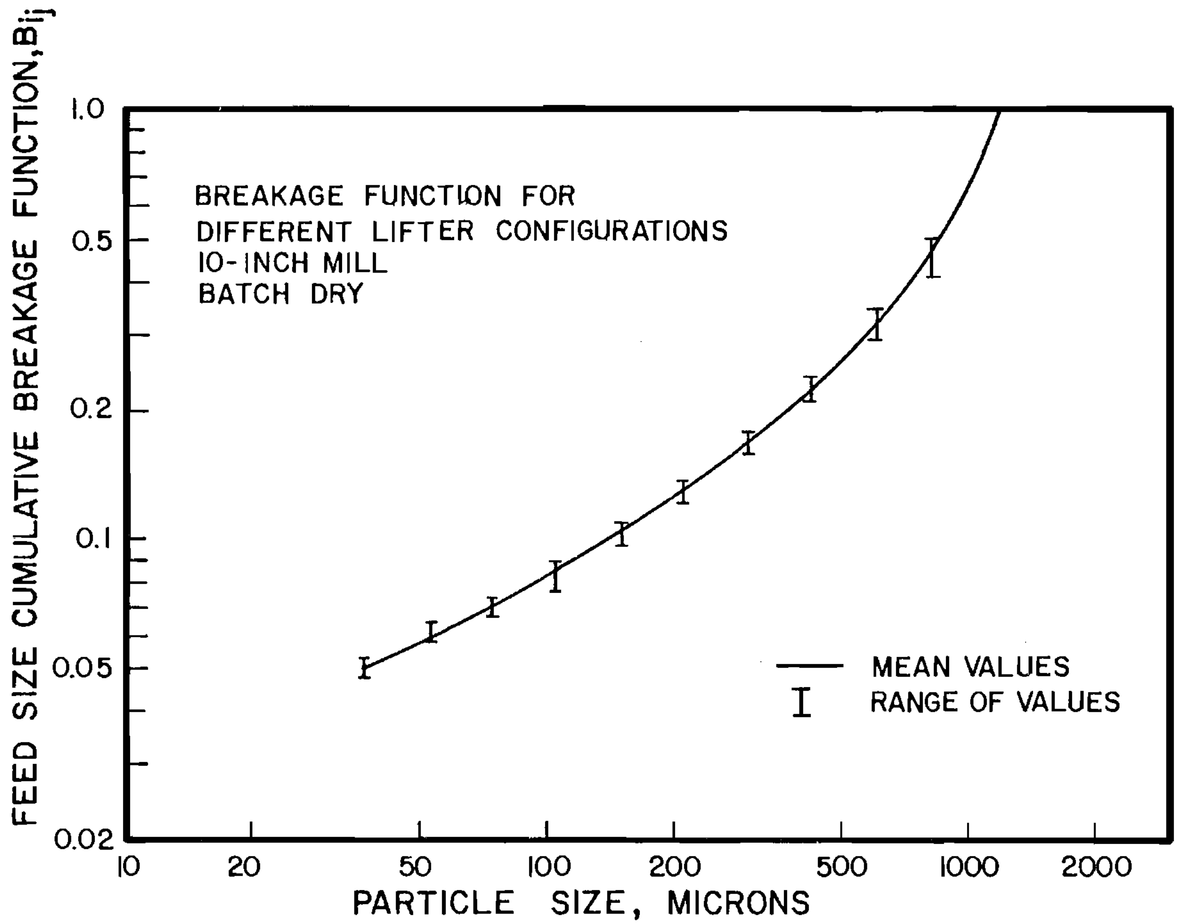


Figure 15. Feed size breakage functions for dry grinding in 10-in. diameter mill with square, ramp and without lifters configuration (N^*=0.6, M_B^*=0.5).

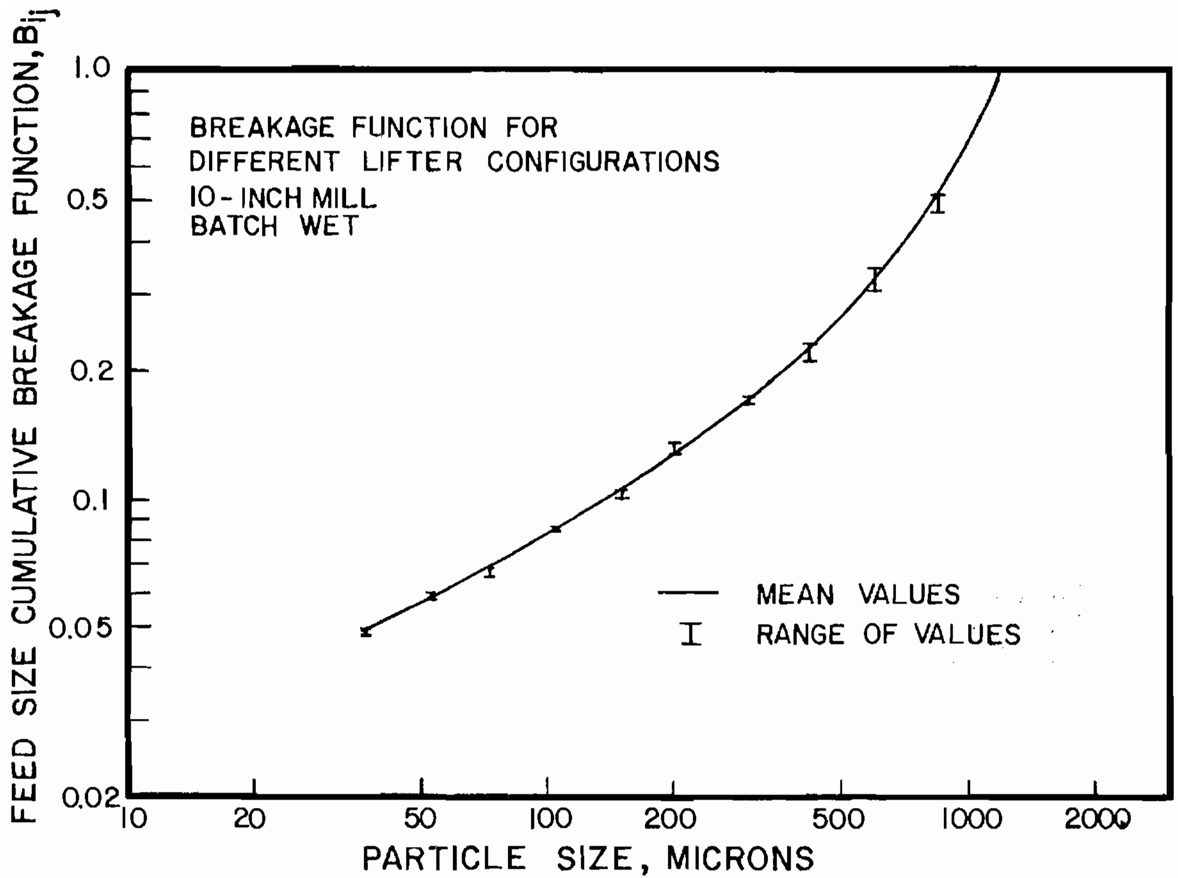


Figure 16. Feed size breakage functions for wet grinding in 10-in. diameter mill with square and ramp lifters configuration (N^*=0.6, M_B^*=0.5, F=0.6).

decided to perform experiments to determine the effect of lifter configuration on the breakage and selection functions. In the 10-inch diameter mill, experiments were performed with square lifters, ramp lifters and without lifters (Figure 2). It was found that the breakage functions were to a good approximation independent of lifter configuration in dry and wet systems. Figures 15 and 16 illustrate the range of variance of feed size cumulative breakage functions with different lifter configurations for dry and wet grinding respectively. These results supported the results already demonstrated in Figures 11 and 12 for the simulation purposes that the breakage functions are to a good approximation independent of mill diameter.

Figure 17 illustrates the specific power correlation for selection functions normalizibility is valid and independent of the lifter configuration and that the kinetics of breakage for the top size interval are normalized with respect to the specific energy input to the mill.

The results presented so far have demonstrated that the breakage functions and the specific selection functions are independent of mill diameter, ball load, and lifter configuration. These results suggest that a scale-up scheme based on the specific energy input to the mill would be a useful criterion for predictive simulation in the larger mills from the data obtained from the laboratory scale mill. In order to further confirm this hypothesis the predictive capability of the linear normalized model (equation II-8) should be tested for different mill sizes in dry and wet systems. In the wet system in which inherent nonlinearity exists the linear model has

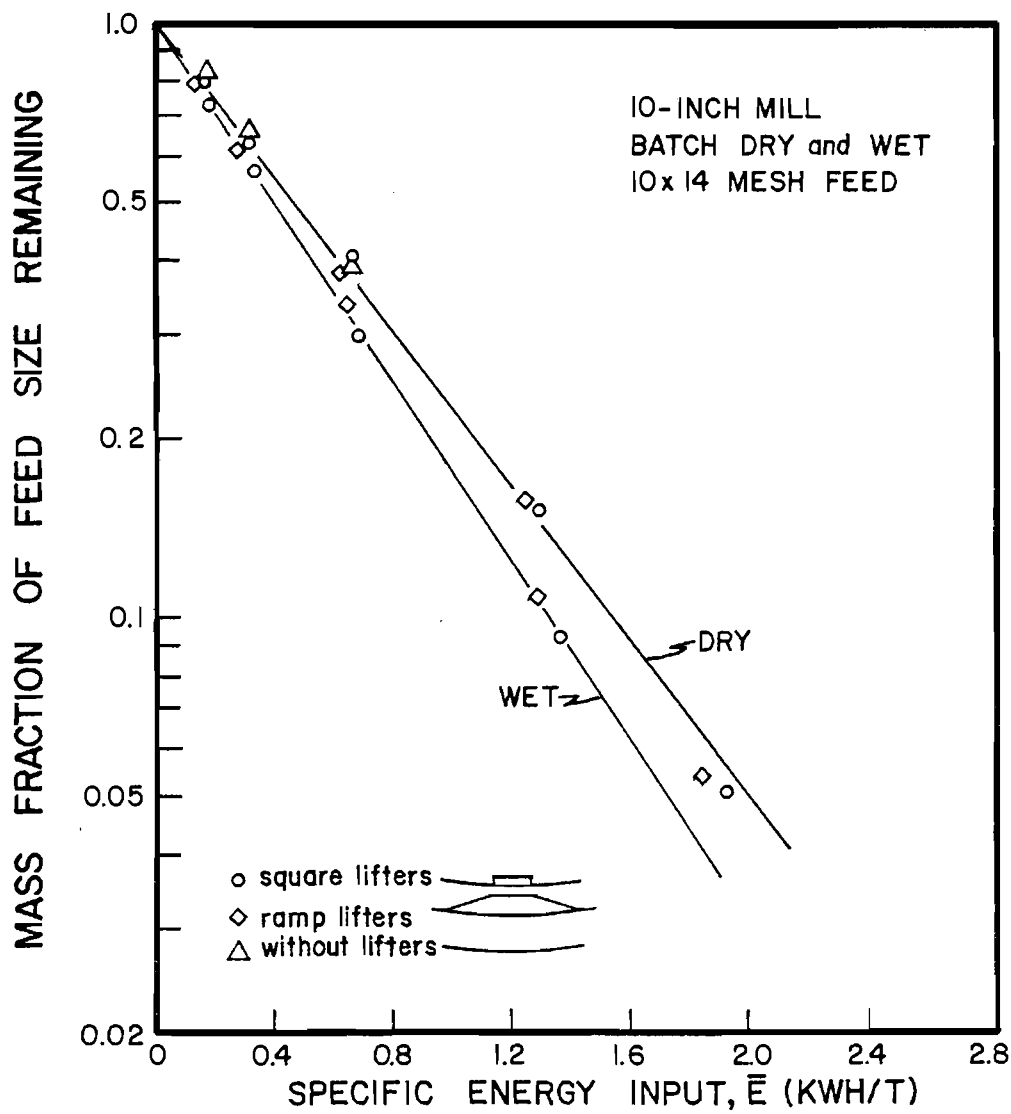


Figure 17. Normalized feed size disappearance plot for dry and wet grinding in 10-in. diameter mill with square, ramp and without lifters configuration. (N^*=0.6, M_B^*=0.5, F=0.6).

its limitations and should be applied over narrow ranges of specific energy input where the kinetics would be "nearly linear". The non-linearity phenomena is briefly discussed below.

Unlike dry grinding, wet grinding is inherently nonlinear. The spatial distribution of material in the mill plays an important role. Apparently in wet grinding system the fine particles tend to suspend in the water while the coarse particles are settled in the ball mass resulting in an increased probability of breakage of the coarse particles[4,8]. This phenomenon is termed as "preferential breakage". As the grind is extended more and more fine particles are produced resulting in the increased rate of breakage of coarse particles while the rate of breakage of fine particles decreases. It demonstrates that the selection function/specific selection function strongly depend upon the size distribution in the mill. This effect of nonlinearity is illustrated in Figure 18.

The linear model in its general form is not valid for simulation of nonlinear wet system. But simulations with estimates from 10-inch mill for similar fineness of grind are in good agreement with experimental product size distributions. The rationale for this approach is discussed in Chapter V.

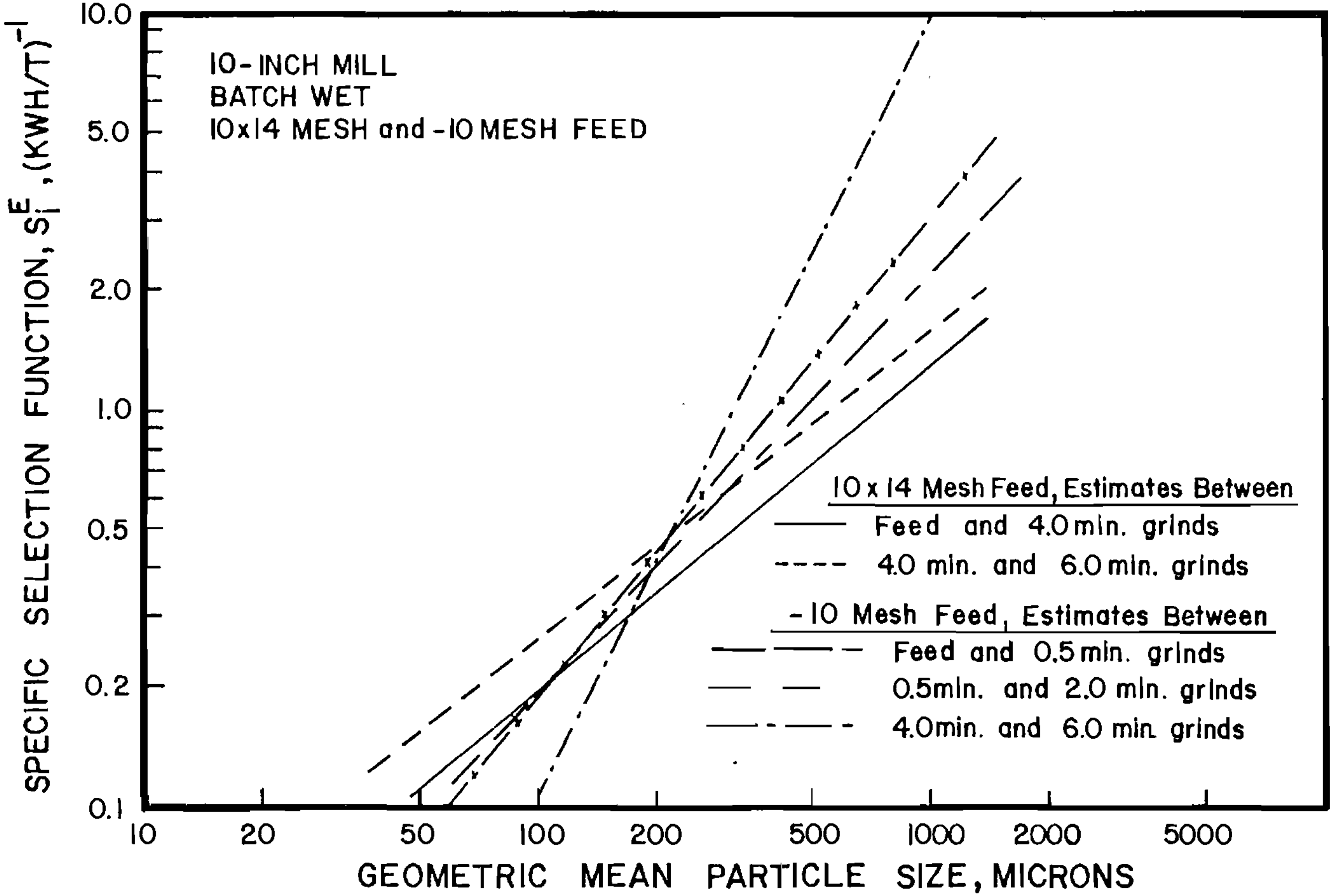


Figure 18. Dependence of specific selection functions on the particle size distribution in the ball mill, for 10x14 mesh feed and -10 mesh feed in the wet grinding, showing pronounced nonlinearity.

CHAPTER V

SCALE-UP PREDICTIONS

In this chapter the parameter estimation procedure used for predictive simulation is outlined and the application of the normalized linear model (equation II-8) to the scale-up predictions is discussed. The criterion for the correlation of kinetic parameters, selection and breakage functions, for scale-up design is established. A scheme to predict the behavior of grinding in the larger mills from the data obtained in the 10-inch batch mill is presented for the dry and wet systems. Accurate predictions of dry grinding behavior are achieved with the linearized model. Difficulties associated with the extension of the linear model to the prediction of wet system behavior are discussed. A 'similar fineness of grind approach' for estimation-prediction with the linear model is shown to give rise to good scale-up predictions in the wet system.

Parameter Estimation for Scale-up

For a complete simulation of batch and/or open circuit grinding, not only the feed size breakage parameters but also those of the smaller size fractions must be known. Batch tests are by far the simplest type of tests to use to obtain kinetic parameters experimentally. The results obtained from a batch test are easier to interpret quantitatively because they do not contain the complications of residence time[14,18]. Several methods for estimating the selection and breakage functions have been presented in the literature[8,12,14,15,17,18,20,24-26]. A direct measurement approach to estimation, similar to that described in the last section, could be used in conjunction with the grinding of a suite of single size fraction feeds or by using tracers. This approach involves a very large experimental effort. Instead for this thesis an indirect estimation scheme has been used involving nonlinear regression using initial estimates obtained by a method described by Herbst and Fuerstenau[17]. This indirect approach was developed within the framework of the linear model (equation II-1) which was formulated in terms of a continuous time variable and a discretized size variable. Estimates obtained by this method minimize the deviations between model predictions and experimentally observed size distributions[4,6,7,8]. This estimation procedure is as follows:

1. Selection function: Feed size selection function can be evaluated by equation IV-1. The remaining initial selection functions estimates (S_i, i = 2, ...,n-1) can be obtained by an expression[17]:

$$S_i = S_1 \left(\frac{\sqrt{x_i x_{i+1}}}{\sqrt{x_1 x_2}} \right)^{\alpha} \qquad \text{(V-1)}$$

where S_1 is evaluated from equation IV-1 and α is the distribution modulus of the breakage function.† For the dry

† The distribution modulus α is obtained by plotting the feed size cumulative functions B_{i1} versus particle size on a log-log plot. The slope of the plot in the fine particle size range is α (Figures 11, 12, 15, 16).

and wet grinding the value of α in the 10-inch mill was 0.56. Therefore the specific grinding rate function dependence on particle size can be approximated by a simple power law, obtained by combining equations (II-12) and (V-1):

$$S_i^E = S_1^E \left(\frac{x_i x_{i+1}}{x_1 x_2} \right)^{0.56} \qquad \text{(V-2)}$$

2. Breakage function: The breakage function estimation is based on the fundamental assumption that the size-discretized breakage function is normalizable, i.e.,[17]

$$B_{ij} = B_{i-j+1,1} \qquad \text{(V-3)}$$

where B_{ij} can be evaluated by a restrictive relationship between the breakage and selection functions:

$$B_{ij}\, S_j = F_i \qquad \text{(V-4)}$$

where S_j is obtained by equation (V-1) and F_i is the initial slope of the fines production plots (Figure 10).

3. Parameter improvement: Parameter estimation for a grinding model involves a large number of adjustable constants (for a 12 size fraction feed, 11 selection functions and 65 breakage functions) which makes the estimation problem difficult. In linear systems, the number of adjustable parameters can be reduced by adopting functional forms for

the selection and breakage functions[27].

$$S_i = S_1 \exp\left[\left\{\xi_1\left(\ln \frac{\sqrt{x_i x_{i+1}}}{\sqrt{x_1 x_2}}\right) + \xi_2\left(\ln \frac{\sqrt{x_i x_{i+1}}}{\sqrt{x_1 x_2}}\right)^2 + \ldots\right\}\right] \qquad \text{(V-5)}$$

$$B_{ij} = \phi \left(\frac{x_i}{x_{j+1}}\right)^{\alpha_1} + (1-\phi)\left(\frac{x_i}{x_{j+1}}\right)^{\alpha_2} \qquad \text{(V-6)}$$

where S_i is the selection function and B_{ij} is the cumulative breakage functions for the ith size interval and ξ_1, ξ_2, ϕ, α_1 and α_2 are adjustable parameters. In this case the number of adjustable parameters can be reduced to five or six these values can be estimated quite simply from batch data using a modified Gauss Newton nonlinear regression program[27]. This linear estimation approach can also be applied to nonlinear systems as special case for narrow ranges of values of specific energy input[8].

Dry Grinding

As discussed in Chapter IV, grinding kinetics are said to be linear when breakage parameters (selection and breakage functions) are environment independent. In dry grinding the selection and breakage functions are independent of grind time and feed size distribution. Figure 8 illustrates that in all the three mills the feed size selection function (S_1) is independent of the extent of grinding. In order to estimate the remaining S_i values the 10 x 14 mesh feed data obtained for the first four grind times (0.5, 1.0, 2.0, 4.0 minutes) in the 10-inch diameter mill were used to obtain initial estimates of the breakage parameters. These parameters values were then improved by the functional forms (equations (V-5) and (V-6) described earlier. As has been illustrated in Figures 13 and 17, for the dry system the specific selection function is independent of mill diameter, size distribution in the mill and lifter configuration. The set of selection functions obtained was then transformed to the specific selection functions by dividing by the specific power input (P/H) according to equation (II-5). The specific power input (P/H), values for the three different mills are tabulated in Table (II-1), Appendix II. The specific selection functions and the corresponding breakage functions are tabulated in Table (II-3), Appendix II. These values of the kinetic parameters were used in conjunction with equation (II-8) tosimulate the grinding behavior in the 15-inch and 30-inch diameter mills using mono-size (10 x 14 mesh) and 'natural' (-10 mesh) feeds.

Wet Grinding

In contrast to dry grinding, wet grinding is inherently nonlinear. As discussed previously, the nonlinearity appears to occur as a result of the 'preferential breakage' of coarse particles which are classified due to the suspension of finer particles in the fluid (water in this case) and settling of coarse particles in the ball mass. This phenomenon resulting in an increased probability of breakage of coarse particles and decreased probability of breakage of fine particles. This environment dependence is reflected in the selection functions. The nonlinearity is illustrated in Figure 9 in which it is observed that with a 10 x 14 mesh feed the nonlinearity occurs when the fraction of feed remaining is below 0.1 after about 4.0 minutes of grinding in the 10-inch mill. This effect is shown even more dramatically in Figure 18 which shows specific selection functions estimated for various fineness of grind in the wet 10-inch mill. As the fineness of the product in the mill increases the top size specific selection functions increase and the fine size specific selection functions decrease.

The specific selection function dependence on product fineness depicted in Figure 18 was obtained in the following way using the linear model. The initial estimates of breakage parameters were obtained from the first four grind times (0.5, 1.0, 2.0, 4.0 minutes), which constitute the coarse grind or 'nearly linear' region in the wet grinding system. The parameters were improved by the functional forms described earlier. For the wet grinding system it was assumed that all of the specific selection functions are independent of mill diameter, ball load and lifter configuration (Figures 14 and 17). The breakage functions and the specific selection functions obtained are tabulated in Table (II-3), Appendix II and the corresponding specific power (P/H, KW/T) and energy (Pt/H, KWH/T) values are tabulated in Table (II-2), Appendix II. Predictions for the 15 and 30-inch mills which are based on the estimates obtained for the region where the breakage kinetics are 'nearly linear', are referred to as Method I predictions in the discussion which follows.

Since it has been established that the kinetics of breakage are highly nonlinear for extended wet grinding it was anticipated that method I predictions may be inappropriate for fine grinding. Thus an alternative set of parameter estimates termed Method II estimates were also obtained from the 10-inch mill wet data.

The rational for this alternative estimation scheme is as follows:

a) The linear model is not strictly valid for wet grinding, particularly for extended grinding[4,8]. However, the linear model can be fitted to wet grinding data in the nonlinear range and the kinetic parameters obtained can be used to predict the grinding behavior in the "neighborhood" of the data used for estimation. The size of the "neighborhood" for which accurate predictions can be made with this linearization technique is determined by the extent of nonlinearity[4,8,21].

b) It appears that in wet systems the specific selection functions (S_i^E) values are independent of mill design and operating variables, they are only functions of the size distribution in the mills, i.e., $S_i^E(m_i,\ i = 1,2,3....n)$. In this context the equation (II-8) can be written as

$$\frac{d\,m_i}{d\bar{E}} = -\ S_i^E(\underline{m})\ m_i + \sum_{j=1}^{i-1} b_{ij}\ S_j^E(\underline{m})\ m_j \qquad \text{(V-7)}$$

and in the neighborhood of a $\underline{m}^*$, reference set of mass fractions, the equation assumes the form:

$$\frac{d\,m_i}{d\bar{E}} = -\ S_i^E(\underline{m}^*)\ m_i + \sum_{j=1}^{i-1} b_{ij}\ S_j^E(\underline{m}^*)\ m_j \qquad \text{(V-8)}$$

where $S_i^E(\underline{m}^*)$ can be taken as constant.

c) Based on (a) and (b) above it seems likely that if parameter estimates of specific selection functions and breakage functions are obtained in the 10-inch mill for a "similar fineness of grind" to that for which predictions are required in the larger mills, these values should allow accurate predictions of large mill behavior in the "neighborhood" of the size distribution used for estimation.

Based on this scheme the breakage parameters were estimated from data obtained within narrow ranges of values of specific energy input to the 10-inch mill using mono-size (10 x 14 mesh) feed. Three ranges were selected for estimation using (i) 2.0 and 4.0 minute products, (ii) 4.0 and 5.0 minute products, and (iii) 5.0 and 6.0 minute products, the corresponding specific energy values input to the 10-inch mill were obtained from Table (II-2), Appendix II. The values of specific selection functions thus obtained are tabulated in Table (II-4), Appendix II. For a 'natural' (-10 mesh) feed ground in the 10-inch mill, four ranges were selected for estimation using (i) 0.5, 2.0, 4.0 and 6.0 minute products, (ii) 0.5 and 2.0 minute products, (iii) 2.0 and 4.0 minute products, and (iv) 4.0 and 6.0 minute products, the corresponding specific energy input to the 10-inch mill values were obtained from Table (II-2), Appendix II. The specific selection functions thus obtained are tabulated in Table (II-5), Appendix II. The specific selection functions obtained using 10 x 14 mesh feed and -10 mesh feed were used to simulate grinding behavior in the 15-inch and 30-inch mills for grinding the corresponding feed with the corresponding energy input within the range for which the estimates were obtained.

Predictive Simulation

In this section, the data obtained in the 10-inch diameter mill were used to test the predictive capability of the normalized linear model equation (II-8) for dry and wet grinding. In the earlier section the data obtained in the three different mills have been shown to be highly correlated with the net power drawn by the mill. Empirical relationships between the parameters of the phenomenological model and the specific power input to the mill have been developed. In addition, the energy input per unit mass of material being ground has been found to be the independent variable which normalizes the linear model to a form which is independent of mill design and operating variables. The cumulative feed-size breakage functions have also been found to be independent of operating variables and the nonfeed size breakage functions were assumed to be normalizable. The two sets of breakage parameters (specific selection functions and breakage functions) formed the basis for predicting grinding behavior in the larger mills for the corresponding specific energy input to the mill for dry and wet grinding systems.

Dry Grinding

For a complete simulation of batch grinding, the data for the breakage parameters was obtained experimentally in the 10-inch diameter mill using mono-size (10 x 14 mesh) feed at $N^*=0.6$, $M_B^*=0.5$ and $M_P^*=1.0$.

Since the breakage functions and the specific selection functions are environment independent, the breakage kinetics are said to be linear. Therefore the breakage parameters obtained for a mono-size (10 x 14 mesh) feed were used to predict product size distribution even for the 'natural' (-10 mesh) feeds. The predictions of breakage behavior in 15-inch and 30-inch mills are in good agreement with the experimental product size distribution.

Figure 19 shows a comparison of the experimental product size distribution with the normalized fittings of the model using the initial estimates of S_i^E and B_{ij} from the 10-inch mill shown in Table (II-3), Appendix II. Figure 20 shows the normalized predictions for 10-inch mill using a 'natural' feed. This confirms that the linear model is applicable and that the breakage functions and specific selections functions are independent of size consist in the mill for a dry grinding system. Figures 21 and 22 show the normalized predictions for the 15-inch mill using mono-size (10 x 14 mesh) and 'natural' (-10 mesh) feeds, respectively. The experimental and predicted product size distributions are in good agreement. Figures 23 and 24 show such predictions for the 30-inch mill.

Figure 25 shows a comparison of the dry and wet specific selection functions using 10 x 14 mesh feeds, indicating that under the same operating conditions and feed the two specific selection functions are not the same.

Wet Grinding

Unlike dry grinding, wet grinding was inherently nonlinear. The specific selection functions strongly depended on the size consist in the mill (Figure 18). It is observed in Figure 18 that geometric mean particle size of about 200 microns was more or less a pivotal point indicating that particles finer than that size had higher probability of suspension in the water resulting in nonlinearity in the

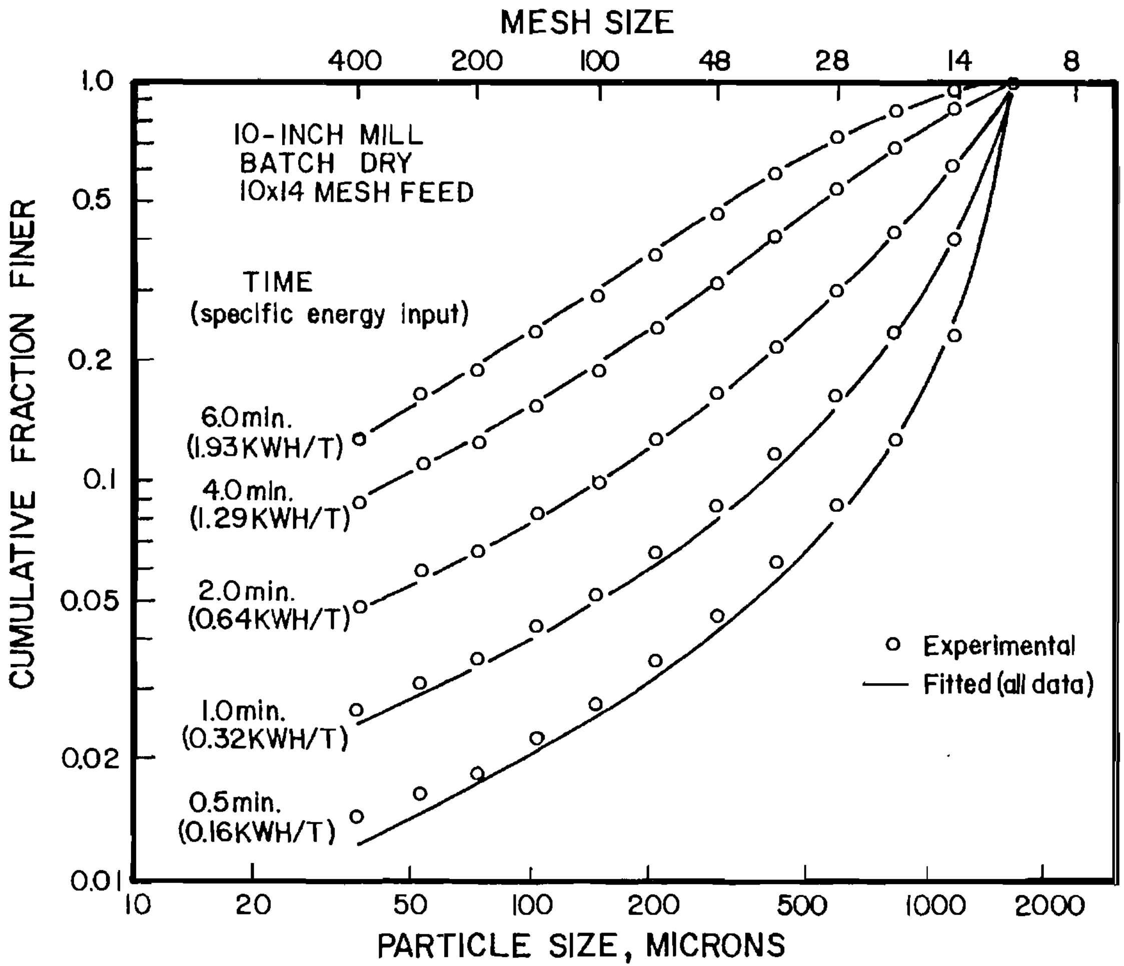


Figure 19. Comparison of experimental product size distribution for dry grinding in 10-in. diameter mill (10x14 mesh feed) and normalized fittings with initial S_i^E and B_{ij} estimates shown in Table (II-3), Appendix II.

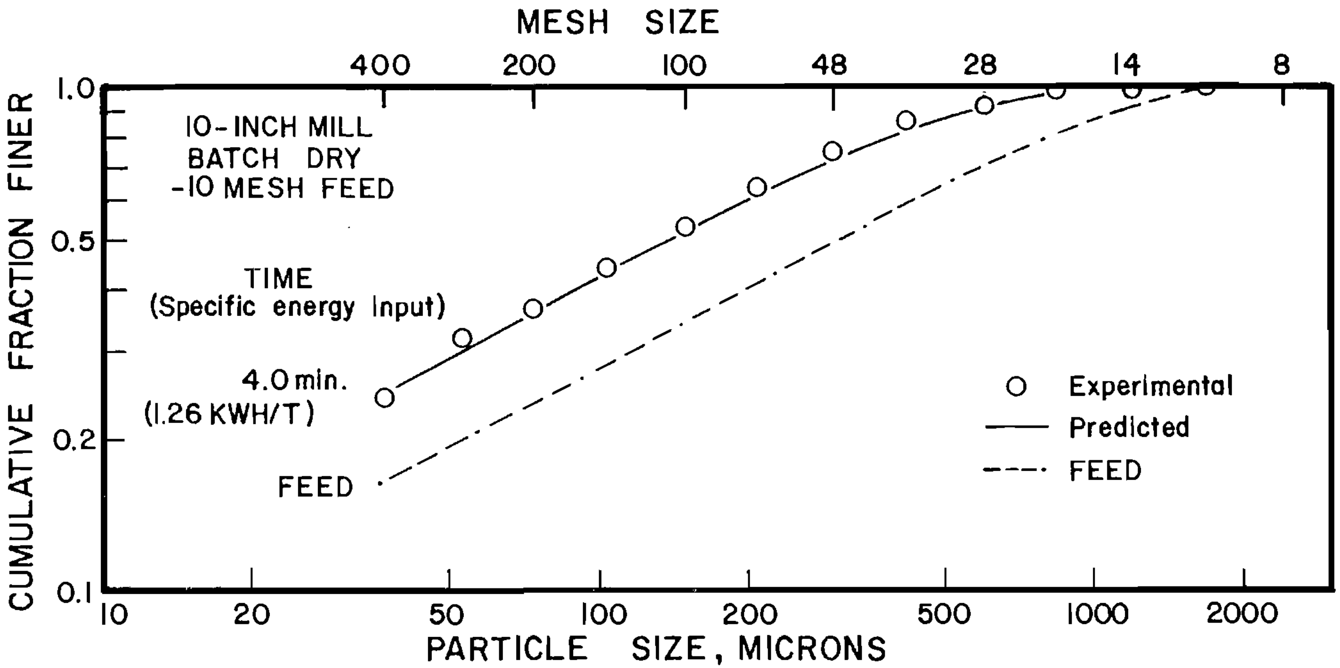


Figure 20. Comparison of experimental product size distribution for dry grinding in 10-in. diameter mill (-10 mesh feed) and normalized predictions with initial S_i^E and B_{ij} estimates shown in Table (II-3), Appendix II.

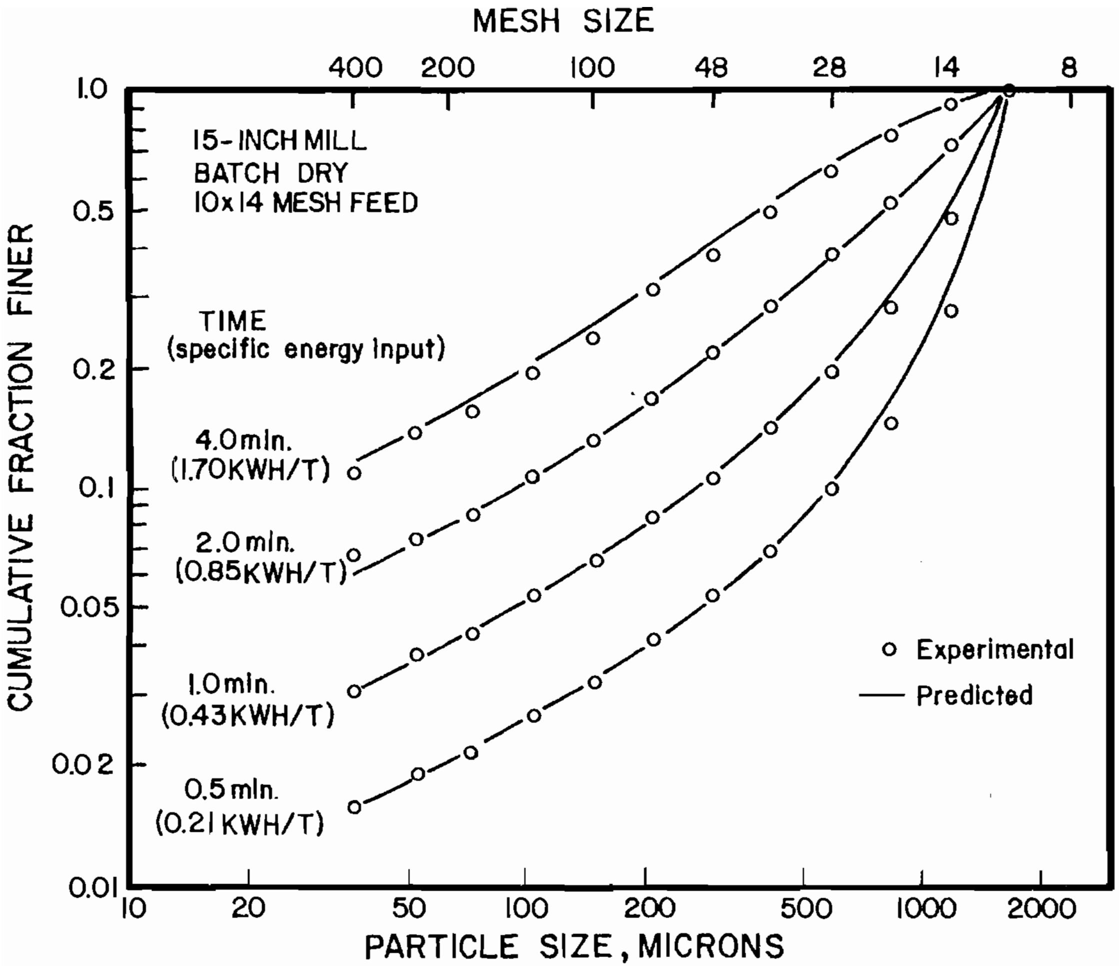


Figure 21. Comparison of experimental product size distribution for dry grinding in 15-in. diameter mill (10x14 mesh feed) and normalized predictions with initial S_i^E and B_{ij} estimates obtained from 10-in. diameter mill, shown in Table (II-3), Appendix II.

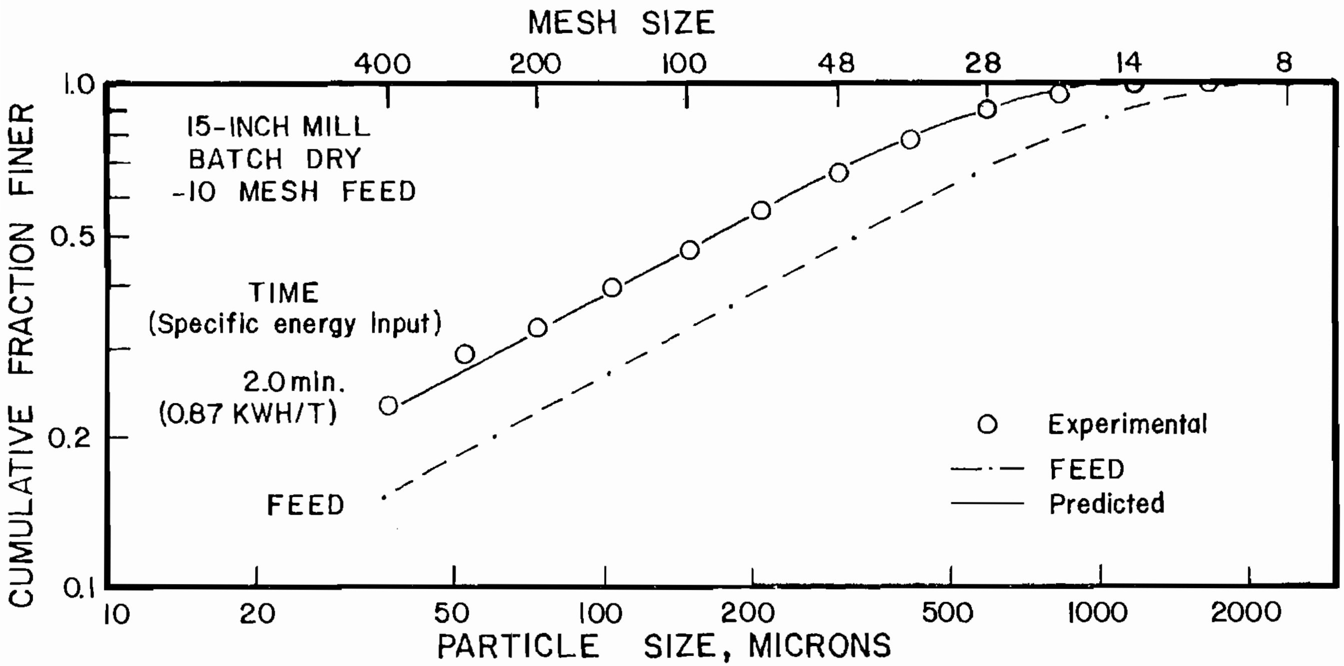


Figure 22. Comparison of experimental product size distribution for dry grinding in 15-in. diameter mill (-10 mesh feed) and normalized predictions with initial S_i^E and B_{ij} estimates obtained from 10-in. diameter mill, shown in Table (II-3), Appendix II.

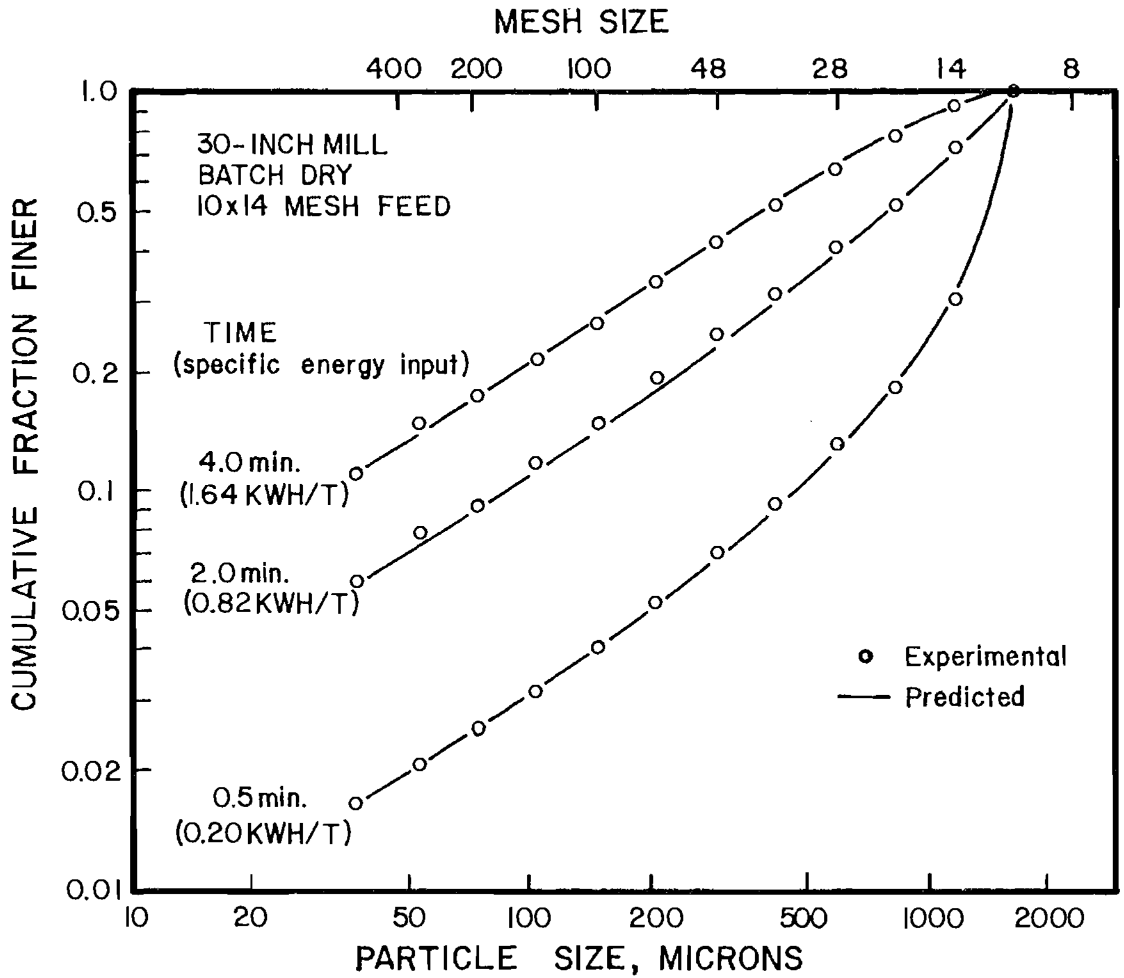


Figure 23. Comparison of experimental product size distribution for dry grinding in 30-in. diameter mill (10x14 mesh feed) and normalized predictions with initial S_i^E and B_{ij} estimates obtained from 10-in. diameter mill, shown in Table (II-3), Appendix II.

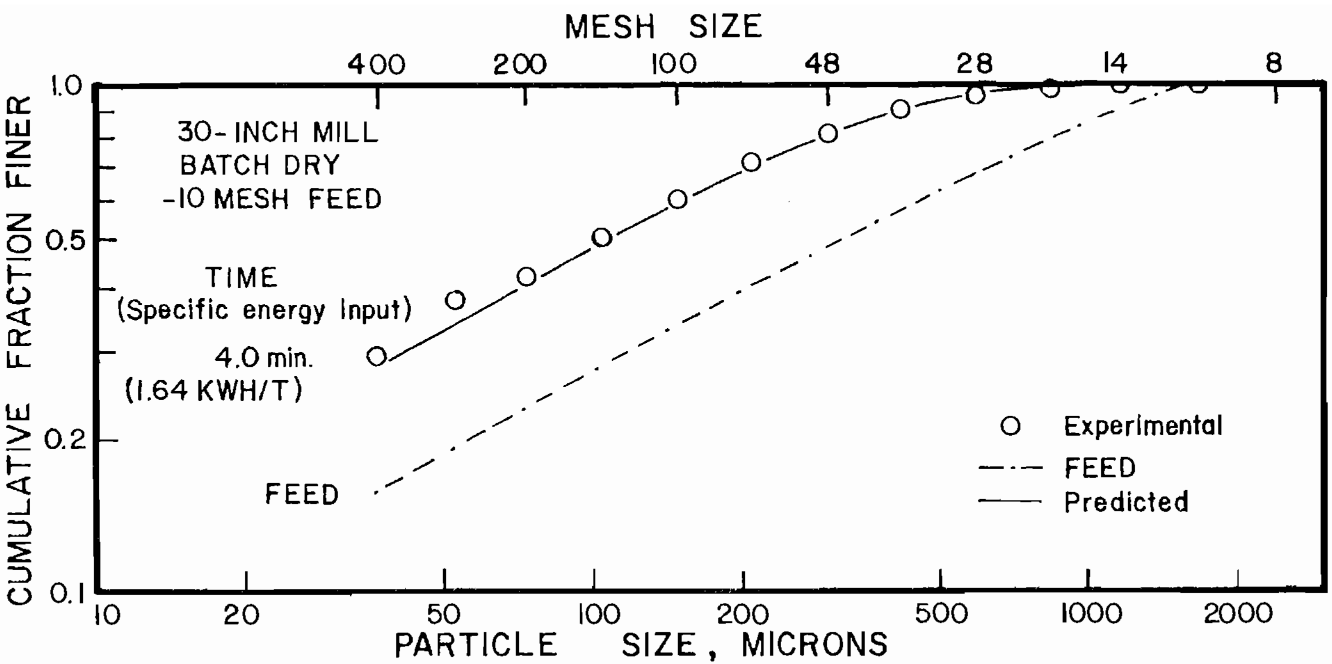


Figure 24. Comparison of experimental product size distribution for dry grinding in 30-in. diameter mill (-10 mesh feed) and normalized predictions with initial S_i^E and B_{ij} estimates obtained from 10-in. diameter mill, shown in Table (II-3), Appendix II.

system. In the absence of replicate experimental data it was not possible to test whether the pivotal point could serve as a fixed point to determine set of specific selection functions by knowing the top size selection function. However, this observation is open to further investigations. The effect of nonlinearity was strong when a 'natural' feed ($\simeq$ 40% below 200 microns) was ground.

To obtain breakage parameters experimentally in the 10-inch diameter mill using mono-size and natural feeds experiments were performed at N^*=0.6, M_B^*=0.5, M_P^*=1.0 and F=0.6 (percent solids). The breakage parameters obtained for 'nearly linear' region (0.0→0.5, 1.0, 2.0, 4.0 minutes) are tabulated in Table (II-3), Appendix II. The specific energy input to the mill values are tabulated in Table (II-2), Appendix II. Figure 26 shows the fitting of these estimates to the normalized model for the 10-inch mill. The agreement is good as the region was a 'nearly linear' region.

Since the breakage functions were independent of the environment, the specific selection functions were the parameters to be estimated for various predictions. Two different approaches were used to estimate the specific selection functions.

Method I: This approach is identical to the one used in dry grinding and assumed wet grinding kinetics to be linear. The linearized estimates (Table II-3), Appendix II obtained for the 'nearly linear' region were used for simulation in this method. The predictions by this method were not expected to agree with the experimental product size distribution, particularly for finely ground products. Figure 27 shows the predictions (firm lines) for the 10-inch mill

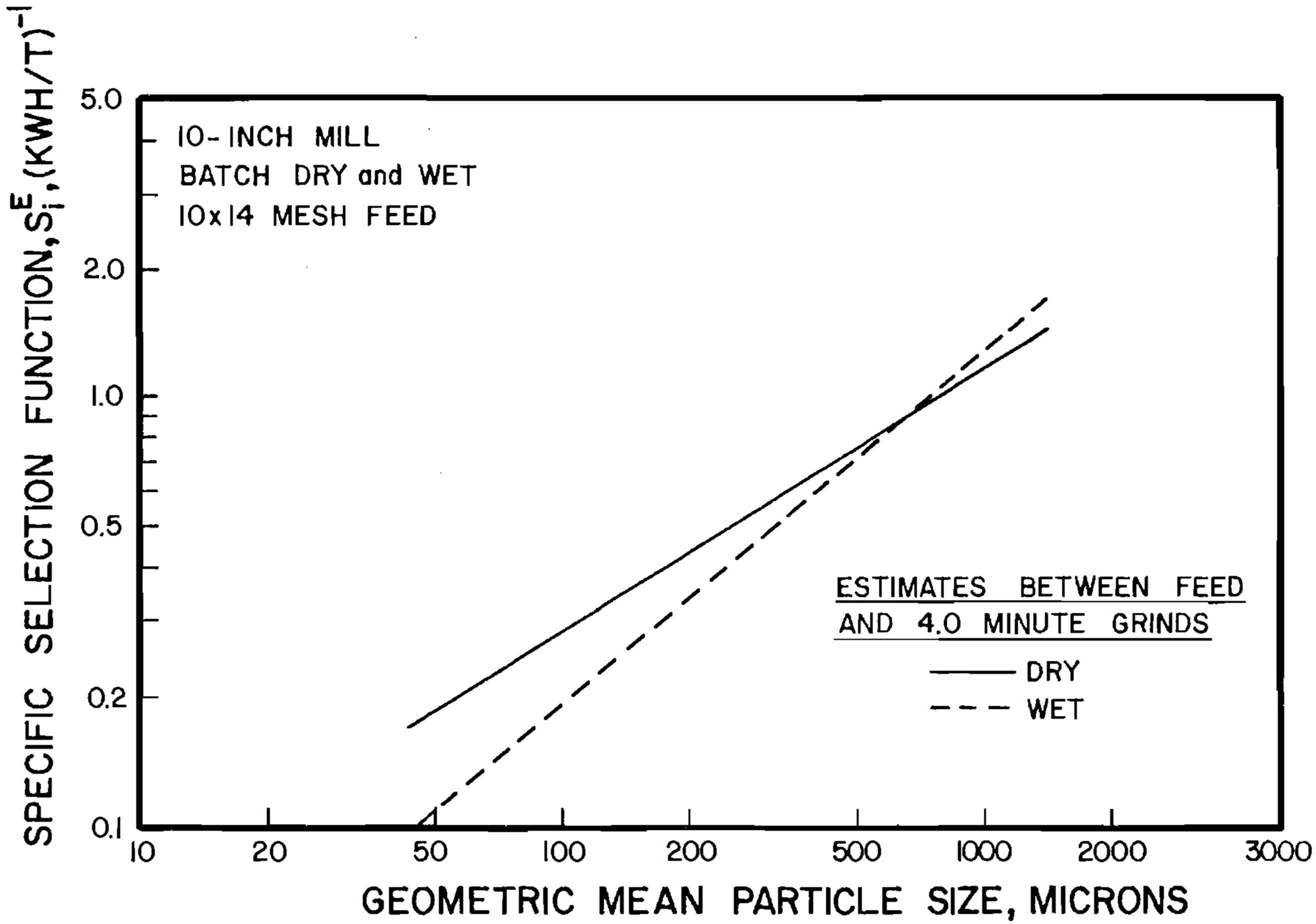


Figure 25. The comparison of dry and wet specific selection functions, S_i^E, (KWH/T) obtained between 10x14 mesh feed and first four grinds.

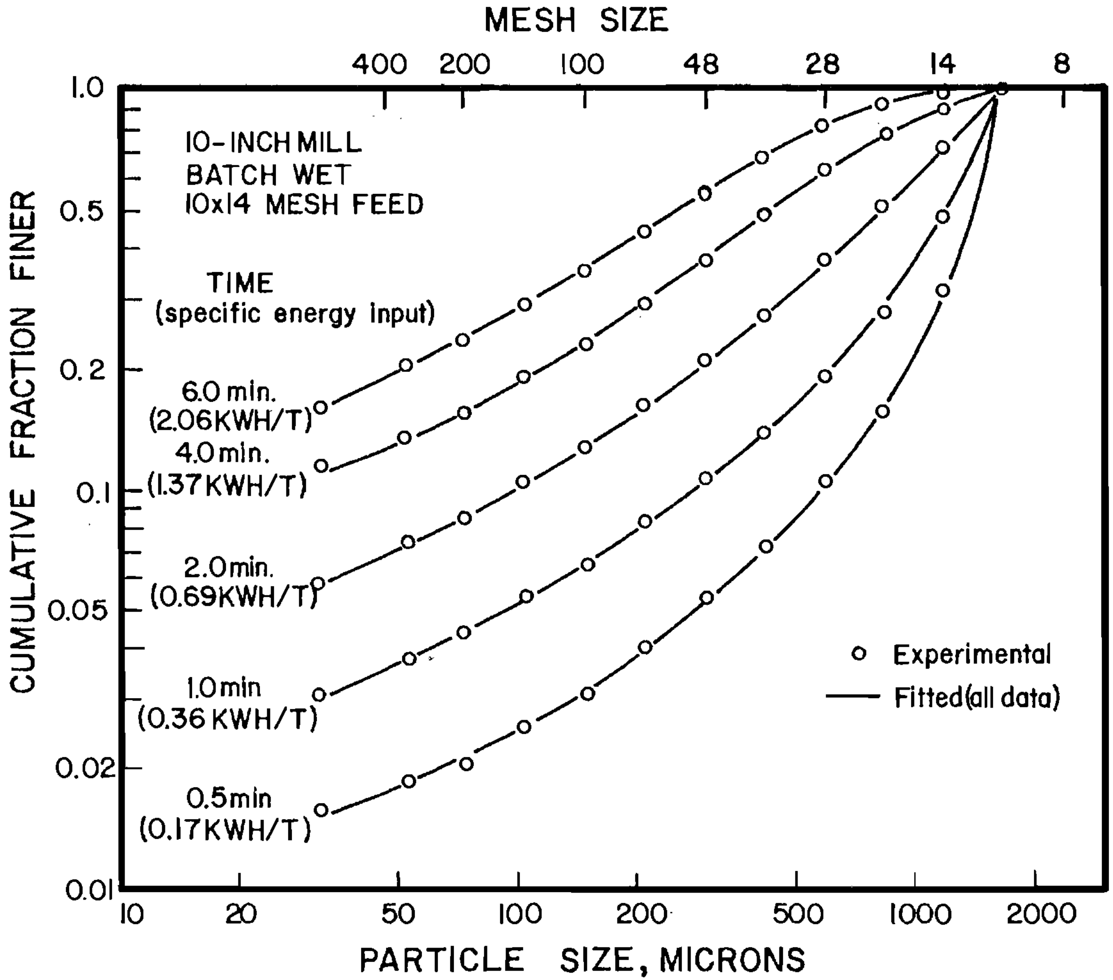


Figure 26. Comparison of experimental product size distribution for wet grinding in 10-in. diameter mill (10x14 mesh feed) and normalized fittings with initial S_i^E and B_{ij} estimates, shown in Table (II-3), Appendix II.

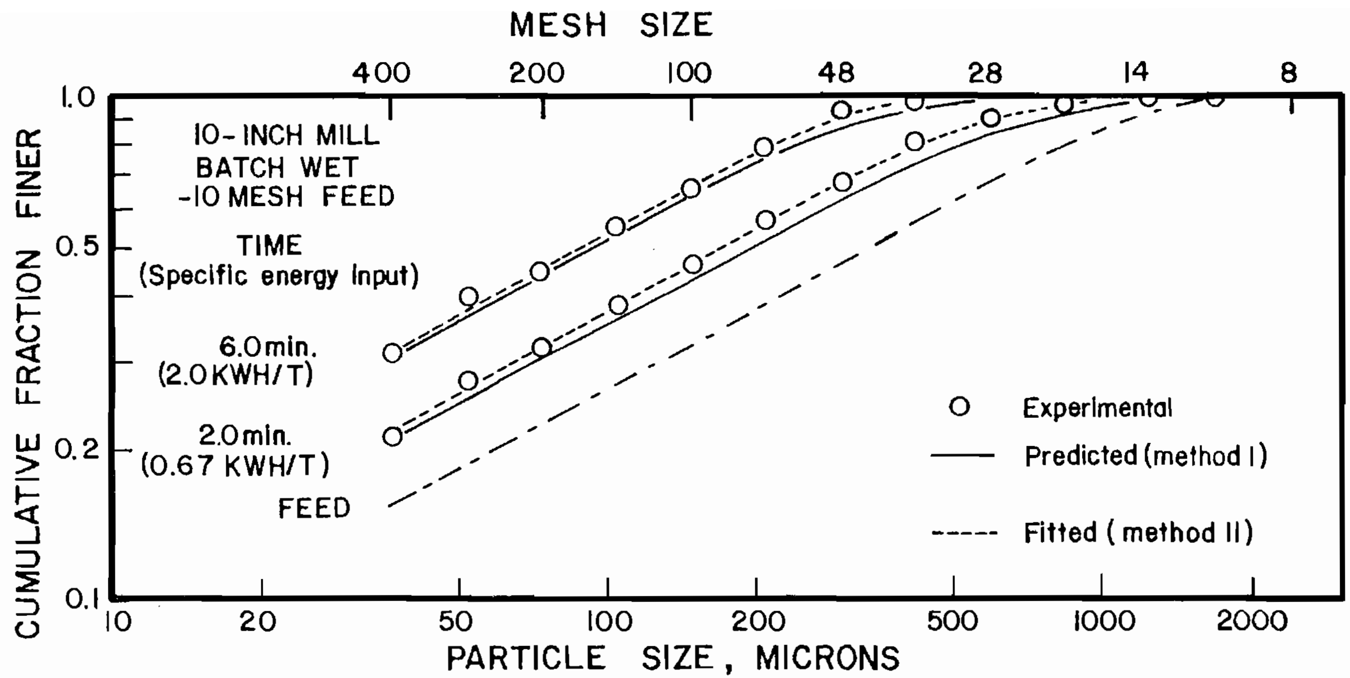


Figure 27. Comparison of experimental product size distribution for wet grinding in 10-in. diameter mill, (-10 mesh feed) and normalized predictions with S_i^E estimates with method I (firm lines) and method II (dotted lines).

product size distributions using a 'natural' (-10 mesh) feed. The predictions do not agree with the product size distribution confirming the dependence of specific selection functions on the size consist in the mill. Figures 28 and 29 show noramlized predictions (firm lines) in the 15-inch and 30-inch mills using 10 x 14 mesh feeds. The predictions agree in the 'nearly linear' region but do not agree when nonlinearity becomes evident. Figures 30 and 31 show normalized predictions (firm lines) in the 15-inch and 30-inch mills using a 'natural' (-10 mesh) feeds. Here the Method I predictions are even worse. These results suggested that Method I predictions are inappropriate for extended wet grinding.

Method II: Simulations with this method were based on the specific selection functions estimated in the 10-inch mill over a narrow specific energy range corresponding to the same specific energy input to the larger mills. The method of estimation of breakage parameters using mono-size and natural feeds have been explained earlier in the chapter and the values obtained are tabulated in Tables (II-4) and (II-5), Appendix II. Figure 27 shows the normalized predictions (dotted lines) in the 10-inch mill for specific energy input values of 0.67 and 2.0 KWH/T using separate estimates of the specific selection functions obtained for the corresponding specific energy range. These predictions closely agree with the experimental product size distributions. Figures 28 and 29 show the normalized predictions (dotted lines) for the 15-inch and the 30-inch mills respectively using 10 x 14 mesh feed with narrow range estimates. Figures 30 and 31 show the similar predictions (dotted lines) for

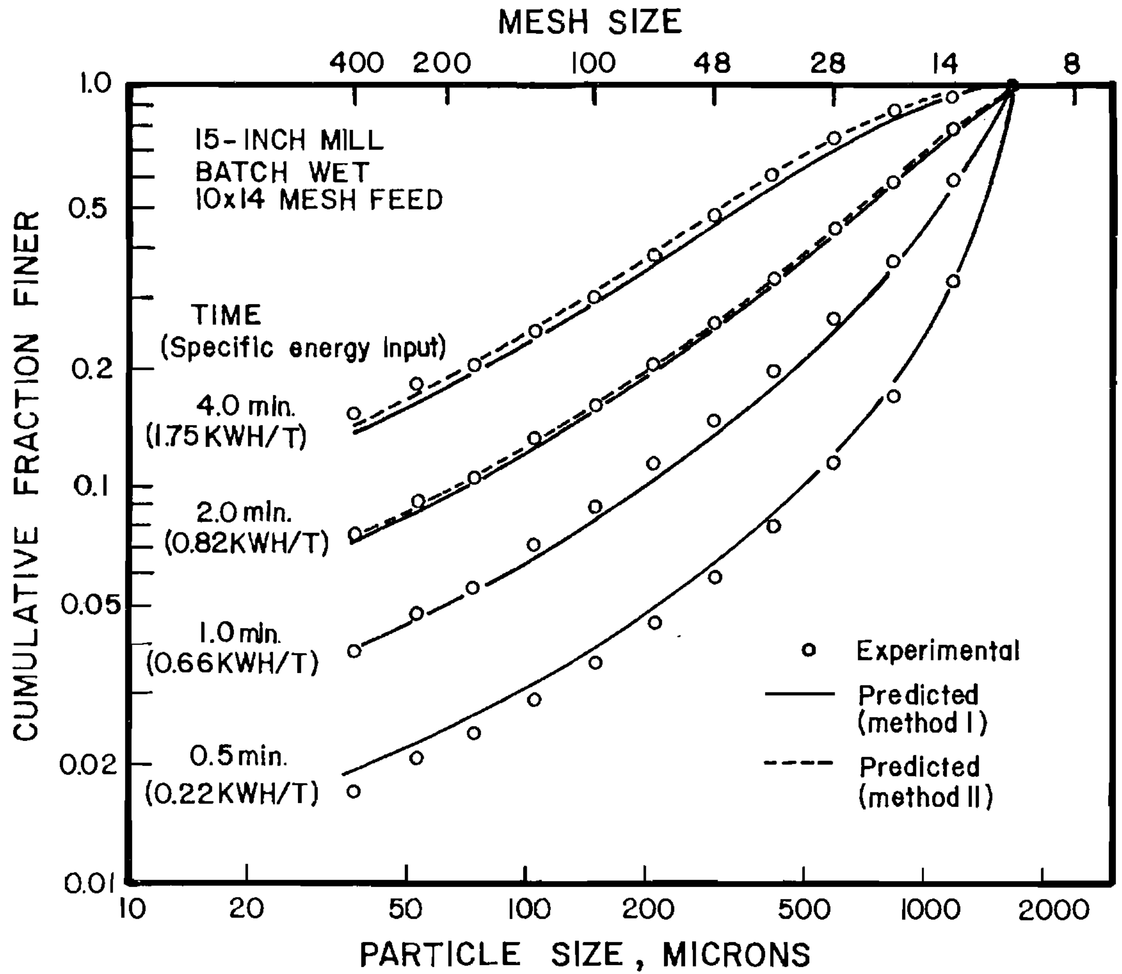


Figure 28. Comparison of experimental product size distribution for wet grinding in 15-in. diameter mill (10x14 mesh feed) and normalized predictions with S_i^E estimates obtained from 10-in. diameter mill with method I (firm lines) and method II (dotted lines).

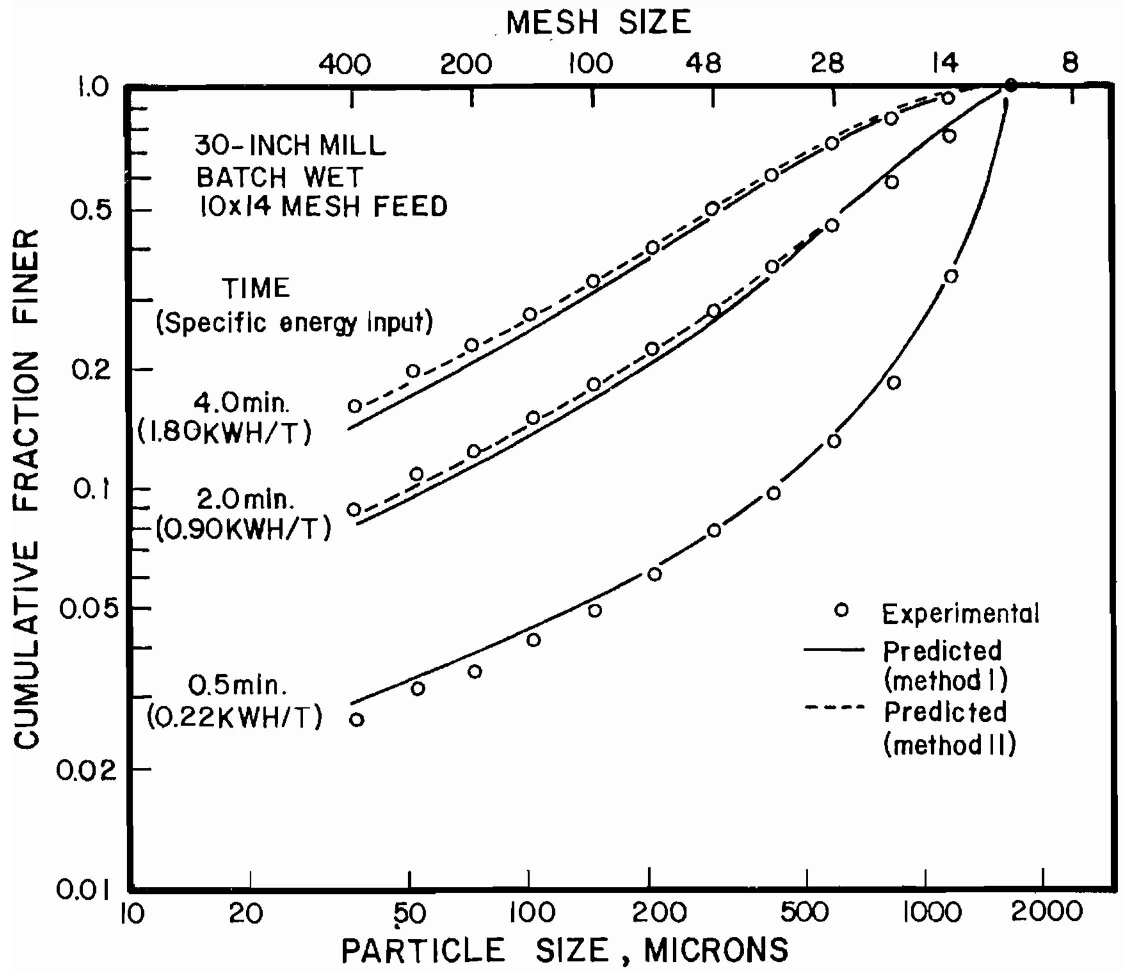


Figure 29. Comparison of experimental product size distribution for wet grinding in 30-in. diameter mill (10x14 mesh feed) and normalized predictions with S_i^E estimates obtained from 10-in. diameter mills with method I (firm lines) and method II (dotted lines).

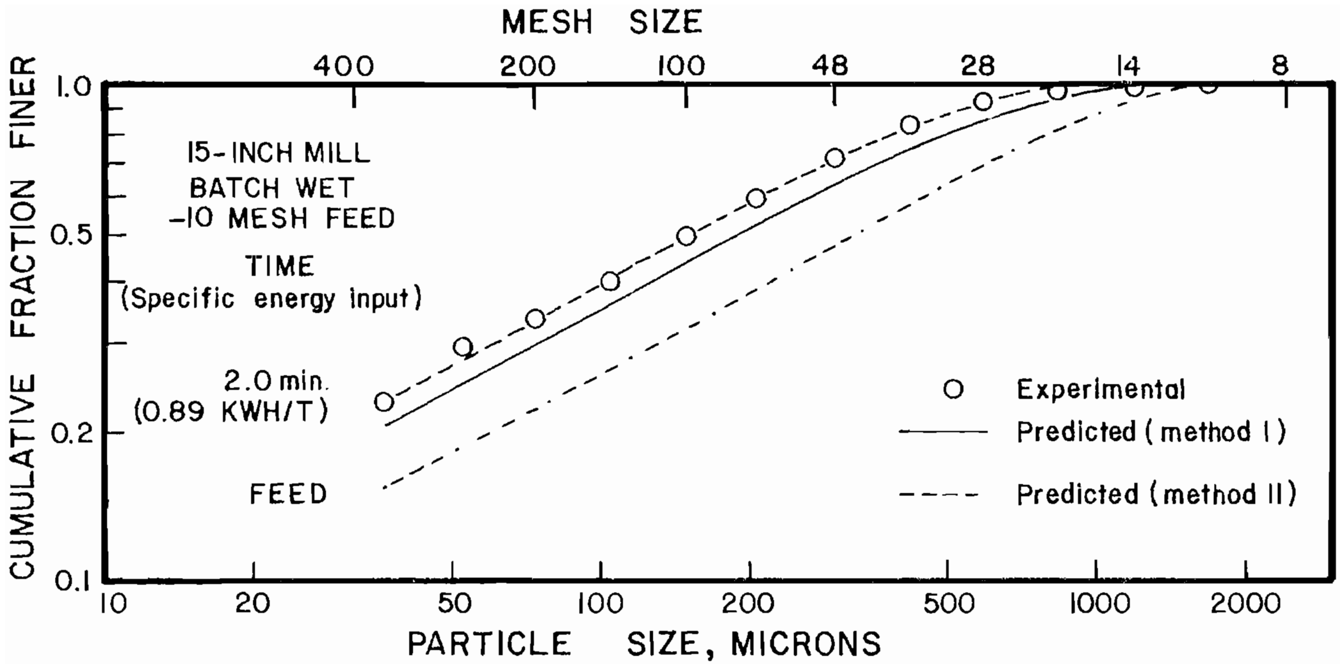


Figure 30. Comparison of experimental product size distribution for wet grinding in 15-in. diameter mill (-10 mesh feed) and normalized predictions with S_i^E estimates obtained from 10-in. diameter mills with method I (firm lines) and method II (dotted lines).

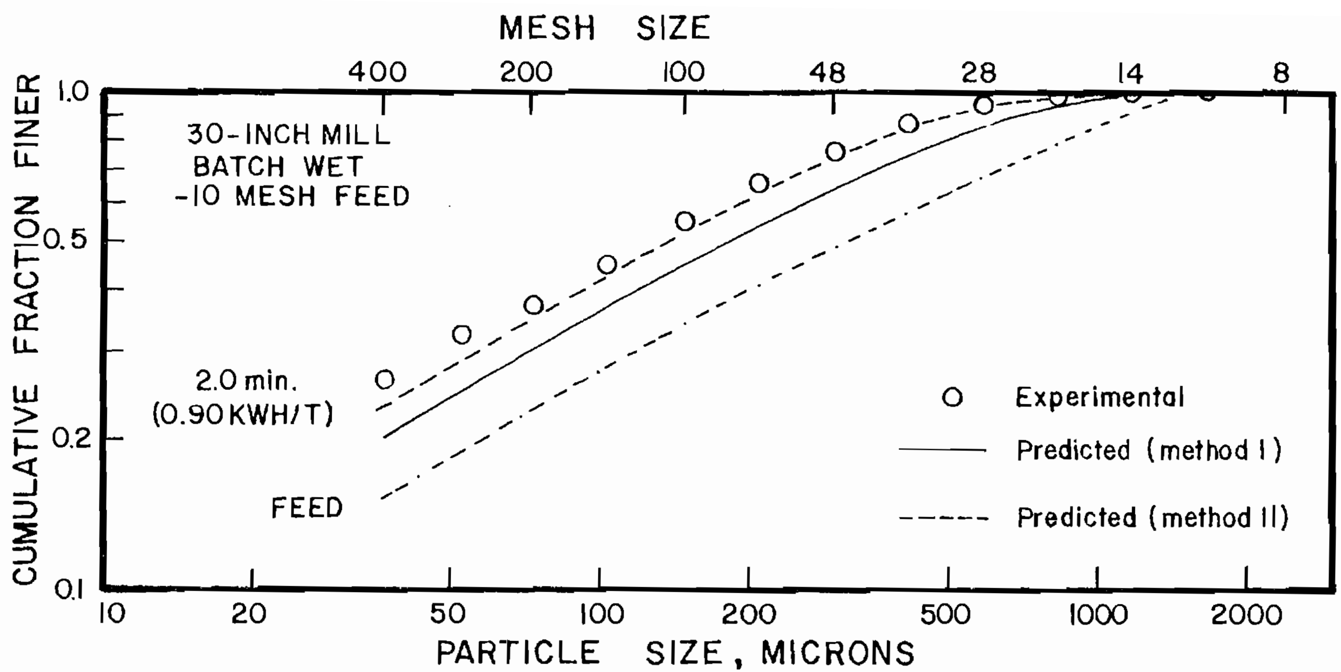


Figure 31. Comparison of experimental product size distribution for wet grinding in 30-in. diameter mill (-10 mesh feed) and normalized predictions with S_i^E estimates obtained from 10-in. diameter mills with method I (firm lines) and method II (dotted lines).

the 15-inch and 30-inch mills respectively using -10 mesh feed with narrow range estimates. In all cases the agreement with experimental product size distributions is good.

A slight deviation in the predictions for batch tests in the 30-inch mill may be due to the magnitude of experimental error associated with charging and discharging of the large mill and variation of efficiency factor (0.84 + 0.07) for the computation of the specific energy input.

From a practical standpoint the true test of a scale-up design procedure in an evaluation of its ability to predict the continuous grinding behavior of large mills based on data obtained in a laboratory scale batch mill.

Figure 32 shows such a prediction for an open circuit wet grinding in the 30-inch diameter mill using a natural (-10 mesh) feed with estimates obtained in the 10-inch diameter mill. The experimental test was run at a constant feed rate of 1040 pounds per hour. All the transients were let to die and steady state was attained. For the purpose of predicting the steady state behavior it was assumed that 1) the mill behaved as a single perfect mixer 2) all particle sizes share a common residence time distribution during the transport through the mill, i.e., $E(\theta)=e^{-\theta}$ in equation (II-4A).

Since the mean residence time of material in the mill is given by the holdup, H divided by the steady-state mass flowrate M_F and the selection functions are given by $S_i=S_i^E(\frac{P}{H})$, the elements of the model matrix for continuous grinding (equation II-4A) are given by

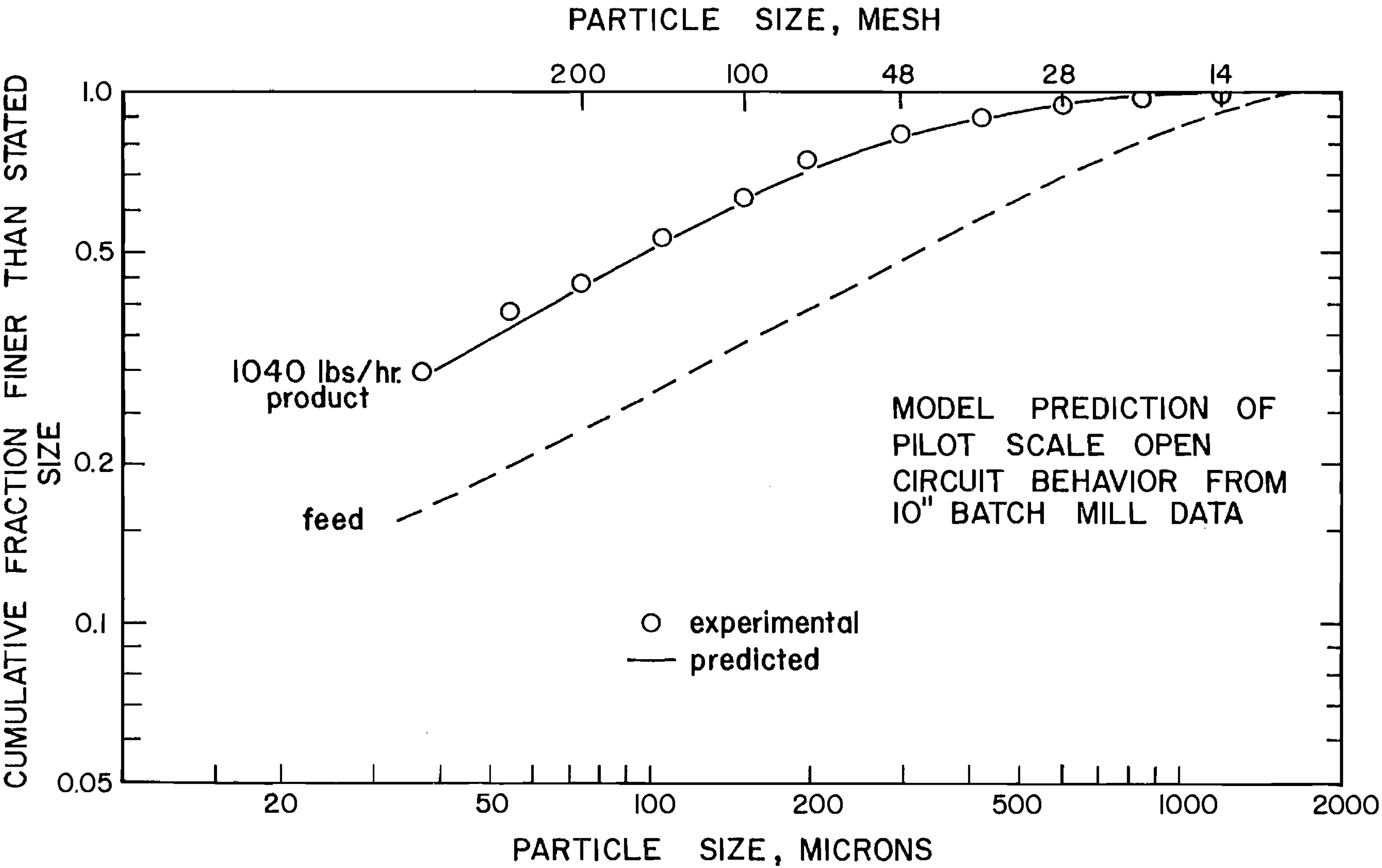


Figure 32. Comparison of experimental product size distribution for an open circuit wet grinding in 30-in. diameter mill (-10 mesh feed) and normalized predictions with S_i^E estimates obtained from 10-in. diameter mills with method II.

$$J_i = \int_0^\infty \exp\,[-(S_i\tau)\theta]E(\theta)d\theta$$

$$J_i = \int_0^\infty \exp\,[-(S_i^E(P/H)H/M_F\cdot\theta)]\ \exp\,[-\theta]\ d\theta$$

$$J_i = \int_0^\infty \exp - [S_i^E(P/M_F)\cdot\theta]\ \exp\,[-\theta]\ d\theta$$

$$J_i = \frac{1}{(1+S_i^E\ \bar{E})} \qquad \text{(V-9)}$$

where $\bar{E}$ is the continuous specific energy input to the product, P/M_F. For this case P=36.5 KW and M_F=1040 lbs./hr. so that $\bar{E}$=2.11 KWH/T. The specific selection functions estimates chosen for prediction were those obtained in the 10-inch batch mill for the corresponding range of energy input. These values along with wet breakage functions (Table II-3, Appendix II) were used in conjunction with equation (II-4A) to provide the predictions shown in Figure 32. The excellent agreement between the predicted and experimental product size distribution shown in the figure provides a very convincing demonstration of the validity of this scale-up design procedure for the range of mill sizes examined.

CHAPTER VI

SUMMARY AND CONCLUSIONS

In this investigation detailed experimentation has been carried out in dry and wet ball mill systems to determine the appropriateness of using linear population balance models for mill scale-up design. Three mills were involved in this study: two at the laboratory scale (10-inch and 15-inch diameters) and one at the pilot plant scale (30-inch diameter). The data was obtained in the three mills under the same operating conditions (N^*=0.6, M_B^*=0.5, M_P^*=1.0, F=0.6) except in the 30-inch mill for wet grinding where M_B^* was 0.4. The maximum ball size was 1 1/2 inch and the ball distribution corresponded to the "equilibrium charge distribution". The batch tests were performed in the three mills and an open circuit test in the 30-inch mill was performed.

The kinetic data obtained in the batch mill were analyzed in the context of the batch grinding model. The results confirmed the validity of the linear population balance model for dry grinding and the specific power correlation previously observed by Herbst and Fuerstenau(5) and Malghan and Fuerstenau(6). The breakage function was found to be invariant over the range of operating variables for the three mills. The specific selection functions (S_i^E=S_i(P/H)) were found to be independent of the operating conditions and mill design variables. Once the breakage parameters (S_i^E, B_{ij}) were obtained for dry grinding from the 10-inch mill data using 10 x 14 mesh feed, the grinding behavior could be predicted in the 15-inch and 30-inch mills for all operating conditions for 10 x 14 mesh or -10 mesh feed.

In the case of wet grinding the inherent nonlinearity reported by other investigators was also observed here. The specific selection functions were found to be strongly dependent on the size consist in the mill. This was due to the preferential breakage of coarse particles and suspension of fines in the water. The phenomena has been explained in Chapter V. This dependence of specific selection functions on the size consist in the mill generated some problems in the application of the linear population balance model. But the problem was overcome, to a good approximation, by estimating parameters for 'nearly linear' regions over narrow ranges of values of specific energy input to the mills. Since the specific selection functions were sensitive to the size consist in the mill, the predictions for the larger mills were made from the parameters obtained in the 10-inch mill for an identical feed and similar fineness of product (approximately the same specific energy).

The conclusions are summarized below:

1. In dry and wet grinding systems the breakage functions were to a good approximation independent of mill diameter, particle load, ball load, lifter configuration and the size consist in the mill.
2. In dry and wet grinding system the selection functions were found to be proportional to the specific power draft of the mill.

3. In dry ball milling the specific selection functions were independent of mill diameter, lifter configuration, particle load and the size consist in the mill.
4. In dry ball milling, where the kinetics were linear, the linear normalized model was found to be adequate for scale-up predictions for the 15-inch and 30-inch mills using the data obtained from the 10-inch mill.
5. In wet ball milling the specific selection functions were strongly dependent on the particle size distribution in the mill but were independent of mill diameter, particle load, ball load and lifter configuration.
6. The linear model was found to be valid in wet ball milling only if the predictions were made for a narrow range of the specific energy input to the larger mills using parameters obtained in the laboratory scale mill for the corresponding energy range and similar fineness of feeds.
7. In an open circuit prediction, the linear model was valid with the same restrictions as applied to the wet batch grinding case using parameters obtained in a laboratory scale batch test.

The results obtained in this investigation have important practical implications for commercial mill scale-up design. The work should be followed with similar studies in still larger mills and subsequently commercial size mills. Specific areas which require further study are: 1) closed circuit grinding--evaluate the accuracy of scale-up predictions for pilot scale and full scale operation 2) ball size effects--study the effect of ball size on the breakage kinetics, as the ball size used in the commercial scale mills is much larger than used in the laboratory scale mills and 3) material transport effects--study the effect of mill operating variables and length to diameter ratio on material residence time distribution. The suggested work based on the investigation results presented in this thesis promises to provide a truly accurate basis for commercial ball mill scale-up design.

SYMBOL TABLE

b_{ij}, B_{ij}, B	Size-discretized breakage functions, cumulative breakage function.
d_B	Ball diameter.
D	Mill diameter.
exp	Exponential
$E(\theta)$	An arbitrary residence time distribution.
$\overline{E}$	Specific energy input to the mill (P/H).
F	Fraction by weight of solids in the pulp in mill.
G	Reaction force to the force exerted by the mill on the lever arm of the prony brake.
H	Mass holdup of material in the mill.
$\underline{\underline{I}}$	Identity matrix
$\underline{\underline{J}}, \underline{\underline{J_c}}$	The modal matrices.
J_{ij}, J_{cij}	Elements of modal matrices.
l	Length of the lever arm of prony brake.
L	Mill length.
$m_i(t), m_i(\overline{E}), \underline{m}$	Mass fraction of material in the ith size interval at time t, at specific energy input $\overline{E}$ and column vector of mass fractions respectively.
M_B, M_{BC}, M_B^*	Mass of balls, mass of balls required to completely fill struck volume of mill and dimensionless ball load (M_B/M_{BC}).
m_{MF}, m_{MP}	The steady-state size distribution of the mill feed and mill product.

M_P, M_{PC}, M_P^*	Mass hold-up, mass of particles that completely fills the interstices between the balls, the dimensionless particle mass (M_P/M_{PC}).
N, N_c, N^*	Mill speed, critical mill speed and dimensionless mill speed (N/Nc).
P	Net power drawn by the mill.
q^*	Dimensionless ball size and lifter geometry variable.
$S_i(t), S_i^E, \underline{\underline{S}}$	Size discretized selection function, specific selection function and diagonal matrix of selection functions respectively.
T	Torque
$T_{ij}, \underline{\underline{T}}$	Elements of eigenvectors of $[\underline{\underline{I}}-\underline{\underline{B}}]\underline{\underline{S}}$ and matrix of eigenvectors.
τ	The mean residence time.
t	Time
w	Weight of lever arm of the prony brake.
α_1, α_2, ϕ	Adjustable parameters in the functional form for breakage functions.
δ	Power index showing power input dependence on the mill diameter.
ξ_1, ξ_2	Adjustable parameters in the functional form for selection functions.
θ	Dimensionless time variable t/τ.

REFERENCES

1. Herbst, J.A., Grandy, G.A. and Fuerstenau, D.W., "Population Balance Models for the Design of Continuous Grinding Mills," paper 19, X International Mineral Processing Congress, London (1973).

2. Blasket, K.S., "Estimation of the Power Consumption in Grinding Mills," in mineral processing and extractive metallurgy (London: IMM, 1970), 631-49.

3. Rose, H.E. and Sullivan, R.M.E., "A Treatise on the Internal Mechanics of Ball, Tube and Rod Mills," 1958, Chemical Publishing Co., Inc., 212 Fifth Avenue, New York.

4. Herbst, J.A., "Batch Ball Mill Simulation: An Approach for Wet Systems," D. Eng. Dissertation,University of California, Berkeley, California, 1971.

5. Herbst, J.A. and Fuerstenau, D.W., "Mathematical Simulation of Dry Ball Milling Using Specific Power Information," Trans. AIME, Vol. 254, 1973, p. 343

6. Malghan, S.G., Fuerstenau, D.W., "The Scale-up of Ball Mills Using Population Balance Models and Specific Power Input," Symposium Zerkleinern, Dechma-Monographien, Vol. 79, Part II, No. 1586, 1976, p. 613.

7. Malghan, S.G., "The Scale-up of Ball Mills Using Population Balance Models," D. Eng. Dissertation, University of California, Berkeley, California.

8. Kim, J.H., "A Normalized Model for Wet Batch Ball Milling," Ph.D. Dissertation, University of Utah, Salt Lake City, Utah, 1974.

9. Bond, F.C., "Crushing and Grinding Calculations," Canadian Min. and Met. Bulletin, Vol. 47, 1954, p. 466.

10. Bond, F.C., "More Accurate Grinding Calculations," Cement Lime and Gravel, 1963, p. 69.

11. Pryor, E.J., "Mineral Processing," 3rd Edition Elsevier Publishing Co. Ltd., Amsterdam-London-New York, 1965.

12. Reid, K.J., "A Solution to the Batch Grinding Equations," Chem. Eng. Sci., Vol. 20, 1965, p. 953.

13. Austin, L.G., "Understanding Ball Mill Sizing," Ind. Eng. Chem. Process Design and Development, Vol. 12, No. 2, 1973, p. 121.

14. Austin, L.G. and Luckie, P.T., "Methods for Determination of Breakage Distribution Parameter," Powder Technology, Vol. 5, 1971-72, p. 215.

15. Grandy, G.A. and Fuerstenau, D.W., "Simulation of Non-Linear Grinding Systems: Rod-Mill Grinding," Trans. AIME, Vol. 247 1970, p. 348.

16. Gaudin, A.M. and Meloy, T.P., "Model and Comminution Distribution Equations for Repeated Fracture," Trans. AIME, Vol. 223, 1962, p. 43.

17. Herbst, J.A. and Fuerstenau, D.W., "The Zero Order Production of Fine Sizes in Comminution and its Implications in Simulation," Trans. AIME, Vol. 241, 1968, p. 538.

18. Austin, L.G. and Bhatia, V.K., "Experimental Methods for Grinding Studies in Laboratory Mills," Powder Technology, Vol. 5, 1971-72, p. 261.

19. Herbst, J.A., Grandy, G.A. and Mika, T.S., "On the Development and Use of Lumped Parameter Model for Continuous Open-and Closed-Circuit Grinding Systems," Trans. IMM, Vol. 80, 1971, p. c193.

20. Herbst, J.A. and Kim, J.H., "Nearly Equal Selection Functions in Linear Size-Discretized Grinding Models," Trans. IMM, Vol. 82, 1973, p. c169.

21. Herbst, J.A. and Mika, T.S., "Linearization of Tumbling Mill Models Involving Nonlinear Breakage Phenomena," Eleventh International Symposium on Computer Applications in Mineral Industries, Tucson, Arizona, April 1973.

22. Tanaka, T., "Scale-up Formula For Grinding Equipment Using Selection Function," Journal of Chemical Engineering of Japan, Vol. 5, No. 3, 1972.

23. Yang, D.C., Mempel, G. and Fuerstenau, D.W., "A Laboratory Mill For Batch Grinding Experimentation," Trans. AIME, Vol. 238, 1967, p. 273.

24. Grandy, G.A., Gumtz, G.D., Herbst, J.A., Mika, T.S. and Fuerstenau, D.W., "Computer Techniques in the Analysis of Laboratory Grinding Tests," Dept. of Mat. Sci. and Eng., University of California, Berkeley, California.

25. Klimpel, R.R. and Austin, A.G., "Determination of Selection-for Breakage Functions in the Batch Grinding Equation by Nonlinear Optimization," Ind. and Eng. Chem. Fundamentals, Vol. 9(2), p. 230 (1970).

26. Herbst, J.A., Grandy, G.A., Mika, T.S. and Fuerstenau, D.W., "An Approach to the Estimation of the Parameter of Lumped Parameter Grinding Models from On-Line Measurements," Zerkleinern, 3rd European Symposium on Comminution, Cannes, 1971, Dechma-Monographien, Vol. 69, 1972.

27. Herbst, J.A. and Rajamani, K., manuscript in preparation.

28. Herbst, J.A., Grandy, G.A., Mika, T.S. and Fuerstenau, D.W., "An Approach to the Estimation of the Parameter of Lumped Parameter Grinding Models from On-Line Measurements," Dechma-Monographien, Vol. 69, 1972.

29. Diederichs and Andrae, "Experimental Mechanical Engineering," Vol. 1, Engineering Instruments, John Wiley and Sons, New York, 1930.

APPENDIX I

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS FOR CALCITIC LIMESTONE GRINDING

TABLE I-1

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 10" Medium: Dry

Feed Size: 10 x 14 mesh batch

Average Net Torque: 110.31 in. lbs.

H = 3300 grams $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	0.5	1.0	2.0	4.0	6.0
- 10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
- 14	0.0450	0.2273	0.3999	0.6170	0.8560	0.9522
- 20	0.0000	0.1254	0.2352	0.4140	0.6815	0.8539
- 28	0.0000	0.0853	0.1624	0.2997	0.5372	0.7324
- 35	0.0000	0.0609	0.1142	0.2186	0.4086	0.5909
- 48	0.0000	0.0456	0.0854	0.1644	0.3101	0.4645
- 65	0.0000	0.0348	0.0656	0.1278	0.2429	0.3705
-100	0.0000	0.0272	0.0513	0.0998	0.1897	0.2910
-150	0.0000	0.0224	0.0427	0.0821	0.1546	0.2353
-200	0.0000	0.0182	0.0350	0.0665	0.1253	0.1891
-270	0.0000	0.0162	0.0307	0.0587	0.1105	0.1649
-400	0.0000	0.0142	0.0261	0.0480	0.0882	0.1283

TABLE I-2

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 10" Medium: Dry

Feed Size: 'natural' (-10 mesh) batch

Average Net Torque: 108.00 in. lbs.

H = 3300 grams $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	1.0	4.0
- 10	1.0000	1.0000	1.0000
- 14	0.9698	0.9844	0.9989
- 20	0.8158	0.8916	0.9817
- 28	0.6955	0.7892	0.9380
- 35	0.5828	0.6735	0.8493
- 48	0.4904	0.5640	0.7360
- 65	0.4090	0.4726	0.6264
-100	0.3379	0.3904	0.5235
-150	0.2842	0.3265	0.4391
-200	0.2347	0.2671	0.3604
-270	0.2071	0.2341	0.3184
-400	0.1663	0.1803	0.2403

TABLE I-3

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 15" Medium: Dry

Feed Size: 10 x 14 mesh batch

Average Net Torque: 420.98 in. lbs.

H = 7425.0 grams $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	0.5	1.0	2.0	4.0
- 10	1.0000	1.0000	1.0000	1.0000	1.0000
- 14	0.0450	0.2754	0.4707	0.7193	0.9082
- 20	0.0000	0.1435	0.2806	0.5102	0.7632
- 28	0.0000	0.0981	0.1966	0.3842	0.6259
- 35	0.0000	0.0695	0.1415	0.2859	0.4913
- 48	0.0000	0.0533	0.1067	0.2173	0.3801
- 65	0.0000	0.0408	0.0829	9.1691	0.3016
-100	0.0000	0.0318	0.0647	0.1310	0.2370
-150	0.0000	0.0263	0.0534	0.1063	0.1929
-200	0.0000	0.0213	0.0431	0.0846	0.1562
-270	0.0000	0.0189	0.0376	0.0734	0.1370
-400	0.0000	0.0157	0.0306	0.0677	0.1082

TABLE I-4

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 15" Medium: Dry

Feed Size: 'natural' (-10 mesh) batch

Average Net Torque: 430.00 in. lbs.

H = 7425.0 grams $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	0.5	2.0
- 10	1.0000	1.0000	1.0000
- 14	0.9651	0.9830	0.9975
- 20	0.8065	0.8619	0.9544
- 28	0.6856	0.7453	0.8809
- 35	0.5758	0.6283	0.7733
- 48	0.4745	0.5274	0.6588
- 65	0.3957	0.4419	0.5575
-100	0.3269	0.3653	0.4645
-150	0.2736	0.3077	0.3929
-200	0.2241	0.2519	0.3268
-270	0.1986	0.2217	0.2897
-400	0.1570	0.1738	0.2302

TABLE I-5

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 30" Medium: Dry

Feed Size: 10 x 14 mesh batch

Average Net Torque: 3551.2 in. lbs.

H = 46.5 kilograms $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	0.5	2.0	4.0
- 10	1.0000	1.0000	1.0000	1.0000
- 14	0.0450	0.3024	0.7009	0.9020
- 20	0.0330	0.1803	0.5175	0.7668
- 28	0.0250	0.1279	0.4043	0.6415
- 35	0.0200	0.0918	0.3099	0.5186
- 48	0.0150	0.0695	0.2469	0.4130
- 65	0.0140	0.0525	0.1905	0.3313
-100	0.0075	0.0399	0.1468	0.2616
-150	0.0050	0.0318	0.1172	0.2113
-200	0.0040	0.0248	0.0906	0.1701
-270	0.0025	0.0212	0.0771	0.1452
-400	0.0020	0.0162	0.0583	0.1097

TABLE I-6

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 30" Medium: Dry

Feed Size: 'natural' (-10 mesh) batch

Average Net Torque: 3551.2 in. lbs.

H = 46.5 kilograms $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	2.0	4.0
- 10	1.0000	1.0000	1.0000
- 14	0.9225	0.9845	0.9970
- 20	0.7764	0.9278	0.9832
- 28	0.6693	0.8540	0.9540
- 35	0.5672	0.7552	0.8901
- 48	0.4838	0.6551	0.7939
- 65	0.4061	0.5623	0.6924
-100	0.3371	0.4721	0.5899
-150	0.2813	0.3964	0.4976
-200	0.2291	0.3264	0.4145
-270	0.1995	0.2860	0.3705
-400	0.1534	0.2191	0.2939

TABLE I-7

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 10" Medium: Wet-60% Solids

Feed Size: 10 x 14 mesh batch

Average Net Torque: 117.29 in. lbs.

H = 3300 grams $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	0.5	1.0	2.0	4.0	5.0	5.5	6.0
- 10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
- 14	0.0450	0.3116	0.4702	0.7180	0.9133	0.9605	0.9690	0.9759
- 20	0.0000	0.1562	0.2726	0.4983	0.7657	0.8596	0.8841	0.9068
- 28	0.0000	0.1040	0.1895	0.3681	0.6253	0.7410	0.7750	0.8125
- 35	0.0000	0.0708	0.1373	0.2722	0.4847	0.6009	0.6374	0.6801
- 48	0.0000	0.0521	0.1039	0.2048	0.3722	0.4746	0.5082	0.5524
- 65	0.0000	0.0392	0.0816	0.1612	0.2920	0.3767	0.4045	0.4396
-100	0.0000	0.0304	0.0640	0.1260	0.2299	0.2984	0.3212	0.3502
-150	0.0000	0.0249	0.0527	0.1027	0.1894	0.2452	0.2638	0.2882
-200	0.0000	0.0203	0.0427	0.0827	0.1546	0.1972	0.2134	0.2333
-270	0.0000	0.0182	0.0375	0.0725	0.1362	0.1718	0.1873	0.2038
-400	0.0000	0.0155	0.0302	0.0570	0.1137	0.1339	0.1455	0.1587

TABLE I-8

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 10" Medium: Wet-60% Solids

Feed Size: 'natural' (-10 mesh) batch

Average Net Torque: 113.34 in. lbs.

H = 3300 grams $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	0.5	2.0	4.0
- 10	1.0000	1.0000	1.0000	1.0000
- 14	0.9651	0.9817	0.9980	0.9995
- 20	0.8065	0.8684	0.9657	0.9928
- 28	0.6856	0.7566	0.9047	9.9710
- 35	0.5758	0.6394	0.8008	0.9131
- 48	0.4745	0.5339	0.6751	0.8111
- 65	0.3957	0.4465	0.5647	0.6868
-100	0.3269	0.3690	0.4635	0.5666
-150	0.2736	0.3092	0.3862	0.4728
-200	0.2241	0.2545	0.3163	0.3892
-270	0.1986	0.2236	0.2773	0.3433
-400	0.1570	0.1735	0.2101	0.2673

TABLE I-9

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 15" Medium: Wet-60% Solids

Feed Size: 'natural' (-10 mesh) batch

Average Net Torque: 434.25 in. lbs.

H = 7425.0 grams $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	0.5	1.0	2.0	4.0
- 10	1.0000	1.0000	1.0000	1.0000	1.0000
- 14	0.0450	0.3295	0.5843	0.7926	0.9620
- 20	0.0000	0.1709	0.3703	0.5801	0.8710
- 28	0.0000	0.1148	0.2685	0.4447	0.7559
- 35	0.0000	0.0799	0.1977	0.3359	0.6146
- 48	0.0000	0.0590	0.1480	0.2599	0.4810
- 65	0.0000	0.0456	0.1153	0.2044	0.3832
-100	0.0000	0.0355	0.0890	0.1611	0.3019
-150	0.0000	0.0290	0.0718	0.1321	0.2480
-200	0.0000	0.0236	0.0560	0.1060	0.2049
-270	0.0000	0.0207	0.0478	0.0923	0.1830
-400	0.0000	0.0168	0.0381	0.0758	0.1538

TABLE I-10

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 15" Medium: Wet-60% Solids

Feed Size: 'natural' (-10 mesh) batch

Average Net Torque: 441.50 in. lbs.

H = 7425.0 grams $M_P^* = 1.0$

$M_B^* = 0.5$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	0.5	2.0
- 10	1.0000	1.0000	1.0000
- 14	0.9651	0.9887	0.9992
- 20	0.8065	0.8874	0.9774
- 28	0.6856	0.7830	0.9278
- 35	0.5758	0.6644	0.8321
- 48	0.4745	0.5580	0.7127
- 65	0.3957	0.4659	0.5960
-100	0.3269	0.3857	0.4910
-150	0.2736	0.3259	0.4104
-200	0.2241	0.2709	0.3371
-270	0.1986	0.2409	0.2967
-400	0.1570	0.1941	0.2319

TABLE I-11

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 30" Medium: Wet-60% Solids

Feed Size: 10 x 14 mesh batch

Average Net Torque: 3141.7 in. lbs.

H = 37.5 kilograms $M_P^* = 1.0$

$M_B^* = 0.4$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	0.5	2.0	4.0
- 10	1.0000	1.0000	1.0000	1.0000
- 14	0.0450	0.3361	0.7584	0.9377
- 20	0.0420	0.1806	0.5617	0.8294
- 28	0.0250	0.1291	0.4445	0.7201
- 35	0.0125	0.0951	0.3488	0.5980
- 48	0.0115	0.0774	0.2769	0.4907
- 65	0.0110	0.0604	0.2221	0.4000
-100	0.0100	0.0484	0.1788	0.3248
-150	0.0100	0.0405	0.1490	0.2727
-200	0.0100	0.0340	0.1225	0.2257
-270	0.0100	0.0306	0.1083	0.1995
-400	0.0100	0.0261	0.0877	0.1597

TABLE I-12

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 30" Medium: Wet-60% Solids

Feed Size: 'natural' (-10 mesh) batch

Average Net Torque: 3141.7 in. lbs.

H = 37.5 kilograms $M_P^* = 1.0$

$M_B^* = 0.4$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Grind Time (Min.) 0.0	2.0	4.0
- 10	1.0000	1.0000	1.0000
- 14	0.9225	0.9968	0.9996
- 20	0.7764	0.9791	0.9982
- 28	0.6693	0.9415	0.9950
- 35	0.5672	0.8646	0.9700
- 48	0.4838	0.7609	0.9112
- 65	0.4061	0.6487	0.8090
-100	0.3371	0.5411	0.6874
-150	0.2813	0.4464	0.5724
-200	0.2291	0.3679	0.4692
-270	0.1995	0.3220	0.4127
-400	0.1534	0.2612	0.3257

TABLE I-13

EXPERIMENTAL PRODUCT SIZE DISTRIBUTIONS

Mill Size: 30" Medium: Wet

Feed Size: 'natural' (-10 mesh) open circuit

Feed Rate: 1040 lbs./hr

H = 80 lbs. Average Net Torque = 3096.0 in. lbs.

$M_B^* = 0.4$ $N^* = 0.6$

Cumulative Mass Fraction Passing Stated Size

Size (Mesh)	Feed	Mill Contents	Mill Discharge
- 10	1.0000	1.0000	1.0000
- 14	0.9225	0.9930	0.9950
- 20	0.7764	0.9700	0.9850
- 28	0.6693	0.9417	0.9490
- 35	0.5672	0.8957	0.9099
- 48	0.4838	0.8299	0.8350
- 65	0.4061	0.7436†	0.7984
-100	0.3371	0.6377	0.6888
-150	0.2813	0.5299	0.5766
-200	0.2291	0.4354	0.4765
-270	0.1995	0.3850	0.4247
-400	0.1534	0.2997	0.3337

† differences in mill contents and mill discharge distributions may be due to selective losses of fines in discharging the mill.

APPENDIX II

COMPUTATIONS OF THE SPECIFIC ENERGY INPUT TO THE MILL

Energy Calculations

During the experimentation the torque exerted by the mill has directly been measured in the case of the 10-inch and 15-inch mills and on the pinion shaft in the case of the 30-inch mill. The specific energy input (KWH/T) to the mill can be computed from the torque measured. The computational method and principle involved was identical to the one used by Kim[10]. He has derived the equation used from the fundamental principles. The final equation thus obtained is given below:

$$P = 1.18 \times 10^{-5}\ N \cdot T \cdot \qquad (KW) \qquad \text{(AII-1)}$$

where P is the power in kilowatts drawn by the mill, N the revolutions per minutes of the mill and T is the torque recorded directly by the sensor in inch-pounds. Then the specific energy input, $\overline{E}$, to the mill is computed from the relationship:

$$\overline{E} = \frac{Pt}{H} \qquad (KWH/T) \qquad \text{(AII-2)}$$

where t is the grind time in hours and H is the particle mass hold-up in the mill in short tons.

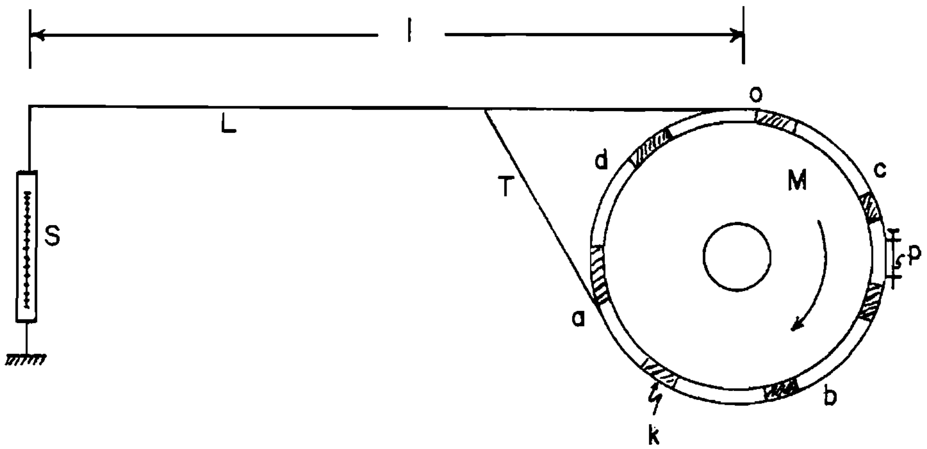
In the case of 30-inch mill the specific energy input to the mill was measured from the pinion shaft by a torque sensor as shown in figure 7, page 23. The actual energy drawn by the mill is computed by multiplying the recorded energy by an efficiency factor accounting for the losses in the transmission of energy from the pinion shaft to the mill.

Actual Energy = Efficiency x Energy measured (AII-3)

The efficiency factor was unknown and for the determination a prony brake was designed. The description of the brake is given in the next section of this Appendix.

Prony Brake Measurements

The necessity of these tests arose out of the fact that the actual power drawn by the 30-inch mill was lower than that recorded by the torque meter at the pinion shaft. Therefore the goal was to determine the efficiency of the mechanical set up (efficiency factor). A prony brake was designed to measure the efficiency[22]. The prony brake is a crude type of absorption dynamometer (sketched below) and is shown on the mill surface in figure 6, page 21.



This is a simple device which measures the power transmitted by the mill M to the steel band abcd. The band is provided with wood blocks K. The grip of the band on the machined mill surface can be adjusted by a screw p. As the grip of the band tightens on the rotating mill the lever arm L, which is welded to the band and supported by the tie rod T, tends to rotate in the direction of the mill rotation. Since the lever arm end is fastened to the ground through a spring balance S records the tangential force exerted G (in pounds) on the lever arm. Then taking moment around point O the total force exerted by the mill tangentially on the steel strap is given by:

$$Gl \ + wl/2 \qquad \text{(BII-1)}$$

where l is the length of the lever arm L in inches and w is the weight (in pounds) of the lever arm and tie rod, assuming to be acting downwards at a distance l/2 from the point O. With increasing tension in the band abcd the value of G increases.

The tests were mostly carried out in the range of power where the actual power drawn by the pinion shaft, at the time of grinding, falls. The mill was packed with material so that it can be treated as a solid mass for prony brake measurements. Various readings were taken simultaneously by the prony brake and torquemeter at the pinion shaft for different tensions applied to the band. The revolutions per minute were also recorded. Then the power recorded by torquemeter was plotted against the prony brake readings (Figure 33). The efficiency of the mill was then given by the slope of the line which was 0.84 ± 0.07 within 95% confidence interval.

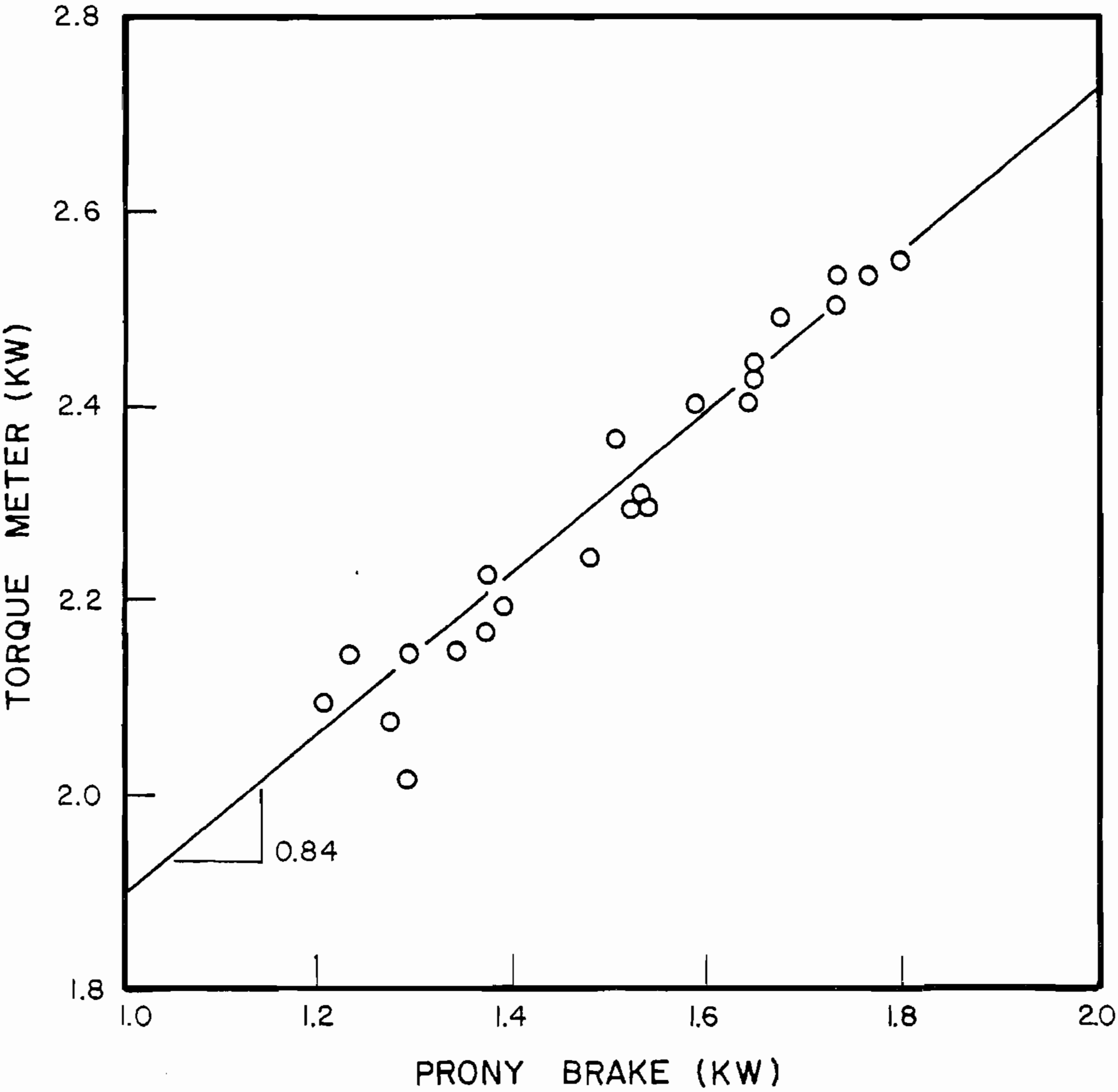


Figure 33. Determination of efficiency factor for the 30-inch mill using torque sensor attached to the pinion shaft and prony brake on the periphery of the mill.

TABLE II-1

SPECIFIC ENERGY INPUT

BATCH DRY

$\overline{E}$(KWH/T)

10-IN MILL

Feed (Mesh)	P/H (KW/T)	Grind Times (Minutes)				
		0.5	1.0	2.0	4.0	6.0
10x14	19.34	0.161	0.322	0.645	1.289	1.934
- 10	18.94	--	0.3156	--	1.262	--

15-IN MILL

Feed (Mesh)	P/H (KW/T)	Grind Times (Minutes)				
		0.5	1.0	2.0	4.0	6.0
10x14	25.51	0.213	0.425	0.851	1.701	--
- 10	26.06	0.217	--	0.869	--	--

30 -IN MILL

Feed (Mesh)	P/H (KW/T)	Grind Times (Minutes)				
		0.5	1.0	2.0	4.0	6.0
10x14 -10	24.53	0.204	--	0.818	1.635	--

TABLE II-2

SPECIFIC ENERGY INPUT

BATCH WET

$\overline{E}$(KWH/T)

10-IN MILL

Feed (Mesh)	P/H (KW/T)	Grind Times (Minutes)						
		0.5	1.0	2.0	4.0	5.0	5.5	6.0
10x14	20.56	0.171	0.343	0.685	1.371	1.714	1.885	2.056
- 10	19.97	0.166	--	0.666	1.334	--	--	1.997

15-IN MILL

Feed (Mesh)	P/H (KW/T)	Grind Times (Minutes)			
		0.5	1.0	2.0	4.0
10x14	26.32	0.219	0.439	0.877	1.755
- 10	26.76	0.223	--	0.892	--

30-IN MILL

Feed (Mesh)	P/H (KW/T)	Grind Times (Minutes)			
		0.5	1.0	2.0	4.0
10x14 -10	26.96	0.225	--	0.899	1.797

TABLE II-3

ESTIMATED FEED SIZE BREAKAGE FUNCTIONS

AND SPECIFIC SELECTION FUNCTIONS

10-IN MILL

INDEX i	DRY		WET	
	B_{ij}	$S_i^E(KWH/T)^{-1}$	B_{ij}	$S_i^E(KWH/T)^{-1}$
1	1.0000	1.438	1.0000	1.714
2	1.0000	1.162	1.0000	1.288
3	0.5551	0.939	0.5285	0.968
4	0.3390	0.759	0.3287	0.727
5	0.2276	0.613	0.2320	0.545
6	0.1665	0.495	0.1778	0.410
7	0.1295	0.400	0.1421	0.309
8	0.1050	0.324	0.1162	0.232
9	0.0868	0.261	0.0955	0.174
10	0.0728	0.211	0.0788	0.131
11	0.0620	0.171	0.0658	0.099
12	0.0523	0.000	0.0541	0.000

TABLE II-4

ESTIMATED SPECIFIC SELECTION FUNCTIONS FOR

STATED NARROW ENERGY INPUT RANGES

$S_i^E (KWH/T)^{-1}$

10-IN MILL | BATCH WET | 10x14 MESH FEED

Index i	1 0.0→2.0,4.0 min grinds	2 0.0→4.0,5.0 min grinds	3 0.0→5.0,6.0 min grinds
1	1.763	1.777	1.860
2	1.322	1.358	1.407
3	0.991	1.038	1.065
4	0.743	0.793	0.805
5	0.557	0.605	0.608
6	0.418	0.463	0.461
7	0.314	0.354	0.349
8	0.235	0.270	0.264
9	0.176	0.206	0.199
10	0.132	0.158	0.151
11	0.099	0.120	0.114
12	0.000	0.000	0.000

TABLE II-5

ESTIMATED SPECIFIC SELECTION FUNCTIONS FOR

STATED NARROW ENERGY INPUT RANGES

$S_i^E (KWH/T)^{-1}$

10-IN MILL BATCH WET -10 MESH FEED

Index i	1 0.0→0.5,2.0,4.0,6.0 min grinds	2 0.0→0.5,2.0 min grinds	3 0.0→2.0,4.0 min grinds	4 0.0→4.0,6.0 min grinds
1	4.095	3.905	4.132	4.722
2	2.791	2.646	2.791	3.119
3	1.902	1.793	1.884	2.060
4	1.297	1.215	1.273	1.361
5	0.882	0.822	0.858	0.897
6	0.602	0.558	0.580	0.593
7	0.411	0.379	0.392	0.393
8	0.280	0.256	0.265	0.259
9	0.190	0.173	0.178	0.170
10	0.130	0.118	0.121	0.113
11	0.088	0.079	0.081	0.074
12	0.000	0.000	0.000	0.000

VITA

NAME:	Mohammad Siddique
BIRTHPLACE:	Chaman, Pakistan
BIRTHDATE:	December 9, 1946
HIGH SCHOOL:	Islamia High School Quetta, Pakistan
COLLEGE:	Government College Quetta, Pakistan
UNIVERSITY:	Engineering University Lahore, Pakistan 1963-67, B. Sci. (Mining Engineering)
	University of Utah Salt Lake City, Utah 1974-77, M. S. (Metallurgy)
PROFESSIONAL QUALIFICATIONS:	Second Class Mine Managers' Competency Certificate, Government of Baluchistan, Quetta, Pakistan, 1972
PROFESSIONAL ORGANIZATIONS:	Associate Member, Institute of Mining Engineers of Pakistan
	Society of Mining Engineers of AIME
PROFESSIONAL POSITIONS:	Mining Engineer, Pakistan Industrial Development Corporation, 1968-73
	Quarry Manager, State Cement Corp. Iskanderabad, Pakistan
	Research Assistant, University of Utah, Salt Lake City, Utah 1974-77