# STAGE-GATE CONTRACTS FOR NEW PRODUCT DEVELOPMENT

by

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# ABSTRACT

Given the trend of globalization, more and more firms are outsourcing their Research and Development (R&D) projects to a second party overseas or domestically. Through outsourcing, firms not only save costs but also build strategic capabilities such as tapping global talents, building partnerships, boosting innovation, and maintaining a lean and flexible operation. These capabilities help shorten the duration of R&D projects and mitigate the risk of failures. However, the complexity of collaborative relationship in outsourcing and risks inherent in an R&D project pose challenges to both the firm who is doing the outsourcing (referred to as the principal) and the firm that the project is outsourced to (referred to as the agent). It is likely that either or both parties have private information regarding their capabilities as well as the likelihood of the success of the project. In addition, the efforts of the firm that the project is outsourced to may be unobservable to the firm who is doing outsourcing. In the dissertation, I investigate whether stage-gate contracts can help firms manage the outsourcing of R&D projects and determine the optimal form of the stage-gate contract when information asymmetry (adverse selection) and unobservable effort (moral hazard) exist.

In Chapter 1, I explore the use of stage-gate contracts in the case where the agent has private information and his effort is unobservable. The principal offers multiple contracts to "screen" the agent. The main tool of the analysis is the screening model in the principal-agent problem.

In Chapter 2, I examine the opposite case, the one where the principle is the firm with the private information (the agent's effort is again unobservable). In this situation a principal may use the stage-gate contract to signal her private information with regard to the new product development project. The main tool of the analysis is the signaling games.

In Chapter 3, I investigate the case of bilateral asymmetric information, namely, both the principal and the agent have their own private information on the project. The main tool of the analysis is the screening model and the signaling games.

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# CHAPTER 1

# USING STAGE-GATE CONTRACTS TO SCREEN AN AGENT WITH INSIDE INFORMATION

## 1.1 Introduction

Over the first decade of the new millennium, the global R&D spending increased rapidly from \$0.753 trillion in 2001 to \$1.051 trillion in 2006 and then to \$1.435 trillion in 2011, averaging 6.4% annually over the first 5-year period and 6.7% annually over the 10-year period (National Science Board, 2014). In contrast, the global GDP grew annually at the rate of 4% from 2004 to 2007 and at a rate less than 3% from 2007 to 2013 because of the Great Recession and slow recovery afterwards (World Bank, 2014). Across all the industries, pharmaceutical has one of the highest levels of R&D expenditures. In 2012, 7 of the top 20 companies by R&D spending are pharmaceutical companies, such as Roche, Novartis, Merck US, Johnson & Johnson, Pfizer, Sanofi-Aventis and Glaxosmithkline (PwC, 2014). For the whole pharmaceutical industry, the worldwide total R&D spending increased at the impressive rate of 11% from about \$83 billion in 2004 to about \$136 billion in 2008 and then slowed its pace reaching about \$140 billion in 2012 due to the Great Recession (Nature Medicine, August 6, 2013).

Another noticeable trend is that the outsourcing of R&D grew at a faster pace than R&D spending. For instance, for the pharmaceutical industry, over the period from 2008 to 2013, the global drug discovery grew at the annual rate of 10.5%, accounting for about 10% of the total global drug R&D spending in 2013. It is projected that with the annual growth rate of 10.5%, the market value of the global drug discovery outsourcing will likely reach \$25 billion by 2018, nearly half of the R&D spending in pharmaceutical industry at that time (Research and Markets, September, 2013). One of the major reasons a firm might pursue outsourcing of R&D is to slow the expenditure growth; the cost of developing a new drug increased almost eightfold from \$199 million in 1979 to $1,506$ in 2012 (European Federation of Pharmaceutical Industry and Association, 2014). However, lack of measures of prequalification and effective monitoring could result in the loss of quality management in outsourced R&D, especially for the new product development process in the knowledge-intense pharmaceutical industry. The use of a stage-gate contract could help facilitate such monitoring, which is another reason that we are interested in analyzing the type of contract appropriate when using the stage-gate framework.

R&D is not only costly but also lengthy and risky. Related to the development of a new medicine, the annual report from European Federation of Pharmaceutical Industry and Association (2014) points out that for a synthesized active substance to reach the market, on average, it has to go through a six-year process of patent application and preclinical development (acute toxicity, phamarcology and chronic toxicity), four years of clinic trials and two to three years of administrative procedures (registration, marketing, price etc.). In the end, only one or two of every 10,000 synthesized substances passes through all these stages to get commercialized. Strategic capabilities built through outsourcing such as tapping global talent, building partnerships, boosting innovation, maintaining a lean and flexible operation etc. (Hauser, Tellis, and Griffin, 2006) help shorten the duration of R&D projects and mitigate the risk of failures.

R&D projects are also highly variable as to their potential for success. In 2005, the global survey by Arthur D. Little (2005) of more than 800 companies across all major industries on innovation excellence found that the top quartile of innovators have 10 times higher returns from innovation investment and 2.5 times higher sales of new products than the ones in the bottom quartile. Its survey in 2012 (Thuriaux-Alemán, Eagar, and Johansson, 2013) showed that the top quartile innovators achieve 13% more profit from new products and services and 30% shorter time-to-break-even.

Adding to R&D outsourcing risks is the uncertainty as to the capabilities of the party to whom the project is outsourced to (we refer to this party as the agent, and will use the male pronoun). There is a distinct possibility that the agent has inside information regarding his capabilities and/or regarding the likely success of the project - this may not be fully observable to the principal (the firm doing the outsourcing, who will be referred to as being female). This is commonly referred to as a problem of "adverse selection." We address the setting where the principal has inside information regarding the project's potential in Chapter 2. In addition, the effort exerted by the agent is usually unobservable - this is a problem involving "moral hazard." In summary, R&D projects are very costly, extremely risky, and outsourcing involves additional risks caused by information asymmetry and unobservable effort. Accordingly, our general research question is: "*How can a principal best manage the outsourcing of costly, risky, and uncertain R&D projects to an agent with* *inside information and whose efforts are unobservable*?"

We help answer this question by showing that a type of contract known as a stage-gate contract can be used to incentivize the agent to exert a high level of effort even though the principal cannot directly observe his effort, and can at the same time be used by the principal to "screen" the agent as to whether he is of a "high-type" with a high level of competency or of a "low-type" with lesser competency (equivalently, whether the agent knows the project to be of a "high-type" with a high chance of success or of a "low-type" with a lesser chance of success). While previous work has illustrated that an offering of a menu of contracts can be used as a screening mechanism, and has shown that moral hazard can be mitigated through contracting, our contribution is to explore how the combination of these concepts plays out within the context of an outsourced development project, providing closed-form mathematical results.

The stage-gate contract (Cooper 2001, 2008; Hauser, Tellis, and Griffin 2006) gains its name from the fact that it breaks a project down into multiple phases or "stages," with a "gate" at the end of each stage which either lets the project continue (a "go" decision) or terminates the project (a "no go" decision). Hauser, Tellis, and Griffin (2006) highlight the stage-gate process as a formal process that has the attributes of increasing success and reducing times for R&D and new product development. However a contribution of ours is to explicitly show how a stage-gate contract can act as a screening tool in a setting where the agent has private information and prefers to "shirk," and how the contract thereby increases the principal's profit.

Our principal-agent model is set up as follows: 1) the success of the project depends on the agent's effort, but this effort is costly to the agent and the agent's effort is unobservable by the principal (this is a classic "moral hazard" problem); 2) the agent to whom the project is being outsourced may be of either a "high-type" or of a "low-type," with the agent's "type" known only to the agent (this is a classic "adverse selection" problem); 3) the project consists of two periods (stages), with each stage's probability of success dependent on the agent's type and his effort, and with the cost to the agent at each stage dependent on the agent's effort; 4) if the project is successful at stage two (which means, by definition, it was also successful at stage one) then the principal receives some predetermined value; and 5) the principal has control over devising a contract (or pair of contracts) that the agent can choose from (or reject entirely) before initiating work on the project which stipulates the money transfers prior to initiation of work and upon success or failure at each stage.

In addition to showing the screening benefit of the stage-gate contract, we explore: 1) the extent to which the principal suffers from not knowing the agent's type a priori; 2) the roles of the money transfers before project initiation after success at the intermediate stage; 3) whether offering the agent the choice between two different contracts is better than providing one contract; and 4) whether it is sufficient for a contract to only include money transfers upon success, or whether it should also allow for penalties upon failing.

The remainder of the paper is organized as follows. In 1.2 we provide a literature review, and in 1.3 we establish the model setup. In 1.4 we discuss the optimal solution for the different types of stage-gate contracts (i.e., with and without an intermediate payment, etc.), as well as the potential impact of penalties, and in 1.5 we provide managerial insights and concluding remarks.

## 1.2 Literature Review

Our model is related to management of R&D projects, and to principal-agent theory. An empirical study by Girotra, Terwiesch, and Ulrich (2007) on portfolios of R&D projects shows that a significant decrease in firm value can be caused by a late-stage failure of a project. Similarly, Chao, Kavadias, and Gaimon (2009) study how funding authority and incentives impact new product portfolio balance and Chan, Nickerson, and Owan (2007) investigate the important roles played by the state of a firm's pipeline, the magnitude of adjustment, and the transaction costs in the decision making of R&D pipeline management. While these papers explore the optimal portfolio, we focus on a single outsourced project and investigate how a principal can maximize her expected profit from each given project.

Different types of outsourcing (R&D) contracts have been discussed in the literature. For example, Bhaskaran and Krishnan (2009) study mechanisms such as revenue sharing, investment sharing and innovation sharing. Xiao and Xu (2012) propose a model that examines the impact of royalty revision in a two-stage R&D alliance with the possibility of renegotiation, where both the marketer and innovator exert efforts in each of two periods. Modeling joint effort can also be seen in Iyer, Schwarz, and Zenios (2005) on production outsourcing. Alternatively, Savva and Scholtes (2014) show that, without assuming moral hazard and hidden information, co-development with opt-out options has advantages over either pure co-development or licensing. Furthermore, Crama, Reyck, and Degraeve (2008) consider a licensing contract model where the licensee has private information on the probability of technical success and whose action over the R&D phase is unobservable.

One key way in which our paper differs is that our model doesn't involve co-development, revenue sharing or licensing and falls into the category of contract research agreement in which upon the completion of the project, the research party will hand over all of the project results to the paying party. Binns and Driscoll (1998) illustrate the main features of three different types of R&D contracts - contract research agreement, collaborative research agreement and joint collaboration agreement. Modeling an innovation search process, Kim and Lim (2015) study a firm's R&D outsourcing decision and the design of R&D contest in an innovation-driven supply chain. With simultaneous sampling and/or sequential sampling from any probability distribution, Poblete and Spulber (2013) show that the optimal incentive contract for R&D takes the form of an option. Finally, the empirical study of Hermosilla and Qian (2013) shows that adverse selection is the main cause of the success gap between in-house compounds that outperform out-licensed ones. An important insight from our work is that we effectively find that this success gap can be reduced by the use of a stage-gate contract.

Turning to principal-agent theory, Levitt and Snyder (1997) consider a setting in which the agent, whose effort is not observable, reports an observed intermediate signal to the principal who can then decide to continue or cancel the project. Bhattacharya, Gaba, and Hasija (2012) study a model on milestone-based contracts, which has three stages: 1) research, 2) regulation verification and 3) development, with an investment from the agent in the research stage, another investment from the principal in the development stage if regulatory verification leads to approval, and an option-based contract to be exercised in the research stage if the intermediate verifiable signal is successful or not. Chao, Lichtendahl, and Grushka-Cockayne (2014) study a stage-gate process where the agent has private information on the idea quality resulting from the first stage, usually referred to as the research stage. The agent's effort in the second stage, usually referred to as the development stage, is also not observable to the principal, while the principal has the right to make a go/no-go decision based on the agent's report of idea quality. While both their models are similar to ours, in our model there is no issue of reporting a signal (either truthfully or not), and the principal and the agent have a consensus on whether each stage succeeds or fails. An additional feature of our model is that both adverse selection and moral hazard exist in each stage.

Our principal-agent model deals with a setting involving hidden action (moral hazard) and hidden information for which Mas-Colell, Whinston, Green, et al. (1995) provide a concise introduction to the general principal-agent framework. Research on information asymmetry in supply chain management is abundant, and Cachon (2003) and Chen (2003) provide a broad review of the literature in this field. For models involving adverse selection, the mechanism design denoted as the "revelation principle" in Myerson (1979) and Myerson (1982) is a very useful tool. The revelation principle states that a mechanism exists to entice the agent to truthfully reveal his type while still providing the principal as good an outcome as can be achieved given the information asymmetry. Chao, Lichtendahl, and Grushka-Cockayne (2014) apply the revelation principle to the optimal contract design problem in a dual-channel supply chain where the retailer has private information that is unknown to the manufacturer. We apply this tool to our model in which both moral hazard and adverse selection exist.

Crama, De Reyck, and Degraeve (2013) use a similar setup to our approach, but there are some distinct differences: in their model, the agent is assumed to buy the project, whereas we allow the payments to flow in either direction and we find that the first payment in fact does flow from the agent to the principal. Furthermore, in their model the probability that a stage is a success is exogenous, whereas in our model it depends on the agent's endogenous effort. Finally, their agent is differentiated on his belief of the value of the project (which is uncertain and can be improved by exerting a marketing effort in the second stage), while ours are differentiated on capabilities and the project's payoff upon success is fixed (although the expected value of the project changes as a function of the effort levels and probabilities of succeeding to the next stage).

## 1.3 Model Setup

We consider a setting in which a principal ("she") wants to outsource a R&D project to an agent ("he"). We are primarily interested in situations where the agent's ability is not known to the principal (a situation of incomplete information), but for reference we compare to the situation where the agent's ability is known by the principal (the case of complete information). We study a two-period project with stage-gate features, meaning the project either succeeds (and continues) or fails (and ends) at the end of each period. Figure 1.1, which also shows the parameters and variables we discuss later, shows that before the start of the project, the principal offers the agent a contract, which specifies the payments that will be made to the agent. We study various possible contract forms, but a general form, and our "baseline" model, is the case where the contract allows for money transfers (the principal may pay the agent or vice versa) at three points - at the start of the project (before period 1), after period 1 contingent on success in this period, and after period 2 again contingent on this period 2's success. The agent can enter the second period only after the first period is successful; otherwise the project stops (the agent exits). The agent exerts effort within each period; the higher the effort the higher the agent's cost but also the higher the probability of success. The agent's ability also affects the likelihood of success in each period. We assume the agent is either of high ability (i.e., high-type) or low ability (i.e., low-type). Equivalently, rather than the agent himself being of high or low-type, it may be that the agent has private information as to the prospects of the project (it may be the project itself that is of high or low-type). We assume that while the principal does not know the agent's type, she knows the probability that the agent will be of high-type.

The principal designs a contract mechanism that maximizes her expected profit. This optimal contract mechanism could take various forms: it could include two different contracts each of which specifies the menu of money transfers at each point (with one contract intended for a high-type agent and the other intended for a low-type), with the agent having the choice of contract. In this case, the revelation principle applies (Myerson, 1979, 1982) - the contract offered by the principal will be such that the agent will truthfully reveal his type by picking the contract that has been designed for his type. We call this a "screening model" because the contract screens the agent as to his type. Alternately, the principal could offer only one contract (one menu of money transfers) to the agent, intending that the agent accept regardless of his type (no screening). Finally, she could offer only one contract, of a kind that only a high-type agent would be willing to accept (the project will "die" if the agent is of low-type) - this again constitutes screening, since acceptance (or not) divulges agent type. In section 4 we present results for the baseline model and the possible variations as just discussed, along with an extension which allows for penalties to the agent after each period if the project fails (in addition to rewards if it succeeds).

We assume that the project returns a value V if successful at the end of period 2, with V known to all parties. The agent is of low-type with probability $\lambda$ , with $0 < \lambda < 1$, and of high-type with probability $1 - \lambda$ . The effort levels that the agent exerts in the two periods if of low-type and high-type are denoted by $\{e_{1L}, e_{2L}\}$ and $\{e_{1H}, e_{2H}\}$, respectively, which all take values in the $[0, 1]$ interval. The parameters $\alpha_L$ and $\alpha_H$ describe agent type, with the logical constraint that $0 < \alpha_L < \alpha_H < 1$. The probabilities of success for the first and the second periods for the agent if of low-type are $\alpha_L e_{1L}$ and $\alpha_L e_{2L}$. Likewise, $\alpha_H e_{1H}$ and $\alpha_H e_{2H}$ denote the corresponding probabilities of success for the agent if of high-type. Thus, while V is a fixed and known amount, the expected value of the project endogenously depends on the effort levels and unobservable agent capabilities.

The costs that the agent incurs in the respective periods are quadratic in effort exerted; $ke_{1L}^2$ and $ke_{2L}^2$ for the agent if of low-type or $ke_{1H}^2$ and $ke_{2H}^2$ if of high-type, where $k$ is the cost coefficient (assumed to be the same regardless of agent type). We denote the upfront, intermediate (after period 1) and final (after period 2) money transfers to the agent by $m_{0L}$,

$m_{1L}$, and $m_{2L}$, if of low-type and $m_{0H}$, $m_{1H}$, and $m_{2H}$ if of high-type. We do not constrict these transfers to be positive. For example, a negative upfront payment is effectively a participation fee paid by the agent. Finally, we assume the intermediate and final transfers occur only when the first and second periods are successful, respectively, and we also briefly discuss the situation where penalties are allowed in case a period fails.

The principal's expected profit is:

$$\begin{aligned} & \lambda \left[\alpha_L e_{1L} \left(\alpha_L e_{2L}(V - m_{2L}) - m_{1L}\right) - m_{0L}\right] \\ + \quad & (1 - \lambda) \left[\alpha_H e_{1H} \left(\alpha_H e_{2H}(V - m_{2H}) - m_{1H}\right) - m_{0H}\right] \end{aligned} \tag{1.1}$$

Given the offered contract(s) and the agent's type (known to the agent), the agent chooses a contract and effort levels in period 1 and (if the project succeeds in period 1) in period 2 that maximize its expected profit (or declines all contracts if none offers a positive expected value). The agent's expected profit (if of low-type and high-type, respectively) is:

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \tag{1.2}$$

and

$$m_{0H} - ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \tag{1.3}$$

This is a Stackleberg game, solved by backwards induction. Notice that the model with "complete information" consists of (1.1), (1.2) and (1.3). But for the model with "incomplete information," two incentive compatibility constraints have to be included, because the principal wants to prevent one type agent from mimicking the other (e.g., if the agent is low-type, its expected profit when accepting the contract intended for a low-type agent must exceed its expected profit when accepting the contract intended for a high-type agent). Let $\{\tilde{e}_{1L}, \tilde{e}_{2L}\}$ and $\{\tilde{e}_{1H}, \tilde{e}_{2H}\}$ denote the efforts exerted by a low-type agent and a high-type agent, respectively, when the agent is dishonest about its type and pretends to be the other type. Thus the incentive compatibility constraints are:

$$\begin{aligned} & m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \\ \geq \quad & m_{0H} - k\tilde{e}_{1L}^2 + \alpha_L \tilde{e}_{1L} m_{1H} - \alpha_L \tilde{e}_{1L} k \tilde{e}_{2L}^2 + \alpha_L^2 \tilde{e}_{1L} e_{2L} m_{2H} \end{aligned} \tag{1.4}$$

and

$$\begin{aligned} & m_{0H} - ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \\ \geq \quad & m_{0L} - k\tilde{e}_{1H}^2 + \alpha_H \tilde{e}_{1H} m_{1L} - \alpha_H \tilde{e}_{1H} k \tilde{e}_{2H}^2 + \alpha_H^2 \tilde{e}_{1H} \tilde{e}_{2H} m_{2L} \end{aligned} \tag{1.5}$$

We chose this setup to show that the stage gate contract is meaningful in the presence of asymmetric information even in the absence of rick aversion (note that risk-neutrality is assumed for both the principal and agent, and the effort exerted by the agent does not contain a direct investment from the principal).

## 1.4 Results

We first (in 1.4.1) analyze the baseline model, where intermediate and end money transfers depend on the success of the previous stage, while the upfront transfer does not. We compare the complete and incomplete information cases to determine the extent of the information rent. In 1.4.2 we then compare this baseline to the case where there is either no upfront and/or no intermediate money transfer, to determine the benefit (if any) of the stage-gate process. Next, in 1.4.3, we show that the principal is better off by offering two contracts (one intended for a high-type agent and the other intended for a low-type) as opposed to only offering one contract. Finally, in 1.4.4 we show that the full flexibility case where the principal also has the ability to penalize the agent when he fails at a stage does not impact her profit.

For all out analyses, we denote $V < 2k/H$ as the maximum value condition. As long as the value for the project falls below this maximum value, then there is an interior solution (the agent, regardless of his type, trades off the cost of effort versus the desire to increase probability of success). Throughout the analysis, this interior region will be the focus of our discussion, since a value V greater than the maximum value condition would simply result in 100% effort exerted by the agent since the possible reward is so high.

### 1.4.1 The baseline model and the extent of the information rent

When upfront money transfers and intermediate and end money transfers (contingent on success) are all included in the contract (i.e., the baseline model), we have the results as shown in Theorem 1 under complete information and Theorem 2 under incomplete information. All proofs are given in the Appendix.

**Theorem 1** *Under complete information, the outcomes with the baseline model are as follows:*

1. *Money transfers if the agent is of low-type are* $m^*_{0L} = -\frac{\alpha_L^6 V^4}{64k^3}$, $m^*_{1L} = 0$ *and* $m^*_{2L} = V$, *resulting in effort levels* $e^*_{2L} = \frac{\alpha_L m^*_{2L}}{2k}$ *and* $e^*_{1L} = \frac{\alpha_L m^*_{1L} + \frac{\alpha_L^3 m^{*\,2}_{2L}}{4k}}{2k}$ *and zero expected profit for the agent.*

2. *Money transfers if the agent is of high-type are* $m^*_{0H} = -\frac{\alpha_H^6 V^4}{64k^3}$, $m^*_{1H} = 0$ *and* $m^*_{2H} = V$,

*resulting in effort levels* $e^*_{2H} = \frac{\alpha_H m^*_{2H}}{2k}$ *and* $e^*_{1H} = \frac{\alpha_H m^*_{1L} + \frac{\alpha_H^3 m^{*\,2}_{2H}}{4k}}{2k}$ *and zero expected profit for the agent.*

3. *The principal's expected profit if* $\lambda \frac{\alpha_L^6 V^4}{64k^3} + (1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}$.

Theorem 1 states that under complete information, there is no need for adding intermediate money transfers in helping the principal achieve the maximal profit. The expressions for the optimal effort levels show that the effort in period two is proportional to the money transfer following period two, and the effort in period one is proportional to the squared money transfer in period two. As such, there is no influence of the intermediate payment.

Note that the upfront payment is negative, meaning the agent pays to join the project and then if the project is successful the agent receives the full final payment $m^*_{2L} = m^*_{2H} = V$. In addition, it can be shown that $e^*_{1L} < \frac{\alpha_L}{2} e^*_{2L}$ and $e^*_{1H} < \frac{\alpha_H}{2} e^*_{2H}$. In other words, the effort level in the second period is higher than in the first period. Furthermore, the upfront money transfer is used by the principal to extract all profit from the agent. Finally, we see that effort levels increase as the value of the project increases, and decrease as the cost coefficient $k$ increases.

**Theorem 2** *Under incomplete information, the outcomes with the baseline model are as follows: The unique, closed form solution for the menu of money transfers and firm profits is shown in the Appendix. The solution shows:*

1. *The principal's expected profit is strictly lower than in the complete information case in Theorem 1.*

2. *If the agent is low-type he receives no profit and if he is high-type he receives strictly positive profit.*

3. *The money transfers satisfies the following:* $0 > m^*_{0L} > -\frac{\alpha_L^6 V^4}{64k^3}$, $m^*_{1L} > 0$ *and* $0 < m^*_{2L} < V$, *along with* $0 > m^*_{0H} > -\frac{\alpha_H^6 V^4}{64k^3}$, $m^*_{1H} = 0$, $m^*_{2H} = V$.

4. *The effort levels* $e^*_{1L}$, $e^*_{2L}$, $e^*_{1H}$ *and* $e^*_{2H}$ *have the same expression as in Theorem 1.*

Thus, under incomplete information, the agent's upfront payment (i.e., the buy-in) is smaller than under complete information. Also, the principal won't set $m^*_{1L} = 0$ in the optimal menu of money transfers $(m^*_{0L}, m^*_{1L}, m^*_{2L})$ to the agent if he is a low-type, although $m^*_{1H}$ is 0 in the optimal menu of money transfers $(m^*_{0H}, m^*_{1H}, m^*_{2H})$ to the high-type agent. Theorem 2 thus states that including the intermediate money transfers provides more expected profit to the principal because the optimal intermediate payment to the agent if of low-type is always greater than zero. So we find that intermediate money transfers play an important role: they help the principal to identify the type of agent and through this screening she is able to retain more expected profit. The agent (regardless of his type) pays less upfront to participate in the project as compared to the full-information case. The principal still extracts all the profits from the agent if he is a low-type, but the high-type agent expects a strictly positive profit which effectively reflects an "information rent" (Mas-Colell et al. 1995).

Figure 1.2 illustrates the principal's expected profits under complete and incomplete information with the following parameter values: $V = 2$, $k = 1$ and $\alpha_H = 0.8$ satisfying the maximum value condition, $\frac{1-\lambda}{\lambda}$ and $x = \frac{\alpha_H}{\alpha_L} = \frac{0.8}{\alpha_L}$. The difference between the two profit curves represents the information rent caused by information asymmetry.

### 1.4.2 The role of a stage-gate setup and the impact of the money transfers

To determine if the insights hold under variations to the baseline model, we investigate the outcomes under the absence of either the upfront and/or the intermediate money transfers (by "intermediate" money transfer we mean the one that is contingent upon success at stage one). We have the results as shown in Theorem 3 under complete information and Theorem 4 under incomplete information.

**Theorem 3** *Under complete information, the outcomes are as follows, in order of the principal's expected profit (from highest to lowest):*

*Case 1: the baseline case; reference Theorem 1.*

*Case 2: the baseline case minus the possibility of an intermediate money transfer. The principal's expected profit is equal to that of case 1 since all the money transfers are exactly the same, and is strictly higher than in cases 3 and 4.*

*Case 3: the baseline case minus the possibility of an upfront money transfer. The principal's expected profit is* $\frac{1}{2}\left[\lambda\frac{\alpha_L^6 V^4}{64k^3} + (1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}\right]$ *which is strictly higher than in case 4 in this Theorem. The payment scheme to the agent if of low and high-types are* $m_{1L}^* = -\frac{\alpha_L^2 V^2}{8k}$ *and* $m_{2L}^* = V$*, and* $m_{1H}^* = -\frac{\alpha_H^2 V^2}{8k}$ *and* $m_{2H}^* = V$*, respectively.*

*Case 4: the baseline case minus the possibility of either upfront or intermediate money transfers. The principal's expected profit is* $\frac{27}{64}\left[\lambda\frac{\alpha_L^6 V^4}{64k^3} + (1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}\right]$ *with* $m_{2L}^* = m_{2H}^* = \frac{3}{4}V$ .

Theorem 3 then states that under complete information, the removal of the intermediate money transfers yields results equivalent to the baseline model (this was already clear from Theorem 1). Once upfront money transfers are eliminated, the principal's expected profit decreases by exactly 50% and to achieve the optimal profit, the intermediate money transfers are no longer zero. In fact, the money transfers after the first period are now from the agent to the principal and differ depending on the agent's type. In essence, when there is no upfront money transfer, the agent is willing to "buy-in" after the first stage is successful, to continue working on the project in pursuit of the pay-off at the successful completion of the project. The expected profit to the principal is then lower, predominantly because she does not receive the buy-in when the first stage fails.

Finally, when neither upfront nor intermediate money transfers are allowed, the principal's expected profit decreases a further 5/64 (or a total of 37/64 = 57.8% when compared to the baseline), and is actually the same as in the incomplete information case to be discussed shortly. The reason for this is rather straightforward: when there are only end money transfers, upon a successful project completion, it does not matter whether the principal is aware of the agent's type. In fact, both agents receive the same payment upon successfully completing the project. The high level type just has a higher probability of attaining it, which is irrelevant to the principal's expected profit.

**Theorem 4** *Under incomplete information, the strict ordering of the principal's expected profit is (from highest to lowest):*

*Case 1: The principal's expected profit in the baseline case as presented in Theorem 2 is strictly higher than in Cases 2, 3, and 4 in this Theorem.*

*Case 2: The principal's expected profit in a modified baseline case that excludes the possibility of an intermediate money transfer is given in the Appendix and is strictly higher than in Cases 3 and 4 in this Theorem.*

*Case 3: The principal's expected profit in a modified baseline case that excludes the possibility of an upfront money transfer is strictly higher than in Case 4 in this Theorem.*

*Case 4: The principal's expected profit in a modified baseline case that excludes the possibility of either upfront or intermediate money transfers is* $\frac{27}{64}\left[\lambda\frac{\alpha_L^6 V^4}{64k^3} + (1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}\right]$.

Thus, Theorem 4 states that when intermediate money transfers are not allowed in the incomplete information case, the principal's expected profit declines somewhat. The percentage decline in the principal's expected profit from case 1 to case 2 is shown in the Appendix to only depend on the relative ability of a high-type versus a low-type agent, and the ratio of the probability that the agent is of high-type to the probability that he is of low-type. This means that the percentage profit increase and the percentage information rent decrease does not depend on $k$ or $V$. For a project with a large monetary value, the relatively small percentage change in profit upon adding an intermediate stage is meaningful and cannot be ignored.

When instead upfront money transfers are removed from the payment scheme (i.e., case 3), a closed form solution for the principal's expected profit can no longer be determined, but we can prove (in the Appendix) that it lies between the principal's expected profit in cases 2 and 4. As an example for case 3, a numerical calculation shows that $V = m_{2H}^* > m_{2L}^* > 0$, and $0 > m_{1L}^* > m_{1H}^*$. When $V = 2$, $k = 1$, $\alpha_H = 0.8$, $\frac{1-\lambda}{\lambda} = 0.4$, and $x = \frac{\alpha_H}{\alpha_L} = 1.2$, the optimal solution is $m_{2H}^* = 2$, $m_{2L}^* = 1.8173$, $m_{1H}^* = -0.2808$, and $m_{1L}^* = -0.1692$, which provides the principal with a profit of 0.01685. Notice again that in this case without upfront money transfers the intermediate money transfers flow from the agent to the principal. Results over a wider range for both the complete and incomplete cases are shown in Figure 1.3. In Figure 1.3 the expected profit line for only end money transfers is the same under complete and incomplete information, since the agents only get paid when the second stage was successful, at which point it does not matter if he was of low or high-type. The parameter values are the same as in Figure 1.2.

Figure 1.3 illustrates the difference of the principal's expected profits under complete and incomplete information for each of the four cases as discussed in Theorems 3 and 4. From this we can see that adding an upfront money transfer (Case 2) to the most basic model with only an end money transfer (Case 4) can dramatically increase the principal's expected profit, while adding an intermediate money transfer (Case 3) adds a less substantial but possibly very significant amount (compare Case 3 to Case 4, and compare Case 1 to Case 2). Information rent can also be determined by comparing the similar cases under complete (upper) and incomplete (lower) information (note the different scales on the y-axes).

Figures 1.4, 1.5, 1.6, 1.7 and 1.8 show the effort levels of the agent under complete and incomplete information. We can see that adding more money transfers doesn't necessarily imply that the first or the second effort level will increase.

### 1.4.3 The effect of a menu of multiple contracts

So far our discussion has centered on models in which the agent can choose his preferred menu of money transfers based on his type, and is willing to participate regardless of his type. The principal used these different menus to screen whether the agent was of low-type or of high-type, while the participation of the agent regardless of his type implies that his profit is always nonnegative. But it is possible that such screening that includes the participation of the agent if of low-type is costly and thereby hurts the principal's expected profit. In other words, it is possible that contracts that exclude the agent if of low-type or that offer the agent the same money transfer regardless of his type could be more advantageous to the principal in terms of profit. The following two theorems give an answer to this question. Both obviously deal with the incomplete information case, since in the complete information case, no screening is necessary.

**Theorem 5** *For the baseline model (Case 1) under incomplete information, by offering two different menus of money transfers to the agent, the principal can realize a strictly higher expected profit than when she offers a contract with one menu of money transfers offered to the agent such that he would be willing to participate regardless of his type.*

**Theorem 6** *For the baseline model (Case 1) under incomplete information, by offering two different menus of money transfers to the agent, the principal can realize a strictly higher expected profit than when she offers a contract with one menu of money transfers to the agent such that he would only be willing to participate if of high-type.*

Theorem 5 and 6 combined state that offering a menu of two contracts increases the principal's expected profit as compared to offering just one contract. The reason that two contracts perform better than one in Theorem 5 is that if the agent is of the high-type, the screening contract tailored toward this agent entices him to exert more efforts in striving to succeed in both periods, thus helping to create more value. On the other hand, if the agent is of low-type, the principal will not lose anything by offering him a contract tailored for a low-type - the principal can in this case appropriate all the agent's profit. Theorem 6 states that excluding the agent if of low-type is not an optimal strategy either, because in this case the principal would forgo any profit if the agent is of low-type (the project would "die" at the outset if the agent is of low-type). By offering two contracts, the profit that the principal expects due to the possibility that the agent is of low-type more than offsets the fact that the principal receives a lower profit if the agent is of high-type (as compared to what the principal would get if she offered only a contract that only a high-type agent would accept).

Taken together, these two theorems combined tell us that in the incomplete information case, it is in the best interest of the principal to offer two different menus of money transfers $(m_{0L}, m_{1L}, m_{2L})$ and $(m_{0H}, m_{1H}, m_{2H})$ such that the agent is willing to participate regardless of his type. In other words, in the analysis to this point, the baseline case (Case 1) offers the highest expected profit to the principal.

### 1.4.4 Full flexibility and the role of possible penalties contingent on failure

Another thing we are interested in is whether the addition of penalties (contingent on failure at each stage, in addition to rewards that accrue upon success at a stage) leads to different profits for the principal. To answer this question, we assume that $U_{0L}$ is the money transfer to the agent if of low-type at the beginning of the project, $R_{1L}$ and $R_{2L}$ are the rewards to the agent if of low-type when the first and the second periods are successful individually, and $P_{1L}$ and $P_{2L}$ are the penalties to the agent if of low-type when the first and the second periods are not successful respectively. Similar meanings of $U_{0H}$, $R_{1H}$, $R_{2H}$, $P_{1H}$ and $P_{2H}$ apply if the agent is of high-type. This is the full flexibility model setup and we call it "Case 5."

The principal chooses $U_{0L}$, $R_{1L}$, $R_{2L}$ , $P_{1L}$ and $P_{2L}$ along with $U_{0H}$, $R_{1H}$ , $R_{2H}$ , $P_{1H}$ and $P_{2H}$ to maximize her expected profit, which is given by:

$$\begin{aligned} & \lambda\left[\alpha_L e_{1L}\left(\alpha_L e_{2L}(V - R_{2L}) + (1-\alpha_L e_{2L})P_{2L} - R_{1L}\right) + (1-\alpha_L e_{1L})P_{1L} - U_{0L}\right] \\ + \quad & (1-\lambda)\left[\alpha_H e_{1H}\left(\alpha_H e_{2H}(V - R_{2H}) + (1-\alpha_H e_{2L})P_{2H} - R_{1H}\right)\right] \\ + \quad & (1-\lambda)\left[(1-\alpha_H e_{1H})P_{1H} - U_{0H}\right] \end{aligned} \tag{1.6}$$

In turn the agent, given his type and given $U_{0L}$, $R_{1L}$, $R_{2L}$, $P_{1L}$,$P_{2L}$ and $U_{0H}$, $R_{1H}$, $R_{2H}$, $P_{1H}$,and $P_{2H}$, maximizes his expected profit by choosing the most profitable contract along with his effort level. The expected profits for a low-type agent and high-type agent are, respectively:

$$\begin{aligned} & U_{0L} - ke_{1L}^2 + \alpha_L e_{1L} R_{1L} - (1-\alpha_L e_{1L})P_{1L} - \alpha_L e_{1L} k e_{2L}^2 \\ + \quad & \alpha_L^2 e_{1L} e_{2L} R_{2L} - \alpha_L e_{1L}(1-\alpha_L e_{2L})P_{2L} \end{aligned} \tag{1.7}$$

and

$$\begin{aligned} & U_{0H} - ke_{1H}^2 + \alpha_H e_{1H} R_{1H} - (1 - \alpha_H e_{1H}) P_{1H} - \alpha_H e_{1H} k e_{2H}^2 \\ + \quad & \alpha_H^2 e_{1H} e_{2H} R_{2H} - \alpha_H e_{1H} (1 - \alpha_H e_{2H}) P_{2H} \end{aligned} \tag{1.8}$$

These, again, need to be nonnegative in order for either type of agent to be willing to participate. Equations (1.6), (1.7) and (1.8) form the full flexibility model with "complete information." For the full flexibility model with "incomplete information," the incentive compatibility constraints are:

$$\begin{aligned} & U_{0L} - ke_{1L}^2 + \alpha_L e_{1L} R_{1L} - (1 - \alpha_L e_{1L}) P_{1L} - \alpha_L e_{1L} k e_{2L}^2 \\ & + \alpha_L^2 e_{1L} e_{2L} R_{2L} - \alpha_L e_{1L} (1 - \alpha_L e_{2L}) P_{2L} \\ \geq & \\ & U_{0H} - k\tilde{e}_{1L}^2 + \alpha_L \tilde{e}_{1L} R_{1H} - (1 - \alpha_L \tilde{e}_{1L}) P_{1H} - \alpha_L \tilde{e}_{1L} k \tilde{e}_{2L}^2 \\ & + \alpha_L^2 \tilde{e}_{1L} \tilde{e}_{2L} R_{2H} - \alpha_L \tilde{e}_{1L} (1 - \alpha_L \tilde{e}_{2L}) P_{2H} \end{aligned} \tag{1.9}$$

and

$$\begin{aligned} & U_{0H} - ke_{1H}^2 + \alpha_H e_{1H} R_{1H} - (1 - \alpha_H e_{1H}) P_{1H} - \alpha_H e_{1H} k e_{2H}^2 \\ & + \alpha_H^2 e_{1H} e_{2H} R_{2H} - \alpha_H e_{1H} (1 - \alpha_H e_{2H}) P_{2H} \\ \geq & \\ & U_{0L} - k\tilde{e}_{1H}^2 + \alpha_H \tilde{e}_{1H} R_{1L} - (1 - \alpha_H \tilde{e}_{1H}) P_{1L} - \alpha_H \tilde{e}_{1H} k \tilde{e}_{2H}^2 \\ & + \alpha_H^2 \tilde{e}_{1H} \tilde{e}_{2H} R_{2L} - \alpha_H \tilde{e}_{1H} (1 - \alpha_H \tilde{e}_{2L}) P_{2H} \end{aligned} \tag{1.10}$$

Similar to the original models with incomplete information, $\{\tilde{e}_{1L}, \tilde{e}_{2L}\}$ and $\{\tilde{e}_{1H}, \tilde{e}_{2H}\}$ represent the efforts exerted by the agent if of low-type and the agent if of high-type, respectively, when the agent deviates from its actual type and pretends to be the other type. While the upfront money transfers $U_{0L}$ and $U_{0H}$ are still not restricted to be either positive or negative, we restrict the rewards and penalties $R_{1L}$, $R_{2L}$, $P_{1L}$, $P_{2L}$ , $R_{1H}$, $R_{2H}$, $P_{1H}$ and $P_{2H}$ to be nonnegative. Equations(1.6) through(1.10), representing the full flexibility model, can be rewritten as

$$\begin{aligned} & \lambda \left[ \alpha_L e_{1L} \left( \alpha_L e_{2L} (V - (R_{2L} + P_{2L})) - (R_{1L} + P_{1L} - P_{2L}) \right) + P_{1L} - U_{0L} \right] \\ + \quad & (1 - \lambda) \left[ \alpha_H e_{1H} \left( \alpha_H e_{2H} (V - (R_{2H} + P_{2H})) - (R_{1H} + P_{1H} - P_{2H}) \right) \right] \\ + \quad & (1 - \lambda) \left[ P_{1H} - U_{0H} \right] \end{aligned} \tag{1.11}$$

subject to

$$\begin{aligned}(U_{0L} - P_{1L}) - ke_{1L}^2 + \alpha_L e_{1L}(R_{1L} + P_{1L} - P_{2L})&\\ -\alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L}(R_{2L} + P_{2L}) &\geq 0 \end{aligned} \tag{1.12}$$

$$\begin{aligned}(U_{0H} - P_{1H}) - ke_{1H}^2 + \alpha_H e_{1H}(R_{1H} + P_{1H} - P_{2H})&\\ -\alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H}(R_{2H} + P_{2H}) &\geq 0 \end{aligned} \tag{1.13}$$

$$\begin{aligned}&(U_{0L} - P_{1L}) - ke_{1L}^2 + \alpha_L e_{1L}(R_{1L} + P_{1L} - P_{2L})\\ &-\alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L}(R_{2L} + P_{2L})\\ \geq&\\ &(U_{0H} - P_{1H}) - k\tilde{e}_{1L}^2 + \alpha_L \tilde{e}_{1L}(R_{1H} + P_{1H} - P_{2H})\\ &-\alpha_L \tilde{e}_{1L} k \tilde{e}_{2L}^2 + \alpha_L^2 \tilde{e}_{1L} \tilde{e}_{2L}(R_{2H} + P_{2H}) \end{aligned} \tag{1.14}$$

and

$$\begin{aligned}&(U_{0H} - P_{1H}) - ke_{1H}^2 + \alpha_H e_{1H}(R_{1H} + P_{1H} - P_{2H})\\ &-\alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H}(R_{2H} + P_{2L})\\ \geq&\\ &(U_{0L} - P_{1L}) - ke_{1L}^2 + \alpha_H \tilde{e}_{1H}(R_{1L} + P_{1L} - P_{2L})\\ &-\alpha_H \tilde{e}_{1H} k \tilde{e}_{2H}^2 + \alpha_H^2 \tilde{e}_{1H} \tilde{e}_{2H}(R_{2L} + P_{2L}) \end{aligned} \tag{1.15}$$

We now discuss a surprising equivalence between the baseline stage-gate contracts and the full flexibility models that include penalties when stage one or two fails, under both complete and incomplete information.

**Theorem 7** *Under complete information, the baseline model (Case 1, with upfront money transfers along with intermediate and end money transfers contingent upon success) provides the principal the same expected profit as full-flexibility contracts (Case 5, with upfront money transfers along with intermediate and end rewards contingent upon success at each respective stage and penalties contingent upon failure at each respective stage).*

**Theorem 8** *Under incomplete information, the baseline model (Case 1) provides the principal the same expected profit as full-flexibility contracts (Case 5). In fact, there are infinitely-many possibly optimal solutions involving full-flexibility contracts (Case 5), including one that specifies upfront money transfers equaling zero.*

Thus, under both complete information and incomplete information, the baseline stage-gate contract gives the principal the maximal profit that is the same as that offered by the full flexibility stage-gate contract where penalties are allowed. But it is not obvious to see that the full flexibility stage-gate contract can do as well as the baseline model. Fortunately, the closed form analytic solutions for the baseline model under both complete and incomplete information offer us a useful tool to tackle this issue for the situation. At least one way to see that these two models are equivalent is by setting the potential penalty after stage one's failure equal to the original model's initial buy-in.

Theorems 7 and 8 thus establish the equivalence between the baseline stage-gate contracts and the full flexibility stage-gate contracts under both complete information and incomplete information. While this equivalence has not been rigorously proven in all the other cases (i.e., Cases 2, 3, and 4 without upfront and/or intermediate payments), our work suggests that the possibility of a penalty if a stage fails can basically replace the buy-in by the agent to the principal that could occur in either the upfront or intermediate stage. As such, the full flexibility model requires one fewer money transfer stage to achieve the same results for the principal. It is thus up to the principal to decide which type of stage-gate contract she prefers: the baseline, where she receives an initial "buy-in" (i.e., the upfront or intermediate money transfer flows in the direction of the principal), or the full flexibility contract where she receives a payment when a stage fails.

## 1.5 Summary

Given the increase in outsourcing of R&D, it is paramount that firms better understand how to develop contracts that appropriately incentivize suppliers. The setting we examine is one where the agent has private information as to the project's potential success (leading to a possible adverse selection problem due to information asymmetry) and must exert costly effort (leading to a possible moral hazard issue). We find that a type of contract known as a "stage-gate" can help mitigate the combination of adverse selection and moral hazard - while the stage-gate framework has long been touted for other reasons, a key contribution of ours is to delineate this aforementioned mitigating potential. We do so with a classic textbook-style model which results in closed-form solutions for a number of model variations.

In our two-stage model, the payment scheme consists of money transfers at the beginning (upfront), after the first period contingent on the success in this period (intermediate), and after the second period contingent on success of the entire project (end). We also look at the role that penalties might play, where success in the current period leads to a reward (a payment from the principal to the agent) while failure leads to a penalty (a payment from the agent to the principal). We focus predominantly on the incomplete information case, where the agent has private information as to his ability or as to the project promise (the agent's "type" is assumed to be high or low). The principal does know the probability that the agent is of high-type and also knows the agent's cost of effort given the agent's possible effort levels. Given this information, the principal can create payment schemes that "screen" the agent, i.e., that entice the agent to truthfully reveal his type. We compare the resulting expected profits to the complete information case where the agent's type is fully known.

Importantly, we find that including the possibility of intermediate money transfers increases the principal's expected profit when she has incomplete information. In other words, a stage-gate contract offers an advantage to the principal by inducing the agent to truthfully reveal his capabilities (or his private knowledge as to the project viability). We also find that the principal increases her expected profit by offering the agent a menu of contracts (as opposed to offering only one contract that the agent would accept regardless of his type; or as opposed to offering only one contract that the agent would accept only if of high-type). In other words, from the principal's perspective, the screening model fully exploits the opportunities that screening offers in terms of preventing the agent, if of high-type, from pretending to be of low-type (and vice versa). In the screening model the agent, if of low-type, pays the principal upfront for the opportunity to participate, which reduces the agent's expected profit to zero, while if of high-type the agent again makes an upfront payment but realizes a (relatively small) expected profit.

We find that a full flexibility stage-gate contract in which the principal can penalize the agent when he fails a stage (in addition to rewarding the agent when he succeeds) does not offer the principal higher expected profit - the full-flexibility contract is equivalent to one including rewards for success (in addition to specifying that the agent pay an initial "buy-in fee"). From a managerial perspective, such equivalence suggests the principal has multiple options in designing the contract menus.

Given the stylized setting that we model, there is of course opportunity for future work to more accurately capture other issues faced in real-life scenarios. For example, one could consider possible risk aversion on the part of the agent, possible financial constraints of the agent, the residual value of a project in case of a failure, and/or allowing second trials after failures. In addition, one might look at the case where it is the principal (rather than, as in our case, the agent) who has private information as to the potential success of the project. In such case, the principal would like to find a way to credibly transmit this private information to the agent, to entice the agent to exert appropriate effort in completion of the project. All these potential research topics are left for future exploration.

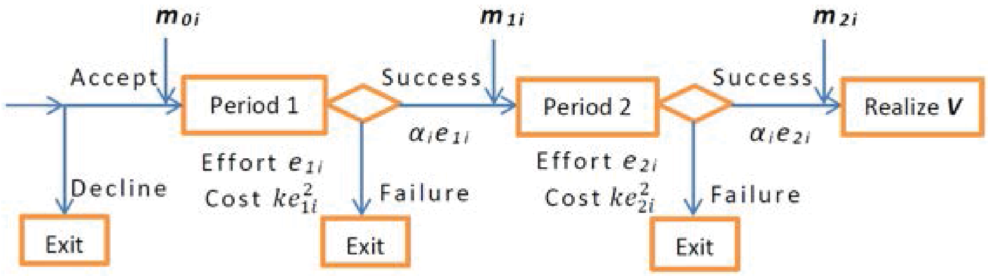



**Figure 1.1**: The Baseline Model (i = L or H)

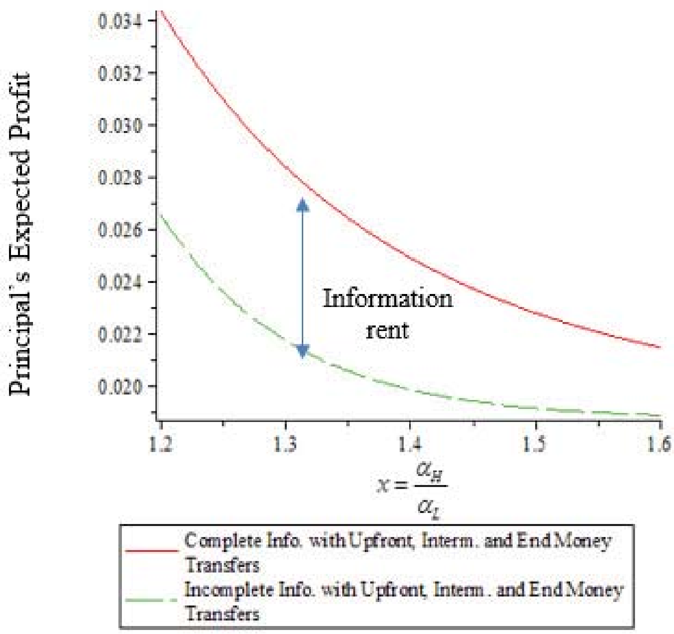
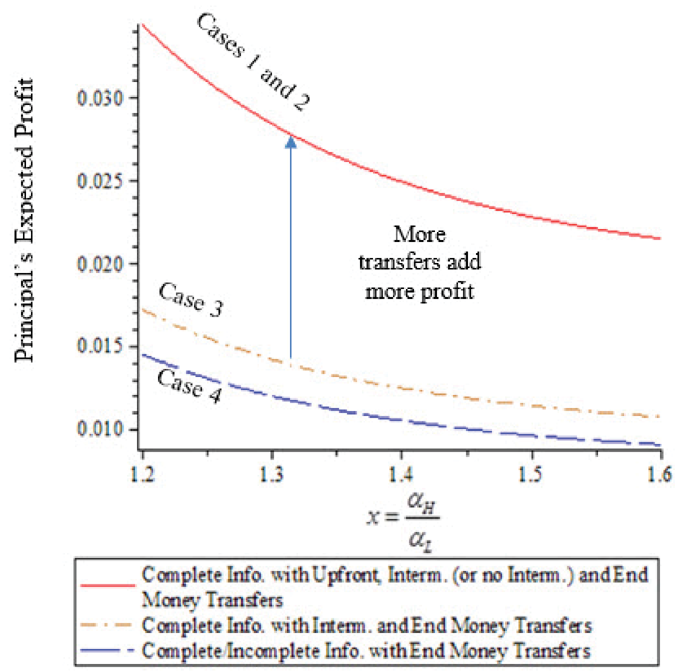



**Figure 1.2**: Incomplete Information Exerts an Information Rent in the Baseline Model



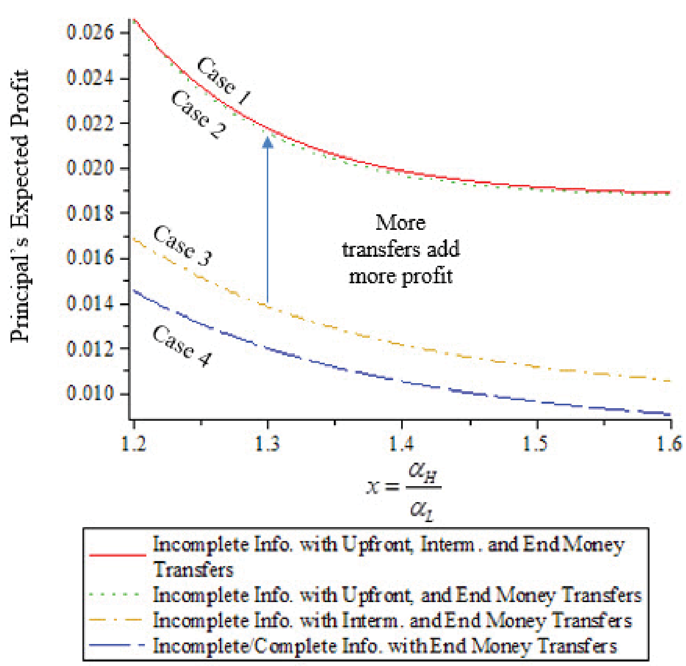




**Figure 1.3**: Adding More Money Transfer Points Increases the Principal's Expected Profit under Both Complete (Upper) and Incomplete (Lower) Information

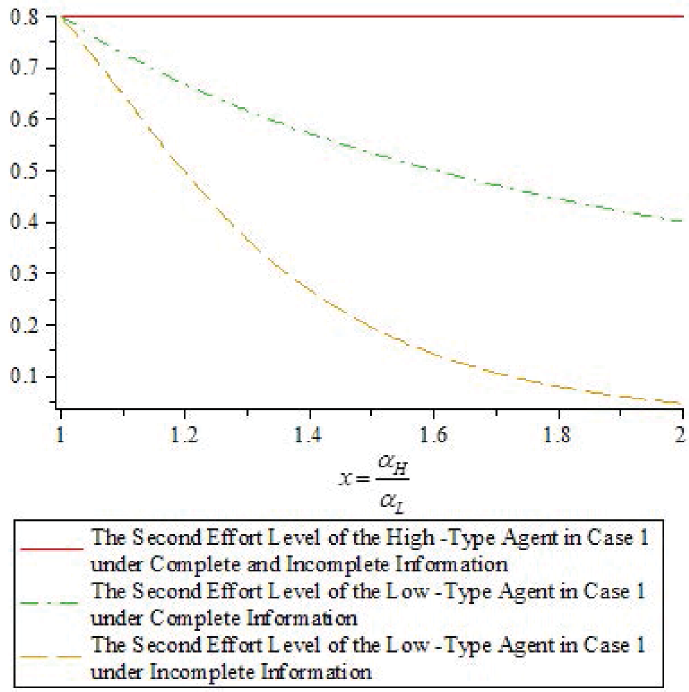



**Figure 1.4**: The Second Effort Levels of the Agents in Case 1 under Complete and Incomplete Information

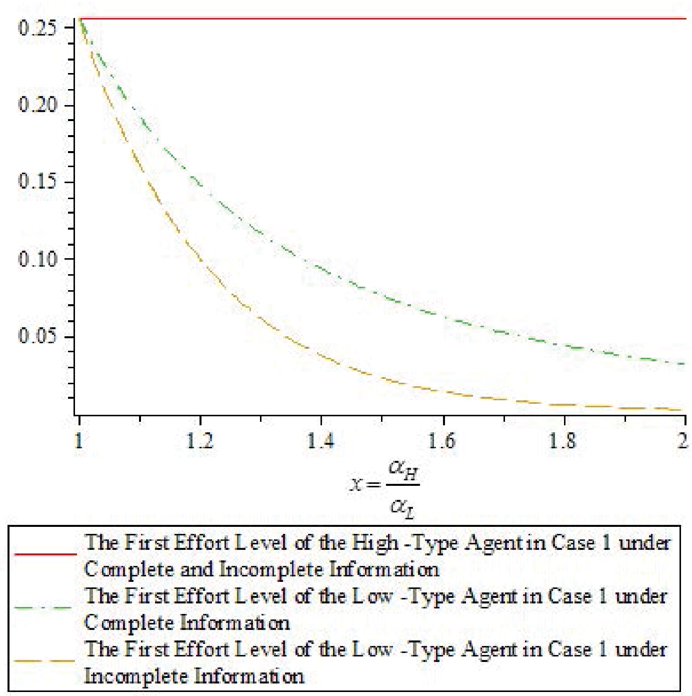



**Figure 1.5**: The First Effort Levels of the Agents in Case 1 under Complete and Incomplete Information

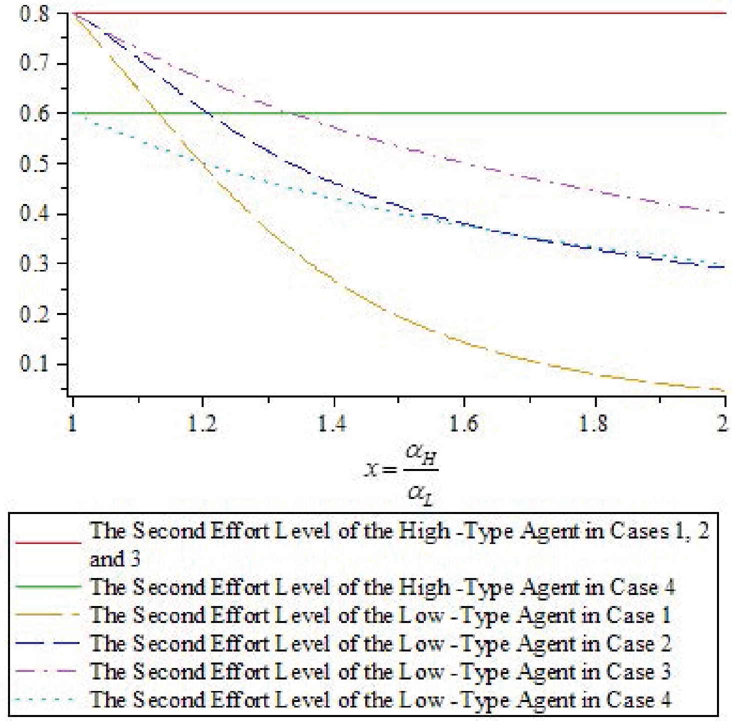



**Figure 1.6**: The Second Effort Levels of the Agents in Cases 1, 2, 3 and 4 under Incomplete Information

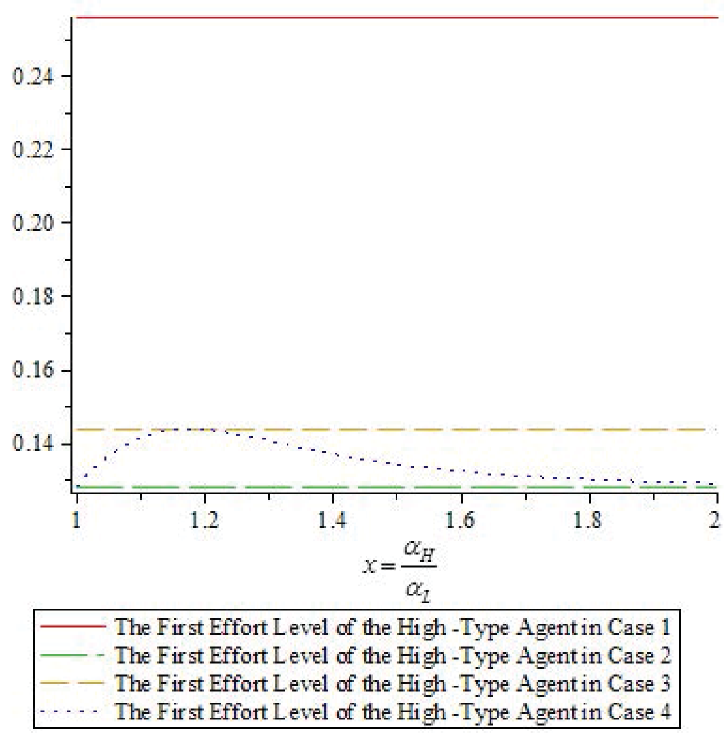



**Figure 1.7**: The First Effort Level of the High-type Agent in Cases 1, 2, 3 and 4 under Incomplete Information

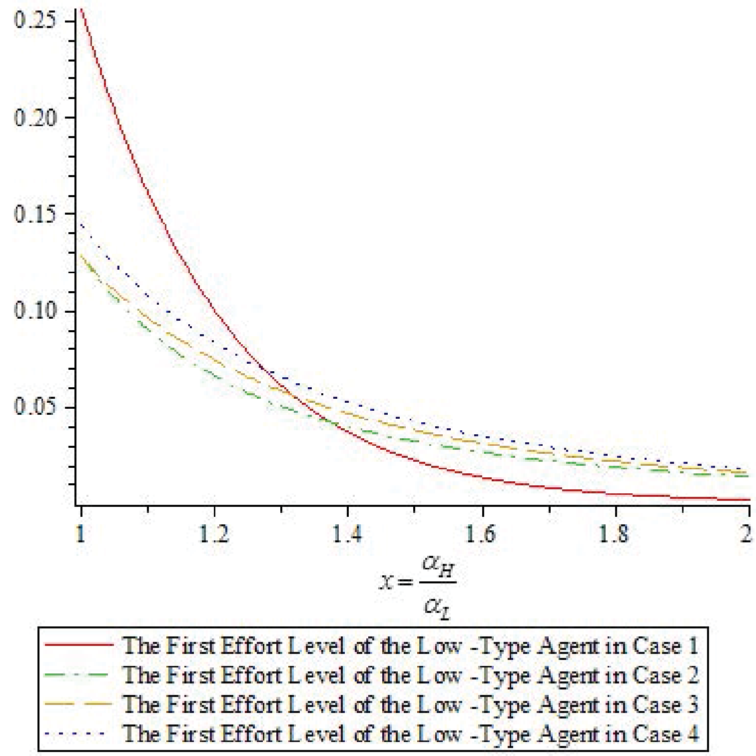



**Figure 1.8**: The First Effort Level of the Low-type Agent in Cases 1, 2, 3 and 4 under Incomplete Information

# CHAPTER 2

# PRINCIPAL'S SIGNALING OF PRIVATE INFORMATION WHEN OUTSOURCING ITS PRODUCT DEVELOPMENT

## 2.1 Introduction

The complexity of collaborative relationship in outsourcing and risks inherent in an R&D project pose challenges to both the principal, the firm who is outsourcing and the agent, the firm to whom the project is outsourced. The principal would strongly prefer to outsource its R&D to a top-tier agent, and on the other side of the coin, an agent would strongly prefer to work for a top-tier principal (see also PwC (2014) on R&D Outsourcing in Hi-tech Industries). But how does the principal know the agent's capabilities - can the principal trust the agent to correctly divulge his private information as to his capabilities as they relate to the project at hand? Conversely, how does the agent know whether the principal is offering a truly-exceptional opportunity for the agent to work on, as opposed to a less-attractive opportunity - can (will) the principal credibly convey her private information to the agent? We explored the first question in Chapter 1, and will tackle the second one in Chapter 2. In both chapters, we also consider a further complicating factor, namely, the fact that the success of the R&D project will likely hinge on the effort exerted by the agent (here we use the term "effort" loosely; it could for example incorporate issues such as the number of resources the agent allocates to the project). The principal would like the agent to exert high effort, but effort is costly to the agent so he will not exert appropriate effort unless sufficiently compensated for this effort (we assume that the principal is not able to directly monitor the effort, and thus the agent's exertion is directly determined by the compensation scheme).

To reiterate, in this chapter, we will address the following research question on this issue: "How can a principal with private information best manage the outsourcing of a costly and risky R&D project to an agent whose efforts are unobservable?" The importance of this research question is highlighted by a survey by PwC (2014) which showed that transparent communication and information sharing were the most important action that the principal and the agent did to make successful relationship. In pursuit of our research question, we examine whether stage-gate contract can (and should) be used by a principal to signal (i.e., reveal) her private information, and at the same time be used to elicit high levels of effort on the part of the agent, in order for the principal to maximize her expected profit. We start by analyzing a "baseline" stage-gate contract involving an upfront money transfer (we allow the payment to go from the principal to the agent or vice versa), an intermediate money transfer (contingent upon success at stage one) and an end money transfer (contingent upon success at stage two). To entice the agent to participate, any contact must offer the agent a positive expected profit. We consider the possibility of either a separating equilibrium or a pooling equilibrium. Under the separating equilibrium, the principal signals to the agent, via the contract, whether she (i.e., the project) is of high or low-type. Under the pooling equilibrium, the principal offers the same money transfer regardless of her type (the contract does not reveal information to the agent regarding the project's type). Of course, the principal will choose either the separating or pooling equilibrium depending on which offers her the higher expected profit.

Since we are interested in determining the advantage (if any) of the stage gate framework, we go on to examine some alternatives to the above baseline case. If the project does not involve stages and gates, then there is no intermediate go/no-go decision so we analyze the case lacking the intermediate money transfer. In addition, we examine the setting lacking an upfront money transfer (with or without an intermediate money transfer) and analyze both the separating and pooling equilibria.

Given that the principal has private information, the setting may involve an information "rent." That is, the principal's expected profit may be reduced from what it would be if there were no private information. We are interested in determining the magnitude of this information rent, and thus in all cases we compare the case of incomplete information to one where the agent knows the principal type (complete information).

The rest of the paper proceeds as follows. In 2.2 we review the relevant literature, and in 2.3 we present the model setup. In 2.4 we establish the main results on stage-gate contracts under separating and pooling equilibria, including the baseline models and those with the absence of upfront and/or intermediate money transfers. In 2.5 we provide managerial insights and concluding remarks.

## 2.2 Literature Review

In Chapter 1, we had a broad review on literature involving new product development and R&D, principal-agent theory, and outsourcing. In Chapter 2, our principal-agent model is a signaling game involving private information of the principal (adverse selection) and unobservable effort of the agent (moral hazard). The extensive literature on signaling games begins with the Spence (1973) education model, where the high-productive employees may be forced to take actions to distinguish themselves from their less able counterparts. Myerson (1983) establishes a general mechanism design for the principal-agent problem with an informed principal. Different from the general setting and the focus of cooperate games in Myerson (1983), Maskin and Tirole (1990) and Maskin and Tirole (1992) consider the principal-agent problem with an informed principal under the framework of noncooperation for two different scenarios, one in which the principal's private information does not directly affect the agent's payoff and the other having the opposite setting. Both their models are one-period and have observable efforts (actions). The main tool used to analyze the games in their models is the Perfect Bayesian Equilibrium, which we will apply to our two-stage (period) model involving both adverse selection (with an informed principal) and moral hazard (with unobservable agent's effort). Research on information asymmetry and signaling games in supply chain management is quite rich; Cachon (2003) and Chen (2003) give a broad review of the literature in this area. One notable article is Lariviere and Padmanabhan (1997) which discusses slotting allowances. In their one-period model, the principal (the manufacturer) signals her private information on demand to the agent whose effort is unobservable through the wholesale price and the slotting allowance. In comparison, the principal in our two-stage (period) model signals her private information on the project to the agent whose effort is unobservable through money transfers stipulated in the contract.

## 2.3 Model Setup

We consider a setting in which a principal ("she") wants to outsource an R&D project to an agent ("he"). The project can either have a relatively higher probability of success (i.e., be of a high-type) or a lower probability (low-type), given some effort level on the part of the agent. This information (the project type and its probability of success as a function of the agent's effort) is known to the principal but unknown to the agent; however, the agent knows the two possible probabilities (as a function of his effort level) and has a prior estimate of the likelihood that the principal is of high-type. When the project is of low-type, the principal may want to fool the agent to get the agent to exert higher effort, which will in turn increase the project's probability of success and its expected profit. Therefore, when the project is of high-type, a decision that the principal faces is whether to signal to the agent (through the contract) that this is in fact a high-type project, or to instead reveal no information to the agent about the quality of the project. The first scenario (revelation of the project type) is called a separating equilibrium and the second is referred to as a pooling equilibrium.

The two-period project has stage-gate features, which means the project can succeed or fail in each period, and the success of the project depends on each period's success. The principal can offer payments at the starting point, after the first period and (if it succeeds in the first period) after the second period of the project. From here on we use the term "money transfers" rather than "payments" to allow for the possibility of cash flows from the agent to the principal, which can be seen as the price of "buy-in" that the agent has to pay. A contract consists of a specified set of three money transfers, the first occurring at the starting point (referred to as an upfront transfer), the second after the first period (the intermediate money transfer) and the third after the second period (the end transfer). The intermediate and end money transfers occur only when the first and second periods are successful, respectively.

We assume the project returns the value of $V$ if it succeeds in both periods. The agent exert efforts $\{e_{1H}, e_{2H}\}$ in the first and second periods if he believes that the project is of high-type, and exerts efforts $\{e_{1H}, e_{2H}\}$ if he believes that it is of low-type, or $\{e_1, e_2\}$ if he is not certain about the exact type of the project; all the efforts take on a value in $[0, 1]$ interval. Correspondingly, costs $\{ke_{1H}^2, ke_{2H}^2\}$ , $\{ke_{1L}^2, ke_{2L}^2\}$ or $\{ke_1^2, ke_2^2\}$ are incurred in the two periods, where $k$ is a (constant) cost coefficient. We let $\alpha_H$ and $\alpha_L$ denote the high-type and low-type projects, respectively, where $0 < \alpha_L < \alpha_H < 1$. Note that in Chapter 1 $\alpha_L$ and $\alpha_H$ refer to agent type while in Chapter 2 they refer to principal type. Probabilities of success in the first period and second periods for a high-type project are $\alpha_H e_{1H}$ and $\alpha_H e_{2H}$ if the agent believes it is a high-type project and therefore exerts high efforts, and $\alpha_H e_{1L}$ and $\alpha_H e_{2L}$ if the agent believes it is a low-type one and therefore exerts low efforts. Similarly, probabilities of success for the first period and second periods for a low-type project are $\alpha_L e_{1H}$ and $\alpha_L e_{2H}$ if the agent believes it is a high-type project and therefore exerts high efforts, and $\alpha_L e_{1L}$ and $\alpha_L e_{2L}$ if the agent believes it is a low-type one and therefore exerts low efforts.

If the agent is uncertain about the project type we let $p$ denote his assessment regarding the probability that the project is of high-type (he believes it is of low-type with probability $1 - p$ ). In this case the agent will exert efforts $e_1$ and $e_2$ in the first period and will infer a probability of success for the first period is $\overline{\alpha} e_1$ , where $\overline{\alpha} = p\alpha_H + (1 - p)\alpha_L$, a weighted average of $\alpha_H$ and $\alpha_L$. If the project is successful in the first period, the agent will update his belief regarding the probability that the project is of high-type, and he will infer a second period probability of success is $\tilde{\alpha}e_2$ , where $\tilde{\alpha} = (p\alpha_H^2 + (1-p)\alpha_L^2)/(p\alpha_H + (1-p)\alpha_L)$ , the Bayesian update of $\overline{\alpha}$. The principal knows the project type and, if it can infer the agent's efforts $e_1$ and $e_2$ then it knows the probabilities of success for the first period and second periods: $\alpha_H e_1$ and $\alpha_H e_2$ for a high-type project and $\alpha_L e_1$ and $\alpha_L e_2$ for a low-type project.

We next proceed to analyze the baseline model involving all three money transfers. Later on we will consider other cases when upfront and/or intermediate money transfers are not included in the contract. For convenience, from here on, we use the term "high-type principal" and "low-type principal" rather than principal with high-type project and principal with low-type project, because the principal has private information regarding the quality of the project.

### 2.3.1 Setup of the separating equilibrium

For the separating equilibrium of the baseline model, we denote the upfront, intermediate, and end money transfers by $m_{0H}$, $m_{1H}$ and $m_{2H}$ for the high-type project, and $m_{0L}$, $m_{1L}$ and $m_{2L}$ for the low-type project (see Figure 2.1).

To find the principal's optimal money transfers, denoted by $(m_{0H}^*, m_{1H}^*, m_{1H}^*)$ if of high-type, and by $(m_{0L}^*, m_{1L}^*, m_{1L}^*)$ if of low-type, we proceed as follows. We consider Perfect Bayesian Equilibria. Regardless of her type, the principal has no incentive to send a wrong signal about the type of the project (this result follows from the revelation principle). Thus the agent interprets the contract accordingly. By choosing the upfront, intermediate and end money transfers $m_{0H}$, $m_{1H}$ and $m_{2H}$, the principal if of high-type maximizes her expected profit:

$$-m_{0H} - \alpha_H e_{1H} m_{1H} + \alpha_H^2 e_{1H} e_{2H} (V - m_{2H}) \tag{2.1}$$

Given $m_{0H}$, $m_{1H}$ and $m_{2H}$, the agent accepts the contract if his efforts in two periods can generate positive expected profit. Otherwise he rejects the contract. The agent maximizes his expected profit, given by:

$$m_{0H} - ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \tag{2.2}$$

Similarly, the principal if of low-type maximizes her expected profit by choosing $m_{0L}$, $m_{1L}$ and $m_{2L}$. Letting $LM$ denote the maximum of the following expected profit of the principal if of low-type:

$$-m_{0L} - \alpha_L e_{1L} m_{1L} + \alpha_L^2 e_{1L} e_{2L} (V - m_{2L}) \tag{2.3}$$

If the principal is of low-type the agent similarly maximizes his expected profit (which must be positive for the agent to accept the contract), with the agent's expected profit given by:

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \tag{2.4}$$

This is a Stackleberg game that can be solved by backwards induction. Since we are solving here for the separating equilibrium, the money transfers selected by a high-type principal differ from those offered by a low-type principal. That is, a principal who is of low-type makes more money by offering the money transfers $m_{0L}$ , $m_{1L}$ and $m_{2L}$ than it makes by mimicking the high-type and offering the money transfers $m_{0H}$ , $m_{1H}$ and $m_{2H}$. This is insured by the constraint:

$$LM \geq -m_{0H} - \alpha_L e_{1H} m_{1H} + \alpha_L^2 e_{1H} e_{2H} (V - m_{2H}) \tag{2.5}$$

In addition, the high-type principal requires higher profit when offering $m_{0H}$ , $m_{1H}$ and $m_{2H}$ than that if she provides any other different money transfers, under the agent's belief that only $m_{0H}$ , $m_{1H}$ and $m_{2H}$ are from the high-type principal while other menus of money transfers are all from the low-type principal. Therefore, money transfers $m_{0H}$ , $m_{1H}$ and $m_{2H}$ satisfy the constraint:

$$-m_{0H} - \alpha_H e_{1H} m_{1H} + \alpha_H^2 e_{1H} e_{2H} (V - m_{2H}) \geq \overline{LM} \tag{2.6}$$

where $\overline{LM}$ is the maximum of the following expected profit of the high-type principal by offering any other contract $\tilde{m}_{0L}$ , $\tilde{m}_{1L}$ and $\tilde{m}_{2L}$:

$$-\tilde{m}_{0L} - \alpha_H e_{1L} \tilde{m}_{1L} + \alpha_H^2 e_{1L} e_{2L} (V - \tilde{m}_{2L}) \tag{2.7}$$

Knowing that money transfers $\tilde{m}_{0L}$ , $\tilde{m}_{1L}$ and $\tilde{m}_{2L}$ are different from $m_{0H}$ , $m_{1H}$ and $m_{2H}$, the agent believes they are provided by the low-type principal and accepts the contract if his efforts in two periods can generate positive expected profit, otherwise he rejects the contract. The agent maximizes his expected profit:

$$\tilde{m}_{0L} - ke_{1L}^2 + \alpha_L e_{1L} \tilde{m}_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \tag{2.8}$$

### 2.3.2 Setup of the pooling equilibrium

For the pooling equilibrium, we denote the upfront, intermediate) and end money transfers by $m_0$, $m_1$ and $m_2$ (see Figure 2.2). Again, the intermediate and end money transfers occur only when the first and second periods are successful respectively. This forms the baseline model of the pooling equilibrium. Later on we will consider other cases when upfront and/or intermediate money transfers are not included in the contract. With the pooling equilibrium, the principal does not reveal her type, offering $m_0^*$, $m_1^*$ and $m_2^*$ regardless of her type. The contract does not offer the agent information regarding the principal type but the agent does update his probability belief in Bayesian fashion after the first period. At the outset the agent believes the principal is of high-type with probability $p$.

By choosing the upfront, intermediate and end money transfers $m_0$, $m_1$ and $m_2$ in the contract to the agent, the high-type principal maximizes her expected profit which is given by:

$$-m_0 - \alpha_H e_1 m_1 + \alpha_H^2 e_1 e_2 (V - m_2) \tag{2.9}$$

Being offered the money transfers $m_0$, $m_1$ and $m_2$, the agent bases his analysis on the contract being from the high-type with probability $p$ and from the low-type with probability $1 - p$. The agent accepts the contract if his efforts in two periods can generate positive expected profit. Otherwise he rejects the contract. The agent maximizes his expected profit, given by:

$$m_0 - ke_1^2 + \overline{\alpha} e_1 m_1 - \overline{\alpha} e_1 k e_2^2 + \overline{\alpha}\tilde{\alpha} e_1 e_2 m_2 \tag{2.10}$$

where $\overline{\alpha}$ equals $p\alpha_H + (1-p)\alpha_L$ , and $\tilde{\alpha}$ is the Bayesian update of $\overline{\alpha}$ and equals $(p\alpha_H^2 + (1-p)\alpha_L^2)/(p\alpha_H + (1-p)\alpha_L)$ with $\overline{\alpha}\tilde{\alpha} = p\alpha_H^2 + (1-p)\alpha_L^2$. This is a Stackleberg game that can be solved by backwards induction.

There are two constraints that need to be considered. One is to prevent a low-type principal from offering different money transfers than $m_0$ , $m_1$ and $m_2$. This constraint is represented by:

$$-m_0 - \alpha_L e_1 m_1 + \alpha_L^2 e_1 e_2 (V - m_2) \geq LM_1 \tag{2.11}$$

where the left side is the low-type principal's expected profit if she offers $m_0$ , $m_1$ and $m_2$, and $LM_1$ is the maximum of the following expected profit of the low-type principal:

$$-m_{0L} - \alpha_L e_{1L} m_{1L} + \alpha_L^2 e_{1L} e_{2L} (V - m_{2L}) \tag{2.12}$$

because any deviation from offering $m_0$ , $m_1$ and $m_2$ is seen by the agent as the signal from the low-type principal. The agent accepts the contract consisting of $m_{0L}$ , $m_{1L}$ and $m_{2L}$ if his efforts in two periods can generate positive expected profit. Otherwise he rejects the contract. The agent maximizes his expected profit:

$$m_{0L} - ke_{1L}^2 + \overline{\alpha} e_{1L} m_{1L} - \overline{\alpha} e_{1L} k e_{L2}^2 + \overline{\alpha}\tilde{\alpha} e_{1L} e_{2L} m_{2L} \tag{2.13}$$

Another constraint is to prevent a high-type principal herself from offering different money transfers than $m_0$, $m_1$ and $m_2$. It is represented by

$$-m_0 - \alpha_H e_1 m_1 + \alpha_H^2 e_1 e_2 (V - m_2) \geq \overline{LM}_1 \tag{2.14}$$

where the left side is the high-type principal's profit if she offers $m_0$ , $m_1$ and $m_2$, and $\overline{LM}_1$ is the maximum of the following expected profit of the high-type principal:

$$-\tilde{m}_{0L} - \alpha_H e_{1L} \tilde{m}_{1L} + \alpha_H^2 e_{1L} e_{2L} (V - \tilde{m}_{2L}) \tag{2.15}$$

because any deviation from offering $m_0$ , $m_1$ and $m_2$ is seen by the agent as the signal from the low-type principal, and therefore efforts $e_{1L}$ and $e_{2L}$ are exerted by the agent. The agent accepts the contract consisting of $\tilde{m}_{0L}$, $\tilde{m}_{1L}$ and $\tilde{m}_{2L}$ if his efforts in two periods can generate positive expected profit. Otherwise he rejects the contract. The agent maximizes his expected profit:

$$\tilde{m}_{0L} - k e_{1L}^2 + \alpha_L e_{1L} \tilde{m}_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \tag{2.16}$$

For all the analysis, we assume that $V \leq 2k/\alpha_H$. This condition ensures that there is an interior solution where the agent trades off the cost of effort versus the desire to increase probability of success. The interior solution is the focus of our discussion, because when efforts reach their up bound, which is 1, any extra amount of effort would be useless in improving the probability of success of the project, and only cause more loss.

### 2.3.3 Separating and pooling equilibria for the baseline model

In 2.3.3 we delineate the pooling and separating equilibria for the baseline model (the case involving upfront, intermediate and end money transfers). Later, in 2.3.4 we compare this baseline to the cases where there is no upfront and/or no intermediate money transfer, to determine the benefit (if any) of the stage-gate process.

As a reference point for interpreting the baseline model, we first consider the case of complete information (the agent knows the principal's type, but the agent's effort is unobservable). Theorem 9 expresses the result under complete information.

**Theorem 9** *Under complete information, the outcomes are as follows:*

1. *Money transfers if the principal is of low-type are* $m_{0L}^* = -\frac{\alpha_L^6 V^4}{64k^3}$, $m_{1L}^* = 0$ *and* $m_{2L}^* = V$, *resulting in effort levels* $e_{2L} = \frac{\alpha_L m_{2L}^*}{2k}$ *and* $e_{1L} = \frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k}$. *The principal's expected profit is* $\frac{\alpha_L^6 V^4}{64k^3}$ *and the agent's expected profit is zero.*

2. *Money transfers if the principal is of high-type are* $m^*_{0H} = -\frac{\alpha_H^6 V^4}{64k^3}$, $m^*_{1H} = 0$ *and* $m^*_{2H} = V$ , *resulting in effort levels* $e_{2H} = \frac{\alpha_H m^*_{2H}}{2k}$ *and* $e_{1H} = \frac{\alpha_H m^*_{1H} + \frac{\alpha_H^3 m^*_{2H}{}^2}{4k}}{2k}$. *The principal's expected profit is* $\frac{\alpha_H^6 V^4}{64k^3}$ *and the agent's expected profit is zero.*

Theorem 9 says that, in the case of complete information, allowing for an intermediate money transfer does not help a principal of either type achieve maximal expected profit (the transfer is zero). The agent's optimal level of the effort in the second period is proportional to the end money transfer and the effort in the first period is proportional to a weighted sum of the intermediate money transfer and the square of the end money transfer. (These effort level equations hold as the best responsive functions of the agent given any intermediate and end money transfers, not only the money transfers representing the principal's profit maximization). The proof of Theorem 9 is the same as that of Theorem 1.

Note that the upfront money transfer is negative, meaning that the agent pays to join the project and then receives the full project value if the project succeeds in both periods. The upfront money transfer is set to yield zero expected profit left for the agent. Also note that the agent's effort level in the second period is higher than in the first period: $e^*_{1L} < \frac{\alpha_L}{2} e^*_{2L}$ and $e^*_{1H} < \frac{\alpha_H}{2} e^*_{2H}$. Finally, a higher $V$ leads to higher effort levels, while a higher cost coefficient results in the opposite.

#### 2.3.3.1 Separating equilibrium for the baseline model

Next we look at the separating equilibrium under incomplete information, when the type of the principal is unknown to the agent. Numerically we find that when $1 < x \lesssim 1.063971$ with $x = \frac{\alpha_H}{\alpha_L}$, the agent earns zero expected profit, $m^*_{0H} < 0$ and $m^*_{1H} < 0$.

**Theorem 10** *The outcomes under the separating equilibrium of the baseline model for are as follows:*

1. *If of low-type, then the principal offers* $m^*_{0L} = -\frac{\alpha_L^6 V^4}{64k^3}$, $m^*_{1L} = 0$ *and* $m^*_{2L} = V$.

2. *If the principal is of high-type:*

   (a) *For* $1.063971 \lesssim x \lesssim 1.335236$, *the agent earns strictly positive expected profit,* $m^*_{0H} < 0$ *and* $m^*_{1H} > 0$.

   (b) *For* $1.335263 \lesssim x < 2$, *the agent earns strictly positive expected profit,* $m^*_{0H} > 0$ *and* $m^*_{1H} > 0$.

   (c) *For* $x = 2$ , *the agent earns strictly positive expected profit,* $m^*_{0H} > 0$ *and* $m^*_{1H} = 0$.

*(d) For $x > 2$ , the agent earns strictly positive expected profit, $m^*_{0H} > 0$ and $m^*_{1H} < 0$.*

We can see that when $x(= \alpha_H/\alpha_L)$ approaches 1, the high-type principal's profit tends to that under complete information, because of the diminishing information asymmetry. Unlike the outcomes under complete information, for the separating equilibrium, adding the intermediate money transfer does bring more profit to the high-type principal for all $x(= \alpha_H/\alpha_L)$, except at $x = 2$ where the intermediate money transfer plays no role. Also, the agent earns positive expected profit for almost all $x$, except for those that are very close to 1. This reflects the fact that when the ability of the high-type principal is close to that of the low-type principal, differentiating herself from the low-type one is very costly for the high-type principal, whose profit decreases dramatically as $x$ increases from 1 to 1.063971 (Figure 2.3). The high-type principal has to decrease its end money transfer and thereby the probability of success for the second the period decreases as $x$ increases from 1 to 1.063971 (Figure 2.4 and 2.5). To shore up the probability of success for the first period, the high-type principal chooses positive intermediate money transfer and increases it. But such measure is not enough to compensate for the loss in profit resulting from the falling effort level in the second period. At the same time, the upfront money transfer is increasing, doesn't have direct influence on the probabilities of success and only makes more payout. Therefore this leads to the drop of the high-type agent's profit. Notice that in Figure 2.3, 2.4 and 2.5 and those that follow, we set $\alpha_H = 0.9$, $V = 2$ and $k = 1$. In fact, the statement of Theorem 10 and others and the features of graphs in all the figures are independent of these parameters' values. The proof is provided in the Appendix.

On the other hand, as the ability of the low-type principal departs from that of the high-type to certain degree ($x \gtrsim 1.063971$), it becomes less costly for the high-type principal to differentiate herself from the low-type one. The high-type principal increases the end money transfer to enhance the probability of success in the second period. The agent starts to earn positive profit. Although the probability of success in the first period is still decreasing, because of the decline in the intermediate payment, overall, the high-type principal's profit increases.

#### 2.3.3.2 Pooling equilibrium for the baseline model

Next we investigate the pooling equilibrium of the baseline model under incomplete information; regardless of her type, the principal offers the same menu of upfront, intermediate and end money transfers to the agent and has no incentive to deviate. The agent's prior belief is that the principal is of high-type with probability $p$ and of low-type with probability $1-p$, and if the contract differs from that in the pooling equilibrium location, the contract comes from a low-type principal.

**Theorem 11** *For $p \gtrsim 0.05$, a unique pooling equilibrium exists as shown in the Appendix. The solution has the following attributes:*

1. $m_2 = \frac{\alpha_H V}{2\alpha_H - \tilde{\alpha}}$ *and* $m_0 < 0$ .
2. $m_1 > 0$ *if* $2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha} > 0$, $m_1 < 0$ *if* $2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha} < 0$ *and* $m_1 = 0$ *if* $2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha} = 0$.

From Theorem 11, we can see that adding the intermediate money transfer does improve the high-type principal's profit when $2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha} \neq 0$ . Furthermore, as $x$ is close to 1 , both $\tilde{\alpha}$ and $\tilde{\alpha}$ are close to $\alpha_H$ and thereby the end money transfer approaches $V$ and the intermediate money transfer tends to zero. In other words, as $\alpha_L$ gets close to $\alpha_H$, information asymmetry between the low-type and high-type principals is disappearing and the outcome in the pooling equilibrium approaches that under complete information. On the other hand, as $x$ goes to infinity, which means that $\alpha_L$ tends to zero, information gets most severely skewed, with $\tilde{\alpha}$ increasing to $\alpha_H$ and $\overline{\alpha}$ decreasing to $p\alpha_H$. The reason why $\tilde{\alpha}$ increases to $\alpha_H$ is that with very low value of $\alpha_L$, after the first period's success, the agent knows that the project is very likely from a high-type principal and thereby has a high probability (close to $\alpha_H$ ) of succeeding in the second period. This process is the Bayesian update of the agent's belief. As a result, the end money transfer increases to $V$.

As to the effect of parameter $p$, since $\overline{\alpha}$ equals $p\alpha_H + (1-p)\alpha_L$, and $\tilde{\alpha}$ is $p\alpha_H^2 + (1-p)\alpha_L^2/(p\alpha_H + (1-p)\alpha_L)$, both $\overline{\alpha}$ and $\tilde{\alpha}$ are increasing functions of $p$ . Since the end money transfer $m_2$ is an increasing function of $\tilde{\alpha}$, $m_2$ is an increasing function of $p$, which implies that the effort in the second period is also an increasing function of $p$. Despite the fact that the effort in the first period drops as $p$ increases because $m_1$ is a decreasing function of $p$ (numerically shown), overall, the high-type principal's profit increases as $p$ increases. Regarding the effect of $x$ : as a function of $x$, $\tilde{\alpha}$ quickly decreases and then slowly increases as $x$ increases from 1 to infinity. As a increasing function of $\tilde{\alpha}$, $m_2$ shows the same pattern and therefore the effort level in the second period has the same pattern. Although the effort in the first period drops as $x$ increases, eventually, the impact from the effort in the second period will outweigh the one in the first period. As a result, the high-type principal's profit drops and then increases very slowly as $x$ increases. This is shown in Figure 2.6.

To sustain the pooling equilibrium, the principal, regardless of her type, has to earn at least the same profit as in the situations if she deviates. As pointed out earlier, a high-type principal tends to deviate only if she is treated as the low-type one. This can be seen in Figure 2.7. The pooling equilibrium exists for the most part in the space of $x$ and $p$ , except a narrow zone bounded by the solid line and the $x$ axis, with $p < 0.015$. The area between the dash line and the solid line represents the region that a low-type principal's incentive constraint binds while a high-type principal's incentive constraint doesn't bind.

#### 2.3.3.3 The dominant equilibrium

Comparing the separating equilibrium of Theorem 10 with the pooling equilibrium of Theorem 11, we would like to know when one of them dominates the other in terms of the high-type principal's profit. We find that when $p$ is below 0.7, the separating equilibrium mainly dominates and gives the high-type principal a higher profit than the pooling equilibrium, except for a small area in the space of $x$ and $p$ . When $p$ is greater than 0.7, the pooling equilibrium dominates and delivers more profit to the high-type principal. Figure 2.8 illustrates the dominating areas for the equilibria. This phenomenon reflects the fact that when the agent believes that the principal is likely to be of low-type, it would be less expensive for the high-type principal to separate her contract from that which would be offered if she were of low-type as compared to "pooling" her contract with that which would be offered if she were of low-type. On the other hand, when the agent believes that the principal is likely to be of high-type, it would be more costly for the high-type principal to separate her contract. Therefore, the high-type principal chooses the separating or pooling equilibrium depending on the value of $p$ and $x$, yielding the profit shown in Figure 2.9.

### 2.3.4 The role of the upfront and intermediate money transfers

To see the benefits of upfront and intermediate money transfers for separating and pooling equilibria, we investigate the outcomes when one or both of the two money transfers are absent from the baseline models. For comparison, there are four cases for consideration: 1) the baseline model that includes upfront, intermediate and end money transfers; 2) only upfront and intermediate money transfers; 3) only intermediate and end money transfers; and 4) only end money transfer is included in the menu.

#### 2.3.4.1 The separating equilibrium

Results are given in Theorem 12 for the separating equilibrium.

**Theorem 12** *For the separating equilibrium, the high-type principal's profit in case 1 is strictly greater than that in case 2 except at the point where two profits are equal, the* *high-type principal's profit in case 1 is strictly greater than that in case 3, the high-type principal's profit in case 2 is strictly greater than that in case 4, and the high-type principal's profit in case 3 is strictly greater than that in case 4.*

Theorem 12 says that for the separating equilibrium, adding an intermediate money transfer (case 1) as compared to having only upfront and end money transfers included (case 2 ), or as compared to having only an end money transfer (case 4) weakly increases the profit of the high-type principal. Since at $x = 2$ (i.e., $\alpha_H = 2\alpha_L$ ) the intermediate payment becomes zero in case 1, there is no advantage for adding it at this point when compared with case. Otherwise the profit is strictly greater. Figure 2.10 shows the difference between the profit of the high-type principal in case 1 and case 2. We see that when $x$ is small, the profit of a high-type principal in case 1 is not significantly higher than that in case 3. However, for larger and larger $x$, the impact of an intermediate money transfer becomes more and more conspicuous. This says that it is beneficial for a high-type principal to add an intermediate money transfer to differentiate herself from a low-type one who has fairly poor ability.

In fact, adding an intermediate money transfer as compared to having only an end money transfer (i.e., case 4) has a much more significant impact than the previous scenario. This can be seen from Figure 2.11. In other words, without the presence of an upfront money transfer, the intermediate money transfer plays a bigger role in increasing a high-type principal's profit. Figure 2.11 also shows that adding an upfront money transfer to the menu with intermediate and end money transfers (i.e., case 3) brings a sizable amount of profit to a high-type principal, and has an even more significant effect when added to the contract with only an end money transfer. Another observation is that the profit curves in cases 1, 2 and 3 have the same shape, the feature of a decrease followed by an increase, while the one in case 4 is monotonically decreasing. This reflects the insight that by using intermediate and end money transfers to differentiate herself, a high -type principal can create more profit, especially when her ability is high as compared to what if would be if she were of low-type.

Notice that Theorem 12 does not give a comparison between case 2) and case 3), because the closed form solution of case 3) is not available. But Figure 2.11 (based on numerical analysis) shows that a high-type principal's profit in case 2) is higher than in case 3). This again says that both the upfront and intermediate money transfers are valuable, but the upfront money transfer is most effective.

#### 2.3.4.2 The pooling equilibrium

Results are given in Theorem 13 for the pooling equilibrium.

**Theorem 13** *For the pooling equilibrium, when the principal, regardless of her type, earns more profit than in the situations if she deviates, the closed form solution of the menu of money transfers at the equilibrium for each of the four cases is obtained and shown in the Appendix and the following comparison holds: the high-type principal's profit in case 1) is greater than that in case 2) except on a segment of a curve* $2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha} = 0$ *where the two profits are equal; the high-type principal's profit in case 1 is greater than that in case 3; the high-type principal's profit in case 2 is greater than that in case 4; and the high-type principal's profit in case 3 is greater than that in case 4 except on a segment of a curve* $3\tilde{\alpha} = 2\alpha_H$ *where the two profits are equal. For each of the four cases, the agent earns zero profit.*

Theorem 13 tells us that for the pooling equilibrium, adding an intermediate money transfer to the menu (case 1) as compared to only upfront and end money transfers (case 2), or to the menu with only end money transfers included (case 4) adds profit for a high-type principal for almost every combination of $x$ and probability $p$. When $2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha} = 0$, the intermediate money transfer in case 1 equals zero, and therefore there is no advantage for adding it at these locations when compared with case 2. When $3\tilde{\alpha} = 2\alpha_H$, the intermediate money transfer in case 3) equals zero and thus there is no advantage for adding it at these locations when compared with case 4. Figure 2.12 shows that the difference between the profits of the high-type principal in case 1 versus case 2 is so insignificant that the impact of the intermediate money transfer is negligible.

Comparing the profits in cases 1, 2, 3 and 4, our numerical results not only verify Theorem 13, but also show that the profit in case 2 is higher than that in case 3. In addition, as $p$ (the probability of the principal of being of high-type) increases, the profit in each case increases. Furthermore, when $p$ is small, the principal's profit in the four cases tends to converge together for increasing values of $x$ up to x = 2, but for increasing values of $p$ , the differences among the profits in cases 2, 3 and 4 get larger, although profits in case 1 and case 2 remain close to each other. This says that as the principal is more likely to be of the high-type one, the benefit of adding an upfront money transfer (to a contract with intermediate and end money transfers or with only an end money transfer) becomes bigger. Figure 2.13, 2.14 and 2.15 illustrate these phenomena.

The dominating regions of the separating and pooling equilibria for case 1 were shown previously in Figure 2.8. Similarly, for case 2, the dominating regions for the separating and pooling equilibria are plotted in Figure 2.16. The two figures are very similar, except that the region for the separating equilibrium is smaller in Figure 2.16 for higher values of $x$. This is because as $x$ increases, the difference between the two separating profits (case 1 versus case 2) for a high-type principal gets larger and larger, while the two pooling profits (case 1 versus case 2) for a high-type principal stay very close. Like case 1, there is a small area in case 2 that has no pooling equilibrium, as shown in Figure 2.17.

For case 3, the dominating regions of the separating and pooling equilibria are shown in Figure 2.18. We can see that compared with case 1 and case 2, for higher values of $x$ at higher values of $p$ , the separating equilibrium generates more profit than the pooling equilibrium. Unlike case 1 and case 2, a pooling equilibrium exists for all $p$ and $x$.

For case 4, the pooling equilibrium always gives a high-type principal a higher profit than the separating equilibrium (for any $x$ and $p$, the pooling equilibrium dominates). A high-type principal's profit under the dominating equilibrium is compared across the various cases in Figures 2.19, 2.20 and 2.21 for $p = 0.3$, 0.7, and 0.9, respectively. First of all, for case 4, as we pointed out, the pooling equilibrium always gives a higher profit than the separating equilibrium. Furthermore, when $p = 0.3$, the separating equilibrium dominates in cases 1, 2 and 3 for higher values of $x$. This reflects the fact that for cases 1, 2, and 3, the separating equilibrium dominates for low $p$ and for $x$ values that are not too close to 1. When $p = 0.7$, for higher values of $x$, the pooling equilibrium dominates in cases 1 and 2, while the separating equilibrium dominates in Case 3. This is because unlike cases 1 and 2, for $p = 0.7$, the separating equilibrium gives a high-type principal more profit for higher values of $x$. When $p = 0.9$, the pooling equilibrium dominates each of cases 1, 2 and 3 for $x$ that are not close to 1. This can be seen from the dominating regions depicted in Figures 2.8, 2.16 and 2.18. Another phenomenon is that for higher values of $p$, a high-type principal's profit in each case is also higher as long as the pooling equilibrium dominates. In addition, the difference among the profits in cases 2, 3 and 4 tends to become bigger with higher values of $p$.

## 2.4 Managerial Insights and Concluding Remarks

When firms outsource their R&D projects, they often face two issues. The first is that the firm outsourcing the project may possess information regarding the attractiveness of the project, and may find it difficult to convey this information to the supplier in a credible way. For example, if the principal offers the supplier the opportunity to work on what the supplier claims is a highly desirable project, the supplier may think the outsourcing firm is inflating the desirability of the project in order to elicit unduly high effort on the part of the supplier. This scenario also alludes to the second issue faced by the outsourcing firm, which is that the supplier's effort level is unobservable. We explore the issue of how to best design a contract that mitigates the adverse effects of these two issues.

More specifically, we are interested in whether a stage-gate contract is superior to a non-stage-gate contract in the presence of adverse selection (hidden information) and moral hazard (unobservable effort). While stage-gate processes have been widely applied in new product development, there is little known about how they can be incorporated in contract design. Our research is one of the few research works that explore this new area. That is, we investigate whether a firm with a high-quality project can use a stage-gate contract to signal her private information regarding the project (in order to differentiate herself from the situation in which the project is of low quality project) and thereby achieve higher expected profit, or whether the firm should instead choose to not reveal her inside information and instead offer the same contract as if the project were of low quality.

To study this question, we analyze signaling models in the context of a two-stage project with stage-gate features. In our baseline model, the contract consists of an upfront money transfer (at the beginning), an intermediate money transfer (contingent upon success of the first stage), and an end money transfer (contingent upon success of second stage following success after the first stage). We consider two types of models, one involving a separating equilibrium and the other a pooling equilibrium. Under the separating equilibrium, to differentiate herself from the contract that she would offer if her project were of low-quality, the principal with a high quality project uses money transfers to signal her private information regarding the project. Meanwhile, the money transfers induce the agent (supplier) to exert appropriate effort; a higher effort level means a higher probability of success but more cost to the agent. Under the pooling equilibrium, the principal with a high-quality contract "pools" the contract with the contract that she would offer if her project were instead of low quality (i.e, the principal's contracts are the same regardless of the project quality); the money transfers are such that the principal has no incentive to deviate from this pooling equilibrium regardless of the project type. Again, the money transfers are also aimed to elicit the agent (supplier) to exert effort.

A key result of our model is that, compared to the case of complete information (where the quality of the project is known to both the principal and the agent), the intermediate money transfer offered by the principal with a high quality project is not generally zero, and by offering this intermediate payment the principal increases her expected profit. Another way of saying this is to say that the stage gate contract is superior to a non-stage-gate contract, particularly when the quality difference between the high and low quality projects is big. This shows that intermediate money transfer is especially useful when the information asymmetry is large and the principal has a high quality project and wants to separate herself from what would be the result if the project were instead of low quality. Furthermore, adding an intermediate money transfer to the menu with only end money transfer brings significantly more profit to the principal with a high quality project under both the separating and pooling equilibria. This says that without the presence of an upfront money transfer, the intermediate money transfer plays a big role in increasing the profit for a principal with a high quality project. Regarding the upfront money transfer, its effect is conspicuous, because it can add even higher profit to the principal with a high quality project than does the intermediate money transfer under both equilibria.

For models involving a separating equilibrium, although costly differentiation results in a dramatic drop in the profit of the principal with a high quality project when the quality gap between the low and high quality project is narrow, both an upfront money transfer and an intermediate money transfer help increase the profit of the principal with a high quality project significantly as the gap becomes bigger. In contrast, the contract with only an end money transfer is unable to increase the principal's profit as the quality gap between the low and high quality project gets bigger. For models involving a pooling equilibrium, with a higher probability of the project being of high quality project (we call this the "show-up probability"), the agent exerts more effort in the second period. As a result, the principal with high quality project obtains higher profit.

As to which of separating and pooling equilibria would deliver more profit to the principal with a high quality project, we find that for almost all models except the ones with only end money transfers for separating and pooling equilibria, when the show-up probability of the principal with a high quality project is not high and the quality difference between the high and low quality projects is not too narrow, the separating equilibrium generates higher profit than the pooling equilibrium. This shows that it is less costly to differentiate herself than pooling herself for a principal with a high quality project in this scenario. On the other hand, when the show-up probability of the principal with a high quality project is high or the difference between the high and low quality projects is narrow, the pooling equilibrium generates higher profit than the separating equilibrium. The reason is that separating becomes more costly than pooling for the principal with a high quality project.

In addition, we point out that all effort levels exerted by the agent are "interior solutions" (i.e., neither 0 nor 1). This reflects the fact that the agent strikes a subtle balance between exerting more effort to achieve higher probability of success and avoiding bigger cost incurred. Moreover, the results we obtained only depend on the quality ratio of the high quality project and the low quality project, independent of the value of the project, the cost coefficient and the quality of either project. This would allow our findings to apply to various practical situations.

Finally, we recognize that due to the simple structure of the models, our results only serve as a theoretic guidance on how stage-gate contracts with multiple money transfers can be used to either signal or not reveal the private information of the principal with a high quality project and help increase her profit. To reflect the real situations in practice, one could include more features in a model of stage-gate contract. For examples, both principal and agent can have their own inside information, the principal may not only provide money transfers but also exert her own effort in the project, the agent may have different (erroneous) beliefs about the show-up probability of the principal with a high quality project, the agent may be given a second chance after the failure in one stage, and the project may have positive residual value left even after the failure in a stage. These are the dimensions that may be worth future exploration in this promising research area.

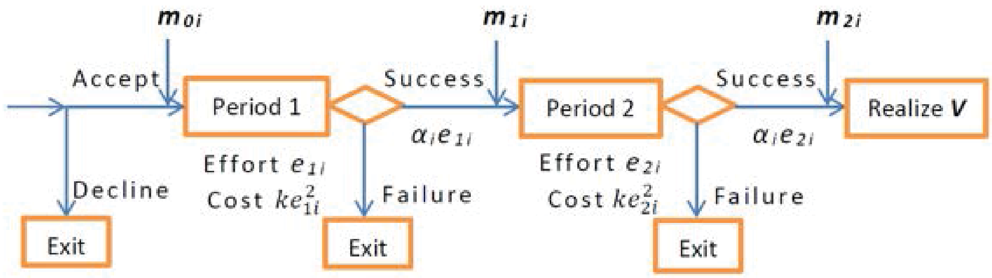



**Figure 2.1**: The Baseline Model for the Separating Equilibrium (i = L or H)

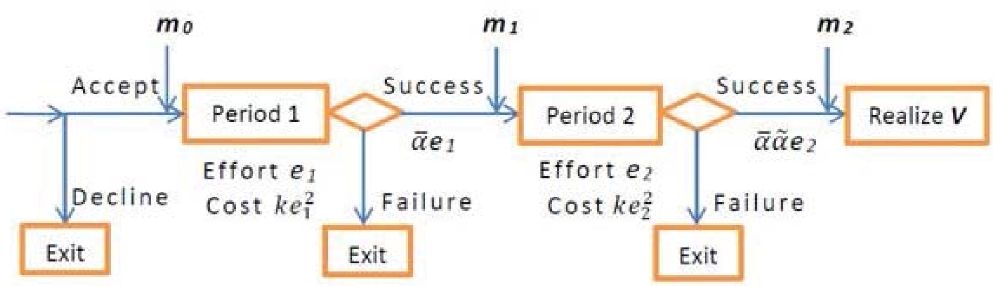



**Figure 2.2**: The Baseline Model for the Pooling Equilibrium

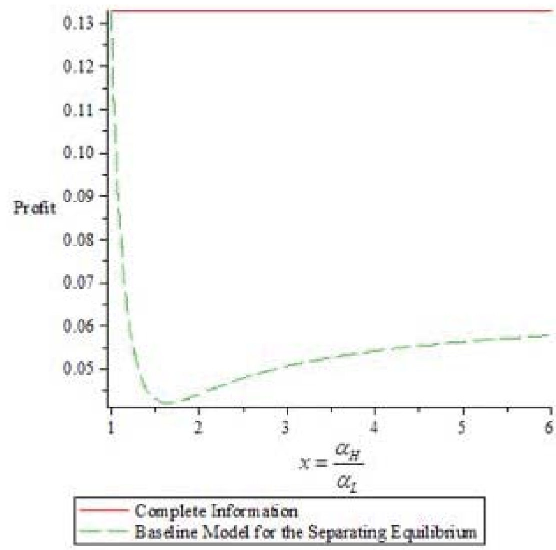



**Figure 2.3**: Profit of a High-type Principal (Separating Equilibrium)

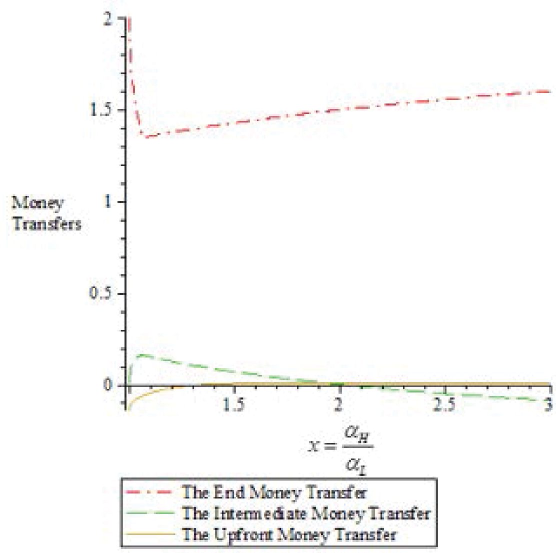



**Figure 2.4**: Money Transfers Offered by a High-Type Principal (Separating Equilibrium)

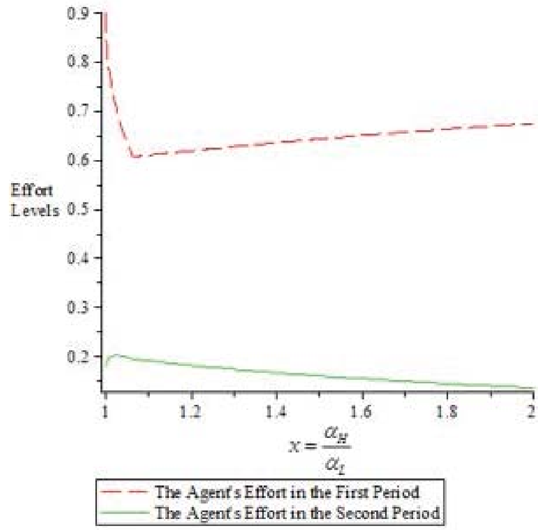



**Figure 2.5**: Agent's Efforts for an Offer from a High-Type Principal (Separating Equilibrium)

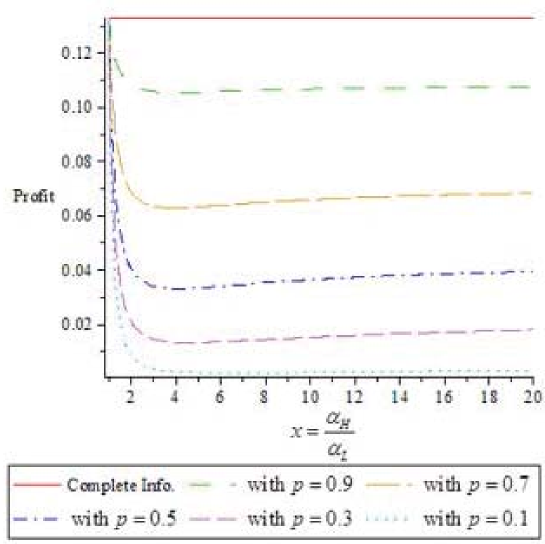



**Figure 2.6**: Profit of a High-Type Principal (Pooling Equilibrium)

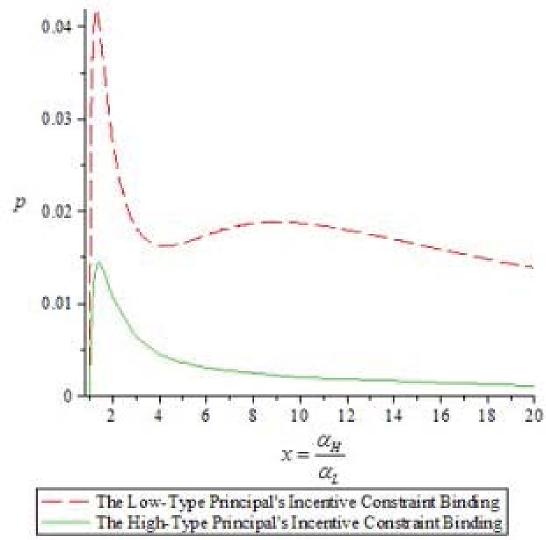



**Figure 2.7**: In the Region below the Lower Curve, the High-Type Principal's Incentive Constraint Is Violated and There Is No Pooling Equilibrium)

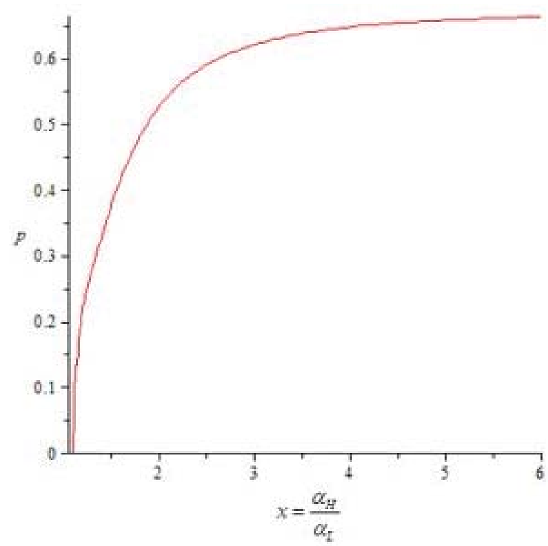



**Figure 2.8**: The Dominating Equilibrium (Separating Below the Line, Pooling Above)

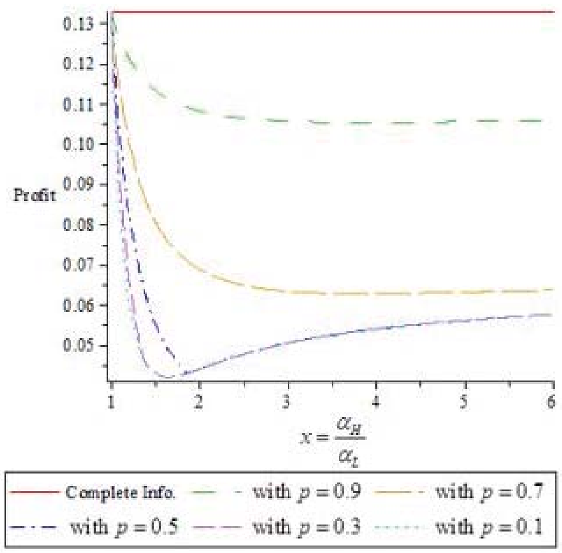



**Figure 2.9**: Profit of a High-type Principal Under the Dominating Equilibrium

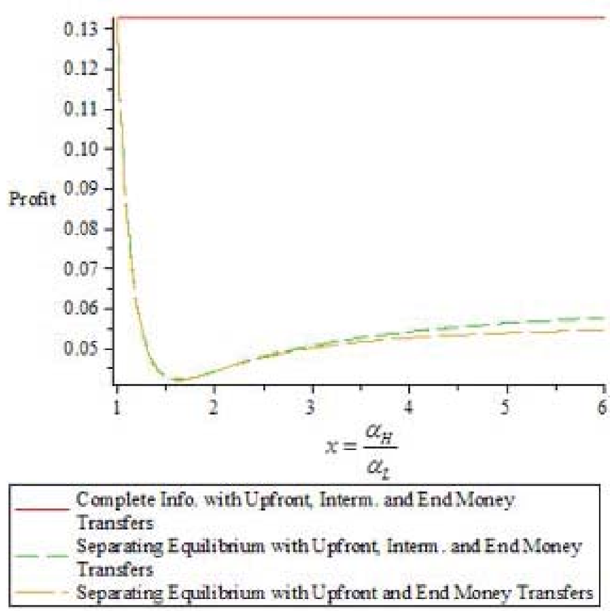



**Figure 2.10**: Profit for a High-type Principal in Case 1 and Case 2

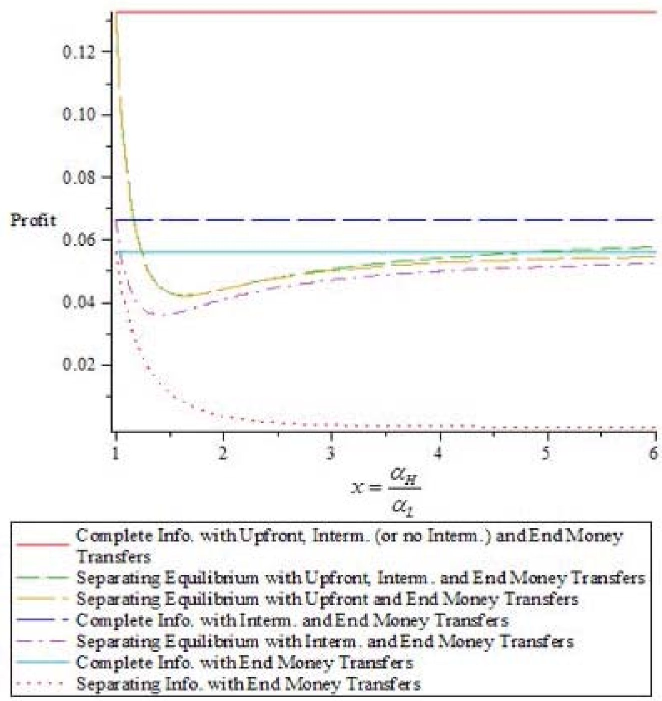



**Figure 2.11**: Profit for a High-type Principal Under the Separating Equilibrium for Four Cases

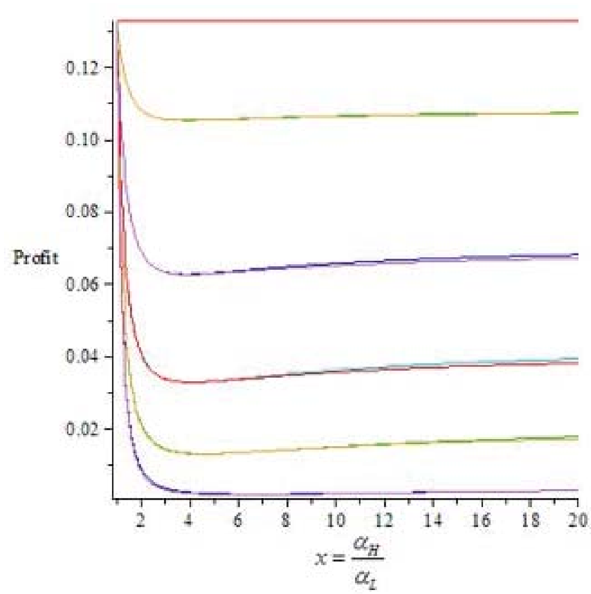



**Figure 2.12**: From Top to Bottom: Profit of a High-Type Principal Under Complete Information, Then Pairs of Case 1 and Case 2 with $p = 0.9, 0.7, 0.5, 0.3, 0.1$

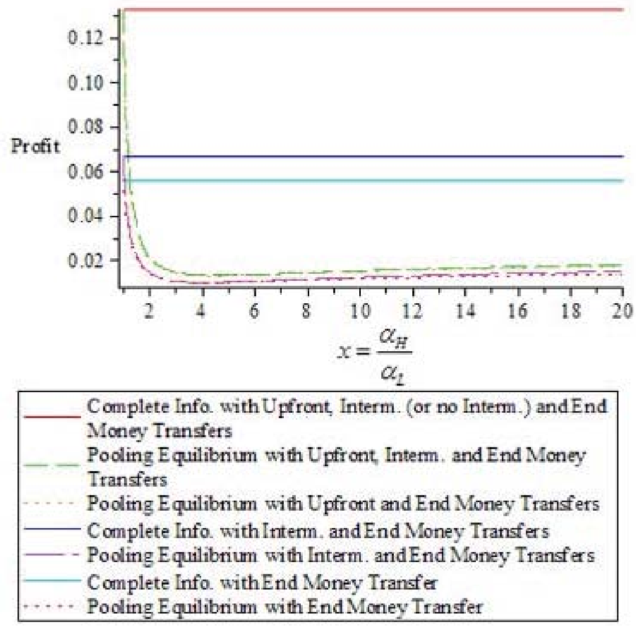



**Figure 2.13**: Profits for a High-type Principal (Pooling Equilibrium) with $p = 0.3$

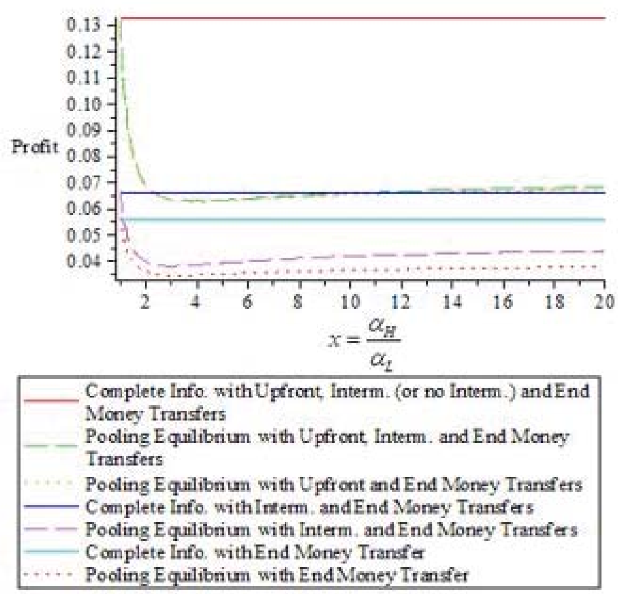



**Figure 2.14**: Profits for a High-type Principal (Pooling Equilibrium) with $p = 0.7$

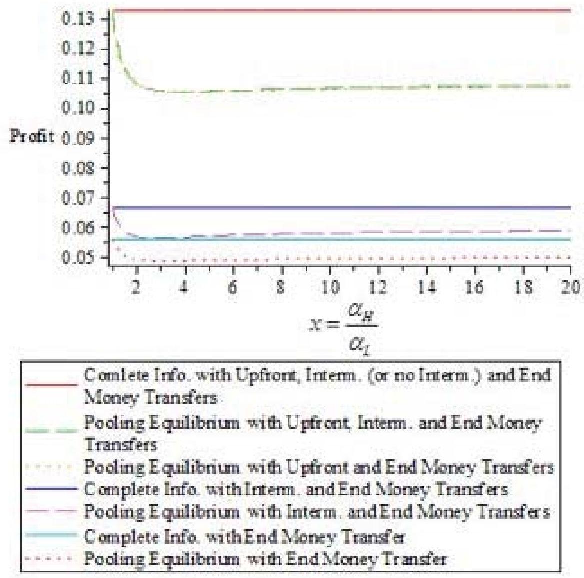



**Figure 2.15**: Profits for a High-type Principal (Pooling Equilibrium) with $p = 0.9$

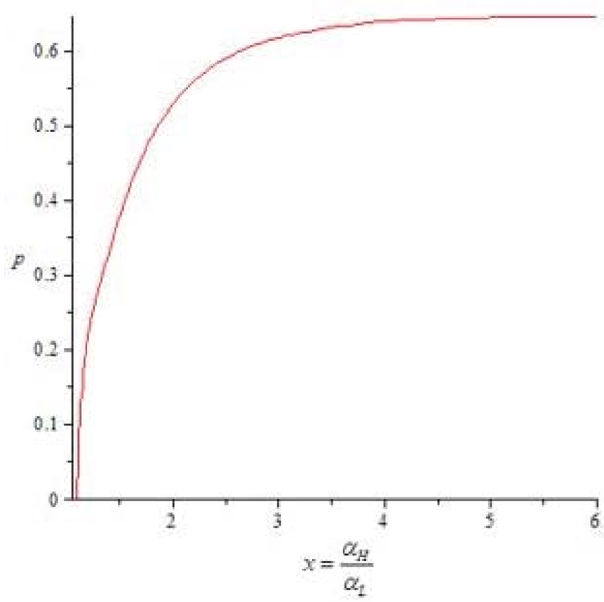



**Figure 2.16**: In Case 2, Separating Equilibrium Dominates below the Solid (Pooling Above)

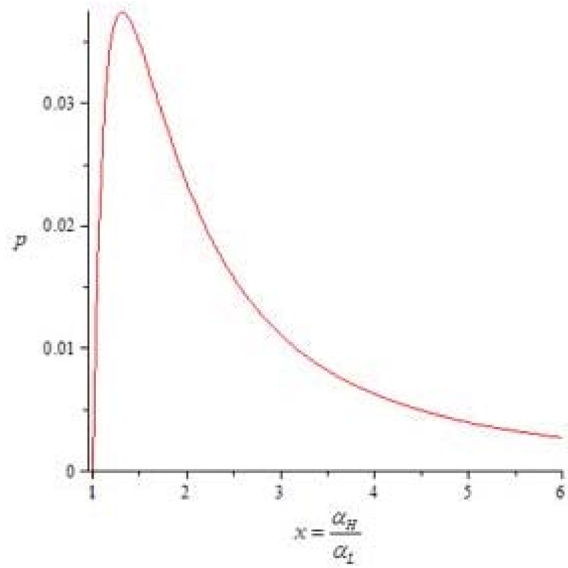



**Figure 2.17**: There Is No Pooling Equilibrium for the Area under the Solid line in Case 2

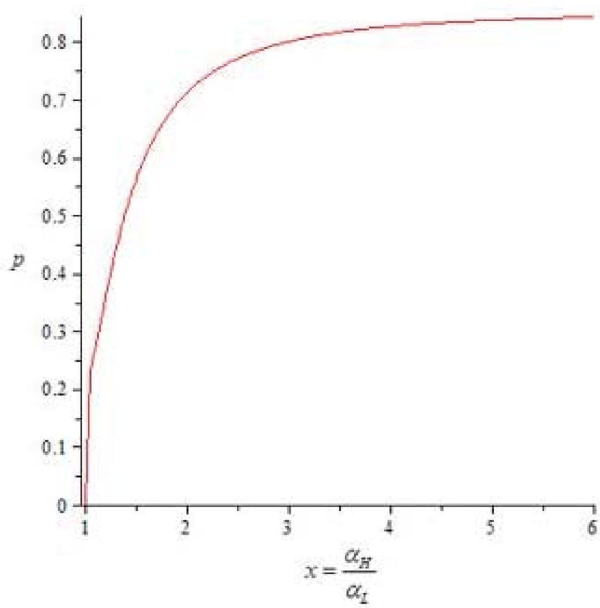



**Figure 2.18**: In Case 3, Separating Equilibrium Dominates below the Solid (Pooling Above)

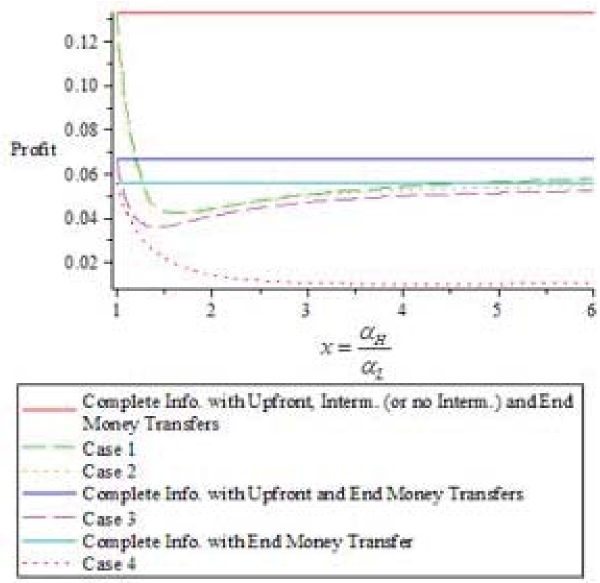



**Figure 2.19**: Profit for a High-type Principal Under the Dominating Equilibrium, $p = 0.3$

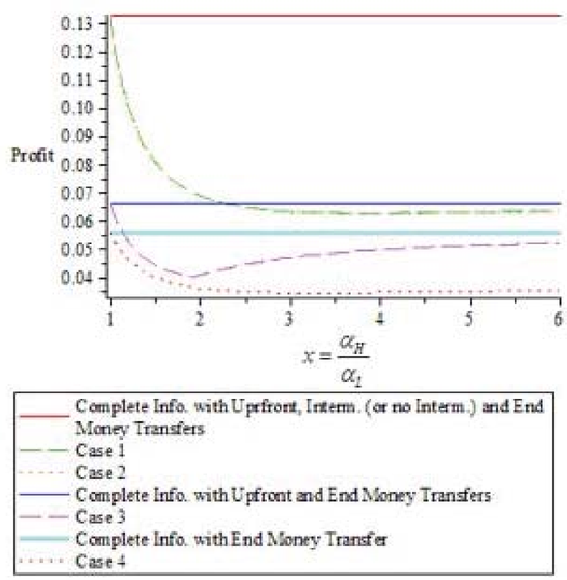



**Figure 2.20**: Profit for a High-type Principal Under the Dominating Equilibrium, $p = 0.7$

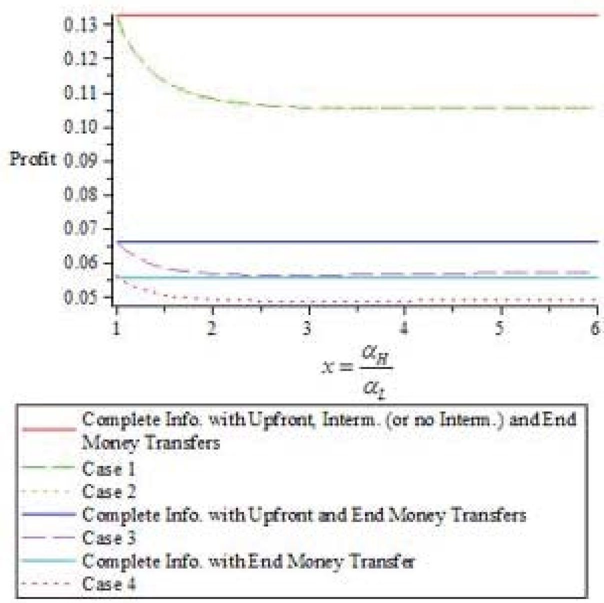



**Figure 2.21**: Profit for a High-type Principal Under the Dominating Equilibrium, $p = 0.9$

# CHAPTER 3

# BOTH PRINCIPAL AND AGENT HAVE INSIDE INFORMATION

In practice, it is often the case that principal and agent have their own inside information at the same time. Maskin and Tirole (1992) pointed out that this is a quite interesting scenario that is worthy of study. In this chapter, we investigate stage-gate contracts when both principal and agent have private information. To achieve the maximal expected profit, the high-type principal not only needs to screen the low-type agent from the high-type one, but also needs to consider whether she should differentiate her type through signalling or pool her type with the low-type principal without signalling. In the following, we outline the framework and model setup for this research.

We assume that there are two types of principals - high-type and low-type, and two types of agents - high-type and low-type. For both types of principals, the project returns the same value $V$ if successful at the end of period 2. For each type of principal, the agent is of low-type with the same probability $\lambda$, with $0 < \lambda < 1$, and of high-type with the same probability $1 - \lambda$. When the principal is of high-type, the effort levels that the agent exerts in the two periods if of low-type and high-type are denoted by $\{\alpha_L^H e_{1L}^H, \alpha_L^H e_{2L}^H\}$ and $\{\alpha_H^H e_{1H}^H, \alpha_H^H e_{2H}^H\}$, respectively, which all take values in the $[0, 1]$ interval. The parameters $\alpha_L^H$ and $\alpha_H^H$ describe agent type, with the logical constraint that $0 < \alpha_L^H < \alpha_H^H < 1$. The probabilities of success for the first and the second periods for the agent if of low-type are $\alpha_L^H e_{1L}^H$ and $\alpha_L^H e_{2L}^H$. Likewise, $\alpha_H^H e_{1H}^H$ and $\alpha_H^H e_{2H}^H$ denote the corresponding probabilities of success for the agent if of high-type. When the principal is of low-type, the effort levels that the agent exerts in the two periods if of low-type and high-type are denoted by $\{\alpha_L^L e_{1L}^L, \alpha_L^L e_{2L}^L\}$ and $\{\alpha_H^L e_{1H}^L, \alpha_H^L e_{2H}^L\}$ respectively, which all take values in the $[0, 1]$ interval. The parameters $\alpha_L^L$ and $\alpha_H^L$ describe agent type, with the constraint that $0 < \alpha_L^L < \alpha_H^L < 1$. The probabilities of success for the first and the second periods for the agent if of low-type are $\alpha_L^L e_{1L}^L$ and $\alpha_L^L e_{2L}^L$. Likewise, $\alpha_H^L e_{1H}^L$ and $\alpha_H^L e_{2H}^L$ denote the corresponding probabilities of success for the agent if of high-type. As to the order among the parameters $\alpha_L^H$, $\alpha_H^H$, $\alpha_L^L$ and $\alpha_H^L$, we assume that $\alpha_L^H > \alpha_L^L$ and $\alpha_H^H > \alpha_H^L$ to reflect the fact that the project has higher probability of success if the principal is of high-type.

When the principal is of high-type, the costs that the agent incurs in the two periods are $k{e_{1L}^H}^2$ and $k{e_{2L}^H}^2$ for the agent if of low-type or $k{e_{1H}^H}^2$ and $k{e_{2H}^H}^2$ if of high-type, where $k$ is the cost coefficient. The upfront, intermediate and final money transfers to the agent are denoted by $m_{0L}^H$, $m_{1L}^H$ and $m_{2L}^H$ if of low-type and $m_{0H}^H$, $m_{1H}^H$ and $m_{2H}^H$ if of high-type. Similar to what we assume in Chapter 1 and Chapter 2, these transfers can be either positive or negative, depending on whether the principal pays the agent or the agent pays the principal. The intermediate and final transfers occur only when the first and second periods are successful, respectively.

Likewise, when the principal is of low-type, the costs that the agent incurs in the two periods are $k{e_{1L}^L}^2$ and $k{e_{2L}^L}^2$ for the agent if of low-type or $k{e_{1H}^L}^2$ and $k{e_{2H}^L}^2$ if of high-type, where $k$ is the cost coefficient. The upfront, intermediate and final money transfers to the agent are denoted by $m_{0L}^L$, $m_{1L}^L$ and $m_{2L}^L$ if of low-type and $m_{0H}^L$, $m_{1H}^L$ and $m_{2H}^L$ if of high-type. Similarly, these transfers can be either positive or negative, depending on whether the principal pays the agent or the agent pays the principal. The intermediate and final transfers occur only when the first and second periods are successful, respectively.

We consider two types of equilibria: separating and pooling. First we examine the setting of the separating equilibrium.

The high-type principal's expected profit is

$$\begin{aligned} & \lambda\left[\alpha_L^H e_{1L}^H \left(\alpha_L^H e_{2L}^H (V - m_{2L}^H) - m_{1L}^H\right) - m_{0L}^H\right] \\ + \quad & (1-\lambda)\left[\alpha_H^H e_{1H}^H \left(\alpha_H^H e_{2H}^H (V - m_{2H}^H) - m_{1H}^H\right) - m_{0H}^H\right] \end{aligned} \tag{3.1}$$

To ensure the participation of both types of agents, the following two constraints have to be satisfied:

$$m_{0L}^H - k{e_{1L}^H}^2 + \alpha_L^H e_{1L}^H m_{1L}^H - \alpha_L^H e_{1L}^H k{e_{2L}^H}^2 + {\alpha_L^H}^2 e_{1L}^H e_{2L}^H m_{2L}^H \geq 0 \tag{3.2}$$

and

$$m_{0H}^H - k{e_{1H}^H}^2 + \alpha_H^H e_{1H}^H m_{1H}^H - \alpha_H^H e_{1H}^H k{e_{2H}^H}^2 + {\alpha_H^H}^2 e_{1H}^H e_{2H}^H m_{2H}^H \geq 0 \tag{3.3}$$

where (3.2) is for the low-type agent and (3.3) is for the high-type agent.

Likewise, the low-type principal's expected profit is

$$\begin{aligned} & \lambda\left[\alpha_L^L e_{1L}^L \left(\alpha_L^L e_{2L}^L (V - m_{2L}^L) - m_{1L}^L\right) - m_{0L}^L\right] \\ + \quad & (1-\lambda)\left[\alpha_H^L e_{1H}^L \left(\alpha_H^L e_{2H}^L (V - m_{2H}^L) - m_{1H}^L\right) - m_{0H}^L\right] \end{aligned} \tag{3.4}$$

and the participation constraints for the low-type and high-type agents are

$$m_{0L}^{L} - k{e_{1L}^{L}}^{2} + \alpha_{L}^{L} e_{1L}^{L} m_{1L}^{L} - \alpha_{L}^{L} e_{1L}^{L} k {e_{2L}^{L}}^{2} + {\alpha_{L}^{L}}^{2} e_{1L}^{L} e_{2L}^{L} m_{2L}^{L} \geq 0 \tag{3.5}$$

and

$$m_{0H}^{L} - k{e_{1H}^{L}}^{2} + \alpha_{H}^{L} e_{1H}^{L} m_{1H}^{L} - \alpha_{H}^{L} e_{1H}^{L} k {e_{2H}^{L}}^{2} + {\alpha_{H}^{L}}^{2} e_{1H}^{L} e_{2H}^{L} m_{2H}^{L} \geq 0 \tag{3.6}$$

In the separating equilibrium, the high-type principal needs not only to screen the high-type agent from the low-type agent, but also to prevent the low-type principal from mimicking her, and herself from mimicking the low-type principal.

To screen the high-type agent from the low-type agent, money transfers $\{m_{0L}^{H}, m_{1L}^{H}, m_{2L}^{H}\}$ and $\{m_{0H}^{H}, m_{1H}^{H}, m_{2H}^{H}\}$ have to be set in such a way that the low-type agent won't mimic the high-type agent and the high-type agent won't mimic the low-type agent, namely two incentive compatibility constraints have to be satisfied:

$$\begin{aligned} & m_{0L}^{H} - k{e_{1L}^{H}}^{2} + \alpha_{L}^{H} e_{1L}^{H} m_{1L}^{H} - \alpha_{L}^{H} e_{1L}^{H} k {e_{2L}^{H}}^{2} + {\alpha_{L}^{H}}^{2} e_{1L}^{H} e_{2L}^{H} m_{2L}^{H} \\ \geq \quad & m_{0H}^{H} - k\tilde{e}_{1L}^{H2} + \alpha_{L}^{H} \tilde{e}_{1L}^{H} m_{1H}^{H} - \alpha_{L}^{H} \tilde{e}_{1L}^{H} k \tilde{e}_{2L}^{H2} + {\alpha_{L}^{H}}^{2} \tilde{e}_{1L}^{H} \tilde{e}_{2L}^{H} m_{2H}^{H} \end{aligned} \tag{3.7}$$

and

$$\begin{aligned} & m_{0H}^{H} - k{e_{1H}^{H}}^{2} + \alpha_{H}^{H} e_{1H}^{H} m_{1H}^{H} - \alpha_{H}^{H} e_{1H}^{H} k {e_{2H}^{H}}^{2} + {\alpha_{H}^{H}}^{2} e_{1H}^{H} e_{2H}^{H} m_{2H}^{H} \\ \geq \quad & m_{0L}^{H} - k\tilde{e}_{1H}^{H2} + \alpha_{H}^{H} \tilde{e}_{1H}^{H} m_{1L}^{H} - \alpha_{H}^{H} \tilde{e}_{1H}^{H} k \tilde{e}_{2H}^{H2} + {\alpha_{H}^{H}}^{2} \tilde{e}_{1H}^{H} \tilde{e}_{2H}^{H} m_{2L}^{H} \end{aligned} \tag{3.8}$$

where $\tilde{e}_{1L}^{H}$ and $\tilde{e}_{2L}^{H}$ are the efforts that the low-type agent incurs when he mimics the high-type agent, and $\tilde{e}_{1H}^{H}$ and $\tilde{e}_{2H}^{H}$ are the efforts that the high-type agent incurs when he mimics the low-type agent.

To prevent the low-type principal from mimicking her, and herself from mimicking the low-type principal, the high-type principal has to make money transfers $\{m_{0H}^{H}, m_{1H}^{H}, m_{2H}^{H}\}$ to satisfy two incentive compatibility constraints:

$$LM \geq -m_{0H}^{H} - \alpha_{H}^{L} e_{1H}^{H} m_{1H}^{H} + {\alpha_{H}^{L}}^{2} e_{1H}^{H} e_{2H}^{H} (V - m_{2H}^{H}) \tag{3.9}$$

and

$$-m_{0H}^{H} - \alpha_{H}^{H} e_{1H}^{H} m_{1H}^{H} + {\alpha_{H}^{H}}^{2} e_{1H}^{H} e_{2H}^{H} (V - m_{2H}^{H}) \geq \overline{LM} \tag{3.10}$$

where $LM$ is the maximal expected profit of the low-type principal when she does her own profit maximization profit, and $\overline{LM}$ is the maximal expected profit of the high-type principal when she pretends to be the low-type principal.

More specifically, to obtain $LM$, the low-type principal maximizes her expected profit:

$$\begin{aligned} & \lambda\left[\alpha_L^L e_{1L}^L \left(\alpha_L^L e_{2L}^L (V - m_{2L}^L) - m_{1L}^L\right) - m_{0L}^L\right] \\ + \quad & (1-\lambda)\left[\alpha_H^L e_{1H}^L \left(\alpha_H^L e_{2H}^L (V - m_{2H}^L) - m_{1H}^L\right) - m_{0H}^L\right] \end{aligned} \tag{3.11}$$

with the participation constraints for the low-type and high-type agents

$$m_{0L}^L - k{e_{1L}^L}^2 + \alpha_L^L e_{1L}^L m_{1L}^L - \alpha_L^L e_{1L}^L k {e_{2L}^L}^2 + {\alpha_L^L}^2 e_{1L}^L e_{2L}^L m_{2L}^L \geq 0 \tag{3.12}$$

and

$$m_{0H}^L - k{e_{1H}^L}^2 + \alpha_H^L e_{1H}^L m_{1H}^L - \alpha_H^L e_{1H}^L k {e_{2H}^L}^2 + {\alpha_H^L}^2 e_{1H}^L e_{2H}^L m_{2H}^L \geq 0 \tag{3.13}$$

and two incentive compatibility constraints - one preventing the low-type agent mimicking the high-type agent and the other preventing the high-type agent mimicking the low-type agent:

$$\begin{aligned} & m_{0L}^L - k{e_{1L}^L}^2 + \alpha_L^L e_{1L}^L m_{1L}^L - \alpha_L^L e_{1L}^L k {e_{2L}^L}^2 + {\alpha_L^L}^2 e_{1L}^L e_{2L}^L m_{2L}^L \\ \geq \quad & m_{0H}^L - k\tilde{e}^2 L_{1L} + \alpha_L^L \tilde{e}_{1L}^L m_{1H}^L - \alpha_L^L \tilde{e}_{1L}^L k \tilde{e}_{2L}^{L^2} + \alpha^{L^2}{}_L \tilde{e}_{1L}^L \tilde{e}_{2L}^L m_{2H}^L \end{aligned} \tag{3.14}$$

and

$$\begin{aligned} & m_{0H}^L - k{e_{1H}^L}^2 + \alpha_H^L e_{1H}^L m_{1H}^L - \alpha_H^L e_{1H}^L k {e_{2H}^L}^2 + {\alpha_H^L}^2 e_{1H}^L e_{2H}^L m_{2H}^L \\ \geq \quad & m_{0L}^L - k\tilde{e}_{1H}^{L^2} + \alpha_H^L \tilde{e}_{1H}^L m_{1L}^L - \alpha_H^L \tilde{e}_{1H}^L k \tilde{e}_{2H}^{L^2} + \alpha^{L^2}{}_H \tilde{e}_{1H}^L \tilde{e}_{2H}^L m_{2L}^L \end{aligned} \tag{3.15}$$

where $\tilde{e}_{1L}^L$ and $\tilde{e}_{2L}^L$ are the efforts that the low-type agent incurs when he mimics the high-type agent, and $\tilde{e}_{1H}^L$ and $\tilde{e}_{2H}^L$ are the efforts that the high-type agent incurs when he mimics the low-type agent. It is clear that the solution of the optimization problem in the screen model in Chapter 1 would give the value of $LM$ if $\alpha_L$ and $\alpha_H$ are replaced with $\alpha_L^L$ and $\alpha_H^L$.

As to $\overline{LM}$, the high-type principal maximizes her expected profit while pretending to be the low-type principal:

$$\begin{aligned} & \lambda\left[\alpha_L^H e_{1L}^L \left(\alpha_L^H e_{2L}^L (V - m_{2L}^L) - m_{1L}^L\right) - m_{0L}^L\right] \\ + \quad & (1-\lambda)\left[\alpha_H^H e_{1H}^L \left(\alpha_H^H e_{2H}^L (V - m_{2H}^L) - m_{1H}^L\right) - m_{0H}^L\right] \end{aligned} \tag{3.16}$$

with the same participation constraints for the low-type and high-type agents (3.12) and (3.13), and the same incentive compatibility constraints (3.14) and (3.15). It is not difficult to see that by the same way of solving the maximization problem of the screen model in Chapter 1, we can solve the above optimization problem and find $\overline{LM}$.

Notice that when neither (3.9) nor (3.10) holds, the profit maximization problem for the high-type principal is the same as the one of the screen model in Chapter 1, and thereby has the same solution. However, when either of (3.9) and (3.10) binds, the solution of the screen model in Chapter 1 no longer applies to the profit maximization problem for the high-type principal here. In fact, we are able to obtain the solution of the problem for the case in which (3.9) binds but (3.10) does not bind. According to the results in Chapter 2, this case seems to hold for most values of parameters and be the most interesting case. But the closed form solution does not give good insights, because of complicated expressions involving quite a few parameters. So numerical plotting would be an more intuitive way that can lead to insights.

Next we examine the setting of the pooling equilibrium. In this equilibrium, both the high-type and low-type principals offer the same contract with money transfers $m_{0L}$, $m_{1L}$ and $m_{2L}$ to the low-type agent and $m_{0H}$, $m_{1H}$ and $m_{2H}$ to the high-type agent. We assume that the agent believes that the principal is of high-type with probability $p$ and of low-type with probability $1-p$.

The high-type principal maximizes her expected profit:

$$\begin{aligned} & \lambda\left[\alpha_L^H e_{1L}\left(\alpha_L^H e_{2L}(V-m_{2L})-m_{1L}\right)-m_{0L}\right] \\ + \quad & (1-\lambda)\left[\alpha_H^H e_{1H}\left(\alpha_H^H e_{2H}(V-m_{2H})-m_{1H}\right)-m_{0H}\right] \end{aligned} \tag{3.17}$$

with the participation constraints of the low-type and high-type agents:

$$m_{0L}-ke_{1L}^2+\overline{\alpha}_L e_{1L}m_{1L}-\overline{\alpha}_L e_{1L}ke_{2L}^2+\overline{\alpha}_L\tilde{\alpha}_L e_{1L}e_{2L}m_{2L}\geq 0 \tag{3.18}$$

and

$$m_{0H}-ke_{1H}^2+\overline{\alpha}_H e_{1H}m_{1H}-\overline{\alpha}_H e_{1H}ke_{2H}^2+\overline{\alpha}_H\tilde{\alpha}_H e_{1H}e_{2H}m_{2H}\geq 0 \tag{3.19}$$

where

$$\begin{aligned} \overline{\alpha}_L &= p\alpha_L^H+(1-p)\alpha_L^L \\ \tilde{\alpha}_L &= \frac{p{\alpha_L^H}^2}{p\alpha_L^H+(1-p)\alpha_L^L}+\frac{(1-p){\alpha_L^L}^2}{p\alpha_L^H+(1-p)\alpha_L^L} \\ \overline{\alpha}_H &= p\alpha_H^H+(1-p)\alpha_H^L \\ \tilde{\alpha}_H &= \frac{p{\alpha_H^H}^2}{p\alpha_H^H+(1-p)\alpha_H^L}+\frac{(1-p){\alpha_H^L}^2}{p\alpha_H^H+(1-p)\alpha_H^L} \end{aligned} \tag{3.20}$$

and with two incentive compatibility constraints for the low-type and high-type agents:

$$\begin{aligned} & m_{0L} - ke_{1L}^2 + \overline{\alpha}_L e_{1L} m_{1L} - \overline{\alpha}_L e_{1L} k e_{2L}^2 + \overline{\alpha}_L \tilde{\alpha}_L e_{1L} e_{2L} m_{2L} \\ \geq \quad & m_{0H} - k\tilde{e}_{1L}^2 + \overline{\alpha}_L \tilde{e}_{1L} m_{1H} - \overline{\alpha}_L \tilde{e}_{1L} k \tilde{e}_{2L}^2 + \overline{\alpha}_L \tilde{\alpha}_L \tilde{e}_{1L} \tilde{e}_{2L} m_{2H} \end{aligned}$$

and

$$\begin{aligned} & m_{0H} - ke_{1H}^2 + \overline{\alpha}_H e_{1H} m_{1H} - \overline{\alpha}_H e_{1H} k e_{2H}^2 + \overline{\alpha}_H \tilde{\alpha} e_{1H} e_{2H} m_{2H} \\ \geq \quad & m_{0L} - k\tilde{e}_{1H}^2 + \overline{\alpha}_H \tilde{e}_{1H} m_{1L} - \overline{\alpha}_H \tilde{e}_{1H} k \tilde{e}_{2H}^2 + \overline{\alpha}_H \tilde{\alpha}_H \tilde{e}_{1H} \tilde{e}_{2H} m_{2L} \end{aligned} \tag{3.21}$$

where $\{\tilde{e}_{1L}, \tilde{e}_{1L}\}$ and $\{\tilde{e}_{1H}, \tilde{e}_{2H}\}$ are the efforts exerted by the low-type and high-type agents respectively when they are dishonest about their types and pretend to be the other type. In addition, the incentive compatibility constraints for the low-type and high-type principals has to be satisfied:

$$\begin{aligned} & \lambda\left[\alpha_L^L e_{1L}\left(\alpha_L^L e_{2L}(V - m_{2L}) - m_{1L}\right) - m_{0L}\right] \\ & +(1-\lambda)\left[\alpha_H^L e_{1H}\left(\alpha_H^L e_{2H}(V - m_{2H}) - m_{1H}\right) - m_{0H}\right] \quad \geq \quad LM_1 \end{aligned} \tag{3.22}$$

and

$$\begin{aligned} & \lambda\left[\alpha_L^H e_{1L}\left(\alpha_L^H e_{2L}(V - m_{2L}) - m_{1L}\right) - m_{0L}\right] \\ & +(1-\lambda)\left[\alpha_H^H e_{1H}\left(\alpha_H^H e_{2H}(V - m_{2H}) - m_{1H}\right) - m_{0H}\right] \quad \geq \quad \overline{LM}_1 \end{aligned} \tag{3.23}$$

where $LM_1$ is the maximal expected profit of the low-type principal when her type is known, and $\overline{LM}_1$ is the maximal expected profit of the high-type principal when she pretends to be the low-type one. As we observe from the results of Chapter 2, the most interesting case seems to be the one when neither (3.22) nor (3.23) binds. We are able to obtain the closed form solution of this case. But due to its complicated expression involving various parameters, it is not easy to derive insights. So numerical plotting would be a good way to exhibit properties and features of this model.

In the above, we discussed the settings of the separating and pooling equilibria when money transfers consists of upfront, intermediate and end transfers. We can also look at the following cases: 1) only intermediate and end transfers are included. 2) only upfront and end transfers are included. 3) only end transfer is included. We can compare the profits of the high-type principal in all the cases and explore the benefits of adding upfront and/or intermediate transfers. In addition, we can compare the profits of the high-type principal in Chapter 3 with those in Chapter 1 to see how much profit would be lost when the high-type principal wants to reveal or hide her private information.

# APPENDIX A

# CHAPTER 1 PROOFS

## A.1 Proof of Theorem 1

We will examine the baseline model (i.e., upfront, intermediate and end money transfers are all included in the contract) under complete information.

By offering the menu of money transfers $(m_{0L}, m_{1L}, m_{2L})$ to the low-type agent and $(m_{0H}, m_{1H}, m_{2H})$ to the high-type agent, the principal wants to maximize her expected profit:

$$\lambda\left[\alpha_L e_{1L}\left(\alpha_L e_{2L}(V-m_{2L})-m_{1L}\right)-m_{0L}\right]+(1-\lambda)\left[\alpha_H e_{1H}\left(\alpha_H e_{2H}(V-m_{2H})-m_{1H}\right)-m_{0H}\right] \tag{A.1}$$

Taking into account $(m_{0L}, m_{1L}, m_{2L})$ and $(m_{0H}, m_{1H}, m_{2H})$, the low-type and high-type agents want to maximize their following expected profits, respectively:

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \tag{A.2}$$

and

$$m_{0H} - ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \tag{A.3}$$

where $0 \le e_{1L} \le 1$, $0 \le e_{2L} \le 1$, $0 \le e_{1H} \le 1$ and $0 \le e_{2H} \le 1$. To ensure both agents' participation, (A.2) and (A.3) have to be nonnegative.

Since the model is of complete information, there are no incentive compatibility constraints involved. Thus the principal can maximize her expected profit from the part on the low-type agent and the part from the high-type agent separately. In other words, the original optimization problem can be decomposed into two independent optimization problems, one involving the principal and the low-type agent and the other involving the principal and the high-type agent.

For the first optimization problem, by providing $(m_{0L}, m_{1L}, m_{2L})$ to the low-type agent, the principal wants to maximize her expected profit from the part on the low-type agent:

$$\lambda\left[\alpha_L e_{1L}\left(\alpha_L e_{2L}(V-m_{2L})-m_{1L}\right)-m_{0L}\right] \tag{A.4}$$

Considering $(m_{0L}, m_{1L}, m_{2L})$, the low-type agent would like to maximize his expected profit:

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \tag{A.5}$$

where $0 \le e_{1L} \le 1$ and $0 \le e_{2L} \le 1$ and (B.88) $\ge 0$ as the participation constraint.

For the second one, by offering $(m_{0H}, m_{1H}, m_{2H})$ to the high-type agent, the principal wants to maximize her expected profit from the part on the high-type agent:

$$\lambda\left[\alpha_H e_{1H}\left(\alpha_H e_{2H}(V - m_{2H}) - m_{1H}\right) - m_{0H}\right] \tag{A.6}$$

Regarding $(m_{0H}, m_{1H}, m_{2H})$, the high-type agent would like to maximize his expected profit:

$$m_{0H} - ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \tag{A.7}$$

where $0 \le e_{1H} \le 1$ and $0 \le e_{2H} \le 1$ and (A.7) $\ge 0$ as the participation constraint.

We can assume $e_{1L}$, $e_{2L}$, $e_{1H}$ and $e_{2H}$ are positive and don't need to consider the scenarios when some of them are 0. The reason is the following.

If $e_{1L} = 0$, the low-type agent's expected profit (B.88) becomes $m_{0L}$, which has to be nonnegative in order for the low-type agent to participate. Thus the part of the principal's expected profit (B.87) from the low-type agent $\lambda\left[\alpha_L e_{1L}\left(\alpha_L e_{2L}(V - m_{2L}) - m_{1L}\right) - m_{0L}\right] = \lambda[-m_{0L}] \le 0$. Clearly, this can't be the maximum location the principal anticipates.

If $e_{2L} = 0$, the low-type agent's expected profit (B.88) becomes $m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L}$, which is nonnegative to ensure the participation of the low-type agent. Therefore the part of the principal's expected profit (B.87) from the low-type agent $\lambda\left[-\alpha_L e_{1L} m_{2L} - m_{0L}\right] \le \lambda\left[-ke_{1L}^2\right] \le 0$. It is clear that this won't be the maximum location the principal looks for.

Similar argument goes for the scenarios when $e_{1H} = 0$ or $e_{2H} = 0$.

Next we will find the expressions of the optimal efforts $e_{1L}^*$ and $e_{2L}^*$ for given $(m_{0L}, m_{1L}, m_{2L})$, corresponding to (B.88).

Notice that the Lagrangian for the maximization problem of the low-type agent's expected profit with $e_{1L}$ and $e_{2L}$ as the decision variables is

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} + \lambda_1(1 - e_{1L}) + \lambda_2(1 - e_{2L}) \tag{A.8}$$

where $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ are Lagrangian multipliers.

The first order conditions of (B.89) with respect to $e_{1L}$ and $e_{2L}$ as follows:

$$-2ke_{1L} + \alpha_L m_{1L} - \alpha_L k e_{2L}^2 + \alpha_L^2 e_{2L} m_{2L} - \lambda_1 = 0 \tag{A.9}$$

$$\alpha_L e_{1L}\left(-2ke_{2L} + \alpha_L m_{2L}\right) - \lambda_2 = 0 \tag{A.10}$$

which lead to the optimal efforts $e_{1L}^*$ and $e_{2L}^*$ satisfying

$$e_{2L}^* = \frac{\alpha_L m_{2L} - \frac{\lambda_2}{\alpha_L e_{1L}^*}}{2k} \tag{A.11}$$

$$e_{1L}^* = \frac{\alpha_L m_{1L} - \alpha_L k {e_{2L}^*}^2 + \alpha_L^2 e_{2L}^* m_{2L} - \lambda_1}{2k} \tag{A.12}$$

where $0 < e_{2L}^* \le 1$ and $0 < e_{1L}^* \le 1$.

There are four situations for consideration:

1. When $\lambda_1 > 0$ and $\lambda_2 > 0$, $e^*_{1L} = 1$ and $e^*_{2L} = 1$. By (B.88), the low-type agent's expected profit is

$$m_{0L} - k + \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \tag{A.13}$$

Since $\lambda_1 > 0$, $\lambda_2 > 0$, $e^*_{1L} = 1$ and $e^*_{2L} = 1$, (B.92) and (B.93) imply that $\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} > 1$ and $\frac{\alpha_L m_{2L}}{2k} > 1$.

2. When $\lambda_1 > 0$ and $\lambda_2 = 0$, $e^*_{1L} = 1$ and $e^*_{2L} = \frac{\alpha_L m_{2L}}{2k} \leq 1$. By (B.88), the low-type agent's expected profit is

$$\begin{aligned} & m_{0L} - k + \alpha_L m_{1L} - \alpha_L k \left(\frac{\alpha_L m_{2L}}{2k}\right)^2 + \alpha_L^2 \frac{\alpha_L m_{2L}}{2k} m_{2L} \\ = \; & m_{0L} - k + \alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k} \end{aligned} \tag{A.14}$$

Since $\lambda_1 > 0$ and $e^*_{1L} = 1$, (B.93) implies that $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} > 1$.

3. When $\lambda_1 = 0$ and $\lambda_2 > 0$, $e^*_{1L} = \frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} \leq 1$ and $e^*_{2L} = 1$, namely, $2ke^*_{1L} = \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}$. By (B.88), the low-type agent's expected profit is

$$\begin{aligned} & m_{0L} - k{e^*_{1L}}^2 + \alpha_L e^*_{1L} m_{1L} - \alpha_L e^*_{1L} k {e^*_{2L}}^2 + \alpha_L^2 e^*_{1L} e^*_{2L} m_{2L} \\ = \; & m_{0L} - k{e^*_{1L}}^2 + \alpha_L e^*_{1L} m_{1L} - \alpha_L e^*_{1L} k + \alpha_L^2 e^*_{1L} m_{2L} \\ = \; & m_{0L} - k{e^*_{1L}}^2 + e^*_{1L} \left[\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}\right] \\ = \; & m_{0L} - k{e^*_{1L}}^2 + e^*_{1L} 2k e^*_{1L} \\ = \; & m_{0L} + k{e^*_{1L}}^2 \\ = \; & m_{0L} + k \left(\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2 \end{aligned} \tag{A.15}$$

Since $\lambda_2 > 0$, (B.92) implies that $\frac{\alpha_L m_{2L}}{2k} > 1$.

4. When $\lambda_1 = 0$ and $\lambda_2 = 0$, by (B.92) and (B.93), $e^*_{2L} = \frac{\alpha_L m_{2L}}{2k} \leq 1$ and

$$\begin{aligned} e^*_{1L} & = \frac{\alpha_L m_{1L} - \alpha_L k {e^*_{2L}}^2 + \alpha_L^2 e^*_{2L} m_{2L}}{2k} \\ & = \frac{\alpha_L m_{1L} - \alpha_L k \left(\frac{\alpha_L m_{2L}}{2k}\right)^2 + \alpha_L^2 \frac{\alpha_L m_{2L}}{2k} m_{2L}}{2k} \\ & = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \end{aligned} \tag{A.16}$$

which is less than or equal to 1.

By (B.88), the low-type agent's expected profit is

$$\begin{aligned}
& m_{0L} - k{e_{1L}^*}^2 + \alpha_L e_{1L}^* m_{1L} - \alpha_L e_{1L}^* k {e_{2L}^*}^2 + \alpha_L^2 e_{1L}^* e_{2L}^* m_{2L} \\
= \quad & m_{0L} - k{e_{1L}^*}^2 + \alpha_L e_{1L}^* m_{1L} - \alpha_L e_{1L}^* k + \alpha_L^2 e_{1L}^* m_{2L} \\
= \quad & m_{0L} - k{e_{1L}^*}^2 + e_{1L}^* \left[\alpha_L m_{1L} - \alpha_L k {e_{2L}^*}^2 + \alpha_L^2 e_{2L}^* m_{2L}\right] \\
= \quad & m_{0L} - k{e_{1L}^*}^2 + e_{1L}^* 2k e_{1L}^* \\
= \quad & m_{0L} + k{e_{1L}^*}^2 \\
= \quad & m_{0L} + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \qquad \text{(A.17)}
\end{aligned}$$

Similarly, for the expressions of the optimal efforts $e_{1H}^*$ and $e_{2H}^*$ for given $(m_{0H}, m_{1H}, m_{2H})$, corresponding to (A.7), we have that both are positive and

1. $e_{1H}^* = 1$ and $e_{2H}^* = 1$. The high-type agent's expected profit is

$$m_{0H} - k + \alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \qquad \text{(A.18)}$$

   and $\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} > 1$ as well as $\frac{\alpha_H m_{2H}}{2k} > 1$.

2. $e_{1H}^* = 1$ and $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k} \leq 1$. The high-type agent's expected profit is

$$m_{0H} - k + \alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k} \qquad \text{(A.19)}$$

   and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} > 1$.

3. $e_{1H}^* = \frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} \leq 1$ and $e_{2H}^* = 1$. The high-type agent's expected profit is

$$m_{0H} + k\left(\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)^2 \qquad \text{(A.20)}$$

   and $\frac{\alpha_H m_{2H}}{2k} > 1$.

4. $e_{1H}^* = \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \leq 1$ and $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k} \leq 1$. The high-type agent's expected profit is

$$m_{0H} + k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 \qquad \text{(A.21)}$$

With the availability of the expressions of the optimal efforts $e_{1L}^*$ and $e_{2L}^*$ for given $(m_{0L}, m_{1L}, m_{2L})$, and $e_{1H}^*$, $e_{2H}^*$ for given $(m_{0H}, m_{1H}, m_{2H})$, we are able to solve two principal's expected profit maximization problems, one consisting of (B.87) and (B.88) and the other consisting of (A.6) and (A.7). It is clear that once one of them is solved, the solution for the other can be easily obtained, due to the symmetric structure. Next, we will solve the maximization problem consisting of (A.6) and (A.7), which involves the principal and the high-type agent.

Notice that (A.7) equals 0, instead of being just nonnegative. To show it, using the fact that the optimal efforts $e^*_{1H}$, $e^*_{2H}$ depend only on $m_{1H}$ and $m_{2H}$, we can write $e^*_{1H}$ and $e^*_{2H}$ as $f_1(m_{1H}, m_{2H})$ and $f_2(m_{1H}, m_{2H})$ respectively, functions of $m_{1H}$ and $m_{2H}$. Thus the first order condition with respect to $m_{0H}$ of the Lagrangian for the principal's expected profit maximization problem consisting (A.6) and (A.7) leads to the positivity of the Lagrangian multiplier for the participation constraint–(A.7) ≥ 0. This means that (A.7) equals 0, namely, the participation constraint is binding.

Using the binding participation constraint–(A.7) equals 0–to replace $m_{0H}$ in the expression (A.6) and taking into account the four scenarios we discussed in (A.18) through (A.21), we have the following four scenarios for consideration for the the principal's expected profit maximization problem consisting (A.6) and (A.7).

1. when $m_{2H} \le \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \le 1$, i.e., $e^*_{2H} \le 1$ and $e^*_{1H} \le 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$\begin{aligned}
&(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right)\right] \\
&+(1-\lambda)k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 \\
&+\lambda_1\left(\frac{2k}{\alpha_H} - m_{2H}\right) + \lambda_2\left(1 - \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)
\end{aligned} \tag{A.22}$$

where $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ are the Lagrangian multipliers.

The first order conditions of (B.99) with respect to $m_{1H}$ and $m_{2H}$ lead to

$$(1-\lambda)\frac{\alpha_H^2}{2k}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) - \lambda_2 \frac{\alpha_H}{2k} = 0 \tag{A.23}$$

$$\begin{aligned}
&(1-\lambda)\left[\alpha_H^4 \frac{2m_{2H}}{8k^2}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right)\right] \\
&+(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V - m_{2H})\right)\right] \\
&-\lambda_1 - \lambda_2 \alpha_H^3 \frac{2m_{2H}}{4k^2} = 0
\end{aligned} \tag{A.24}$$

Multiplying (B.100) by $\alpha_H^3 \frac{2m_{2H}}{4k^2}$ and subtracting the product from (B.101) gives

$$(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V - m_{2H})\right)\right] - \lambda_1 = 0 \tag{A.25}$$

There are three cases for consideration:

(a) When $\lambda_1 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$. The Lagrangian equals

$$(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} + \alpha_H k}{2k}\right)\left(\alpha_H\left(V - \frac{2k}{\alpha_H}\right) - m_{1H}\right) + k\left(\frac{\alpha_H m_{1H} + \alpha_H k}{2k}\right)^2\right]$$
$$+\tilde{\lambda}_1\left(1 - \frac{\alpha_H m_{1H} + \alpha_H k}{2k}\right) \tag{A.26}$$

where $\tilde{\lambda}_1 \geq 0$ is a Lagrangian multiplier.

The first order condition of (B.103) with respect to $m_{1H}$ is

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\alpha_H\left(V - \frac{2k}{\alpha_H}\right) - m_{1H}\right)\right] - \tilde{\lambda}_1 \frac{\alpha_H}{2k} = 0 \tag{A.27}$$

When $\tilde{\lambda}_1 = 0$, $m_{1H} = \alpha_H\left(V - \frac{2k}{\alpha_H}\right)$. So the principal's expected profit equals

$$(1-\lambda)k\left[\frac{\alpha_H^2\left(V - \frac{2k}{\alpha_H}\right) + \alpha_H k}{2k}\right]^2 = (1-\lambda)k\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \tag{A.28}$$

Notice that $\frac{\alpha_H^2\left(V - \frac{2k}{\alpha_H}\right) + \alpha_H k}{2k} \leq 1$.

When $\tilde{\lambda}_1 > 0$, $\frac{\alpha_H m_{1H} + \alpha_H k}{2k} = 1$, namely $m_{1H} = \frac{2k}{\alpha_H} - k$. Thus the principal's expected profit equals

$$(1-\lambda)\left[\alpha_H\left(\alpha_H\left(V - \frac{2k}{\alpha_H}\right) + k - \frac{2k}{\alpha_H}\right)\right] = (1-\lambda)\left(\alpha_H^2 V - \alpha_H k - 2k\right) \tag{A.29}$$

It is easy to see that (B.105) is greater than (B.106).

(b) When $\lambda_2 > 0$, $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} = 1$ which implies that $m_{1H} = \frac{2k}{\alpha_H} - \frac{\alpha_H^2 m_{2H}^2}{4k}$. The Lagrangian equals

$$(1-\lambda)\left[\alpha_H\left(\alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - \frac{2k}{\alpha_H} + \frac{\alpha_H^2 m_{2H}^2}{4k}\right) + k\right] + \tilde{\lambda}_2\left(\frac{\alpha_H}{2k} - m_{2H}\right) \tag{A.30}$$

where $\tilde{\lambda}_2 \geq 0$ is a Lagrangian multiplier.

The first order condition of (B.107) gives

$$(1-\lambda)\frac{\alpha_H^3}{2k}(V-m_{2H})-\tilde{\lambda}_2=0 \tag{A.31}$$

When $\tilde{\lambda}_2>0$, $m_{2H}=\frac{\alpha_H}{2k}$, which means that $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}=1$ is equivalent to $\frac{\alpha_H m_{1H}+\alpha_H k}{2k}=1$. Thus the principal's expected profit has the same value as in (B.106), which is less than or equal to the value in (B.105).

When $\tilde{\lambda}_2=0$, $V=m_{2H}$. Thus the principal's expected profit equals

$$(1-\lambda)\left(-k+\frac{\alpha_H^3 V^2}{4k}\right) \tag{A.32}$$

which is $\leq -k+\alpha_H k<0$, since $V=m_{2H}\leq\frac{2k}{\alpha_H}$. Thus (B.109) can't be the local maximum, compared with (B.105).

(c) When $\lambda_1=0$ and $\lambda_2=0$, from (B.102) and (B.100), we have $m_{2H}=V$ and $m_{1H}=0$. Since $m_{2H}\leq\frac{2k}{\alpha_H}$, $V\leq\frac{2k}{\alpha_H}$. The principal's expected profit equals

$$(1-\lambda)k\left(\frac{\frac{\alpha_H^3 V^2}{4k}}{2k}\right)^2 \tag{A.33}$$

Notice that comparing (B.110) with (B.105), we have

$$(1-\lambda)k\left(\frac{\frac{\alpha_H^3 V^2}{4k}}{2k}\right)^2\geq(1-\lambda)k\left(\frac{\alpha_H^2 V-\alpha_H k}{2k}\right)^2 \tag{A.34}$$

where the equality holds only when $V=\frac{2k}{\alpha_H}$, because $\frac{\alpha_H^3 V^2}{4k}\geq\alpha_H^2 V-\alpha_H k$.

One more thing we need to show is that the expected profit obtained from the above discussion is local maximal. The reason is the following.

$0\leq e_{2H}^*=\frac{\alpha_H m_{2H}}{2k}\leq 1$ and $0\leq e_{1H}^*=\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\leq 1$ ensure that $m_{1H}$ and $m_{2H}$ are bounded in absolute value. This means that the expression of the principal's expected profit in (B.99) is also bounded in absolute value. Thus, the maximum of the principal's expected profit exists and the unique solution of the first order conditions of the Lagrangian above provides the only candidate for the location of the maximum. Therefore the expected profit obtained is local maximal.

2. When $m_{2H} \leq \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \geq 1$, i.e., $e_{2H}^* \leq 1$ and $e_{1H}^* = 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$(1-\lambda)\left[\alpha_H \cdot 1 \cdot \left(\alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) + \left(-k + \alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}\right)\right]$$
$$+\lambda_1 \left(\frac{2k}{\alpha_H} - m_{2H}\right) + \lambda_2 \left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} - 1\right) \tag{A.35}$$

The first order condition of (B.112) with respect to $m_{1H}$ and $m_{2H}$ are

$$\lambda_2 \frac{\alpha_H}{2k} = 0 \tag{A.36}$$
$$(1-\lambda)\left[\frac{\alpha_H^3}{2k}(V - m_{2H})\right] - \lambda_1 + \lambda_2 \alpha_H^3 \frac{m_{2H}}{4k^2} = 0 \tag{A.37}$$

where (B.113) gives $\lambda_2 = 0$. Thus (B.114) becomes

$$(1-\lambda)\left[\frac{\alpha_H^3}{2k}(V - m_{2H})\right] - \lambda_1 = 0 \tag{A.38}$$

When $\lambda_1 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$. Thus (B.115) implies that $V$ must be greater than $\frac{2k}{\alpha_H}$. The principal's expected profit equals

$$(1-\lambda)\left[\alpha_H \cdot 1 \cdot \left(\alpha_H \left(V - \frac{2k}{\alpha_H}\right) - m_{1H}\right) + (-k + \alpha_H m_{1H} + \alpha_H k)\right]$$
$$= \alpha_H^2 V - \alpha_H k - k \tag{A.39}$$

When $\lambda_1 = 0$, $m_{2H} = V$. Thus the principal's expected profit equals

$$(1-\lambda)\left(-k + \frac{\alpha_H^3 V^2}{4k}\right) \tag{A.40}$$

which is less than or equal to $-k + \alpha_H k < 0$, because $m_{2H} \leq \frac{2k}{\alpha_H}$.

To show the expected profit obtained above is local maximal, we notice that with $0 \leq e_{2H}^* = \frac{\alpha_H m_{2H}}{2k} \leq 1$, the expression of the principal's expected profit in (B.112) as a function of $m_{2H}$ (with $m_{1H}$ being eliminated) is bounded in absolute value, and thereby has a maximum. Thus the unique solution of the first order conditions must be the location of the local maximal expected profit.

3. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \leq 2k$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* \leq 1$. The Lagrangian for the maximum of the principal's expected profit is

$$\begin{aligned}
&(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)(\alpha_H(V - m_{2H}) - m_{1H})\right] \\
&+(1-\lambda)\left[k\left(\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)^2\right] \\
&+\lambda_1\left(m_{2H} - \frac{2k}{\alpha_H}\right) + \lambda_2(2k - \alpha_H m_{1H} + \alpha_H k - \alpha_H^2 m_{2H})
\end{aligned} \tag{A.41}$$

The first order conditions of (B.118) with respect to $m_{1H}$ and $m_{2H}$ are

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\alpha_H\left(V - m_{2H}\right) - m_{1H}\right)\right] - \lambda_2 \alpha_H = 0 \tag{A.42}$$

$$(1-\lambda)\left[\frac{\alpha_H^3}{2k}\left(\alpha_H\left(V - m_{2H}\right) - m_{1H}\right)\right] + \lambda_1 - \lambda_2 \alpha_H^2 = 0 \tag{A.43}$$

Multiplying (B.119) by $\alpha_H$ and subtracting the product from (B.120) gives $\lambda_1 = 0$.

When $\lambda_2 > 0$, $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} = 2k$, namely, $\alpha_H m_{2H} + m_{1H} = \frac{2k}{\alpha_H} + k$. Then the principal's expected profit becomes

$$\alpha_H\left(\alpha_H V - \frac{2k}{\alpha_H} - k\right) + k = \alpha_H^2 V - \alpha_H k - k \tag{A.44}$$

When $\lambda_2 = 0$, (B.119) implies that $\alpha_H(V - m_{2H}) - m_{1H} = 0$, which means that $\alpha_H m_{2H} + m_{1H} = \alpha_H V$. The principal's expected profit equals

$$(1-\lambda)k\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \tag{A.45}$$

which is greater than or equal to $(1-\lambda)\left(\alpha_H^2 V - \alpha_H k - k\right)$, with the equality holding when $\alpha_H^2 V - \alpha_H k = 2k$. Notice that the constraint $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \leq 2k$ implies that $\alpha_H^2 V - \alpha_H k \leq 2k$, because $\alpha_H m_{2H} + m_{1H} = \alpha_H V$. Therefore, when $\alpha_H^2 V - \alpha_H k \leq 2k$,

$$(1-\lambda)k\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \geq (1-\lambda)\left(\alpha_H^2 V - \alpha_H k - k\right) \tag{A.46}$$

where the equality holds only when $\alpha_H^2 V - \alpha_H k = 2k$.

To show the expected profit obtained above is local maximal, we notice that with $0 \leq \frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} \leq 1$, the expression of the principal's expected profit in (B.118) as a function of $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}$ is bounded in absolute value, and thereby has a maximum. Thus the unique solution of the first order conditions must be the location of the local maximal expected profit.

4. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \geq 2k$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* = 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$\begin{aligned}&(1-\lambda)\left[\alpha_H\left(\alpha_H(V-m_{2H})-m_{1H}\right)+\left(-k+\alpha_H m_{1H}-\alpha_H k+\alpha_H^2 m_{2H}\right)\right]\\&+\lambda_1\left(m_{2H}-\frac{2k}{\alpha_H}\right)+\lambda_2\left(\alpha_H m_{1H}-\alpha_H k+\alpha_H^2 m_{2H}-2k\right)\end{aligned} \tag{A.47}$$

It is easy to see that the first order conditions of (B.124) give $\lambda_1 = 0$ and $\lambda_2 = 0$. Notice that the principal's expected profit equals

$$\begin{aligned}&(1-\lambda)\left[\alpha_H\left(\alpha_H(V-m_{2H})-m_{1H}\right)+\left(-k+\alpha_H m_{1H}-\alpha_H k+\alpha_H^2 m_{2H}\right)\right]\\=\;&(1-\lambda)\left(\alpha_H^2 V-\alpha_H k-k\right)\end{aligned} \tag{A.48}$$

The constancy of the principal's expected profit implies that it is also local maximum.

In summary, the optimal money transfers $(m_{0H}^*, m_{1H}^*, m_{2H}^*)$ offered to the high-type agent and the principal's expected profit satisfies:

1. When $V \leq \frac{2k}{\alpha_H}$, $m_{2H}^* = V$, $m_{1H}^* = 0$,

$$m_{0H}^* = -\frac{\alpha_H^6 V^4}{64k^3} \tag{A.49}$$

and the principal's expected profit equals

$$(1-\lambda)\frac{\alpha_H^6 V^4}{64k^3} \tag{A.50}$$

2. When $V \geq \frac{2k}{\alpha_H}$ and $\alpha_H^2 V - \alpha_H k \leq 2k$, $m_{2H}^* \geq \frac{2k}{\alpha_H}$, $\alpha_H m_{2H}^* + m_{1H}^* = \alpha_H V$.

$$m_{0H}^* = -k\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \tag{A.51}$$

and the principal's expected profit equals

$$(1-\lambda)k\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \tag{A.52}$$

3. When $V \geq \frac{2k}{\alpha_H}$ and $\alpha_H^2 V - \alpha_H k \geq 2k$, $m_{2H}^* \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H}^* - \alpha_H k + \alpha_H^2 m_{2H}^* \geq 2k$.

$$m_{0H}^* = -k + \alpha_H m_{1H}^* - \alpha_H k + \alpha_H^2 m_{2H}^* \geq k \tag{A.53}$$

and the principal's expected profit equals

$$(1-\lambda)(\alpha_H^2 V - \alpha_H k - k) \tag{A.54}$$

Similar argument can apply to the principal's expected profit maximization problem consisting of (B.87) and (B.88). The optimal money transfers $(m_{0L}^*, m_{1L}^*, m_{2L}^*)$ offered to the low-type agent and the principal's expected profit satisfy

1. When $V \le \frac{2k}{\alpha_L}$, $m_{2L}^* = V$, $m_{1L}^* = 0$,

$$m_{0L}^* = -\frac{\alpha_L^6 V^4}{64k^3} \tag{A.55}$$

and the principal's expected profit equals

$$\lambda \frac{\alpha_L^6 V^4}{64k^3} \tag{A.56}$$

2. When $V \ge \frac{2k}{\alpha_L}$ and $\alpha_L^2 V - \alpha_L k \le 2k$, $m_{2L}^* \ge \frac{2k}{\alpha_L}$, $\alpha_L m_{2L}^* + m_{1L}^* = \alpha_L V$.

$$m_{0L}^* = -k\left(\frac{\alpha_L^2 V - \alpha_L k}{2k}\right)^2 \tag{A.57}$$

and the principal's expected profit equals

$$\lambda k\left(\frac{\alpha_L^2 V - \alpha_L k}{2k}\right)^2 \tag{A.58}$$

3. When $V \ge \frac{2k}{\alpha_L}$ and $\alpha_L^2 V - \alpha_L k \ge 2k$, $m_{2L}^* \ge \frac{2k}{\alpha_L}$ and $\alpha_L m_{1L}^* - \alpha_L k + \alpha_L^2 m_{2L}^* \ge 2k$.

$$m_{0L}^* = -k + \alpha_L m_{1L}^* - \alpha_L k + \alpha_L^2 m_{2L}^* \ge k \tag{A.59}$$

and the principal's expected profit equals

$$\lambda(\alpha_L^2 V - \alpha_L k - k) \tag{A.60}$$

Notice that when $V \le \frac{2k}{\alpha_H}$, which implies $V < \frac{2k}{\alpha_L}$, the principal's expected profit from both the low-type and high-type agents is

$$\lambda \frac{\alpha_L^6 V^4}{64k^3} + (1-\lambda)\frac{\alpha_H^6 V^4}{64k^3} \tag{A.61}$$

This concludes the proof of Theorem 1.

## A.2 Proof of Theorem 2

The proof consists of three parts. In the first part, for given menu of money transfers $(m_{0L}, m_{1L}, m_{2L})$ and $(m_{0H}, m_{1H}, m_{2H})$, we establish the expressions of optimal efforts of both types of agents and the corresponding expressions of expected profits. In the second part, we solve the principal's expected profit maximization problem for a particular region in which the money transfers take values, using the technique of decomposing the problem into two independent problems with each of them only associated to one type of agent. In the third part, we show that when $V \le \frac{2k}{\alpha_H}$, the local maximum obtained in the second part is the global maximum by ruling out the possible local maxima in other regions.

### A.2.1 Expressions of optimal efforts

The principal maximizes her following expected profit by offering $(m_{0L}, m_{1L}, m_{2L})$ to the low-type agent and $(m_{0H}, m_{1H}, m_{2H})$ to the high-type agent

$$\lambda\left[\alpha_L e_{1L}\left(\alpha_L e_{2L}(V - m_{2L}) - m_{1L}\right) - m_{0L}\right] + (1-\lambda)\left[\alpha_H e_{1H}\left(\alpha_H e_{2H}(V - m_{2H}) - m_{1H}\right) - m_{0H}\right] \tag{A.62}$$

For given $(m_{0L}, m_{1L}, m_{2L})$ and $(m_{0H}, m_{1H}, m_{2H})$, the low-type and high-type agents maximize their following expected profits, respectively:

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \tag{A.63}$$

and

$$m_{0H} - ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \tag{A.64}$$

where $0 \le e_{1L} \le 1$, $0 \le e_{2L} \le 1$, $0 \le e_{1H} \le 1$ and $0 \le e_{2H} \le 1$.

To ensure both agents' participation and prevent each agent from mimicking the other, the following participation constraints and incentive compatibility constraints have to be satisfied:

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \ge 0 \tag{A.65}$$

$$m_{0H} - ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \ge 0 \tag{A.66}$$

$$\begin{aligned} & m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \\ \ge\ & m_{0H} - k\tilde{e}_{1L}^2 + \alpha_L \tilde{e}_{1L} m_{1H} - \alpha_L \tilde{e}_{1L} k \tilde{e}_{2L}^2 + \alpha_L^2 \tilde{e}_{1L} \tilde{e}_{2L} m_{2H} \end{aligned} \tag{A.67}$$

$$\begin{aligned} & m_{0H} - ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \\ \ge\ & m_{0L} - k\tilde{e}_{1H}^2 + \alpha_H \tilde{e}_{1H} m_{1L} - \alpha_H \tilde{e}_{1H} k \tilde{e}_{2H}^2 + \alpha_H^2 \tilde{e}_{1H} \tilde{e}_{2H} m_{2L} \end{aligned} \tag{A.68}$$

where (A.65) and (A.66) are the low-type and high-type agents' participation constraints, and (A.67) and (A.68) are the low-type and high-type agents' incentive compatibility constraints. Notice $\tilde{e}_{1L}$ and $\tilde{e}_{2L}$ are the efforts when the low-type agent pretends to be the high one, while $\tilde{e}_{1H}$ and $\tilde{e}_{2H}$ are the efforts when the high-type agent pretends to be the low-type one.

In fact, we can assume that $e_{1L}$, $e_{2L}$, $e_{1H}$ and $e_{2H}$ are all positive and don't need to consider the scenarios when some of them are 0. The reason is the same as that in the proof of Theorem 1.

Therefore the Lagrangian for the maximization problem of the low-type agent's expected profit with $e_{1L}$ and $e_{2L}$ as the decision variables is

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} + \lambda_1(1 - e_{1L}) + \lambda_2(1 - e_{2L}) \tag{A.69}$$

where $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ are Lagrangian multipliers.

Let $e^*_{1L}$, $e^*_{2L}$, $e^*_{1H}$, and $e^*_{2H}$ denote the optimal efforts of the low-type and high-type agents. Using the same argument in the proof of Theorem 1, we have four situations:

1. When $\lambda_1 > 0$ and $\lambda_2 > 0$, $e^*_{1L} = 1$ and $e^*_{2L} = 1$. The low-type agent's expected profit is

$$m_{0L} - k + \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \tag{A.70}$$

   with $\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} > 1$ and $\frac{\alpha_L m_{2L}}{2k} > 1$.

2. When $\lambda_1 > 0$ and $\lambda_2 = 0$, $e^*_{1L} = 1$ and $e^*_{2L} = \frac{\alpha_L m_{2L}}{2k} \leq 1$. The low-type agent's expected profit is

$$m_{0L} - k + \alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k} \tag{A.71}$$

   with $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} > 1$.

3. When $\lambda_1 = 0$ and $\lambda_2 > 0$, $e^*_{1L} = \frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} \leq 1$ and $e^*_{2L} = 1$. The low-type agent's expected profit is

$$m_{0L} + k\left(\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2 \tag{A.72}$$

   with $\frac{\alpha_L m_{2L}}{2k} > 1$.

4. When $\lambda_1 = 0$ and $\lambda_2 = 0$, $e^*_{2L} = \frac{\alpha_L m_{2L}}{2k} \leq 1$ and

$$e^*_{1L} = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \tag{A.73}$$

   which is less than or equal to 1. The low-type agent's expected profit is

$$m_{0L} + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \tag{A.74}$$

Similarly, for the high-type agent, we have $e^*_{1H} > 0$ and $e^*_{2H} > 0$ and there are four situations:

1. $e^*_{1H} = 1$ and $e^*_{2H} = 1$. The high-type agent's expected profit is

$$m_{0H} - k + \alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \tag{A.75}$$

   and $\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} > 1$ as well as $\frac{\alpha_H m_{2H}}{2k} > 1$.

2. $e^*_{1H} = 1$ and $e^*_{2H} = \frac{\alpha_H m_{2H}}{2k} \leq 1$. The high-type agent's expected profit is

$$m_{0H} - k + \alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k} \tag{A.76}$$

   and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} > 1$.

3. $e^*_{1H} = \frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} \le 1$ and $e^*_{2H} = 1$. The high-type agent's expected profit is

$$m_{0H} + k\left(\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)^2 \tag{A.77}$$

and $\frac{\alpha_H m_{2H}}{2k} > 1$.

4. $e^*_{1H} = \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \le 1$ and $e^*_{2H} = \frac{\alpha_H m_{2H}}{2k} \le 1$. The high-type agent's expected profit is

$$m_{0H} + k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 \tag{A.78}$$

In addition, because of incentive constraints involved in the model, we have to consider two scenarios where each type agent is not honest about his true type: one is that the low-type agent pretends to be the high-type agent, and the other is that the high-agent pretends to be the low-type agent.

When the low-type agent pretends to be the high-type agent, if successful, his expected profit would be

$$m_{0H} - k\tilde{e}_{1L}^2 + \alpha_L \tilde{e}_{1L} m_{1H} - \alpha_L \tilde{e}_{1L} k \tilde{e}_{2L}^2 + \alpha_L^2 \tilde{e}_{1L} \tilde{e}_{2L} m_{2H} \tag{A.79}$$

with $\lambda\left[\alpha_L \tilde{e}_{1L}\left(\alpha_L \tilde{e}_{2L}(V - m_{2H}) - m_{1H}\right) - m_{0H}\right]$ as the corresponding expected profit for the principal. Thus by similar argument we did above, $\tilde{e}^*_{1L} > 0$ and $\tilde{e}^*_{2L} > 0$ and there are four situations:

1. $\tilde{e}^*_{1L} = 1$ and $\tilde{e}^*_{2L} = 1$. The high-type agent's expected profit is

$$m_{0H} - k + \alpha_L m_{1H} - \alpha_L k + \alpha_L^2 m_{2H} \tag{A.80}$$

and $\frac{\alpha_L m_{1H} - \alpha_L k + \alpha_L^2 m_{2H}}{2k} > 1$ as well as $\frac{\alpha_L m_{2H}}{2k} > 1$.

2. $\tilde{e}^*_{1L} = 1$ and $\tilde{e}^*_{2L} = \frac{\alpha_L m_{2H}}{2k} \le 1$. The high-type agent's expected profit is

$$m_{0H} - k + \alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k} \tag{A.81}$$

and $\frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k} > 1$.

3. $\tilde{e}^*_{1L} = \frac{\alpha_L m_{1H} - \alpha_L k + \alpha_L^2 m_{2H}}{2k} \le 1$ and $\tilde{e}^*_{2L} = 1$. The high-type agent's expected profit is

$$m_{0H} + k\left(\frac{\alpha_L m_{1H} - \alpha_L k + \alpha_L^2 m_{2H}}{2k}\right)^2 \tag{A.82}$$

and $\frac{\alpha_L m_{2H}}{2k} > 1$.

4. $\tilde{e}^*_{1L} = \frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k} \leq 1$ and $\tilde{e}^*_{2L} = \frac{\alpha_L m_{2H}}{2k} \leq 1$. The high-type agent's expected profit is

$$m_{0H} + k\left(\frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k}\right)^2 \tag{A.83}$$

When the high-type agent pretends to be the low-type agent, if successful, his expected profit would be

$$m_{0L} - k\tilde{e}_{1H}^2 + \alpha_H \tilde{e}_{1H} m_{1L} - \alpha_H \tilde{e}_{1H} k \tilde{e}_{2H}^2 + \alpha_H^2 \tilde{e}_{1H} \tilde{e}_{2H} m_{2L} \tag{A.84}$$

with $\lambda\left[\alpha_H \tilde{e}_{1H}\left(\alpha_H \tilde{e}_{2H}(V - m_{2L}) - m_{1L}\right) - m_{0L}\right]$ as the corresponding expected profit for the principal. Thus by similar argument we did above, $\tilde{e}^*_{1H} > 0$ and $\tilde{e}^*_{2H} > 0$ and there are four situations:

1. $\tilde{e}^*_{1H} = 1$ and $\tilde{e}^*_{2H} = 1$. The high-type agent's expected profit is

$$m_{0L} - k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \tag{A.85}$$

   and $\frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k} > 1$ as well as $\frac{\alpha_H m_{2L}}{2k} > 1$.

2. $\tilde{e}^*_{1H} = 1$ and $\tilde{e}^*_{2H} = \frac{\alpha_H m_{2L}}{2k} \leq 1$. The high-type agent's expected profit is

$$m_{0L} - k + \alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k} \tag{A.86}$$

   and $\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} > 1$.

3. $\tilde{e}^*_{1H} = \frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k} \leq 1$ and $\tilde{e}^*_{2H} = 1$. The high-type agent's expected profit is

$$m_{0L} + k\left(\frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2 \tag{A.87}$$

   and $\frac{\alpha_H m_{2L}}{2k} > 1$.

4. $\tilde{e}^*_{1H} = \frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \leq 1$ and $\tilde{e}^*_{2H} = \frac{\alpha_H m_{2L}}{2k} \leq 1$. The high-type agent's expected profit is

$$m_{0L} + k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2 \tag{A.88}$$

### A.2.2 The local maximum of a particular region

In the following we will solve the principal's expected profit maximization problem in a particular described below, considering that the principal wants to prevent the low-type agent from mimicking the high-type agent and the high-type agent from mimicking the low-type agent, meanwhile ensuring both types participation.

Besides the positivity of $e^*_{1H}$, $e^*_{2H}$, $\tilde{e}^*_{1H}$, $\tilde{e}^*_{2H}$, $e^*_{1L}$, $e^*_{2L}$, $\tilde{e}^*_{1L}$ and $\tilde{e}^*_{2L}$, we assume that $e^*_{1H} = \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}}{4k}}{2k} \leq 1$, $e^*_{2H} = \frac{\alpha_H m_{2H}}{2k} \leq 1$, $\tilde{e}^*_{1H} = \frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}}{4k}}{2k} \leq 1$, and $\tilde{e}^*_{2H} = \frac{\alpha_H m_{2L}}{2k} \leq 1$, which imply $\tilde{e}^*_{1L} = \frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}}{4k}}{2k} < 1$, $\tilde{e}^*_{2L} = \frac{\alpha_L m_{2H}}{2k} < 1$, $e^*_{1L} = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}}{4k}}{2k} < 1$, and $e^*_{2L} = \frac{\alpha_L m_{2L}}{2k} < 1$, because $\alpha_L < \alpha_H$. This describes a particular region where we will find the local interior maximum for the principal's expected profit maximization problem. In the next subsection we will show that this local interior maximal solution is also a unique global maximal solution when $V \leq \frac{2k}{\alpha_H}$.

For this region, to screen the two different types of agent and maximize her expected profit at the same time, the principal faces the following optimization problem:

$$\max_{\substack{(m_{0L}, m_{1L}, m_{2L}) \\ (m_{0H}, m_{1H}, m_{2H})}} \lambda \Pi_{L,P} + (1-\lambda)\Pi_{H,P} \tag{A.89}$$

where $\lambda \Pi_{L,P}$ and $(1-\lambda)\Pi_{H,P}$ are the expected profits obtained from the low-type and the high-type agents, respectively, satisfying

$$\Pi_{L,P} = \alpha_L \left[ \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right] \left[ \alpha_L \frac{\alpha_L m_{2L}}{2k} (V - m_{2L}) - m_{1L} \right] - m_{0L} \tag{A.90}$$

$$\Pi_{H,P} = \alpha_H \left[ \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right] \left[ \alpha_H \frac{\alpha_H m_{2H}}{2k} (V - m_{2H}) - m_{1H} \right] - m_{0H} \tag{A.91}$$

subject to

$$m_{0L} + k \left[ \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right]^2 \geq m_{0H} + k \left[ \frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k} \right]^2 \tag{A.92}$$

$$m_{0L} + k \left[ \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right]^2 \geq 0 \tag{A.93}$$

$$m_{0H} + k \left[ \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right]^2 \geq m_{0L} + k \left[ \frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \right]^2 \tag{A.94}$$

$$m_{0H} + k \left[ \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right]^2 \geq 0 \tag{A.95}$$

We claim that constraint (A.95) can be implied by constraints (A.93) and (A.94), i.e., (A.95) is redundant. To show this statement, first note that

$$\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} = \alpha_H \frac{m_{1L} + \frac{\alpha_H^2 m_{2L}^2}{4k}}{2k} > \alpha_L \frac{m_{1L} + \frac{\alpha_L^2 m_{2L}^2}{4k}}{2k} > 0 \tag{A.96}$$

because $0 < e_{2L}$ and $0 < e_{1L}$ ensure that $m_{2L} \neq 0$ and $\frac{m_{1L}+\frac{\alpha_L^2 m_{2L}^2}{4k}}{2k} > 0$. Using (A.96), we have

$$m_{0L} + k\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]^2 > m_{0L} + k\left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]^2 \tag{A.97}$$

It is clear that (A.94), (A.97) and (A.93) imply that

$$m_{0H} + k\left[\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right]^2 > 0 \tag{A.98}$$

which shows that constraint (A.95) is redundant.

Therefore the optimization problem consisting of (A.89), (A.92), (A.93), (A.94) and (A.95) is equivalent to the optimization problem consisting of (A.89), (A.92), (A.93), and (A.94), namely, without (A.95). In fact, constraint (A.92) is also redundant. To show this, we use the following approach: first show that the optimization problem consisting of (A.89), (A.93), and (A.94) can be solved, and then show that the solution set satisfies (A.92).

Now we look at the optimization problem consisting of (A.89), (A.93), and (A.94). The associated Lagrangian is

$$\begin{aligned}
&\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_L \frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right) - m_{0L}\right] \\
&+(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) - m_{0H}\right] \\
&+\lambda_1\left[m_{0L} + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2\right] \\
&+\lambda_2\left[m_{0H} + k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 - m_{0L} - k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2\right]
\end{aligned} \tag{A.99}$$

where $\lambda_1$ and $\lambda_2$ are the Lagrangian multipliers.

The first order conditions of (A.99) with respect to $m_{0L}$, $m_{1L}$, $m_{2L}$, $m_{0H}$, $m_{1H}$, and $m_{2H}$ are

$$-\lambda + \lambda_1 - \lambda_2 = 0 \tag{A.100}$$

$$\lambda\left[\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right) + \alpha_L\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)(-1)\right] + \lambda_1 \cdot 2k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\frac{\alpha_L}{2k} - \lambda_2 \cdot 2k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)\frac{\alpha_H}{2k} = 0 \tag{A.101}$$

$$\lambda\left[\frac{\alpha_L^4 \frac{2m_{2L}}{4k}}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right)\right] + \lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V - 2m_{2L})\right)\right] + \lambda_1 \cdot 2k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\frac{\alpha_L^3 \frac{2m_{2L}}{4k}}{2k} - \lambda_2 \cdot 2k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)\frac{\alpha_H^3 \frac{2m_{2L}}{4k}}{2k} = 0 \tag{A.102}$$

$$-(1-\lambda) + \lambda_2 = 0 \tag{A.103}$$

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) + \alpha_H\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)(-1)\right] + \lambda_2 \cdot 2k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\frac{\alpha_H}{2k} = 0 \tag{A.104}$$

$$(1-\lambda)\left[\frac{\alpha_H^4 \frac{2m_{2H}}{4k}}{2k}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right)\right] + (1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V - 2m_{2H})\right)\right] + \lambda_2 \cdot 2k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\frac{\alpha_H^3 \frac{2m_{2H}}{4k}}{2k} = 0 \tag{A.105}$$

From (A.100) and (A.103), we know that $\lambda_2 = 1 - \lambda > 0$, and $\lambda_1 = 1$, which means that constraints (A.93) and (A.94) are binding. Substituting them into (A.101), (A.102), (A.104) and (A.105), we have

$$\lambda\left[\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right]+\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)(-1)$$
$$+1\cdot 2k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\frac{\alpha_L}{2k}-(1-\lambda)\cdot 2k\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)\frac{\alpha_H}{2k} = 0 \tag{A.106}$$

$$\lambda\left[\frac{\alpha_L^4\frac{2m_{2L}}{4k}}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right]$$
$$+\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-2m_{2L})\right)\right]$$
$$+1\cdot 2k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\frac{\alpha_L^3\frac{2m_{2L}}{4k}}{2k}-(1-\lambda)\cdot 2k\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)\frac{\alpha_H^3\frac{2m_{2L}}{4k}}{2k} = 0 \tag{A.107}$$

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)+\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)(-1)\right]$$
$$+(1-\lambda)\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\alpha_H = 0 \tag{A.108}$$

$$(1-\lambda)\left[\frac{\alpha_H^4\frac{2m_{2H}}{4k}}{2k}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right]$$
$$+(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V-2m_{2H})\right)\right]$$
$$+(1-\lambda)\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\alpha_H^3\frac{2m_{2H}}{4k} = 0 \tag{A.109}$$

which can be simplified as

$$\lambda\left[\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right]$$
$$+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L-\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)\alpha_H\right] = 0 \tag{A.110}$$

$$\lambda\left[\frac{\alpha_L^4\frac{2m_{2L}}{4k}}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right]$$
$$+\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)\right]$$
$$+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L^3\frac{2m_{2L}}{4k}-\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)\alpha_H^3\frac{2m_{2L}}{4k}\right] = 0 \tag{A.111}$$

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right] = 0 \tag{A.112}$$

$$(1-\lambda)\left[\frac{\alpha_H^4\frac{2m_{2H}}{4k}}{2k}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right]$$
$$+(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V-m_{2H})\right)\right] = 0 \tag{A.113}$$

It is easy to see that (A.112) leads to

$$\left(\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)=0 \tag{A.114}$$

Substituting (A.114) into (A.113), we obtain

$$(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V-m_{2H})\right)\right]=0 \tag{A.115}$$

One thing we would like to point out is that the binding constraints (A.93) and (A.94) make the expression of the principal's expected profit in (A.89) become the sum of two functions, one on $(m_{1L}, m_{2L})$ and the other on $(m_{1H}, m_{2H})$. With the assumption that $0 \le e_{1H}^* = \frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}}{4k}}{2k} \le 1$, $0 \le e_{2H}^* = \frac{\alpha_H m_{2H}}{2k} \le 1$, $0 \le \tilde{e}_{1H}^* = \frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}}{4k}}{2k} \le 1$, and $0 \le \tilde{e}_{2H}^* = \frac{\alpha_H m_{2L}}{2k} \le 1$, variables $m_{1L}$, $m_{2L}$, $m_{1H}$ and $m_{2H}$ are bounded in absolute value. This means that the expression of the principal's expected profit in (A.89) is bounded and thereby has a local maximum. The uniqueness of the solution of the first order conditions of the Lagrangian associated with (A.89) would ensure that this solution is the location of the local maximum.

Let $(m^*_{0L}, m^*_{1L}, m^*_{2L})$ and $(m^*_{0H}, m^*_{1H}, m^*_{2H})$ be the location of the local maximum. Then (A.115) and (A.114) imply that $m^*_{2H} = V$ and $m^*_{1H} = 0$.

As for $m^*_{1L}$ and $m^*_{2L}$, we have to rely on (A.110) and (A.111). First, we examine some features of $m_{1L}$ and $m_{2L}$. Note that $0 < e^*_{1L} = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$ implies that

$$0 < \frac{m_{1L} + \frac{\alpha_L^2 m_{2L}^2}{4k}}{2k} < \frac{m_{1L} + \frac{\alpha_H^2 m_{2L}^2}{4k}}{2k} \tag{A.116}$$

which leads to

$$\alpha_L^2 \left( \frac{m_{1L} + \frac{\alpha_L^2 m_{2L}^2}{4k}}{2k} \right) - \alpha_H^2 \left( \frac{m_{1L} + \frac{\alpha_H^2 m_{2L}^2}{4k}}{2k} \right) < 0 \tag{A.117}$$

namely,

$$\left( \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right) \alpha_L - \left( \frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \right) \alpha_H < 0 \tag{A.118}$$

From (A.118) and (A.110), we can see that

$$\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) - m_{1L} > 0 \tag{A.119}$$

On the other hand, if we multiply (A.110) by $\alpha_L^2 \frac{2m_{2L}}{4k}$, then it becomes

$$\begin{aligned} &\lambda \left[ \frac{\alpha_L^4 \frac{2m_{2L}}{4k}}{2k} \left( \frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) - m_{1L} \right) \right] \\ &+(1-\lambda) \left[ \left( \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right) \alpha_L^3 \frac{2m_{2L}}{4k} - \left( \frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \right) \alpha_H \alpha_L^2 \frac{2m_{2L}}{4k} \right] = 0 \end{aligned} \tag{A.120}$$

Subtracting (A.120) from (A.111), we have

$$\lambda \left[ \alpha_L \left( \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right) \left( \frac{\alpha_L^2}{2k}(V - m_{2L}) \right) \right] + (1-\lambda) \left[ \left( \frac{m_{1L} + \frac{\alpha_H^2 m_{2L}^2}{4k}}{2k} \right) \left( \alpha_H^2 \alpha_L^2 - \alpha_H^4 \right) \frac{2m_{2L}}{4k} \right] = 0 \tag{A.121}$$

Using (A.116) and the fact that $\frac{m_{2L}}{4k} > 0$, which results from the positivity of $e^*_{2L} = \frac{\alpha_L m_{2L}}{4k}$, we obtain

$$V - m_{2L} > 0 \tag{A.122}$$

Combining this with the fact that $m_{2L} > 0$, we have

$$0 < m_{2L} < V \tag{A.123}$$

We will use (A.119) and (A.122) later to show that the redundancy of (A.92). Now we want to find the expression of $m_{1L}$ in terms of $m_{2L}$, when they satisfy both (A.110) and (A.111). Note that (A.110) can be written as

$$\lambda\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)+(1-\lambda)\left(\frac{\alpha_L^2-\alpha_H^2}{2k}\right)m_{1L}+(1-\lambda)\left(\frac{\alpha_L^4-\alpha_H^4}{8k^2}\right)m_{2L}^2=0 \tag{A.124}$$

With the terms of $m_{1L}$ being collected on one side of the equal sign and the ones of $m_{2L}$ on the other side, (A.124) becomes

$$\lambda\frac{\alpha_L^4}{4k^2}m_{2L}(V-m_{2L})+(1-\lambda)\left(\frac{\alpha_L^4-\alpha_H^4}{8k^2}\right)m_{2L}^2=\left(\lambda\frac{\alpha_L^2}{2k}+(1-\lambda)\left(\frac{\alpha_H^2-\alpha_L^2}{2k}\right)\right)m_{1L} \tag{A.125}$$

Therefore,

$$m_{1L}=\frac{\lambda\frac{\alpha_L^4}{4k^2}m_{2L}(V-m_{2L})+(1-\lambda)\left(\frac{\alpha_L^4-\alpha_H^4}{8k^2}\right)m_{2L}^2}{\lambda\frac{\alpha_L^2}{2k}+(1-\lambda)\left(\frac{\alpha_H^2-\alpha_L^2}{2k}\right)} \tag{A.126}$$

Substituting (A.126) into (A.121), we obtain an equation involving only unknown variable $m_{2L}$, which can be solved.

Now we temporarily leave the discussion of solving for $m_{1L}$ and $m_{2L}$, and turn to proving that the solution set to the optimization problem consisting of (A.89), (A.93), and (A.94), which we just discussed in the above, satisfies the constraint (A.92), namely, (A.92) is redundant.

Note that for the optimization problem consisting of (A.89), (A.93), and (A.94), we showed that both (A.93) and (A.94) are binding. This fact makes (A.92) equivalent to

$$k\left[\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right]^2-k\left[\frac{\alpha_L m_{1H}+\frac{\alpha_L^3 m_{2H}^2}{4k}}{2k}\right]^2\geq k\left[\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]^2-k\left[\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]^2 \tag{A.127}$$

Substituting $m_{2H}=V$ and $m_{1H}=0$ into (A.127), we have

$$\frac{\alpha_H^6 V^4}{64k^3}-\frac{\alpha_L^6 V^4}{64k^3}\geq k\left[\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]^2-k\left[\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]^2 \tag{A.128}$$

which suggests that if we can show that the maximum of the function on the right side is never bigger than the value on the left side, then we can prove (A.128).

Now we consider the following optimization problem:

$$\max_{m_{1L},m_{2L}} k\left[\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]^2-k\left[\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]^2 \tag{A.129}$$

subject to

$$\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) - m_{1L} \geq 0 \tag{A.130}$$

$$0 \leq m_{2L} \tag{A.131}$$

$$0 \leq V - m_{2L} \tag{A.132}$$

$$0 \leq \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \tag{A.133}$$

Note that when $m_{1L}$ and $m_{2L}$ satisfies (A.119) and (A.123), they also satisfy (A.130), (A.131) and (A.132). When $0 < e_{1L}^* = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$, then $e_{1L}^*$ also satisfies (A.133).

Let $\beta_1$, $\beta_2$, $\beta_3$ and $\beta_4$ be the Lagrangian multipliers of (A.130), (A.131), (A.132) and (A.133). The Lagrangian function for the optimization problem consisting of (A.129), (A.130), (A.131), (A.132) and (A.133) is

$$k\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]^2 - k\left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]^2 + \beta_1\left(\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right)$$
$$+\beta_2 m_{2L} + \beta_3(V - m_{2L}) + \beta_4\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right) \tag{A.134}$$

The first order conditions of the Lagrangian function (A.134) with respect to $m_{1L}$ and $m_{2L}$ are

$$k \cdot 2\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]\frac{\alpha_H}{2k} - k \cdot 2\left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]\frac{\alpha_L}{2k} - \beta_1 + \beta_4\frac{\alpha_L}{2k} = 0 \tag{A.135}$$

$$k \cdot 2\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]\frac{\alpha_H^3 \frac{2m_{2L}}{4k}}{2k} - k \cdot 2\left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]\frac{\alpha_L^3 \frac{2m_{2L}}{4k}}{2k}$$
$$+\beta_1\frac{\alpha_L^2}{2k}(V - 2m_{2L}) + \beta_2 - \beta_3 + \beta_4\frac{\alpha_L^3 m_{2L}}{4k} = 0 \tag{A.136}$$

If $m_{1L}$ and $m_{2L}$ as the solution of the optimal problem consisting of(A.129), (A.130), (A.131), (A.132) and (A.133) satisfy

$$\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) - m_{1L} > 0 \tag{A.137}$$

$$0 < m_{2L} \tag{A.138}$$

$$0 < V - m_{2L} \tag{A.139}$$

$$0 < \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \tag{A.140}$$

namely interior points, then $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ and (A.135) and (A.136) become

$$\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]\alpha_H - \left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]\alpha_L = 0 \quad (A.141)$$

$$\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]\alpha_H^3 \frac{2m_{2L}}{4k} - \left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]\alpha_L^3 \frac{2m_{2L}}{4k} = 0 \quad (A.142)$$

namely,

$$\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]\alpha_H = \left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]\alpha_L \quad (A.143)$$

$$\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right]\alpha_H^3 \frac{2m_{2L}}{4k} = \left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right]\alpha_L^3 \frac{2m_{2L}}{4k} \quad (A.144)$$

Dividing the both sides of (A.144) by the corresponding sides of (A.143), we have

$$\alpha_H^2 \frac{2m_{2L}}{4k} = \alpha_L^2 \frac{2m_{2L}}{4k} \quad (A.145)$$

Since $0 < m_{2L}$, (A.145) implies that $\alpha_H^2 = \alpha_L^2$, which can't happen. Therefore, the optimal solution $m_{1L}$ and $m_{2L}$ won't satisfy all of (A.137), (A.138) and (A.139). Therefore, they must be located at the boundaries. There are three possibilities:

1. $\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) - m_{1L} = 0$.

   Thus $m_{1L} = \frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L})$ and the objective function (A.129) becomes

$$k\frac{\alpha_H^2}{4k^2}\left[\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) + \alpha_H^2 \frac{m_{2L}^2}{4k}\right]^2 - k\frac{\alpha_L^2}{4k^2}\left[\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) + \alpha_L^2 \frac{m_{2L}^2}{4k}\right]^2 \quad (A.146)$$

   The derivative of (A.146) with respect to $m_{2L}$ is

$$\begin{aligned} & k\frac{\alpha_H^2}{4k^2}\left[\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) + \alpha_H^2 \frac{m_{2L}^2}{4k}\right] \cdot 2\left[\frac{\alpha_L^2}{2k}(V - 2m_{2L}) + \alpha_H^2 \frac{m_{2L}}{2k}\right] \\ - \; & k\frac{\alpha_L^2}{4k^2}\left[\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) + \alpha_H^2 \frac{m_{2L}^2}{4k}\right] \cdot 2\left[\frac{\alpha_L^2}{2k}(V - 2m_{2L}) + \alpha_L^2 \frac{m_{2L}}{2k}\right] \end{aligned} \quad (A.147)$$

   It is easy to see that when $0 \le m_{2L}$ and $0 \le (V - m_{2L})$, (A.147) equals 0 for $m_{2L} = 0$ and always greater than 0 for $0 < m_{2L} \le V$. When $m_{2L} = 0$, $m_{1L} = 0$. This leads to (A.129) being 0.

2. $m_{2L} = 0$. Thus the objective function (A.129) becomes

$$k\left[\frac{\alpha_H m_{1L}}{2k}\right]^2 - k\left[\frac{\alpha_L m_{1L}}{2k}\right]^2 = k\left[\frac{\alpha_H^2 - \alpha_L^2}{4k^2}\right]m_{1L}^2 \quad (A.148)$$

   Note that when $m_{2L} = 0$, (A.130) implies that $0 \le m_{1L}$, while (A.133) gives $m_{1L} \ge 0$. This means that $m_{1L} = 0$, which leads to (A.148) being 0. Since 0 obviously can't be the maximum, this scenario can't happen.

3. $m_{2L} = V$. It is easy to see that (A.130) implies that $m_{1L} \leq 0$, and (A.133) gives $m_{1L} \geq -\alpha_L^2 \frac{V^2}{4k}$. Under the condition of $m_{2L} = V$, the objective function (A.129) becomes

$$k\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 V^2}{4k}}{2k}\right]^2 - k\left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 V^2}{4k}}{2k}\right]^2 \tag{A.149}$$

The first order condition of (A.149) gives

$$k\left[\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 V^2}{4k}}{2k}\right] \cdot 2\frac{\alpha_H}{2k} - k\left[\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 V^2}{4k}}{2k}\right] \cdot 2\frac{\alpha_L}{2k} \tag{A.150}$$

namely,

$$\left(\alpha_H^2 - \alpha_L^2\right) m_{1L} + \left(\alpha_H^4 - \alpha_L^4\right) \frac{V^2}{4k} = 0 \tag{A.151}$$

which implies that $m_{1L} = -\left(\alpha_H^2 + \alpha_L^2\right) \frac{V^2}{4k}$. But this value does not satisfy $-\alpha_H^2 \frac{V^2}{4k} \leq m_{1L} \leq 0$. Therefore, the maximum of the objective function (A.149) must be attained at the endpoint $m_{1L} = 0$ or $m_{1L} = -\alpha_H^2 \frac{V^2}{4k}$.

When $m_{1L} = 0$, the objective function (A.149) achieves the value $\frac{\alpha_H^6 V^4}{64k^3} - \frac{\alpha_L^6 V^4}{64k^3}$.

When $m_{1L} = -\alpha_H^2 \frac{V^2}{4k}$, the objective function (A.149) has the value $\alpha_H^2 \frac{\left(\alpha_H^2 - \alpha_L^2\right)^2 V^4}{64k^3}$.

To find which value is bigger, we use the following result:

$$\begin{aligned}
& \frac{\alpha_H^6 V^4}{64k^3} - \frac{\alpha_L^6 V^4}{64k^3} - \alpha_H^2 \frac{\left(\alpha_H^2 - \alpha_L^2\right)^2 V^4}{64k^3} \\
= \; & \left[\left(\alpha_H^2 - \alpha_L^2\right)\left(\alpha_H^4 + \alpha_H^2\alpha_L^2 + \alpha_L^4\right)\right] \frac{V^4}{64k^3} - \left[\alpha_H^2\left(\alpha_H^2 - \alpha_L^2\right)^2\right] \frac{V^4}{64k^3} \\
= \; & \left[\alpha_H^4 + \alpha_H^2\alpha_L^2 + \alpha_L^4 - \alpha_H^2\left(\alpha_H^2 - \alpha_L^2\right)\right]\left(\alpha_H^2 - \alpha_L^2\right) \frac{V^4}{64k^3} \\
= \; & \left[2\alpha_H^2\alpha_L^2 + \alpha_L^4\right]\left(\alpha_H^2 - \alpha_L^2\right) \frac{V^4}{64k^3} \\
> \; & 0
\end{aligned} \tag{A.152}$$

This shows that the value of the objective function (A.149) is bigger when $m_{2L} = V$ and $m_{1L} = 0$.

4. $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 V^2}{4k}}{2k} = 0$. The objective function equals

$$k\left[\frac{\alpha_H\left(\alpha_H^2 - \alpha_L^2\right)\frac{m_{2L}^2}{4k}}{2k}\right]^2 \tag{A.153}$$

which has the maximum $\alpha_H^2 \frac{\left(\alpha_H^2 - \alpha_L^2\right)^2 V^4}{64k^3}$. As we showed above, this can't be the maximum of the objective function (A.129).

To summarize, we find that the maximum of the optimization problem consisting of (A.129), (A.130), (A.131), (A.132), and (A.133) is attained at $m_{2L} = V$ and $m_{1L} = 0$, with the value $\frac{\alpha_H^6 V^4}{64k^3} - \frac{\alpha_L^6 V^4}{64k^3}$. This shows that (A.128) holds for the solution set of the optimization problem consisting (A.89), (A.93) and (A.94). Since (A.128) is equivalent to (A.92) when (A.93) and (A.94) are binding, (A.92) holds for them the same solution set. This shows that (A.92) is redundant to the optimization problem consisting of (A.89), (A.93) and (A.94).

Now we return to the discussion of solving for $m_{1L}$ and $m_{2L}$ which satisfy both (A.110) and (A.111). As shown in (A.126), we already found the expression of $m_{1L}$ in term of $m_{2L}$. One thing worthy of mentioning is that we can easily show that $m_{1L} \neq 0$. The proof goes as follows.

Suppose $m_{1L} = 0$, then (A.110) and (A.111) become

$$\lambda\left[\frac{\alpha_L^4}{4k} m_{2L}(V - m_{2L})\right] = (1-\lambda)\left[\frac{\alpha_H^4 m_{2L}^2}{8k^2} - \frac{\alpha_L^4 m_{2L}^2}{8k^2}\right] \tag{A.154}$$

$$\lambda\left[\frac{\alpha_L^6 m_{2L}^2}{8k^3}\frac{3}{2}(V - m_{2L})\right] = (1-\lambda)\frac{\left(\alpha_H^6 - \alpha_L^6\right) m_{2L}^3}{16k^3} \tag{A.155}$$

Dividing both sides of (A.155) by the corresponding sides of (A.154), we obtain

$$\frac{\alpha_L^2 m_{2L}}{2k^2}\frac{3}{2} = \frac{\left(\alpha_H^6 - \alpha_L^6\right) m_{2L}}{2k^2\left(\alpha_H^4 - \alpha_L^4\right)} \tag{A.156}$$

Using the fact that $m_{2L} > 0$, (A.156) can be simplified to

$$\alpha_L^2 \frac{3}{2} = \frac{\alpha_H^6 - \alpha_L^6}{\alpha_H^4 - \alpha_L^4} \tag{A.157}$$

which clearly is not true. This results in a contradiction. Therefore $m_{1L} \neq 0$.

Next we will find the value of $m_{2L}$, using the expression of $m_{1L}$ in terms of $m_{2L}$.

For convenience, we introduce a new notation $\rho$ and define it as $\frac{1-\lambda}{\lambda}$. Thus, (A.126) can be written as

$$m_{1L} = \frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{8k^2}\right) m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)} \tag{A.158}$$

In addition, (A.121) can be written as

$$\left[\alpha_L\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V - m_{2L})\right)\right] + \rho\left[\left(\frac{m_{1L} + \frac{\alpha_H^2 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_H^2\alpha_L^2 - \alpha_H^4\right)\frac{2m_{2L}}{4k}\right] = 0 \tag{A.159}$$

Using (A.158), we have

$$\begin{aligned}
&\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \\
= \quad & \frac{\alpha_L}{2k}\left[m_{1L} + \alpha_L^2 \frac{m_{2L}^2}{4k}\right] \\
= \quad & \frac{\alpha_L}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{8k^2}\right) m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)} + \alpha_L^2 \frac{m_{2L}^2}{4k}\right] \\
= \quad & \frac{\alpha_L}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^4}{8k^2} m_{2L}^2 + \rho \frac{\alpha_L^2(\alpha_H^2 - \alpha_L^2)}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right] \\
= \quad & \frac{\alpha_L}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_H^2 \alpha_L^2 - \alpha_H^4}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^4}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right], \qquad \text{(A.160)}
\end{aligned}$$

$$\begin{aligned}
&\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \\
= \quad & \frac{\alpha_H}{2k}\left[m_{1L} + \alpha_H^2 \frac{m_{2H}^2}{4k}\right] \\
= \quad & \frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{8k^2}\right) m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)} + \alpha_H^2 \frac{m_{2L}^2}{4k}\right] \\
= \quad & \frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^2 \alpha_H^2}{8k^2} m_{2L}^2 + \rho \frac{\alpha_H^2(\alpha_H^2 - \alpha_L^2)}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right] \\
= \quad & \frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^2 \alpha_L^2}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^2 \alpha_H^2}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right], \qquad \text{(A.161)}
\end{aligned}$$

and

$$
\begin{aligned}
& \alpha_L \left[ \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right] \left( \frac{\alpha_L^2}{2k} (V - m_{2L}) \right) \\
= & \frac{\alpha_L^4}{4k^2} (V - m_{2L}) \left[ m_{1L} + \alpha_L^2 \frac{m_{2L}^2}{4k} \right] \\
= & \frac{\alpha_L^4}{4k^2} (V - m_{2L}) \left[ \frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho \left( \frac{\alpha_H^2 \alpha_L^2 - \alpha_H^4}{8k^2} \right) m_{2L}^2 + \frac{\alpha_L^4}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho \left( \frac{\alpha_H^2 - \alpha_L^2}{2k} \right)} \right] \\
= & \frac{m_{2L}}{4k^2} \left[ \frac{\frac{\alpha_L^8}{4k^2} (V - m_{2L})^2 + \rho \left( \frac{\alpha_H^2 \alpha_L^6 - \alpha_L^4 \alpha_H^4}{8k^2} \right) m_{2L}(V - m_{2L}) + \frac{\alpha_L^8}{8k^2} m_{2L}(V - m_{2L})}{\frac{\alpha_L^2}{2k} + \rho \left( \frac{\alpha_H^2 - \alpha_L^2}{2k} \right)} \right]
\end{aligned}
\tag{A.162}
$$

Therefore, by (A.161) and (A.162), we can write (A.159) as

$$
\begin{aligned}
& \frac{m_{2L}}{4k^2} \left[ \frac{\frac{\alpha_L^8}{4k^2} (V - m_{2L})^2 + \rho \left( \frac{\alpha_H^2 \alpha_L^6 - \alpha_L^4 \alpha_H^4}{8k^2} \right) m_{2L}(V - m_{2L}) + \frac{\alpha_L^8}{8k^2} m_{2L}(V - m_{2L})}{\frac{\alpha_L^2}{2k} + \rho \left( \frac{\alpha_H^2 - \alpha_L^2}{2k} \right)} \right] \\
= & \rho \frac{m_{2L}}{4k^2} \left( \alpha_H^4 - \alpha_H^2 \alpha_L^2 \right) \left[ \frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho \left( \frac{\alpha_L^4 - \alpha_H^2 \alpha_L^2}{8k^2} \right) m_{2L}^2 + \frac{\alpha_L^2 \alpha_H^2}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho \left( \frac{\alpha_H^2 - \alpha_L^2}{2k} \right)} \right]
\end{aligned}
\tag{A.163}
$$

Note that $m_{2L} > 0$, because $e_{2L}^* = \frac{\alpha_L m_{2L}}{2k} > 0$. We can cancel a pair of $m_{2L}$ from both sides of (A.163). Thus (A.163) is equivalent to

$$
\begin{aligned}
& 2\alpha_L^8 (V - m_{2L})^2 + \rho \left( \alpha_H^2 \alpha_L^6 - \alpha_L^4 \alpha_H^4 \right) m_{2L}(V - m_{2L}) + \alpha_L^8 m_{2L}(V - m_{2L}) \\
= & \rho \left( \alpha_H^4 - \alpha_H^2 \alpha_L^2 \right) \left[ 2\alpha_L^4 m_{2L}(V - m_{2L}) + \rho \left( \alpha_L^4 - \alpha_H^2 \alpha_L^2 \right) m_{2L}^2 + \alpha_L^2 \alpha_H^2 m_{2L}^2 \right]
\end{aligned}
\tag{A.164}
$$

which is equivalent to the following quadratic equation in $m_{2L}$:

$$
A m_{2L}^2 + B m_{2L} + C = 0 \tag{A.165}
$$

with

$$
\begin{aligned}
A = & \; 2\alpha_L^8 - \rho \left( \alpha_H^2 \alpha_L^6 - \alpha_L^4 \alpha_H^4 \right) - \alpha_L^8 \\
& + 2\rho \alpha_L^4 \left( \alpha_H^4 - \alpha_H^2 \alpha_L^2 \right) - \rho^2 \left( \alpha_H^4 - \alpha_H^2 \alpha_L^2 \right) \left( \alpha_L^4 - \alpha_H^2 \alpha_L^2 \right) - \rho \alpha_L^2 \alpha_H^2 \left( \alpha_H^4 - \alpha_H^2 \alpha_L^2 \right) \\
= & \; \alpha_L^8 + 3\rho \left( \alpha_L^4 \alpha_H^4 - \alpha_L^6 \alpha_H^2 \right) - \rho^2 \left( 2\alpha_H^4 \alpha_L^4 - \alpha_H^6 \alpha_L^2 - \alpha_L^6 \alpha_H^2 \right) - \rho \left( \alpha_H^6 \alpha_L^2 - \alpha_L^4 \alpha_H^4 \right)
\end{aligned}
\tag{A.166}
$$

$$
\begin{aligned}
B = & \left[ -4\alpha_L^8 + \rho \left( \alpha_H^2 \alpha_L^6 - \alpha_L^4 \alpha_H^4 \right) + \alpha_L^8 - 2\rho \left( \alpha_H^4 - \alpha_H^2 \alpha_L^2 \right) \alpha_L^4 \right] V \\
= & \left[ 3\rho \left( \alpha_H^2 \alpha_L^6 - \alpha_L^4 \alpha_H^4 \right) - 3\alpha_L^8 \right] V
\end{aligned}
\tag{A.167}
$$

$$
C = 2\alpha_L^8 V^2 \tag{A.168}
$$

For convenience, we introduce a new notation $X$ to denote $\frac{\alpha_H}{\alpha_L}$, then (A.165) is equivalent to the quadratic equation

$$\tilde{A}m_{2L}^2 + \tilde{B}m_{2L} + \tilde{C} = 0 \tag{A.169}$$

with

$$\tilde{A} = 1 + 3\rho\left(X^4 - X^2\right) - \rho^2\left(2X^4 - X^6 - X^2\right) - \rho\left(X^6 - X^4\right) \tag{A.170}$$

$$\tilde{B} = \left[3\rho\left(X^2 - X^4\right) - 3\right]V \tag{A.171}$$

$$\tilde{C} = 2V^2 \tag{A.172}$$

The expression of the determinant

$$\begin{aligned}
& \tilde{B}^2 - 4\tilde{A}\tilde{C} \\
= & \left[3\rho\left(X^2 - X^4\right) - 3\right]^2 V^2 \\
& -4\left[1 + 3\rho\left(X^4 - X^2\right) - \rho^2\left(2X^4 - X^6 - X^2\right) - \rho\left(X^6 - X^4\right)\right] \cdot 2V^2 \\
= & \left[9\rho^2(X^2 - X^4)^2 - 18\rho\left(X^2 - X^4\right) + 9\right]V^2 \\
& -\left[8 + 24\rho\left(X^4 - X^2\right) - 8\rho^2\left(2X^4 - X^6 - X^2\right) - 8\rho\left(X^6 - X^4\right)\right]V^2 \\
= & \left[9\rho^2\left(X^4 - 2X^6 + X^8\right) - 18\rho\left(X^2 - X^4\right) + 9\right]V^2 \\
& -\left[8 + 24\rho\left(X^4 - X^2\right) - 8\rho^2\left(2X^4 - X^6 - X^2\right) - 8\rho\left(X^6 - X^4\right)\right]V^2 \\
= & 1 \cdot V^2 + \rho^2\left[25X^4 - 26X^6 + 9X^8 - 8X^2\right]V^2 + \rho\left[-14X^4 + 6X^2 + 8X^6\right]V^2
\end{aligned} \tag{A.173}$$

Note that

$$\begin{aligned}
& 25X^4 - 26X^6 + 9X^8 - 8X^2 \\
= & 25X^4 - 25X^6 - X^6 + X^8 + 8X^8 - 8X^2 \\
= & 25X^4\left(1 - X^2\right) + X^6\left(X^2 - 1\right) + 8X^2\left(X^6 - 1\right) \\
= & \left[-25X^4 + X^6\right]\left(X^2 - 1\right) + 8X^2\left(X^4 + X^2 + 1\right)\left(X^2 - 1\right) \\
= & \left[-25X^4 + X^6 + 8X^6 + 8X^4 + 8X^2\right]\left(X^2 - 1\right) \\
= & \left[9X^6 - 17X^4 + 8X^2\right]\left(X^2 - 1\right) \\
= & \left[9X^4 - 17X^2 + 8\right]X^2\left(X^2 - 1\right) \\
= & \left[9X^4 - 9X^2 - 8X^2 + 8\right]X^2\left(X^2 - 1\right) \\
= & \left[9X^2\left(X^2 - 1\right) - 8\left(X^2 - 1\right)\right]X^2\left(X^2 - 1\right) \\
= & \left[9X^2 - 8\right]\left(X^2 - 1\right)X^2\left(X^2 - 1\right)
\end{aligned} \tag{A.174}$$

which is greater than 0, because $X = \frac{\alpha_H}{\alpha_L} > 1$.

In addition

$$\begin{aligned} & -14X^4 + 6X^2 + 8X^6 \\ = & -6X^4 + 6X^2 - 8X^4 + 8X^6 \\ = & 6X^2\left(-X^2 + 1\right) + 8X^4\left(X^2 - 1\right) \\ = & \left(8X^4 - 6X^2\right)\left(X^2 - 1\right) \\ = & \left(8X^2 - 6\right)X^2\left(X^2 - 1\right) \end{aligned} \tag{A.175}$$

which is also greater than 0, because $X = \frac{\alpha_H}{\alpha_L} > 1$.

Therefore the determinant $\tilde{B}^2 - 4\tilde{A}\tilde{C}$ is always greater than 0. This means that when $\tilde{A} \neq 0$, there always exist two real number roots for equation (A.169). The expressions of these two roots are

$$m_{2L}^* = \frac{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}}{2\tilde{A}} \tag{A.176}$$

and

$$m_{2L}^* = \frac{-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}}{2\tilde{A}} \tag{A.177}$$

It is clear that (A.176) is equivalent to

$$\begin{aligned} m_{2L}^* &= \frac{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}}{2\tilde{A}} \cdot \frac{-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}}{-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} \\ &= \frac{\left(-\tilde{B}\right)^2 - \left(\tilde{B}^2 - 4\tilde{A}\tilde{C}\right)}{2\tilde{A}\left(-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}\right)} \\ &= \frac{4\tilde{A}\tilde{C}}{2\tilde{A}\left(-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}\right)} \\ &= \frac{2\tilde{C}}{-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} \\ &= \frac{4V^2}{-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} \end{aligned} \tag{A.178}$$

Similarly, (A.177) is equivalent to

$$\begin{aligned} m_{2L}^* &= \frac{-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}}{2\tilde{A}} \cdot \frac{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}}{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} \\ &= \frac{2\tilde{C}}{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} \\ &= \frac{4V^2}{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} \end{aligned} \tag{A.179}$$

Note that if $\tilde{A} < 0$, then $-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}} < 0$, as $C > 0$. Thus (A.178) is less than zero, which means that such $m_{2L}^*$ does not satisfy $0 < m_{2L} < V$, i.e., (A.123). On the other hand, if $\tilde{A} > 0$, then $-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}} > 0$.

In the following, we will show that when (A.110) holds and $\tilde{A} > 0$, (A.178) does not satisfy (A.158) and $0 < e_{1L}^* = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$ at the same time.

The proof goes as follows.

From (A.160), we know that

$$\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} = \frac{\alpha_L}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_H^2\alpha_L^2 - \alpha_H^4}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^4}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right] \tag{A.180}$$

which means that $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} > 0$ is equivalent to

$$\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_H^2\alpha_L^2 - \alpha_H^4}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^4}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)} > 0 \tag{A.181}$$

Multiplying (A.181) by $\frac{8k^2}{\alpha_L^4}\left(\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)\right)$ and using the notation $X$ for $\frac{\alpha_H}{\alpha_L}$ , we have

$$2m_{2L}\left(V - m_{2L}\right) + \rho\left(X^2 - X^4\right) m_{2L}^2 + m_{2L}^2 > 0 \tag{A.182}$$

Dividing both sides of (A.182) by $m_{2L}$ which is $> 0$ and rearranging terms leads to

$$2V > \left(1 + \rho\left(X^4 - X^2\right)\right) m_{2L} \tag{A.183}$$

namely,

$$m_{2L} < \frac{2V}{1 + \rho\left(X^4 - X^2\right)} \tag{A.184}$$

On the other hand, (A.174) gives

$$\begin{aligned} & 25X^4 - 26X^6 + 9X^8 - 8X^2 \\ = \; & \left[9X^2 - 8\right]\left(X^2 - 1\right) X^2 \left(X^2 - 1\right) \\ > \; & X^2\left(X^2 - 1\right) X^2 \left(X^2 - 1\right) \end{aligned} \tag{A.185}$$

and (A.175) results in

$$\begin{aligned} & -14X^4 + 6X^2 + 8X^6 \\ = \; & \left(8X^2 - 6\right) X^2 \left(X^2 - 1\right) \\ > \; & 2X^2\left(X^2 - 1\right) \end{aligned} \tag{A.186}$$

Therefore, by (A.172), we have

$$\begin{aligned}
&\tilde{B}^2 - 4\tilde{A}\tilde{C} \\
= \;& 1 \cdot V^2 + \rho^2 \left[25X^4 - 26X^6 + 9X^8 - 8X^2\right] V^2 + \rho\left[-14X^4 + 6X^2 + 8X^6\right] V^2 \\
> \;& 1 \cdot V^2 + \rho^2 X^2 \left(X^2 - 1\right) X^2 \left(X^2 - 1\right) V^2 + 2\rho X^2 \left(X^2 - 1\right)
\end{aligned} \tag{A.187}$$

which is equal to $\left[1 + \rho X^2 \left(X^2 - 1\right)\right]^2 V^2$.

This shows that

$$\begin{aligned}
-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}} \;&<\; \left[3 + 3\rho\left(X^4 - X^2\right)\right] V - \left[1 + \rho X^2\left(X^2 - 1\right)\right] V \\
&=\; \left[2 + 2\rho\left(X^4 - X^2\right)\right] V
\end{aligned} \tag{A.188}$$

which means that when $\tilde{A} > 0$, (A.178) satisfies

$$\frac{4V^2}{-\tilde{B} - \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} > \frac{4V^2}{\left[2 + 2\rho\left(X^4 - X^2\right)\right] V} = \frac{2V}{1 + \rho\left(X^4 - X^2\right)} \tag{A.189}$$

This implies that the expression of $m_{2L}^*$ given by (A.178) violates (A.184). Therefore the root represented by (A.178) can be ruled out for consideration.

Next, we show that the expression of $m_{2L}^*$ given by (A.179) satisfies (A.184). The proof goes as follows.

Using (A.187), we have

$$\begin{aligned}
-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}} \;&>\; \left[3 + 3\rho\left(X^4 - X^2\right)\right] V + \left[1 + \rho X^2\left(X^2 - 1\right)\right] V \\
&=\; \left[4 + 4\rho\left(X^4 - X^2\right)\right] V
\end{aligned} \tag{A.190}$$

Thus

$$\frac{4V^2}{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} < \frac{4V^2}{\left[4 + 4\rho\left(X^4 - X^2\right)\right] V} = \frac{V}{1 + \rho\left(X^4 - X^2\right)} \tag{A.191}$$

This means that the expression of $m_{2L}^*$ given by (A.179) satisfies (A.184), and $m_{2L}^* < V$.

On the other hand, it is clear that $m_{2L}$ of (A.179) is greater than 0, because both $-\tilde{B}$ and $\sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}$ are greater than 0. Therefore, $m_{2L}$ of (A.179) is the root we want.

Up to now, we have discussed the roots of quadratic equation (A.169) of $m_{2L}$ when the coefficient $\tilde{A} \neq 0$. As for the degenerate case when $\tilde{A} = 0$, it is easy to see that

$$m_{2L}^* = -\frac{\tilde{C}}{\tilde{B}} = \frac{2V^2}{\left[3 + 3\rho\left(X^4 - X^2\right)\right] V} < \frac{V}{1 + \rho\left(X^4 - X^2\right)} \tag{A.192}$$

which means that $0 < m_{2L} < V$, and (A.184) is satisfied, namely, $0 < e_{1L}^* = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$ is satisfied. Therefore, $m_{2L}$ of (A.192) is the root we want when $\tilde{A} = 0$.

It is clear that with $\tilde{A} = 0$, $m_{2L}^* = \frac{4V^2}{-\tilde{B}+\sqrt{\tilde{B}^2-4\tilde{A}\tilde{C}}}$ is the same as $m_{2L}^* = -\frac{\tilde{C}}{\tilde{B}} = \frac{2V^2}{[3+3\rho(X^4-X^2)]V}$. Thus we can use the former as the unified expression without specifying whether $\tilde{A} = 0$ or $\neq 0$.

In the following, we will show that when $V \leq \frac{2k}{\alpha_H}$, $m_{2L} \leq \frac{2k}{\alpha_H}$, and $m_{1L}$ and $m_{2L}$ satisfy (A.158), then

$$\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} < 1, \tag{A.193}$$

i.e., $\tilde{e}_{1H}^* < 1$.

From (A.161) we know that

$$\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} = \frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^2\alpha_L^2}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^2\alpha_H^2}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right], \tag{A.194}$$

where $\rho\left(\frac{\alpha_L^4 - \alpha_H^2\alpha_L^2}{8k^2}\right) < 0$ and $m_{2L}(V - m_{2L}) \leq \frac{V^2}{4}$. Thus

$$\begin{aligned}
&\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^2\alpha_L^2}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^2\alpha_H^2}{8k^2} m_{2L}^2 \\
\leq\ & \frac{\alpha_L^4}{4k^2}\frac{V^2}{4} + \frac{\alpha_L^2\alpha_H^2}{8k^2} m_{2L}^2 \\
\leq\ & \frac{\alpha_L^4}{4k^2}\frac{4k^2}{4\alpha_H^2} + \frac{\alpha_L^2\alpha_H^2}{8k^2}\frac{4k^2}{\alpha_H^2} \\
=\ & \frac{\alpha_L^4}{4\alpha_H^2} + \frac{\alpha_L^2}{2}
\end{aligned} \tag{A.195}$$

It is clear that

$$\alpha_H\left[\frac{\alpha_L^4}{4\alpha_H^2} + \frac{\alpha_L^2}{2}\right] < \frac{3}{4}\alpha_L^2 \leq \frac{3}{4}\left[\alpha_L^2 + \rho\left(\alpha_H^2 - \alpha_L^2\right)\right] \tag{A.196}$$

Therefore

$$\frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^2\alpha_L^2}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^2\alpha_H^2}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right] < \frac{3}{4} < 1 \tag{A.197}$$

which means that (A.193) holds. This result will be used later.

In summary, by the above procedure, we have the location of the local interior maximum as follows:

$$
\begin{aligned}
m_{2H}^* &= V \\
m_{1H}^* &= 0 \\
m_{2L}^* &= \frac{4V^2}{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} \\
m_{1L}^* &= \frac{\frac{\alpha_L^4}{4k^2} m_{2L}^*(V - m_{2L}^*) + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{8k^2}\right) {m_{2L}^*}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)} \\
m_{0L}^* &= -k\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k}\right]^2 \\
m_{0H}^* &= -k\left[\frac{\alpha_H m_{1H}^* + \frac{\alpha_H^3 {m_{2H}^*}^2}{4k}}{2k}\right]^2 - k\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k}\right]^2 \\
&\quad + k\left[\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 {m_{2L}^*}^2}{4k}}{2k}\right]^2
\end{aligned} \tag{A.198}
$$

with

$$
\begin{aligned}
\tilde{A} &= 1 + 3\rho\left(X^4 - X^2\right) - \rho^2\left(2X^4 - X^6 - X^2\right) - \rho\left(X^6 - X^4\right) \\
\tilde{B} &= \left[3\rho\left(X^2 - X^4\right) - 3\right] V \\
\tilde{C} &= 2V^2 \\
\tilde{B}^2 - 4\tilde{A}\tilde{C} &= V^2 + \rho^2\left[25X^4 - 26X^6 + 9X^8 - 8X^2\right] V^2 + \rho\left[-14X^4 + 6X^2 + 8X^6\right] V^2 \\
\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k} &= \frac{\alpha_L}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}^*(V - m_{2L}^*) + \rho\left(\frac{\alpha_H^2\alpha_L^2 - \alpha_H^4}{8k^2}\right) {m_{2L}^*}^2 + \frac{\alpha_L^4}{8k^2} {m_{2L}^*}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right] \\
\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 {m_{2L}^*}^2}{4k}}{2k} &= \frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}^*(V - m_{2L}^*) + \rho\left(\frac{\alpha_L^4 - \alpha_H^2\alpha_L^2}{8k^2}\right) {m_{2L}^*}^2 + \frac{\alpha_L^2\alpha_H^2}{8k^2} {m_{2L}^*}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right]
\end{aligned} \tag{A.199}
$$

where $25X^4 - 26X^6 + 9X^8 - 8X^2 > 0$ and $-14X^4 + 6X^2 + 8X^6 > 0$.

There are several observations about the above solution:

1. $m_{1L}^* > 0$ when $\alpha_L < \alpha_H$. The proof goes as follows:

   It is clear that $m_{1L}^* > 0$ is equivalent to

$$
\frac{\alpha_L^4}{4k^2} m_{2L}^*(V - m_{2L}^*) + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{8k^2}\right) {m_{2L}^*}^2 > 0 \tag{A.200}
$$

   namely,

$$
\alpha_L^4(V - m_{2L}^*) + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{2}\right) m_{2L}^* > 0 \tag{A.201}
$$

because $m_{2L}^* > 0$. (A.201) is equivalent to

$$\alpha_L^4 > \left[\alpha_L^4 + \rho\left(\frac{\alpha_H^4 - \alpha_L^4}{2}\right)\right] m_{2L}^* \tag{A.202}$$

Using the expression of $m_{2L}^*$ in (A.198), (A.202) is equivalent to

$$\left[-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}\right]\alpha_L^4 V > \left[\alpha_L^4 + \rho\left(\frac{\alpha_H^4 - \alpha_L^4}{2}\right)\right] \cdot 4V^2 \tag{A.203}$$

which is equivalent to

$$\left[-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}\right] > \left[1 + \rho\left(\frac{X^4 - 1}{2}\right)\right] \cdot 4V \tag{A.204}$$

Since

$$-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}} > \left[4 + 4\rho\left(X^4 - X^2\right)\right] V \tag{A.205}$$

and

$$4\left[\alpha_L^4 + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{2}\right)\right] < \left[4 + 2\rho\left(X^4 - 1\right)\right] \tag{A.206}$$

to show (A.203) holds, it is sufficient to show the following inequality holds:

$$\left[4 + 4\rho\left(X^4 - X^2\right)\right] > \left[4 + 2\rho\left(X^4 - 1\right)\right] \tag{A.207}$$

which is equivalent to $X^4 - 2X^2 + 1 > 0$, which is true when $\alpha_L < \alpha_H$. This shows that $m_{1L}^* > 0$.

2. $m_{0H}^* > -\frac{\alpha_H^6 V^4}{64k^3}$. This is because

$$m_{0H}^* - \left[-\frac{\alpha_H^6 V^4}{64k^3}\right] = k\left[\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 m_{2L}^{*\,2}}{4k}}{2k}\right]^2 - k\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k}\right]^2 \tag{A.208}$$

and

$$k\left[\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 m_{2L}^{*\,2}}{4k}}{2k}\right]^2 > k\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k}\right]^2 \tag{A.209}$$

3. $m_{0L}^* > -\frac{\alpha_L^6 V^4}{64k^3}$. The proof goes as follows.

By (A.160), we have

$$
\begin{aligned}
& \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \\
= \quad & \frac{\alpha_L}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \rho\left(\frac{\alpha_H^2\alpha_L^2 - \alpha_H^4}{8k^2}\right) m_{2L}^2 + \frac{\alpha_L^4}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right] \\
< \quad & \frac{\alpha_L}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2} m_{2L}(V - m_{2L}) + \frac{\alpha_L^4}{8k^2} m_{2L}^2}{\frac{\alpha_L^2}{2k}}\right] \qquad \text{(A.210)}
\end{aligned}
$$

which has a unique maximum $\frac{\alpha_L^3 V^2}{8k^2}$ at $m_{2L} = V$.

4. Using the fact that $e_{1L}^* = \frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k}$, $e_{1H}^* = \frac{\alpha_H m_{1H}^* + \frac{\alpha_H^3 m_{2H}^{*\,2}}{4k}}{2k}$ and $\tilde{e}_{1H}^* = \frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 m_{2L}^{*\,2}}{4k}}{2k}$, we have

$$
\begin{aligned}
m_{0L}^* &= -k{e_{1L}^*}^2 \\
m_{0H}^* &= -k{e_{1H}^*}^2 - k{e_{1L}^*}^2 + k{\tilde{e}_{1H}^*}^2 \\
\text{Expected Profit from the High-type Agent} &= (1-\lambda)\Pi_{H,P} \\
&= (1-\lambda)\left(k{e_{1H}^*}^2 + k{e_{1L}^*}^2 - k{\tilde{e}_{1H}^*}^2\right) \\
\text{Expected Profit from the Low-type Agent} &= \lambda\Pi_{L,P} \\
&= \lambda\left(2\rho k{e_{1L}^*}^2 - 2k\rho e_{1L}^*\tilde{e}_{1H}^* + k{e_{1L}^*}^2\right) \\
\text{The High-type Agent's Expected Profit} &= (1-\lambda)\left(k{\tilde{e}_{1H}^*}^2 - k{e_{1L}^*}^2\right) \qquad \text{(A.211)}
\end{aligned}
$$

where $e_{1L}^*$, $e_{1H}^*$ and $\tilde{e}_{1H}^*$ are the first period efforts for the low-type agent, the high-type agent and the high-type agent when pretending be the low-type, and $k{e_{1L}^*}^2$, $k{e_{1H}^*}^2$ and $k{\tilde{e}_{1H}^*}^2$ are the corresponding costs incurred.

Next we will show that this local maximal solution is actually the global maximal solution when $V \leq \frac{2k}{\alpha_H}$.

### A.2.3 The global maximum when $V \leq \frac{2k}{\alpha_H}$

One important observation we had from the procedure of solving the local maximization problem in previous part is that the maximization problem consisting of (A.89) through (A.95) can be written as

$$
\max_{\substack{(m_{0L}, m_{1L}, m_{2L}) \\ (m_{0H}, m_{1H}, m_{2H})}} \lambda\Pi_{L,P} + (1-\lambda)\Pi_{H,P} \qquad \text{(A.212)}
$$

where

$$\Pi_{L,P} = \alpha_L \left[ \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right] \left[ \alpha_L \frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L} \right] + k \left[ \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right]^2 \quad \text{(A.213)}$$

$$\Pi_{H,P} = \alpha_H \left[ \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right] \left[ \alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H} \right] + k \left[ \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right]^2$$
$$+k \left[ \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right]^2 - k \left[ \frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \right]^2 \quad \text{(A.214)}$$

because constraints (A.93) and (A.94) are binding, while constraints (A.92) and (A.95) are not binding.

We can rewrite $\lambda \Pi_{L,P} + (1 - \lambda)\Pi_{H,P}$ as $\overline{\Pi}_{L,P} + \overline{\Pi}_{H,P}$, where

$$\overline{\Pi}_{L,P} = \lambda \left[ \alpha_L \left( \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right) \left( \alpha_L \frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L} \right) + k \left( \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right)^2 \right]$$
$$+(1 - \lambda) \left[ k \left( \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \right)^2 - k \left( \frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \right)^2 \right] \quad \text{(A.215)}$$

$$\overline{\Pi}_{H,P} = (1 - \lambda) \left[ \alpha_H \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right) \left( \alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H} \right) \right]$$
$$+(1 - \lambda) \left[ k \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right)^2 \right] \quad \text{(A.216)}$$

We can see that $\overline{\Pi}_{L,P}$ is a function of $m_{1L}$ and $m_{2L}$ and $\overline{\Pi}_{H,P}$ is a function of $m_{1H}$ and $m_{2H}$.

Thus the maximization problem consisting of (A.89) through (A.95) is equivalent to the maximization problem

$$\max_{(m_{1L}, m_{2L})} \overline{\Pi}_{L,P} + \max_{(m_{1H}, m_{2H})} \overline{\Pi}_{H,P} \quad \text{(A.217)}$$

In fact, it is easy to check that the first order conditions of $\overline{\Pi}_{L,P}$ with respect to $m_{1L}$ and $m_{2L}$ give (A.110) and (A.111), and the ones of $\overline{\Pi}_{H,P}$ with respect to $m_{1H}$ and $m_{2H}$ give (A.112) and (A.113). In other words, we decompose the maximization problem consisting of (A.89) through (A.95) into two separate maximization problems, with one about $(m_{1L}, m_{2L})$ and the other about $(m_{1H}, m_{2H})$. We will apply the same procedure to the scenarios when $(m_{1L}, m_{2L})$, $(m_{1H}, m_{2H})$ belong to different regions. Notice that for these regions, the expressions of $\overline{\Pi}_{H,P}$ and $\overline{\Pi}_{L,P}$ may take different forms.

First we look at $\overline{\Pi}_{H,P}$. By taking into account of all possible expressions which $m_{1H}$ and $m_{2H}$ can have, we will find the optimal payments of $m_{1H}$ and $m_{2H}$ to maximize $\overline{\Pi}_{H,P}$ when $V \le \frac{2k}{\alpha_H}$.

There are four scenarios for consideration:

1. When $m_{2H} \le \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \le 1$, i.e., $e^*_{2H} \le 1$ and $e^*_{1H} \le 1$. The Lagrangian for the maximum of $\overline{\Pi}_{H,P}$ equals

$$
\begin{aligned}
&(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right)\right] \\
&+(1-\lambda)k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 + \lambda_1\left(\frac{2k}{\alpha_H} - m_{2H}\right) + \lambda_2\left(1 - \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)
\end{aligned}
\tag{A.219}
$$

where $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ are the Lagrangian multipliers.

The first order conditions of (A.218) with respect to $m_{1H}$ and $m_{2H}$ lead to

$$
(1-\lambda)\frac{\alpha_H^2}{2k}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) - \lambda_2\frac{\alpha_H}{2k} = 0 \tag{A.220}
$$

$$
\begin{aligned}
&(1-\lambda)\left[\alpha_H^4\frac{2m_{2H}}{8k^2}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right)\right] \\
&+(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V - m_{2H})\right)\right] - \lambda_1 - \lambda_2\alpha_H^3\frac{2m_{2H}}{4k^2} = 0
\end{aligned}
\tag{A.221}
$$

Multiplying (A.220) by $\alpha_H^2\frac{2m_{2H}}{4k}$ and subtracting the product from (A.221) gives

$$
(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V - m_{2H})\right)\right] - \lambda_1 = 0 \tag{A.222}
$$

There are three cases for consideration:

(a) When $\lambda_1 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$. The Lagrangian for the maximum of $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H}+\alpha_H k}{2k}\right)\left(\alpha_H\left(V-\frac{2k}{\alpha_H}\right)-m_{1H}\right)+k\left(\frac{\alpha_H m_{1H}+\alpha_H k}{2k}\right)^2\right]$$
$$+\tilde{\lambda}_1\left(1-\frac{\alpha_H m_{1H}+\alpha_H k}{2k}\right) \quad \text{(A.223)}$$

where $\tilde{\lambda}_1 \geq 0$ is a Lagrangian multiplier.

The first order condition of (A.223) with respect to $m_{1H}$ is

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\alpha_H\left(V-\frac{2k}{\alpha_H}\right)-m_{1H}\right)\right]-\tilde{\lambda}_1\frac{\alpha_H}{2k}=0 \quad \text{(A.224)}$$

When $\tilde{\lambda}_1 = 0$, $m_{1H} = \alpha_H\left(V-\frac{2k}{\alpha_H}\right)$. So $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)k\left[\frac{\alpha_H^2\left(V-\frac{2k}{\alpha_H}\right)+\alpha_H k}{2k}\right]^2=(1-\lambda)k\left(\frac{\alpha_H^2 V-\alpha_H k}{2k}\right)^2 \quad \text{(A.225)}$$

Notice that $\frac{\alpha_H^2\left(V-\frac{2k}{\alpha_H}\right)+\alpha_H k}{2k} \leq 1$.

When $\tilde{\lambda}_1 > 0$, $\frac{\alpha_H m_{1H}+\alpha_H k}{2k} = 1$, namely $m_{1H} = \frac{2k}{\alpha_H} - k$. Thus $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)\left[\alpha_H\left(\alpha_H\left(V-\frac{2k}{\alpha_H}\right)+k-\frac{2k}{\alpha_H}\right)\right]=(1-\lambda)\left(\alpha_H^2 V-\alpha_H k-2k\right) \quad \text{(A.226)}$$

It is easy to see that (A.225) is greater than (A.226).

(b) When $\lambda_2 > 0$, $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} = 1$ which implies that $m_{1H} = \frac{2k}{\alpha_H} - \frac{\alpha_H^2 m_{2H}^2}{4k}$. The Lagrangian for the maximum of $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)\left[\alpha_H\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-\frac{2k}{\alpha_H}+\frac{\alpha_H^2 m_{2H}^2}{4k}\right)+k\right]+\tilde{\lambda}_2\left(\frac{\alpha_H}{2k}-m_{2H}\right) \quad \text{(A.227)}$$

where $\tilde{\lambda}_2 \geq 0$ is a Lagrangian multiplier.

The first order condition of (A.227) gives

$$(1-\lambda)\frac{\alpha_H^3}{2k}(V-m_{2H})-\tilde{\lambda}_2=0 \quad \text{(A.228)}$$

When $\tilde{\lambda}_2 > 0$, $m_{2H} = \frac{\alpha_H}{2k}$, which means that $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} = 1$ is equivalent to $\frac{\alpha_H m_{1H}+\alpha_H k}{2k} = 1$. Thus $\overline{\Pi}_{H,P}$ has the same value as in (A.226), which is less than or equal to the value in (A.225).

When $\tilde{\lambda}_2 = 0$, $V = m_{2H}$. Thus $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)\left(-k+\frac{\alpha_H^3 V^2}{4k}\right) \quad \text{(A.229)}$$

which is $\leq -k+\alpha_H k < 0$, since $V = m_{2H} \leq \frac{2k}{\alpha_H}$. Thus (A.229) can't be the local maximum.

(c) When $\lambda_1 = 0$ and $\lambda_2 = 0$, from (A.222) and (A.220), we have $m_{2H} = V$ and $m_{1H} = 0$. Since $m_{2H} \leq \frac{2k}{\alpha_H}$, $V \leq \frac{2k}{\alpha_H}$. $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)k\left(\frac{\frac{\alpha_H^3 V^2}{4k}}{2k}\right)^2 = (1-\lambda)\frac{\alpha_H^6 V^4}{64k^3} \tag{A.230}$$

Notice that comparing (A.230) with (A.225), we have

$$(1-\lambda)k\left(\frac{\frac{\alpha_H^3 V^2}{4k}}{2k}\right)^2 \geq (1-\lambda)k\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \tag{A.231}$$

where the equality holds only when $V = \frac{2k}{\alpha_H}$, because $\frac{\alpha_H^3 V^2}{4k} \geq \alpha_H^2 V - \alpha_H k$.

2. When $m_{2H} \leq \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \geq 1$, i.e., $e_{2H}^* \leq 1$ and $e_{1H}^* = 1$. The Lagrangian for the maximum of $\overline{\Pi}_{H,P}$ equals

$$\begin{aligned}&(1-\lambda)\left[\alpha_H \cdot 1 \cdot \left(\alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) + \left(-k + \alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}\right)\right] \\ &+\lambda_1\left(\frac{2k}{\alpha_H} - m_{2H}\right) + \lambda_2\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} - 1\right)\end{aligned} \tag{A.232}$$

The first order condition of (A.232) with respect to $m_{1H}$ and $m_{2H}$ are

$$\lambda_2 \frac{\alpha_H}{2k} = 0 \tag{A.233}$$

$$(1-\lambda)\left[\frac{\alpha_H^3}{2k}(V - m_{2H})\right] - \lambda_1 + \lambda_2 \alpha_H^3 \frac{m_{2H}}{4k^2} = 0 \tag{A.234}$$

where (A.233) gives $\lambda_2 = 0$. Thus (A.234) becomes

$$(1-\lambda)\left[\frac{\alpha_H^3}{2k}(V - m_{2H})\right] - \lambda_1 = 0 \tag{A.235}$$

When $\lambda_1 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$. Thus (A.235) implies that $V$ must be greater than $\frac{2k}{\alpha_H}$. $\overline{\Pi}_{H,P}$ equals

$$\begin{aligned}&(1-\lambda)\left[\alpha_H \cdot 1 \cdot \left(\alpha_H\left(V - \frac{2k}{\alpha_H}\right) - m_{1H}\right) + (-k + \alpha_H m_{1H} + \alpha_H k)\right] \\ =\ & \alpha_H^2 V - \alpha_H k - k\end{aligned} \tag{A.236}$$

When $\lambda_1 = 0$, $m_{2H} = V$. Thus $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)\left(-k + \frac{\alpha_H^3 V^2}{4k}\right) \tag{A.237}$$

which is less than or equal to $-k + \alpha_H k < 0$, because $m_{2H} \leq \frac{2k}{\alpha_H}$.

3. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \leq 2k$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* \leq 1$. The Lagrangian for the maximum of $\overline{\Pi}_{H,P}$ is

$$(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)(\alpha_H(V - m_{2H}) - m_{1H})\right]$$
$$+(1-\lambda)\left[k\left(\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)^2\right]$$
$$+\lambda_1\left(m_{2H} - \frac{2k}{\alpha_H}\right) + \lambda_2(2k - \alpha_H m_{1H} + \alpha_H k - \alpha_H^2 m_{2H}) \quad \text{(A.238)}$$

The first order conditions of (A.238) with respect to $m_{1H}$ and $m_{2H}$ are

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\alpha_H\left(V - m_{2H}\right) - m_{1H}\right)\right] - \lambda_2\alpha_H = 0 \quad \text{(A.239)}$$
$$(1-\lambda)\left[\frac{\alpha_H^3}{2k}\left(\alpha_H\left(V - m_{2H}\right) - m_{1H}\right)\right] + \lambda_1 - \lambda_2\alpha_H^2 = 0 \quad \text{(A.240)}$$

Multiplying (A.239) by $\alpha_H$ and subtracting the product from (A.240) gives $\lambda_1 = 0$.

When $\lambda_2 > 0$, $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} = 2k$, namely, $\alpha_H m_{2H} + m_{1H} = \frac{2k}{\alpha_H} + k$. Then $\overline{\Pi}_{H,P}$ becomes

$$\alpha_H\left(\alpha_H V - \frac{2k}{\alpha_H} - k\right) + k = \alpha_H^2 V - \alpha_H k - k \quad \text{(A.241)}$$

When $\lambda_2 = 0$, (A.239) implies that $\alpha_H(V - m_{2H}) - m_{1H} = 0$, which means that $\alpha_H m_{2H} + m_{1H} = \alpha_H V$. $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)k\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \quad \text{(A.242)}$$

which is greater than or equal to $(1-\lambda)\left(\alpha_H^2 V - \alpha_H k - k\right)$, with the equality holding when $\alpha_H^2 V - \alpha_H k = 2k$. Notice that the constraint $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \leq 2k$ implies that $\alpha_H^2 V - \alpha_H k \leq 2k$, because $\alpha_H m_{2H} + m_{1H} = \alpha_H V$. Therefore, when $\alpha_H^2 V - \alpha_H k \leq 2k$,

$$(1-\lambda)k\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \geq (1-\lambda)\left(\alpha_H^2 V - \alpha_H k - k\right) \quad \text{(A.243)}$$

where the equality holds only when $\alpha_H^2 V - \alpha_H k = 2k$.

4. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \geq 2k$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* = 1$. The Lagrangian for the maximum of $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)\left[\alpha_H\left(\alpha_H(V - m_{2H}) - m_{1H}\right) + \left(-k + \alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}\right)\right]$$
$$+\lambda_1\left(m_{2H} - \frac{2k}{\alpha_H}\right) + \lambda_2\left(\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} - 2k\right) \quad \text{(A.244)}$$

It is easy to see that the first order conditions of (A.244) give $\lambda_1 = 0$ and $\lambda_2 = 0$. Notice that $\overline{\Pi}_{H,P}$ equals

$$\begin{aligned} & (1-\lambda)\left[\alpha_H\left(\alpha_H(V-m_{2H})-m_{1H}\right)+\left(-k+\alpha_H m_{1H}-\alpha_H k+\alpha_H^2 m_{2H}\right)\right] \\ = \; & (1-\lambda)\left(\alpha_H^2 V-\alpha_H k-k\right) \end{aligned} \tag{A.245}$$

One thing we would like to point out is that using the similar argument in the proof of Theorem 1, we can show that the expressions of the principal's expected profit in all cases above are bounded and thereby have local maximum. The uniqueness of the solution of the first order conditions in each case would ensure that the solution is the location of the local maximum.

In summary, the optimal money transfers provided by the principal to the high-type agent and the principal's expected profit are

1. When $V \leq \frac{2k}{\alpha_H}$, $m_{2H}^* = V$, $m_{1H}^* = 0$ and $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)k\left(\frac{\frac{\alpha_H^3 V^2}{4k}}{2k}\right)^2 \tag{A.246}$$

2. When $V \geq \frac{2k}{\alpha_H}$ and $\alpha_H^2 V - \alpha_H k \leq 2k$, $m_{2H}^* \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{2H}^* + m_{1H}^* = \alpha_H V$. $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)k\left(\frac{\alpha_H^2 V-\alpha_H k}{2k}\right)^2 \tag{A.247}$$

3. When $V \geq \frac{2k}{\alpha_H}$ and $\alpha_H^2 V - \alpha_H k \geq 2k$, $m_{2H}^* \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H}^* - \alpha_H k + \alpha_H^2 m_{2H}^* \geq 2k$. $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)(\alpha_H^2 V-\alpha_H k-k) \tag{A.248}$$

In the following, we look at the maximum of $\overline{\Pi}_{L,P}$. There are three scenarios and each scenario with several cases for consideration:

1. When $m_{2L} \leq \frac{2k}{\alpha_H}$, i.e., effort $\tilde{e}_{2H}^* \leq 1$, there are three cases:

(a) $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \leq 1$, which means that effort $\tilde{e}_{2H}^* \leq 1$. The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right)\right]$$
$$+\lambda\left[k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2\right]$$
$$+(1-\lambda)\left[k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 - k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2\right] + \lambda_1\left(\frac{2k}{\alpha_H} - m_{2L}\right)$$
$$+\lambda_2\left(1 - \frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right) \tag{A.249}$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are Lagrangian multipliers. Notice that $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq 1$, because $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq \frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}$. Thus there is no need to consider the situation when $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \geq 1$.

In fact, the second constraint $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \leq 1$ is redundant when $m_{2L} \leq \frac{2k}{\alpha_H}$ and $V \leq \frac{2k}{\alpha_H}$. This was proved in (A.193) through (A.197).

(b) $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \geq 1$ and $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq 1$, which mean that efforts $\tilde{e}_{1H}^* = 1$ and $e_{1L}^* \leq 1$. Notice that when $m_{2L} < \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \geq 1$, $\tilde{e}_{2H}^* = \frac{\alpha_H m_{2L}}{2k} < 1$ and $\tilde{e}_{1H}^* = 1$. Therefore, when pretending to be the low-type one, the high-type agent has the expected profit

$$m_{0L} - k + \alpha_H m_{1L} - \alpha_L k {\tilde{e}_{2H}^*}^2 + \alpha_H^2 \tilde{e}_{2H}^* m_{2L} \tag{A.250}$$

which has the maximum occurring at $\tilde{e}_{2H}^* = \frac{\alpha_H m_{2L}}{2k}$ and the optimal expected profit is

$$m_{0L} - k + \alpha_H m_{1L} - \frac{\alpha_H^3 m_2^2}{4k} + \frac{\alpha_H^3 m_{2L}^2}{2k} = m_{0L} - k + \alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k} \tag{A.251}$$

The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_L m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right]$$
$$+\lambda\left[k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2\right]$$
$$+(1-\lambda)\left[k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2-\left(-k+\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}\right)\right]+\lambda_1\left(\frac{2k}{\alpha_H}-m_{2L}\right)$$
$$+\lambda_2\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}-1\right)+\lambda_3\left(1-\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right) \tag{A.252}$$

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$, are Lagrangian multipliers.

(c) $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \geq 1$ and $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \geq 1$, which mean that efforts $\tilde{e}_{1H}^* = 1$ and $e_{1L}^* = 1$. As we pointed out in previous case, when pretending to be the low-type one, the high-type agent has the optimal expected profit $m_{0L}-k+\alpha_H m_{1L}+\frac{\alpha_H^3 m_2^2}{4k}$. Similarly, the low-type agent has the optimal expected profit $m_{0L}-k+\alpha_L m_{1L}+\frac{\alpha_L^3 m_2^2}{4k}$. Notice that constraint $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \geq 1$ becomes redundant, because $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq \frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}$. The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\lambda\left[\alpha_L\cdot 1\cdot\left(\alpha_L\frac{\alpha_L m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)+\left(-k+\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}\right)\right]$$
$$+(1-\lambda)\left[\left(-k+\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}\right)-\left(-k+\alpha_H m_{1L}+\frac{\alpha_H^3 m_2^2}{4k}\right)\right]$$
$$+\lambda_1\left(\frac{2k}{\alpha_H}-m_{2L}\right)+\lambda_2\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}-1\right) \tag{A.253}$$

where $\lambda_1 \geq 0$, and $\lambda_2 \geq 0$ are Lagrangian multipliers.

2. When $\frac{2k}{\alpha_H} \leq m_{2L} \leq \frac{2k}{\alpha_L}$, i.e., effort $\tilde{e}_{2H}^* = 1$ and effort $e_{2L}^* \leq 1$, there are four following cases. Notice that when $\tilde{e}_{2H}^* = 1$, pretending to be the low one, the high-type agent has expected profit:

$$m_{0L}-k\tilde{e}_{1H}^{*\,2}+\alpha_H\tilde{e}_{1H}^* m_{1L}-\alpha_H\tilde{e}_{1H}^*+\alpha_H^2 m_{2L} \tag{A.254}$$

with the optimal location at $\tilde{e}_{1H}^* = \frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}$ and the optimal expected profit $k\tilde{e}_{1H}^{*\,2} = k\left(\frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)^2$.

(a) $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \le 2k$, and $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \le 1$, which means $\tilde{e}_{1H}^* \le 1$ and $e_{1L}^* \le 1$. the Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\begin{aligned}
&\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_L \frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right)\right] \\
&+\lambda\left[k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2\right] \\
&+(1-\lambda)\left[k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 - k\left(\frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2\right] \\
&+\lambda_1\left(m_{2L} - \frac{2k}{\alpha_H}\right) + \lambda_2\left(\frac{2k}{\alpha_L} - m_{2L}\right) + \lambda_3\left(2k - \alpha_H m_{1L} + \alpha_H k - \alpha_H^2 m_{2L}\right) \\
&+\lambda_4\left(1 - \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)
\end{aligned} \tag{A.255}$$

where $\lambda_1 \ge 0$, $\lambda_2 \ge 0$, $\lambda_3 \ge 0$ and $\lambda_4 \ge 0$ are Lagrangian multipliers.

(b) $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \le 2k$, and $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \ge 1$, which means $\tilde{e}_{1H}^* \le 1$ and $e_{1L}^* = 1$. As we pointed out previously, $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \ge 1$ implies that the low-type agent has the optimal expected profit $m_{0L} - k + \alpha_L m_{1L} + \frac{\alpha_L^3 m_2^2}{4k}$. The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\begin{aligned}
&\lambda\left[\alpha_L \cdot 1 \cdot \left(\alpha_L \frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right) + \left(-k + \alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}\right)\right] \\
&+(1-\lambda)\left[\left(-k + \alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}\right) - k\left(\frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2\right] \\
&+\lambda_1\left(m_{2L} - \frac{2k}{\alpha_H}\right) + \lambda_2\left(\frac{2k}{\alpha_L} - m_{2L}\right) + \lambda_3\left(2k - \alpha_H m_{1L} + \alpha_H k - \alpha_H^2 m_{2L}\right) \\
&+\lambda_4\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} - 1\right)
\end{aligned} \tag{A.256}$$

where $\lambda_1 \ge 0$, $\lambda_2 \ge 0$, $\lambda_3 \ge 0$ and $\lambda_4 \ge 0$ are Lagrangian multipliers.

(c) $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \ge 2k$, and $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \le 1$, which means $\tilde{e}_{1H}^* = 1$ and $e_{1L}^* \le 1$. Notice that the high-type agent's expected profit when pretending to be the low-type is $m_{0L} - k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}$, because $\tilde{e}_{1H}^* = 1$ and $\tilde{e}_{2H}^* = 1$. The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_L m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right]$$
$$+\lambda\left[k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2\right]$$
$$+(1-\lambda)\left[k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2-\left(-k+\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}\right)\right]$$
$$+\lambda_1\left(m_{2L}-\frac{2k}{\alpha_H}\right)+\lambda_2\left(\frac{2k}{\alpha_L}-m_{2L}\right)+\lambda_3\left(\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}-2k\right)$$
$$+\lambda_4\left(1-\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right) \tag{A.257}$$

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, $\lambda_3 \geq 0$ and $\lambda_4 \geq 0$ are Lagrangian multipliers.

(d) $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \geq 2k$, and $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \geq 1$, which means $\tilde{e}_{1H}^* = 1$ and $e_{1L}^* = 1$. As we pointed out previously,the high-type agent's expected profit when pretending to be the low-type is $m_{0L} - k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}$, and $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \geq 1$ implies that the low-type agent has the optimal expected profit $m_{0L} - k + \alpha_L m_{1L} + \frac{\alpha_L^3 m_2^2}{4k}$. The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\lambda\left[\alpha_L\cdot 1\cdot\left(\alpha_L\frac{\alpha_L m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)+\left(-k+\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}\right)\right]$$
$$+(1-\lambda)\left[\left(-k+\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}\right)-\left(-k+\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}\right)\right]$$
$$+\lambda_1\left(m_{2L}-\frac{2k}{\alpha_H}\right)+\lambda_2\left(\frac{2k}{\alpha_L}-m_{2L}\right)+\lambda_3\left(\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}-2k\right)$$
$$+\lambda_4\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}-1\right) \tag{A.258}$$

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, $\lambda_3 \geq 0$ and $\lambda_4 \geq 0$ are Lagrangian multipliers.

3. When $m_{2L} \geq \frac{2k}{\alpha_L}$, $m_{2L} \geq \frac{2k}{\alpha_H}$. This means that $\tilde{e}_{2H}^* = 1$ and $e_{2L}^* = 1$. Similar to what we discussed before, $\tilde{e}_{1H}^* = \frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}$ if $\frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k} \leq 1$, i.e., $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \leq 2k$, otherwise $\tilde{e}_{1H}^* = 1$, and $e_{1L}^* = \frac{\alpha_L m_{1L}-\alpha_L k+\alpha_L^2 m_{2L}}{2k}$ if $\frac{\alpha_L m_{1L}-\alpha_L k+\alpha_H^2 m_{2L}}{2k} \leq 1$, i.e., $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \geq 2k$ , otherwise $e_{1L}^* = 1$. There are three cases.

(a) $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \le 2k$. Notice that $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \le 2k$ implies $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \le 2k$, because $0 \le \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \le \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}$. The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\begin{aligned} &\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)(\alpha_L(V - m_{2L}) - m_{1L})\right] \\ &+\lambda\left[k\left(\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2\right] \\ &+(1-\lambda)\left[k\left(\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2 - k\left(\frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2\right] \\ &+\lambda_1\left(\frac{2k}{\alpha_L} - m_{2L}\right) + \lambda_2\left(2k - \alpha_H m_{1L} + \alpha_H k - \alpha_H^2 m_{2L}\right) \end{aligned} \tag{A.259}$$

where $\lambda_1 \ge 0$, and $\lambda_2 \ge 0$ are Lagrangian multipliers.

(b) $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \ge 2k$, but $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \le 2k$. As we mentioned earlier, when $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \ge 2k$, the high-type agent's expected profit when pretending to be the low-type one is $m_{0L} - k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}$. The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\begin{aligned} &\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)(\alpha_L(V - m_{2L}) - m_{1L})\right] \\ &+\lambda\left[k\left(\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2\right] \\ &+(1-\lambda)\left[k\left(\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2 - \left(-k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}\right)\right] \\ &+\lambda_1\left(\frac{2k}{\alpha_L} - m_{2L}\right) + \lambda_2\left(2k - \alpha_L m_{1L} + \alpha_L k - \alpha_L^2 m_{2L}\right) \\ &+\lambda_3\left(\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} - 2k\right) \end{aligned} \tag{A.260}$$

where $\lambda_1 \ge 0$, $\lambda_2 \ge 0$, and $\lambda_3 \ge 0$ are Lagrangian multipliers.

(c) When $m_{2L} \ge \frac{2k}{\alpha_L}$ and $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \ge 2k$. Notice that $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \ge 2k$ implies $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \ge 2k$, because $0 \le \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \le \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}$. The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\begin{aligned} &\lambda\left[\alpha_L\left(\alpha_L(V - m_{2L}) - m_{1L}\right) + \left(-k + \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}\right)\right] \\ &+(1-\lambda)\left[\left(-k + \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}\right) - \left(-k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}\right)\right] \\ &+\lambda_1\left(m_{2L} - \frac{2k}{\alpha_L}\right) + \lambda_2\left(\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} - 2k\right) \end{aligned} \tag{A.261}$$

where $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ are Lagrangian multipliers.

One thing worthy of mentioning is that using the similar argument in the proof of Theorem 1, we can show that the expressions of the principal's expected profit in all cases above are bounded and thereby have local maximum. The uniqueness of the solution of the first order conditions in each case would ensure that the solution is the location of the local maximum.

Next we examine each scenario with their cases in detail.

1. For the first scenario, we don't need to add constraints $m_{2L} \geq 0$, $\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \geq 0$, and $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \geq 0$. The reason is the following:

   When $m_{2L} = 0$,

   $\overline{\Pi}_{L,P}$ in the first case becomes

$$\begin{aligned} & \lambda\left[\frac{\alpha_L^2 m_{1L}}{2k}(-m_{1L}) + k\frac{\alpha_L^2 m_{1L}^2}{4k^2}\right] + (1-\lambda)\left(\frac{\alpha_L^2 m_{1L}^2}{4k} - \frac{\alpha_L^2 m_{1L}^2}{4k}\right) \\ = \quad & -\lambda\frac{\alpha_L^2 m_{1L}^2}{4k} + (1-\lambda)\left(\alpha_L^2 - \alpha_H^2\right)\frac{m_{1L}^2}{4k} \\ \leq \quad & 0 \end{aligned} \tag{A.262}$$

   $\overline{\Pi}_{L,P}$ in the second case becomes

$$\begin{aligned} & \lambda\left[\frac{\alpha_L^2 m_{1L}}{2k}(-m_{1L}) + k\frac{\alpha_L^2 m_{1L}^2}{4k^2}\right] + (1-\lambda)\left(\frac{\alpha_L^2 m_{1L}^2}{4k} - (-k + \alpha_H m_{1L})\right) \\ \leq \quad & -\lambda\frac{\alpha_L^2 m_{1L}^2}{4k} \\ \leq \quad & 0 \end{aligned} \tag{A.263}$$

   because $\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} = 1$, $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq 1$ and $m_{2L}$ lead to $m_{1L} \leq \frac{2k}{\alpha_L}$ and $m_{1L} \geq \frac{2k}{\alpha_H}$, which gives $\frac{\alpha_L^2 m_{1L}^2}{4k} - (-k + \alpha_H m_{1L}) \leq k - k = 0$.

   $\overline{\Pi}_{L,P}$ in the third case becomes

$$(1-\lambda)(\alpha_L - \alpha_H)m_{1L} < 0 \tag{A.264}$$

   because $\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \geq 1$ implies $m_{1L} \geq \frac{2k}{\alpha_H} > 0$.

   When $\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} = 0$, In the first case, $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} = 0$, because $0 \leq \frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq \frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}$. Therefore, $\overline{\Pi}_{L,P}$ becomes

$$-(1-\lambda)k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2 \leq 0 \tag{A.265}$$

As to the second and third cases, since $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \geq 1$, they can't happen.

When $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} = 0$,

$\overline{\Pi}_{L,P}$ in the first case has the same expression as (A.265).

$\overline{\Pi}_{L,P}$ in the second case becomes

$$-(1-\lambda)\left(-k + \alpha_H m_{1L} + \frac{\alpha_H^3 m_2^2}{4k}\right) \leq 0 \tag{A.266}$$

because $-k + \alpha_H m_{1L} + \frac{\alpha_H^3 m_2^2}{4k} \geq 0$.

As to the third case, since $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \geq 1$, it can't happen.

On the other hand, we can show that for any $V > 0$, the global maximum of $\overline{\Pi}_{L,P}$ is greater than zero. The proof as follows:

Fixing $m_{1L} = 0$, $\overline{\Pi}_{L,P}$ in the first case becomes

$$\begin{aligned} & \lambda\left[\frac{\alpha_L^4 m_{2L}^2}{8k^2}\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L}) + k\frac{\alpha_L^4 m_{2L}^4}{64k^4}\right] + (1-\lambda)k\frac{\left(\alpha_L^6 - \alpha_H^6\right) m_{2L}^4}{64k^4} \\ = \; & \frac{m_{2L}^3}{16k^3}\left[\lambda\alpha_L^6(V - m_{2L}) + \lambda k\frac{\alpha_L^4 m_{2L}}{4k} + (1-\lambda)k\frac{\left(\alpha_L^6 - \alpha_H^6\right) m_{2L}}{4k}\right] \end{aligned} \tag{A.267}$$

It is easy to see that for any $V > 0$, we can find a small positive $m_{2L}$ such that (A.267) is greater than zero. Thus the maximum of $\overline{\Pi}_{L,P}$ in the first case of the first scenario is greater than zero, which means that the global maximum of $\overline{\Pi}_{L,P}$ is greater than zero.

Now we investigate each of the three cases in the first scenario.

(a) For the first case, first we ignore the term $\lambda_2\left(1-\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)$ in the Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ (see (A.249)). The corresponding first order conditions with respect to $m_{1L}$ and $m_{2L}$ give

$$\begin{aligned} &\lambda\left[\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right] \\ &+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L-\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)\alpha_H\right] = 0 \end{aligned} \tag{A.268}$$

$$\begin{aligned} &\lambda\left[\frac{\alpha_L^4\frac{2m_{2L}}{4k}}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right] \\ &+\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)\right]-\lambda_1 \\ &+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L^3\frac{2m_{2L}}{4k}-\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)\alpha_H^3\frac{2m_{2L}}{4k}\right] = 0 \end{aligned} \tag{A.269}$$

Multiplying (A.268) by $\alpha_L^2\frac{2m_{2L}}{4k}$, and subtracting this product from (A.269) gives

$$\begin{aligned} &\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)\right] \\ &+(1-\lambda)\left[\left(\frac{m_{1L}+\frac{\alpha_H^2 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_H^2\alpha_L^2-\alpha_H^4\right)\frac{2m_{2L}}{4k}\right]-\lambda_1\alpha_L^2\frac{2m_{2L}}{4k}=0 \end{aligned} \tag{A.270}$$

This means that when $V\le\frac{2k}{\alpha_H}$, $m_{2L}=\frac{2k}{\alpha_H}$ can't be the solution of (A.270), because otherwise the left side of (A.270) would be negative instead of zero. This shows that when $V\le\frac{2k}{\alpha_H}$, the maximum occurs when $m_{2L}<\frac{2k}{\alpha_H}$, i.e., interior solution.

Notice that in (A.193) through (A.197), we proved that when $V\le\frac{2k}{\alpha_H}$ and $m_{2L}\le\frac{2k}{\alpha_H}$ $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}<1$. This shows that the term $\lambda_2\left(1-\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)$ in (A.249) is redundant. Notice that (A.179) and (A.192) imply that at the location of the maximum, $m_{2L}$ is an increasing function of $V$.

(b) For the second case, the first order conditions of the Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ (see (A.252)) with respect to $m_{1L}$ and $m_{2L}$ give

$$\begin{aligned} \lambda\left[\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right] \\ +(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L-\alpha_H\right]+\lambda_2\frac{\alpha_H}{2k}-\lambda_3\frac{\alpha_L}{2k} &= 0 \end{aligned} \tag{A.271}$$

$$\begin{aligned} \lambda\left[\frac{\alpha_L^4\frac{2m_{2L}}{4k}}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right] \\ +\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)\right] \\ +(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L^3\frac{2m_{2L}}{4k}-\alpha_H^3\frac{2m_{2L}}{4k}\right] \\ -\lambda_1+\lambda_2\frac{\alpha_H^3\frac{2m_{2L}}{4k}}{2k}-\lambda_3\frac{\alpha_L^3\frac{2m_{2L}}{4k}}{2k} &= 0 \end{aligned} \tag{A.272}$$

Multiplying (A.271) by $\alpha_L^2\frac{2m_{2L}}{4k}$, and subtracting this product from (A.272) gives

$$\begin{aligned} \lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)\right] \\ +(1-\lambda)\left[\left(\alpha_H\alpha_L^2-\alpha_H^3\right)\frac{2m_{2L}}{4k}\right]-\lambda_1+\lambda_2\left(\alpha_H^3-\alpha_L^2\alpha_H\right)\frac{m_{2L}^2}{4k^2} = 0 \end{aligned} \tag{A.273}$$

When $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}=1$, it implies that

$$-k+\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}=k=k\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2$$

Thus $\overline{\Pi}_{L,P}$ in this case has the same expression as the one in the first case. In other words, the situation goes back to the first case.

When $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}>1$, $\lambda_2=0$. Under this condition and $\lambda_2=0$, (A.271) becomes

$$\begin{aligned} \lambda\left[\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right] \\ +(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L-\alpha_H\right]-\lambda_3\frac{\alpha_L}{2k} &= 0 \end{aligned} \tag{A.274}$$

Since $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} < 1$ and $\lambda_3 \geq 0$, we have

$$\lambda\left[\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right] + (1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L-\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_H\right] \geq 0 \quad \text{(A.275)}$$

which leads to

$$m_{1L} \leq \frac{\frac{\alpha_L^4}{4k^2}m_{2L}(V-m_{2L})+\rho\frac{\alpha_L^3 m_{2L}^2}{8k^2}(\alpha_L-\alpha_H)}{\frac{\alpha_L^2}{2k}+\rho\frac{\alpha_L(\alpha_H-\alpha_L)}{2k}} \quad \text{(A.276)}$$

where $\rho = \frac{1-\lambda}{\lambda}$. Thus

$$\begin{aligned}
&\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \\
\leq\ & \frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2}m_{2L}(V-m_{2L})+\rho\frac{\alpha_L^3 m_{2L}^2}{8k^2}(\alpha_L-\alpha_H)}{\frac{\alpha_L^2}{2k}+\rho\frac{\alpha_L(\alpha_H-\alpha_L)}{2k}}+\frac{\alpha_H^2 m_{2L}^2}{4k}\right] \\
=\ & \frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2}m_{2L}(V-m_{2L})+\rho\frac{\alpha_L^3(\alpha_L-\alpha_H)m_{2L}^2}{8k^2}+\frac{\alpha_H^2\alpha_L^2 m_{2L}^2}{8k^2}+\rho\frac{\alpha_L\alpha_H^2(\alpha_H-\alpha_L)m_{2L}^2}{8k^2}}{\frac{\alpha_L^2}{2k}+\rho\frac{\alpha_L(\alpha_H-\alpha_L)}{2k}}\right] \\
=\ & \frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2}m_{2L}(V-m_{2L})+\frac{\alpha_H^2\alpha_L^2 m_{2L}^2}{8k^2}+\rho\frac{(\alpha_L\alpha_H^2-\alpha_L^3)(\alpha_H-\alpha_L)m_{2L}^2}{8k^2}}{\frac{\alpha_L^2}{2k}+\rho\frac{\alpha_L(\alpha_H-\alpha_L)}{2k}}\right] \qquad \text{(A.277)}
\end{aligned}$$

Notice that when $V \leq \frac{2k}{\alpha_H}$ and $m_{2L} \leq \frac{2k}{\alpha_H}$,

$$\begin{aligned}
&\frac{\alpha_L^4}{4k^2}m_{2L}(V-m_{2L})+\frac{\alpha_H^2\alpha_L^2 m_{2L}^2}{8k^2} \\
\leq\ & \frac{\alpha_L^4}{4k^2}\frac{V^2}{4}+\frac{\alpha_H^2\alpha_L^2 m_{2L}^2}{8k^2} \\
<\ & \frac{\alpha_L^4}{4k^2}\left(\frac{2k}{\alpha_H}\right)^2\frac{1}{4}+\frac{\alpha_H^2\alpha_L^2\left(\frac{2k}{\alpha_H}\right)^2}{8k^2} \\
=\ & \left(\frac{\alpha_L}{\alpha_H}\right)^4\frac{1}{4}+\frac{\alpha_L^2}{2} \\
<\ & \alpha_L^2 \qquad \text{(A.278)}
\end{aligned}$$

and

$$\begin{aligned}
&\rho\frac{(\alpha_L\alpha_H^2-\alpha_L^3)(\alpha_H-\alpha_L)m_{2L}^2}{8k^2} \\
\leq\ & \rho(\alpha_L\alpha_H^2-\alpha_L^3)(\alpha_H-\alpha_L)\frac{4k^2}{\alpha_H^2}\frac{1}{8k^2} \\
\leq\ & \rho\frac{1}{\alpha_H^2}(\alpha_L\alpha_H^2-\alpha_L^3)(\alpha_H-\alpha_L)\frac{1}{2} \\
<\ & \rho\alpha_L(\alpha_H-\alpha_L) \qquad \text{(A.279)}
\end{aligned}$$

Therefore (A.277) is less than $\alpha_H$, which is less than 1. This means that $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} < 1$, which contradicts the assumption that $\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} > 1$. Thus, this situation is ruled out.

(c) For the third case, the first order condition of the Lagrangian of the maximum of $\overline{\Pi}_{L,P}$ with respect to $m_{1L}$ gives

$$(1-\lambda)(\alpha_L - \alpha_H) + \lambda_2 \frac{\alpha_L}{2k} = 0 \tag{A.280}$$

This shows that $\lambda_2 > 0$, which means that $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} = 1$, which means that $-k+\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k} = k = k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2$. The situation belongs to the second case.

2. For the second scenario, $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$ can't be zero in the first case, because otherwise $\overline{\Pi}_{L,P}$ becomes

$$-(1-\lambda)k\left(\frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2 \le 0 \tag{A.281}$$

In the third case, $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$ can't be zero, because otherwise $\overline{\Pi}_{L,P}$ becomes

$$-(1-\lambda)\left(-k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}\right) \le 0 \tag{A.282}$$

In the first and second cases, $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}$ can't be zero, because if $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} = 0$, then $m_{2L} = k - \alpha_H m_{2L}$. Therefore

$$\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} = \frac{\alpha_L(k - \alpha_H m_{2L}) + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \le \frac{\alpha_L\left(k - \alpha_H \frac{2k}{\alpha_H}\right) + \frac{\alpha_L^3\left(\frac{2k}{\alpha_L}\right)^2}{4k}}{2k} \le 0 \tag{A.283}$$

which causes $\overline{\Pi}_{L,P} \le 0$ in the first case.

Notice that we already showed that for any $V > 0$, the global maximum of $\overline{\Pi}_{L,P}$ is greater than zero.

Next we investigate the four cases in the second scenario.

(a) For the first case, the first order conditions of (A.255) with respect to $m_{1L}$ and $m_{2L}$ give

$$\begin{aligned}
&\lambda\left[\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right] \\
&+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L-\left(\frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)\alpha_H\right] \\
&-\lambda_3\alpha_H-\lambda_4\frac{\alpha_L}{2k} \quad = \quad 0
\end{aligned} \tag{A.284}$$

$$\begin{aligned}
&\lambda\left[\frac{\alpha_L^4\frac{2m_{2L}}{4k}}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right] \\
&+\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)\right] \\
&+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L^3\frac{2m_{2L}}{4k}-\left(\frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)\alpha_H^2\right] \\
&+\lambda_1-\lambda_2-\lambda_3\alpha_H^2-\lambda_4\frac{\alpha_L^3 m_{2L}}{4k^2} \quad = \quad 0
\end{aligned} \tag{A.285}$$

Multiplying (A.284) by $\alpha_L^2\frac{2m_{2L}}{4k}$, and subtracting this product from (A.285) gives

$$\begin{aligned}
&\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)\right] \\
&+(1-\lambda)\left[\left(\frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)\left(\alpha_H\alpha_L^2\frac{2m_{2L}}{4k}-\alpha_H^2\right)\right] \\
&+\lambda_1-\lambda_2-\lambda_3\left(\alpha_H^2-\alpha_H\alpha_L^2\frac{2m_{2L}}{4k}\right)=0
\end{aligned} \tag{A.286}$$

where $\alpha_H\alpha_L^2\frac{2m_{2L}}{4k}-\alpha_H^2<0$, and $\alpha_H^2-\alpha_H\alpha_L^2\frac{2m_{2L}}{4k}>0$, because $m_{2L}\leq\frac{2k}{\alpha_L}$. When $V<\frac{2k}{\alpha_H}$, $V-m_{2L}<0$, because $m_{2L}\geq\frac{2k}{\alpha_H}$. Thus, we must have $\lambda_1>0$ for $V<\frac{2k}{\alpha_H}$, which means that $m_{2L}=\frac{2k}{\alpha_H}$ is the location of the maximum of $\overline{\Pi}_{L,P}$. When $V=\frac{2k}{\alpha_H}$, if $m_{2L}>\frac{2k}{\alpha_H}$, we also have $\lambda_1>0$ for $V<\frac{2k}{\alpha_H}$, which implies $m_{2L}=\frac{2k}{\alpha_H}$ is still the location of the maximum of $\overline{\Pi}_{L,P}$. Notice that when $m_{2L}=\frac{2k}{\alpha_H}$, $\frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}=\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}$, which means that this situation belongs to the first case of the first scenario.

(b) For the second case, the first order condition of (A.256) with respect to $m_{1L}$ gives

$$(1-\lambda)(\alpha_L-\alpha_H)-\lambda_3\alpha_H+\lambda_4\frac{\alpha_L}{2k}=0 \tag{A.287}$$

This means that $\lambda_4>0$.

When $\lambda_4>0$, $\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}=1$, which means that $-k+\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}=k=k\left(\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}\right)^2$. Thus, the situation goes to the first case of the second scenario.

(c) For the third case, the first order conditions of (A.257) with respect to $m_{1L}$ and $m_{2L}$ give

$$\begin{aligned}\lambda\left[\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right]&\\+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L-\alpha_H\right]&\\+\lambda_3\alpha_H-\lambda_4\frac{\alpha_L}{2k}&=0\end{aligned} \tag{A.288}$$

$$\begin{aligned}\lambda\left[\frac{\alpha_L^4\frac{2m_{2L}}{4k}}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)\right]&\\+\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)\right]&\\+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\alpha_L^3\frac{2m_{2L}}{4k}-\alpha_H^2\right]&\\+\lambda_1-\lambda_2+\lambda_3\alpha_H^2-\lambda_4\frac{\alpha_L^3 m_{2L}}{4k^2}&=0\end{aligned} \tag{A.289}$$

Multiplying (A.288) by $\alpha_L^2\frac{2m_{2L}}{4k}$, and subtracting this product from (A.289) gives

$$\begin{aligned}&\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)\right]+(1-\lambda)\left[\alpha_H\alpha_L^2\frac{2m_{2L}}{4k}-\alpha_H^2\right]\\&+\lambda_1-\lambda_2+\lambda_3\left(\alpha_H^2-\alpha_H\alpha_L^2\frac{2m_{2L}}{4k}\right)=0\end{aligned} \tag{A.290}$$

where $\alpha_H\alpha_L^2\frac{2m_{2L}}{4k}-\alpha_H^2<0$, and $\alpha_H^2-\alpha_H\alpha_L^2\frac{2m_{2L}}{4k}>0$, because $m_{2L}\leq\frac{2k}{\alpha_L}$. When $V\leq\frac{2k}{\alpha_H}$, $V-m_{2L}<0$, because $m_{2L}\geq\frac{2k}{\alpha_H}$. Thus, we must have $\lambda_1>0$ or $\lambda_3>0$

for $V \leq \frac{2k}{\alpha_H}$. Notice that $\lambda_1 > 0$ means that $m_{2L} = \frac{2k}{\alpha_H}$ which implies that $-k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} = -k + \alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}$. Thus $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} \geq 2k$ implies $\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \geq 1$. The situation goes back to the second case of the first scenario. $\lambda_3 > 0$ means that $-k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} = k\left(\frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2$. The situation goes to the first case of the second scenario.

(d) For the fourth case, the first order conditions of (A.258) with respect to $m_{1L}$ and $m_{2L}$ give

$$(1-\lambda)(\alpha_L - \alpha_H) + \lambda_3 \alpha_H + \lambda_4 \frac{\alpha_L}{2k} = 0 \tag{A.291}$$

$$\lambda\left[\frac{\alpha_L^3}{2k}(V - m_{2L})\right] + (1-\lambda)\left(\frac{\alpha_L^3 m_{2L}}{2k} - \alpha_H^2\right) + \lambda_1 - \lambda_2 + \lambda_3 \alpha_H^2 + \lambda_4 \frac{\alpha_L^3 m_{2L}}{4k^2} = 0 \tag{A.292}$$

From (A.291), we can see that either $\lambda_3 > 0$ or $\lambda_4 > 0$.

When $\lambda_3 > 0$, $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} = 2k$, which implies that $-k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} = k = k\left(\frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2$. The situation goes to the second case of the second scenario.

When $\lambda_4 > 0$, $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} = 1$. This belongs to the third case of the second scenario.

3. For the third scenario, $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}$ can't be zero in the first case, because otherwise $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} = 0$, which would lead to $\overline{\Pi}_{L,P} = 0$. Here we use the fact that $0 \leq \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \leq \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}$ .

   In the second case, $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}$ can't be zero, because otherwise $\overline{\Pi}_{L,P}$ becomes

$$-(1-\lambda)(-k + \alpha_H m_{1L} - \alpha_H k + \alpha_H m_{2L}) \leq 0 \tag{A.293}$$

   Notice that we already showed that for any $V > 0$, the global maximum of $\overline{\Pi}_{L,P}$ is greater than zero.

   Next we investigate the three cases in the third scenario.

(a) For the first case, the first order conditions of (A.259) with respect to $m_{1L}$ and $m_{2L}$ are

$$\begin{aligned} &\lambda\left[\frac{\alpha_L^2}{2k}\left(\alpha_L(V-m_{2L})-m_{1L}\right)\right] \\ &+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)\alpha_L-\left(\frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)\alpha_H\right] \\ &-\lambda_2\alpha_H = 0 \end{aligned} \tag{A.294}$$

$$\begin{aligned} &\lambda\left[\frac{\alpha_L^3}{2k}\left(\alpha_L(V-m_{2L})-m_{1L}\right)\right] \\ &+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)\alpha_L^2-\left(\frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)\alpha_H^2\right] \\ &+\lambda_1-\lambda_2\alpha_H^2 = 0 \end{aligned} \tag{A.295}$$

Multiplying (A.294) by $\alpha_L$, and subtracting this product from (A.295) gives

$$(1-\lambda)\left(\frac{\alpha_H m_{1L}-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)\left(\alpha_L\alpha_H-\alpha_H^2\right)+\lambda_1+\lambda_2\left(\alpha_L\alpha_H-\alpha_H^2\right)=0 \tag{A.296}$$

This means that $\lambda_1 > 0$, namely, $m_{2L} = \frac{2k}{\alpha_L}$ is the location of the maximum of $\overline{\Pi}_{L,P}$. Notice that when $m_{2L} = \frac{2k}{\alpha_L}$, $\frac{\alpha_L m_{1L}-\alpha_L k+\alpha_L^2 m_{2L}}{2k} = \frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$, which means that the situation goes to the first case of the second scenario.

(b) For the second case, the first order conditions of (A.260) with respect to $m_{1L}$ and $m_{2L}$ are

$$\begin{aligned} &\lambda\left[\frac{\alpha_L^2}{2k}\left(\alpha_L(V-m_{2L})-m_{1L}\right)\right] \\ &+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)\alpha_L-\alpha_H\right]-\lambda_2\alpha_L+\lambda_3\alpha_H = 0 \end{aligned} \tag{A.297}$$

$$\begin{aligned} &\lambda\left[\frac{\alpha_L^3}{2k}\left(\alpha_L(V-m_{2L})-m_{1L}\right)\right] \\ &+(1-\lambda)\left[\left(\frac{\alpha_L m_{1L}-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)\alpha_L^2-\alpha_H^2\right]+\lambda_1-\lambda_2\alpha_L^2+\lambda_3\alpha_H^2 = 0 \end{aligned} \tag{A.298}$$

Multiplying (A.297) by $\alpha_L$ and subtracting the product from (A.298) gives

$$(1-\lambda)\left(\alpha_L\alpha_H-\alpha_H^2\right)+\lambda_1+\lambda_3\left(\alpha_H^2-\alpha_H\alpha_L\right)=0 \tag{A.299}$$

This means that either $\lambda_1 > 0$ or $\lambda_3 > 0$.

When $\lambda_1 > 0$, $m_{2L} = \frac{2k}{\alpha_L}$, which implies that $\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$. Thus the situation goes to the third case of the second scenario.

When $\lambda_3 > 0$, $\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} = 2k$, which means that $-k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L} = k\left(\frac{\alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2$. Thus the situation goes to the first case of the third scenario.

(c) For the third case, the first order conditions of (A.261) with respect to $m_{1L}$ and $m_{2L}$ are

$$(1-\lambda)(\alpha_L - \alpha_H) + \lambda_2 \alpha_L = 0 \tag{A.300}$$

$$(1-\lambda)\left(\alpha_L^2 - \alpha_H^2\right) + \lambda_1 + \lambda_2 \alpha_L^2 = 0 \tag{A.301}$$

Solving the above two equations, we have $\lambda_2 = (1-\lambda)\left(\frac{\alpha_H}{\alpha_L} - 1\right) > 0$ and $\lambda_1 = (1-)\left(\alpha_H^2 - \alpha_H \alpha_L\right) > 0$. This means that $m_{2L} = \frac{2k}{\alpha_L}$ and $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} = 2k$ which implies that $m_{1L} = \frac{2k}{\alpha_L} - k$.

Notice that when $m_{2L} = \frac{2k}{\alpha_L}$ and $m_{1L} = \frac{2k}{\alpha_L} - k$, $\overline{\Pi}_{L,P}$ equals

$$\begin{aligned}
& \lambda\left[\alpha_L\left(\alpha_L(V - m_{2L}) - m_{1L}\right) + \left(-k + \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}\right)\right] \\
& +(1-\lambda)\left[\left(-k + \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}\right) - \left(-k + \alpha_H m_{1L} - \alpha_H k + \alpha_H^2 m_{2L}\right)\right] \\
= \; & \lambda\left(\alpha_L^2 V - \alpha_L k - k\right) + (1-\lambda)\left[(\alpha_L - \alpha_H)(m_{1L} - k) + \left(\alpha_L^2 - \alpha_H^2\right) m_{2L}\right] \\
= \; & \lambda\left(\alpha_L^2 V - \alpha_L k - k\right) + (1-\lambda)\left[(\alpha_L - \alpha_H)\left(\frac{2k}{\alpha_L} - 2k\right) + \left(\alpha_L^2 - \alpha_H^2\right)\frac{2k}{\alpha_L}\right]
\end{aligned} \tag{A.302}$$

It is clear that when $V \le \frac{2k}{\alpha_H}$, the above value of $\overline{\Pi}_{L,P}$ is less than zero. So the global maximum can't occur here.

The following Table A.1 summarizes the discussion for $V \le \frac{2k}{\alpha_H}$.

From the above, we can see that when $V \le \frac{2k}{\alpha_H}$, eventually the first case of the first scenario has the highest value of $\overline{\Pi}_{L,P}$ than any other case except for the third case of the third scenario. Since the value of $\overline{\Pi}_{L,P}$ in the third case of the third scenario is less than zero and we already showed that the highest value of $\overline{\Pi}_{L,P}$ in the first case of the first scenario is positive, $\overline{\Pi}_{L,P}$ takes the global maximum in the first case of the first scenario.

Therefore when $V \le \frac{2k}{\alpha_H}$, the global optimal solution for the problem

$$\max_{\substack{(m_{0L}, m_{1L}, m_{2L}) \\ (m_{0H}, m_{1H}, m_{2H})}} \lambda \Pi_{L,P} + (1-\lambda)\Pi_{H,P} \tag{A.303}$$

**Table A.1**

| Scenarios | Cases | Status |
|---|---|---|
| 1 | a) | local maximum achieved interiorly |
| | b) | this case is ruled out |
| | c) | goes to b) of the 1st scenario |
| 2 | a) | goes to a) of the 1st scenario |
| | b) | goes to a) of the 2nd scenario |
| | c) | goes to b) of the 1st scenario or goes to a) of the 2nd scenario |
| | d) | goes to c) of the 1st scenario or goes to b) of the 2nd scenario |
| 3 | a) | goes to a) of the 2nd scenario |
| | b) | goes to c) of the 2nd scenario or goes to a) of the 3rd scenario |
| | c) | local maximum achieved at the boundary |

is the following:

$$
\begin{aligned}
m^*_{2H} &= V \\
m^*_{1H} &= 0 \\
m^*_{2L} &= \frac{4V^2}{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}} \\
m^*_{1L} &= \frac{\frac{\alpha_L^4}{4k^2} m^*_{2L}(V - m^*_{2L}) + \rho\left(\frac{\alpha_L^4 - \alpha_H^4}{8k^2}\right) {m^*}^2_{2L}}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)} \\
m^*_{0L} &= -k\left[\frac{\alpha_L m^*_{1L} + \frac{\alpha_L^3 {m^*_{2L}}^2}{4k}}{2k}\right]^2 \\
m^*_{0H} &= -k\left[\frac{\alpha_H m^*_{1H} + \frac{\alpha_H^3 {m^*_{2H}}^2}{4k}}{2k}\right]^2 - k\left[\frac{\alpha_L m^*_{1L} + \frac{\alpha_L^3 {m^*_{2L}}^2}{4k}}{2k}\right]^2 \\
&\quad +k\left[\frac{\alpha_H m^*_{1L} + \frac{\alpha_H^3 {m^*_{2L}}^2}{4k}}{2k}\right]^2 \\
\Pi_{H,P} &= -m^*_{0H}
\end{aligned}
$$

$$\begin{aligned}
\Pi_{L,P} &= \alpha_L\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k}\right]\left[\alpha_L\frac{\alpha_L m_{2L}^*}{2k}(V-m_{2L}^*) - m_{1L}^*\right] - m_{0L}^* \\
&= 2k\rho\frac{\alpha_L}{\alpha_H}\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k}\right]\left(\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 {m_{2L}^*}^2}{4k}}{2k}\right) \\
&\quad -2k\rho\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k}\right]\left(\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 {m_{2L}^*}^2}{4k}}{2k}\right)\alpha_L \\
&\quad +k\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k}\right]^2
\end{aligned}$$

$$\begin{aligned}
\text{The Principal's Expected Profit} &= \lambda\Pi_{L,P} + (1-\lambda)\Pi_{H,P} \\
\text{The Low-type Agent's Expected Profit} &= 0 \\
\text{The High-type Agent's Expected Profit} &= k\left[\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 {m_{2L}^*}^2}{4k}}{2k}\right]^2 - k\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k}\right]^2
\end{aligned} \tag{A.304}$$

with

$$\begin{aligned}
\tilde{A} &= 1 + 3\rho\left(X^4 - X^2\right) - \rho^2\left(2X^4 - X^6 - X^2\right) - \rho\left(X^6 - X^4\right) \\
\tilde{B} &= \left[3\rho\left(X^2 - X^4\right) - 3\right]V \\
\tilde{C} &= 2V^2
\end{aligned}$$

$$\begin{aligned}
&\tilde{B}^2 - 4\tilde{A}\tilde{C} \\
&= V^2 + \rho^2\left[25X^4 - 26X^6 + 9X^8 - 8X^2\right]V^2 + \rho\left[-14X^4 + 6X^2 + 8X^6\right]V^2 \\
&\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k} \\
&= \frac{\alpha_L}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2}m_{2L}^*(V-m_{2L}^*) + \rho\left(\frac{\alpha_H^2\alpha_L^2 - \alpha_H^4}{8k^2}\right){m_{2L}^*}^2 + \frac{\alpha_L^4}{8k^2}{m_{2L}^*}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right] \\
&\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 {m_{2L}^*}^2}{4k}}{2k} \\
&= \frac{\alpha_H}{2k}\left[\frac{\frac{\alpha_L^4}{4k^2}m_{2L}^*(V-m_{2L}^*) + \rho\left(\frac{\alpha_L^4 - \alpha_H^2\alpha_L^2}{8k^2}\right){m_{2L}^*}^2 + \frac{\alpha_L^2\alpha_H^2}{8k^2}{m_{2L}^*}^2}{\frac{\alpha_L^2}{2k} + \rho\left(\frac{\alpha_H^2 - \alpha_L^2}{2k}\right)}\right]
\end{aligned} \tag{A.305}$$

where $25X^4 - 26X^6 + 9X^8 - 8X^2 > 0$ and $-14X^4 + 6X^2 + 8X^6 > 0$ (see (A.179), (A.158), (A.160), (A.161), (A.170) through (A.175)). Notice that the second expression of $\Pi_{L,P}$ is obtained by using (A.110).

We pointed out at the end of previous subsection that $m^*_{1L} > 0$, $m^*_{0H} > -\frac{\alpha_H^6 V^4}{64k^3}$ and $m^*_{0L} > -\frac{\alpha_L^6 V^4}{64k^3}$.

As we mentioned, the reason why in (A.217) we can decompose the the problem

$$\max_{\substack{(m_{0L},m_{1L},m_{2L})\\(m_{0H},m_{1H},m_{2H})}} \lambda\Pi_{L,P} + (1-\lambda)\Pi_{H,P} \tag{A.306}$$

as $\max_{(m_{1L},m_{2L})} \overline{\Pi}_{L,P} + \max_{(m_{1H},m_{2H})} \overline{\Pi}_{H,P}$ is that we used the fact that the low-type agent's participation constraint and the high-type agent's incentive constraint are binding, which can be easily shown by the first order conditions of the Lagrangian with respect to $m_{0L}$. It is easy to show that the high-type's participation constraint is satisfied. As for the the low-type's incentive constraint, we showed that it holds for the first case of the first scenario. It is not difficult to show that it is true for other cases in various scenarios. We leave the verification to the readers. Notice that when $V \le \frac{2k}{\alpha_H}$, we don't need to go through this process for every case in all the three scenarios mentioned above, because the interior solution of the first case in the first scenario not only satisfies both constraints but also gives the global maximum when those two constraints are not considered, and thereby attains global maximum when the two constraints are taken into account.

This concludes the whole proof of Theorem 2.

## A.3 Proof of Theorem 3

In the proof of Theorem 1, we obtained the principal's expected profit in case 1) (i.e., the baseline model ) under complete information. The principal's expected profit in case 2) (i.e., no intermediate money transfer) under complete information is the same as that in case 1), since the optimal intermediate money transfer equaled 0.

In the following part, we will find the principal's expected profit in case 3) and case 4).

First, we look at the principal's expected profit maximization problem in case 3) under complete information.

It is clear that because of complete information, the principal's expected profit maximization problem can be decoupled into two separate problems, one involving the principal and the high-type agent and the other involving the principal and the low-type agent. Solving either of would easily lead to solving the other. In the following, we will solve the problem faced by the principal and the high-type agent.

The principal chooses optimal money transfers on $(m_{1H}, m_{2H})$ to maximize her expected profit:

$$(1-\lambda)\alpha_H e_{1H}\left(\alpha_H e_{2H}(V-m_{2H})-m_{1H}\right) \tag{A.307}$$

such that the high-type agent is willing to participate, namely

$$-ke_{1H}^2+\alpha_H e_{1H}m_{1H}-\alpha_H e_{1H}ke_{2H}^2+\alpha_H^2 e_{1H}e_{2H}m_{2H}\geq 0 \tag{A.308}$$

For given money transfers $(m_{1H}, m_{2H})$, the high-type agent chooses optimal efforts on $e_{1H}$ and $e_{2H}$ to maximize his expected profit, which is the right side of (A.308).

To solve this problem, same as what we will discuss in Theorem 1 for a similar problem which includes upfront money transfers, there are four scenarios for consideration in terms of various regions that $m_{1H}$ and $m_{2H}$ belong to.

1. When $m_{2H}\leq\frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\leq 1$, $e_{2H}^*\leq 1$ and $e_{1H}^*\leq 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$(1-\lambda)\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)+\lambda_1\left(\frac{2k}{\alpha_H}-m_{2H}\right)$$
$$+\lambda_2\left(1-\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right) \tag{A.309}$$

where $\lambda_1\geq 0$ and $\lambda_2\geq 0$ are the Lagrangian multipliers.

The first order conditions of with respect to $m_{1H}$ and $m_{2H}$ result in

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)-\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\right]-\lambda_2\frac{\alpha_H}{2k} = 0 \tag{A.310}$$

$$(1-\lambda)\alpha_H^4\frac{2m_{2H}}{8k^2}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)$$
$$+(1-\lambda)\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V-2m_{2H})\right)-\lambda_1-\lambda_2\alpha_H^3\frac{m_{2H}}{4k^2} = 0 \tag{A.311}$$

Multiplying (A.310) by $\alpha_H^2\frac{2m_{2H}}{4k}$ and subtracting the product from (A.311) leads to

$$(1-\lambda)\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2}{2k}(V-m_{2H})\right)-\lambda_1=0 \tag{A.312}$$

There are three cases for consideration:

(a) When $\lambda_1 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$. The Lagrangian for the maximum of the principal's expected profit equals

$$(1-\lambda)\alpha_H\left(\frac{\alpha_H m_{1H}+\alpha_H k}{2k}\right)\left(\alpha_H\left(V-\frac{2k}{\alpha_H}\right)-m_{1H}\right)+\tilde{\lambda}_1\left(1-\frac{\alpha_H m_{1H}+\alpha_H k}{2k}\right) \tag{A.313}$$

where $\tilde{\lambda}_1 \geq 0$ is a Lagrangian multiplier.

The first order condition of (A.313) with respect to $m_{1H}$ is

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\alpha_H\left(V-\frac{2k}{\alpha_H}\right)-m_{1H}\right)-\alpha_H\left(\frac{\alpha_H m_{1H}+\alpha_H k}{2k}\right)\right]-\tilde{\lambda}_1\frac{\alpha_H}{2k}=0 \tag{A.314}$$

When $\tilde{\lambda}_1 = 0$, $m_{1H} = \frac{1}{2}\alpha_H\left(V-\frac{3k}{\alpha_H}\right)$. So the principal's expected profit equals

$$(1-\lambda)\frac{k}{2}\left(\frac{\alpha_H^2 V-\alpha_H k}{2k}\right)^2 \tag{A.315}$$

When $\tilde{\lambda}_1 > 0$, $\frac{\alpha_H m_{1H}+\alpha_H k}{2k} = 1$, namely $m_{1H} = \frac{2k}{\alpha_H} - k$. This means the principal's expected profit equals

$$(1-\lambda)\alpha_H\left(\alpha_H\left(V-\frac{2k}{\alpha_H}\right)+k-\frac{2k}{\alpha_H}\right) \tag{A.316}$$

(b) When $\lambda_2 > 0$, $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} = 1$ which implies that $m_{1H} = \frac{2k}{\alpha_H} - \frac{\alpha_H^2 m_{2H}^2}{4k}$. The Lagrangian for the maximum of the principal's expected profit equals

$$(1-\lambda)\alpha_H\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-\frac{2k}{\alpha_H}+\frac{\alpha_H^2 m_{2H}^2}{4k}\right)+\tilde{\lambda}_2\left(\frac{2k}{\alpha_H}-m_{2H}\right) \tag{A.317}$$

where $\tilde{\lambda}_2 \geq 0$ is a Lagrangian multiplier.

The first order condition of (A.317) gives

$$(1-\lambda)\frac{\alpha_H^3}{2k}(V-m_{2H})-\tilde{\lambda}_2=0 \tag{A.318}$$

When $\tilde{\lambda}_2 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$, which means that $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} = 1$ is equivalent to $\frac{\alpha_H m_{1H}+\alpha_H k}{2k} = 1$. Thus the principal's expected profit has the same value as in (A.316).

When $\tilde{\lambda}_2 = 0$, $V = m_{2H}$. Thus the principal's expected profit equals

$$(1-\lambda)\left(\frac{\alpha_H^3 V^2}{4k}-2k\right) \tag{A.319}$$

which is $\leq -2k+\alpha_H k < 0$, since $V = m_{2H} \leq \frac{2k}{\alpha_H}$. Thus (A.229) can't be the local maximum, compared with (A.315).

(c) $\lambda_1 = 0$ and $\lambda_2 = 0$, from (A.312) and (A.310), we have $m_{2H} = V$ and $m_{1H} = -\frac{\alpha_H^2 V^2}{8k}$. Since $m_{2H} \le \frac{2k}{\alpha_H}$, $V \le \frac{2k}{\alpha_H}$. The principal's expected profit equals

$$(1-\lambda)\frac{k}{2}\left(\frac{\frac{\alpha_H^3 V^2}{4k}}{2k}\right)^2 = (1-\lambda)\frac{\alpha_H^6 V^4}{128k^3} \tag{A.320}$$

Comparing (A.320) with (A.315), we have

$$(1-\lambda)\frac{k}{2}\left(\frac{\frac{\alpha_H^3 V^2}{4k}}{2k}\right)^2 \ge (1-\lambda)\frac{k}{2}\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \tag{A.321}$$

where the equality holds only when $V = \frac{2k}{\alpha_H}$, because $\frac{\alpha_H^3 V^2}{4k} \ge \alpha_H^2 V - \alpha_H k$.

2. When $m_{2H} \le \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \ge 1$, $e_{2H}^* \le 1$ and $e_{1H}^* = 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$(1-\lambda)\alpha_H \cdot 1 \cdot \left(\alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) + \lambda_1\left(\frac{2k}{\alpha_H} - m_{2H}\right)$$
$$+\lambda_2\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} - 1\right) \tag{A.322}$$

where $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ are Lagrangian multipliers.

The first order condition of (A.322) with respect to $m_{1H}$ and $m_{2H}$ is

$$-(1-\lambda) + \lambda_2 \frac{\alpha_H}{2k} = 0 \tag{A.323}$$

$$(1-\lambda)\left[\frac{\alpha_H^3}{2k}(V - 2m_{2H})\right] - \lambda_1 + \lambda_2 \alpha_H^3 \frac{m_{2H}}{4k^2} = 0 \tag{A.324}$$

where (A.323) leads to $\lambda_2 > 0$, because $1-\lambda > 0$. This means that $m_{1H} = \frac{2k}{\alpha_H} - \frac{\alpha_H^2 m_{2H}^2}{4k}$. Thus the Lagrangian for the maximum of the principal's expected profit becomes

$$(1-\lambda)\alpha_H \cdot 1 \cdot \left(\alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}^2}{4k} - \frac{2k}{\alpha_H}\right) + \tilde{\lambda}_1\left(\frac{2k}{\alpha_H} - m_{2H}\right) \tag{A.325}$$

where $\tilde{\lambda}_1 \ge$ is a Lagrangian multiplier.

The first order condition of (A.325) gives

$$(1-\lambda)\left[\frac{\alpha_H^3}{2k}(V - m_{2H})\right] - \tilde{\lambda}_1 = 0 \tag{A.326}$$

When $\tilde{\lambda}_1 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} = \frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}$. This belongs to the fourth scenario that will be discussed.

When $\tilde{\lambda}_1 = 0$, (A.326) implies that $m_{2H} = V$. Thus the principal's expected profit is

$$(1-\lambda)\left(\frac{\alpha_H^3 V^2}{4k} - 2k\right) \tag{A.327}$$

Since $m_{2H} \leq \frac{2k}{\alpha_H}$ and $m_{2H} = V$, (A.327) is less than zero.

3. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \leq 2k$, $e_{2H}^* = 1$ and $e_{1H}^* \leq 1$. The Lagrangian for the maximum of the principal's expected profit is

$$(1-\lambda)\alpha_H\left(\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)(\alpha_H(V - m_{2H}) - m_{1H}) + \lambda_1\left(m_{2H} - \frac{2k}{\alpha_H}\right)$$
$$+\lambda_2(2k - \alpha_H m_{1H} + \alpha_H k - \alpha_H^2 m_{2H}) \tag{A.328}$$

The first order conditions of (A.328) with respect to $m_{1H}$ and $m_{2H}$ are

$$(1-\lambda)\left[\frac{\alpha_H^2}{2k}\left(\alpha_H\left(V - m_{2H}\right) - m_{1H}\right) - \alpha_H\left(\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)\right]$$
$$-\lambda_2\alpha_H = 0 \tag{A.329}$$

$$(1-\lambda)\left[\frac{\alpha_H^3}{2k}\left(\alpha_H\left(V - m_{2H}\right) - m_{1H}\right) - \alpha_H^2\left(\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)\right]$$
$$+\lambda_1 - \lambda_2\alpha_H^2 = 0 \tag{A.330}$$

Multiplying (A.329) by $\alpha_H$ and subtracting the product from (A.330) gives $\lambda_1 = 0$.

When $\lambda_2 > 0$, $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} = 2k$, namely, $\alpha_H m_{2H} + m_{1H} = \frac{2k}{\alpha_H} + k$. The principal's expected profit becomes

$$\alpha_H\left(\alpha_H V - \frac{2k}{\alpha_H} - k\right) = \alpha_H^2 V - \alpha_H k - 2k \tag{A.331}$$

When $\lambda_2 = 0$, (A.329) implies that $\alpha_H(V - m_{2H}) - m_{1H} = m_{1H} - k + \alpha_H m_{2H}$, which means that $\alpha_H m_{2H} + m_{1H} = \frac{\alpha_H V + k}{2}$. The principal's expected profit equals

$$(1-\lambda)\frac{k}{2}\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \tag{A.332}$$

4. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \geq 2k$, $e_{2H}^* = 1$ and $e_{1H}^* = 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$(1-\lambda)\alpha_H\left(\alpha_H(V-m_{2H})-m_{1H}\right)+\lambda_1\left(m_{2H}-\frac{2k}{\alpha_H}\right)+\lambda_2\left(\alpha_H m_{1H}-\alpha_H k+\alpha_H^2 m_{2H}-2k\right) \tag{A.333}$$

The first order condition of (A.333) with respect to $m_{1H}$ gives

$$-(1-\lambda)+\lambda_2\alpha_H = 0 \tag{A.334}$$

which implies $\lambda_2 > 0$. Thus $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} = 2k$, namely, $\alpha_H m_{1H} + \alpha_H^2 m_{2H} = 2k + \alpha_H k$. Thus the principal's expected profit is

$$(1-\lambda)(\alpha_H^2 V - \alpha_H k - 2k) \tag{A.335}$$

One thing worthy of mentioning is that for all cases above, the expression of the principal's expected profit is bounded in absolute value and thereby has the maximum. The uniqueness of the solution of the first order conditions of the associated Lagrangian ensures the solution is the location of the local maximum.

In summary, the optimal money transfers $(m_{1H}^*, m_{2H}^*)$ offered by the principal to the high-type agent and the principal's expected profit satisfy

1. When $V \leq \frac{2k}{\alpha_H}$, $m_{2H}^* = V$, $m_{1H}^* = -\frac{\alpha_H^2 V^2}{8k}$. The principal's expected profit equals

$$(1-\lambda)\frac{\alpha_H^6 V^4}{128k^3} \tag{A.336}$$

2. When $V \geq \frac{2k}{\alpha_H}$ and $\frac{k}{2}\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \geq \alpha_H^2 V - \alpha_H k - 2k$, $m_{2H}^* \geq \frac{2k}{\alpha_H}$, $\alpha_H m_{1H}^* - \alpha_H k + \alpha_H^2 m_{2H}^* \leq 2k$ and $\alpha_H m_{2H}^* + m_{1H}^* = \frac{\alpha_H V + k}{2}$ . The principal's expected profit equals

$$(1-\lambda)\frac{k}{2}\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \tag{A.337}$$

3. When $V \geq \frac{2k}{\alpha_H}$ and $\frac{k}{2}\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 \leq \alpha_H^2 V - \alpha_H k - 2k$, $m_{2H}^* \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H}^* - \alpha_H k + \alpha_H^2 m_{2H}^* = 2k$. The principal's expected profit equals

$$(1-\lambda)(\alpha_H^2 V - \alpha_H k - 2k) \tag{A.338}$$

As to the problem faced by the principal and the low-type agent, the principal chooses optimal money transfers on $(m_{1L}, m_{2L})$ to maximize her expected profit:

$$(1-\lambda)\alpha_L e_{1L}\left(\alpha_L e_{2L}(V - m_{2L}) - m_{1L}\right) \tag{A.339}$$

such that the low-type agent is willing to participate, namely

$$-ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \geq 0 \tag{A.340}$$

For given payments $(m_{1L}, m_{2L})$, the low-type agent chooses optimal efforts on $e_{1L}$ and $e_{2L}$ to maximize his expected profit, which is the right side of (A.340).

Similar to what we did in the above for the problem faced by the principal and the high-type agent, the optimal money transfers $(m_{1L}^*, m_{2L}^*)$ offered by the principal to the low-type agent and the principal's expected profit satisfy

1. When $V \leq \frac{2k}{\alpha_L}$, $m_{2L}^* = V$, $m_{1L}^* = -\frac{\alpha_L^2 V^2}{8k}$. The principal's expected profit equals

$$\lambda \frac{\alpha_L^6 V^4}{128k^3} \tag{A.341}$$

2. When $V \geq \frac{2k}{\alpha_L}$ and $\frac{k}{2}\left(\frac{\alpha_L^2 V - \alpha_L k}{2k}\right)^2 \geq \alpha_L^2 V - \alpha_L k - 2k$, $m_{2L}^* \geq \frac{2k}{\alpha_L}$, $\alpha_L m_{1L}^* - \alpha_L k + \alpha_L^2 m_{2L}^* \leq 2k$ and $\alpha_L m_{2L}^* + m_{1L}^* = \frac{\alpha_L V + k}{2}$ . The principal's expected profit equals

$$\lambda \frac{k}{2}\left(\frac{\alpha_L^2 V - \alpha_L k}{2k}\right)^2 \tag{A.342}$$

3. When $V \geq \frac{2k}{\alpha_L}$ and $\frac{k}{2}\left(\frac{\alpha_L^2 V - \alpha_L k}{2k}\right)^2 \leq \alpha_L^2 V - \alpha_L k - 2k$, $m_{2L}^* \geq \frac{2k}{\alpha_L}$ and $\alpha_L m_{1L}^* - \alpha_L k + \alpha_L^2 m_{2L}^* = 2k$. The principal's expected profit equals

$$\lambda(\alpha_L^2 V - \alpha_L k - 2k) \tag{A.343}$$

Notice that the principal's expected profit from both the high-type and low-type agents is the sum of two terms, with one from (A.336), (A.337) or (A.338), and the other from (A.341), (A.342) or (A.343). In particular, when $V \leq \frac{2k}{\alpha_H}$, which implies $V < \frac{2k}{\alpha_L}$, the principal's expected profit from both the low-type and high-type agents is

$$\lambda \frac{\alpha_L^6 V^4}{128k^3} + (1-\lambda)\frac{\alpha_H^6 V^4}{128k^3} \tag{A.344}$$

which is exactly half of the principal's expected profit in (B.131) when upfront money transfer is included for both types of agent in the proof of Theorem 1.

Next we will look at the principal's expected profit maximization problem in case 4) (i.e., only end money transfers included) under complete information. We will show that when $V \le \frac{2k}{\alpha_H}$, the optimal solution consisting of $m_{2L}^*$ and $m_{2H}^*$ for this model is also the optimal solution for the model under incomplete information with only end money transfers included, because $m_{2L}^* = m_{2H}^*$ implies that the incentive compatibility constraints for both types of agents are satisfied.

As we argued before, due to complete information, the principal's expected profit maximization problem can be decomposed into two separate problems, one involving the principal and the high-type agent and the other involving the principal and the low-type agent. Solving either of them will easily lead to solving the other. In the following, we will solve the problem faced by the principal and the high-type agent.

The principal chooses optimal money transfer on $m_{2H}$ to maximize her expected profit:

$$(1-\lambda)\alpha_H e_{1H}\left(\alpha_H e_{2H}(V - m_{2H})\right) \tag{A.345}$$

such that the high-type agent is willing to participate, namely

$$-ke_{1H}^2 - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \ge 0 \tag{A.346}$$

For given money transfer $m_{2H}$, the high-type agent chooses optimal efforts on $e_{1H}$ and $e_{2H}$ to maximize his expected profit, which is the right side of (A.346).

Notice that with $m_{2H} \le \frac{2k}{\alpha_H}$ and $m_{1H} = 0$, $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \le \frac{\frac{\alpha_H^3 \left(\frac{2k}{\alpha_H}\right)^2}{4k}}{2k} = \frac{\alpha_H}{2} < 1$, which means $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \ge 1$ can't happen. Thus, to solve the principal's expected profit maximization problem, there are only three scenarios for consideration in terms of various regions that $m_{2H}$ belongs to.

1. When $m_{2H} \le \frac{2k}{\alpha_H}$, $e_{2H}^* \le 1$ and $e_{1H}^* = \frac{\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \le \frac{\frac{\alpha_H^3 \left(\frac{2k}{\alpha_H}\right)^2}{4k}}{2k} = \frac{\alpha_H}{2} < 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$(1-\lambda)\alpha_H \left(\frac{\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H})\right) + \lambda_1\left(\frac{2k}{\alpha_H} - m_{2H}\right) \tag{A.347}$$

where $\lambda_1 \ge 0$ is a Lagrangian multiplier.

The first order conditions of with respect to $m_{2H}$ gives

$$(1-\lambda)\frac{\alpha_H^6}{16k^3}\left(3m_{2H}^2 V - 4m_{2H}^3\right) - \lambda_1 = 0 \tag{A.348}$$

When $\lambda_1 = 0$, i.e., $m_{2H} \leq \frac{2k}{\alpha_H}$, (A.348) leads to $m_{2H} = \frac{3}{4}V$, which results in the principal's expected profit as

$$(1-\lambda)\frac{27}{32}\frac{\alpha_H^6 V^4}{128k^3} \tag{A.349}$$

Since $m_{2H} \leq \frac{2k}{\alpha_H}$ and $m_{2H} = \frac{3}{4}V$, $V \leq \frac{8k}{3\alpha_H}$.

When $\lambda_1 > 0$, i.e., $m_{2H} = \frac{2k}{\alpha_H}$, the principal's expected profit becomes

$$(1-\lambda)\alpha_H \left( \frac{\frac{\alpha_H^3 \left(\frac{2k}{\alpha_H}\right)^2}{4k}}{2k} \right) \left( \alpha_H \frac{\alpha_H \frac{2k}{\alpha_H}}{2k} \left( V - \frac{2k}{\alpha_H} \right) \right) = (1-\lambda)\frac{\alpha_H^3}{2}\left( V - \frac{2k}{\alpha_H} \right) \tag{A.350}$$

To compare (A.349) and (A.350), we let $F(V) = \frac{27}{32}\frac{\alpha_H^6 V^4}{128k^3} - \frac{\alpha_H^3}{2}\left( V - \frac{2k}{\alpha_H} \right)$. We can see that $F(0) = \alpha_H^2 k > 0$ and the first order condition $F'(V) = 0$ gives $V = \frac{8k}{3\alpha_H}$. Since the second derivative $F''(V) > 0$, $F(V)$ is a strict convex function with the global minimum at $V = \frac{8k}{3\alpha_H}$. Notice that at $V = \frac{8k}{3\alpha_H}$, $F(V) = \frac{\alpha_H^3}{2} - \frac{\alpha_H^3}{2} = 0$. Therefore (A.349) is greater than (A.350) when $0 \leq V < \frac{8k}{3\alpha_H}$ and (A.349) equals (A.350) when $V = \frac{8k}{3\alpha_H}$.

2. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\frac{-\alpha_H k + \alpha_H^2 m_{2H}}{2k} \leq 1$, $e_{2H}^* = 1$ and $e_{1H}^* \leq 1$. The Lagrangian for the maximum of the principal's expected profit is

$$(1-\lambda)\alpha_H \left( \frac{-\alpha_H k + \alpha_H^2 m_{2H}}{2k} \right) (\alpha_H (V - m_{2H})) + \lambda_1 \left( m_{2H} - \frac{2k}{\alpha_H} \right) + \lambda_2 (2k + \alpha_H k - \alpha_H^2 m_{2H}) \tag{A.351}$$

where $\lambda_1$ and $\lambda_2$ are Lagrangian multipliers. Notice that since $m_{2H} \geq \frac{2k}{\alpha_H}$, to have a positive expected profit for the principal, $V \geq \frac{2k}{\alpha_H}$ is needed.

The first order condition of with respect to $m_{2H}$ gives

$$(1-\lambda)\left[ \frac{\alpha_H^3}{2k} (\alpha_H (V - m_{2H})) - \alpha_H^2 \left( \frac{-\alpha_H k + \alpha_H^2 m_{2H}}{2k} \right) \right] + \lambda_1 - \lambda_2 \alpha_H^2 = 0 \tag{A.352}$$

When $\lambda_1 = 0$ and $\lambda_2 = 0$, (A.352) gives

$$\frac{\alpha_H^4}{2k} V + \frac{\alpha_H^3}{2} - \frac{\alpha_H^4}{k} m_{2H} = 0 \tag{A.353}$$

Thus $m_{2H} = \frac{V}{2} + \frac{k}{2\alpha_H}$, which means the principal's expected profit is

$$(1-\lambda)\frac{k}{2}\left( \frac{\alpha_H^2 V - \alpha_H k}{2k} \right)^2 \tag{A.354}$$

Since $m_{2H} = \frac{V}{2} + \frac{k}{2\alpha_H}$, $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\frac{-\alpha_H k + \alpha_H^2 m_{2H}}{2k} \leq 1$, $\frac{3k}{\alpha_H} \leq V \leq \frac{\alpha_H k + 4k}{\alpha_H^2}$.

When $\lambda_1 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$. The principal's expected profit is

$$(1-\lambda)\frac{\alpha_H^3}{2}\left(V - \frac{2k}{\alpha_H}\right) \tag{A.355}$$

which is the same as (A.350).

To compare (A.354) with (A.355), we let $G(V) = \frac{k}{2}\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 - \frac{\alpha_H^3}{2}\left(V - \frac{2k}{\alpha_H}\right)$. We can see that $G(0) = \frac{9}{8}k\alpha_H^2 > 0$. The first order condition $G'(V) = 0$ leads to $V = \frac{3k}{\alpha_H}$, which is the global minimum because $G(V)$ is a strict convex function with the second derivative $G''(V) > 0$. At this global minimum, $G = \frac{\alpha_H^2}{2}k - \frac{\alpha_H^2}{2}k = 0$. This shows that (A.354) is greater than (A.355) when $V \neq \frac{3k}{\alpha_H}$ and $V \leq \frac{\alpha_H k + 4k}{\alpha_H^2}$ and (A.354) equals (A.355) when $V = \frac{3k}{\alpha_H}$.

When $\lambda_2 > 0$, $-\alpha_H k + \alpha_H^2 m_{2H} = 2k$, which implies $m_{2H} = \frac{k}{\alpha_H} + \frac{2k}{\alpha_H^2}$. The principal's expected profit becomes

$$(1-\lambda)\alpha_H^2(V - m_{2H}) = (1-\lambda)\alpha_H^2\left(V - \frac{k}{\alpha_H} - \frac{2k}{\alpha_H^2}\right) = (1-\lambda)(\alpha_H^2 V - \alpha_H k - 2k) \tag{A.356}$$

To compare (A.354) with (A.356), we let $P(V) = \frac{k}{2}\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2 - (\alpha_H^2 V - \alpha_H k - 2k)$. We can see that $P(0) = \frac{1}{8}k\alpha_H^2 + \alpha_H k + 2k > 0$. The first order condition $P'(V) = 0$ results in $V = \frac{\alpha_H k + 4k}{\alpha_H^2}$, which is the global minimum because $P(V)$ is a strict convex function with the second derivative $P''(V) > 0$. At this global minimum, $P = 2k - 2k = 0$. This shows that (A.354) is greater than (A.356) when $V < \frac{\alpha_H k + 4k}{\alpha_H^2}$ and (A.354) equals (A.356) when $V = \frac{\alpha_H k + 4k}{\alpha_H^2}$.

Therefore compared with (A.354) and (A.356), when $\frac{8k}{3\alpha_H} \leq V \leq \frac{\alpha_H k + 4k}{\alpha_H^2}$, (A.354) gives the principal the highest expected profit.

3. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\frac{-\alpha_H k + \alpha_H^2 m_{2H}}{2k} \geq 1$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* = 1$. Notice that $\frac{-\alpha_H k + \alpha_H^2 m_{2H}}{2k} \geq 1$ implies $m_{2H} \geq \frac{2k}{\alpha_H^2} + \frac{k}{\alpha_H}$. Thus $m_{2H} > \frac{2k}{\alpha_H}$.

   The Lagrangian for the maximum of the principal's expected profit equals

$$(1-\lambda)\alpha_H^2(V - m_{2H}) + \lambda_1\left(-\alpha_H k + \alpha_H^2 m_{2H} - 2k\right) \tag{A.357}$$

   where $\lambda_1$ is a Lagrangian multiplier.

The first order condition of (A.357) with respect to $m_{1H}$ gives

$$-(1-\lambda)+\lambda_1\alpha_H^2=0 \tag{A.358}$$

which implies $\lambda_1>0$. Thus $-\alpha_H k+\alpha_H^2 m_{2H}=2k$, namely, $m_{2H}=\frac{k}{\alpha_H}+\frac{2k}{\alpha_H^2}$ which gives the principal's expected profit as

$$(1-\lambda)(\alpha_H^2 V-\alpha_H k-2k) \tag{A.359}$$

which is the same as (A.356).

To know which one of (A.355) and (A.359) is bigger when $V\le\frac{3k}{\alpha_H}$ , we let $T(V)=(\alpha_H^2 V-\alpha_H k-2k)-\frac{\alpha_H^3}{2}\left(V-\frac{2k}{\alpha_H}\right)$. At $V=\frac{3k}{\alpha_H}$, $T=2k\alpha_H-2k-\frac{\alpha_H^2 k}{2}<0$. Notice that the first derivative $T'(V)=\alpha_H^2-\frac{\alpha_H^3}{2}>0$. Thus (A.355) is greater than (A.359) when $V\le\frac{3k}{\alpha_H}$.

In summary, the optimal money transfer $m_{2H}^*$ offered by the principal to the high-type agent and the principal's expected profit satisfy

1. When $V\le\frac{8k}{3\alpha_H}$, $m_{2H}^*=\frac{3}{4}V$. The principal's expected profit equals

$$(1-\lambda)\frac{27}{32}\frac{\alpha_H^6 V^4}{128k^3} \tag{A.360}$$

2. When $\frac{8k}{3\alpha_H}\le V\le\frac{3k}{\alpha_H}$, $m_{2H}^*=\frac{2k}{\alpha_H}$. The principal's expected profit equals

$$(1-\lambda)\frac{\alpha_H^3}{2}\left(V-\frac{2k}{\alpha_H}\right) \tag{A.361}$$

3. When $\frac{3k}{\alpha_H}\le V\le\frac{\alpha_H k+4k}{\alpha_H^2}$, $m_{2H}^*=\frac{V}{2}+\frac{k}{2\alpha_H}$. The principal's expected profit equals

$$(1-\lambda)\frac{k}{2}\left(\frac{\alpha_H^2 V-\alpha_H k}{2k}\right)^2 \tag{A.362}$$

4. When $V\ge\frac{\alpha_H k+4k}{\alpha_H^2}$, $m_{2H}^*=\frac{k}{\alpha_H}+\frac{2k}{\alpha_H^2}$ which gives the principal's expected profit as

$$(1-\lambda)(\alpha_H^2 V-\alpha_H k-2k) \tag{A.363}$$

One thing we would like to point out is that for all cases above, the expression of the principal's expected profit is bounded in absolute value and thereby has the maximum. The uniqueness of the solution of the first order conditions of the associated Lagrangian ensures the solution is the location of the local maximum.

As to the problem faced by the principal and the low-type agent, the principal chooses optimal money transfer on $m_{2L}$ to maximize her expected profit:

$$(1-\lambda)\alpha_L e_{1L}\left(\alpha_L e_{2L}(V-m_{2L})\right) \tag{A.364}$$

such that the low-type agent is willing to participate, namely

$$-ke_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \geq 0 \tag{A.365}$$

For given payment $m_{2L}$, the low-type agent chooses optimal efforts on $e_{1L}$ and $e_{2L}$ to maximize his expected profit, which is the right side of (A.365).

Similar to what we did in the above for the problem faced by the principal and the high-type agent, the optimal payment $m_{2L}^*$ offered by the principal to the low-type agent and the principal's expected profit satisfy

1. When $V \leq \frac{8k}{3\alpha_L}$, $m_{2L}^* = \frac{3}{4}V$. The principal's expected profit equals
$$(1-\lambda)\frac{27}{32}\frac{\alpha_L^6 V^4}{128k^3} \tag{A.366}$$
2. When $\frac{8k}{3\alpha_L} \leq V \leq \frac{3k}{\alpha_L}$, $m_{2L}^* = \frac{2k}{\alpha_L}$. The principal's expected profit equals
$$(1-\lambda)\frac{\alpha_L^3}{2}\left(V - \frac{2k}{\alpha_L}\right) \tag{A.367}$$
3. When $\frac{3k}{\alpha_L} \leq V \leq \frac{\alpha_L k + 4k}{\alpha_L^2}$, $m_{2L}^* = \frac{V}{2} + \frac{k}{2\alpha_L}$. The principal's expected profit equals
$$(1-\lambda)\frac{k}{2}\left(\frac{\alpha_L^2 V - \alpha_L k}{2k}\right)^2 \tag{A.368}$$
4. When $V \geq \frac{\alpha_L k + 4k}{\alpha_L^2}$, $m_{2L} = \frac{k}{\alpha_L} + \frac{2k}{\alpha_L^2}$ which gives the principal's expected profit as
$$(1-\lambda)(\alpha_L^2 V - \alpha_L k - 2k) \tag{A.369}$$

From the above, we can see that when $V \leq \frac{2k}{\alpha_H}$, which implies $V < \frac{2k}{\alpha_L}$, $m_{2H}^* = m_{2L}^* = \frac{3}{4}V$. This means that $m_{2H}^* = m_{2L}^* = \frac{3}{4}V$ is also the optimal solution for the model under incomplete information with only end money transfer included, because the incentive compatibility constraints for agents will be satisfied.

It is clear that the principal's expected profit from both the high-type and low-type agents is the sum of two terms, with one from (A.360), (A.361), (A.362) or (A.363) and the other from (A.366), (A.367), (A.368), or (A.369). In particular, when $V \leq \frac{2k}{\alpha_H}$, the principal's expect profit from both the low-type and high-type agents is

$$\lambda\frac{27}{32}\frac{\alpha_L^6 V^4}{128k^3} + (1-\lambda)\frac{27}{32}\frac{\alpha_H^6 V^4}{128k^3} \tag{A.370}$$

This concludes the proof of Theorem 3.

## A.4 Proof of Theorem 4

This section has two parts. In part one, we prove the statement of Theorem 4. In part two, we find the value of the principal's expected profit in case 2), which is used in the argument of part one.

### A.4.1 Main result

First we show that the principal's expected profit in case 1) (i.e., the baseline ) is higher than that in case 2) (i.e., no intermediate money transfer).

Recall that in case 1), the principal maximizes her following expected profit by offering $(m_{0L}, m_{1L}, m_{2L})$ to the low-type agent and $(m_{0H}, m_{1H}, m_{2H})$ to the high-type agent

$$\lambda\left[\alpha_L e_{1L}\left(\alpha_L e_{2L}(V-m_{2L})-m_{1L}\right)-m_{0L}\right]+(1-\lambda)\left[\alpha_H e_{1H}\left(\alpha_H e_{2H}(V-m_{2H})-m_{1H}\right)-m_{0H}\right] \tag{A.371}$$

For given $(m_{0L}, m_{1L}, m_{2L})$ and $(m_{0H}, m_{1H}, m_{2H})$, the low-type and high-type agents maximize their following expected profits, respectively:

$$m_{0L}-ke_{1L}^2+\alpha_L e_{1L}m_{1L}-\alpha_L e_{1L}ke_{2L}^2+\alpha_L^2 e_{1L}e_{2L}m_{2L} \tag{A.372}$$

and

$$m_{0H}-ke_{1H}^2+\alpha_H e_{1H}m_{1H}-\alpha_H e_{1H}ke_{2H}^2+\alpha_H^2 e_{1H}e_{2H}m_{2H} \tag{A.373}$$

where $0 \le e_{1L} \le 1$, $0 \le e_{2L} \le 1$, $0 \le e_{1H} \le 1$ and $0 \le e_{2H} \le 1$.

To ensure both agents' participation and prevent each agent from imitating the other, the following participation constraints and incentive compatibility constraints have to be satisfied:

$$m_{0L}-ke_{1L}^2+\alpha_L e_{1L}m_{1L}-\alpha_L e_{1L}ke_{2L}^2+\alpha_L^2 e_{1L}e_{2L}m_{2L} \ge 0 \tag{A.374}$$

$$m_{0H}-ke_{1H}^2+\alpha_H e_{1H}m_{1H}-\alpha_H e_{1H}ke_{2H}^2+\alpha_H^2 e_{1H}e_{2H}m_{2H} \ge 0 \tag{A.375}$$

$$\begin{aligned} & m_{0L}-ke_{1L}^2+\alpha_L e_{1L}m_{1L}-\alpha_L e_{1L}ke_{2L}^2+\alpha_L^2 e_{1L}e_{2L}m_{2L} \\ \ge\ & m_{0H}-k\tilde{e}_{1L}^2+\alpha_L \tilde{e}_{1L}m_{1H}-\alpha_L \tilde{e}_{1L}k\tilde{e}_{2L}^2+\alpha_L^2 \tilde{e}_{1L}\tilde{e}_{2L}m_{2H} \end{aligned} \tag{A.376}$$

$$\begin{aligned} & m_{0H}-ke_{1H}^2+\alpha_H e_{1H}m_{1H}-\alpha_H e_{1H}ke_{2H}^2+\alpha_H^2 e_{1H}e_{2H}m_{2H} \\ \ge\ & m_{0L}-k\tilde{e}_{1H}^2+\alpha_H \tilde{e}_{1H}m_{1L}-\alpha_H \tilde{e}_{1H}k\tilde{e}_{2H}^2+\alpha_H^2 \tilde{e}_{1H}\tilde{e}_{2H}m_{2L} \end{aligned} \tag{A.377}$$

where (A.374) and (A.375) are the low-type and high-type agents' participation constraints, and (A.376) and (A.377) are the low-type and high-type agents' incentive compatibility constraints. In addition, $\tilde{e}_{1L}$ and $\tilde{e}_{2L}$ are the efforts when the low-type agent pretends to be the high one, while $\tilde{e}_{1H}$ and $\tilde{e}_{2H}$ are the efforts when the high-type agent pretends to be the low-type one.

We know that by eliminating intermediate money transfers $m_{1L}$ and $m_{1H}$ from expressions (A.371) through (A.377), the principal maximizes her expected profit in case 2). Assuming that $(\overline{m}^*_{0L}, \overline{m}^*_{2L})$ and $(\overline{m}^*_{0H}, \overline{m}^*_{2H})$ is the solution for the menu of money transfers in case 2), by adding 0 as intermediate money transfer, the menu of money transfers $(\overline{m}^*_{0L}, 0, \overline{m}^*_{2L})$ and $(\overline{m}^*_{0H}, 0, \overline{m}^*_{2H})$ satisfy (A.374), (A.375), (A.376) and (A.377) in case 1) and allow the principal to achieve the same expected profit in case 2). On the other hand, in the proof of Theorem 2, we showed that the solution of the principal's expected profit maximization problem consisting of (A.371) through (A.377) has the feature that the intermediate money transfer to the low-type agent $m^*_{1L} > 0$. Therefore, the principal's expected profit in case 1) must be greater than her expected profit in case 2).

Next we show that the principal's expected profit in case 2) (i.e., no intermediate money transfer) is greater than that in case 3) (i.e., no upfront money transfer). Instead of using a direct approach, we are going to adopt a different way, namely, proving that the principal's expected profit in case 2) (i.e., no intermediate money transfer) is greater than $\frac{1}{2}\left[\frac{\alpha_L^6 V^4}{64k^3} + \frac{\alpha_H^6 V^4}{64k^3}\right]$ - the principal's expected profit in case 3) (i.e., no upfront money transfer) under complete information in Theorem 3, which serves as the upper bound of the principal's expected profit in case 3) under incomplete information.

Notice that with $X$ denoting $\frac{\alpha_H}{\alpha_L} \geq 1$, and $\rho$ denoting $\frac{1-\lambda}{\lambda} > 0$, using similar argument in the proof of Theorem 2, we can show that the principal's expected profit in case 2) is

$$\begin{aligned} & \lambda\left[\frac{\alpha_L^6}{16k^3}\left(\frac{3V}{\rho\left(X^6-1\right)+3}\right)^3\left(\frac{\rho\left(X^6-1\right)V}{\rho\left(X^6-1\right)+3}\right)\right] + \lambda\left[\frac{\alpha_L^6}{64k^3}\left(\frac{3V}{\rho\left(X^6-1\right)+3}\right)^4\right] \\ + \quad & (1-\lambda)\left[\frac{\alpha_H^6 V^4}{64k^3} + \frac{\left(\alpha_L^6 - \alpha_H^6\right)}{64k^3}\left(\frac{3V}{\rho\left(X^6-1\right)+3}\right)^4\right] \end{aligned} \tag{A.378}$$

The verification of (A.378) is allocated to the next subsection, due to its length.

We rewrite (A.378) as

$$\begin{aligned} & \lambda\alpha_L^6\left[\frac{1}{64k^3}\frac{108\rho\left(X^6-1\right)+81}{\left(\rho\left(X^6-1\right)+3\right)^4} + \frac{\rho X^6}{64k^3} + \frac{\rho\left(1-X^6\right)}{64k^3}\frac{81}{\left(\rho\left(X^6-1\right)+3\right)^4}\right]V^4 \\ = \quad & \lambda\frac{\alpha_L^6}{64k^3}\left[\frac{27\rho\left(X^6-1\right)+81}{\left(\rho\left(X^6-1\right)+3\right)^4} + \rho X^6\right]V^4 \end{aligned} \tag{A.379}$$

Denoting $\rho\left(X^6-1\right)$ by $Y$, (A.379) can be written as

$$\lambda\frac{\alpha_L^6}{64k^3}\left[\frac{27Y+81}{(Y+3)^4}+Y+\rho\right]V^4 \tag{A.380}$$

On the other hand,

$$\begin{aligned}&\frac{1}{2}\left[\frac{\alpha_L^6 V^4}{64k^3}+\frac{\alpha_H^6 V^4}{64k^3}\right]\\ =&\ \lambda\frac{\alpha_L^6}{64k^3}\left[\frac{1}{2}+\frac{1}{2}\rho X^6\right]\\ =&\ \lambda\frac{\alpha_L^6}{64k^3}\left[\frac{1}{2}+\frac{1}{2}Y+\frac{1}{2}\rho\right]\end{aligned} \tag{A.381}$$

Thus to show the principal's expected profit in case 2) is greater than $\frac{1}{2}\left[\frac{\alpha_L^6 V^4}{64k^3}+\frac{\alpha_H^6 V^4}{64k^3}\right]$ is equivalent to show

$$\frac{27Y+81}{(Y+3)^4}+Y+\rho>\frac{1}{2}+\frac{1}{2}Y+\frac{1}{2}\rho \tag{A.382}$$

i.e.,

$$\frac{2\,(27Y+81)}{(Y+3)^4}+Y+\rho>1 \tag{A.383}$$

i.e.,

$$2\,(27Y+81)-(1-Y)(Y+3)^4+\rho(Y+3)^4>0 \tag{A.384}$$

i.e.,

$$(Y+3)\left[54-(1-Y)(Y+3)^3\right]+\rho(\rho Y+3)^4>0 \tag{A.385}$$

Let $F(Y)=54-(1-Y)(Y+3)^3$, the derivative of $F(Y)$ with respect to $Y$ is

$$\begin{aligned}&(Y+3)^3-(1-Y)\cdot 3(Y+3)^2\\ =&\ (Y+3)^2\,[Y+3-3+3Y]\\ =&\ (Y+3)^2\cdot 4Y\\ \geq&\ 0\end{aligned} \tag{A.386}$$

because $Y\geq 0$. In fact the derivative of $F(Y)$ is strictly $>0$ when $Y>0$ i.e., $X^6>1$. Since $F(0)=54-27=27>0$, $F(Y)$ must be positive for any $Y\geq 0$. Therefore (A.385) holds for any $Y\geq 0$. This means that the principal's expected profit in case 2) is greater than $\frac{1}{2}\left[\frac{\alpha_L^6 V^4}{64k^3}+\frac{\alpha_H^6 V^4}{64k^3}\right]$. Therefore, we proved that the principal's expected profit in case 2) (i.e., no intermediate money transfer) is greater than that in case 3) (i.e., no upfront money transfer).

Finally, we show that the principal's expected profit in case 3) (i.e., no intermediate money transfer) is greater than that in case 4) (i.e., only end money transfer).

Recall that by the end of the proof of Theorem 3, we mentioned that the principal's expected profit in case 4) under incomplete information is $\frac{27}{64}\left[\frac{\alpha_L^6 V^4}{64k^3} + \frac{\alpha_H^6 V^4}{64k^3}\right]$, with $m_{2L}^* = m_{2H}^* = \frac{3}{4}V$. Considering the following principal's expected profit maximization problem

$$\lambda\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_L \frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right)$$
$$+ \quad (1-\lambda)\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) \qquad \text{(A.387)}$$

with $m_{2L}$ and $m_{2H}$ being set as $\frac{3}{4}V$. Then (A.387) is a function of $m_{1L}$ and $m_{1H}$. The first order conditions with respect to $m_{1L}$ and $m_{1H}$ are:

$$\frac{\alpha_L}{2k}\left(\alpha_L^2 \frac{3}{32k}V^2 - m_{1L}\right) - \frac{\alpha_L m_{1L}}{2k} - \frac{\alpha_L^3 \frac{9}{64}V^2}{2k} = 0 \qquad \text{(A.388)}$$

$$\frac{\alpha_H}{2k}\left(\alpha_H^2 \frac{3}{32k}V^2 - m_{1H}\right) - \frac{\alpha_H m_{1H}}{2k} - \frac{\alpha_H^3 \frac{9}{64}V^2}{2k} = 0 \qquad \text{(A.389)}$$

which gives $m_{1L}^* = -\frac{3\alpha_L^2}{128}V^2$ and $m_{1H}^* = -\frac{3\alpha_L^2}{128}V^2$. It is clear that this is the location of maximum, because the concavity of the part of (A.387) on $m_{1L}$ and the part of (A.387) on $m_{1H}$. With $m_{1L} = m_{1H} = -\frac{3\alpha_L^2}{128}V^2$ and $m_{2L} = m_{2H} = \frac{3}{4}V$, (A.387) yields an expected profit of $\frac{225}{512}\left[\lambda\frac{\alpha_L^6 V^4}{64k^3} + (1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}\right]$, which is greater than $\frac{27}{64}\left[\lambda\frac{\alpha_L^6 V^4}{64k^3} + (1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}\right]$, the principal's expected profit in case 4).

On the other hand, it is clear that with $e_{1L} = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$, $e_{2L} = \frac{\alpha_L m_{2L}}{2k}$, $e_{1H} = \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$, $e_{2H} = \frac{\alpha_H m_{2H}}{2k}$, $\tilde{e}_{1L} = \frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k}$, $\tilde{e}_{2L} = \frac{\alpha_L m_{2H}}{2k}$, $\tilde{e}_{1H} = \frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}$, and $\tilde{e}_{2H} = \frac{\alpha_H m_{2L}}{2k}$, the menu of money transfers $(m_{1L}, m_{2L})$ and $(m_{1H}, m_{2H})$ with $m_{1L} = m_{1H} = -\frac{3\alpha_L^2}{128}V^2$ and $m_{2L} = m_{2H} = \frac{3}{4}V$ satisfy (A.374), (A.375), (A.376), and (A.377), when $m_{0L}$ and $m_{0H}$ are eliminated from them. This means that the principal's expected profit in case 3) must be greater or equal to $\frac{225}{512}\left[\lambda\frac{\alpha_L^6 V^4}{64k^3} + (1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}\right]$. Therefore the principal's expected profit in case 3) is greater than the principal's expected profit in case 4).

### A.4.2 The principal's expected profit in case 2)

In this subsection, we show that the principal's expected profit in case 2) is (A.378), which was used in the previous subsection. Similar to the proof of Theorem 2, the discussion consists of three progressive parts. In the first part, for given money transfers $(m_{0L}, m_{2L})$ and $(m_{0H}, m_{2H})$, we establish the expressions of optimal efforts of both types of agents and their profits. In the second part, we solve the principal's expected profit maximization problem for a particular region where the money transfers take values, using the technique of decomposing the problem into two independent problems with each of them only associated to one type of agent. In the third part, we show that when $V \leq \frac{2k}{\alpha_H}$, the local maximum obtained in the second part is the global maximum by ruling out the possible local maxima in other regions.

### A.4.2.1 Expressions of optimal efforts

By offering two different menus of money transfers $(m_{0L}, m_{2L})$ and $(m_{0H}, m_{2H})$ to the low-type and high-type agents separately, the principal maximizes her expected profit:

$$\lambda\left[\alpha_L e_{1L}\alpha_L e_{2L}(V - m_{2L}) - m_{0L}\right] + (1-\lambda)\left[\alpha_H e_{1H}\alpha_H e_{2H}(V - m_{2H}) - m_{0H}\right] \tag{A.390}$$

For given money transfers $(m_{0L}, m_{2L})$ and $(m_{0H}, m_{2H})$, the low-type and high-type agents maximize their following expected profits, respectively

$$m_{0L} - ke_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \tag{A.391}$$

and

$$m_{0H} - ke_{1H}^2 - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \tag{A.392}$$

where $0 \leq e_{1L} \leq 1$, $0 \leq e_{2L} \leq 1$, $0 \leq e_{1H} \leq 1$ and $0 \leq e_{2H} \leq 1$. (A.391) and (A.392) have to be nonnegative in order for both agents to participate.

Using a similar argument as in the proof of Theorem 2, we can assume the positivity of $e_{1L}$, $e_{2L}$, $e_{1H}$ and $e_{2H}$ and have following situations for the high-type and low-type agents, respectively.

For the high-type agent, there are three situations:

1. $m_{2H} \leq \frac{2k}{\alpha_H}$ and $\frac{\alpha_H^3 m_{2H}^2}{8k^2} \leq 1$, i.e., $e_{2H}^* \leq 1$ and $e_{1H}^* \leq 1$. The high-type agent's expected profit is
$$m_{0H} + k\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)^2 \tag{A.393}$$

2. $m_{2H} > \frac{2k}{\alpha_H}$ and $-\alpha_H k + \alpha_H^2 m_{2H} \leq 2k$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* \leq 1$. The high-type agent's expected profit is
$$m_{0H} + k\left(\frac{-\alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)^2 \tag{A.394}$$

3. $m_{2H} > \frac{2k}{\alpha_H}$ and $-\alpha_H k + \alpha_H^2 m_{2H} > 2k$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* = 1$. The high-type agent's expected profit is
$$m_{0H} - k - \alpha_H k + \alpha_H^2 m_{2H} \tag{A.395}$$

Notice that the case where $m_{2H} \le \frac{2k}{\alpha_H}$ and $\frac{\alpha_H^3 m_{2H}^2}{8k^2} > 1$ does not exist, because $m_{2H} \le \frac{2k}{\alpha_H}$ leads to $\frac{\alpha_H^3 m_{2H}^2}{8k^2} \le \frac{\alpha_H}{2} < 1$.

For the low-type agent, there are three situations:

1. $m_{2L} \le \frac{2k}{\alpha_L}$ and $\frac{\alpha_L^3 m_{2L}^2}{8k^2} \le 1$, i.e., $e_{2L}^* \le 1$ and $e_{1L}^* \le 1$. The low-type agent's expected profit is

$$m_{0L} + k\left(\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right)^2 \tag{A.396}$$

2. $m_{2L} > \frac{2k}{\alpha_L}$ and $-\alpha_L k + \alpha_L^2 m_{2L} \le 2k$, i.e., $e_{2L}^* = 1$ and $e_{1L}^* \le 1$. The low-type agent's expected profit is

$$m_{0L} + k\left(\frac{-\alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2 \tag{A.397}$$

3. $m_{2L} > \frac{2k}{\alpha_L}$ and $-\alpha_L k + \alpha_L^2 m_{2L} > 2k$, i.e., $e_{2L}^* = 1$ and $e_{1L}^* = 1$. The high-type agent's expected profit is

$$m_{0L} - k - \alpha_L k + \alpha_L^2 m_{2L} \tag{A.398}$$

Since $m_{2L} \le \frac{2k}{\alpha_L}$ leads to $\frac{\alpha_L^3 m_{2L}^2}{8k^2} \le \frac{\alpha_L}{2}$, there is no case in which $m_{2L} \le \frac{2k}{\alpha_L}$ and $\frac{\alpha_L^3 m_{2L}^2}{8k^2} > 1$.

In addition, because of incentive constraints involved in the model, it is necessary to consider two scenarios when each type agent is not honest about his true type: one is that the low-type agent pretends to be the high-type agent, and the other is that the high-agent pretends to be the low-type agent.

When the low-type agent pretends to be the high-type agent, if successful, his expected profit would be

$$m_{0H} - k\tilde{e}_{1L}^2 - \alpha_L \tilde{e}_{1L} k \tilde{e}_{2L}^2 + \alpha_L^2 \tilde{e}_{1L} \tilde{e}_{2L} m_{2H} \tag{A.399}$$

There are three situations:

1. $m_{2H} \le \frac{2k}{\alpha_L}$ and $\frac{\alpha_L^3 m_{2H}^2}{8k^2} \le 1$, i.e., $\tilde{e}_{2L}^* \le 1$ and $\tilde{e}_{1L}^* \le 1$. The low-type agent's expected profit is

$$m_{0H} + k\left(\frac{\alpha_L^3 m_{2H}^2}{8k^2}\right)^2 \tag{A.400}$$

2. $m_{2H} > \frac{2k}{\alpha_L}$ and $-\alpha_L k + \alpha_L^2 m_{2H} \le 2k$, i.e., $\tilde{e}_{2L}^* = 1$ and $\tilde{e}_{1L}^* \le 1$. The low-type agent's expected profit is

$$m_{0H} + k\left(\frac{-\alpha_L k + \alpha_L^2 m_{2H}}{2k}\right)^2 \tag{A.401}$$

3. $m_{2H} > \frac{2k}{\alpha_L}$ and $-\alpha_L k + \alpha_L^2 m_{2H} > 2k$, i.e., $\tilde{e}_{2L}^* = 1$ and $\tilde{e}_{1L}^* = 1$ . The low-type agent's expected profit is

$$m_{0H} - k - \alpha_L k + \alpha_L^2 m_{2H} \tag{A.402}$$

When the high-type agent pretends to be the low-type agent, if successful, his expected profit would be

$$m_{0L} - k\tilde{e}_{1H}^2 - \alpha_H \tilde{e}_{1H} k \tilde{e}_{2H}^2 + \alpha_H^2 \tilde{e}_{1H} \tilde{e}_{2H} m_{2L} \tag{A.403}$$

There are three situations:

1. $m_{2L} \leq \frac{2k}{\alpha_H}$ and $\frac{\alpha_H^3 m_{2L}^2}{8k^2} \leq 1$, i.e., $\tilde{e}_{2H}^* \leq 1$ and $\tilde{e}_{1H}^* \leq 1$. The high-type agent's expected profit is

$$m_{0L} + k\left(\frac{\alpha_H^3 m_{2L}^2}{8k^2}\right)^2 \tag{A.404}$$

2. $m_{2L} > \frac{2k}{\alpha_H}$ and $-\alpha_H k + \alpha_H^2 m_{2L} \leq 2k$, i.e., $\tilde{e}_{2H}^* = 1$ and $\tilde{e}_{1H}^* \leq 1$. The high-type agent's expected profit is

$$m_{0L} + k\left(\frac{-\alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2 \tag{A.405}$$

3. $m_{2L} > \frac{2k}{\alpha_H}$ and $-\alpha_H k + \alpha_H^2 m_{2L} > 2k$, i.e., $\tilde{e}_{2H}^* = 1$ and $\tilde{e}_{1H}^* = 1$ . The high-type agent's expected profit is

$$m_{0L} - k - \alpha_H k + \alpha_H^2 m_{2L} \tag{A.406}$$

In the following we will solve the principal's expected profit maximization problem for the region where $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k} \leq 1$ $\tilde{e}_{2H}^* = \frac{\alpha_H m_{2L}}{2k} \leq 1$, which imply that $e_{1H}^* = \frac{\frac{\alpha_H^3 m_{2H}}{4k}}{2k} < 1$, $\tilde{e}_{1H}^* = \frac{\frac{\alpha_H^3 m_{2L}}{4k}}{2k} < 1$, $e_{1L}^* = \frac{\frac{\alpha_L^3 m_{2L}}{4k}}{2k} < 1$, $e_{2L}^* = \frac{\alpha_L m_{2L}}{2k} < 1$, $\tilde{e}_{1L}^* = \frac{\frac{\alpha_L^3 m_{2H}}{4k}}{2k} < 1$, and $\tilde{e}_{2L}^* = \frac{\alpha_L m_{2H}}{2k} < 1$. This means that for given money transfers, we have the best response functions of efforts, which can be used in the optimization problem.

#### A.4.2.2 The local maximum of a particular region

Similar to what we did in the proof of Theorem 2, we can assume the positivity of $e_{1H}^*$, $e_{2H}^*$, $\tilde{e}_{1H}^*$, $\tilde{e}_{2H}^*$, $e_{1L}^*$, $e_{2L}^*$, $\tilde{e}_{1L}^*$ and $\tilde{e}_{2L}^*$. There are four constraints for consideration: the participation constraints and the incentive compatibility constraints for for both types of agents. We will find the local interior maximum for the region described above. Later on we will show that this local interior maximal solution is also a unique global maximal solution when $V \leq \frac{2k}{\alpha_H}$.

To differentiate the two different types of agent and meanwhile maximize her expected profit, the principal faces the following optimization problem:

$$\max_{\substack{(m_{0L},m_{2L})\\(m_{0H},m_{2H})}} \lambda\Pi_{L,P} + (1-\lambda)\Pi_{H,P} \tag{A.407}$$

where $\lambda\Pi_{L,P}$ and $(1-\lambda)\Pi_{H,P}$ are the expected profits obtained from the low-type and the high-type agents, respectively, satisfying

$$\Pi_{L,P} = \frac{\alpha_L^4 m_{2L}^2}{8k^2}\left[\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L})\right] - m_{0L} \tag{A.408}$$

$$\Pi_{H,P} = \frac{\alpha_H^4 m_{2H}^2}{8k^2}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H})\right] - m_{0H} \tag{A.409}$$

subject to

$$m_{0L} + k\left[\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right]^2 \geq m_{0H} + k\left[\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right]^2 \tag{A.410}$$

$$m_{0L} + k\left[\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right]^2 \geq 0 \tag{A.411}$$

$$m_{0H} + k\left[\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right]^2 \geq m_{0L} + k\left[\frac{\alpha_H^3 m_{2L}^2}{8k^2}\right]^2 \tag{A.412}$$

$$m_{0H} + k\left[\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right]^2 \geq 0 \tag{A.413}$$

It is clear that (A.413) is redundant, because (A.411) and (A.412) imply that

$$m_{0H} + k\left[\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right]^2 \geq m_{0L} + k\left[\frac{\alpha_H^3 m_{2L}^2}{8k^2}\right]^2 \geq m_{0L} + k\left[\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right]^2 \tag{A.414}$$

Therefore the optimization problem consisting of (A.407), (A.410), (A.411), (A.412) and (A.413) is equivalent to the optimization problem consisting of (A.407), (A.410), (A.411), and (A.412), namely, excluding (A.413). In fact, constraint (A.410) is also redundant. To show this, we first show that the optimization problem consisting of (A.407), (A.411), and (A.412) can be solved and then prove that the solution set satisfies (A.410).

Now we solve the optimization problem consisting of (A.407), (A.411), and (A.412). The corresponding Lagrangian is

$$\begin{aligned}
&\lambda\left[\frac{\alpha_L^4 m_{2L}^2}{8k^2}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L})\right) - m_{0L}\right] \\
&+(1-\lambda)\left[\frac{\alpha_H^4 m_{2H}^2}{8k^2}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H})\right) - m_{0H}\right] \\
&+\lambda_1\left[m_{0L} + k\left(\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right)^2\right] + \lambda_2\left[m_{0H} + k\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)^2 - m_{0L} - k\left(\frac{\alpha_H^3 m_{2L}^2}{8k^2}\right)^2\right]
\end{aligned} \tag{A.415}$$

The first order conditions of (A.415) with respect to $m_{0L}$, $m_{0H}$, $m_{2L}$ and $m_{2H}$ are

$$-\lambda + \lambda_1 - \lambda_2 = 0 \tag{A.416}$$

$$-(1-\lambda) + \lambda_2 = 0 \tag{A.417}$$

$$\lambda\left[\frac{\alpha_L^6 3m_{2L}^2}{16k^3}(V - m_{2L}) - \frac{\alpha_L^6 m_{2L}^3}{16k^3}\right] + \lambda_1 \frac{\alpha_L^6 m_{2L}^3}{16k^3} - \lambda_2 \frac{\alpha_H^6 m_{2L}^3}{16k^3} = 0 \tag{A.418}$$

$$(1-\lambda)\left[\frac{\alpha_H^6 3m_{2H}^2}{16k^3}(V - m_{2H}) - \frac{\alpha_H^6 m_{2H}^3}{16k^3}\right] + \lambda_2 \frac{\alpha_H^6 m_{2H}^3}{16k^3} = 0 \tag{A.419}$$

where (A.417) and (A.416) imply that $\lambda_2 = 1-\lambda$ and $\lambda_1 = 1$. Substituting $1-\lambda$ for $\lambda_2$ in (A.419), we have

$$(1-\lambda)\frac{\alpha_H^6 3m_{2H}^2}{16k^3}(V - m_{2H}) = 0 \tag{A.420}$$

which means that $m_{2H}^* = V$, because $e_{2H}^* > 0$ implies $m_{2H} > 0$. Thus when $V \le \frac{2k}{\alpha_H}$, $\frac{\alpha_H m_{2H}^*}{2k} < 1$ and $\frac{\alpha_L m_{2H}^*}{2k} < 1$.

Substituting 1 for $\lambda_1$ and $1-\lambda$ for $\lambda_2$ in (A.418) and rearranging the terms gives

$$\lambda\left[\frac{\alpha_L^6 3m_{2L}^2}{16k^3}(V - m_{2L})\right] = (1-\lambda)\left(\frac{\alpha_H^6 - \alpha_L^6}{16k^3}\right) m_{2L}^3 \tag{A.421}$$

namely,

$$3\alpha_L^3 (V - m_{2L}) = \rho\left(\alpha_H^6 - \alpha_L^6\right) m_{2L} \tag{A.422}$$

where $\rho = \frac{1-\lambda}{\lambda}$. Solving (A.422) leads to

$$m_{2L}^* = \frac{3V}{\rho\,(X^6 - 1) + 3} \tag{A.423}$$

where $X = \frac{\alpha_H}{\alpha_L} > 1$. This means that

$$V - m_{2L}^* = \frac{\rho\left(X^6 - 1\right) V}{\rho\,(X^6 - 1) + 3} \tag{A.424}$$

It is clear that $0 < m_{2L}^* < V$. Thus when $V \le \frac{2k}{\alpha_H}$, $\frac{\alpha_H m_{2L}^*}{2k} < 1$ and $\frac{\alpha_L m_{2L}^*}{2k} < 1$, namely, $m_{2L}^*$ is an interior solution.

Since $\lambda_1 > 0$ and $\lambda_2 > 0$, constraints (A.411) and (A.412) are binding. Therefore

$$m_{0L}^* = -k\left[\frac{\alpha_L^3 {m_{2L}^*}^2}{8k^2}\right]^2 = -\frac{\alpha_L^6}{64k^3}\left[\frac{3V}{\rho\,(X^6 - 1) + 3}\right]^4 \tag{A.425}$$

and

$$m_{0H}^* = -\frac{\alpha_L^6 m_{2L}^4}{64k^3} + \frac{\alpha_H^6 m_{2L}^4}{64k^3} - \frac{\alpha_H^6 m_{2H}^4}{64k^3} = \left[\frac{\alpha_H^6 - \alpha_L^6}{64k^3}\right]\left[\frac{3V}{\rho\,(X^6 - 1) + 3}\right]^4 - \frac{\alpha_H^6}{64k^3} V^4 \tag{A.426}$$

In this situation, the principal's expected profit equals

$$\lambda\left[\frac{\alpha_L^6}{16k^3}\left(\frac{3V}{\rho\left(X^6-1\right)+3}\right)^3\left(\frac{\rho\left(X^6-1\right)V}{\rho\left(X^6-1\right)+3}\right)\right]+\lambda\left[\frac{\alpha_L^6}{64k^3}\left(\frac{3V}{\rho\left(X^6-1\right)+3}\right)^4\right]$$
$$+\quad(1-\lambda)\left[\frac{\alpha_H^6V^4}{64k^3}+\frac{\left(\alpha_L^6-\alpha_H^6\right)}{64k^3}\left(\frac{3V}{\rho\left(X^6-1\right)+3}\right)^4\right] \tag{A.427}$$

Now we verify that (A.410) is redundant. Notice that using binding constraints (A.411) and (A.412), (A.410) is equivalent to

$$0\geq-k\left[\frac{\alpha_H^3m_{2H}^2}{8k^2}\right]^2-k\left[\frac{\alpha_L^3m_{2H}^2}{8k^2}\right]^2+k\left[\frac{\alpha_H^3m_{2L}^2}{8k^2}\right]^2+k\left[\frac{\alpha_L^3m_{2L}^2}{8k^2}\right]^2 \tag{A.428}$$

namely,

$$\left(\alpha_H^6-\alpha_L^6\right)m_{2H}^4\geq\left(\alpha_H^6-\alpha_L^6\right)m_{2L}^4 \tag{A.429}$$

Clearly, the interior solution set $m_{2H}^*=V$ and $m_{2L}^*=\frac{3V}{\rho(X^6-1)+3}<V$ satisfy (A.429). This shows that (A.410) is redundant.

Notice that the maximization problem consisting of (A.407) through (A.413) is equivalent to

$$\max_{\substack{(m_{0L},m_{2L})\\(m_{0H},m_{2H})}}\lambda\Pi_{L,P}+(1-\lambda)\Pi_{H,P} \tag{A.430}$$

where

$$\Pi_{L,P}\quad=\quad\frac{\alpha_L^4m_{2L}^2}{8k^2}\left[\frac{\alpha_L^2m_{2L}}{2k}(V-m_{2L})\right]+k\left[\frac{\alpha_L^3m_{2L}^2}{8k^2}\right]^2 \tag{A.431}$$

$$\Pi_{H,P}\quad=\quad\frac{\alpha_H^4m_{2H}^2}{8k^2}\left[\frac{\alpha_H^2m_{2H}}{2k}(V-m_{2H})\right]+k\left[\frac{\alpha_H^3m_{2H}^2}{8k^2}\right]^2+k\left[\frac{\alpha_L^3m_{2L}^2}{8k^2}\right]^2-k\left[\frac{\alpha_H^3m_{2L}^2}{8k^2}\right]^2 \tag{A.432}$$

because constraints (A.411) and (A.412) are binding, while constraints (A.410) and (A.413) are not binding.

We can rewrite $\lambda\Pi_{L,P}+(1-\lambda)\Pi_{H,P}$ as $\overline{\Pi}_{L,P}+\overline{\Pi}_{H,P}$, where

$$\overline{\Pi}_{L,P}\quad=\quad\lambda\left[\frac{\alpha_L^4m_{2L}^2}{8k^2}\left(\frac{\alpha_L^2m_{2L}}{2k}(V-m_{2L})\right)+k\left(\frac{\alpha_L^3m_{2L}^2}{8k^2}\right)^2\right.$$
$$\left.+(1-\lambda)\left[k\left(\frac{\alpha_L^3m_{2L}^2}{8k^2}\right)^2-k\left(\frac{\alpha_H^3m_{2L}^2}{8k^2}\right)^2\right]\right. \tag{A.433}$$

$$\overline{\Pi}_{H,P}\quad=\quad(1-\lambda)\left[\frac{\alpha_H^4m_{2H}^2}{8k^2}\left(\frac{\alpha_H^2m_{2H}}{2k}(V-m_{2H})\right)+k\left(\frac{\alpha_H^3m_{2H}^2}{8k^2}\right)^2\right] \tag{A.434}$$

Thus the maximization problem consisting of (A.407) through (A.413) is equivalent to the maximization problem

$$\max_{m_{2L}} \overline{\Pi}_{L,P} + \max_{m_{2H}} \overline{\Pi}_{H,P} \tag{A.435}$$

In other words, we decompose the maximization problem consisting of (A.407) through (A.413) into two separate maximization problems, with one about $m_{2L}$ and the other about $m_{2H}$. We will apply the same procedure to the scenarios when $m_{2H}$ and $m_{2L}$ belong to different regions.

#### A.4.2.3 The global maximum when $V \leq \frac{2k}{\alpha_H}$

Notice that for different regions, the expressions of $\overline{\Pi}_{H,P}$ and $\overline{\Pi}_{L,P}$ may take different forms.

First we look at the maximization problem of $\overline{\Pi}_{H,P}$. Considering all possible expressions which $m_{1H}$ and $m_{2H}$ can take, we will find the expressions of optimal money transfers $m_{1H}$ and $m_{2H}$ to maximize $\overline{\Pi}_{H,P}$.

Like what we discussed in (A.393), (A.394) and (A.395), there are three scenarios for consideration:

1. $m_{2H} \leq \frac{2k}{\alpha_H}$ and $\frac{\alpha_H^3 m_{2H}^2}{8k^2} \leq 1$, i.e., $e_{2H}^* \leq 1$ and $e_{1H}^* \leq 1$. The Lagrangian for the maximum of $\overline{\Pi}_{H,P}$ is

$$(1-\lambda)\left[\frac{\alpha_H^4 m_{2H}^2}{8k^2}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H})\right) + k\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)^2\right] + \lambda_1\left(\frac{2k}{\alpha_H} - m_{2H}\right) \tag{A.436}$$

The first order condition with respect to $m_{2H}$ of (A.436) gives

$$(1-\lambda)\frac{\alpha_H^6 3m_{2H}^2}{16k^3}(V - m_{2H}) - \lambda_1 = 0 \tag{A.437}$$

When $\lambda_1 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$. It is clear that to make (A.438) hold, $V$ has to be greater than $\frac{2k}{\alpha_H}$. Thus $\overline{\Pi}_{H,P}$ equals

$$\begin{aligned}
& (1-\lambda)\left[\frac{\alpha_H^4 m_{2H}^2}{8k^2}\left(\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H})\right) + k\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)^2\right] \\
= \;& (1-\lambda)\left[\frac{\alpha_H^{(\frac{2k}{\alpha_H})^2}}{8k^2}\left(\frac{\alpha_H^2(\frac{2k}{\alpha_H})}{2k}(V - (\frac{2k}{\alpha_H}))\right) + k\left(\frac{\alpha_H^3(\frac{2k}{\alpha_H})^2}{8k^2}\right)^2\right] \\
= \;& (1-\lambda)\left[\frac{\alpha_H^3}{2}\left(V - \frac{2k}{\alpha_H}\right) + \frac{\alpha_H^2}{4}k\right]
\end{aligned} \tag{A.438}$$

When $\lambda_1 = 0$, (A.438) implies $V = m_{2H} \le \frac{2k}{\alpha_H}$. Thus $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)k\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)^2 = (1-\lambda)k\left(\frac{\alpha_H^3(\frac{2k}{\alpha_H})^2}{8k^2}\right)^2 = (1-\lambda)\frac{\alpha_H^2}{4}k \qquad \text{(A.439)}$$

2. $m_{2H} > \frac{2k}{\alpha_H}$ and $-\alpha_H k + \alpha_H^2 m_{2H} \le 2k$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* \le 1$. The Lagrangian for the maximum of $\overline{\Pi}_{H,P}$ is

$$(1-\lambda)\left[\alpha_H\left(\frac{-\alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)(\alpha_H(V - m_{2H})) + k\left(\frac{-\alpha_H k + \alpha_H^2 m_{2H}}{2k}\right)^2\right]$$
$$+\lambda_1\left(\frac{2k}{\alpha_H} - m_{2H}\right) + \lambda_2\left(2k + \alpha_H k - \alpha_H^2 m_{2H}\right) \qquad \text{(A.440)}$$

The first order condition of (A.440) with respect to $m_{2H}$ gives

$$(1-\lambda)\frac{\alpha_H^4}{2k}(V - m_{2H}) + \lambda_1 - \lambda_3\alpha_H^2 = 0 \qquad \text{(A.441)}$$

When $\lambda_1 > 0$, $m_{2H} = \frac{2k}{\alpha_H}$, which implies that $-\alpha_H k + \alpha_H^2 m_{2H} = \alpha_H k < 2k$. So constraint $-\alpha_H k + \alpha_H^2 m_{2H} \le 2k$ is not binding. This means that $\lambda_3 = 0$. Therefore (A.441) implies $V < m_{2H} = \frac{2k}{\alpha_H}$. $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)\left[\alpha_H\left(\frac{-\alpha_H k + \alpha_H^2\frac{2k}{\alpha_H}}{2k}\right)\left(\alpha_H\left(V - \frac{2k}{\alpha_H}\right)\right) + k\left(\frac{-\alpha_H k + \alpha_H^2\frac{2k}{\alpha_H}}{2k}\right)^2\right]$$
$$= (1-\lambda)\left[\frac{\alpha_H^3}{2}\left(V - \frac{2k}{\alpha_H}\right) + \frac{\alpha_H^2}{4}k\right] \qquad \text{(A.442)}$$

which is less than $(1-\lambda)\frac{\alpha_H^2}{4}k$ for $V < m_{2H} = \frac{2k}{\alpha_H}$.

When $\lambda_1 = 0$ and $\lambda_2 > 0$, $-\alpha_H k + \alpha_H^2 m_{2H} = 2k$ which means that $m_{2H} = \frac{2k+\alpha_H k}{\alpha_H^2} > \frac{2k}{\alpha_H}$. Thus $\overline{\Pi}_{H,P}$ equals

$$\begin{aligned} & (1-\lambda)\left[\alpha_H \cdot 1 \cdot \alpha_H(V - m_{2H}) + k\right] \\ = \; & (1-\lambda)\left[\alpha_H^2\left(V - \frac{2k+\alpha_H k}{\alpha_H^2}\right) + k\right] \\ = \; & (1-\lambda)\left[\alpha_H^2 V - \alpha_H k - k\right] \end{aligned} \qquad \text{(A.443)}$$

Notice that if $V \le \frac{2k}{\alpha_H}$, (A.443) is less than or equal to $2\alpha_H k - \alpha_H k - k$ which is less than 0.

When $\lambda_1 = 0$ and $\lambda_2 = 0$, (A.442) gives $V = m_{2H} \ge \frac{2k}{\alpha_H}$. Since $-\alpha_H k + \alpha_H^2 m_{2H} \le 2k$, $m_{2H} \le \frac{2k+\alpha_H k}{\alpha_H^2}$. Thus $V \le \frac{2k+\alpha_H k}{\alpha_H^2}$, i.e., $\alpha_H^2 V - \alpha_H k - 2k$. $\overline{\Pi}_{H,P}$ equals

$$(1-\lambda)k\left(\frac{-\alpha_H k+\alpha_H^2 V}{2k}\right)^2 \tag{A.444}$$

Notice that

$$k\left(\frac{-\alpha_H k+\alpha_H^2 V}{2k}\right)^2 \geq \alpha_H^2 V-\alpha_H k-k \tag{A.445}$$

with the equality holding only when $\alpha_H^2 V-\alpha_H k=2k$. To show it, let $x=\alpha_H^2 V-\alpha_H k$ and use the inequality $\frac{x^2}{4k}\geq x-k$.

On the other hand, we have

$$k\left(\frac{-\alpha_H k+\alpha_H^2 V}{2k}\right)^2 \geq \left[\frac{\alpha_H^3}{2}\left(V-\frac{2k}{\alpha_H}\right)+\frac{\alpha_H^2}{4}k\right] \tag{A.446}$$

because (A.446) is equivalent to

$$\frac{\alpha_H^2\left(\alpha_H^2 V^2-2\alpha_H V k+k^2\right)}{4k} \geq \left[\frac{\alpha_H^3}{2}\left(V-\frac{2k}{\alpha_H}\right)+\frac{\alpha_H^2}{4}k\right] \tag{A.447}$$

which is equivalent to

$$\frac{\alpha_H V\left(V-\frac{2k}{\alpha_H}\right)}{4k} \geq \frac{1}{2}\left(V-\frac{2k}{\alpha_h}\right) \tag{A.448}$$

It is clear that when $V=\frac{2k}{\alpha_H}$, (A.448) holds with equality, and when $V>\frac{2k}{\alpha_H}$, (A.448) is equivalent to $\frac{\alpha_H V}{4k}\geq\frac{1}{2}$ which is true and in fact the strict inequality holds.

3. $m_{2H}>\frac{2k}{\alpha_H}$ and $-\alpha_H k+\alpha_H^2 m_{2H}>2k$, i.e., $e_{2H}^*=1$ and $e_{1H}^*=1$. $\overline{\Pi}_{H,P}$ is

$$\begin{aligned} & (1-\lambda)\left[\alpha_H^2(V-m_{2H})-k-\alpha_H k+\alpha_H^2 m_{2H}\right] \\ = \quad & (1-\lambda)\left[\alpha_H^2 V-\alpha_H k-k\right] \end{aligned} \tag{A.449}$$

which gives the highest value of $\overline{\Pi}_{H,P}$ when $V>\frac{2k+\alpha_H k}{\alpha_H^2}$, because (A.445) only holds when $V\leq\frac{2k+\alpha_H k}{\alpha_H^2}$ i.e., $\alpha_H^2 V-\alpha_H k\leq 2k$.

One thing worthy of mentioning is that for all cases above, the expression of $\overline{\Pi}_{H,P}$ is bounded in absolute value and thereby has the maximum. The uniqueness of the solution of the first order conditions of the Lagrangian associated to $\overline{\Pi}_{H,P}$ ensures the solution is the location of the local maximum.

In summary,

1. When $V\leq\frac{2k}{\alpha_H}$, $m_{2H}^*=V$, and $\overline{\Pi}_{H,P}=(1-\lambda)\frac{\alpha_H^2}{4}k$.

2. When $V>\frac{2k}{\alpha_H}$ and $\alpha_H^2 V-\alpha_H k\leq 2k$, $m_{2H}^*=V$, and $\overline{\Pi}_{H,P}=(1-\lambda)k\left(\frac{-\alpha_H k+\alpha_H^2 V}{2k}\right)^2$.

3. When $V > \frac{2k}{\alpha_H}$ and $\alpha_H^2 V - \alpha_H k > 2k$, $-\alpha_H k + \alpha_H^2 m_{2H}^* \geq 2k$ which implies $m_{2H}^* \geq \frac{2k+\alpha_H k}{\alpha_H^2} > \frac{2k}{\alpha_H}$, $m_{2H}^* = -k - \alpha_H k + \alpha_H^2 m_{2H}^* \geq k$ and $\overline{\Pi}_{H,P} = (1-\lambda)(\alpha_H^2 V - \alpha_H k - k)$.

In the following, we look at the maximization problem of $\overline{\Pi}_{L,P}$. There are three scenarios and each scenario with several cases for consideration:

1. When $m_{2L} \leq \frac{2k}{\alpha_H}$, $\tilde{e}_{2H}^* \leq 1$. It is clear that $\tilde{e}_{1H}^* = \frac{\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} = \frac{\alpha_H^3 m_{2L}^2}{8k^2} \leq \frac{\alpha_H^3 \left(\frac{2k}{\alpha_H}\right)^2}{8k^2} = \frac{\alpha_H}{2} < 1$, and $e_{1L}^* = \frac{\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq \frac{\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq \frac{\alpha_L}{2} < 1$. The Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\lambda\left[\alpha_L \frac{\alpha_L^3 m_{2L}^2}{8k^2}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V - m_{2L})\right) + k\left(\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right)^2\right]$$
$$+(1-\lambda)\left[k\left(\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right)^2 - k\left(\frac{\alpha_H^3 m_{2L}^2}{8k^2}\right)^2\right] + \lambda_1\left(\frac{2k}{\alpha_H} - m_{2L}\right) \tag{A.450}$$

where $\lambda_1$ is a Lagrangian multiplier.

The first order condition of (A.450) with respect to $m_{2L}$ gives

$$\lambda\left[\frac{\alpha_L^6 3 m_{2L}^2}{16k^3}(V - m_2 L)\right] + (1-\lambda)\left[\frac{\alpha_L^6 m_{2L}^3}{16k^3} - \frac{\alpha_H^6 m_{2L}^3}{16k^3}\right] - \lambda_1 = 0 \tag{A.451}$$

When $\lambda_1 > 0$, $m_{2L} = \frac{2k}{\alpha_H}$. It is clear that to make (A.451) hold, $V$ has to be greater than $\frac{2k}{\alpha_H}$. $\overline{\Pi}_{L,P}$ equals

$$\lambda_1\left[\alpha_L \frac{\alpha_L^3}{\alpha_H^2}\frac{1}{2}\left(\frac{\alpha_L^2}{\alpha_H}\left(V - \frac{2k}{\alpha_H}\right)\right) + k\frac{\alpha_L^6}{4\alpha_H^4}\right] + (1-\lambda)\left[k\frac{\alpha_L^6}{4\alpha_H^4} - k\frac{\alpha_H^2}{4}\right] \tag{A.452}$$

When $\lambda_1 = 0$, (A.451) becomes

$$\lambda\left[\frac{\alpha_L^6 3 m_{2L}^2}{16k^3}(V - m_2 L)\right] + (1-\lambda)\left[\frac{\alpha_L^6 m_{2L}^3}{16k^3} - \frac{\alpha_H^6 m_{2L}^3}{16k^3}\right] = 0 \tag{A.453}$$

which is equivalent to

$$3m_{2L}(V - m_{2L}) + \rho\left(1 - X^6\right) m_{2L}^2 = 0 \tag{A.454}$$

where $\rho = \frac{1-\lambda}{\lambda}$ and $X = \frac{\alpha_H}{\alpha_L}$. Since $m_{2L} > 0$, (A.454) is equivalent to

$$3(V - m_{2L}) + \rho\left(1 - X^6\right) m_{2L} = 0 \tag{A.455}$$

which gives

$$m_{2L}^* = \frac{3V}{\rho\left(X^6 - 1\right) + 3} \tag{A.456}$$

It is the exactly same interior solution given by (A.423).

2. When $\frac{2k}{\alpha_H} \le m_{2L} \le \frac{2k}{\alpha_L}$, $\tilde{e}^*_{2H} = 1$. $\tilde{e}^*_{1H} = \frac{-\alpha_H k + \alpha_H^2 m_{2L}}{2k}$ if $\frac{-\alpha_H k + \alpha_H^2 m_{2L}}{2k} \le 1$ and $\tilde{e}^*_{1H} = 1$ if $\frac{-\alpha_H k + \alpha_H^2 m_{2L}}{2k} \ge 1$. It is clear that when $m_{2L} \le \frac{2k}{\alpha_L}$, $e^*_{1L} = \frac{\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \le \frac{\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \le \frac{\alpha_L}{2} < 1$.

   Therefore there are two cases for consideration: (1) $-\alpha_H k + \alpha_H^2 m_{2L} \le 2k$; (2) $-\alpha_H k + \alpha_H^2 m_{2L} \ge 2k$.

   For the first case in which $-\alpha_H k + \alpha_H^2 m_{2L} \le 2k$ , the Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\begin{aligned}&\lambda\left[\alpha_L\frac{\alpha_L^3 m_{2L}^2}{8k^2}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})\right)+k\left(\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right)^2\right]\\&+(1-\lambda)\left[k\left(\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right)^2-k\left(\frac{-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)^2\right]+\lambda_1\left(m_{2L}-\frac{2k}{\alpha_H}\right)\\&+\lambda_2\left(\frac{2k}{\alpha_L}-m_{2L}\right)+\lambda_3\left(2k+\alpha_H k-\alpha_H^2 m_{2L}\right)\end{aligned} \tag{A.457}$$

   where $\lambda_1 \ge 0$, $\lambda_2 \ge 0$ and $\lambda_3 \ge 0$ are Lagrangian multipliers.
   The first order condition of (A.457) with respect to $m_{2L}$ gives

$$\lambda\left[\frac{\alpha_L^6 3m_{2L}^2}{16k^3}(V-m_{2L})\right]+(1-\lambda)\left[\frac{\alpha_L^6 m_{2L}^3}{16k^3}-\left(\frac{-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)\alpha_H^2\right]+\lambda_1-\lambda_2-\lambda_3\alpha_H^2=0 \tag{A.458}$$

   Notice that when $\frac{2k}{\alpha_H} \le m_{2L} \le \frac{2k}{\alpha_L}$,

$$\frac{\alpha_L^6 m_{2L}^3}{16k^3} < \left(\frac{-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)\alpha_H^2 \tag{A.459}$$

   because (A.459) holds when $m_{2L} = \frac{2k}{\alpha_H}$ and $\frac{2k}{\alpha_L}$, and $\frac{\alpha_L^6 m_{2L}^3}{16k^3} - \left(\frac{-\alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)\alpha_H^2$ is a convex function which has a negative second order derivative with respect to $m_{2L}$.

   When $\lambda_1 > 0$, $m_{2L} = \frac{2k}{\alpha_H}$. This means that $m_{2L} < \frac{2k}{\alpha_L}$ and $-\alpha_H k + \alpha_H^2 m_{2L} = \alpha_H k = \frac{\alpha_H^3 m_{2L}^2}{8k^2}$. This situation goes back to the first scenario.

   When $\lambda_2 > 0$, $m_{2L} = \frac{2k}{\alpha_L}$ which implies $m_{2L} > \frac{2k}{\alpha_H}$. Therefore $\lambda_1 = 0$. Clearly, to make (A.458) hold, $V$ has to be greater than $\frac{2k}{\alpha_L}$. $\overline{\Pi}_{L,P}$ equals

$$\lambda\left[\alpha_L\frac{\alpha_L}{2}\left(\alpha_L\left(V-\frac{2k}{\alpha_L}\right)\right)+k\frac{\alpha_L^2}{4}\right]+(1-\lambda)\left[k\frac{\alpha_L^2}{4}-k\left(\frac{-\alpha_H k+\frac{\alpha_H^2}{\alpha_L}2k}{2k}\right)^2\right] \tag{A.460}$$

When $\lambda_3 > 0$, $-\alpha_H k + \alpha_H m_{2H} = 2k$ which means that $m_{2L} = \frac{2k+\alpha_H k}{\alpha_H^2} > \frac{2k}{\alpha_H}$. Therefore $\lambda_1 = 0$ and $V$ has to be greater than $\frac{2k}{\alpha_L}$ to make (A.458) hold. $\overline{\Pi}_{L,P}$ equals

$$\begin{aligned}&\lambda\left[\alpha_L\frac{\alpha_L^3\left(\frac{2k+\alpha_H k}{\alpha_H^2}\right)^2}{8k^2}\left(\frac{\alpha_L^2\left(\frac{2k+\alpha_H k}{\alpha_H^2}\right)}{2k}\left(V-\left(\frac{2k+\alpha_H k}{\alpha_H^2}\right)\right)\right)+k\left(\frac{\alpha_L^3\left(\frac{2k+\alpha_H k}{\alpha_H^2}\right)^2}{8k^2}\right)^2\right]\\&+(1-\lambda)\left[k\left(\frac{\alpha_L^3\left(\frac{2k+\alpha_H k}{\alpha_H^2}\right)^2}{8k^2}\right)^2-k\right]\end{aligned} \tag{A.461}$$

When $\lambda_1 = \lambda_2 = \lambda_3 = 0$, (A.458) is a cubic equation of $m_{2L}$. To make (A.458) hold, $V$ has to be greater than $m_{2L}$ which is no less than $\frac{2k}{\alpha_H}$, namely $V$ has to be greater than $\frac{2k}{\alpha_H}$.

For the second case in which $-\alpha_H k + \alpha_H^2 m_{2L} \geq 2k$, the Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\begin{aligned}&\lambda\left[\alpha_L\frac{\alpha_L^3 m_{2L}^2}{8k^2}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})\right)+k\left(\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right)^2\right]\\&+(1-\lambda)\left[k\left(\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right)^2-(-k-\alpha_H k+\alpha_H^2 m_{2L})\right]\\&+\lambda_1\left(\frac{2k}{\alpha_L}-m_{2L}\right)+\lambda_2\left(2k+\alpha_H k-\alpha_H^2 m_{2L}\right)\end{aligned} \tag{A.462}$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are Lagrangian multipliers. Notice that $m_{2L} > \frac{2k}{\alpha_H}$, namely, not binding, because $-\alpha_H k + \alpha_H^2 m_{2L} \geq 2k$ implies $m_{2L} \geq \frac{2k+\alpha_H k}{\alpha_H^2} > \frac{2k}{\alpha_H}$.

The first order condition of (A.462) with respect to $m_{2L}$ gives

$$\lambda\left[\frac{\alpha_L^6 3m_{2L}^2}{16k^3}(V-m_{2L})\right]+(1-\lambda)\left[\frac{\alpha_L^6 m_{2L}^3}{16k^3}-\alpha_H^2\right]-\lambda_1+\lambda_2\alpha_H^2=0 \tag{A.463}$$

When $\lambda_1 > 0$ and $\lambda_2 = 0$, $m_{2L} = \frac{2k}{\alpha_L}$. To make (A.463) hold, $V$ has to be greater than $\frac{2k}{\alpha_L}$ because $\frac{\alpha_L^6 m_{2L}^3}{16k^3} - \alpha_H^2$ when $m_{2L} = \frac{2k}{\alpha_L}$. $\overline{\Pi}_{L,P}$ equals

$$\lambda\left[\alpha_L\frac{\alpha_L}{2}\left(\alpha_L\left(V-\frac{2k}{\alpha_L}\right)\right)+k\frac{\alpha_L^2}{4}\right]+(1-\lambda)\left[k\frac{\alpha_L^2}{4}-\left(-k-\alpha_H k+\alpha_H^2\left(\frac{2k}{\alpha_H}\right)\right)\right] \tag{A.464}$$

When $\lambda_2 > 0$, $-\alpha_H k + \alpha_H^2 m_{2L} = 2k$. This situation goes back to the first case.

When $\lambda_1 = \lambda_2 = 0$, (A.463) is a cubic equation of $m_{2L}$. It is clear that $V$ has to be greater than $m_{2L}$ which is no less than $\frac{2k}{\alpha_H}$. Thus $V$ has to be greater than $\frac{2k}{\alpha_H}$.

3. When $m_{2L} \geq \frac{2k}{\alpha_L}$, $\tilde{e}^*_{2H} = 1$. $\tilde{e}^*_{1H} = \frac{-\alpha_H k+\alpha_H^2 m_{2L}}{2k}$ if $\frac{-\alpha_H k+\alpha_H^2 m_{2L}}{2k} \leq 1$ and $\tilde{e}^*_{1H} = 1$ if $\frac{-\alpha_H k+\alpha_H^2 m_{2L}}{2k} \geq 1$. $e^*_{1L} = \frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k}$ if $\frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k} \leq 1$ and $e^*_{1L} = 1$ if $\frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k} \geq 1$.

   There are three cases for consideration: (1) $-\alpha_H k + \alpha_H^2 m_{2L} \leq 2k$; (2) $-\alpha_H k + \alpha_H^2 m_{2L} \geq 2k$ but $-\alpha_L k + \alpha_L^2 m_{2L} \leq 2k$; (3) $-\alpha_L k + \alpha_L^2 m_{2L} \geq 2k$. Notice that $-\alpha_H k + \alpha_H^2 m_{2L} \leq 2k$ implies $-\alpha_L k + \alpha_L^2 m_{2L} \leq 2k$, because $-\alpha_L k + \alpha_L^2 m_{2L} < -\alpha_H k + \alpha_H^2 m_{2L}$ by $m_{2L} > 0$.

   For the first case, the Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\lambda\left[\alpha_L\left(\frac{-\alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)(\alpha_L(V - m_{2L})) + k\left(\frac{-\alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2\right]$$
$$+(1-\lambda)\left[k\left(\frac{-\alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2 - k\left(\frac{-\alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)^2\right]$$
$$+\lambda_1\left(\frac{2k}{\alpha_L} - m_{2L}\right) + \lambda_2(2k + \alpha_H k - \alpha_H^2 m_{2L}) \tag{A.465}$$

   where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are Lagrangian multipliers.

   The first order condition of (A.465) with respect to $m_{2L}$ gives

$$\lambda\left[\frac{\alpha_L^4}{2k}(V - m_{2L})\right] + (1-\lambda)\left[\left(\frac{-\alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)\alpha_L^2 - \left(\frac{-\alpha_H k + \alpha_H^2 m_{2L}}{2k}\right)\alpha_H^2\right] - \lambda_1 - \lambda_2\alpha_H^2 \tag{A.466}$$

   Notice that $\left(\frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)\alpha_L^2 < \left(\frac{-\alpha_H k+\alpha_H^2 m_{2L}}{2k}\right)\alpha_H^2$, because $-\alpha_L k + \alpha_L^2 m_{2L} < -\alpha_H k + \alpha_H^2 m_{2L}$.

   It is clear that to make (A.466) hold, $V$ has to be greater than $m_{2L}$ which is greater than $\frac{2k}{\alpha_L}$. Thus $V$ has to be greater than $\frac{2k}{\alpha_L}$.

   When $\lambda_1 > 0$, $m_{2L} = \frac{2k}{\alpha_L}$. $\overline{\Pi}_{L,P}$ has the same expression as in (A.464).
   When $\lambda_2 > 0$, $-\alpha_H k + \alpha_H^2 m_{2L} = 2k$, i.e., $m_{2L} = \frac{2k+\alpha_H k}{\alpha_H^2}$. $\overline{\Pi}_{L,P}$ equals

$$\lambda\left[\alpha_L\left(\frac{-\alpha_L k + \alpha_L^2\left(\frac{2k+\alpha_H k}{\alpha_H^2}\right)}{2k}\right)\left(\alpha_L\left(V - \frac{2k + \alpha_H k}{\alpha_H^2}\right)\right) + k\left(\frac{-\alpha_L k + \alpha_L^2\left(\frac{2k+\alpha_H k}{\alpha_H^2}\right)}{2k}\right)^2\right]$$
$$+(1-\lambda)\left[k\left(\frac{-\alpha_L k + \alpha_L^2\left(\frac{2k+\alpha_H k}{\alpha_H^2}\right)}{2k}\right)^2 - k\right] \tag{A.467}$$

   When $\lambda_1 = \lambda_2 = 0$, (A.466) is a cubic function of $m_{2L}$.

For the second case, the Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\lambda\left[\alpha_L\left(\frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)(\alpha_L(V-m_{2L}))+k\left(\frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)^2\right]$$
$$+(1-\lambda)\left[k\left(\frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)^2-(-k-\alpha_H k+\alpha_H^2 m_{2L})\right]$$
$$+\lambda_1\left(\frac{2k}{\alpha_L}-m_{2L}\right)+\lambda_2(-\alpha_H k+\alpha_H^2 m_{2L}-2k)+\lambda_3(2k+\alpha_L k-\alpha_L^2 m_{2L}) \quad (A.468)$$

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$ are Lagrangian multipliers.

The first order condition of (A.465) with respect to $m_{2L}$ gives

$$\lambda\left[\frac{\alpha_L^4}{2k}(V-m_{2L})\right]+(1-\lambda)\left[\left(\frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)\alpha_L^2-\alpha_H^2\right]-\lambda_1+\lambda_2\alpha_H^2-\lambda_3\alpha_L^2=0 \quad (A.469)$$

Notice that $\left(\frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k}\right)\alpha_L^2<\alpha_H^2$ because $0 \leq \frac{-\alpha_L k+\alpha_L^2 m_{2L}}{2k} \leq 1$.

When $\lambda_2 > 0$, $-\alpha_H k+\alpha_H^2 m_{2L}=2k$. This situation belongs to the first case (see the situation related to (A.467)).

When $\lambda_2 = 0$, (A.469) implies that $V$ has to be greater than $m_{2L}$ which is no less than $\frac{2k}{\alpha_L}$, in order to make (A.469) holds.

There are three situations for $\lambda_2 = 0$:

(a) When $\lambda_1 > 0$, then $m_{2L} = \frac{2k}{\alpha_L}$, $\overline{\Pi}_{L,P}$ equals

$$\lambda\left[\alpha_L\frac{1}{2}\alpha_L\left(V-\frac{2k}{\alpha_L}\right)+\frac{k}{4}\right]+(1-\lambda)\left[\frac{k}{4}-\left(-k-\alpha_H k+\alpha_H^2\frac{2k}{\alpha_L}\right)\right] \quad (A.470)$$

(b) When $\lambda_3 > 0$, $-\alpha_L k+\alpha_L^2 m_{2L}=2k$, i.e., $m_{2L}=\frac{2k+\alpha_L k}{\alpha_L^2}$. $\overline{\Pi}_{L,P}$ equals

$$\begin{aligned}&\lambda\left[\alpha_L^2\left(V-\frac{2k+\alpha_L k}{\alpha_L^2}\right)+k\right]\\&+(1-\lambda)\left[k-\left(-k-\alpha_H k+\alpha_H^2\left(\frac{2k+\alpha_L k}{\alpha_L^2}\right)\right)\right]\\=\;&\lambda\left[\alpha_L^2 V-\alpha_L k-k\right]+(1-\lambda)\left[\alpha_H k-\frac{\alpha_H^2}{\alpha_L^2}2k-\frac{\alpha_H^2}{\alpha_L}k\right]\end{aligned} \quad (A.471)$$

(c) When $\lambda_1 = \lambda_3 = 0$, (A.469) is a cubic function of $m_{2L}$.

For the third case, the Lagrangian for the maximum of $\overline{\Pi}_{L,P}$ is

$$\lambda\left[\alpha_L\cdot 1\cdot(\alpha_L(V-m_{2L}))+(-k-\alpha_L k+\alpha_L^2 m_{2L})\right]$$
$$+(1-\lambda)\left[(-k-\alpha_L k+\alpha_L^2 m_{2L})-(-k-\alpha_H k+\alpha_H^2 m_{2L})\right]$$
$$+\lambda_1(-\alpha_L k+\alpha_L^2 m_{2L}-2k) \quad (A.472)$$

where $\lambda_1 \geq 0$ is a Lagrangian multiplier.

The first order condition of (A.472) with respect to $m_{2L}$ gives

$$(1-\lambda)\left(\alpha_L^2 - \lambda_H^2\right) + \lambda_1 \alpha_L^2 = 0 \tag{A.473}$$

which implies that $\lambda_1 > 0$. Thus $-\alpha_L k + \alpha_L^2 m_{2L} = 2k$. This belongs to the second case (see the situation related to (A.471)).

One thing we would like to point out is that for all cases above, the expression of $\overline{\Pi}_{L,P}$ is bounded in absolute value and thereby has the maximum. The uniqueness of the solution of the first order conditions of the Lagrangian associated to $\overline{\Pi}_{L,P}$ ensures the solution is the location of the local maximum.

From the above discussion, we can see that when $V \le \frac{2k}{\alpha_H}$, the global maximum of $\overline{\Pi}_{H,P}$ is obtained at $m_{2H}^* = V$ and the global maximum of $\overline{\Pi}_{L,P}$ is obtained at $m_{2L}^* = \frac{3V}{\rho(X^6-1)+3}$. Because of the equivalence between the maximization of $\lambda\Pi_{L,P} + (1-\lambda)\Pi_{H,P}$ and the maximization of $\overline{\Pi}_{L,P} + \overline{\Pi}_{H,P}$, the global maximum of $\lambda\Pi_{L,P} + (1-\lambda)\Pi_{H,P}$ is also achieved at $m_{2H}^* = V$ and $m_{2L}^* = \frac{3V}{\rho(X^6-1)+3}$. Using the binding participation constraint for the low-type agent and the incentive compatibility constraint for the high-type (see (A.425) and (A.426)),

$$\begin{aligned} m_{0L}^* &= -\frac{\alpha_L^6}{64k^3}\left[\frac{3V}{\rho(X^6-1)+3}\right]^4 \\ m_{0H}^* &= \left[\frac{\alpha_H^6-\alpha_L^6}{64k^3}\right]\left[\frac{3V}{\rho(X^6-1)+3}\right]^4 - \frac{\alpha_H^6}{64k^3}V^4 \end{aligned} \tag{A.474}$$

and the principal's expected profit (see (A.427)) equals

$$\begin{aligned} &\lambda\left[\frac{\alpha_L^6}{16k^3}\left(\frac{3V}{\rho(X^6-1)+3}\right)^3\left(\frac{\rho\left(x^6-1\right)V}{\rho(x^6-1)+3}\right)\right] + \lambda\left[\frac{\alpha_L^6}{64k^3}\left(\frac{3V}{\rho(X^6-1)+3}\right)^4\right] \\ +\quad &(1-\lambda)\left[\frac{\alpha_H^6V^4}{64k^3} + \frac{\left(\alpha_L^6-\alpha_H^6\right)}{64k^3}\left(\frac{3V}{\rho(X^6-1)+3}\right)^4\right] \end{aligned} \tag{A.475}$$

with

$$\begin{aligned} \overline{\Pi}_{L,P} &= \frac{\alpha_L^6}{16k^3}\left(\frac{3V}{\rho(X^6-1)+3}\right)^3\left(\frac{\rho\left(x^6-1\right)V}{\rho(x^6-1)+3}\right) + \frac{\alpha_L^6}{64k^3}\left(\frac{3V}{\rho(X^6-1)+3}\right)^4 \\ \overline{\Pi}_{H,P} &= \frac{\alpha_H^6V^4}{64k^3} + \frac{\left(\alpha_L^6-\alpha_H^6\right)}{64k^3}\left(\frac{3V}{\rho(X^6-1)+3}\right)^4 \end{aligned} \tag{A.476}$$

This concludes the proof of Theorem 4.

## A.5 Proof of Theorem 5

Notice that if we add constraints $m_{0L} = m_{0H}$, $m_{1L} = m_{1H}$ and $m_{2L} = m_{2H}$ to the principal's expected maximization problem for the baseline model in which two different menus $(m_{0L}, m_{1L}, m_{2L})$ and $(m_{0H}, m_{1H}, m_{2H})$ are offered to two different types of agents such that both agents are willing to participate, then we obtain the principal's expected profit maximization problem for the model in which only one menu $(m_0, m_1, m_2)$ is offered to two different types of agents such that both agents are willing to participate. Therefore the principal's expected profit of the former model is at least as big as the one of the latter model. For the former model, we know that when $V \le \frac{2k}{\alpha_H}$, $m_{1L}^* \neq m_{1H}^*$ in the optimal solution. Therefore in this situation, the principal would obtain higher expected profit in the former model than in the latter.

This concludes the proof of Theorem 5.

## A.6 Proof of Theorem 6

We will show that under incomplete information, when $V \le \frac{2k}{\alpha_H}$ and upfront, intermediate and end money transfers are all included, it is always better for the principal to offer two menus of money transfers to the high-type and low-type agents separately than to offer one menu to the high-type agent with the exclusion of the low-type agent.

We assume that $\frac{\alpha_H m_{2L}}{2k} \le 1$ and $\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k} \le 1$. Thus $\frac{\alpha_L m_{2L}}{2k} \le 1$ and $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \le 1$. As a result, the low-type agent's expected profit when he is honest about his type is

$$m_{0L} + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \tag{A.477}$$

and the high-type agent's expected profit when he pretends to be the low-type one is

$$m_{0L} + k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2 \tag{A.478}$$

We set $m_{2H} = V$ and $m_{1H} = 0$. Thus $\frac{\alpha_H m_{2H}}{2k} \le 1$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} = \frac{\alpha_H^3 V^2}{8k^2} \le \frac{\alpha_H}{2} < 1$ and $\frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k} < 1$, because $\frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k} \le \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$. According to (A.78) and (A.83), the high-type agent's expected profit when being honest about his type is

$$m_{0H} + k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 = m_{0H} + \frac{\alpha_H^6 V^4}{64k^3} \tag{A.479}$$

and the low-type agent's expected profit when he pretends to be the high-type one is

$$m_{0H} + k\left(\frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k}\right)^2 = m_{0H} + \frac{\alpha_L^6 V^4}{64k^3} \tag{A.480}$$

Notice that the participation constraints for both types of agents are

$$m_{0L} + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \geq 0 \tag{A.481}$$

$$m_{0H} + k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 \geq 0 \tag{A.482}$$

and the incentive compatibility constraints are

$$m_{0L} + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \geq m_{0H} + k\left(\frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k}\right)^2 \tag{A.483}$$

$$m_{0H} + k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 \geq m_{0L} + k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2 \tag{A.484}$$

Choosing appropriate $m_{0H}$ such that the high-type agent's incentive compatibility constraint (A.484) is binding, then

$$m_{0H} = -k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 + m_{0L} + k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2 \tag{A.485}$$

Thus the low-type agent's incentive compatibility constraint (A.483) becomes

$$\begin{aligned} & m_{0L} + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \\ \geq\ & m_{0L} - k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 + k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2 + k\left(\frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k}\right)^2 \end{aligned} \tag{A.486}$$

which is equivalent to

$$\begin{aligned} & k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \\ \geq\ & -k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 + k\left(\frac{\alpha_H m_{1L} + \frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2 + k\left(\frac{\alpha_L m_{1H} + \frac{\alpha_L^3 m_{2H}^2}{4k}}{2k}\right)^2 \end{aligned} \tag{A.487}$$

namely,

$$k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \geq -\frac{\alpha_H^6 V^4}{64k^3}+\frac{\alpha_L^6 V^4}{64k^3}+k\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2 \tag{A.488}$$

Here we use the fact that $m_{2H}=V$ and $m_{1H}=0$.

Therefore, for any $V>0$, which satisfies $V\leq\frac{2k}{\alpha_H}$, there exists a small $\epsilon_1>0$ such that as long as $0\leq m_{2L}<\epsilon_1$ and $m_{1L}=0$, (A.488) holds. This means that the low-type incentive compatibility constraint is satisfied. We can set $m_{0L}=-k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2$, which implies the low-type's participation constraint is satisfied. The binding of the high-type agent's incentive compatibility constraint means that the high-type agent's incentive compatibility constraint is satisfied. Notice that it is easy to show that the high-type agent's participation constraint is also satisfied, using the low-type agent's participation constraint and the high-type agent's incentive compatibility constraint. According to (A.215), (A.216) and (A.217), with $m_{2H}=V$, $m_{1H}=0$ and $m_{1L}=0$, the principal's expected profit is

$$\begin{aligned}
&\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_L m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)+k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2\right]\\
&+(1-\lambda)\left[k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2-k\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2\right]\\
&+(1-\lambda)\left[\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right]\\
&+(1-\lambda)\left[k\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2\right]\\
=\ &\lambda\left[\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_L m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)+k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2\right]\\
&+(1-\lambda)\left[k\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2-k\left(\frac{\alpha_H m_{1L}+\frac{\alpha_H^3 m_{2L}^2}{4k}}{2k}\right)^2\right]+(1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}\\
=\ &\lambda\frac{\alpha_L^6 m_{2L}^3}{16k^3}\left[(V-m_{2L})+\frac{m_{2L}}{4}+\left(\frac{1-\lambda}{\lambda}\right)\frac{m_{2L}}{4}-\left(\frac{1-\lambda}{\lambda}\right)\frac{\alpha_H^6}{\alpha_L^6}\frac{m_{2L}}{4}\right]+(1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}
\end{aligned} \tag{A.489}$$

where for any $V>0$, the first term involving $m_{2L}$ must be greater than 0, as long as $m_{2L}$ is positive and less than a positive real number $\epsilon_2$. Therefore when $0<m_{2L}<\epsilon_2$, the principal's expected profit is greater than $(1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}$.

Now we see that when $0 < m_{2L} < \min(\epsilon_1, \epsilon_2)$, all the participation constraints and incentive compatibility constraints are satisfied and the principal's expected profit is greater than $(1-\lambda)\frac{\alpha_H^6 V^4}{64k^3}$, which is greater than or equal to the expected profit for the principal when one menu is offered to the high-type agent with exclusion of the low-type one. This shows that in this scenario when two menus are given to the high-type and low-type agents, the expected profit for the principal is bigger than that when one menu is offered to the high-type agent without including the low-type one.

This concludes the proof of Theorem 6.

## A.7 Proof of Theorem 7 and Theorem 8

In this section we will show two things: 1) under complete information, the full flexibility model (i.e., upfront money transfer, intermediate and end rewards are included) can achieve the same expected profit for the principal as the baseline model (i.e., upfront, intermediate and end money transfers are included), no matter what value $V$ takes; 2) under incomplete information, when $V \le \frac{2k}{\alpha_H}$, the full flexibility model can attain the same expected profit for the principal as the one of the baseline model does. Notice that the optimal solutions obtained in the proofs of Theorem 1 and Theorem 2 will be used here.

We assume that the baseline model has optimal solution made up of of $m_{0L}^*$, $m_{1L}^*$, $m_{2L}^*$, $m_{0H}^*$, $m_{1H}^*$, and $m_{2H}^*$, which would take different values under complete information and under incomplete information, respectively. The full flexibility model has upfront money transfers, intermediate and end rewards and penalties $U_{0L}$, $R_{1L}$, $P_{1L}$, $R_{2L}$, $P_{2L}$ for the low-type agent and $U_{0H}$, $R_{1H}$, $P_{1H}$, $R_{2H}$, $P_{2H}$ for the high-type agent.

For both complete information and incomplete information cases, we use the following common procedure at the beginning: set $\overline{m}_{0L} = m_{0L}^*$, $\overline{m}_{1L} = m_{1L}^*$, $\overline{m}_{2L} = m_{2L}^*$, $\overline{m}_{0H} = m_{0H}^*$, $\overline{m}_{1H} = m_{1H}^*$, and $\overline{m}_{2H} = m_{2H}^*$. Recall that we introduced the transformations $\overline{m}_{0L} = U_{0L} - P_{1L}$, $\overline{m}_{1L} = R_{1L} + P_{1L} - P_{2L}$, $\overline{m}_{2L} = R_{2L} + P_{2L}$, $\overline{m}_{0H} = U_{0H} - P_{1H}$, $\overline{m}_{1H} = R_{1H} + P_{1H} - P_{2H}$ and $\overline{m}_{2H} = R_{2H} + P_{2H}$. Thus $U_{0L} - P_{1L} = m_{0L}^*$, $R_{1L} + P_{1L} - P_{2L} = m_{1L}^*$, $R_{2L} + P_{2L} = m_{2L}^*$, $U_{0H} - P_{1H} = m_{0H}^*$, $R_{1H} + P_{1H} - P_{2H} = m_{1H}^*$, and $R_{2H} + P_{2H} = m_{2H}^*$.

1. For the complete information case, according to the part involving (A.49) through (B.131) in the proof of Theorem 1, we can set $m_{1L}^* = 0$ and $m_{1H}^* = 0$. Let us first look at the upfront money transfer, intermediate and end rewards and penalties related to the high-type agent in the full flexibility model. There are two scenarios, according to $m_{0H}^* \le 0$ or $m_{0H}^* \ge 0$.

(a) If $m_{0H}^* \le 0$, we set $U_{0H} = 0$. Thus $-P_{1H} = m_{0H}^*$. Substituting $P_{1H} = -m_{0H}^*$ into $R_{1H}+P_{1H}-P_{2H} = m_{1H}^*$ gives $R_{1H}+(-m_{0H}^*)-P_{2H} = m_{1H}^*$. Setting $R_{1H} = 0$ leads to $P_{2H} = -m_{0H}^*$, because $m_{1H}^* = 0$. Substituting $P_{2H} = -m_{0H}^*$ into $R_{2H} + P_{2H} = m_{2H}^*$ gives $R_{2H} = m_{2H}^* - (-m_{0H}^*)$. If we can show that $m_{2H}^* - (-m_{0H}^*) \ge 0$, then $R_{2H} \ge 0$. Thus $R_{1H}$, $P_{1H}$, $R_{2H}$ and $P_{2H}$ are all nonnegative.

In the following, we show that $m_{2H}^* - (-m_{0H}^*) \ge 0$.

From the part involving (A.49) through (A.54) in the proof of Theorem 1, we know that

i. When $V \le \frac{2k}{\alpha_H}$, $m_{2H}^* = V$, $m_{1H}^* = 0$ and $m_{0H}^* = -\frac{\alpha_H^6 V^4}{64k^3}$, which implies

$$-m_{0H}^* \le \frac{\alpha_H^6 V^4}{64k^3} \le \frac{\alpha_H^6 \left(\frac{2k}{\alpha_H}\right)^3 V}{64k^3} = \frac{\alpha_H^3 V}{8} < \frac{1}{8}V = \frac{1}{8}m_{2H}^* \tag{A.490}$$

which means $\frac{1}{8}m_{2H}^* - (-m_{0H}^*) > 0$. Thus $m_{2H}^* - (-m_{0H}^*) \ge 0$.

ii. When $V \ge \frac{2k}{\alpha_H}$ and $\alpha_H^2 V - \alpha_H k \le 2k$, $m_{2H}^* \ge \frac{2k}{\alpha_H}$, $\alpha_H m_{2H}^* + m_{1H}^* = \alpha_H V$ and $m_{0H}^* = -k\left(\frac{\alpha_H^2 V - \alpha_H k}{2k}\right)^2$. It is clear that $-m_{0H}^* \le k$. We can set $m_{1H}^* = 0$, then $m_{2H}^* = V$, which is $\ge \frac{2k}{\alpha_H}$. This means $\frac{1}{2}m_{2H}^* - (-m_{0H}^*) > 0$. Thus $m_{2H}^* - (-m_{0H}^*) \ge 0$.

iii. When $V \ge \frac{2k}{\alpha_H}$ and $\alpha_H^2 V - \alpha_H k \ge 2k$, $m_{2H}^* \ge \frac{2k}{\alpha_H}$, $\alpha_H m_{1H}^* - \alpha_H k + \alpha_H^2 m_{2H}^* \ge 2k$ and $m_{0H}^* = -k + \alpha_H m_{1H}^* - \alpha_H k + \alpha_H^2 m_{2H}^* \ge k$. Since $m_{0H}^* > 0$, we don't need to consider this situation.

(b) When $m_{0H}^* \ge 0$, we set $U_{0H} = m_{0H}^*$, then $P_{1H} = 0$. Substituting $P_{1H} = 0$ into $R_{1H} + P_{1H} - P_{2H} = m_{1H}^*$ gives $R_{1H} - P_{2H} = m_{1H}^*$. Setting $R_{1H} = 0$ leads to $P_{2H} = 0$, because $m_{1H}^* = 0$. Substituting $P_{2H} = 0$ into $R_{2H} + P_{2H} = m_{2H}^*$ gives $R_{2H} = m_{2H}^* \ge 0$. Thus $R_{1H}$, $P_{1H}$, $R_{2H}$ and $P_{2H}$ are all nonnegative.

Similarly, we can show that there exist $U_{0L}$, $R_{1L} \ge 0$, $P_{1L} \ge 0$, $R_{2L} \ge 0$ and $P_{2L} \ge 0$ such that $U_{0L} - P_{1L} = m_{0L}^*$, $R_{1L} + P_{1L} - P_{2L} = m_{1L}^*$ and $R_{2L} + P_{2L} = m_{2L}^*$.

Therefore such $U_{0L}$, $R_{1L}$, $P_{1L}$, $R_{2L}$ and $P_{2L}$ for the low-type agent and $U_{0H}$, $R_{1H}$, $P_{1H}$, $R_{2H}$ and $P_{2H}$ for the high-type agent in the full flexibility model will give the principal the same expected profit as the expected profit given in the baseline model. On the other hand, using transformations $\overline{m}_{0L}$, $\overline{m}_{1L}$, $\overline{m}_{2L}$, $\overline{m}_{0H}$, $\overline{m}_{1H}$, and $\overline{m}_{2H}$, it is easy to see that the expected profit given in the baseline model is greater or equal to that given in the full flexibility model.

This concludes the proof of Theorem 7.

2. For the incomplete information case, when $V \le \frac{2k}{\alpha_H}$, the optimal solution for the principal's maximization problem of the baseline model is given by (A.304) and (A.305) in the proof of Theorem 2. Notice that $m_{1L}^* > 0$ and $m_{1H}^* = 0$. Set $R_{1L} = m_{1L}^*$ and $R_{1H} = 0$, then both $R_{1L}$ and $R_{1H}$ satisfy the nonnegativity requirement. Thus $P_{2L} = -m_{0L}^* > 0$ and $P_{2H} = -m_{0H}^* > 0$. Substituting them into $R_{2L} + P_{2L} = m_{2L}^*$ and $R_{2H} + P_{2H} = m_{2H}^*$ leads to $R_{2L} = m_{2L}^* - (-m_{0L}^*)$ and $R_{2H} = m_{2H}^* - (-m_{0H}^*)$. If we can show that $m_{2L}^* - (-m_{0L}^*) \ge 0$ and $m_{2H}^* - (-m_{0H}^*) \ge 0$, then such choices for $R_{2L}$ and $R_{2H}$ will satisfy the nonnegativity requirement, and thereby what are established in the above for $U_{0L}$, $U_{0H}$, $P_{1L}$, $P_{1H}$, $R_{2L}$, $R_{2H}$, $P_{2L}$ and $P_{2H}$ form a solution that gives the principal the same expected profit as the expected profit of the baseline model when $V \le \frac{2k}{\alpha_H}$. Notice that using transformations $\overline{m}_{0L}$, $\overline{m}_{1L}$, $\overline{m}_{2L}$, $\overline{m}_{0H}$, $\overline{m}_{1H}$, and $\overline{m}_{2H}$, it is easy to see that the expected profit given in the baseline model is greater or equal to that given in the full flexibility model.

In the following, we show that $m_{2L}^* - (-m_{0L}^*) \ge 0$ and $m_{2H}^* - (-m_{0H}^*) \ge 0$.

According to (A.304) and (A.305) in the proof of Theorem 2, when $V \le \frac{2k}{\alpha_H}$,

$$k\left(\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 {m_{2L}^*}^2}{4k}}{2k}\right) = \alpha_L \frac{\frac{\alpha_L^4}{4k^2} m_{2L}^*(V - m_{2L}^*) + \rho\left(\frac{\alpha_H^2\alpha_L^2 - \alpha_H^4}{8k^2}\right){m_{2L}^*}^2 + \frac{\alpha_L^4}{8k^2}{m_{2L}^*}^2}{\frac{\alpha_L^2}{k} + \rho\left(\frac{\alpha_L^2 - \alpha_H^2}{k}\right)} \tag{A.491}$$

We want to show the left side of (A.491) is less than $\frac{1}{2}m_{2L}^*$. This is equivalent to show the right side of (A.491) is less than $\frac{1}{2}m_{2L}^*$, namely,

$$\alpha_L \frac{\frac{\alpha_L^4}{4k^2} m_{2L}^*(V - m_{2L}^*) + \rho\left(\frac{\alpha_H^2\alpha_L^2 - \alpha_H^4}{8k^2}\right){m_{2L}^*}^2 + \frac{\alpha_L^4}{8k^2}{m_{2L}^*}^2}{\frac{\alpha_L^2}{k} + \rho\left(\frac{\alpha_L^2 - \alpha_H^2}{k}\right)} \le \frac{1}{2}m_{2L}^* \tag{A.492}$$

which is equivalent to

$$\frac{\alpha_L^5}{4k^2}\left(V - \frac{1}{2}m_{2L}^*\right) + \rho\left(\frac{\alpha_H^2\alpha_L^2 - \alpha_H^4}{8k^2}\right)m_{2L}^* < \frac{1}{2}\frac{\alpha_L^2}{k} + \frac{1}{2}\left(\frac{\alpha_L^2 - \alpha_H^2}{k}\right) \tag{A.493}$$

because $m_{2L}^* > 0$ is a common factor on both sides can can be canceled. To show (A.493), it is sufficient to show

$$\frac{\alpha_L^5}{4k}\left(V - \frac{1}{2}m_{2L}^*\right) < \frac{1}{2}\alpha_L^2 \tag{A.494}$$

Since $0 < m_{2L}^* < V$ and $V \le \frac{2k}{\alpha_H}$,

$$\frac{\alpha_L^5}{4k}\left(V - \frac{1}{2}m_{2L}^*\right) < \frac{\alpha_L^5}{4k}V < \frac{\alpha_L^5}{4k}\frac{2k}{\alpha_H} < \frac{\alpha_L^2}{2} \tag{A.495}$$

Therefore

$$k\left(\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k}\right) \le \frac{1}{2} m_{2L}^* \tag{A.496}$$

On the other hand, by (A.197) in the proof of Theorem 2,

$$\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 m_{2L}^{*\,2}}{4k}}{2k} < \frac{3}{4} \tag{A.497}$$

Since $\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k} \le \frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 m_{2L}^{*\,2}}{4k}}{2k}$,

$$\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k} \le \frac{3}{4} \tag{A.498}$$

Combining (A.498) with (A.496), we have

$$k\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k}\right]^2 \le \frac{3}{8} m_{2L}^* \tag{A.499}$$

namely, $\frac{3}{8} m_{2L}^* - (-m_{0L}^*) \ge 0$. Therefore $m_{2L}^* - (-m_{0L}^*) \ge 0$.

As to the verification of $m_{2H}^* - (-m_{0H}^*) \ge 0$, notice that when $V \le \frac{2k}{\alpha_H}$,

$$m_{0H}^* = -k\left[\frac{\alpha_H m_{1H}^* + \frac{\alpha_H^3 m_{2H}^{*\,2}}{4k}}{2k}\right]^2 - k\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k}\right]^2 + k\left[\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 m_{2L}^{*\,2}}{4k}}{2k}\right]^2 \tag{A.500}$$

with and $m_{1H}^* = 0$ and $m_{2H}^* = V$.

We know that

$$\frac{\alpha_H^6 V^4}{64k^3} - \frac{\alpha_L^6 V^4}{64k^3} \ge k\left[\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 m_{2L}^{*\,2}}{4k}}{2k}\right]^2 - k\left[\frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k}\right]^2 \tag{A.501}$$

using the fact that $\frac{\alpha_H m_{1L}^* + \frac{\alpha_H^3 m_{2L}^{*\,2}}{4k}}{2k} \ge \frac{\alpha_L m_{1L}^* + \frac{\alpha_L^3 m_{2L}^{*\,2}}{4k}}{2k}$.

Thus to show $m_{2H}^* - (-m_{0H}^*) \ge 0$, it is sufficient to show that

$$m_{2H}^* - k\left[\frac{\alpha_H m_{1H}^* + \frac{\alpha_H^3 m_{2H}^{*\,2}}{4k}}{2k}\right]^2 \ge 0 \tag{A.502}$$

which is equivalent to

$$V - \frac{\alpha_H^6 V^4}{64k^3} \ge 0 \tag{A.503}$$

Since $V \le \frac{2k}{\alpha_H}$,

$$\frac{\alpha_H^6 V^4}{64k^3} < \frac{\alpha_H^6 V^3}{64k^3} V < \frac{\alpha_H^6 \left(\frac{2k}{\alpha_H}\right)^3}{64k^3} V = \frac{\alpha_H^6}{8} V < \frac{V}{8} \tag{A.504}$$

This shows that $\frac{1}{8} m_{2H}^* - (-m_{0H}^*) \ge 0$. Therefore $m_{2H}^* - (-m_{0H}^*) \ge 0$.

In conclusion, when $V \le \frac{2k}{\alpha_H}$, $m^*_{2H} - (-m^*_{0H}) \ge 0$ and $m^*_{2L} - (-m^*_{0L}) \ge 0$ hold.

It is clear that when $V \le \frac{2k}{\alpha_H}$, there are infinitely many solutions because $U_{0L} - P_{1L} = m^*_{0L}$ and $U_{0H} - P_{1H} = m^*_{0H}$. One of them is to let $U_{0L} = 0$, $P_{1L} = -m^*_{0L}$, $U_{0H} = 0$ and $P_{1H} = -m^*_{0H}$, and $R_{1L}$, $R_{2L}$, $P_{2L}$, $R_{1H}$, $R_{2H}$ and $P_{2H}$ take values specified above.

This concludes the proof of Theorem 8.

# APPENDIX B

# CHAPTER 2 PROOFS

## B.1 Proof of Theorem 10

We examine the separating equilibrium when upfront, intermediate and end money transfers are all included in the menu.

For the high-type principal, her maximal profit satisfies

$$\max_{(m_{0H},m_{1H},m_{2H})}\{-m_{0H}-\alpha_H e_{1H}m_{1H}+\alpha_H^2 e_{1H}e_{2H}(V-m_{2H})\} \tag{B.1}$$

subject to

$$LM \geq -m_{0H}-\alpha_L e_{1H}m_{1H}+\alpha_L^2 e_{1H}e_{2H}(V-m_{2H}) \tag{B.2}$$

and

$$-m_{0H}-\alpha_H e_{1H}m_{1H}+\alpha_H^2 e_{1H}e_{2H}(V-m_{2H}) \geq \overline{LM} \tag{B.3}$$

with the agent's profit satisfying:

$$\max_{(e_{1H},e_{2H})} m_{0H}-ke_{1H}^2+\alpha_H e_{1H}m_{1H}-\alpha_H e_{1H}ke_{2H}^2+\alpha_H^2 e_{1H}e_{2H}m_{2H} \tag{B.4}$$

and

$$m_{0H}-ke_{1H}^2+\alpha_H e_{1H}m_{1H}-\alpha_H e_{1H}ke_{2H}^2+\alpha_H^2 e_{1H}e_{2H}m_{2H} \geq 0 \tag{B.5}$$

where

$$LM = \max_{(m_{0L},m_{1L},m_{2L})}\{-m_{0L}-\alpha_L e_{1L}m_{1L}+\alpha_L^2 e_{1L}e_{2L}(V-m_{2L})\} \tag{B.6}$$

as the low-type principal's maximal profit, and

$$\overline{LM} = \max_{(\tilde{m}_{0L},\tilde{m}_{1L},\tilde{m}_{2L})}\{-\tilde{m}_{0L}-\alpha_H e_{1L}\tilde{m}_{1L}+\alpha_H^2 e_{1L}e_{2L}(V-\tilde{m}_{2L})\} \tag{B.7}$$

with the agent satisfying:

$$\max_{(e_{1L},e_{2L})} \tilde{m}_{0L}-ke_{1L}^2+\alpha_L e_{1L}\tilde{m}_{1L}-\alpha_L e_{1L}ke_{2L}^2+\alpha_L^2 e_{1L}e_{2L}\tilde{m}_{2L} \tag{B.8}$$

and

$$\tilde{m}_{0L}-ke_{1L}^2+\alpha_L e_{1L}\tilde{m}_{1L}-\alpha_L e_{1L}ke_{2L}^2+\alpha_L^2 e_{1L}e_{2L}\tilde{m}_{2L} \geq 0 \tag{B.9}$$

where (B.2) is the constraint of preventing the low-type principal from mimicking the high-type one, (B.3) is the constraint of preventing the high-type principal from mimicking the low-type one, (B.5) is the agent's participation constraint for the high-type principal's offer.

From the supplemental part allocated at the end, we know that $LM = \frac{\alpha_L^6 V^4}{64k^3}$ and $\overline{LM} = \frac{\alpha_L^6 x^6}{64k^3(2x-1)^3}$ with $x = \frac{\alpha_H}{\alpha_L}$. Since $LM$ equals to the profit that the low-type principal earns under complete information, which means that the agent knows her type, the optimal menu offered by the low-type principal is $m_{0L} = -\frac{\alpha_L^6 V^4}{64k^3}$, $m_{1L} = 0$ and $m_{2L} = V$.

The proof consists of two parts. First we look at the high-type principal's profit maximization problem when the agent earns positive profit and the low-type principal does not want to mimic the high-type one, i.e., (B.2) binding, but (B.5) not binding. This is the first scenario. Then we study the high-type principal's profit maximization problem when the agent earns zero profit and the low-type principal wants to mimic the high-type one, i.e., both (B.2) binding and (B.5) binding. This is the second scenario.

There are four cases for consideration:

1. when $m_{2H} \le \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \le 1$, i.e., $e_{2H}^* \le 1$ and $e_{1H}^* \le 1$.
2. when $m_{2H} \le \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \ge 1$, i.e., $e_{2H}^* \le 1$ and $e_{1H}^* = 1$.
3. when $m_{2H} \ge \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} \le 1$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* \le 1$.
4. when $m_{2H} \ge \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} \ge 1$, i.e., $e_{2H}^* = 1$ and $e_{1H}^* = 1$.

where $e_{1H}^*$ and $e_{2H}^*$ are the solution of the agent's profit maximization problem (B.4), when the payments are $(m_{0H}, m_{1H}, m_{2H})$. In fact we only need to consider case 1, namely effort levels in two periods take the forms $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k}$ and $e_{1H}^* = \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$. The reason is allocated to the supplemental part.

Next we study the first scenario of the high-type principal's profit maximization problem in which the agent earns positive profit and the constraint to prevent the low-type principal from mimicking the high-type one binds, i.e., (B.2) binding, but (B.5) not binding.

### B.1.1 The first scenario

It is clear that when $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k}$ and $e_{1H}^* = \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$, (B.4) is solved and (B.5) can be written as

$$m_{0H} + k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 \ge 0 \tag{B.10}$$

Therefore, we can study the high-type principal's profit maximization problem consisting of (B.1), (B.2) and (B.10).

In fact, it will be shown later that the binding condition of (B.2) implies (B.10) under certain conditions. Thus, we focus on the high-type principal's profit maximization problem consisting of (B.1) and (B.2). Its Lagrangian is

$$
\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)-m_{0H}
$$
$$
+\quad \lambda_1\left[\frac{\alpha_L^6 V^4}{64k^3}+m_{0H}-\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right] \tag{B.11}
$$

The first order condition of (B.11) with respect to $m_{0H}$ gives

$$
-1+\lambda_1=0 \tag{B.12}
$$

which leads to $\lambda_1=1$.

With $\lambda_1=1$, the first order conditions of (B.12) with respect to $m_{1H}$ and $m_{2H}$ result in

$$
\frac{\alpha_H^2}{2k}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]-\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)
$$
$$
-\quad\frac{\alpha_H\alpha_L}{2k}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]+\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)=0 \tag{B.13}
$$

and

$$
\frac{\alpha_H^4 m_{2H}}{4k^2}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]+\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\frac{\alpha_H^2}{2k}(V-2m_{2H})\right]
$$
$$
-\quad\frac{\alpha_L\alpha_H^3 m_{2H}}{4k^2}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]
$$
$$
-\quad\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\frac{\alpha_L\alpha_H}{2k}(V-2m_{2H})\right]=0 \tag{B.14}
$$

From (B.13), we have

$$
\frac{\alpha_H^4}{4k^2}m_{2H}(V-m_{2H})-\frac{\alpha_H^2}{2k}m_{1H}-\frac{\alpha_H^2}{2k}m_{1H}-\frac{\alpha_H^4 m_{2H}^2}{8k^2}
$$
$$
-\quad\frac{\alpha_H^2\alpha_L^2}{4k^2}m_{2H}(V-m_{2H})+\frac{\alpha_H\alpha_L}{2k}m_{1H}+\frac{\alpha_H\alpha_L}{2k}m_{1H}+\frac{\alpha_L\alpha_H^3 m_{2H}^2}{8k^2}=0 \tag{B.15}
$$

Simplifying (B.15) gives

$$
\frac{\left(\alpha_H^4-\alpha_H^2\alpha_L^2\right)}{4k^2}m_{2H}(V-m_{2H})-\frac{\left(\alpha_H^4-\alpha_L\alpha_H^3\right)m_{2H}^2}{8k^2}=\frac{\left(\alpha_H^2-\alpha_H\alpha_L\right)m_{1H}}{k} \tag{B.16}
$$

Dividing both sides of (B.16) by $\alpha_H^2-\alpha_H\alpha_L$ leads to

$$
\frac{\left(\alpha_H^2+\alpha_H\alpha_L\right)}{4k^2}m_{2H}(V-m_{2H})-\frac{\alpha_H^2 m_{2H}^2}{8k^2}=\frac{m_{1H}}{k} \tag{B.17}
$$

namely,

$$\frac{\left(\alpha_H^2 + \alpha_H\alpha_L\right)}{4k} m_{2H}(V - m_{2H}) - \frac{\alpha_H^2 m_{2H}^2}{8k} = m_{1H} \tag{B.18}$$

which implies

$$\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k} = \frac{\left(\alpha_H^3 + \alpha_H^2\alpha_L\right)}{4k} m_{2H}(V - m_{2H}) + \frac{\alpha_H^3 m_{2H}^2}{8k} \tag{B.19}$$

On the other hand, dividing (B.14) leads to

$$\begin{aligned} & m_{2H}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right] + \left(m_{1H} + \frac{\alpha_H^2 m_{2H}^2}{4k}\right)(V - 2m_{2H}) \\ - \; & \frac{\alpha_L}{\alpha_H} m_{2H}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right] - \left(\frac{\alpha_L}{\alpha_H}\right)^2 \left(m_{1H} + \frac{\alpha_H^2 m_{2H}^2}{4k}\right)(V - 2m_{2H}) = 0 \end{aligned} \tag{B.20}$$

namely,

$$\begin{aligned} & m_{2H}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right] + \left[1 - \left(\frac{\alpha_L}{\alpha_H}\right)^2\right]\left(m_{1H} + \frac{\alpha_H^2 m_{2H}^2}{4k}\right)(V - 2m_{2H}) \\ - \; & \frac{\alpha_L}{\alpha_H} m_{2H}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right] = 0 \end{aligned} \tag{B.21}$$

Substituting (B.19) into (B.21) and canceling $m_{2H}$ from both sides of the equation gives

$$\begin{aligned} & \left[\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right] \\ + \; & \left[1 - \left(\frac{\alpha_L}{\alpha_H}\right)^2\right]\left[\frac{\left(\alpha_H^2 + \alpha_H\alpha_L\right)}{4k}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}}{8k}\right](V - 2m_{2H}) \\ - \; & \frac{\alpha_L}{\alpha_H}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right] = 0 \end{aligned} \tag{B.22}$$

Substituting the expression of $m_{1H}$ in into (B.22) gives

$$\begin{aligned} & \frac{\left(\alpha_H^2 - \alpha_H\alpha_L\right)}{4k} m_{2H}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}^2}{8k} \\ + \; & \left[1 - \left(\frac{\alpha_L}{\alpha_H}\right)^2\right]\left[\frac{\left(\alpha_H^2 + \alpha_H\alpha_L\right)}{4k}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}}{8k}\right](V - 2m_{2H}) \\ - \; & \frac{\alpha_L}{\alpha_H}\left[\frac{\left(\alpha_H\alpha_L - \alpha_H^2\right)}{4k} m_{2H}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}^2}{8k}\right] = 0 \end{aligned} \tag{B.23}$$

which implies

$$\begin{aligned} & \left(1 + \frac{\alpha_L}{\alpha_H}\right)\frac{\left(\alpha_H^2 - \alpha_H\alpha_L\right)}{4k} m_{2H}(V - m_{2H}) + \left(1 - \frac{\alpha_L}{\alpha_H}\right)\frac{\alpha_H^2 m_{2H}^2}{8k} \\ + \; & \left[1 - \left(\frac{\alpha_L}{\alpha_H}\right)^2\right]\left[\frac{\left(\alpha_H^2 + \alpha_H\alpha_L\right)}{4k}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}}{8k}\right](V - 2m_{2H}) = 0 \end{aligned} \tag{B.24}$$

namely,

$$
\begin{aligned}
& (\alpha_H + \alpha_L)\frac{(\alpha_H - \alpha_L)}{4k} m_{2H}(V - m_{2H}) + (\alpha_H - \alpha_L)\frac{\alpha_H m_{2H}^2}{8k} \\
+ \quad & \left(\alpha_H^2 - \alpha_L^2\right)\left[\frac{\left(1 + \frac{\alpha_L}{\alpha_H}\right)}{4k}(V - m_{2H}) + \frac{m_{2H}}{8k}\right](V - 2m_{2H}) = 0
\end{aligned} \tag{B.25}
$$

Multiplying both sides of (B.25) by $8k\frac{\alpha_H}{\alpha_H - \alpha_L}$ gives

$$
\begin{aligned}
& 2\alpha_H(\alpha_H + \alpha_L)m_{2H}(V - m_{2H}) + \alpha_H^2 m_{2H}^2 \\
+ \quad & (\alpha_H + \alpha_L)\left[2(\alpha_H + \alpha_L)(V - m_{2H}) + \alpha_H m_{2H}\right](V - 2m_{2H}) = 0
\end{aligned} \tag{B.26}
$$

which is

$$
\begin{aligned}
& 2\alpha_H(\alpha_H + \alpha_L)m_{2H}V - 2\alpha_H(\alpha_H + \alpha_L)m_{2H}^2 + \alpha_H^2 m_{2H}^2 \\
+ \quad & 2(\alpha_H + \alpha_L)^2\left(V^2 - 3m_{2H}V + 2m_{2H}^2\right) + \alpha_H(\alpha_H + \alpha_L)m_{2H}V - 2(\alpha_H + \alpha_L)\alpha_H m_{2H}^2 = 0
\end{aligned} \tag{B.27}
$$

namely,

$$
\begin{aligned}
& \left[3\alpha_H(\alpha_H + \alpha_L) - 6(\alpha_H + \alpha_L)^2\right] m_{2H}V \\
+ \quad & \left[4(\alpha_H + \alpha_L)^2 - 4\alpha_H(\alpha_H + \alpha_L) + \alpha_H^2\right] m_{2H}^2 + 2(\alpha_H + \alpha_L)^2 V^2 = 0
\end{aligned} \tag{B.28}
$$

Notice that (B.28) can be simplified as

$$
(\alpha_H + 2\alpha_L)^2 m_{2H}^2 - 3(\alpha_H + \alpha_L)(\alpha_H + 2\alpha_L)m_{2H}V + 2(\alpha_H + \alpha_L)^2 V^2 = 0 \tag{B.29}
$$

namely,

$$
\left[(\alpha_H + 2\alpha_L)m_{2H} - (\alpha_H + \alpha_L)V\right]\left[(\alpha_H + 2\alpha_L)m_{2H} - 2(\alpha_H + \alpha_L)V\right] = 0 \tag{B.30}
$$

The roots of (B.30) are

$$
m_{2H} = \left(\frac{\alpha_H + \alpha_L}{\alpha_H + 2\alpha_L}\right)V \tag{B.31}
$$

and

$$
m_{2H} = \left(\frac{2\alpha_H + 2\alpha_L}{\alpha_H + 2\alpha_L}\right)V \tag{B.32}
$$

Notice that when $m_{2H} = \left(\frac{2\alpha_H + 2\alpha_L}{\alpha_H + 2\alpha_L}\right)V$,

$$
\begin{aligned}
m_{1H} + \frac{\alpha_H^2 m_{2H}^2}{4k} &= \frac{\alpha_H^2 + \alpha_H\alpha_L}{4k} m_{2H}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}^2}{8k} \\
&= \left[\frac{\alpha_H^2 + \alpha_H\alpha_L}{4k}\left(\frac{-\alpha_H}{\alpha_H + 2\alpha_L}\right)V + \frac{\alpha_H^2}{4k}\frac{(\alpha_H + \alpha_L)}{\alpha_H + 2\alpha_L}V\right] m_{2H} \\
&= 0
\end{aligned} \tag{B.33}
$$

which implies $e_{1H}^* = 0$. Clearly this won't give the high-type principal the maximal profit. Therefore this root can be ruled out.

When $m_{2H} = \left(\frac{\alpha_H+\alpha_L}{\alpha_H+2\alpha_L}\right)V$,

$$\begin{aligned} m_{1H} + \frac{\alpha_H^2 m_{2H}^2}{4k} &= \frac{\alpha_H^2 + \alpha_H\alpha_L}{4k}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}^2}{8k} \\ &= \left[\frac{\alpha_H^2 + \alpha_H\alpha_L}{4k}\left(\frac{\alpha_L}{\alpha_H + 2\alpha_L}\right)V + \frac{\alpha_H^2}{8k}\frac{(\alpha_H + \alpha_L)}{\alpha_H + 2\alpha_L}V\right]\left(\frac{\alpha_H + \alpha_L}{\alpha_H + 2\alpha_L}\right)V \\ &> 0 \end{aligned} \tag{B.34}$$

which means $e_{1H}^* > 0$.

For this value of $m_{2H}$,

$$\begin{aligned} m_{1H} &= \frac{\alpha_H^2 + \alpha_H\alpha_L}{4k}m_{2H}(V - m_{2H}) - \frac{\alpha_H^2 m_{2H}^2}{8k} \\ &= \left[\frac{\alpha_H^2 + \alpha_H\alpha_L}{4k}\left(\frac{\alpha_L}{\alpha_H + 2\alpha_L}\right)V - \frac{\alpha_H^2}{8k}\frac{(\alpha_H + \alpha_L)}{\alpha_H + 2\alpha_L}V\right]\left(\frac{\alpha_H + \alpha_L}{\alpha_H + 2\alpha_L}\right)V \\ &= \left[\frac{\alpha_H^2\alpha_L + 2\alpha_H\alpha_L^2 - \alpha_H^3}{8k(\alpha_H + 2\alpha_L)}\right]\left(\frac{\alpha_H + \alpha_L}{\alpha_H + 2\alpha_L}\right)V^2 \end{aligned} \tag{B.35}$$

Next we show that when $V \le \frac{2k}{\alpha_H}$, $m_{2H} < 1$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} < 1$.

Note that

$$m_{2H} = \left(\frac{\alpha_H + \alpha_L}{\alpha_H + 2\alpha_L}\right)V < V \le \frac{2k}{\alpha_H} \tag{B.36}$$

and

$$\begin{aligned} \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} &= \frac{\alpha_H}{2k}\left[\frac{\alpha_H^2 + \alpha_H\alpha_L}{4k}\left(\frac{\alpha_L}{\alpha_H + 2\alpha_L}\right)V + \frac{\alpha_H^2}{8k}\frac{(\alpha_H + \alpha_L)}{\alpha_H + 2\alpha_L}V\right]\left(\frac{\alpha_H + \alpha_L}{\alpha_H + 2\alpha_L}\right)V \\ &\le \frac{\alpha_H}{2k}\left[\frac{\alpha_H^2 + \alpha_H\alpha_L}{4k}\left(\frac{\alpha_L}{\alpha_H + 2\alpha_L}\right) + \frac{\alpha_H^2}{8k}\frac{(\alpha_H + \alpha_L)}{\alpha_H + 2\alpha_L}\right]\left(\frac{\alpha_H + \alpha_L}{\alpha_H + 2\alpha_L}\right)\frac{4k^2}{\alpha_H^2} \\ &< \frac{\alpha_H + \alpha_L}{2}\frac{1}{3} + \frac{\alpha_H}{4} \\ &\le \frac{7}{12} \\ &< 1 \end{aligned} \tag{B.37}$$

Next we want to find when the agent's participation constraint (B.5) i.e., (B.10) is binding.

Notice that (B.5) is binding means that

$$m_{0H} = \alpha_L\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) - \frac{\alpha_L^6 V^4}{64k^3} \tag{B.38}$$

Thus

$$
\begin{aligned}
& m_{0H} + k\left[\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right]^2 \\
= \;& \alpha_L \left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) \\
& -\frac{\alpha_L^6 V^4}{64k^3} + k\left[\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right]^2 \\
= \;& \left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\alpha_L^2 \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - \alpha_L m_{1H} + \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2}\right] \\
& -\frac{\alpha_L^6 V^4}{64k^3}
\end{aligned}
\tag{B.39}
$$

Using (B.17), (B.18) and (B.31), we have

$$
\begin{aligned}
& \alpha_L^2 \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - \alpha_L m_{1H} + \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \\
= \;& \frac{\alpha_L\left(\alpha_H \alpha_L - \alpha_H^2\right)}{4k} m_{2H}(V - m_{2H}) + \frac{\alpha_L \alpha_H^2 m_{2H}^2}{8k} \\
& + \frac{\alpha_H^3 + \alpha_H^2 \alpha_L}{4k} m_{2H}(V - m_{2H}) + \frac{\alpha_H^3 m_{2H}^2}{16k} \\
= \;& m_{2H} \frac{\alpha_L\left(\alpha_H \alpha_L - \alpha_H^2\right)}{4k}\left(\frac{\alpha_L}{\alpha_H + 2\alpha_L} V\right) + m_{2H} \frac{\alpha_L \alpha_H^2(\alpha_L + \alpha_H)}{8k(\alpha_H + 2\alpha_L)} V \\
& + m_{2H} \frac{\alpha_H^3 + \alpha_H^2 \alpha_L}{8k}\left(\frac{\alpha_L}{\alpha_H + 2\alpha_L} V\right) + m_{2H} \frac{\alpha_L^3(\alpha_L + \alpha_H)}{16k(\alpha_H + 2\alpha_L)} V \\
= \;& m_{2H} \frac{2\alpha_L\left(\alpha_L \alpha_H - \alpha_H^2\right)\alpha_L V + \alpha_L \alpha_H^2\left(\alpha_L + \alpha_H\right)V + \left(\alpha_H^3 \alpha_L + \alpha_H^2 \alpha_L^2\right)V}{8k(\alpha_H + 2\alpha_L)} \\
& + m_{2H} \frac{\frac{1}{2}\alpha_H^3(\alpha_L + \alpha_H)V}{8k(\alpha_H + 2\alpha_L)} \\
= \;& m_{2H} \frac{\left(2\alpha_L^3 \alpha_H - 2\alpha_H^2 \alpha_L^2\right)V + \left(\alpha_L^2 \alpha_H^2 + \alpha_L \alpha_H^3\right)V + \left(\alpha_H^3 \alpha_L + \alpha_H^2 \alpha_L^2\right)V}{8k(\alpha_H + 2\alpha_L)} \\
& + m_{2H} \frac{\frac{1}{2}\alpha_H^3(\alpha_L + \alpha_H)V}{8k(\alpha_H + 2\alpha_L)} \\
= \;& m_{2H} \frac{2\alpha_L^3 \alpha_H V + 2\alpha_H^3 \alpha_L V + \frac{1}{2}\alpha_H^3(\alpha_L + \alpha_H)V}{8k(\alpha_H + 2\alpha_L)}
\end{aligned}
\tag{B.40}
$$

On the other hand, we have

$$
\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} = \frac{\left(\alpha_H^3 + \alpha_H^2 \alpha_L\right)}{8k^2} \frac{\alpha_L(\alpha_L + \alpha_H)}{(\alpha_H + 2\alpha_L)^2} V^2 + \frac{\alpha_H^3}{16k^2} \frac{(\alpha_H + \alpha_L)^2}{(\alpha_H + 2\alpha_L)^2} V^2 \tag{B.41}
$$

It is clear that expression (B.40) is greater or equal to $\frac{5\alpha_L^3}{24}V$, expression (B.41) is greater than or equal to $\frac{\alpha_L^3}{12}V^2$ and $m_{2H}$ is greater than or equal to $\frac{2}{3}V$. Therefore (B.39) is greater than or equal to

$$\frac{5\alpha_L^6}{144 \times 3}V^4 - \frac{\alpha_L^6}{64}V^4 \tag{B.42}$$

which is negative. This means that when $\alpha_L$ and $\alpha_H$ are close to each other, (B.39) is negative. It is clear to see that when when the difference between $\alpha_L$ and $\alpha_H$ is large, (B.39) is positive.

Notice that

$$\alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H} = m_{2H}\frac{\alpha_L\left(\alpha_H\alpha_L - \alpha_H^2\right)}{4k}\left(\frac{\alpha_L}{\alpha_H + 2\alpha_L}V\right) + m_{2H}\frac{\alpha_L\alpha_H^2(\alpha_L + \alpha_H)}{8k(\alpha_H + 2\alpha_L)}V \tag{B.43}$$

Using the (B.21), (B.38) and (B.43), we can see that

$$\alpha_L\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) - \frac{\alpha_L^6 V^4}{64k^3} > 0 \tag{B.44}$$

when the difference between $\alpha_L$ and $\alpha_H$ is large.

Denote $\frac{\alpha_H}{\alpha_L}$ by $x$, then $m_{2H} = \left(\frac{\alpha_H + \alpha_L}{\alpha_H + 2\alpha_L}\right)V$ can be expressed as $m_{2H} = \left(\frac{x+1}{x+2}\right)V$.

By expression (B.34), we have

$$m_{1H} + \frac{\alpha_H^2 m_{2H}^2}{4k} = \alpha_L^2\left(\frac{x^2 + x}{4k(x+2)} + \frac{x^2(x+1)}{8k(x+2)}\right)\left(\frac{x+1}{x+2}\right)V^2 \tag{B.45}$$

Thus

$$\alpha_H\left(\frac{m_{1H} + \frac{\alpha_H^2 m_{2H}^2}{4k}}{2k}\right) = \frac{\alpha_L^3 x}{2k}\left(\frac{x^2 + x}{4k(x+2)} + \frac{x^2(x+1)}{8k(x+2)}\right)\left(\frac{x+1}{x+2}\right)V^2 \tag{B.46}$$

Using expression (B.35), i.e.,

$$m_{1H} = \frac{\alpha_H^2 + \alpha_H\alpha_L}{4k}m_{2H}(V - m_{2H}) - \frac{\alpha_H^2 m_{2H}^2}{8k} \tag{B.47}$$

we have

$$\begin{aligned} & \alpha_H\frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H} \\ = \; & \frac{\alpha_H^2 - \alpha_H\alpha_L}{4k}m_{2H}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}^2}{8k} \\ = \; & \alpha_L^2\left(\frac{x^2 - x}{4k(x+2)} + \frac{x^2(x+1)}{8k(x+2)}\right)\left(\frac{x+1}{x+2}\right)V^2 \end{aligned} \tag{B.48}$$

and

$$
\begin{aligned}
& \alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H} \\
= & \frac{\alpha_H \alpha_L - \alpha_H^2}{4k} m_{2H}(V - m_{2H}) + \frac{\alpha_H^2 m_{2H}^2}{8k} \\
= & \alpha_L^2 \left( \frac{x - x^2}{4k(x+2)} + \frac{x^2(x+1)}{8k(x+2)} \right) \left( \frac{x+1}{x+2} \right) V^2 \qquad \text{(B.49)}
\end{aligned}
$$

Notice that (B.38) gives

$$
m_{0H} = \alpha_L \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right) \left( \alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H} \right) - \frac{\alpha_L^6 V^4}{64k^3} \qquad \text{(B.50)}
$$

Therefore

$$
\begin{aligned}
& m_{0H} + \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right)^2 \\
= & \alpha_L \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right) \left( \alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H} \right) - \frac{\alpha_L^6 V^4}{64k^3} + \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right)^2 \\
= & \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right) \left( \alpha_L \frac{\alpha_H \alpha_L - \alpha_H^2}{4k} m_{2H}(V - m_{2H}) + \alpha_L \alpha_H^2 \frac{m_{2H}^2}{8k} + \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2} \right) \\
& - \frac{\alpha_L^6 V^4}{64k^3} \qquad \text{(B.51)}
\end{aligned}
$$

where $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$ can be expressed as (B.46), and

$$
\begin{aligned}
& \alpha_L \frac{\alpha_H \alpha_L - \alpha_H^2}{4k} m_{2H}(V - m_{2H}) + \alpha_L \alpha_H^2 \frac{m_{2H}^2}{8k} + \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2} \\
= & \alpha_L^3 \frac{(x - x^2)(x+1)}{4k(x+2)^2} V^2 + \alpha_L^3 \frac{x^2(x+1)^2}{8k(x+2)^2} V^2 + \alpha_L^3 \frac{x^2(x+1)^2}{8k(x+2)^2} V^2 + \alpha_L^3 \frac{x^3(x+1)^2}{16k(x+2)^2} V^2 \\
= & \alpha_L^3 \frac{(x - x^2)(x+1)}{4k(x+2)^2} V^2 + \alpha_L^3 \frac{x^2(x+1)^2}{4k(x+2)^2} V^2 + \alpha_L^3 \frac{x^3(x+1)^2}{16k(x+2)^2} V^2 \\
= & \alpha_L^3 \frac{(x + x^3)(x+1)}{4k(x+2)^2} V^2 + \alpha_L^3 \frac{x^3(x+1)^2}{16k(x+2)^2} V^2 \qquad \text{(B.52)}
\end{aligned}
$$

where for the first equality, we apply (B.46).

It is easy to show that (B.46) and (B.52) are increasing functions in $x$ which equals $\frac{\alpha_H}{\alpha_L} \geq 1$, because $\frac{x+1}{x+2}$ and $\frac{x^2+1}{(x+2)^2}$ are increasing functions in $x$. Let $x = 1$, then

$$
\begin{aligned}
& m_{0H} + \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right)^2 \\
= \quad & \frac{\alpha_L^3}{2k} \left( \frac{2}{4k}\frac{1}{3} + \frac{1}{8k}\frac{2}{3} \right) \left( \frac{2}{3} \right) V^2 \cdot \left( \alpha_L^3 \frac{1}{9k} + \frac{1}{36k} \right) V^2 - \frac{\alpha_L^6 V^4}{64k^3} \\
= \quad & \alpha_L^6 \frac{1}{12} \cdot \frac{5}{36} V^2 - \frac{\alpha_L^6 V^4}{64k^3} \qquad \text{(B.53)}
\end{aligned}
$$

which is less than 0. We can see that as long as $x$ is greater than a critical value, the (B.10) is no longer binding. In fact, we can find this critical value by solving the following equation:

$$
\frac{x}{2} \left( \frac{(x^2+x)(x+1)}{4(x+2)^2} + \frac{x^2(x+1)^2}{8(x+2)^2} \right) \cdot \left( \frac{(x+x^3)(x+1)}{4(x+2)^2} + \frac{x^3(x+1)^2}{16(x+2)^2} \right) = \frac{1}{64} \qquad \text{(B.54)}
$$

namely,

$$
x\left(2(x^2+x)(x+1) + x^2(x+1)^2\right)\left(4(x+x^3)(x+1) + x^3(x+1)^2\right) - 4(x+2)^4 = 0 \qquad \text{(B.55)}
$$

Numerical result shows that $x \approx 1.063971$ is the only real root of (B.55) which is great than 1. Therefore this the critical point we want to find. This means that as long as $\alpha_H > 1.063971\alpha_L$, the agent will participate when the high-type principal offers the contract.

Next, we want to know then $m_{0H} > 0$.

By (B.38), we know that

$$
\begin{aligned}
m_{0H} \quad = \quad & \alpha_L \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right) \left( \alpha_L \frac{\alpha_H m_{2H}}{2k} (V - m_{2H}) - m_{1H} \right) - \frac{\alpha_L^6 V^4}{64k^3} \\
= \quad & \frac{\alpha_L^6 x}{2k} \left( \frac{(x^2+x)(x+1)}{4k(x+2)^2} + \frac{x^2(x+1)^2}{8k(x+2)^2} \right) \cdot \left( \frac{(x-x^2)(x+1)}{4k(x+2)^2} + \frac{x^2(x+1)^2}{8k(x+2)^2} \right) V^4 - \frac{\alpha_L^6 V^4}{64k^3} \\
& \qquad \text{(B.56)}
\end{aligned}
$$

Thus $m_{0H} > 0$ is equivalent to

$$
x\left(2(x^2+x)(x+1) + x^2(x+1)^2\right)\left(2(x-x^2)(x+1) + x^2(x+1)^2\right) - 2(x+2)^4 > 0 \qquad \text{(B.57)}
$$

Numerical result shows that when $x > 1.335236$, (B.57) holds, while $1 \le x < 1.335236$,(B.57) is violated and equality does not hold. This means that as long as $\alpha_H > 1.335236\alpha_L$, $m_{0H} > 0$.

The high-type principal's profit is

$$
\begin{aligned}
&\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)-m_{0H}\\
= &\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\alpha_L^3\left(\frac{x(x^2-x)(x+1)}{4k(x+2)^2}+\frac{x^3(x+1)^2}{8k(x+2)^2}-\frac{(x-x^2)(x+1)}{4k(x+2)^2}\right)V^4\\
&-\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\alpha_L^3\left(\frac{x^2(x+1)^2}{8k(x+2)^2}\right)V^4+\frac{\alpha_L^6 V^4}{64k^3}\\
= &\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\alpha_L^3\left(\frac{(x^3-x)(x+1)}{4k(x+2)^2}+\frac{(x^3-x^2)(x+1)^2}{8k(x+2)^2}\right)V^4+\frac{\alpha_L^6 V^4}{64k^3}\\
= &\frac{\alpha_L^3 x}{2k}\left(\frac{(x^2+x)(x+1)}{4k(x+2)^2}+\frac{x^2(x+1)^2}{8k(x+2)^2}\right)\cdot\alpha_L^3\left(\frac{(x^3-x)(x+1)}{4k(x+2)^2}\right)V^4\\
&+\frac{\alpha_L^3 x}{2k}\left(\frac{(x^2+x)(x+1)}{4k(x+2)^2}+\frac{x^2(x+1)^2}{8k(x+2)^2}\right)\cdot\alpha_L^3\left(\frac{(x^3-x^2)(x+1)^2}{8k(x+2)^2}\right)V^4+\frac{\alpha_L^6 V^4}{64k^3}\\
= &\frac{\alpha_L^6\left[x^2\left(2(x+1)^2+x(x+1)^2\right)x\left((x^2-1)(x+1)+\frac{1}{2}(x^2-x)(x+1)^2\right)+(x+2)^4\right]}{64k^3(x+2)^4}
\end{aligned}
\tag{B.58}
$$

It is clear that (B.58) is an increasing function in $x$, namely the high-type principal's profit increases as $x$ increases when $\alpha_L$ being held constant. On the other hand, the high-type principal's profit increases as $\alpha_L$ increases when $x$ being held constant.

On thing we want to show is that when (B.2) binds and (B.5) does not bind, (B.3) is redundant.

In order to show that (B.147) is less than or equal to (B.58), it is equivalent to show that

$$
\begin{aligned}
&(4x^8-4x^7+x^6)(x+2)^4\\
\le &\left[x^2\left(2(x+1)^2+x(x+1)^2\right)x\left((x^2-1)(x+1)+\frac{1}{2}(x^2-x)(x+1)^2\right)+(x+2)^4\right](2x-1)^5
\end{aligned}
\tag{B.59}
$$

where

$$
4x^8-4x^7+x^6=x^6(4x^2-4x+1)=x^6(2x-1)^2 \tag{B.60}
$$

Thus, to show (B.59), it is equivalent to show

$$
\begin{aligned}
&x^6(x+2)^4\\
\le &\left[x^2\left(2(x+1)^2+x(x+1)^2\right)x\left((x^2-1)(x+1)+\frac{1}{2}(x^2-x)(x+1)^2\right)+(x+2)^4\right](2x-1)^3
\end{aligned}
\tag{B.61}
$$

namely,

$$
\begin{aligned}
& x^6(x+2)^4 \\
\leq \; & \left[x^3\left(x^3+4x^2+5x+2\right)\left(\frac{1}{2}x^4+\frac{3}{2}x^3+\frac{1}{2}x^2-\frac{3}{2}x-1\right)+(x+2)^4\right](2x-1)^3
\end{aligned}
\tag{B.62}
$$

Denote $F(x)$ be the expression

$$
\left[x^3\left(x^3+4x^2+5x+2\right)\left(\frac{1}{2}x^4+\frac{3}{2}x^3+\frac{1}{2}x^2-\frac{3}{2}x-1\right)+(x+2)^4\right](2x-1)^3-x^6(x+2)^4 \tag{B.63}
$$

Using Maple we can find that polynomial $F(x)$ has only a real root at $x=1$, when $x \geq 0$. Since $F(x)$ increases to positive infinity as $x$ increases, this means $F(x)$ is positive when $x>1$ and equals 0 when $x=1$. This shows that (B.63) always holds when $x \geq 1$.

For the separating equilibrium, one interesting question is whether $m_{1H}>0$ or $<0$.

Applying $x=\frac{\alpha_H}{\alpha_L}$ to (B.35), we have

$$
\begin{aligned}
m_{1H} &= \frac{\alpha_H^2+\alpha_H\alpha_L}{4k}m_{2H}(V-m_{2H})-\frac{\alpha_H^2 m_{2H}^2}{8k} \\
&= \left(\frac{\alpha_H^2+\alpha_H\alpha_L}{4k}\frac{\alpha_L}{\alpha_H+2\alpha_L}-\frac{\alpha_H^2(\alpha_H+\alpha_L)}{8k(\alpha_H+2\alpha_L)}\right)\frac{\alpha_H+\alpha_L}{\alpha_H+2\alpha_L} \\
&= \left(\frac{2(\alpha_H^2+\alpha_H\alpha_L)\alpha_L-\alpha_H^3-\alpha_H^2\alpha_L}{8k(\alpha_H+2\alpha_L)}\right)\frac{\alpha_H+\alpha_L}{\alpha_H+2\alpha_L} \\
&= \left(\frac{\alpha_H^2\alpha_L+2\alpha_H\alpha_L^2-\alpha_H^3}{8k(\alpha_H+2\alpha_L)}\right)\frac{\alpha_H+\alpha_L}{\alpha_H+2\alpha_L} \\
&= \alpha_L^2\left(\frac{x^2+2x-x^3}{8k(x+2)}\right)\frac{x+1}{x+2} \\
&= \alpha_L^2\frac{x(2-x)(x+1)^2}{8k(x+2)^2}
\end{aligned}
\tag{B.64}
$$

Thus, $m_{1H}>0$ when $0 \leq x<2$, and $m_{1H}<0$ when $x>2$.

Taking the derivative of (B.64) with respect to $x$ gives

$$
\begin{aligned}
& \frac{\left((2-2x)(x+1)^2+(2x-x^2)\cdot 2(x+1)\right)(x+2)^2-(2x-x^2)(x+1)^2\cdot 2(x+2)}{8k(x+2)^4} \\
= \; & \frac{\left[\left((2-2x)(x+1)+(2x-x^2)\cdot 2\right)(x+2)-(2x-x^2)(x+1)\cdot 2\right](x+1)(x+2)}{8k(x+2)^4} \\
= \; & \frac{\left[(2+4x-4x^2)(x+2)-(4x-2x^2)(x+1)\right](x+1)(x+2)}{8k(x+2)^4} \\
= \; & \frac{\left(2x+4x^2-4x^3+4+8x-8x^2-4x^2+2x^3-4x+2x^2\right)(x+1)(x+2)}{8k(x+2)^4} \\
= \; & \frac{\left(-2x^3-6x^2+6x+4\right)(x+1)(x+2)}{8k(x+2)^4}
\end{aligned}
\tag{B.65}
$$

Numerical result shows that (B.65) is positive when $1 \le x < 1.145103$, and negative when $x > 1.145103$. This means that $m_{1H}$ is an increasing function in $x$ when $1 \le x < 1.145103$, and a decreasing function in $x$ when $x > 1.145103$.

Next we study the second scenario of the high-type principal's profit maximization problem in which the agent earns zero profit and the constraint to prevent the low-type principal from mimicking the high-type one binds, i.e., both (B.2) binding and (B.5) binding.

### B.1.2 The second scenario

In the second scenario, the Lagrangian for the high-type principal's profit maximization problem is

$$\begin{aligned}
&\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)-m_{0H}\\
+\quad&\lambda_1\left[\frac{\alpha_L^6V^4}{64k^3}+m_{0H}-\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right]\\
&+\lambda_2\left[m_{0H}+k\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2\right] \qquad \text{(B.66)}
\end{aligned}$$

Since (B.10) is binding, we have

$$m_{0H}=-k\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 \qquad \text{(B.67)}$$

Using this, the Lagrangian of the high-type principal's profit maximization problem becomes

$$\begin{aligned}
&\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)+k\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2\\
+\quad&\lambda_1\left[\frac{\alpha_L^6V^4}{64k^3}-k\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2\right]\\
-\quad&\lambda_1\left[\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right]
\end{aligned}$$

(B.68)

We claim that (B.3) does not bind, i.e., redundant. Notice that constraint (B.3) can be written as

$$\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)+k\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2$$
$$\geq \frac{\alpha_L^6 x^6 V^4}{64k^3(2x-1)^3} \tag{B.69}$$

where the right side is $\overline{LM}$ (see (B.147)).

Since both (B.2) and (B.5) are binding, from the expression of the constraint in (B.66), we have

$$\frac{\alpha_L^6 V^4}{64k^3}=k\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2-\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right) \tag{B.70}$$

Let

$$m_{2H}=\frac{1}{x^{1.5}}V \tag{B.71}$$

with $x=\frac{\alpha_H}{\alpha_L}$ and

$$m_{1H}=\alpha_L\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})=\alpha_L^2\frac{x\frac{1}{x^{1.5}}V}{2k}\left(V-\frac{1}{x^{1.5}}V\right) \tag{B.72}$$

Substituting the expressions of $m_{1H}$ and $m_{2H}$ into the left side of (B.69) results in an expression in $x$. Denoting it by $F(x)$. Thus to show (B.69) does not bind is equivalent to show

$$F(x)>\frac{\alpha_L^6 x^6 V^4}{64k^3(2x-1)^3} \tag{B.73}$$

when $x>1$. Notice that $\alpha_L^6$ and $V^4$ can be canceled from both sides of (B.73). Using Maple, either directly calculating the difference between the right side and the left side of (B.73) or calculating the real roots of an polynomial equation which is equivalent to (B.73) with equality holding, we find that (B.73) always holds for $x>1$. This shows that (B.69) does not bind, namely, (B.3) is redundant.

Next we look at the first order conditions of the Lagrangian (B.68).

The first order condition of (B.68) with respect to $m_{1H}$ gives

$$\frac{\alpha_H^2}{2k}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]-\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)+\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)$$
$$+\lambda_1\left[-(\alpha_H-\alpha_L)\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)-\frac{\alpha_H\alpha_L}{2k}\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right]=0 \tag{B.74}$$

which means that

$$\frac{\alpha_H^4}{4k^2}m_{2H}(V-m_{2H})-\frac{\alpha_H^2}{2k}m_{1H}-\lambda_1\frac{\left(\alpha_H^2-2\alpha_H\alpha_L\right)}{2k}m_{1H}$$
$$-\lambda_1\frac{\left(\alpha_H^4-\alpha_L\alpha_H^3\right)m_{2H}^2}{8k^2}-\lambda_1\frac{\alpha_H^2\alpha_L^2}{4k^2}m_{2H}(V-m_{2H}) \quad = \quad 0 \tag{B.75}$$

Simplifying (B.75) leads to

$$\frac{\alpha_H^2+\lambda_1\alpha_H^2-2\lambda_1\alpha_H\alpha_L}{2k}m_{1H}=\frac{\left(\alpha_H^4-\lambda_1\alpha_H^2\alpha_L^2\right)}{4k^2}m_{2H}(V-m_{2H})-\frac{\lambda_1\left(\alpha_H^4-\alpha_L\alpha_H^3\right)}{8k^2}m_{2H}^2 \tag{B.76}$$

which gives

$$m_{1H}=\frac{\left(\alpha_H^4-\lambda_1\alpha_H^2\alpha_L^2\right)}{2k\left(\alpha_H^2+\lambda_1\alpha_H^2-2\lambda_1\alpha_H\alpha_L\right)}m_{2H}(V-m_{2H})-\frac{\lambda_1\left(\alpha_H^4-\alpha_L\alpha_H^3\right)}{4k\left(\alpha_H^2+\lambda_1\alpha_H^2-2\lambda_1\alpha_H\alpha_L\right)}m_{2H}^2 \tag{B.77}$$

The first order condition with respect to $m_{2H}$ gives

$$\frac{\alpha_H^4 m_{2H}}{4k^2}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]+\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\frac{\alpha_H^2}{2k}(V-m_{2H})\right]$$
$$+\lambda_1\left[-\frac{\alpha_H^3 m_{2H}}{2k}\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)-\frac{\alpha_L\alpha_H^3 m_{2H}}{4k^2}\left(\frac{\alpha_H\alpha_L m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right]$$
$$-\lambda_1\left[\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\frac{\alpha_L\alpha_H}{2k}(V-2m_{2H})\right)\right] \quad = \quad 0 \tag{B.78}$$

Notice that

$$\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$$
$$=\frac{\left(\alpha_H^5-\lambda_1\alpha_H^3\alpha_L^2\right)}{4k^2\left(\alpha_H^2+\lambda_1\alpha_H^2-2\lambda_1\alpha_H\alpha_L\right)}m_{2H}(V-m_{2H})-\frac{\lambda_1\left(\alpha_H^5-\alpha_L\alpha_H^4\right)}{8k^2\left(\alpha_H^2+\lambda_1\alpha_H^2-2\lambda_1\alpha_H\alpha_L\right)}m_{2H}^2$$
$$+\frac{\alpha_H^3 m_{2H}^2}{8k^2}$$
$$=\frac{\left(\alpha_H^5-\lambda_1\alpha_H^3\alpha_L^2\right)}{4k^2\left(\alpha_H^2+\lambda_1\alpha_H^2-2\lambda_1\alpha_H\alpha_L\right)}m_{2H}(V-m_{2H})+\frac{\left(\alpha_H^5-\lambda_1\alpha_L\alpha_H^4\right)}{8k^2\left(\alpha_H^2+\lambda_1\alpha_H^2-2\lambda_1\alpha_H\alpha_L\right)}m_{2H}^2 \tag{B.79}$$

$$\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}$$
$$=\frac{\left(\lambda_1\alpha_H^4-2\lambda_1\alpha_H^3\alpha_L+\lambda_1\alpha_H^2\alpha_L^2\right)}{2k\left(\alpha_H^2+\lambda_1\alpha_H^2-2\lambda_1\alpha_H\alpha_L\right)}m_{2H}(V-m_{2H})+\frac{\lambda_1\left(\alpha_H^4-\alpha_L\alpha_H^3\right)}{4k\left(\alpha_H^2+\lambda_1\alpha_H^2-2\lambda_1\alpha_H\alpha_L\right)}m_{2H}^2 \tag{B.80}$$

and

$$\frac{\alpha_L \alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}$$
$$= \frac{\left(\alpha_H^3 \alpha_L + \lambda_1 \alpha_H^3 \alpha_L - \lambda_1 \alpha_H^2 \alpha_L^2 - \alpha_H^4\right)}{2k\left(\alpha_H^2 + \lambda_1 \alpha_H^2 - 2\lambda_1 \alpha_H \alpha_L\right)} m_{2H}(V - m_{2H}) + \frac{\lambda_1\left(\alpha_H^4 - \alpha_L \alpha_H^3\right)}{4k\left(\alpha_H^2 + \lambda_1 \alpha_H^2 - 2\lambda_1 \alpha_H \alpha_L\right)} m_{2H}^2 \tag{B.81}$$

Substituting (B.79), (B.80) and (B.81) into (B.78) and multiplying it by $16k^3$ leads to

$$\begin{aligned}
&2\alpha_H^4\left(\lambda_1 \alpha_H^4 - 2\lambda_1 \alpha_H^3 \alpha_L + \lambda_1 \alpha_H^2 \alpha_L^2\right) m_{2H}^2 (V - m_{2H}) + \lambda_1 \alpha_H^4\left(\alpha_H^4 - \alpha_L \alpha_H^3\right) m_{2H}^3 \\
&+2\alpha_H^3\left(\alpha_H^5 - \lambda_1 \alpha_H^3 \alpha_L^2\right) m_{2H}(V - m_{2H})^2 + \alpha_H^3\left(\alpha_H^5 - \lambda_1 \alpha_L \alpha_H^4\right) m_{2H}^2 (V - m_{2H}) \\
&-2\lambda_1 \alpha_H^3\left(\alpha_H^5 - \lambda_1 \alpha_H^3 \alpha_L^2\right) m_{2H}^2 (V - m_{2H}) - \lambda_1 \alpha_H^3\left(\alpha_H^5 - \lambda_1 \alpha_L \alpha_H^4\right) m_{2H}^3 \\
&-\alpha_L \alpha_H^3\left(\alpha_H^3 \alpha_L + \lambda_1 \alpha_H^3 \alpha_L - 2\lambda_1 \alpha_H^2 \alpha_L^2 - \alpha_H^4\right) m_{2H}^2 (V - m_{2H}) \\
&+\lambda_1 \alpha_L \alpha_H^3\left(\alpha_H^4 - \alpha_L \alpha_H^3\right) m_{2H}^3 \\
&-2\lambda_1 \alpha_L^2 \alpha_H\left(\alpha_H^5 - \lambda_1 \alpha_H^3 \alpha_L^2\right) m_{2H}(V - m_{2H})(V - 2m_{2H}) \\
&-\lambda_1 \alpha_L^2 \alpha_H\left(\alpha_H^5 - \lambda_1 \alpha_L \alpha_H^4\right) m_{2H}^2 (V - 2m_{2H}) \\
&= 0
\end{aligned} \tag{B.82}$$

It is clear that one $m_{2H}$ can be factored out of the equation (B.82). Thus we can obtain a quadratic equation of $m_{2H}$ with unknown parameter $\lambda_1$. Notice that the binding constraint that appears in (B.68) forms another equation of $m_{2H}$ and $\lambda_1$, namely

$$\frac{\alpha_L^6 V^4}{64k^3} - k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 - \alpha_L\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) = 0 \tag{B.83}$$

where $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$ and $\alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}$ have expressions (B.79) and (B.81). Thus with these two equations of $m_{2H}$ and $\lambda_1$, we are able to solve for them.

We claim that as the solution of (B.82) and (B.83), $m_{2H}$ and $\lambda_1$ are functions of $x = \frac{\alpha_H}{\alpha_L}$ and $V$.

By (B.79) and (B.80), we have

$$\begin{aligned}
&\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \\
&= \frac{\left(\alpha_H^5 - \lambda_1 \alpha_H^3 \alpha_L^2\right)}{4k^2\left(\alpha_H^2 + \lambda_1 \alpha_H^2 - 2\lambda_1 \alpha_H \alpha_L\right)} m_{2H}(V - m_{2H}) + \frac{\left(\alpha_H^5 - \lambda_1 \alpha_L \alpha_H^4\right)}{8k^2\left(\alpha_H^2 + \lambda_1 \alpha_H^2 - 2\lambda_1 \alpha_H \alpha_L\right)} m_{2H}^2 \\
&= \frac{\alpha_L^3\left(x^5 - \lambda_1 x^3\right)}{4k^2\left(x^2 + \lambda_1 x^2 - 2\lambda_1 x\right)} m_{2H}(V - m_{2H}) + \frac{\alpha_L^3\left(x^5 - \lambda_1 x^4\right)}{8k^2\left(x^2 + \lambda_1 x^2 - 2\lambda_1 x\right)} m_{2H}^2
\end{aligned} \tag{B.84}$$

and

$$
\begin{aligned}
&\frac{\alpha_H^2 m_{2H}}{2k}(V - m_{2H}) - m_{1H} \\
&= \frac{\left(\lambda_1\alpha_H^4 - 2\lambda_1\alpha_H^3\alpha_L + \lambda_1\alpha_H^2\alpha_L^2\right)}{2k\left(\alpha_H^2 + \lambda_1\alpha_H^2 - 2\lambda_1\alpha_H\alpha_L\right)} m_{2H}(V - m_{2H}) + \frac{\lambda_1\left(\alpha_H^4 - \alpha_L\alpha_H^3\right)}{4k\left(\alpha_H^2 + \lambda_1\alpha_H^2 - 2\lambda_1\alpha_H\alpha_L\right)} m_{2H}^2 \\
&= \frac{\alpha_L^2\left(\lambda_1 x^4 - 2\lambda_1 x^3 + \lambda_1 x^2\right)}{2k\left(x^2 + \lambda_1 x^2 - 2\lambda_1 x\right)} m_{2H}(V - m_{2H}) + \frac{\alpha_L^2\lambda_1\left(x^4 - x^3\right)}{4k\left(x^2 + \lambda_1 x^2 - 2\lambda_1 x\right)} m_{2H}^2
\end{aligned}
\tag{B.85}
$$

Substituting (B.84) and (B.85) into (B.83) and dividing it by $\frac{\alpha_L^6}{k^3}$, we obtain an equation in $m_{2H}$, $\lambda_1$, $V$ and $x$.

On the other hand, (B.82) can be rewritten as

$$
\begin{aligned}
&2\alpha_L^8\left(\lambda_1 x^8 - 2\lambda_1 x^7 + \lambda_1 x^6\right) m_{2H}^2(V - m_{2H}) + \lambda_1\alpha_L^8\left(x^8 - x^7\right) m_{2H}^3 \\
&+2\alpha_L^8\left(x^8 - \lambda_1 x^6\right) m_{2H}(V - m_{2H})^2 + \alpha_L^8\left(x^8 - \lambda_1 x^7\right) m_{2H}^2(V - m_{2H}) \\
&-2\lambda_1\alpha_L^8\left(x^8 - \lambda_1 x^6\right) m_{2H}^2(V - m_{2H}) - \lambda_1\alpha_L^8\left(x^8 - \lambda_1 x^7\right) m_{2H}^3 \\
&-\alpha_L^8\left(x^6 + \lambda_1 x^6 - 2\lambda_1 x^5 - x^7\right) m_{2H}^2(V - m_{2H}) + \lambda_1\alpha_L^8\left(x^7 - x^6\right) m_{2H}^3 \\
&-2\lambda_1\alpha_L^8\left(x^6 - \lambda_1 x^4\right) m_{2H}(V - m_{2H})(V - 2m_{2H}) - \lambda_1\alpha_L^8\left(x^6 - \lambda_1 x^5\right) m_{2H}^2(V - 2m_{2H}) \\
&= 0
\end{aligned}
\tag{B.86}
$$

Dividing (B.86) by $\alpha_L^8$ gives another equation in $m_{2H}$, $\lambda_1$, $V$ and $x$.

Therefore the solution for $m_{2H}$ and $\lambda$ of these two new equations would only depends on $x$ and $V$. It is easy to see that the high-type principal's maximum profit can be written as $\frac{\alpha_L}{k^3} f(x, V)$, where $f(x, V)$ is a function only depends on $x$ and $V$.

### B.1.3 Supplements

In this part, we will calculate $LM$ and $\overline{LM}$ and show that we only need to consider the situation in which effort levels in two periods take the forms $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k}$ and $e_{1H}^* = \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$.

First we show that $LM = \frac{\alpha_L^6 V^4}{64k^3}$.

Under complete information, which means that the low-type principal's type is known to the agent, by offering the menu of money transfers $(m_{0L}, m_{1L}, m_{2L})$ to the low agent and $(m_{0H}, m_{1H}, m_{2H})$ to the high-type agent, the low-type principal wants to maximize her expected profit:

$$
\alpha_L e_{1L}\left(\alpha_L e_{2L}(V - m_{2L}) - m_{1L}\right) - m_{0L} \tag{B.87}
$$

Considering $(m_{0L}, m_{1L}, m_{2L})$, the low-type agent would like to maximize his expected profit:

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \tag{B.88}$$

where $0 \le e_{1L} \le 1$ and $0 \le e_{2L} \le 1$ and (B.88) $\ge 0$ as the participation constraint.

We can assume $e_{1L}$ and $e_{2L}$ are positive and don't need to consider the scenarios when some of them are 0. The reason is the following.

If $e_{1L} = 0$, the agent's expected profit (B.88) becomes $m_{0L}$, which has to be nonnegative in order for the agent to participate. Thus the principal's expected profit (B.87) satisfies $\alpha_L e_{1L} (\alpha_L e_{2L}(V - m_{2L}) - m_{1L}) - m_{0L} = -m_{0L} \le 0$. Clearly, this can't be the maximum location the principal anticipates.

If $e_{2L} = 0$, the agent's expected profit (B.88) becomes $m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L}$, which is nonnegative to ensure the participation of the agent. Therefore the principal's expected profit (B.87) satisfies $-\alpha_L e_{1L} m_{2L} - m_{0L} \le -ke_{1L}^2 \le 0$. It is clear that this won't be the maximum location the principal looks for.

Next we will find the expressions of the optimal efforts $e_{1L}^*$ and $e_{2L}^*$ for given $m_{0L}$, $m_{1L}$ and $m_{2L}$, corresponding to (B.88).

Notice that the Lagrangian for the maximization problem of the low-type expected profit with $e_{1L}$ and $e_{2L}$ as the decision variables is

$$m_{0L} - ke_{1L}^2 + \alpha_L e_{1L} m_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} + \lambda_1 (1 - e_{1L}) + \lambda_2 (1 - e_{2L}) \tag{B.89}$$

where $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ are Lagrangian multipliers.

The first order conditions of (B.89) with respect to $e_{1L}$ and $e_{2L}$ as follows:

$$-2ke_{1L} + \alpha_L m_{1L} - \alpha_L k e_{2L}^2 + \alpha_L^2 e_{2L} m_{2L} - \lambda_1 = 0 \tag{B.90}$$

$$\alpha_L e_{1L} (-2ke_{2L} + \alpha_L m_{2L}) - \lambda_2 = 0 \tag{B.91}$$

which lead to the optimal efforts $e_{1L}^*$ and $e_{2L}^*$ satisfying

$$e_{2L}^* = \frac{\alpha_L m_{2L} - \frac{\lambda_2}{\alpha_L e_{1L}^*}}{2k} \tag{B.92}$$

$$e_{1L}^* = \frac{\alpha_L m_{1L} - \alpha_L k {e_{2L}^*}^2 + \alpha_L^2 e_{2L}^* m_{2L} - \lambda_1}{2k} \tag{B.93}$$

where $0 < e_{2L}^* \le 1$ and $0 < e_{1L}^* \le 1$.

There are four situations for consideration:

1. When $\lambda_1 > 0$ and $\lambda_2 > 0$, $e^*_{1L} = 1$ and $e^*_{2L} = 1$. By (B.88), the low-type agent's expected profit is

$$m_{0L} - k + \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \tag{B.94}$$

Since $\lambda_1 > 0$, $\lambda_2 > 0$, $e^*_{1L} = 1$ and $e^*_{2L} = 1$, (B.92) and (B.93) imply that

$$\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} > 1$$

and $\frac{\alpha_L m_{2L}}{2k} > 1$.

2. When $\lambda_1 > 0$ and $\lambda_2 = 0$, $e^*_{1L} = 1$ and $e^*_{2L} = \frac{\alpha_L m_{2L}}{2k} \leq 1$. By (B.88), the low-type agent's expected profit is

$$\begin{aligned} & m_{0L} - k + \alpha_L m_{1L} - \alpha_L k \left(\frac{\alpha_L m_{2L}}{2k}\right)^2 + \alpha_L^2 \frac{\alpha_L m_{2L}}{2k} m_{2L} \\ = \; & m_{0L} - k + \alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k} \end{aligned} \tag{B.95}$$

Since $\lambda_1 > 0$ and $e^*_{1L} = 1$, (B.93) implies that $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} > 1$.

3. When $\lambda_1 = 0$ and $\lambda_2 > 0$, $e^*_{1L} = \frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} \leq 1$ and $e^*_{2L} = 1$, namely, $2ke^*_{1L} = \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}$. By (B.88), the low-type agent's expected profit is

$$\begin{aligned} & m_{0L} - k{e^*_{1L}}^2 + \alpha_L e^*_{1L} m_{1L} - \alpha_L e^*_{1L} k {e^*_{2L}}^2 + \alpha_L^2 e^*_{1L} e^*_{2L} m_{2L} \\ = \; & m_{0L} - k{e^*_{1L}}^2 + \alpha_L e^*_{1L} m_{1L} - \alpha_L e^*_{1L} k + \alpha_L^2 e^*_{1L} m_{2L} \\ = \; & m_{0L} - k{e^*_{1L}}^2 + e^*_{1L} \left[\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}\right] \\ = \; & m_{0L} - k{e^*_{1L}}^2 + e^*_{1L} 2k e^*_{1L} \\ = \; & m_{0L} + k{e^*_{1L}}^2 \\ = \; & m_{0L} + k\left(\frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k}\right)^2 \end{aligned} \tag{B.96}$$

Since $\lambda_2 > 0$, (B.92) implies that $\frac{\alpha_L m_{2L}}{2k} > 1$.

4. When $\lambda_1 = 0$ and $\lambda_2 = 0$, by (B.92) and (B.93), $e^*_{2L} = \frac{\alpha_L m_{2L}}{2k} \leq 1$ and

$$\begin{aligned} e^*_{1L} &= \frac{\alpha_L m_{1L} - \alpha_L k {e^*_{2L}}^2 + \alpha_L^2 e^*_{2L} m_{2L}}{2k} \\ &= \frac{\alpha_L m_{1L} - \alpha_L k \left(\frac{\alpha_L m_{2L}}{2k}\right)^2 + \alpha_L^2 \frac{\alpha_L m_{2L}}{2k} m_{2L}}{2k} \\ &= \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \end{aligned} \tag{B.97}$$

which is less than or equal to 1.

By (B.88), the low-type agent's expected profit is

$$\begin{aligned} & m_{0L} - k{e_{1L}^*}^2 + \alpha_L e_{1L}^* m_{1L} - \alpha_L e_{1L}^* k {e_{2L}^*}^2 + \alpha_L^2 e_{1L}^* e_{2L}^* m_{2L} \\ = \;& m_{0L} - k{e_{1L}^*}^2 + \alpha_L e_{1L}^* m_{1L} - \alpha_L e_{1L}^* k + \alpha_L^2 e_{1L}^* m_{2L} \\ = \;& m_{0L} - k{e_{1L}^*}^2 + e_{1L}^* \left[\alpha_L m_{1L} - \alpha_L k {e_{2L}^*}^2 + \alpha_L^2 e_{2L}^* m_{2L}\right] \\ = \;& m_{0L} - k{e_{1L}^*}^2 + e_{1L}^* 2k e_{1L}^* \\ = \;& m_{0L} + k{e_{1L}^*}^2 \\ = \;& m_{0L} + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \end{aligned} \tag{B.98}$$

With the availability of the expressions of the optimal efforts $e_{1L}^*$ and $e_{2L}^*$ for given $(m_{0L}, m_{1L}, m_{2L})$, we are able to solve two principal's expected profit maximization problem consisting of (B.87) and (B.88). Next, we will solve the maximization problem consisting of (B.87) and (B.88).

Notice that (B.88) equals 0, instead of being just nonnegative. To show it, using the fact that the optimal efforts $e_{1L}^*$, $e_{2L}^*$ depend only on $m_{1L}$ and $m_{2L}$, we can write $e_{1L}^*$ and $e_{2L}^*$ as $f_1(m_{1L}, m_{2L})$ and $f_2(m_{1L}, m_{2L})$, respectively, functions of $m_{1L}$ and $m_{2L}$. Thus the first order condition with respect to $m_{0L}$ of the Lagrangian for the principal's expected profit maximization problem consisting of (B.87) and (B.88) leads to the positivity of the Lagrangian multiplier for the participation constraint–(B.88) $\geq 0$. This means that (B.88) equals 0, namely, the participation constraint is binding.

Using the binding participation constraint–(B.88) equals 0–to replace $m_{0L}$ in the expression (B.87) and taking into account the four scenarios we discussed in (B.94) through (B.98), we have the following four scenarios for consideration for the the principal's expected profit maximization problem consisting (B.87) and (B.88).

1. When $m_{2L} \leq \frac{2k}{\alpha_L}$ and $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq 1$, i.e., $e_{2L}^* \leq 1$ and $e_{1L}^* \leq 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$\begin{aligned} & \alpha_L\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_L \frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right) + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \\ & + \lambda_1\left(\frac{2k}{\alpha_L} - m_{2L}\right) + \lambda_2\left(1 - \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right) \end{aligned} \tag{B.99}$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are the Lagrangian multipliers.

The first order conditions of (B.99) with respect to $m_{1L}$ and $m_{2L}$ lead to

$$\frac{\alpha_L^2}{2k}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)-\lambda_2\frac{\alpha_L}{2k} = 0 \quad \text{(B.100)}$$

$$\alpha_L^4\frac{2m_{2L}}{8k^2}\left(\frac{\alpha_L^2 m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)$$
$$+\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)-\lambda_1-\lambda_2\alpha_L^3\frac{2m_{2L}}{4k^2} = 0 \quad \text{(B.101)}$$

Multiplying (B.100) by $\alpha_L^3\frac{2m_{2L}}{4k^2}$ and subtracting the product from (B.101) gives

$$\alpha_L\left(\frac{\alpha_L m_{1L}+\frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_L^2}{2k}(V-m_{2L})\right)-\lambda_1=0 \quad \text{(B.102)}$$

There are three cases for consideration:

(a) When $\lambda_1>0$, $m_{2L}=\frac{2k}{\alpha_L}$. The Lagrangian equals

$$\alpha_L\left(\frac{\alpha_L m_{1L}+\alpha_L k}{2k}\right)\left(\alpha_L\left(V-\frac{2k}{\alpha_L}\right)-m_{1L}\right)+k\left(\frac{\alpha_L m_{1L}+\alpha_L k}{2k}\right)^2$$
$$+\tilde{\lambda}_1\left(1-\frac{\alpha_L m_{1L}+\alpha_L k}{2k}\right) \quad \text{(B.103)}$$

where $\tilde{\lambda}_1\geq 0$ is a Lagrangian multiplier.

The first order condition of (B.103) with respect to $m_{1L}$ is

$$\frac{\alpha_L^2}{2k}\left(\alpha_L\left(V-\frac{2k}{\alpha_L}\right)-m_{1L}\right)-\tilde{\lambda}_1\frac{\alpha_L}{2k}=0 \quad \text{(B.104)}$$

When $\tilde{\lambda}_1=0$, $m_{1L}=\alpha_L\left(V-\frac{2k}{\alpha_L}\right)$. So the principal's expected profit equals

$$k\left[\frac{\alpha_L^2\left(V-\frac{2k}{\alpha_L}\right)+\alpha_L k}{2k}\right]^2=k\left(\frac{\alpha_L^2 V-\alpha_L k}{2k}\right)^2 \quad \text{(B.105)}$$

Notice that $\frac{\alpha_L^2\left(V-\frac{2k}{\alpha_L}\right)+\alpha_L k}{2k}\leq 1$.

When $\tilde{\lambda}_1>0$, $\frac{\alpha_L m_{1L}+\alpha_L k}{2k}=1$, namely $m_{1L}=\frac{2k}{\alpha_L}-k$. Thus the principal's expected profit equals

$$\alpha_L\left(\alpha_L\left(V-\frac{2k}{\alpha_L}\right)+k-\frac{2k}{\alpha_L}\right)=\alpha_L^2 V-\alpha_L k-2k \quad \text{(B.106)}$$

It is easy to see that (B.105) is greater than (B.106).

(b) When $\lambda_2 > 0$, $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} = 1$ which implies that $m_{1L} = \frac{2k}{\alpha_L} - \frac{\alpha_L^2 m_{2L}^2}{4k}$. The Lagrangian equals

$$\alpha_L \left( \alpha_L \frac{\alpha_L m_{2L}}{2k} (V - m_{2L}) - \frac{2k}{\alpha_L} + \frac{\alpha_L^2 m_{2L}^2}{4k} \right) + k + \tilde{\lambda}_2 \left( \frac{\alpha_L}{2k} - m_{2L} \right) \tag{B.107}$$

where $\tilde{\lambda}_2 \geq 0$ is a Lagrangian multiplier.

The first order condition of (B.107) gives

$$\frac{\alpha_L^3}{2k}(V - m_{2L}) - \tilde{\lambda}_2 = 0 \tag{B.108}$$

When $\tilde{\lambda}_2 > 0$, $m_{2L} = \frac{\alpha_L}{2k}$, which means that $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} = 1$ is equivalent to $\frac{\alpha_L m_{1L} + \alpha_L k}{2k} = 1$. Thus the principal's expected profit has the same value as in (B.106), which is less than or equal to the value in (B.105).

When $\tilde{\lambda}_2 = 0$, $V = m_{2L}$. Thus the principal's expected profit equals

$$-k + \frac{\alpha_L^3 V^2}{4k} \tag{B.109}$$

which is $\leq -k + \alpha_L k < 0$, since $V = m_{2L} \leq \frac{2k}{\alpha_L}$. Thus (B.109) can't be the local maximum, compared with (B.105).

(c) When $\lambda_1 = 0$ and $\lambda_2 = 0$, from (B.102) and (B.100), we have $m_{2L} = V$ and $m_{1L} = 0$. Since $m_{2L} \leq \frac{2k}{\alpha_L}$, $V \leq \frac{2k}{\alpha_L}$. The principal's expected profit equals

$$k \left( \frac{\frac{\alpha_L^3 V^2}{4k}}{2k} \right)^2 \tag{B.110}$$

Notice that comparing (B.110) with (B.105), we have

$$k \left( \frac{\frac{\alpha_L^3 V^2}{4k}}{2k} \right)^2 \geq k \left( \frac{\alpha_L^2 V - \alpha_L k}{2k} \right)^2 \tag{B.111}$$

where the equality holds only when $V = \frac{2k}{\alpha_L}$, because $\frac{\alpha_L^3 V^2}{4k} \geq \alpha_L^2 V - \alpha_L k$.

One more thing we need to show is that the expected profit obtained from the above discussion is local maximal. The reason is the following.

$0 \leq e_{2L}^* = \frac{\alpha_L m_{2L}}{2k} \leq 1$ and $0 \leq e_{1L}^* = \frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \leq 1$ ensure that $m_{1L}$ and $m_{2L}$ are bounded in absolute value. This means that the expression of the principal's expected profit in (B.99) is also bounded in absolute value. Thus, the maximum of the principal's expected profit exists and the unique solution of the first order conditions of the Lagrangian above provides the only candidate for the location of the maximum. Therefore the expected profit obtained is local maximal.

2. When $m_{2L} \leq \frac{2k}{\alpha_L}$ and $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} \geq 1$, i.e., $e^*_{2L} \leq 1$ and $e^*_{1L} = 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$\alpha_L \cdot 1 \cdot \left(\alpha_L \frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right) + \left(-k + \alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}\right)$$
$$+\lambda_1 \left(\frac{2k}{\alpha_L} - m_{2L}\right) + \lambda_2 \left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k} - 1\right) \tag{B.112}$$

The first order condition of (B.112) with respect to $m_{1L}$ and $m_{2L}$ are

$$\lambda_2 \frac{\alpha_L}{2k} = 0 \tag{B.113}$$
$$\frac{\alpha_L^3}{2k}(V - m_{2L}) - \lambda_1 + \lambda_2 \alpha_L^3 \frac{m_{2L}}{4k^2} = 0 \tag{B.114}$$

where (B.113) gives $\lambda_2 = 0$. Thus (B.114) becomes

$$\frac{\alpha_L^3}{2k}(V - m_{2L}) - \lambda_1 = 0 \tag{B.115}$$

When $\lambda_1 > 0$, $m_{2L} = \frac{2k}{\alpha_L}$. Thus (B.115) implies that $V$ must be greater than $\frac{2k}{\alpha_L}$. The principal's expected profit equals

$$\alpha_L \cdot 1 \cdot \left(\alpha_L \left(V - \frac{2k}{\alpha_L}\right) - m_{1L}\right) + (-k + \alpha_L m_{1L} + \alpha_L k)$$
$$= \alpha_L^2 V - \alpha_L k - k \tag{B.116}$$

When $\lambda_1 = 0$, $m_{2L} = V$. Thus the principal's expected profit equals

$$-k + \frac{\alpha_L^3 V^2}{4k} \tag{B.117}$$

which is less than or equal to $-k + \alpha_L k < 0$, because $m_{2L} \leq \frac{2k}{\alpha_L}$.

To show the expected profit obtained above is local maximal, we notice that with $0 \leq e^*_{2L} = \frac{\alpha_L m_{2L}}{2k} \leq 1$, the expression of the principal's expected profit in (B.112) as a function of $m_{2L}$ (with $m_{1L}$ being eliminated) is bounded in absolute value, and thereby has a maximum. Thus the unique solution of the first order conditions must be the location of the local maximal expected profit.

3. When $m_{2L} \geq \frac{2k}{\alpha_L}$ and $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \leq 2k$, i.e., $e_{2L}^* = 1$ and $e_{1L}^* \leq 1$. The Lagrangian for the maximum of the principal's expected profit is

$$\begin{aligned} &\alpha_L \left( \frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} \right) (\alpha_L (V - m_{2L}) - m_{1L}) \\ &+k \left( \frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} \right)^2 \\ &+\lambda_1 \left( m_{2L} - \frac{2k}{\alpha_L} \right) + \lambda_2 (2k - \alpha_L m_{1L} + \alpha_L k - \alpha_L^2 m_{2L}) \end{aligned} \tag{B.118}$$

The first order conditions of (B.118) with respect to $m_{1L}$ and $m_{2L}$ are

$$\frac{\alpha_L^2}{2k} (\alpha_L (V - m_{2L}) - m_{1L}) - \lambda_2 \alpha_L = 0 \tag{B.119}$$

$$\frac{\alpha_L^3}{2k} (\alpha_L (V - m_{2L}) - m_{1L}) + \lambda_1 - \lambda_2 \alpha_L^2 = 0 \tag{B.120}$$

Multiplying (B.119) by $\alpha_L$ and subtracting the product from (B.120) gives $\lambda_1 = 0$.

When $\lambda_2 > 0$, $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} = 2k$, namely, $\alpha_L m_{2L} + m_{1L} = \frac{2k}{\alpha_L} + k$. Then the principal's expected profit becomes

$$\alpha_L \left( \alpha_L V - \frac{2k}{\alpha_L} - k \right) + k = \alpha_L^2 V - \alpha_L k - k \tag{B.121}$$

When $\lambda_2 = 0$, (B.119) implies that $\alpha_L (V - m_{2L}) - m_{1L} = 0$, which means that $\alpha_L m_{2L} + m_{1L} = \alpha_L V$. The principal's expected profit equals

$$k \left( \frac{\alpha_L^2 V - \alpha_L k}{2k} \right)^2 \tag{B.122}$$

which is greater than or equal to $\alpha_L^2 V - \alpha_L k - k$, with the equality holding when $\alpha_L^2 V - \alpha_L k = 2k$. Notice that the constraint $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \leq 2k$ implies that $\alpha_L^2 V - \alpha_L k \leq 2k$, because $\alpha_L m_{2L} + m_{1L} = \alpha_L V$. Therefore, when $\alpha_L^2 V - \alpha_L k \leq 2k$,

$$k \left( \frac{\alpha_L^2 V - \alpha_L k}{2k} \right)^2 \geq \alpha_L^2 V - \alpha_L k - k \tag{B.123}$$

where the equality holds only when $\alpha_L^2 V - \alpha_L k = 2k$.

To show the expected profit obtained above is local maximal, we notice that with $0 \leq \frac{\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}}{2k} \leq 1$, the expression of the principal's expected profit in (B.118) as a function of $\alpha_L m_{1L} - \alpha_H k + \alpha_L^2 m_{2L}$ is bounded in absolute value, and thereby has a maximum. Thus the unique solution of the first order conditions must be the location of the local maximal expected profit.

4. When $m_{2L} \geq \frac{2k}{\alpha_L}$ and $\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} \geq 2k$, i.e., $e_{2L}^* = 1$ and $e_{1L}^* = 1$. The Lagrangian for the maximum of the principal's expected profit equals

$$\begin{aligned} & \alpha_L \left(\alpha_L (V - m_{2L}) - m_{1L}\right) + \left(-k + \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}\right) \\ & + \lambda_1 \left(m_{2L} - \frac{2k}{\alpha_L}\right) + \lambda_2 \left(\alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L} - 2k\right) \end{aligned} \tag{B.124}$$

It is easy to see that the first order conditions of (B.124) give $\lambda_1 = 0$ and $\lambda_2 = 0$. Notice that the principal's expected profit equals

$$\begin{aligned} & \alpha_L \left(\alpha_L (V - m_{2L}) - m_{1L}\right) + \left(-k + \alpha_L m_{1L} - \alpha_L k + \alpha_L^2 m_{2L}\right) \\ = \; & \alpha_L^2 V - \alpha_L k - k \end{aligned} \tag{B.125}$$

The constancy of the principal's expected profit implies that it is also local maximum.

In summary, the optimal money transfers $(m_{0L}^*, m_{1L}^*, m_{2L}^*)$ offered to the agent by the low-type principal and the principal's expected profit satisfy

1. When $V \leq \frac{2k}{\alpha_L}$, $m_{2L}^* = V$, $m_{1L}^* = 0$,

$$m_{0L}^* = -\frac{\alpha_L^6 V^4}{64k^3} \tag{B.126}$$

and the principal's expected profit equals

$$\frac{\alpha_L^6 V^4}{64k^3} \tag{B.127}$$

2. When $V \geq \frac{2k}{\alpha_L}$ and $\alpha_L^2 V - \alpha_L k \leq 2k$, $m_{2L}^* \geq \frac{2k}{\alpha_L}$, $\alpha_L m_{2L}^* + m_{1L}^* = \alpha_L V$.

$$m_{0L}^* = -k\left(\frac{\alpha_L^2 V - \alpha_L k}{2k}\right)^2 \tag{B.128}$$

and the principal's expected profit equals

$$k\left(\frac{\alpha_L^2 V - \alpha_L k}{2k}\right)^2 \tag{B.129}$$

3. When $V \geq \frac{2k}{\alpha_L}$ and $\alpha_L^2 V - \alpha_L k \geq 2k$, $m_{2L}^* \geq \frac{2k}{\alpha_L}$ and $\alpha_L m_{1L}^* - \alpha_L k + \alpha_L^2 m_{2L}^* \geq 2k$.

$$m_{0L}^* = -k + \alpha_L m_{1L}^* - \alpha_L k + \alpha_L^2 m_{2L}^* \geq k \tag{B.130}$$

and the principal's expected profit equals

$$\alpha_L^2 V - \alpha_L k - k \tag{B.131}$$

Using the same procedure as that of finding $LM$ above , we can show that $\overline{LM}$ is the maximum of the following expression

$$\alpha_H\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_L\tilde{m}_{2L}}{2k}(V-\tilde{m}_{2L})-\tilde{m}_{1L}\right)+k\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)^2 \quad \text{(B.132)}$$

In the following, we calculate $\overline{LM}$.

The first order condition of (B.132) with respect to $\tilde{m}_{1L}$ is

$$\frac{\alpha_H\alpha_L}{2k}\left(\alpha_H\frac{\alpha_L\tilde{m}_{2L}}{2k}(V-\tilde{m}_{2L})-\tilde{m}_{1L}\right)+(\alpha_L-\alpha_H)\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)=0 \quad \text{(B.133)}$$

The first order condition of (B.132) with respect to $\tilde{m}_{2L}$ is

$$\frac{\alpha_H\alpha_L^3m_2}{4k^2}\left(\alpha_H\frac{\alpha_L\tilde{m}_{2L}}{2k}(V-\tilde{m}_{2L})-\tilde{m}_{1L}\right)+\alpha_H\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_H\alpha_L}{2k}(V-2\tilde{m}_{2L})\right)$$

$$+\quad\frac{\alpha_L^3\tilde{m}_{2L}}{2k}\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)=0 \quad \text{(B.134)}$$

Notice that (B.133) is equivalent to

$$\frac{\alpha_H^2\alpha_L^2}{4k}\tilde{m}_{2L}(V-\tilde{m}_{2L})+\frac{\alpha_L^4\tilde{m}_{2L}^2}{8k^2}-\frac{\alpha_H\alpha_L^3\tilde{m}_{2L}^2}{8k^2}-\frac{\alpha_H\alpha_L}{k}\tilde{m}_{1L}+\frac{\alpha_L^2}{2k}\tilde{m}_{1L}=0 \quad \text{(B.135)}$$

namely

$$\frac{\alpha_H^2\alpha_L^2}{4k}\tilde{m}_{2L}(V-\tilde{m}_{2L})+\frac{\alpha_L^4\tilde{m}_{2L}^2}{8k^2}-\frac{\alpha_H\alpha_L^3\tilde{m}_{2L}^2}{8k^2}=\frac{2\alpha_H\alpha_L-\alpha_L^2}{2k}\tilde{m}_{1L} \quad \text{(B.136)}$$

On the other hand, multiplying (B.133) by $\frac{\alpha_L^2\tilde{m}_{2L}}{2k}$ and subtracting it from (B.134) gives

$$\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)\left(\frac{\alpha_H^2\alpha_L}{2k}(V-2\tilde{m}_{2L})+\frac{\alpha_L^2}{2k}\tilde{m}_{2L}-(\alpha_L-\alpha_H)\frac{\alpha_L^2}{2k}\tilde{m}_{2L}\right)=0 \quad \text{(B.137)}$$

which means that

$$V=\left(2-\frac{\alpha_L}{\alpha_H}\right)\tilde{m}_{1L} \quad \text{(B.138)}$$

because $\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)$ as the effort in the first stage has to be positive. Thus (B.138) gives

$$\tilde{m}_{2L}=\frac{\alpha_H}{2\alpha_H-\alpha_L}V \quad \text{(B.139)}$$

Substituting (B.139) into (B.136) gives

$$\frac{\alpha_H^2\alpha_L^2\alpha_H(\alpha_H-\alpha_L)V^2}{4k^2(2\alpha_H-\alpha_L)^2}+\frac{\left(\alpha_L^4-\alpha_H\alpha_L^3\right)\alpha_H^2V^2}{8k^2(2\alpha_H-\alpha_L)^2}=\frac{2\alpha_H\alpha_L-\alpha_L^2}{2k}\tilde{m}_{1L} \quad \text{(B.140)}$$

namely

$$\frac{2\alpha_H^2\alpha_L^2\alpha_H(\alpha_H-\alpha_L)V^2+\left(\alpha_L^4-\alpha_H\alpha_L^3\right)\alpha_H^2V^2}{8k^2(2\alpha_H-\alpha_L)^2}=\frac{2\alpha_H\alpha_L-\alpha_L^2}{2k}\tilde{m}_{1L} \tag{B.141}$$

which is equivalent to

$$\tilde{m}_{1L}=\frac{\left(2\alpha_H^4\alpha_L-3\alpha_H^3\alpha_L^2+\alpha_L^3\alpha_H^2\right)V^2}{4k(2\alpha_H-\alpha_L)^3} \tag{B.142}$$

Therefore, we have

$$\begin{aligned}
&\alpha_H\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)\\
=\;&\frac{\alpha_H\alpha_L}{2k}\left(\tilde{m}_{1L}+\frac{\alpha_L^2\tilde{m}_{2L}^2}{4k}\right)\\
=\;&\frac{\alpha_H\alpha_L}{2k}\left[\frac{\left(2\alpha_H^4\alpha_L-3\alpha_H^3\alpha_L^2+\alpha_L^3\alpha_H^2\right)V^2}{4k(2\alpha_H-\alpha_L)^3}+\frac{\alpha_L^2\alpha_H^2V^2}{4k(2\alpha_H-\alpha_L)^2}\right]\\
=\;&\frac{\alpha_H\alpha_L}{2k}\frac{\left(2\alpha_H^4\alpha_L-3\alpha_H^3\alpha_L^2+\alpha_L^3\alpha_H^2+\alpha_L^2\alpha_H^2(2\alpha_H-\alpha_L)\right)V^2}{4k(2\alpha_H-\alpha_L)^3}\\
=\;&\frac{\alpha_H^4\alpha_L^2(2\alpha_H-\alpha_L)V^2}{8k^2(2\alpha_H-\alpha_L)^3}\\
=\;&\frac{\alpha_H^4\alpha_L^2V^2}{8k^2(2\alpha_H-\alpha_L)^2}
\end{aligned} \tag{B.143}$$

$$\begin{aligned}
&\alpha_H\frac{\alpha_L\tilde{m}_{2L}}{2k}(V-\tilde{m}_{2L})-\tilde{m}_{1L}\\
=\;&\alpha_H\alpha_L\frac{\alpha_H}{2k(2\alpha_H-\alpha_L)}\left(\frac{\alpha_H-\alpha_L}{2\alpha_H-\alpha_L}\right)V^2-\frac{\left(2\alpha_H^4\alpha_L-3\alpha_H^3\alpha_L^2+\alpha_L^3\alpha_H^2\right)V^2}{4k(2\alpha_H-\alpha_L)^3}\\
=\;&\frac{2\alpha_H^2\alpha_L(\alpha_H-\alpha_L)(2\alpha_H-\alpha_L)-2\alpha_H^4\alpha_L+3\alpha_H^3\alpha_L^2-\alpha_L^3\alpha_H^2}{4k(2\alpha_H-\alpha_L)^3}V^2\\
=\;&\frac{\left(2\alpha_H^4\alpha_L-3\alpha_H^3\alpha_L^2+\alpha_L^3\alpha_H^2\right)V^2}{4k(2\alpha_H-\alpha_L)^3}
\end{aligned} \tag{B.144}$$

and

$$k\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)^2=k\left(\frac{\alpha_H^3\alpha_L^2V^2}{8k^2(2\alpha_H-\alpha_L)^2}\right)^2=\frac{\alpha_H^6\alpha_L^4V^4}{64k^3(2\alpha_H-\alpha_L)^4} \tag{B.145}$$

Thus

$$\begin{aligned}
&\alpha_H\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_L\tilde{m}_{2L}}{2k}(V-\tilde{m}_{2L})-\tilde{m}_{1L}\right)+k\left(\frac{\alpha_L\tilde{m}_{1L}+\frac{\alpha_L^3\tilde{m}_{2L}^2}{4k}}{2k}\right)^2\\
=\;&\frac{\alpha_H^4\alpha_L^2V^2}{8k^2(2\alpha_H-\alpha_L)^2}\frac{\left(2\alpha_H^4\alpha_L-3\alpha_H^3\alpha_L^2+\alpha_L^3\alpha_H^2\right)V^2}{4k(2\alpha_H-\alpha_L)^3}+\frac{\alpha_H^6\alpha_L^4V^4}{64k^3(2\alpha_H-\alpha_L)^4}\\
=\;&\frac{2\alpha_H^4\alpha_L^2\left(2\alpha_H^4\alpha_L-3\alpha_H^3\alpha_L^2+\alpha_L^3\alpha_H^2\right)+\alpha_H^6\alpha_L^4(2\alpha_H-\alpha_L)}{64k^3(2\alpha_H-\alpha_L)^5}V^4\\
=\;&\frac{4\alpha_H^8\alpha_L-4\alpha_H^7\alpha_L^4+\alpha_H^6\alpha_L^5}{64k^3(2\alpha_H-\alpha_L)^5}V^4 \qquad \text{(B.146)}
\end{aligned}$$

Denoting $\frac{\alpha_H}{\alpha_L}$ by $x$, (B.146) can be expressed as

$$\frac{\alpha_L^6\left(4x^8-4x^7+x^6\right)V^4}{64k^3(2x-1)^5}=\frac{\alpha_L^6V^4}{64k^3(2x-1)^3} \qquad \text{(B.147)}$$

Next we show that for the discussion of the separating equilibrium, we only need to consider the situation in which effort levels in two periods take the forms $e_{2H}^*=\frac{\alpha_H m_{2H}}{2k}$ and $e_{1H}^*=\frac{\alpha_H m_{1H}+\frac{\alpha_H^3m_{2H}^2}{4k}}{2k}$.

As we mentioned, there are four cases depending on the different ranges that $m_{2H}$ and $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3m_{2H}^2}{4k}}{2k}$ take.

1. When $m_{2H}\le\frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3m_{2H}^2}{4k}}{2k}\le 1$, namely $e_{2H}^*=1$ and $e_{1H}^*=1$.

   Assuming (B.10) is not binding, the Lagrangian of the high-type principal's profit is

$$\begin{aligned}
&\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\\
+\;&\left[\frac{\alpha_L^6V^4}{64k^3}-\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right]\\
+\;&\lambda_1\left(\frac{2k}{\alpha_H}-m_{2H}\right)+\lambda_2\left(1-\frac{\alpha_H m_{1H}+\frac{\alpha_H^3m_{2H}^2}{4k}}{2k}\right) \qquad \text{(B.148)}
\end{aligned}$$

   The first order conditions of (B.148) with respect to $m_{1H}$ and $m_{2H}$ result in

$$\begin{aligned}
&\frac{\alpha_H^2}{2k}\left[\frac{\alpha_H^2m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]-\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3m_{2H}^2}{4k}}{2k}\right)\\
&-\frac{\alpha_H\alpha_L}{2k}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]+\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3m_{2H}^2}{4k}}{2k}\right)-\lambda_2\frac{\alpha_H}{2k}=0
\end{aligned}$$

(B.149)

and

$$\frac{\alpha_H^4 m_{2H}}{4k^2}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]+\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\frac{\alpha_H^2}{2k}(V-2m_{2H})\right]$$
$$-\frac{\alpha_L\alpha_H^3 m_{2H}}{4k^2}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]-\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\frac{\alpha_L\alpha_H}{2k}(V-2m_{2H})\right]$$
$$-\lambda_1-\lambda_2\frac{\alpha_H^3 m_{2H}}{4k^2}=0 \tag{B.150}$$

There are several cases:

(a) When $\lambda_1 = 0$ and $\lambda_2 = 0$, this is the situation we discussed in (B.11), (B.12), (B.13) and (B.14).

(b) When $\lambda_1 > 0$ and $\lambda_2 = 0$, $m_{2H} = \frac{2k}{\alpha_H}$. Then (B.149) becomes

$$\begin{aligned}\frac{\alpha_H^2}{2k}\left[\alpha_H\left(V-\frac{2k}{\alpha_H}\right)-m_{1H}\right]-\alpha_H\left(\frac{\alpha_H m_{1H}+\alpha_H k}{2k}\right)&\\ -\frac{\alpha_L\alpha_H}{2k}\left[\alpha_L\left(V-\frac{2k}{\alpha_H}\right)-m_{1H}\right]+\alpha_L\left(\frac{\alpha_H m_{1H}+\alpha_H k}{2k}\right)&=0\end{aligned} \tag{B.151}$$

The first order condition of (B.151) with respect to $m_{1H}$, which is also the second order condition of (B.148) with respect to $m_{1H}$, is equal to

$$-\frac{\alpha_H^2}{2k}-\frac{\alpha_H^2}{2k}+\frac{\alpha_H\alpha_L}{2k}+\frac{\alpha_H\alpha_L}{2k} \tag{B.152}$$

which is less than 0. Therefore the critical point $m_{2H} = \frac{2k}{\alpha_H}$ and

$$m_{1H}=\frac{(\alpha_H+\alpha_L)}{2}\left(V-\frac{2k}{\alpha_H}\right)-\frac{k}{2}$$

is the local maximum. Notice that

$$\frac{\alpha_H m_{1H}+\alpha_H k}{2k}=\frac{\alpha_H(\alpha_H+\alpha_L)}{2}\left(V-\frac{2k}{\alpha_H}\right)+\frac{\alpha_H k}{2}<2k \tag{B.153}$$

Notice that we can use the argument below (B.66) to show that the solution (the root in the middle) is the location of the global maximum even without the constraints that $m_{2H} \leq \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \leq 1$.

2. When $m_{2H} \leq \frac{2k}{\alpha_H}$ and $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \geq 1$, namely $e_{2H}^* = 1$ and $e_{1H}^* = 1$. The Lagrangian of the high-type principal's profit is

$$\begin{aligned} & \alpha_H \cdot 1 \cdot \left( \alpha_H \frac{\alpha_H m_{2H}}{2k} (V - m_{2H}) - m_{1H} \right) \\ + & \left[ \frac{\alpha_L^6 V^4}{64k^3} - \alpha_L \cdot 1 \cdot \left( \alpha_L \frac{\alpha_H m_{2H}}{2k} (V - m_{2H}) - m_{1H} \right) \right] \\ + & \lambda_1 \left( \frac{2k}{\alpha_H} - m_{2H} \right) + \lambda_2 \left( 1 - \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right) \end{aligned} \quad \text{(B.154)}$$

The first order condition of (B.154) with respect to $m_{1H}$ gives

$$-\alpha_H + \alpha_L + \lambda_2 \frac{\alpha_H}{2k} = 0 \quad \text{(B.155)}$$

which means $\lambda_2 > 0$. Thus $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} = 1$. This boundary situation is included in previous case.

3. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \leq 2k$, namely $e_{2H}^* = 1$ and $e_{1H}^* leq 1$. The Lagrangian of the high-type principal's profit is

$$\begin{aligned} & \alpha_H \left( \frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} \right) (\alpha_H (V - m_{2H}) - m_{1H}) \\ + & \left[ \frac{\alpha_L^6 V^4}{64k^3} - \alpha_L \cdot 1 \cdot \left( \frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} \right) \left( \alpha_L \frac{\alpha_H m_{2H}}{2k} (V - m_{2H}) - m_{1H} \right) \right] \\ + & \lambda_1 \left( m_{2H} - \frac{2k}{\alpha_H} \right) + \lambda_2 \left( 1 - \left( \alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \right) \right) \end{aligned} \quad \text{(B.156)}$$

The first order conditions of (B.156) with respect to $m_{1H}$ and $m_{2H}$ are

$$\begin{aligned} & \frac{\alpha_H^2}{2k} (\alpha_H V - \alpha_H m_{2H} - m_{1H}) - \frac{\alpha_H^2}{2k} (m_{1H} - k + \alpha_H m_{2H}) \\ & - \frac{\alpha_L \alpha_H}{2k} (\alpha_L V - \alpha_L m_{2H} - m_{1H}) + \frac{\alpha_L \alpha_H}{2k} (m_{1H} - k + \alpha_H m_{2H}) - \lambda_2 \alpha_H = 0 \end{aligned} \quad \text{(B.157)}$$

and

$$\begin{aligned} & \frac{\alpha_H^3}{2k} (\alpha_H V - \alpha_H m_{2H} - m_{1H}) - \frac{\alpha_H^3}{2k} (m_{1H} - k + \alpha_H m_{2H}) \\ & - \frac{\alpha_L \alpha_H^2}{2k} (\alpha_L V - \alpha_L m_{2H} - m_{1H}) + \frac{\alpha_L^2 \alpha_H}{2k} (m_{1H} - k + \alpha_H m_{2H}) + \lambda_1 - \lambda_2 \alpha_H^2 = 0 \end{aligned} \quad \text{(B.158)}$$

Notice that multiplying (B.158) by $\alpha_H$ and then subtracting (B.157) from it gives

$$(\alpha_H \alpha_L - \alpha_L^2) \left( \frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} \right) - \lambda_1 = 0 \quad \text{(B.159)}$$

Since $\alpha_H > \alpha_L$ and $e_1^* = \frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} > 0$, $\lambda_1 > 0$. This means that $m_{2H} = \frac{2k}{\alpha_H}$, which implies that $\frac{\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H}}{2k} = \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$. Therefore this situation belongs to the second case.

4. When $m_{2H} \geq \frac{2k}{\alpha_H}$ and $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} \geq 2k$, namely $e_{2H}^* = 1$ and $e_{1H}^* geq 1$. The high-type principal's profit is

$$\alpha_H\left(\alpha_H(V - m_{2H}) - m_{1H}\right) + \left[\frac{\alpha_L^6 V^4}{64k^3} - \alpha_L\left(\alpha_L(V - m_{2H}) - m_{1H}\right)\right]$$
$$+ \quad \lambda_1\left(m_{2H} - \frac{2k}{\alpha_H}\right) + \lambda_2\left(\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} - 2k\right) \tag{B.160}$$

The first order condition of (B.160) with respect to $m_{1H}$ gives

$$-\alpha_H + \alpha_L + \alpha_H \lambda_2 = 0 \tag{B.161}$$

which means that $\lambda_2 = \frac{\alpha_H - \alpha_L}{\alpha_H} > 0$. Thus $\alpha_H m_{1H} - \alpha_H k + \alpha_H^2 m_{2H} = 2k$. This belongs to the boundary situation in previous case.

## B.2 Proof of Theorem 11

We examine the pooling equilibrium when upfront, intermediate and end money transfers are all included in the menu.

We assume that the agent believes that the high-type principal appears with probability $p$ while the low-type principal appears with probability $1-p$. For convenience, we denote $p\alpha_H + (1-p)\alpha_L$ by $\overline{\alpha}$ and $\frac{p\alpha_H^2}{p\alpha_H + (1-p)\alpha_L} + \frac{(1-p)\alpha_L^2}{p\alpha_H + (1-p)\alpha_L}$ by $\tilde{\alpha}$. Notice that $\overline{\alpha}\tilde{\alpha} = p\alpha_H^2 + (1-p)\alpha_L^2$.

In the pooling equilibrium, both high-type and low-type principals offer the same payment menu $(m_0, m_1, m_2)$. The high-type principal's profit satisfies

$$\max_{(m_0, m_1, m_2)} \{-m_0 - \alpha_H e_1 m_1 + \alpha_H^2 e_1 e_2 (V - m_2)\} \tag{B.162}$$

such that the agent's profit satisfies

$$\max_{(e_1, e_2)} m_0 - k e_1^2 + \overline{\alpha} e_1 m_1 - \overline{\alpha} e_1 k e_2^2 + \overline{\alpha}\tilde{\alpha} e_1 e_2 m_2 \tag{B.163}$$

and

$$m_0 - k e_1^2 + \overline{\alpha} e_1 m_1 - \overline{\alpha} e_1 k e_2^2 + \overline{\alpha}\tilde{\alpha} e_1 e_2 m_2 \geq 0 \tag{B.164}$$

There are two other constraints that have to be satisfied:

$$-m_0 - \alpha_L e_1 m_1 + \alpha_L^2 e_1 e_2 (V - m_2) \geq LM_1 \tag{B.165}$$

and

$$-m_0 - \alpha_H e_1 m_1 + \alpha_H^2 e_1 e_2 (V - m_2) \geq \overline{LM}_1 \tag{B.166}$$

where

$$LM_1 = \max_{(m_{0L}, m_{1L}, m_{2L})} \{-m_{0L} - \alpha_L e_{1L} m_{1L} + \alpha_L^2 e_{1L} e_{2L} (V - m_{2L})\} \tag{B.167}$$

and

$$\overline{LM}_1 = \max_{(\tilde{m}_{0L}, \tilde{m}_{1L}, \tilde{m}_{2L})} \{-\tilde{m}_{0L} - \alpha_H e_{1L} \tilde{m}_{1L} + \alpha_H^2 e_{1L} e_{2L} (V - \tilde{m}_{2L})\} \tag{B.168}$$

with the agent satisfying

$$\max_{(e_{1L}, e_{2L})} \tilde{m}_{0L} - k e_{1L}^2 + \alpha_L e_{1L} \tilde{m}_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \tag{B.169}$$

and

$$\tilde{m}_{0L} - k e_{1L}^2 + \alpha_L e_{1L} \tilde{m}_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \geq 0 \tag{B.170}$$

where (B.164) is the agent's participation constraint, (B.166) is the constraint of preventing high-type principal's deviation and (B.165) is the constraint of preventing the low-type principal's deviation.

The proof consists of two parts. First we look at the high-type principal's profit maximization problem when the agent earns zero profit and the low-type principal does not want to deviate, i.e., (B.164) binds, but (B.166) does not bind. In this scenario, we show that the high-type principal does not want to deviate, i.e., (B.165) is redundant. Then we look at the high-type principal's profit maximization problem when the agent earns zero profit and the low-type principal wants to deviate, i.e., both (B.164) and (B.166) bind. In this scenario, there exist two subscenarios, one in which the high-type principal does not want to deviate, i.e., (B.165) does not bind, and the other in which the high-type principal wants to deviate, i.e., (B.165) binds.

Comparing (B.167) and (B.168) with (B.6) and (B.7) in the part for the separating equilibrium, clearly $LM_1 = LM$ and $\overline{LM}_1 = \overline{LM}$. Therefore, $LM_1 = \frac{\alpha_L^6 V^4}{64k^3}$ and $\overline{LM}_1$ is the maximum of expression (B.132), namely

$$\alpha_H \left( \frac{\alpha_L \tilde{m}_{1L} + \frac{\alpha_L^3 \tilde{m}_{2L}^2}{4k}}{2k} \right) \left( \alpha_H \frac{\alpha_L \tilde{m}_{2L}}{2k} (V - \tilde{m}_{2L}) - \tilde{m}_{1L} \right) + k \left( \frac{\alpha_L \tilde{m}_{1L} + \frac{\alpha_L^3 \tilde{m}_{2L}^2}{4k}}{2k} \right)^2 \tag{B.171}$$

We can see that (B.171) has the same expression as that of (B.132). This means that they share the same solution for corresponding maximization problems with $\tilde{m}_{2L} = \frac{\alpha_H}{2\alpha_H - \alpha_L} V$ and $0 < \tilde{m}_{2L} < V$ . Thus $\overline{LM}_1$ equals

$$\frac{\alpha_L^6 V^4}{64k^3 (2x-1)^3} \tag{B.172}$$

Next we study the first scenario in which the agent earns zero profit and the low-type principal does not want to deviate, i.e., (B.164) binds, but (B.166) does not bind.

### B.2.1 The first scenario

Notice that the Lagrangian for the maximization problem of the agent's profit with respect to $e_1$ and $e_2$ as the decision variables is

$$m_0 - ke_1^2 + \overline{\alpha}e_1m_1 - \overline{\alpha}e_1ke_2^2 + \overline{\alpha}\tilde{\alpha}e_1e_2m_2 + \lambda_1(1-e_1) + \lambda_2(1-e_2) \tag{B.173}$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are Lagrangian multipliers. It is clear that we can assume $e_1 > 0$ and $e_2 > 0$, because neither $e_1 = 0$ nor $e_2 = 0$ would lead to the maximal profit for the high-type principal.

The first order condition of (B.163) with respect to $e_1$ and $e_2$ gives

$$-2ke_1 + \overline{\alpha}m_1 - \overline{\alpha}ke_2^2 + \overline{\alpha}\tilde{\alpha}e_2m_2 - \lambda_1 = 0 \tag{B.174}$$

$$-\overline{\alpha}e_1 \cdot 2ke_2 + \overline{\alpha}\tilde{\alpha}e_1m_2 - \lambda_2 = 0 \tag{B.175}$$

which lead to the optimal efforts $e_1^*$ and $e_2^*$ satisfying

$$e_2^* = \frac{\tilde{\alpha}m_2 - \frac{\lambda_2}{\overline{\alpha}e_1^*}}{2k} \tag{B.176}$$

$$e_1^* = \frac{\overline{\alpha}m_1 - \overline{\alpha}ke_2^{*2} + \overline{\alpha}\tilde{\alpha}e_2^*m_2 - \lambda_2}{2k} \tag{B.177}$$

where $0 < e_2^* \leq 1$ and $0 < e_1^* \leq 1$.

There are four situations for consideration:

1. When $\lambda_1 > 0$ and $\lambda_2 > 0$, $e_1^* = 1$ and $e_2^* = 1$. By (B.163), the agent's profit is

$$m_0 - k + \overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2 \tag{B.178}$$

   Since $\lambda_1 > 0$, $\lambda_2 > 0$, $e_1^* = 1$ and $e_2^* = 1$, (B.176) and (B.177) imply that $\frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} > 1$ and $\frac{\tilde{\alpha}m_2}{2k} > 1$.

2. When $\lambda_1 > 0$ and $\lambda_2 = 0$, $e_1^* = 1$ and $e_2^* = \frac{\tilde{\alpha}m_2}{2k} \leq 1$. By (B.163), the agent's profit is

$$\begin{aligned} & m_0 - k + \overline{\alpha}m_1 - \overline{\alpha}k\left(\frac{\tilde{\alpha}m_2}{2k}\right)^2 + \overline{\alpha}\tilde{\alpha}\frac{\tilde{\alpha}m_2}{2k}m_2 \\ = \ & m_0 - k + \overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k} \end{aligned} \tag{B.179}$$

   Since $\lambda_1 > 0$ and $e_1^* = 1$, (B.177) implies that $\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k} > 1$.

3. When $\lambda_1 = 0$ and $\lambda_2 > 0$, $e_1^* = \frac{\overline{\alpha} m_{1L} - \overline{\alpha} k + \overline{\alpha}\tilde{\alpha} m_2}{2k} \leq 1$ and $e_2^* = 1$, namely, $2ke_1^* = \overline{\alpha} m_{1L} - \overline{\alpha} k + \overline{\alpha}\tilde{\alpha} m_2$. By (B.163), the agent's profit is

$$
\begin{aligned}
& m_0 - k{e_1^*}^2 + \overline{\alpha} e_{1L}^* m_{1L} - \overline{\alpha} e_1^* k{e_2^*}^2 + \overline{\alpha}\tilde{\alpha} e_1^* e_2^* m_2 \\
= \; & m_0 - k{e_1^*}^2 + \overline{\alpha} e_{1L}^* m_{1L} - \overline{\alpha} e_1^* k + \overline{\alpha}\tilde{\alpha} e_1^* m_2 \\
= \; & m_{0L} - k{e_1^*}^2 + e_1^* \left[\overline{\alpha} m_{1L} - \overline{\alpha} k + \overline{\alpha}\tilde{\alpha} m_2\right] \\
= \; & m_{0L} - k{e_1^*}^2 + e_1^* 2ke_1^* \\
= \; & m_{0L} + k{e_1^*}^2 \\
= \; & m_{0L} + k\left(\frac{\overline{\alpha} m_{1L} - \overline{\alpha} k + \overline{\alpha}\tilde{\alpha} m_2}{2k}\right)^2
\end{aligned}
\tag{B.180}
$$

Since $\lambda_2 > 0$, (B.176) implies that $\frac{\tilde{\alpha} m_2}{2k} > 1$.

4. When $\lambda_1 = 0$ and $\lambda_2 = 0$, by (B.176) and (B.177), $e_2^* = \frac{\tilde{\alpha} m_2}{2k} \leq 1$ and

$$
\begin{aligned}
e_1^* &= \frac{\overline{\alpha} m_1 - \overline{\alpha} k{e_2^*}^2 + \overline{\alpha}\tilde{\alpha} e_2^* m_2}{2k} \\
&= \frac{\overline{\alpha} m_1 - \overline{\alpha} k\left(\frac{\tilde{\alpha} m_2}{2k}\right)^2 + \overline{\alpha}\tilde{\alpha}\frac{\tilde{\alpha} m_2}{2k} m_2}{2k} \\
&= \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}
\end{aligned}
\tag{B.181}
$$

which is less than or equal to 1.

By (B.163), the agent's profit is

$$
\begin{aligned}
& m_0 - k{e_1^*}^2 + \overline{\alpha} e_1^* m_1 - \overline{\alpha} e_1^* k{e_2^*}^2 + \overline{\alpha}\tilde{\alpha} e_1^* e_2^* m_2 \\
= \; & m_0 - k{e_1^*}^2 + e_1^* \left[\overline{\alpha} m_1 - \overline{\alpha} k{e_2^*}^2 + \overline{\alpha}\tilde{\alpha} e_2^* m_2\right] \\
= \; & m_0 - k{e_1^*}^2 + e_1^* 2ke_1^* \\
= \; & m_0 + k{e_1^*}^2 \\
= \; & m_0 + k\left(\frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}\right)^2
\end{aligned}
\tag{B.182}
$$

Although there are four cases for the expressions of $e_1^*$ and $e_2^*$, we only need to consider the fourth case in which $e_2^* = \frac{\tilde{\alpha} m_2}{2k} \leq 1$ and $e_1^* = \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \leq 1$. We allocate the proof of this statement to the supplemental part at the end.

When $e_2^* = \frac{\tilde{\alpha} m_2}{2k} \leq 1$ and $e_1^* = \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \leq 1$, the Lagrangian of the high-type principal's profit is

$$\alpha_H \left( \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right) \left( \alpha_H \frac{\tilde{\alpha} m_2}{2k} (V - m_2) - m_1 \right) - m_0 + \lambda \left[ m_0 + k \left( \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right)^2 \right] \quad \text{(B.183)}$$

Notice that the boundary conditions for $\frac{\tilde{\alpha} m_2}{2k} \leq 1$ and $\frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \leq 1$ are not included in (B.183). We will show that when $V \leq \frac{2k}{\alpha_H}$, the location of the local maximum of the high-type principal's profit won't occur at the boundaries.

The first order condition of (B.183) with respect to $m_0$ gives

$$-1 + \lambda = 0 \quad \text{(B.184)}$$

This means that $\lambda = 1$ and $m_0 = -k \left( \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right)^2$.

Therefore the high-type principal's profit becomes

$$\alpha_H \left( \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right) \left( \alpha_H \frac{\tilde{\alpha} m_2}{2k} (V - m_2) - m_1 \right) + k \left( \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right)^2 \quad \text{(B.185)}$$

The first order condition of (B.185) with respect to $m_1$ gives

$$\frac{\alpha_H \overline{\alpha}}{2k} \left( \alpha_H \frac{\tilde{\alpha} m_2}{2k} (V - m_2) - m_1 \right) - \alpha_H \left( \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right) + \overline{\alpha} \left( \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right) = 0 \quad \text{(B.186)}$$

which means that

$$\frac{\alpha_H^2 \overline{\alpha}\tilde{\alpha}}{4k} m_2 (V - m_2) - \frac{\alpha_H \overline{\alpha}\tilde{\alpha}^2}{8k^2} m_2^2 + \frac{\overline{\alpha}^2 \tilde{\alpha}^2}{8k^2} m_2^2 = \frac{\left(2\alpha_H \overline{\alpha} - \overline{\alpha}^2\right)}{2k} m_1 \quad \text{(B.187)}$$

namely,

$$m_1 = \frac{\alpha_H^2 \overline{\alpha}\tilde{\alpha} m_2 (V - m_2)}{2k \left(2\alpha_H \overline{\alpha} - \overline{\alpha}^2\right)} - \frac{\left(\alpha_H \overline{\alpha}\tilde{\alpha}^2 - \overline{\alpha}^2 \tilde{\alpha}^2\right) m_2^2}{4k \left(2\alpha_H \overline{\alpha} - \overline{\alpha}^2\right)} \quad \text{(B.188)}$$

The first order condition of (B.185) with respect to $m_2$ gives

$$\frac{\alpha_H \overline{\alpha}\tilde{\alpha}^2}{4k^2} m_2 \left( \alpha_H \frac{\tilde{\alpha} m_2}{2k} (V - m_2) - m_1 \right) + \alpha_H \left( \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right) \left( \frac{\alpha_H \tilde{\alpha}}{2k} (V - 2m_2) \right)$$
$$+ \frac{\overline{\alpha}\tilde{\alpha}^2}{2k} m_2 \left( \frac{\overline{\alpha} m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right) = 0 \quad \text{(B.189)}$$

Using the expression of $m_1$ obtained in (B.188), we have

$$\begin{aligned}
&\frac{\alpha_H\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\\
&=\frac{\alpha_H\tilde{\alpha}m_2}{2k}(V-m_2)-\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}m_2(V-m_2)}{2k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}+\frac{\left(\alpha_H\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2\right)m_2^2}{4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}\\
&=\frac{\left(\alpha_H^2\overline{\alpha}\tilde{\alpha}-\alpha_H\overline{\alpha}^2\tilde{\alpha}\right)}{2k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2(V-m_2)+\frac{\left(\alpha_H\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2\right)}{4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2^2
\end{aligned} \tag{B.190}$$

and

$$\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}=\frac{\overline{\alpha}}{2k}\left(\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}}{2k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2(V-m_2)+\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2}{4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2^2\right) \tag{B.191}$$

Therefore (B.189) becomes

$$\begin{aligned}
&\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2}{4k^2}m_2\left[\frac{\left(\alpha_H^2\overline{\alpha}\tilde{\alpha}-\alpha_H\overline{\alpha}^2\tilde{\alpha}\right)}{2k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2(V-m_2)+\frac{\left(\alpha_H\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2\right)}{4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2^2\right]\\
&+\frac{\alpha_H\overline{\alpha}}{2k}\left[\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}}{2k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2(V-m_2)+\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2}{4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2^2\right]\left(\frac{\alpha_H\tilde{\alpha}}{2k}(V-2m_2)\right)\\
&+\frac{\overline{\alpha}^2\tilde{\alpha}^2}{2k}m_2\left[\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}}{2k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2(V-m_2)+\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2}{4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2^2\right]=0
\end{aligned} \tag{B.192}$$

Multiplying both sides of (B.192) by $\frac{4k^2}{m_2}$ and $4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)$, (B.192) becomes

$$\begin{aligned}
&\alpha_H\overline{\alpha}\tilde{\alpha}^2\left[2\left(\alpha_H^2\overline{\alpha}\tilde{\alpha}-\alpha_H\overline{\alpha}^2\tilde{\alpha}\right)m_2(V-m_2)+\left(\alpha_H\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2\right)m_2^2\right]\\
&+\alpha_H^2\overline{\alpha}\tilde{\alpha}\left[2\alpha_H^2\overline{\alpha}\tilde{\alpha}(V-m_2)+\alpha_H\overline{\alpha}\tilde{\alpha}^2m_2\right](V-2m_2)\\
&+\overline{\alpha}^2\tilde{\alpha}^2\left[2\alpha_H^2\overline{\alpha}\tilde{\alpha}m_2(V-m_2)+\alpha_H\overline{\alpha}\tilde{\alpha}^2m_2^2\right]=0
\end{aligned} \tag{B.193}$$

In equation (B.193), the coefficient of $m_2^2$ equals

$$\begin{aligned}
&-2\alpha_H^3\overline{\alpha}^2\tilde{\alpha}^3+2\alpha_H^2\overline{\alpha}^3\tilde{\alpha}^3+\alpha_H^2\overline{\alpha}^2\tilde{\alpha}^4-\alpha_H\overline{\alpha}^3\tilde{\alpha}^4\\
+\quad&4\alpha_H^4\overline{\alpha}^2\tilde{\alpha}^2-2\alpha_H^3\overline{\alpha}^2\tilde{\alpha}^3-2\alpha_H^2\overline{\alpha}^3\tilde{\alpha}^3+\alpha_H\overline{\alpha}^3\tilde{\alpha}^4
\end{aligned} \tag{B.194}$$

which can be simplified as

$$4\alpha_H^4\alpha^2\tilde{\alpha}^2-4\alpha_H^3\alpha^2\tilde{\alpha}^3+\alpha_H^2\alpha^2\tilde{\alpha}^4 \tag{B.195}$$

The coefficient of $m_2V$ equals

$$2\alpha_H^3\alpha^2\tilde{\alpha}^3-\alpha_H^2\alpha^3\tilde{\alpha}^3-6\alpha_H^4\alpha^2\tilde{\alpha}^2+\alpha_H^3\alpha^2\tilde{\alpha}^3+2\alpha_H^2\alpha^3\tilde{\alpha}^3=3\alpha_H^3\alpha^2\tilde{\alpha}^3-6\alpha_H^4\alpha^2\tilde{\alpha}^2 \tag{B.196}$$

The coefficient of $V^2$ is $\alpha_H^4\alpha^2\tilde{\alpha}^2$. Therefore (B.193) can be simplified as

$$\left(4\alpha_H^4\alpha^2\tilde{\alpha}^2 - 4\alpha_H^3\alpha^2\tilde{\alpha}^3 + \alpha_H^2\alpha^2\tilde{\alpha}^4\right)m_2^2 + \left(3\alpha_H^3\alpha^2\tilde{\alpha}^3 - 6\alpha_H^4\alpha^2\tilde{\alpha}^2\right)m_2V + \alpha_H^4\alpha^2\tilde{\alpha}^2V^2 = 0 \tag{B.197}$$

Dividing both sides of (B.197) by $\alpha_H^2\overline{\alpha}^2\tilde{\alpha}^2$ gives

$$\left(4\alpha_H^2 - 4\alpha_H\tilde{\alpha} + \tilde{\alpha}^2\right)m_2^2 + \left(3\alpha_H\tilde{\alpha} - 6\alpha_H^2\right)m_2V + 2\alpha_H^2V^2 = 0 \tag{B.198}$$

which is

$$\left[\left(2\alpha_H - \tilde{\alpha}\right)m_2 - 2\alpha_HV\right]\left[\left(2\alpha_H - \tilde{\alpha}\right)m_2 - \alpha_HV\right] = 0 \tag{B.199}$$

The roots of (B.199) are

$$m_2 = \left(\frac{2\alpha_H}{2\alpha_H - \tilde{\alpha}}\right)V \tag{B.200}$$

and

$$m_2 = \left(\frac{\alpha_H}{2\alpha_H - \tilde{\alpha}}\right)V \tag{B.201}$$

Notice that $m_2$ is the root we exclude through factorization. The root in (B.200) can be ruled out, using either the reasoning below (B.66) or the following argument.

When $m_2 = \left(\frac{2\alpha_H}{2\alpha_H-\tilde{\alpha}}\right)V$,

$$\begin{aligned}
\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k} &= \frac{\overline{\alpha}}{2k}\left(\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}}{2k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)}m_2(V - m_2) + \frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2}{4k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)}m_2^2\right) \\
&= \frac{\overline{\alpha}}{2k}\left(\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}\cdot 2\alpha_H\left(-\tilde{\alpha}\right)}{2k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)\left(2\alpha_H - \tilde{\alpha}\right)^2} + \frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2\cdot 4\alpha_H^2}{4k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)\left(2\alpha_H - \tilde{\alpha}\right)^2}\right) \\
&= \frac{\overline{\alpha}}{2k}\left(\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}\cdot 4\alpha_H\left(-\tilde{\alpha}\right) + \alpha_H\overline{\alpha}\tilde{\alpha}^2\cdot 4\alpha_H^2}{4k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)\left(2\alpha_H - \tilde{\alpha}\right)^2}\right) \\
&= 0
\end{aligned} \tag{B.202}$$

which means the effort level $e_1^* = 0$. Clearly such $m_2$ can't be the location for the high-type principal's maximal profit.

For the second root of $m_2$, when $V \le \frac{2k}{\alpha_H}$, $m_2 < \frac{2k}{\alpha_H}$, because $\alpha_H > \tilde{\alpha}$ gives $\left(\frac{\alpha_H}{2\alpha_H-\tilde{\alpha}}\right) < 1$. The corresponding $m_1$ satisfies

$$\begin{aligned}
m_1 &= \frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}\alpha_H\left(\alpha_H - \tilde{\alpha}\right)V^2}{2k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)\left(2\alpha_H - \tilde{\alpha}\right)^2} - \frac{\left(\alpha_H\overline{\alpha}\tilde{\alpha}^2 - \overline{\alpha}^2\tilde{\alpha}^2\right)\alpha_H^2V^2}{4k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)\left(2\alpha_H - \tilde{\alpha}\right)^2} \\
&= \frac{2\alpha_H^4\overline{\alpha}\tilde{\alpha} - 3\alpha_H^3\overline{\alpha}\tilde{\alpha}^2 + \alpha_H^2\overline{\alpha}^2\tilde{\alpha}^2}{4k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)\left(2\alpha_H - \tilde{\alpha}\right)^2}V^2 \\
&= \frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}\left(2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha}\right)}{4k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)\left(2\alpha_H - \tilde{\alpha}\right)^2}V^2
\end{aligned} \tag{B.203}$$

Notice that when $V \leq \frac{2k}{\alpha_H}$, (B.203) satisfies

$$\begin{aligned}
\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}\left(2\alpha_H^2-3\alpha_H\tilde{\alpha}+\overline{\alpha}\tilde{\alpha}\right)}{4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)\left(2\alpha_H-\tilde{\alpha}\right)^2}V^2 &\leq \frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}\left(2\alpha_H^2-3\alpha_H\tilde{\alpha}+\overline{\alpha}\tilde{\alpha}\right)}{4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)\left(2\alpha_H-\tilde{\alpha}\right)^2}\cdot\frac{4k^2}{\alpha_H^2} \\
&= \frac{\overline{\alpha}\tilde{\alpha}\left(2\alpha_H^2-3\alpha_H\tilde{\alpha}+\overline{\alpha}\tilde{\alpha}\right)k}{\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)\left(2\alpha_H-\tilde{\alpha}\right)^2} \\
&< \frac{\overline{\alpha}\tilde{\alpha}\left(2\alpha_H^2-3\alpha_H\overline{\alpha}+\overline{\alpha}^2\right)k}{\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)\left(2\alpha_H-\tilde{\alpha}\right)^2} \\
&= \frac{\tilde{\alpha}(\alpha_H-\overline{\alpha})k}{\left(2\alpha_H-\tilde{\alpha}\right)^2} \\
&< k
\end{aligned} \tag{B.204}$$

Therefore, when $V \leq \frac{2k}{\alpha_H}$,

$$\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k} < \frac{\overline{\alpha}k+\frac{\overline{\alpha}\tilde{\alpha}^2\frac{4k^2}{\alpha^2}}{4k}}{2k} < \frac{2\overline{\alpha}k}{2k} = \overline{\alpha} < 1 \tag{B.205}$$

This shows that when $V \leq \frac{2k}{\alpha_H}$, the local maximum is an interior point and does not hit either of the boundaries $\frac{\tilde{\alpha}m_2}{2k} = 1$ and $= \frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k} = 1$.

Notice that we can express $m_2$, $V-m_2$, $m_1$ and other terms in $\alpha_L$, $x$ and $p$.

$$\begin{aligned}
m_2 &= \frac{\alpha_H}{2\alpha_H-\tilde{\alpha}}V \\
&= \frac{\alpha_H/\alpha_L}{2\alpha_H/\alpha_L-\tilde{\alpha}/\alpha_L}V \\
&= \frac{\alpha_H/\alpha_L}{2\alpha_H/\alpha_L-\frac{p\alpha_H^2/\alpha_L+(1-p)\alpha_L^2/\alpha_L}{p\alpha_H+(1-p)\alpha_L}}V \\
&= \frac{\alpha_H/\alpha_L}{2\alpha_H/\alpha_L-\frac{p\alpha_H^2/\alpha_L^2+(1-p)}{p\alpha_H/\alpha_L+(1-p)}}V \\
&= \frac{x}{2x-\frac{px^2+(1-p)}{px+(1-p)}}V \\
&= \frac{px^2+(1-p)x}{px^2+(1-p)(2x-1)}V
\end{aligned} \tag{B.206}$$

$$V-m_2 = \frac{(1-p)(x-1)}{px^2+(1-p)(2x-1)}V \tag{B.207}$$

By (B.203), we have

$$\begin{aligned}
m_1 &= \frac{\alpha_H^2 \overline{\alpha}\tilde{\alpha}\left(2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha}\right)}{4k\left(2\alpha_H\overline{\alpha} - \overline{\alpha}^2\right)\left(2\alpha_H - \tilde{\alpha}\right)^2} V^2 \\
&= \frac{\alpha_H^2\left(p\alpha_H^2 + (1-p)\alpha_L^2\right)\left(2\alpha_H^2 - 3\alpha_H \frac{p\alpha_H^2+(1-p)\alpha_L^2}{p\alpha_H+(1-p)\alpha_L} + p\alpha_H^2 + (1-p)\alpha_L^2\right)}{4k\left(2\alpha_H(p\alpha_H + (1-p)\alpha_L) - (p\alpha_H + (1-p)\alpha_L)^2\right)\left(2\alpha_H - \frac{p\alpha_H^2+(1-p)\alpha_L^2}{p\alpha_H+(1-p)\alpha_L}\right)^2} V^2 \\
&= \frac{\alpha_L^3 x^2\left(px^2 + (1-p)\right)\left(2x^2 - 3x\frac{px^2+(1-p)}{px+(1-p)} + px^2 + (1-p)\right)}{4k\left(2px^2 + 2(1-p)x - (px + (1-p))^2\right)\left(2x - \frac{px^2+(1-p)}{px+(1-p)}\right)^2} V^2
\end{aligned} \tag{B.208}$$

It is clear that $m_1 > 0$ is equivalent to $2x^2 - 3x\frac{px^2+(1-p)}{px+(1-p)} + px^2 + (1-p) > 0$ , namely,

$$\frac{(p^2-p)x^3 + (1-p)(2+p)x^2 + (4p - p^2 - 3)x + (1-p)^2}{px + (1-p)} > 0 \tag{B.209}$$

The denominator of (B.209) is always positive, so (B.209) is positive when its numerator is positive. Plotting implicit function, we obtain the area in the space of $x$ and $p$ that indicates that positivity of (B.209), i.e., the positivity of $m_1$.

In the following, we show that (B.166) is redundant, i.e., the high-type principal has no incentive to deviate from the equilibrium location.

For the maximization problem for (B.171), according to (B.136), the first order condition with respect to $m_{1L}$ gives

$$m_{1L} = \frac{\alpha_H^2\alpha_L^2 m_{2L}(V - m_{2L})}{2k(2\alpha_H\alpha_L - \alpha_L^2)} - \frac{(\alpha_H\alpha_L^3 - \alpha_L^4)m_{2L}^2}{4k(2\alpha_H\alpha_L - \alpha_L^2)} \tag{B.210}$$

Thus

$$\begin{aligned}
&\alpha_H\frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L} \\
= &\frac{\alpha_H\alpha_L(2\alpha_H\alpha_L - \alpha_L^2) - \alpha_H^2\alpha_L^2}{2k(2\alpha_H\alpha_L - \alpha_L^2)} m_{2L}(V - m_{2L}) + \frac{(\alpha_H\alpha_L^3 - \alpha_L^4)m_{2L}^2}{4k(2\alpha_H\alpha_L - \alpha_L^2)} \\
= &\frac{\alpha_H^2\alpha_L^2 - \alpha_H\alpha_L^3}{2k(2\alpha_H\alpha_L - \alpha_L^2)} m_{2L}(V - m_{2L}) + \frac{(\alpha_H\alpha_L^3 - \alpha_L^4)m_{2L}^2}{4k(2\alpha_H\alpha_L - \alpha_L^2)}
\end{aligned} \tag{B.211}$$

Since $m_{2L} = \frac{\alpha_H}{2\alpha_H - \alpha_L}V$ with $0 < m_{2L} < V$, (B.211) is greater than 0.

As to $\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}$, since it represents the effort in the first stage, it must be greater than zero.

On the other hand, by (B.185), the principal's profit function is

$$\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)+k\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)^2 \tag{B.212}$$

By (B.190), we have

$$\frac{\alpha_H\tilde{\alpha}m_2}{2k}(V-m_2)-m_1=\frac{\left(\alpha_H^2\overline{\alpha}\tilde{\alpha}-\alpha_H\overline{\alpha}^2\tilde{\alpha}\right)}{2k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2(V-m_2)+\frac{\left(\alpha_H\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2\right)}{4k\left(2\alpha_H\overline{\alpha}-\overline{\alpha}^2\right)}m_2^2 \tag{B.213}$$

Since (B.201) gives $m_2=\left(\frac{\alpha_H}{2\alpha_H-\tilde{\alpha}}\right)V$, which means $0<m_2<V$. Thus (B.213) is greater than 0. As to $\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}$, since it represents the effort in the first stage, it must be greater than zero.

Replacing variables $m_1$ and $m_2$ with $m_{1L}$ and $m_{2L}$ in (B.212) , then (B.212) becomes

$$\alpha_H\left(\frac{\overline{\alpha}m_{1L}+\frac{\overline{\alpha}\tilde{\alpha}^2m_{2L}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\tilde{\alpha}m_{2L}}{2k}(V-m_{2L})-m_{1L}\right)+k\left(\frac{\overline{\alpha}m_{1L}+\frac{\overline{\alpha}\tilde{\alpha}^2m_{2L}^2}{4k}}{2k}\right)^2 \tag{B.214}$$

Maximize the new profit function (B.214) with respect to $m_{1L}$ and $m_{2L}$, then it will give the same maximum value as that of (B.212). In addition, we have $\alpha_H\frac{\tilde{\alpha}m_{2L}}{2k}(V-m_{2L})-m_{1L}>0$ and $\frac{\overline{\alpha}m_{1L}+\frac{\overline{\alpha}\tilde{\alpha}^2m_{2L}^2}{4k}}{2k}>0$ at the location of the maximum.

Since

$$\alpha_H\frac{\tilde{\alpha}m_{2L}}{2k}(V-m_{2L})-m_{1L}\geq\alpha_H\frac{\alpha_Lm_{2L}}{2k}(V-m_{2L})-m_{1L}>0 \tag{B.215}$$

and

$$\frac{\overline{\alpha}m_{1L}+\frac{\overline{\alpha}\tilde{\alpha}^2m_{2L}^2}{4k}}{2k}\geq\frac{\alpha_Lm_{1L}+\frac{\alpha_L^3m_{2L}^2}{4k}}{2k}>0 \tag{B.216}$$

Therefore (B.214) is greater than or equal to (B.210). This shows that (B.166) always holds, i.e., the high-type principal has no incentive to deviate from the equilibrium location.

Next we look at the scenario when the agent earns zero profit and the low-type principal wants to deviate, i.e., both (B.164) and (B.166) bind.

### B.2.2 The second scenario

In this scenario, there exist subscenarios, one in which the high-type principal does not want to deviate, i.e., (B.165) does not bind, and the other in which the high-type principal wants to deviate, i.e., (B.165) binds.

For the first subscenario, the Lagrangian corresponding to the high-type principal's profit maximization problem is

$$\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)+k\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)^2$$
$$+\lambda_1\left[k\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)^2+\alpha_L\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)-\frac{\alpha_L^6V^4}{64k^3}\right] \tag{B.217}$$

Notice that (B.165) is

$$k\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)^2+\alpha_L\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)\geq\frac{\alpha_L^6V^4}{64k^3} \tag{B.218}$$

(B.166) is

$$k\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)^2+\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)\geq\frac{\alpha_L^6V^4}{64k^3} \tag{B.219}$$

If we choose $m_1 = 0$ and $m_2 = V$, then (B.218) and (B.219) both hold with strict inequality. This shows that (B.219) holds with strict inequality when the high-type principal optimizes her profit, because the left side of (B.219) is the expression for the high-type principal's profit. Thus (B.166) is redundant.

The first order condition of (B.217) with respect to $m_1$ is

$$\frac{\alpha_H\overline{\alpha}}{2k}\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)-\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)+\overline{\alpha}\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)$$
$$+\quad\lambda_1\left[(\overline{\alpha}-\alpha_L)\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)-\frac{\alpha_L\overline{\alpha}}{2k}\left(\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)\right]=0 \tag{B.220}$$

Thus

$$\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}}{4k^2}m_2(V-m_2)+\left(-\frac{\alpha_H\overline{\alpha}}{2k}-\frac{\alpha_H\overline{\alpha}}{2k}+\frac{\overline{\alpha}^2}{2k}\right)m_1-\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2}{8k^2}m_2^2+\frac{\overline{\alpha}^2\tilde{\alpha}^2}{8k^2}m_2^2$$
$$+\quad\lambda_1\frac{\overline{\alpha}^2}{2k}m_1+\lambda_1(\overline{\alpha}-\alpha_L)\frac{\overline{\alpha}\tilde{\alpha}^2}{8k^2}m_2^2-\lambda_1\frac{\alpha_L^2\overline{\alpha}\tilde{\alpha}}{4k^2}m_2(V-m_2)=0 \tag{B.221}$$

which implies

$$m_1=\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}-\lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha}}{2k(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)}m_2(V-m_2)-\frac{\lambda_1(\overline{\alpha}-\alpha_L)\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2+\alpha_H\overline{\alpha}\tilde{\alpha}^2}{4k(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)} \tag{B.222}$$

The first order condition of (B.217) with respect to $m_2$ is

$$\begin{aligned}
&\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2}{4k^2}\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)+\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_H\frac{\tilde{\alpha}}{2k}(V-2m_2)\right)\\
+\quad&\frac{\overline{\alpha}\tilde{\alpha}^2}{2k}\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)m_2+\lambda_1\frac{\overline{\alpha}\tilde{\alpha}^2}{2k}\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)m_2\\
+\quad&\lambda_1\left[\frac{\alpha_L\overline{\alpha}\tilde{\alpha}^2}{4k^2}\left(\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)-\alpha_L\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_L\frac{\tilde{\alpha}}{2k}(V-2m_2)\right)\right]=0
\end{aligned} \tag{B.223}$$

By the expression of $m_1$ in (B.222), we have

$$\begin{aligned}
&\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\\
=\quad&\frac{\overline{\alpha}}{2k}\left(\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}-\lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha}}{2k(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)}m_2(V-m_2)\right)\\
&-\frac{\overline{\alpha}}{2k}\left(\frac{\lambda_1(\overline{\alpha}-\alpha_L)\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2+\alpha_H\overline{\alpha}\tilde{\alpha}^2-\tilde{\alpha}^2(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)}{4k(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)}m_2^2\right)
\end{aligned} \tag{B.224}$$

$$\begin{aligned}
&\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\\
=\quad&\frac{\alpha_H\tilde{\alpha}\left(2\alpha_H\overline{\alpha}-(1+\lambda_1)\overline{\alpha}^2\right)-\alpha_H^2\overline{\alpha}\tilde{\alpha}-\lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha}}{2k(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)}m_2(V-m_2)\\
&+\frac{\lambda_1(\overline{\alpha}-\alpha_L)\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2+\alpha_H\overline{\alpha}\tilde{\alpha}^2}{4k(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)}m_2^2
\end{aligned} \tag{B.225}$$

and

$$\begin{aligned}
&\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\\
=\quad&\frac{\alpha_L\tilde{\alpha}\left(2\alpha_H\overline{\alpha}-(1+\lambda_1)\overline{\alpha}^2\right)-\alpha_H^2\overline{\alpha}\tilde{\alpha}-\lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha}}{2k(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)}m_2(V-m_2)\\
&+\frac{\lambda_1(\overline{\alpha}-\alpha_L)\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2+\alpha_H\overline{\alpha}\tilde{\alpha}^2}{4k(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)}m_2^2
\end{aligned} \tag{B.226}$$

Substituting (B.224), (B.225), and (B.226) into (B.223) and multiplying it by $4k(2\alpha_H\overline{\alpha}-\overline{\alpha}^2-\lambda_1\overline{\alpha}^2)$ and $16k^3$ gives

$$\begin{aligned}
& \alpha_H\overline{\alpha}\tilde{\alpha}^2\left[2\alpha_H\tilde{\alpha}(2\alpha_H\overline{\alpha}-(1+\lambda_1)\overline{\alpha}^2)-(\alpha_H^2\overline{\alpha}\tilde{\alpha}-\lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha})\right]m_2(V-m_2)\\
+ \quad & \alpha_H\overline{\alpha}\tilde{\alpha}^2\left[\lambda_1(\overline{\alpha}-\alpha_L)\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2+\alpha_H\overline{\alpha}\tilde{\alpha}^2\right]m_2^2\\
+ \quad & 2\overline{\alpha}\left(\alpha_H^2\overline{\alpha}\tilde{\alpha}-\lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha}\right)m_2(V-m_2)\left[\alpha_H^2\tilde{\alpha}(V-2m_2)+(1+\lambda_1)\overline{\alpha}\tilde{\alpha}^2m_2\right]\\
- \quad & \overline{\alpha}\left[\lambda_1(\overline{\alpha}-\alpha_L)\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2+\alpha_H\overline{\alpha}\tilde{\alpha}^2\right]\\
- \quad & \overline{\alpha}\left[\tilde{\alpha}^2\left(2\alpha_H\overline{\alpha}-(1+\lambda_1)\overline{\alpha}^2\right)\right]m_2^2\left[\alpha_H^2\tilde{\alpha}(V-2m_2)+(1+\lambda_1)\overline{\alpha}\tilde{\alpha}^2m_2\right]\\
+ \quad & \lambda_1\alpha_L\overline{\alpha}\tilde{\alpha}^2\left[2\alpha_L\tilde{\alpha}(2\alpha_H\overline{\alpha}-(1+\lambda_1)\overline{\alpha}^2)-(\alpha_H^2\overline{\alpha}\tilde{\alpha}-\lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha})\right]m_2(V-m_2)\\
+ \quad & \lambda_1\alpha_L\overline{\alpha}\tilde{\alpha}^2\left[\lambda_1(\overline{\alpha}-\alpha_L)\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2+\alpha_H\overline{\alpha}\tilde{\alpha}^2\right]m_2^2\\
- \quad & 2\lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha}\left(\alpha_H^2\overline{\alpha}\tilde{\alpha}-\lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha}\right)m_2(V-m_2)\\
+ \quad & \lambda_1\alpha_L^2\overline{\alpha}\tilde{\alpha}\left[\lambda_1(\overline{\alpha}-\alpha_L)\overline{\alpha}\tilde{\alpha}^2-\overline{\alpha}^2\tilde{\alpha}^2+\alpha_H\overline{\alpha}\tilde{\alpha}^2-\tilde{\alpha}^2\left(2\alpha_H\overline{\alpha}-(1+\lambda_1)\overline{\alpha}^2\right)\right]m_2^2=0
\end{aligned} \tag{B.227}$$

Using $x=\frac{\alpha_H}{\alpha_L}$, $\alpha=\alpha_L\left(px+(1-p)\right)$ and $\tilde{\alpha}=\alpha_L\frac{px^2+(1-p)}{px+(1-p)}$, we can see that (B.228) can be written as $a_L^8 g_1(m_2,\lambda_1,x,V)$, where $g_1(m_2,\lambda_1,x,V)$ is a function only depending on $m_2$, $\lambda_1$, $x$, and $V$. Thus (B.223) is equivalent to $g_1(m_2,\lambda_1,x,V)=0$.

On the other hand, it is not difficult to see that the binding constraints (B.164) can be written as $\frac{\alpha_L^6}{k^3}g_2(m_2,\lambda_1,x,V)=0$, where $g_2(m_2,\lambda_1,x,V)$ only depends on $m_2$, $\lambda_1$, $x$, and $V$. Therefore, from equations $g_1=0$ and $g_2=0$, we can solve for $m_2$ and $\lambda_1$. The solution of them only depend on $x$ and $V$. Applying the solution of $m_2$ and $\lambda_1$ and the corresponding expression of $m_1$ to the following profit function of the high-type principal

$$\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)+k\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)^2 \tag{B.228}$$

we can see that the high-type principal's profit can be expressed as $\frac{\alpha_L^6}{k^3}g_3(x,V)$, where the function $g_3(x,V)$ only depends on $x$ and $V$. Therefore we prove that at the pooling equilibrium, the high-type agent's maximal profit is a function of $x$ and $V$, in spite of the unavailability of the analytic expression of $m_2$ and $\lambda_1$.

Next, we look at the high-type principal's profit maximization problem with (B.164), (B.166) and (B.165) all binding, i.e., the second subscenario.

In this subscenario, $(m_1,m_2)$ as the location of the high-type principal's profit maximization problem satisfies

$$\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)+k\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)^2=\frac{\alpha_L^6V^4}{64k^3(2x-1)^3} \tag{B.229}$$

and

$$k\left(\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}\right)^2 + \alpha_L\left(\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}\right)\left(\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V - m_2) - m_1\right) = \frac{\alpha_L^6 V^4}{64k^3} \tag{B.230}$$

Numerical result shows that when (B.164), (B.166) and (B.165) all bind, this situation holds on a boundary which encloses an area with $p$ small and $p$ going to zero as $x$ going to positive infinity. Inside the region, there is no solution there.

### B.2.3 Supplements

In the discussion above, the effort levels in two periods take the forms $e_2^* = \frac{\tilde{\alpha}m_2}{2k} \le 1$ and $e_1^* = \frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \le 1$. In fact, there are three other cases. We argue that we don't need to consider them.

1. $m_2 \le \frac{2k}{\tilde{\alpha}}$ and $\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \ge 1$, i.e., $e_2^* \le 1$ and $e_1^* = 1$.

   In this case, the Lagrangian of the high-type principal's profit is

$$\alpha_H \cdot 1 \cdot \left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V - m_2) - m_1\right) - m_0 + \lambda\left[m_0 - k + \overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}\right] + \lambda_1\left(\frac{2k}{\tilde{\alpha}} - m_2\right) + \lambda_2\left(\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} - 1\right) \tag{B.231}$$

   The first order condition with respect to $m_0$ gives $\lambda = 1$. Thus the Lagrangian of the high-type principal's profit becomes

$$\alpha_H \cdot 1 \cdot \left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V - m_2) - m_1\right) + \left[-k + \overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}\right] + \lambda_1\left(\frac{2k}{\tilde{\alpha}} - m_2\right) + \lambda_2\left(\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} - 1\right) \tag{B.232}$$

   The first order condition with respect to $m_1$ gives

$$-\alpha_H + \overline{\alpha} + \lambda_3\frac{\overline{\alpha}}{2k} = 1 \tag{B.233}$$

   Since $\alpha_H > \overline{\alpha}$, (B.233) implies $\lambda_3 > 0$, which means that $\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} = 1$. This belongs to the case we discussed above.

2. $m_2 \geq \frac{2k}{\tilde{\alpha}}$ and $\frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} \leq 1$, i.e., $e_2^* = 1$ and $e_1^* \leq 1$. The Lagrangian of the high-type principal's profit is

$$\alpha_H \left( \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} \right) (\alpha_H (V - m_2) - m_1) - m_0 + \lambda \left[ m_0 + k \left( \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} \right)^2 \right]$$
$$+\lambda_1 \left( \frac{2k}{\tilde{\alpha}} - m_2 \right) + \lambda_2 \left( 1 - \frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right) \tag{B.234}$$

The first order condition with respect to $m_0$ gives $\lambda = 1$. Thus the Lagrangian of the high-type principal's profit is

$$\alpha_H \left( \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} \right) (\alpha_H (V - m_2) - m_1) - m_0 + \left[ m_0 + k \left( \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} \right)^2 \right]$$
$$+\lambda_1 \left( \frac{2k}{\tilde{\alpha}} - m_2 \right) + \lambda_2 \left( 1 - \frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \right) \tag{B.235}$$

The first order condition of (B.235) with respect to $m_1$ gives

$$\frac{\alpha_H \overline{\alpha}}{2k} (\alpha_H (V - m_2) - m_1) + (-\alpha_H + \overline{\alpha}) \left( \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} \right) - \lambda_2 \frac{\overline{\alpha}}{2k} = 0 \tag{B.236}$$

The first order condition of (B.235) with respect to $m_2$ gives

$$\frac{\alpha_H \overline{\alpha}\tilde{\alpha}}{2k} (\alpha_H (V - m_2) - m_1) + \left( -\alpha_H^2 + \overline{\alpha}\tilde{\alpha} \right) \left( \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} \right) - \lambda_1 - \lambda_2 \frac{\overline{\alpha}\tilde{\alpha}}{2k} = 0 \tag{B.237}$$

Multiplying (B.236) by $\tilde{\alpha}$ and subtracting (B.238) from it gives

$$\left( -\alpha_H \tilde{\alpha} + \alpha_H^2 \right) \left( \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} \right) + \lambda_1 = 0 \tag{B.238}$$

Since $e_1^* = \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} > 0$, $\lambda_1 > 0$, which means that $m_2 = \frac{2k}{\tilde{\alpha}}$. Notice that when $m_2 = \frac{2k}{\tilde{\alpha}}$, $\frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} = \frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}$. Thus this belongs to previous case.

3. $m_2 \geq \frac{2k}{\tilde{\alpha}}$ and $\frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} \geq 1$, i.e., $e_2^* = 1$ and $e_1^* \leq 1$. The Lagrangian of the high-type principal's profit is

$$\alpha_H \cdot 1 \cdot (\alpha_H \cdot (V - m_2) - m_1) - m_0 + \lambda \left[ m_0 - k + \overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2 \right] + \lambda_1 \left( \frac{2k}{\tilde{\alpha}} - m_2 \right)$$
$$+\lambda_2 \left( \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} - 1 \right) \tag{B.239}$$

The first order condition with respect to $m_0$ gives $\lambda = 0$. Thus the Lagrangian of the high-type principal's profit becomes

$$\alpha_H \cdot 1 \cdot (\alpha_H \cdot (V - m_2) - m_1) - m_0 + \left[ m_0 - k + \overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2 \right] + \lambda_1 \left( \frac{2k}{\tilde{\alpha}} - m_2 \right)$$
$$+\lambda_2 \left( \frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} - 1 \right) \tag{B.240}$$

The first order condition with respect to $m_1$ gives

$$-\alpha_H + \overline{\alpha} + \lambda_2 \frac{\tilde{\alpha}}{2k} = 0 \tag{B.241}$$

which implies that $\lambda_2 > 0$. Thus $\frac{\overline{\alpha}m_1 - \overline{\alpha}k + \overline{\alpha}\tilde{\alpha}m_2}{2k} = 1$. This belongs to previous case.

## B.3 Proof of Theorem 12

We need to examine the separating equilibrium of case 2, case 3 and case 4, and then compare the high-type principal's profit in the separating equilibria of case 1 (studied in Theorem 10), case 2, case 3 and case 4. First we look at case 2.

### B.3.1 Separating equilibrium when upfront and end money transfers are included in the payment menu

We study the separating equilibrium when intermediate and end money transfers are included from the menu.

For the high-type principal, her maximal profit satisfies

$$\max_{(m_{0H}, m_{2H})} \{-m_{0H} + \alpha_H^2 e_{1H} e_{2H} (V - m_{2H})\} \tag{B.242}$$

subject to

$$LM_2 \geq -m_{0H} + \alpha_L^2 e_{1H} e_{2H} (V - m_{2H}) \tag{B.243}$$

and

$$-m_{0H} + \alpha_H^2 e_{1H} e_{2H} (V - m_{2H}) \geq \overline{LM}_2 \tag{B.244}$$

with the agent's profit satisfying:

$$\max_{(e_{1H}, e_{2H})} m_{0H} - k e_{1H}^2 - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \tag{B.245}$$

and

$$m_{0H} - k e_{1H}^2 - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \geq 0 \tag{B.246}$$

where

$$LM_2 = \max_{(m_{0L}, m_{2L})} \{-m_{0L} + \alpha_L^2 e_{1L} e_{2L} (V - m_{2L})\} \tag{B.247}$$

and

$$\overline{LM}_2 = \max_{(\tilde{m}_{0L}, \tilde{m}_{2L})} \{-\tilde{m}_{0L} + \alpha_H^2 e_{1L} e_{2L} (V - \tilde{m}_{2L})\} \tag{B.248}$$

with the agent satisfying:

$$\max_{(e_{1L}, e_{2L})} \tilde{m}_{0L} - k e_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \tag{B.249}$$

and

$$\tilde{m}_{0L} - ke_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \geq 0 \tag{B.250}$$

where (B.243) is the constraint of preventing the low-type principal from mimicking the high-type one, (B.244) is the constraint of preventing the high-type principal from mimicking the low-type one, (B.246) is the agent's participation constraint for the high-type principal's offer.

The proof consists of two parts. First we look at the high-type principal's profit maximization problem when the low-type principal wants to mimic the high-type one and the agent earns positive profit, i.e., (B.243) binds, but (B.246) does not bind. In this scenario, we show that the high-type principal does not want to deviate, i.e., (B.244) is redundant. Then we look at the high-type principal's profit maximization problem when the low-type principal wants to mimic the high-type one and the agent earns zero profit, i.e., both (B.243) and (B.246) bind. In this scenario, there exist two subscenarios, one in which the high-type principal does not want to deviate, i.e., (B.244) not binding and the other in which the high-type principal wants to deviate, i.e., (B.244) binding.

Before we go to discussion in detail, we calculate $LM_2$ and $\overline{LM}_2$.

As what we did in previous sections, it is easy to see that $LM_2 = \frac{\alpha_L^6 V^4}{64k^3}$ and $\overline{LM}_2$ is the maximum of the following expression:

$$\alpha_H \left( \frac{\alpha_L^3 \tilde{m}_{2L}^2}{8k^2} \right) \left( \alpha_H \frac{\alpha_L \tilde{m}_{2L}}{2k} (V - \tilde{m}_{2L}) \right) + \frac{\alpha_L^6 \tilde{m}_{2L}^4}{64k^3} \tag{B.251}$$

The first order condition of (B.251) with respect to $m_{2L}$ gives

$$\frac{\alpha_H^2 \alpha_L^4}{16k^3} \tilde{m}_{2L}^2 \left( 3V - 4\tilde{m}_{2L} + \frac{\alpha_L^2}{\alpha_H^2} \tilde{m}_{2L} \right) = 0 \tag{B.252}$$

which gives $\tilde{m}_{2L} = \frac{3x^2 V}{4x^2 - 1}$ with $x = \frac{\alpha_H}{\alpha_L}$. It is easy to check that the second derivative of (B.251) is negative at the location. This means that it is local maximum. Since it is a unique critical point, it is the location of the global maximum. Thus $\overline{LM}_2$ equals

$$\frac{\alpha_H \alpha_L^3}{8k^2} \left( \frac{3x^2 V}{4x^2 - 1} \right)^2 \left[ \frac{\alpha_H \alpha_L}{2k} \left( \frac{3x^2 V}{4x^2 - 1} \right) \left( \frac{(x^2 - 1)V}{4x^2 - 1} \right) \right] + \frac{\alpha_L^6}{64k^3} \left( \frac{3x^2 V}{4x^2 - 1} \right)^4 \tag{B.253}$$

Let $y = x^2$ and using $x = \frac{\alpha_H}{\alpha_L}$ , then $\overline{LM}_2$ can be written as

$$\begin{aligned} & \frac{\alpha_L^6 y}{16k^3} \left( \frac{9y^2 V^2}{(4y - 1)^2} \right) \left( \frac{3y(y - 1)V^2}{(4y - 1)^2} \right) + \frac{\alpha_L^6}{64k^3} \left( \frac{3yV}{4y - 1} \right)^4 \\ = \; & \alpha_L^6 \frac{108y^4(y - 1) + 81y^4}{64k^3 (4y - 1)^4} V^4 \end{aligned} \tag{B.254}$$

According to the same reason we discussed in the previous section, the effort levels of the agent for the high-type principal's offer takes the form $e_{1H}^* = \frac{\alpha_H^3 m_{2H}^2}{8k^2}$ and $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k}$.

Next we study the first scenario in which the low-type principal wants to mimic the high-type one and the agent earns positive profit, i.e., (B.243) binds, but (B.246) does not bind.

#### B.3.1.1 The first scenario

The Lagrangian for the high-type principal's profit is

$$\begin{aligned} & \alpha_H\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})\right)-m_{0H} \\ + \quad & \lambda_1\left[\frac{\alpha_L^6V^4}{64k^3}+m_{0H}-\alpha_L\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})\right)\right] \end{aligned} \tag{B.255}$$

The first order conditions with respect to $m_{0H}$ and $m_{2H}$ are

$$-1+\lambda_1=0 \tag{B.256}$$

and

$$\begin{aligned} & \frac{\alpha_H^4 m_{2H}}{4k^2}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})\right]+\alpha_H\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)\left[\frac{\alpha_H^2}{2k}(V-2m_{2H})\right] \\ - \quad & \lambda_1\frac{\alpha_L\alpha_H^3 m_{2H}}{4k^2}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V-m_{2H})\right]-\lambda_1\alpha_L\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)\left[\frac{\alpha_L\alpha_H}{2k}(V-2m_{2H})\right]=0 \end{aligned} \tag{B.257}$$

From (B.256), we obtain $\lambda_1=1$. This means the the high-type principal's profit equals

$$\left(\alpha_H^2-\alpha_L^2\right)\left(\frac{\alpha_H^3 m_{2H}^2}{8k^2}\right)\left(\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})\right)+\frac{\alpha_L^6V^4}{64k^3} \tag{B.258}$$

Substituting $\lambda_1=1$ into (B.257) and dividing the both sides of (B.257) by $\frac{\alpha_H^4}{16k^3}$ gives

$$2\alpha_H^2 m_{2H}^2(V-m_{2H})+\alpha_H^2 m_{2H}^2(V-2m_{2H})-2\alpha_L^2 m_{2H}^2(V-m_{2H})-\alpha_L^2 m_{2H}^2(V-2m_{2H})=0 \tag{B.259}$$

which is equivalent to

$$2\left(\alpha_H^2-\alpha_L^2\right)m_{2H}^2(V-m_{2H})+\left(\alpha_H^2-\alpha_L^2\right)m_{2H}^2(V-2m_{2H})=0 \tag{B.260}$$

namely,

$$\left(\alpha_H^2-\alpha_L^2\right)m_{2H}^2(3V-4m_{2H})=0 \tag{B.261}$$

It is clear that (B.261) has three roots, with two being 0 and the third one being $\frac{3}{4}V$. Thus the high-type principal's profit (B.258) equals

$$\begin{aligned} & \left(\alpha_H^2-\alpha_L^2\right)\alpha_H^4\left(\frac{\left(\frac{3}{4}V\right)^2}{8k^2}\right)\left(\frac{3}{8k}V\cdot\frac{1}{4}V\right)+\frac{\alpha_L^6V^4}{64k^3} \\ = \quad & \alpha_L^6\left(x^6-x^4\right)\left(\frac{27}{64^2k^3}+\frac{81}{4\cdot 64^2k^3}\right)V^2+\frac{\alpha_L^6V^4}{64k^3} \end{aligned} \tag{B.262}$$

with $x=\frac{\alpha_H}{\alpha_L}$.

Substituting $m_{2H} = \frac{3}{4}V$ into the constraint associated with $\lambda_1$ gives

$$\frac{\alpha_L^6 V^4}{64k^3} + m_{0H} - \alpha_L \left[\frac{\alpha_H^3 \left(\frac{3}{4}V\right)^2}{8k^2}\right]\left[\alpha_L \frac{\alpha_H \left(\frac{3}{4}V\right)}{2k}\left(V - \frac{3}{4}V\right)\right] = 0 \tag{B.263}$$

which can be simplified as

$$\frac{\alpha_L^6 V^4}{64k^3} + m_{0H} - \frac{27}{32}\frac{\alpha_L^2 \alpha_H^4 V^4}{128k^3} = 0 \tag{B.264}$$

Thus

$$m_{0H} = \frac{27}{32}\frac{\alpha_L^2 \alpha_H^4 V^4}{128k^3} - \frac{\alpha_L^6 V^4}{64k^3} \tag{B.265}$$

From (B.265), we can see that $m_{0H}$ is negative when $\alpha_L$ is close to $\alpha_H$, and $m_{0H}$ is positive when $\alpha_L$ is far away from $\alpha_H$.

From $m_{2H} = \frac{3}{4}V$ and (B.255), we have

$$m_{0H} + k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 = \frac{27}{32}\frac{\alpha_L^2 \alpha_H^4 V^4}{128k^3} - \frac{\alpha_L^6 V^4}{64k^3} + \frac{\alpha_H^6 V^4}{16 \cdot 16 \cdot 64k^3} \tag{B.266}$$

Therefore (B.266) being zero is equivalent to

$$\frac{27}{32 \cdot 128}x^4 - \frac{1}{64} + \frac{81}{16 \cdot 16 \cdot 64}x^6 = 0 \tag{B.267}$$

Let $F3(x)$ be the left side of (B.267). Using Maple, we find that (B.267) has only one real root $x \approx 1.064048$ for $x \geq 1$. Since $F3(x)$ increases to positive infinity as $x$ increases to positive infinity, $F3(x) > 0$ when $x > 1.064048$, i.e., case 1) and $F3(x) < 0$ when $x < 1.064048$, i.e., case 2).

Next we show that constraint (B.244) is redundant.

Notice that according to (B.254), $LM_2$ can be written as

$$\frac{108y^4(y-1) + 81y^4}{64k^3(4y-1)^4} \tag{B.268}$$

On the other hand, we know that (B.262) can be written as

$$\alpha_L^6\left(y^3 - y^2\right)\left(\frac{27}{64^2k^3} + \frac{81}{4 \cdot 64^2k^3}\right)V^2 + \frac{\alpha_L^6 V^4}{64k^3} \tag{B.269}$$

We want to show that (B.269) is greater or equal to (B.254) when $y \geq 1$, namely $x \geq 1$. We can see that it is equivalent to show that

$$108y^4(y-1) + 81y^4 \leq \left[\left(y^3 - y^2\right)\left(\frac{27}{64} + \frac{81}{4 \cdot 64}\right) + 1\right](4y-1)^4 \tag{B.270}$$

Let $F_1(y)$ be the expression

$$\left[\left(y^3-y^2\right)\left(\frac{27}{64}+\frac{81}{4\cdot 64}\right)+1\right](4y-1)^4-108y^4(y-1)-81y^4 \tag{B.271}$$

Using Maple, we can find that $F_1(y)$ has only one real root at $y=1$ when $y\geq 1$. Since $F_1(y)$ goes to positive infinity when $y$ increases to positive infinity, $F_1(y)\geq 0$ holds for all $y\geq 1$. This means that (B.244) is automatically satisfied, namely redundant.

Next we look at the second scenario in which the low-type principal wants to mimic the high-type one and the agent earns zero profit, i.e., both (B.243) and (B.246) bind.

#### B.3.1.2 The second scenario

In this scenario, there exists subscenarios, one in which the high-type principal does not want to deviate, i.e., (B.244) not binding and the other in which the high-type principal wants to deviate, i.e., (B.244) binding.

We know that binding constraints (B.243) and (B.246) imply that as the location of the maximum of the high-type principal's profit $m_{2H}$ satisfies

$$\frac{\alpha_L^6V^4}{64k^3}=\frac{\alpha_H^6m_{2H}^4}{64k^3}-\alpha_L\left(\frac{\alpha_H^3m_{2H}^2}{8k^2}\right)\left(\alpha_L\frac{\alpha_Hm_{2H}}{2k}(V-m_{2H})\right) \tag{B.272}$$

namely,

$$\frac{\alpha_H^6m_{2H}^4}{64k^3}+\frac{\alpha_L^2\alpha_H^4m_{2H}^3}{16k^3}(V-m_{2H})=\frac{\alpha_L^6V^4}{64k^3} \tag{B.273}$$

Suppose constraint (B.244) is either binding or violated. This means that implies that

$$\alpha_H\left(\frac{\alpha_H^3m_{2H}^2}{8k^2}\right)\left(\alpha_H\frac{\alpha_Hm_{2H}}{2k}(V-m_{2H})\right)+\frac{\alpha_H^6m_{2H}^4}{64k^3}\leq\frac{108y^4(y-1)+81y^4}{64k^3(4y-1)^4}V^4 \tag{B.274}$$

namely

$$\frac{\alpha_H^6m_{2H}^4}{64k^3}+\frac{\alpha_H^6m_{2H}^3}{16k^3}(V-m_{2H})\leq\alpha_L^6\frac{108y^4(y-1)+81y^4}{64k^3(4y-1)^4}V^4 \tag{B.275}$$

The right side of (B.275) are the high-type principal's profit and the right side equals $LM_2$ with $y=x^2=\frac{\alpha_H^2}{\alpha_L^2}$.

Multiplying both sides of (B.273) by $\frac{\alpha_H^2}{\alpha_L}$ and then subtracting (B.275) from this new identity from gives

$$\frac{\left(\frac{\alpha_H^8}{\alpha_L^2}-\alpha_H^6\right)m_{2H}^4}{64k^3}\geq\frac{\alpha_L^4\alpha_H^2V^4}{64k^3}-\alpha_L^6\frac{108y^4(y-1)+81y^4}{64k^3(4y-1)^4}V^4 \tag{B.276}$$

which is equivalent to

$$\left(y^4-y^3\right)m_{2H}^4\geq y-\frac{108y^4(y-1)+81y^4}{(4y-1)^4}V^4 \tag{B.277}$$

i.e.,

$$m_{2H} \geq \left( \frac{y - \frac{108y^4(y-1)+81y^4}{(4y-1)^4}}{(y^4 - y^3)} \right)^{\frac{1}{4}} V^4 \tag{B.278}$$

Using Maple, we can find that polynomial

$$y(4y-1)^4 - \left(108y^4(y-1) + 81y^4\right) - \left(y^4 - y^3\right)(4y-1)^4$$

has only one real root at $y = 1$. Since the polynomial decreases to $-\infty$ as $y$ increases to $+\infty$, this shows that it is less than 1 when $y > 1$ and equals 1 when $y = 1$. This means that the right side of (B.278) is less than 1 when $y > 1$. Letting $m_T$ be the right side of (B.278), it is easy to show that $m_T$ must satisfy the following expression when (B.275) holds.

$$y^3 m_T^4 + 4y^2 m_T^3 (V - m_T) \leq \frac{108y^4(y-1) + 81y^4}{(4y-1)^4} V^4 \tag{B.279}$$

However, using Maple, either directly calculating the difference between the left side and right side of (B.279) or calculating the roots of an equivalent polynomial for (B.279) with equality holding, we find that the left side of (B.279) is always greater than the right side of (B.279) when $y > 1$. We leave the verification to the readers. This shows that (B.275) does not hold. Therefore constraint (B.244) is redundant.

### B.3.2 Separating equilibrium when intermediate and end money transfers are included in the payment menu

We study the separating equilibrium in which intermediate and end money transfers are included in the menu.

For the high-type principal, her maximal profit satisfies

$$\max_{(m_{1H}, m_{2H})} \{-\alpha_H e_{1H} m_{1H} + \alpha_H^2 e_{1H} e_{2H} (V - m_{2H})\} \tag{B.280}$$

subject to

$$LM_4 \geq -\alpha_L e_{1H} m_{1H} + \alpha_L^2 e_{1H} e_{2H} (V - m_{2H}) \tag{B.281}$$

and

$$-\alpha_H e_{1H} m_{1H} + \alpha_H^2 e_{1H} e_{2H} (V - m_{2H}) \geq \overline{LM}_4 \tag{B.282}$$

with the agent's profit satisfying:

$$\max_{(e_{1H}, e_{2H})} -ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \tag{B.283}$$

and

$$-ke_{1H}^2 + \alpha_H e_{1H} m_{1H} - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \geq 0 \tag{B.284}$$

where

$$LM_4 = \max_{(m_{1L}, m_{2L})} \{-\alpha_L e_{1L} m_{1L} + \alpha_L^2 e_{1L} e_{2L} (V - m_{2L})\} \tag{B.285}$$

as the low-type principal's maximal profit, and

$$\overline{LM}_4 = \max_{(\tilde{m}_{1L}, \tilde{m}_{2L})} \{-\alpha_H e_{1L} \tilde{m}_{1L} + \alpha_H^2 e_{1L} e_{2L} (V - \tilde{m}_{2L})\} \tag{B.286}$$

with the agent satisfying:

$$\max_{(e_{1L}, e_{2L})} -ke_{1L}^2 + \alpha_L e_{1L} \tilde{m}_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \tag{B.287}$$

and

$$-ke_{1L}^2 + \alpha_L e_{1L} \tilde{m}_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \geq 0 \tag{B.288}$$

where (B.281) is the constraint of preventing the low-type principal from mimicking the high-type one, (B.282) is the constraint of preventing the high-type principal from mimicking the low-type one, (B.284) is the agent's participation constraint for the high-type principal's offer.

The proof consists of two parts. First we look at the high-type principal's profit maximization problem when the low-type principal wants to mimic the high-type one, i.e., (B.281) binds, but the high-type principal does not want to deviate i.e.,(B.282) does not bind. Then we look at the high-type principal's profit maximization problem when when the low-type principal wants to mimic the high-type one, i.e., (B.281) binds, but the high-type principal wants to deviate, i.e.,(B.282) binds. We will point out that (B.284) always holds, namely, it is redundant.

Before we go to discussion in detail, we calculate $LM_4$ and $\overline{LM}_4$.

Using the same way of calculating $LM$, we can find $LM_4 = \frac{\alpha_L^6 V^4}{128k^3}$. As to $\overline{LM}_4$, it is the maximum of the following expression:

$$\alpha_H \left( \frac{\alpha_L \tilde{m}_{1L} + \frac{\alpha_L^3 \tilde{m}_{2L}^2}{4k}}{2k} \right) \left( \alpha_H \frac{\alpha_L \tilde{m}_{2L}}{2k} (V - \tilde{m}_{2L}) - \tilde{m}_{1L} \right) \tag{B.289}$$

The first order condition of (B.289) with respect to $\tilde{m}_{1L}$ gives

$$\frac{\alpha_H \alpha_L}{2k} \left( \alpha_H \frac{\alpha_L \tilde{m}_{2L}}{2k} (V - \tilde{m}_{2L}) - \tilde{m}_{1L} \right) - \alpha_H \left( \frac{\alpha_L \tilde{m}_{1L} + \frac{\alpha_L^3 \tilde{m}_{2L}^2}{4k}}{2k} \right) = 0 \tag{B.290}$$

and the first order condition of (B.289) with respect to $\tilde{m}_{2L}$ gives

$$\frac{\alpha_H \alpha_L^3 \tilde{m}_{2L}}{4k} \left( \alpha_H \frac{\alpha_L \tilde{m}_{2L}}{2k} (V - \tilde{m}_{2L}) - \tilde{m}_{1L} \right) + \alpha_H \left( \frac{\alpha_L \tilde{m}_{1L} + \frac{\alpha_L^3 \tilde{m}_{2L}^2}{4k}}{2k} \right) \left( \frac{\alpha_H \alpha_L}{2k} (V - 2\tilde{m}_{2L}) \right) \tag{B.291}$$

Multiplying (B.290) by $\frac{\alpha_L^2 \tilde{m}_{2L}}{2k}$ and subtracting it from (B.291), we have

$$\alpha_H \left( \frac{\alpha_L \tilde{m}_{1L} + \frac{\alpha_L^3 \tilde{m}_{2L}^2}{4k}}{2k} \right) \left( \frac{\alpha_H \alpha_L}{2k} \left( V - 2\tilde{m}_{2L} + \frac{\alpha_L}{\alpha_H} \right) \right. \text{(B.292)}$$

Since $\alpha_H \left( \frac{\alpha_L \tilde{m}_{1L} + \frac{\alpha_L^3 \tilde{m}_{2L}^2}{4k}}{2k} \right)$ as the effort level in the first period has to be positive, we have $\tilde{m}_{2L} = \frac{\alpha_H}{2\alpha_H - \alpha_L}$.

Notice that (B.290) is equivalent to

$$\tilde{m}_{1L} = \frac{\alpha_H \alpha_L \tilde{m}_{2L}(V - \tilde{m}_{2L})}{4k} - \frac{\alpha_L^2 \tilde{m}_{2L}^2}{8k} \tag{B.293}$$

Thus

$$\begin{aligned} \tilde{m}_{1L} + \frac{\alpha_L^2 \tilde{m}_{2L}}{4k} &= \frac{\alpha_H \alpha_L \tilde{m}_{2L}(V - \tilde{m}_{2L})}{4k} + \frac{\alpha_L^2 \tilde{m}_{2L}^2}{8k} \\ &= \frac{2\alpha_H \alpha_L \alpha_H (2\alpha_H - \alpha_L)}{8k(2\alpha_H - \alpha_L)^2} \\ &= \frac{\alpha_H^2 \alpha_L}{8k(2\alpha_H - \alpha_L)} \end{aligned} \tag{B.294}$$

and

$$\begin{aligned} \alpha_H \frac{\alpha_L \tilde{m}_{2L}}{2k}(V - \tilde{m}_{2L}) - \tilde{m}_{1L} &= \frac{\alpha_H \alpha_L \tilde{m}_{2L}(V - \tilde{m}_{2L})}{4k} + \frac{\alpha_L^2 \tilde{m}_{2L}^2}{8k} \\ &= \frac{\alpha_H^2 \alpha_L}{8k(2\alpha_H - \alpha_L)} \end{aligned} \tag{B.295}$$

Therefore $\overline{LM}_4$ equals

$$\frac{\alpha_H^5 \alpha_L^3}{128k^3(2\alpha_H - \alpha_L)^2} \tag{B.296}$$

By similar argument in previous sections, the effort levels in two periods take the forms $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k}$ and $e_{1H}^* = \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$, the agent's participation constraint (B.284) when the high-type principal offers a contract can be written as

$$k \left[ \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right]^2 \geq 0 \tag{B.297}$$

Thus (B.284) always holds, namely it is redundant. Therefore, for the two scenarios mentioned above, we don't need to consider this constraint.

Next we look at the first scenario in which the low-type principal wants to mimic the high-type one, i.e., (B.281) binds, but the high-type principal does not want to deviate i.e.,(B.282) does not bind.

#### B.3.2.1 The first scenario

The Lagrangian for the high-type principal's profit is

$$\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)$$
$$+\quad\lambda\left[\frac{\alpha_L^6 V^4}{128k^3}-\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)\right] \quad \text{(B.298)}$$

The first order condition of (B.298) with respect to $m_{1H}$ gives

$$\frac{\alpha_H^2}{2k}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]-\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)$$
$$-\quad\lambda\frac{\alpha_H\alpha_L}{2k}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]+\lambda\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)=0 \quad \text{(B.299)}$$

which means that

$$\frac{\alpha_H^4-\lambda\alpha_H^2\alpha_L^2}{4k^2}m_{2H}(V-m_{2H})+\frac{\left(\lambda\alpha_L\alpha_H^3-\alpha_H^4\right)m_{2H}^2}{8k^2}-\frac{\left(\alpha_H^2-\lambda\alpha_H\alpha_L\right)m_{1H}}{k}=0 \quad \text{(B.300)}$$

This gives

$$m_{1H}=\frac{\alpha_H^4-\lambda\alpha_H^2\alpha_L^2}{4k\left(\alpha_H^2-\lambda_1\alpha_H\alpha_L\right)}m_{2H}(V-m_{2H})+\frac{\left(\lambda\alpha_L\alpha_H^3-\alpha_H^4\right)m_{2H}^2}{8k\left(\alpha_H^2-\lambda_1\alpha_H\alpha_L\right)} \quad \text{(B.301)}$$

The first order condition of (B.298) with respect to $m_{2H}$ gives

$$\frac{\alpha_H^4 m_{2H}}{4k^2}\left[\frac{\alpha_H^2 m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]+\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\frac{\alpha_H^2}{2k}(V-2m_{2H})\right]$$
$$-\quad\lambda\frac{\alpha_L\alpha_H^3 m_{2H}}{4k^2}\left[\frac{\alpha_L\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right]$$
$$-\quad\lambda\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\frac{\alpha_L\alpha_H}{2k}(V-2m_{2H})\right]=0 \quad \text{(B.302)}$$

Multiplying (B.299) by $\frac{\alpha_H^2}{2k}m_{2H}$ and subtracting the product from (B.302) gives

$$\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\frac{\alpha_H^2}{2k}(V-m_{2H})\right]$$
$$=\quad\lambda\alpha_L\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left[\frac{\alpha_L\alpha_H}{2k}\left(V-2m_{2H}+\frac{\alpha_H}{\alpha_L}m_{2H}\right)\right] \quad \text{(B.303)}$$

Since $\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$ as the effort in the first period cannot be zero, (B.303) becomes

$$\alpha_H\left[\frac{\alpha_H^2}{2k}(V-m_{2H})\right]=\lambda\alpha_L\left[\frac{\alpha_L\alpha_H}{2k}\left(V-2m_{2H}+\frac{\alpha_H}{\alpha_L}m_{2H}\right)\right] \quad \text{(B.304)}$$

which implies that

$$\lambda = \frac{\frac{\alpha_H^2}{\alpha_L^2}(V - m_{2H})}{\left[V - \left(2 - \frac{\alpha_H}{\alpha_L}\right)m_{2H}\right]} \tag{B.305}$$

Therefore

$$\begin{aligned} & \alpha_H^4 - \lambda\alpha_H^2\alpha_L^2 \\ = \;& \alpha_H^4 - \frac{\alpha_H^4(V - m_{2H})}{\left[V - \left(2 - \frac{\alpha_H}{\alpha_L}\right)m_{2H}\right]} \\ = \;& \frac{\alpha_H^4\left(\frac{\alpha_H}{\alpha_L} - 1\right)m_{2H}}{\left[V - \left(2 - \frac{\alpha_H}{\alpha_L}\right)m_{2H}\right]} \end{aligned} \tag{B.306}$$

and

$$\begin{aligned} & \alpha_H^2 - \lambda\alpha_H\alpha_L \\ = \;& \alpha_H^2 - \frac{\frac{\alpha_H^3}{\alpha_L}(V - m_{2H})}{\left[V - \left(2 - \frac{\alpha_H}{\alpha_L}\right)m_{2H}\right]} \\ = \;& \frac{\left(\alpha_H^2 - \frac{\alpha_H^3}{\alpha_L}\right)V - \left(2\alpha_H^2 - 2\frac{\alpha_H^3}{\alpha_L}\right)m_{2H}}{\left[V - \left(2 - \frac{\alpha_H}{\alpha_L}\right)m_{2H}\right]}m_{2H}^2 \end{aligned} \tag{B.307}$$

Notice that

$$\frac{(\lambda\alpha_L\alpha_H^3 - \alpha_H^4)m_{2H}^2}{\alpha_H^2 - \lambda\alpha_H\alpha_L} = -\alpha_H^2 m_{2H}^2 \tag{B.308}$$

Applying (B.306), (B.307) and (B.308) to (B.301), we have

$$\begin{aligned} m_{1H} &= \frac{\alpha_H^4\left(\frac{\alpha_H}{\alpha_L} - 1\right)m_{2H}^2(V - m_{2H})}{4k\left[\left(\alpha_H^2 - \frac{\alpha_H^3}{\alpha_L}\right)V - \left(2\alpha_H^2 - 2\frac{\alpha_H^3}{\alpha_L}\right)m_{2H}\right]} - \frac{\alpha_H^2}{8k}m_{2H}^2 \\ &= \frac{\alpha_H^2 m_{2H}^2(V - m_{2H})}{4k(-V + 2m_{2H})} - \frac{\alpha_H^2}{8k}m_{2H}^2 \\ &= \frac{2\alpha_H^2 m_{2H}^2(V - m_{2H})}{8k(-V + 2m_{2H})} - \frac{\alpha_H^2(-V + 2m_{2H})m_{2H}^2}{8k(-V + 2m_{2H})} \\ &= \frac{3\alpha_H^2 m_{2H}^2 V - 4\alpha_H^2 m_{2H}^3}{8k(-V + 2m_{2H})} \end{aligned} \tag{B.309}$$

Thus

$$\begin{aligned} m_{1H} + \frac{\alpha_H^2 m_{2H}^2}{4k} &= \frac{3\alpha_H^2 m_{2H}^2 V - 4\alpha_H^2 m_{2H}^3}{8k(-V + 2m_{2H})} + \frac{\alpha_H^2 m_{2H}^2}{4k} \\ &= \frac{3\alpha_H^2 m_{2H}^2 V - 4\alpha_H^2 m_{2H}^3 + 2\alpha_H^2 m_{2H}^2(-V + 2m_{2H})}{8k(-V + 2m_{2H})} \\ &= \frac{\alpha_H^2 m_{2H}^2 V}{8k(-V + 2m_{2H})} \end{aligned} \tag{B.310}$$

and

$$\begin{aligned}
& \alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H} \\
= \ & \alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - \frac{3\alpha_H^2 m_{2H}^2 V - 4\alpha_H^2 m_{2H}^3}{8k(-V + 2m_{2H})} \\
= \ & \frac{4\alpha_L \alpha_H m_{2H}(V - m_{2H})(-V + 2m_{2H}) - 3\alpha_H^2 m_{2H}^2 V + 4\alpha_H^2 m_{2H}^3}{8k(-V + 2m_{2H})} \qquad \text{(B.311)}
\end{aligned}$$

Since $\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}$ as the effort in the first period has to be positive, (B.310) means that $-V + 2m_{2H} > 0$, namely $m_{2H} > V/2$.

From (B.298), we know that when (B.281) is binding, it can be written as

$$\frac{\alpha_L^6 V^4}{128k^3} = \alpha_L \left( \frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k} \right) \left( \alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H} \right) \qquad \text{(B.312)}$$

Substituting (B.310) and (B.311) into (B.312) gives

$$\begin{aligned}
\frac{\alpha_L^6 V^4}{128k^3} \ = \ & \frac{\alpha_L \alpha_H}{2k} \left[ \frac{\alpha_H^2 m_{2H}^2 V}{8k(-V + 2m_{2H})} \right] \left[ \frac{4\alpha_L \alpha_H m_{2H}(V - m_{2H})(-V + 2m_{2H})}{8k(-V + 2m_{2H})} \right] \\
& + \frac{\alpha_L \alpha_H}{2k} \left[ \frac{\alpha_H^2 m_{2H}^2 V}{8k(-V + 2m_{2H})} \right] \left[ \frac{-3\alpha_H^2 m_{2H}^2 V + 4\alpha_H^2 m_{2H}^3}{8k(-V + 2m_{2H})} \right] \qquad \text{(B.313)}
\end{aligned}$$

Letting $Vy = m_{2H}$ and $x = \frac{\alpha_H}{\alpha_L}$, (B.313) can be simplified as

$$1 = \frac{x^3 y^2 \left[ 4xy(1 - y)(2y - 1) - 3x^2 y^2 + 4x^2 y^3 \right]}{(-1 + 2y)^2} \qquad \text{(B.314)}$$

Notice that (B.314) is equivalent to

$$x^3 y^2 \left[ 4xy(1 - y)(2y - 1) - 3x^2 y^2 + 4x^2 y^3 \right] - (-1 + 2y)^2 \qquad \text{(B.315)}$$

When $x = 1$, (B.315) is equivalent to

$$y^2 \left[ 4y(1 - y)(2y - 1) - 3y^2 + 4y^3 \right] - (-1 + 2y)^2 = 0 \qquad \text{(B.316)}$$

Notice that according to (B.298), the high-type principal's profit equals

$$\frac{\alpha_H \alpha_H}{2k} \left[ \frac{\alpha_H^2 m_{2H}^2 V}{8k(-V + 2m_{2H})} \right] \left[ \frac{4\alpha_H \alpha_H m_{2H}(V - m_{2H})(-V + 2m_{2H}) - 3\alpha_H^2 m_{2H}^2 V + 4\alpha_H^2 m_{2H}^3}{8k(-V + 2m_{2H})} \right] \qquad \text{(B.317)}$$

Next we look at the second scenario in which the low-type principal wants to mimic the high-type one, i.e., (B.281) binds, but the high-type principal wants to deviate i.e.,(B.282) binds.

#### B.3.2.2 The second scenario

Our numerical calculation shows that this does not happen. In other words, when (B.402) holds, the high-type principal's profit in (B.317) is always bigger than $\overline{LM}_4$, which equals

$$\frac{\alpha_H^5 \alpha_L^3}{128k^3(2\alpha_H - \alpha_L)^2} \tag{B.318}$$

(see (B.296) ).

### B.3.3 Separating equilibrium when only end money transfers are included in the payment menu

First we look at the separating equilibrium when only end money transfers are included in the menu .

For the high-type principal, her maximal profit satisfies

$$\max_{m_{2H}}\{\alpha_H^2 e_{1H} e_{2H}(V - m_{2H})\} \tag{B.319}$$

subject to

$$LM_6 \geq \alpha_L^2 e_{1H} e_{2H}(V - m_{2H}) \tag{B.320}$$

and

$$\alpha_H^2 e_{1H} e_{2H}(V - m_{2H}) \geq \overline{LM}_6 \tag{B.321}$$

with the agent's profit satisfying:

$$\max_{(e_{1H}, e_{2H})} -ke_{1H}^2 - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \tag{B.322}$$

and

$$-ke_{1H}^2 - \alpha_H e_{1H} k e_{2H}^2 + \alpha_H^2 e_{1H} e_{2H} m_{2H} \geq 0 \tag{B.323}$$

where

$$LM_6 = \max_{m_{2L}}\{\alpha_L^2 e_{1L} e_{2L}(V - m_{2L})\} \tag{B.324}$$

as the low-type principal's maximal profit, and

$$\overline{LM}_6 = \max_{m_{2L}}\{\alpha_H^2 e_{1L} e_{2L}(V - m_{2L})\} \tag{B.325}$$

with the agent satisfying:

$$\max_{(e_{1L}, e_{2L})} -ke_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \tag{B.326}$$

and

$$-ke_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} m_{2L} \geq 0 \tag{B.327}$$

where (B.320) is the constraint of preventing the low-type principal from mimicking the high-type one, (B.321) is the constraint of preventing the high-type principal from mimicking the low-type one, (B.323) is the agent's participation constraint for the high-type principal's offer.

Before we go to the discussion in detail, we want to find $LM_6$ and $\overline{LM}_6$.

As what we did in previous sections, it is easy to see that when $V \leq \frac{2k}{\alpha_L}$, $LM_6 = \frac{27}{32}\frac{\alpha_L^6 V^4}{128k^3}$. As to $\overline{LM}_6$, it is the maximum of the following expression

$$\alpha_H^2 \frac{\alpha_L^3 m_{2H}^2}{8k^2} \frac{\alpha_L m_{2H}}{2k}(V - m_{2H}) \tag{B.328}$$

with the constraint that

$$\frac{27}{32}\frac{\alpha_L^6 V^4}{128k^3} = \alpha_L^2 \frac{\alpha_H^3 m_{2H}^2}{8k^2} \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) \tag{B.329}$$

Thus $\overline{LM}_6 = \frac{27}{32}\frac{\alpha_L^4 \alpha_H^2 V^4}{128k^3}$.

As we discussed in previous sections, when $V \leq \frac{2k}{\alpha_L}$, $LM = \frac{27}{32}\frac{\alpha_L^6 V^4}{128k^3}$. There are three cases for consideration:

1. when $m_{2H} \leq \frac{2k}{\alpha_H}$, which means that $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k} \leq 1$, $e_{1H}^* = \frac{\alpha_H^3 m_{2H}^2}{8k^2} \leq 1$.
2. when $m_{2H} \geq \frac{2k}{\alpha_H}$, which means that $e_{2H}^* = 1$, $e_{1H}^* = \frac{\alpha_H^3 m_{2H}^2}{8k^2} \leq 1$.
3. when $m_{2H} \geq \frac{2k}{\alpha_H}$, which means that $e_{2H}^* = 1$, $\frac{\alpha_H^3 m_{2H}^2}{8k^2} \geq 1$ and $e_{1H}^* = 1$.

where $e_{2H}^*$ is the solution of the agent's profit maximization problem (B.326) when the payment is $m_{2H}$.

We will focus the discussion on the first one of the three cases, because using a similar argument to what we did for the scenario when upfront, intermediate and end payments are all included, we can show that the global optimal solution won't occur in the second case (area). We leave the proof to readers.

With $e_{2H}^* = \frac{\alpha_H m_{2H}}{2k}$ and $e_{1H}^* = \frac{\alpha_H^3 m_{2H}^2}{8k^2}$, (B.319) and (B.320) become

$$\max_{m_{2H}}\{\alpha_H^2 \frac{\alpha_H^3 m_{2H}^2}{8k^2} \frac{\alpha_H m_{2H}}{2k}(V - m_{2H})\} \tag{B.330}$$

subject to

$$\frac{27}{32}\frac{\alpha_L^6 V^4}{128k^3} \geq \alpha_L^2 \frac{\alpha_H^3 m_{2H}^2}{8k^2} \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) \tag{B.331}$$

It is clear that $m_{2H}$ as the solution to the optimization problem consisting of (B.330) and (B.331) is also the solution of (B.331) when it is binding, i.e.,

$$\frac{27}{32}\frac{\alpha_L^6 V^4}{128k^3} = \alpha_L^2 \frac{\alpha_H^3 m_{2H}^2}{8k^2} \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) \tag{B.332}$$

We can solve (B.332) for $m_{2H}$.

From (B.329), we know that $\overline{LM}_6 = \frac{27}{32}\frac{\alpha_L^4\alpha_H^2V^4}{128k^3}$. On the other hand, by (B.330) and (B.332), the high-type principal's profit equals $\frac{27}{32}\frac{\alpha_L^2\alpha_H^4V^4}{128k^3}$ which is greater than $\overline{LM}_6$. Therefore we showed that (B.321) is redundant.

Notice that the agent's profit for the high-type principal's offer is

$$k\left(\frac{\alpha_L^3 m_{2H}^2}{8k^2}\right)^2 \geq 0 \tag{B.333}$$

which means that the agent's participation constraint (B.323) is redundant.

### B.3.4 Compare the high-type principal's profits of the four cases

Next we prove the claim that for separating equilibrium, the high-type principal's profit in case 1) is greater than that in case 2), the high-type principal's profit in case 1) is greater than that in case 3), and the high-type principal's profit in case 3) is greater than that in case 4).

Recall that case 1) refers to the situation in which upfront, intermediate and end money transfers are all included, case 2) refers to the situation in which the intermediate money transfers are missed, case 3) refers to the situation in which upfront money transfers are missed, and case 4) refers to the situation in which end money transfers are missed.

First we show that the high-type principal's profit in case 1) is greater than that in case 2).

Assume that $(m_{0H}^*, m_{2H}^*)$ and $(m_{0L}^*, m_{2L}^*)$ are the solution for the high-type principal's profit maximization problem in case 2). We know that both $LM$ in case 1) and $LM_2$ in case 2) are equal to $\frac{\alpha_L^6V^4}{64k^3}$, thus $(m_{0H}^*, 0, m_{2H}^*)$ and $(m_{0L}^*, 0, m_{2L}^*)$ satisfy (B.2). It is clear that $(m_{0H}^*, 0, m_{2H}^*)$ and $(m_{0L}^*, 0, m_{2L}^*)$ also satisfy (B.5).

Now we look at (B.3). We know that $\overline{LM}$ in case 1) is the maximum of the following profit function (see (B.132)):

$$\alpha_H\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_L m_{2L}}{2k}(V - m_{2L}) - m_{1L}\right) + k\left(\frac{\alpha_L m_{1L} + \frac{\alpha_L^3 m_{2L}^2}{4k}}{2k}\right)^2 \tag{B.334}$$

and $\overline{LM}_2$ in case 2) is the maximum of the following profit function (see (B.251)):

$$\alpha_H\left(\frac{\alpha_L^3 m_{2L}^2}{8k^2}\right)\left(\alpha_H\frac{\alpha_L m_{2L}}{2k}(V - m_{2L})\right) + \frac{\alpha_L^6 m_{2L}^4}{64k^3} \tag{B.335}$$

Clearly, $LM > LM$. Thus there are two scenarios: one is that $(m_{0H}^*, 0, m_{2H}^*)$ satisfies (B.3), and the other is that $(m_{0H}^*, 0, m_{2H}^*)$ does not satisfy (B.3). When the second scenario occurs, the high-type principal's profit in case 1) must be higher than the high-type principal's profit in case 2). Now we look at the first scenario. We want to show that the high-type principal's profit in case 1) must be higher than the high-type principal's profit in case 2) for this scenario.

Notice that for case 1) there are two situations: one is that (B.2) binds but (B.5) not binds, and the other is that both (B.2) and (B.5) bind. We assume that $(\overline{m}^*_{0H}, \overline{m}^*_{1H}, \overline{m}^*_{2H})$ is the menu of money transfers to the agent when the high-type principal maximizes her profit. In the first situation which occurs when $x = \frac{\alpha_H}{\alpha_L} > 1.063971$ , we showed that

$$\overline{m}^*_{1H} = \alpha_L^2 \frac{x(2-x)(x+1)^2}{8k(x+2)^2} \tag{B.336}$$

and

$$\overline{m}^*_{2H} = \left(\frac{\alpha_H + \alpha_L}{\alpha_H + 2\alpha_L}\right) V \tag{B.337}$$

where $x = \frac{\alpha_H}{\alpha_L}$. (see (B.64) and (B.31) ) We can see that only when $x = 2$, $\overline{m}^*_{1H} = 0$ and $\overline{m}^*_{2H} = \frac{3}{4}V$. According to (B.50), we have the upfront money transfer $\overline{m}^*_{0H} = \frac{27}{32}\frac{\alpha_L^2\alpha_H^4 V^4}{128k^3} - \frac{\alpha_L^6 V^4}{64k^3}$, same as (B.265) -the upfront money transfer in case 2). This says that when $x > 1.063971$ and $x \neq 2$, the intermediate money transfer $\overline{m}^*_{1H}$ in the optimal menu from the high-type principal to the agent is not zero. Therefore, when $x > 1.063971$ and $x \neq 2$, the high-type principal's profit in case 1) is strictly higher than that in case 2).

As for the second situation, namely, $x \leq 1.063971$, both (B.2) and (B.5) bind, we no longer have analytic solution for the location of the high-type principal's profit maximization problem. We will use contrapostion argument to show that the intermediate money transfer can't be zero in case 1), therefore the high-type principal's profit in case 1) is strictly higher than that in case 2).

First we need to show that the high-type principal's profit maximization problem (B.1) with both (B.2) and (B.5) being binding has a solution. In other words, there exist $m_{1H}$ and $m_{2H}$ that satisfy both (B.2) and (B.5) which are binding, and (B.3). It is clear that with (B.5) being binding, the binding constraint (B.2) can be written as

$$\frac{\alpha_L^6 V^4}{64k^3} = k\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2 - \alpha_L\left(\frac{\alpha_H m_{1H} + \frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_L \frac{\alpha_H m_{2H}}{2k}(V - m_{2H}) - m_{1H}\right) \tag{B.338}$$

(see the binding constraint in (B.68)). On the other hand, constraint (B.3) can be written as

$$\alpha_H\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)\left(\alpha_H\frac{\alpha_H m_{2H}}{2k}(V-m_{2H})-m_{1H}\right)+k\left(\frac{\alpha_H m_{1H}+\frac{\alpha_H^3 m_{2H}^2}{4k}}{2k}\right)^2$$
$$\geq \frac{\alpha_L^6 V^4}{64k^3(2x-1)^3} \tag{B.339}$$

(see (B.146) and (B.147) ) where $LM=\frac{\alpha_L^6V^4}{64k^3}$ and $\overline{LM}=\frac{\alpha_L^6V^4}{64k^3(2x-1)^3}$.

Setting $m_{1H}=0$, then (B.338) gives

$$\frac{\alpha_L^6V^4}{64k^3}=\frac{\alpha_H^6 m_{2H}^4}{64k^3}+\frac{\alpha_L^2\alpha_H^4 m_{2H}^3}{8k^2}(V-m_{2H}) \tag{B.340}$$

and (B.339) becomes

$$\frac{m_{2H}^3(V-m_{2H})}{8k^2}+\frac{m_{2H}^4}{64k^3}\geq\frac{V^4}{64k^3(2x-1)^3} \tag{B.341}$$

using the fact that $x=\frac{\alpha_H}{\alpha_L}$. From (B.340), we have

$$\frac{m_{2H}^3(V-m_{2H})}{8k^2}=\frac{1}{\alpha_L^2\alpha_H^4}\left(\frac{\alpha_L^6V^4}{64k^3}-\frac{\alpha_H^6 m_{2H}^4}{64k^3}\right) \tag{B.342}$$

Substituting (B.342) into (B.341) and setting $m_{2H}=\frac{1}{2}V$ gives

$$V^4\left(1-\frac{x^6}{16}+\frac{x^4}{16}-\frac{1}{(2x-1)^3}\right)\geq 0 \tag{B.343}$$

which is equivalent to

$$\left(16-x^6+x^4\right)(2x-1)^3-16\geq 0 \tag{B.344}$$

because $V>0$ and $x\geq 1$.

Let $F_2(x)$ be the right side of the inequality of (B.344). Using Maple we find that $x=1$ and $x\approx 1.684523$ are the only two real roots that are greater than or equal to 1. Since $F_2(x)$ goes to positive infinity as $x$ increases to positive infinity, so $F_2(x)>0$ when $1<x<1.684523$. Thus we showed that $m_{1H}=0$ and $m_{2H}=\frac{1}{2}V$ satisfies both (B.338) and (B.339) when $1\leq x\leq 1.063971$. This means that the high-type principal's profit maximization problem (B.1) with both (B.2) and (B.5) being binding has a solution.

Next we will show that when $1\leq x\leq 1.063971$, the intermediate money transfer $m_{1H}\neq 0$ in the solution of the high-type principal's profit maximization problem. Notice that we already showed that (B.3) is not all binding.

Setting $m_{1H} = 0$, (B.75) can be simplified as

$$\alpha_H^4 m_{2H}(V - m_{2H}) = \lambda_1 \left(\alpha_H^2 \alpha_L^2 (V - m_{2H}) + \left(\alpha_H^4 - \alpha_L \alpha_h^3\right)\right) m_{2H} \tag{B.345}$$

Since $m_{2H}$ cannot be 0, dividing both sides of (B.345) by $m_{2H}$ gives

$$\alpha_H^4 (V - m_{2H}) = \lambda_1 \left(\alpha_H^2 \alpha_L^2 (V - m_{2H}) + \left(\alpha_H^4 - \alpha_L \alpha_H^3\right) m_{2H}\right) \tag{B.346}$$

On the other hand, with $m_{1H} = 0$, (B.78) can be simplified as

$$3\alpha_H^6 m_{2H}^2 (V - m_{2H}) = \lambda_1 \left(-\alpha_H^6 m_{2H} - \alpha_L^2 \alpha_H^4 (3V - 4m_{2H})\right) m_{2H}^2 \tag{B.347}$$

Since $m_{2H}$ cannot be 0, dividing both sides of (B.3.4) $m_{2H}^2$ gives

$$3\alpha_H^6 (V - m_{2H}) = \lambda_1 \left(-\alpha_H^6 m_{2H} - \alpha_L^2 \alpha_H^4 (3V - 4m_{2H})\right) \tag{B.348}$$

Substituting (B.29) into the right side of gives

$$3\alpha_H^2 \lambda_1 \left(\alpha_H^2 \alpha_L^2 (V - m_{2H}) + \left(\alpha_H^4 - \alpha_L \alpha_H^3\right) m_{2H}\right) = \lambda_1 \left(-\alpha_H^6 m_{2H} - \alpha_L^2 \alpha_H^4 (3V - 4m_{2H})\right) \tag{B.349}$$

Since $\lambda_1 > 0$ which means that (B.2) and (B.5) are binding, solving (B.349) for $m_{2H}$ gives $m_{2H} = 0$ which cannot happen, because this would lead to zero effort for the second period. Therefore, this forms a contradiction. This says that $m_{1H} \neq 0$ for the second scenario.

In conclusion, we showed that for separating equilibrium, the high-type principal's profit in case 1) is strictly higher than that in case 2) for all $x \neq 2$, and the two are equal when $x = 2$.

Next we show that the high-type principal's profit in case 1) is strictly greater than that in case 3).

Assume $(m_{0H}, m_{1H}, m_{2H})$ be the location of the high-type principal's profit maximization problem in case 1). We know that $m_{0H} \neq 0$when $x \neq 1.335236$, and $m_{0H} = 0$ when $x = 1.335236$. Thus, we can see that when $x \neq 1.335236$, the high-type principal's profit in case 1) is greater than that in case 3).

As to the case when $x = 1.335236$. Since $1.335236 > 1.063971$, $m_{2H} = \frac{x+1}{2x+1}V$. We claim that $m_{2H} = \frac{x+1}{2x+1}V$ is not part of the location of the high-type principal's maximal profit in case 3). We use contraposition argument. Suppose $m_{2H} = \frac{x+1}{2x+1}V$ satisfies (B.313), namely

$$\begin{aligned}\frac{\alpha_L^6 V^4}{128k^3} &= \frac{\alpha_L \alpha_H}{2k}\left[\frac{\alpha_H^2 m_{2H}^2 V}{8k(-V + 2m_{2H})}\right]\left[\frac{4\alpha_L \alpha_H m_{2H}(V - m_{2H})(-V + 2m_{2H})}{8k(-V + 2m_{2H})}\right] \\ &\quad + \frac{\alpha_L \alpha_H}{2k}\left[\frac{\alpha_H^2 m_{2H}^2 V}{8k(-V + 2m_{2H})}\right]\left[\frac{-3\alpha_H^2 m_{2H}^2 V + 4\alpha_H^2 m_{2H}^3}{8k(-V + 2m_{2H})}\right]\end{aligned} \tag{B.350}$$

Letting $y = m_{2H}/V = \frac{x+1}{2x+1}$, then $y$ satisfies

$$1 = \frac{x^3y^2\left[4xy(1-y)(2y-1) - 3x^2y^2 + 4x^2y^3\right]}{(-1+2y)^2} \tag{B.351}$$

However, Using Maple to either directly calculate the difference between the right side and the left side of (B.351) or the real roots of a polynomial which is equivalent to the (B.351), we find that the right side of (B.351) is always strictly greater than its left side when $y > 0$. This means that $m_{2H} = \frac{x+1}{2x+1}V$ does not satisfy (B.313), and thereby can not be part of the location of the high-type principal's maximal profit in case 3). This shows $x = 1.335236$, the high-type principal's profit in case 1) is greater than that in case 3).

Next we show that the high-type principal's profit in case 3) is strictly greater than that in case 4).

Let $m_{2H}$ be the money transfer for the location of the high-type principal's profit in case 4). According to (B.332), $m_{2H}$ satisfies

$$\frac{27}{32}\frac{\alpha_L^6V^4}{128k^3} = \alpha_L^2\frac{\alpha_H^3m_{2H}^2}{8k^2}\frac{\alpha_Hm_{2H}}{2k}(V - m_{2H}) \tag{B.352}$$

Since $LM_4 = \frac{\alpha_L^6V^4}{128k^3}$, $LM_4 > \frac{27}{32}\frac{\alpha_L^6V^4}{128k^3}$. Let $\overline{m}_{2H} = m_{2H}$ , we have

$$LM_4 > \frac{27}{32}\frac{\alpha_L^6V^4}{128k^3} = \alpha_L^2\frac{\alpha_H^3\overline{m}_{2H}^2}{8k^2}\frac{\alpha_H\overline{m}_{2H}}{2k}(V - \overline{m}_{2H}) \tag{B.353}$$

Let $\overline{m}_{1H} = 0$, then $(m_{1H}, \overline{m}_{2H})$ satisfies (B.281) with strictly inequality holds. We also know that the agent's profit for the high-type principal's money transfers is strictly positive when $\overline{m}_{2H} > 0$ (see (B.333). Thus we can increase $\overline{m}_{2H}$ a little bit such that (B.281) still holds. If $(m_{1H}, \overline{m}_{2H})$ satisfies (B.282), then this shows that the high-type principal's profit in case 3) is greater than that in case 4). If $(m_{1H}, \overline{m}_{2H})$ does not satisfy (B.282), this clearly shows that the high-type principal's profit in case 3) is greater than that in case 4), since the left side of (B.282) equals the high-type principal's profit in case 3).

## B.4 Proof of Theorem 13

We need to examine the pooling equilibrium of case 2, case 3 and case 4, and then compare the high-type principal's profits in case 1 (studied in Theorem 11), case 2, case 3 and case 4.

### B.4.1 Pooling equilibrium when upfront and end money transfers are all included in the payment menu

We study the pooling equilibrium when intermediate money transfers are excluded from the menu. We assume that the agent believes that the high-type principal appears with probability $p$ while the low-type principal appears with probability $1-p$. For convenience, we denote $p\alpha_H + (1-p)\alpha_L$ by $\overline{\alpha}$ and $\frac{p\alpha_H^2}{p\alpha_H+(1-p)\alpha_L} + \frac{(1-p)\alpha_L^2}{p\alpha_H+(1-p)\alpha_L}$ by $\tilde{\alpha}$. Notice that $\overline{\alpha}\tilde{\alpha} = p\alpha_H^2 + (1-p)\alpha_L^2$.

In the pooling equilibrium, both high-type and low-type principals offer the same payment menu $(m_0, m_2)$. The high-type principal's profit satisfies

$$\max_{(m_0,m_2)} \{-m_0 + \alpha_H^2 e_1 e_2 (V - m_2)\} \tag{B.354}$$

such that the agent's profit satisfies

$$\max_{(e_1,e_2)} m_0 - ke_1^2 - \overline{\alpha} e_1 k e_2^2 + \overline{\alpha}\tilde{\alpha} e_1 e_2 m_2 \tag{B.355}$$

and

$$m_0 - ke_1^2 - \overline{\alpha} e_1 k e_2^2 + \overline{\alpha}\tilde{\alpha} e_1 e_2 m_2 \geq 0 \tag{B.356}$$

There are two other constraints that have to be satisfied:

$$-m_0 + \alpha_L^2 e_1 e_2 (V - m_2) \geq LM_3 \tag{B.357}$$

and

$$-m_0 + \alpha_H^2 e_1 e_2 (V - m_2) \geq \overline{LM}_3 \tag{B.358}$$

where

$$LM_3 = \max_{(m_{0L},m_{2L})} \{-m_{0L} + \alpha_L^2 e_{1L} e_{2L} (V - m_{2L})\} \tag{B.359}$$

and

$$\overline{LM}_3 = \max_{(m_{0L},m_{2L})} \{-m_{0L} + \alpha_H^2 e_{1L} e_{2L} (V - m_{2L})\} \tag{B.360}$$

with the agent satisfying:

$$\max_{(e_{1L},e_{2L})} m_{0L} - ke_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tag{B.361}$$

and

$$m_{0L} - ke_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \geq 0 \tag{B.362}$$

where (B.356) is the agent's participation constraint, (B.357) is the constraint of preventing low-type principal's deviation and (B.358) is the constraint of preventing the high-type principal's deviation.

The proof consists of two parts. First we look at the high-type principal's profit maximization problem when the agent earns zero profit and the low-type principal does not want to deviate, i.e., (B.356) binds, but (B.357) does not bind. In this scenario, we show that the high-type principal does not want to deviate, i.e., (B.165) is redundant. Then we look at the high-type principal's profit maximization problem when the agent earns zero profit and the low-type principal wants to deviate, i.e., both (B.356) and (B.357) bind. In this scenario, there exist two subscenarios, one in which the high-type principal does not want to deviate, i.e., (B.358) does not bind, and the other in which the high-type principal wants to deviate, i.e., (B.358) binds.

Before we go to discussion in detail, we calculate $LM_3$ and $\overline{LM}_3$. It is clear that $LM_3 = \frac{\alpha_L^6 V^4}{64k^3}$. As to $\overline{LM}_3$, it is the maximum of the following expression:

$$\alpha_H \left( \frac{\alpha_L^3 \tilde{m}_{2L}^2}{8k^2} \right) \left( \alpha_H \frac{\alpha_L \tilde{m}_{2L}}{2k} (V - \tilde{m}_{2L}) \right) + k \left( \frac{\alpha_L^3 \tilde{m}_{2L}^2}{8k^2} \right)^2 \tag{B.363}$$

It is clear that $\overline{LM}_3$ equals $\overline{LM}_2$, which has the expression

$$\alpha_L^6 \frac{108y^4(y-1) + 81y^4}{64k^3(4y-1)^4} V^4 \tag{B.364}$$

(see (B.254)).

For the same reason we mentioned in previous section, the effort levels in two periods take the forms $e_2^* = \frac{\tilde{\alpha} m_2}{2k} \leq 1$ and $e_1^* = \frac{\overline{\alpha} \tilde{\alpha}^2 m_2^2}{8k^2} \leq 1$.

Next we look at the first scenario in which the agent earns zero profit and the low-type principal does not want to deviate, i.e., (B.356) binds, but (B.357) does not bind.

#### B.4.1.1 The first scenario

The Lagrangian of the high-type principal's profit is

$$\alpha_H \left( \frac{\overline{\alpha} \tilde{\alpha}^2 m_2^2}{8k^2} \right) \left( \alpha_H \frac{\tilde{\alpha} m_2}{2k} (V - m_2) \right) - m_0 + \lambda \left[ m_0 + k \left( \frac{\overline{\alpha} \tilde{\alpha}^2 m_2^2}{8k^2} \right)^2 \right] \tag{B.365}$$

Notice that the boundary conditions for $\frac{\tilde{\alpha} m_2}{2k} \leq 1$ and $\frac{\frac{\overline{\alpha} \tilde{\alpha}^2 m_2^2}{4k}}{2k} \leq 1$ are not included in (B.365). We will show that when $V \leq \frac{2k}{\alpha_H}$, the location of the local maximum of the high-type principal's profit won't occur at the boundaries.

The first order condition of (B.365) with respect to $m_0$ gives

$$-1 + \lambda = 0 \tag{B.366}$$

This means that $\lambda = 1$ and $m_0 = -k \left( \frac{\overline{\alpha} \tilde{\alpha}^2 m_2^2}{8k^2} \right)^2$.

Therefore the high-type principal's profit becomes

$$\alpha_H\left(\frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2}\right)\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)\right)+k\left(\frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2}\right)^2 \tag{B.367}$$

The first order condition of (B.367) with respect to $m_2$ gives

$$\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2 m_2}{4k^2}\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)\right)+\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2}\left(\frac{\alpha_H\tilde{\alpha}}{2k}(V-2m_2)\right)+2k\left(\frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2}\right)\frac{\overline{\alpha}\tilde{\alpha}^2}{4k^2}m_2=0 \tag{B.368}$$

Multiplying both sides of (B.368) by $16k^3$ gives

$$2\alpha_H^2\overline{\alpha}\tilde{\alpha}^3 m_2^2(V-m_2)+\alpha_H^2\overline{\alpha}\tilde{\alpha}^3 m_2^2(V-2m_2)+\overline{\alpha}^2\tilde{\alpha}^4 m_2^3=0 \tag{B.369}$$

namely,

$$\overline{\alpha}\tilde{\alpha}^3 m_2^2\left(\alpha_H^2(3V-4m_2)+\alpha\tilde{\alpha}m_2\right)=0 \tag{B.370}$$

There are three roots for equation (B.370). Two of them are 0 and can be ruled out, because they lead to $e_1=0$ and $e_2=0$. The third one is

$$m_2=\frac{3\alpha_H^2 V}{4\alpha_H^2-\overline{\alpha}\tilde{\alpha}} \tag{B.371}$$

This means that

$$m_0=-\frac{\overline{\alpha}^2\tilde{\alpha}^4 m_2^4}{64k^3} \tag{B.372}$$

Next we will show that (B.358) is redundant.

Recall that $\overline{LM}_3$ is the maximum of the following expression (replace $\tilde{m}_{2L}$ in (B.363) with $m_2$ )

$$\alpha_H\left(\frac{\alpha_L^3 m_2^2}{8k^2}\right)\left(\alpha_H\frac{\alpha_L m_2}{2k}(V-m_2)\right)+k\left(\frac{\alpha_L^3 m_2^2}{8k^2}\right)^2 \tag{B.373}$$

which has $m_2=\frac{3\alpha_H^2 V}{4\alpha_H^2-\alpha_L^2}$ as the location of the maximum. This means that $V-m_2>0$ at the location of the maximum.

On the other hand, from (B.367), we know that the high-type principal's profit is the maximum of

$$\alpha_H\left(\frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2}\right)\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)\right)+k\left(\frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2}\right)^2 \tag{B.374}$$

which has $m_2=\frac{3\alpha_H^2 V}{4\alpha_H^2-\alpha\tilde{\alpha}}$ as the location of the maximum. This means that $V-m_2>0$ at the location of the maximum.

Clearly, (B.374) is greater than or equal to (B.373) when $V-m_2>0$, because $\tilde{\alpha}>\overline{\alpha}>\alpha_L$. This means that the high-type principal's profit is greater than or equal to $\overline{LM}_3$. In other words, (B.358) is redundant.

Next we look at the high-type principal's profit maximization problem when the agent earns zero profit and the low-type principal wants to deviate, i.e., both (B.356) and (B.357) bind.

#### B.4.1.2 The second scenario

In this scenario, there exist two subscenarios, one in which the high-type principal does not want to deviate, i.e., (B.358) does not bind, and the other in which the high-type principal wants to deviate, i.e., (B.358) binds.

The Lagrangian for the high-type principal's profit is

$$\begin{aligned} & \alpha_H \left( \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2} \right) \left( \alpha_H \frac{\tilde{\alpha} m_2}{2k} (V - m_{2H}) \right) + \frac{\overline{\alpha}^2 \tilde{\alpha}^4 m_2^4}{64k^4} \\ + & \lambda_1 \left[ \frac{\overline{\alpha}^2 \tilde{\alpha}^4 m_2^4}{64k^4} + \alpha_L \left( \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2} \right) \left( \alpha_L \frac{\tilde{\alpha} m_2}{2k} (V - m_2) \right) - \frac{\alpha_L^6 V^4}{64k^3} \right] \end{aligned} \tag{B.375}$$

Assuming $m_2$ is the location for the maximum of high-type principal's profit, it satisfies

$$\alpha_L \left( \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2} \right) \left( \frac{\alpha_L \tilde{\alpha}}{2k} (V - 2m_2) \right) = \frac{\overline{\alpha}^2 \tilde{\alpha}^4 m_2^3}{16k^4} \tag{B.376}$$

Numerical result shows that when (B.376) holds, (B.358) does not hold, namely

$$\alpha_H \left( \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{8k^2} \right) \left( \frac{\alpha_H \tilde{\alpha}}{2k} (V - 2m_{2H}) \right) + \frac{\overline{\alpha}^2 \tilde{\alpha}^4 m_2^3}{16k^4} < \overline{LM}_3 \tag{B.377}$$

where $\overline{LM}_3$ has the following expression

$$\alpha_L^6 \frac{108y^4(y-1) + 81y^4}{64k^3(4y-1)^4} V^4 \tag{B.378}$$

In other words, numerical results suggest that there are no pooling equilibria when both (B.356) and (B.357) bind.

### B.4.2 Pooling equilibrium when intermediate and end money transfers are all included in the payment menu

Next we look at the pooling equilibrium when the upfront money transfers are excluded from the menu. We assume that the agent believes that the high-type principal appears with probability $p$ while the low-type principal appears with probability $1-p$. For convenience, we denote $p\alpha_H + (1-p)\alpha_L$ by $\overline{\alpha}$ and $\frac{p\alpha_H^2}{p\alpha_H+(1-p)\alpha_L} + \frac{(1-p)\alpha_L^2}{p\alpha_H+(1-p)\alpha_L}$ by $\tilde{\alpha}$. Notice that $\overline{\alpha}\tilde{\alpha} = p\alpha_H^2 + (1-p)\alpha_L^2$.

In the pooling equilibrium, both high-type and low-type principals offer the same menu $(m_1, m_2)$. The high-type principal's profit satisfies

$$\max_{(m_1,m_2)} \{-\alpha_H e_1 m_1 + \alpha_H^2 e_1 e_2 (V - m_2)\} \tag{B.379}$$

such that the agent's profit satisfies

$$\max_{(e_1,e_2)} -ke_1^2 + \overline{\alpha} e_1 m_1 - \overline{\alpha} e_1 k e_2^2 + \overline{\alpha}\tilde{\alpha} e_1 e_2 m_2 \tag{B.380}$$

and

$$-ke_1^2 + \overline{\alpha} e_1 m_1 - \overline{\alpha} e_1 k e_2^2 + \overline{\alpha}\tilde{\alpha} e_1 e_2 m_2 \geq 0 \tag{B.381}$$

There are two other constraints that have to be satisfied:

$$-\alpha_L e_1 m_1 + \alpha_L^2 e_1 e_2 (V - m_2) \geq LM_5 \tag{B.382}$$

and

$$-\alpha_H e_1 m_1 + \alpha_H^2 e_1 e_2 (V - m_2) \geq \overline{LM}_5 \tag{B.383}$$

where

$$LM_5 = \max_{(m_{1L},m_{2L})} \{-\alpha_L e_{1L} m_{1L} + \alpha_L^2 e_{1L} e_{2L} (V - m_{2L})\} \tag{B.384}$$

as the low-type principal's maximal profit, and

$$\overline{LM}_5 = \max_{(\tilde{m}_{1L},\tilde{m}_{2L})} \{-\alpha_H e_{1L} \tilde{m}_{1L} + \alpha_H^2 e_{1L} e_{2L} (V - \tilde{m}_{2L})\} \tag{B.385}$$

with the agent satisfying:

$$\max_{(e_{1L},e_{2L})} -ke_{1L}^2 + \alpha_L e_{1L} \tilde{m}_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \tag{B.386}$$

and

$$-ke_{1L}^2 + \alpha_L e_{1L} \tilde{m}_{1L} - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tilde{m}_{2L} \geq 0 \tag{B.387}$$

where (B.381) is the agent's participation constraint, (B.382) is the constraint of preventing low-type principal's deviation and (B.383) is the constraint of preventing the high-type principal's deviation.

The proof consists of two parts. First we look at the high-type principal's profit maximization problem when the low-type principal wants to deviate, i.e., (B.382) binds, but the high-type principal does not want to deviate, i.e., (B.383) does not bind. Then we look at the high-type principal's profit maximization problem when the low-type principal wants to deviate, i.e., (B.382) binds, and the high-type principal wants to deviate, i.e., (B.383) binds. We will point out that the gent earn positive profit, i.e., (B.381) is redundant.

It is clear that $LM_5$ in (B.384) and $\overline{LM}_5$ in (B.385) equal $LM_4$ in (B.285) and $\overline{LM}_4$ in (B.286).

As we discussed in the screening model, when $V \leq \frac{2k}{\tilde{\alpha}}$, there are four cases for consideration:

1. when $m_2 \leq \frac{2k}{\tilde{\alpha}}$ and $\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \leq 1$, i.e., $e_2^* \leq 1$ and $e_1^* \leq 1$.
2. when $m_2 \leq \frac{2k}{\tilde{\alpha}}$ and $\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \geq 1$, i.e., $e_2^* \leq 1$ and $e_1^* = 1$.
3. when $m_2 \geq \frac{2k}{\tilde{\alpha}}$ and $\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \leq 1$, i.e., $e_2^* = 1$ and $e_1^* \leq 1$.
4. when $m_2 \geq \frac{2k}{\tilde{\alpha}}$ and $\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k} \geq 1$ , i.e., $e_2^* = 1$ and $e_1^* = 1$.

where $e_1^*$ and $e_2^*$ are the solution of the agent's profit maximization problem (B.380), when the payments are $(m_1, m_2)$.

We will focus the discussion on the first one of the four cases in which the effort levels in two periods take the forms $e_2^* = \frac{\tilde{\alpha}m_2}{2k}$ and $e_1^* = \frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}$, because using a similar argument to what we did for the scenario when upfront, intermediate and end payments are all included, we can show that the global optimal solution won't occur in three other cases (areas). We leave the proof to readers.

Since $e_2^* = \frac{\tilde{\alpha}m_2}{2k}$ and $e_1^* = \frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}$, the agent's participation constraint (B.381) when the high-type principal offers a contract can be written as

$$k\left[\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}\right]^2 \geq 0 \tag{B.388}$$

Thus (B.381) holds true, namely redundant. This means that the high-type principal's profit maximization problem consisting of (B.379) through (B.382) is equivalent to the one consisting of (B.379), (B.380), (B.383), and (B.382).

Next we look at the first scenario in which the low-type principal wants to deviate, i.e., (B.382) binds, but the high-type principal does not want to deviate, i.e., (B.383) does not bind.

### B.4.2.1 The first scenario

The Lagrangian for the high-type principal's profit is

$$\begin{aligned} & \alpha_H\left(\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}\right)\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V - m_2) - m_1\right) \\ + & \lambda\left[\alpha_L\left(\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2 m_2^2}{4k}}{2k}\right)\left(\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V - m_2) - m_1\right) - \frac{\alpha_L^6 V^4}{128k^3}\right] \end{aligned} \tag{B.389}$$

The first order condition of (B.389) with respect to $m_1$ is

$$\frac{\alpha_H\overline{\alpha}}{2k}\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)-\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)$$

$$+\quad\lambda\left[\frac{\alpha_L\overline{\alpha}}{2k}\left(\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)-\alpha_L\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\right]=0 \qquad \text{(B.390)}$$

which can be written as

$$\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}+\lambda\alpha_L^2\overline{\alpha}\tilde{\alpha}}{4k^2}m_2(V-m_2)-\frac{\alpha_H\overline{\alpha}+\lambda\alpha_L\overline{\alpha}}{k}m_1-\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2+\lambda\alpha_L\overline{\alpha}\tilde{\alpha}^2}{8k^2}m_2^2=0 \qquad \text{(B.391)}$$

namely,

$$\begin{aligned} m_1 &= \frac{1}{\alpha_H\overline{\alpha}+\lambda\alpha_L\overline{\alpha}}\left[\frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}+\lambda\alpha_L^2\overline{\alpha}\tilde{\alpha}}{4k^2}m_2(V-m_2)-\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2+\lambda\alpha_L\overline{\alpha}\tilde{\alpha}^2}{8k^2}m_2^2\right] \\ &= \frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}+\lambda\alpha_L^2\overline{\alpha}\tilde{\alpha}}{4k(\alpha_H\overline{\alpha}+\lambda\alpha_L\overline{\alpha})}m_2(V-m_2)-\frac{\tilde{\alpha}^2}{8k}m_2^2 \end{aligned} \qquad \text{(B.392)}$$

The first order condition of (B.389) with respect to $m_2$ is

$$\frac{\alpha_H\overline{\alpha}\tilde{\alpha}^2m_2}{4k^2}\left(\alpha_H\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)+\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\frac{\alpha_H\tilde{\alpha}}{2k}(V-2m_2)\right)$$

$$+\quad\lambda\left[\frac{\alpha_L\overline{\alpha}\tilde{\alpha}^2m_2}{4k^2}\left(\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V-m_2)-m_1\right)+\alpha_L\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\frac{\alpha_L\tilde{\alpha}}{2k}(V-2m_2)\right)\right]=0 \qquad \text{(B.393)}$$

Multiplying (B.391) by $\frac{\tilde{\alpha}^2m_2}{2k}$ and subtracting the product from (B.393) gives

$$\begin{aligned} &\alpha_H\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\frac{\alpha_H\tilde{\alpha}}{2k}\left(V-2m_2+\frac{\tilde{\alpha}}{\alpha_H}m_2\right)\right) \\ &= -\lambda_1\alpha_L\left(\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\frac{\alpha_L\tilde{\alpha}}{2k}\left(V-2m_2+\frac{\tilde{\alpha}}{\alpha_L}m_2\right)\right) \end{aligned} \qquad \text{(B.394)}$$

Since $\frac{\overline{\alpha}m_1+\frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}$ as the effort in the first period is positive, (B.394) implies that

$$\lambda_1=-\frac{-\frac{\alpha_H^2}{\alpha_L^2}\left(V-2m_2+\frac{\tilde{\alpha}}{\alpha_H}m_2\right)}{\left[V-\left(2-\frac{\tilde{\alpha}}{\alpha_L}\right)m_2\right]} \qquad \text{(B.395)}$$

Thus

$$\begin{aligned}
&\alpha_H^2\overline{\alpha}\tilde{\alpha} + \lambda\alpha_L^2\overline{\alpha}\tilde{\alpha} \\
&= \frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}\left[V - \left(2 - \frac{\tilde{\alpha}}{\alpha_L}\right)m_2\right] - \alpha_H^2\overline{\alpha}\tilde{\alpha}\left(V - 2m_2 + \frac{\tilde{\alpha}}{\alpha_H}m_2\right)}{\left[V - \left(2 - \frac{\tilde{\alpha}}{\alpha_L}\right)m_2\right]} \\
&= \frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}\left(\frac{\tilde{\alpha}}{\alpha_L} - \frac{\tilde{\alpha}}{\alpha_H}\right)}{\left[V - \left(2 - \frac{\tilde{\alpha}}{\alpha_L}\right)m_2\right]} \qquad \text{(B.396)}
\end{aligned}$$

and

$$\begin{aligned}
\alpha_H\overline{\alpha} + \lambda\alpha_L\overline{\alpha} &= \frac{\alpha_H\overline{\alpha}\left[V - \left(2 - \frac{\tilde{\alpha}}{\alpha_L}\right)m_2\right] - \frac{\alpha_H^2\overline{\alpha}}{\alpha_L}\left(V - 2m_2 + \frac{\tilde{\alpha}}{\alpha_H}m_2\right)}{\left[V - \left(2 - \frac{\tilde{\alpha}}{\alpha_L}\right)m_2\right]} \\
&= \frac{\left(\alpha_H\overline{\alpha} - \frac{\alpha_H^2\overline{\alpha}}{\alpha_L}\right)V - 2\left(\alpha_H\overline{\alpha} - \frac{\alpha_H^2\overline{\alpha}}{\alpha_L}\right)m_2}{\left[V - \left(2 - \frac{\tilde{\alpha}}{\alpha_L}\right)m_2\right]} \qquad \text{(B.397)}
\end{aligned}$$

Thus according to (B.392)

$$\begin{aligned}
m_1 &= \frac{\alpha_H^2\overline{\alpha}\tilde{\alpha}\left(\frac{\tilde{\alpha}}{\alpha_L} - \frac{\tilde{\alpha}}{\alpha_H}\right)}{4k\left[\left(\alpha_H\overline{\alpha} - \frac{\alpha_H^2\overline{\alpha}}{\alpha_L}\right)V - 2\left(\alpha_H\overline{\alpha} - \frac{\alpha_H^2\overline{\alpha}}{\alpha_L}\right)m_2\right]}m_2^2(V - m_2) - \frac{\tilde{\alpha}^2}{8k^2}m_2^2 \\
&= \frac{\tilde{\alpha}^2}{4k(-V + 2m_2)}m_2^2(V - m_2) - \frac{\tilde{\alpha}^2}{8k^2}m_2^2 \\
&= \frac{3\tilde{\alpha}^2m_2^2V - 4\tilde{\alpha}^2m_2^3}{8k(-V + 2m_2)} \qquad \text{(B.398)}
\end{aligned}$$

which implies that

$$\begin{aligned}
m_1 + \frac{\tilde{\alpha}^2}{4k}m_2^2 &= \frac{3\tilde{\alpha}^2m_2^2V - 4\tilde{\alpha}^2m_2^3}{8k(-V + 2m_2)} + \frac{\tilde{\alpha}^2}{4k}m_2^2 \\
&= \frac{3\tilde{\alpha}^2m_2^2V - 4\tilde{\alpha}^2m_2^3 + 2\tilde{\alpha}^2m_2^2(-V + 2m_2)}{8k(-V + 2m_2)} \\
&= \frac{\tilde{\alpha}^2m_2^2V}{8k(-V + 2m_2)} \qquad \text{(B.399)}
\end{aligned}$$

and

$$\begin{aligned}
\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V - m_2) - m_1 &= \alpha_L\frac{\tilde{\alpha}m_2}{2k}(V - m_2) - \frac{3\tilde{\alpha}^2m_2^2V - 4\tilde{\alpha}^2m_2^3}{8k(-V + 2m_2)} \\
&= \frac{\frac{4\alpha_L\tilde{\alpha}m_2}{2k}(V - m_2)(V - 2m_2) - 3\tilde{\alpha}^2m_2^2V + 4\tilde{\alpha}^2m_2^3}{8k(-V + 2m_2)} \qquad \text{(B.400)}
\end{aligned}$$

From the binding condition that appears in (B.389), we know

$$\frac{\alpha_L^6V^4}{128k^3} = \alpha_L\left(\frac{\overline{\alpha}m_1 + \frac{\overline{\alpha}\tilde{\alpha}^2m_2^2}{4k}}{2k}\right)\left(\alpha_L\frac{\tilde{\alpha}m_2}{2k}(V - m_2) - m_1\right) \qquad \text{(B.401)}$$

Substituting (B.399) and (B.400) into (B.401) gives

$$\frac{\alpha_L^6 V^4}{128k^3} = \frac{\alpha_L\tilde{\alpha}}{2k}\left[\frac{\tilde{\alpha}^2 m_2^2 V}{8k(-V+2m_2)}\right]\left[\frac{4\alpha_L\tilde{\alpha}m_2(V-m_2)(V-2m_2)-3\tilde{\alpha}^2m_2^2V+4\tilde{\alpha}^2m_2^3}{8k(-V+2m_2)}\right] \tag{B.402}$$

Since (B.402) has only one variable - $m_2$, we can solve it for $m_2$.

Notice that according to (B.389), the high-type principal's profit equals

$$\frac{\alpha_H\tilde{\alpha}}{2k}\left[\frac{\tilde{\alpha}^2 m_2^2 V}{8k(-V+2m_2)}\right]\left[\frac{4\alpha_H\tilde{\alpha}m_2(V-m_2)(V-2m_2)-3\tilde{\alpha}^2m_2^2V+4\tilde{\alpha}^2m_2^3}{8k(-V+2m_2)}\right] \tag{B.403}$$

Next we look at the second scenario when the low-type principal wants to deviate, i.e., (B.382) binds, and the high-type principal wants to deviate, i.e., (B.383) binds.

#### B.4.2.2 The second scenario

Our numerical calculation shows that this does not happen. In other words, when (B.402) holds, the high-type principal's profit in (B.403) is always bigger than $\overline{LM}_5$, which equals

$$\frac{\alpha_H^5\alpha_L^3}{128k^3(2\alpha_H-\alpha_L)^2} \tag{B.404}$$

namely, the expression of $\overline{LM}_4$ in (B.296).

Similar to what we did before, with $x=\frac{\alpha_H}{\alpha_L}$, we can prove that the maximal profit of the high-type principal can be written as $\alpha_L^6 g(x,k,V)$, where $g$ is a function of $x$, $k$ and $V$.

### B.4.3 Pooling equilibrium when only end money transfers are included in the payment menu

Next we look at the pooling equilibrium when only end money transfers are included in the menu. We assume that the agent believes that the high-type principal appears with probability $p$ while the low-type principal appears with probability $1-p$. For convenience, we denote $p\alpha_H+(1-p)\alpha_L$ by $\overline{\alpha}$ and $\frac{p\alpha_H^2}{p\alpha_H+(1-p)\alpha_L}+\frac{(1-p)\alpha_L^2}{p\alpha_H+(1-p)\alpha_L}$ by $\tilde{\alpha}$. Notice that $\overline{\alpha}\tilde{\alpha}=p\alpha_H^2+(1-p)\alpha_L^2$.

In the pooling equilibrium, both high-type and low-type principals offer the same payment menu $m_2$. The high-type principal's profit satisfies

$$\max_{m_2}\{\alpha_H^2 e_1 e_2(V-m_2)\} \tag{B.405}$$

such that the agent's profit satisfies

$$\max_{(e_1,e_2)} -ke_1^2-\overline{\alpha}e_1ke_2^2+\overline{\alpha}\tilde{\alpha}e_1e_2m_2 \tag{B.406}$$

and

$$-ke_1^2-\overline{\alpha}e_1ke_2^2+\overline{\alpha}\tilde{\alpha}e_1e_2m_2\geq 0 \tag{B.407}$$

There are two other constraints that have to be satisfied:

$$\alpha_L^2 e_1 e_2 (V - m_2) \geq LM_7 \tag{B.408}$$

and

$$\alpha_H^2 e_1 e_2 (V - m_2) \geq \overline{LM}_7 \tag{B.409}$$

where

$$LM_7 = \max_{m_{2L}} \{\alpha_L^2 e_{1L} e_{2L} (V - m_{2L})\} \tag{B.410}$$

as the low-type principal's maximal profit, and

$$\overline{LM}_7 = \max_{m_{2L}} \{\alpha_H^2 e_{1L} e_{2L} (V - m_{2L})\} \tag{B.411}$$

with the agent satisfying:

$$\max_{(e_{1L}, e_{2L})} -ke_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \tag{B.412}$$

and

$$-ke_{1L}^2 - \alpha_L e_{1L} k e_{2L}^2 + \alpha_L^2 e_{1L} e_{2L} \geq 0 \tag{B.413}$$

where (B.407) is the agent's participation constraint, (B.408) is the constraint of preventing low-type principal's deviation and (B.409) is the constraint of preventing the high-type principal's deviation.

We will show that (B.409) and (B.408) are redundant.

Before we go to the discussion in detail, we want to find $LM_7$ and $\overline{LM}_7$.

It is easy to see that $LM_7 = \frac{27}{32} \frac{\alpha_L^6 V^4}{128k^3}$. Notice that the principal's profit function is

$$\alpha_H^2 \frac{\tilde{\alpha} m_2}{2k} \frac{\alpha \tilde{\alpha}^2 m_2^2}{8k^2} (V - m_2) \tag{B.414}$$

Without any constraint, (B.414) has the maximum at $m_2 = \frac{3}{4}V$. We will show that (B.409) and (B.408) are redundant.

Notice that $LM_7$ is the maximum of the following function

$$\alpha_H^2 \frac{\alpha_L m_{2L}}{2k} \frac{\alpha_L^3 m_{2L}^2}{8k^2} (V - m_{2L}) \tag{B.415}$$

which has $m_{2L} = \frac{3}{4}V$ as the location of the maximum.

On the other hand, $\overline{LM}_7$ is the maximum of the following function

$$\alpha_L^2 \frac{\alpha_L m_{2L}}{2k} \frac{\alpha_L^3 m_{2L}^2}{8k^2} (V - m_{2L}) \tag{B.416}$$

which has $m_{2L} = \frac{3}{4}V$ as the location of the maximum.

Replacing variable $m_{2L}$ with $m_2$ in (B.415) and (B.416), we can see that (B.414) is greater than both of (B.415) and (B.416) when $m_{2L} > 0$, because $\alpha_H > \tilde{\alpha} > \overline{\alpha} > \alpha_L$. This shows that (B.409) and (B.408) are not binding, i.e., they are redundant.

There are three cases for consideration:

1. When $m_2 \le \frac{2k}{\tilde{\alpha}}$, which means that $e_2^* = \frac{\tilde{\alpha}m_2}{2k} \le 1$, $e_1^* = \frac{\alpha\tilde{\alpha}^2 m_2^2}{8k^2} \le 1$.
2. When $m_2 \ge \frac{2k}{\tilde{\alpha}}$, which means that $e_2^* = \frac{\tilde{\alpha}m_2}{2k} \ge 1$, $e_1^* = \frac{\alpha\tilde{\alpha}^2 m_2^2}{8k^2} \le 1$.
3. When $m_2 \ge \frac{2k}{\tilde{\alpha}}$, which means that $e_2^* = \frac{\tilde{\alpha}m_2}{2k} \ge 1$, $e_1^* = \frac{\alpha\tilde{\alpha}^2 m_2^2}{8k^2} \ge 1$.

where $e_1^*$ and $e_2^*$ are the solution of the agent's profit maximization problem (B.326) when the payment is $m_2$.

We will focus the discussion on the first one of the three cases, in which the effort levels in two periods take the forms $e_2^* = \frac{\tilde{\alpha}m_2}{2k}$ and $e_1^* = \frac{\alpha\tilde{\alpha}^2 m_2^2}{8k^2}$, because using a similar argument to what we did for the scenario when upfront, intermediate and end payments are all included, we can show that the global optimal solution won't occur in second case (area). We leave the proof to readers.

With $e_2^* = \frac{\tilde{\alpha}m_2}{2k}$ and $e_1^* = \frac{\alpha\tilde{\alpha}^2 m_2^2}{8k^2}$, the solution to the optimization problem consisting of (B.405) through (B.408) is $m_2^* = \frac{3}{4}V$. The principal's maximal profit equals

$$\frac{27}{32}\frac{\alpha_H^2\overline{\alpha}^2\tilde{\alpha}^2V^4}{128k^3} \tag{B.417}$$

### B.4.4 Compare the high-type principal's profits of the four cases

It is clear that if we restrict $m_1 = 0$ in case 1, we obtain case 2. This means that the high-type principal's profit in case 2 cannot be bigger than that in case 1. We know that only on the segment $2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha} = 0$, the optimal solution of case 1 has $m_1^* = 0$. This shows that the high-type principal's profit in case 1 is strictly greater than that in case 2 except when $2\alpha_H^2 - 3\alpha_H\tilde{\alpha} + \overline{\alpha}\tilde{\alpha} = 0$ where the two profits are equal. Notice that if we restrict $m_0 = 0$, we obtain case 3. But since the optimal solution of case has $m_0^* \ne 0$, the high-type principal's profit in case 1 is strictly bigger than that in case 3. Similarly, if we restrict $m_0 = 0$ and $m_1 = 0$, we obtain case 4. But since the optimal solution of case 4 has $m_0^* \ne 0$, the high-type principal's profit in case 1 strictly bigger than that in case 4. As for the comparison of case 3 and case 4, if we restrict $m_1 = 0$ in case 3, we obtain case 4. But for case 3, the optimal solution has $m_1^* \ne 0$ except on the segment $3\tilde{\alpha} = 2\alpha_H$. Thus the high-type principal's profit in case 3 is strictly bigger than that in case 4 except on the segment $3\tilde{\alpha} = 2\alpha_H$ where the two profits are equal.

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