POLLUTION REGULATION AND PRODUCTION IN IMPERFECT MARKETS

by

François C. Giraud-Carrier

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STATEMENT OF DISSERTATION APPROVAL

The dissertation of **François C. Giraud-Carrier**

has been approved by the following supervisory committee members:

Krishnan S. Anand	, Chair	**11/20/2013** Date Approved
Glen M. Schmidt	, Member	**11/20/2013** Date Approved
Don Wardell	, Member	**11/21/2013** Date Approved
Manu Goyal	, Member	**11/20/2013** Date Approved
Rachel Hayes	, Member	**11/20/2013** Date Approved

and by **William Hesterly** , Chair/Dean of

the Department/College/School of **David Eccles School of Business**

and by David B. Kieda, Dean of The Graduate School.

ABSTRACT

The problem of pollution is not going away. As global Gross Domestic Product (GDP) rises, so does pollution. Due to the existence of environmental externalities, polluting firms lack the incentive to abate their pollution, and without regulations, markets do not adequately control pollution. While regulators are responsible for enacting regulations, the firms ultimately determine the environmental outcomes through their production decisions. Furthermore, polluting industries are typically large and concentrated, raising the concern that market power may be present in these industries. In this dissertation, we study the interactions between powerful, strategic, firms operating under pollution regulations and the regulator when markets are imperfectly competitive.

An important contribution of this work is our integrated pollution-production model, which incorporates the firms' emissions, abatement technologies, the damage from pollution, and three widely-used regulatory mechanisms–Cap, Cap-and-Trade, and Tax. The firms compete with each other and control prices by setting their production quantities. In our model, the firms have many options to comply with the pollution constraints enforced by the regulator, including abating pollution, reducing output, trading in emission allowances, paying emission taxes, investing in abatement innovations, colluding, and combining some of these options. Following the introduction in Chapter 1, we address three broad questions in three separate chapters.

- Chapter 2: What is the effect of the pollution control mechanisms on firms, consumers, and society as a whole? Which mechanisms and policies should regulators use to control pollution in a fair, effective, and practical manner?
- Chapter 3: Does Cap-and-Trade enable collusion? If it does, what are the effects of collusion?
- Chapter 4: Which mechanisms encourage more investments in abatement innovations?

Our results apply to different types of pollutants and market structures. Our research provides guidelines for both policy-makers and regulated firms.

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CHAPTER 1

INTRODUCTION

With the explosion of world GDP comes mounting pressure on the natural ecosystems that support life on earth. Consider the case of climate change. The most recent assessment report of the Intergovernmental Panel on Climate Change (Alexander et al. 2013) unequivocally confirms the warming of the climate system, and reaffirms the human factor as the dominant cause of the warming. It is well-known that the human component of global warming is primarily driven by uncontrolled emissions of greenhouse gases. Because producers lack the incentives to reduce their emissions, government-mandated regulations are required. Since the early 1970s, regulators around the world have used a variety of pollution control mechanisms to enforce pollution reductions, and encourage investments in abating technologies. This dissertation studies three widely-used types of pollution control mechanisms: (i) emission taxes, in which polluters are assessed a fee that is proportional to their emissions; and emission quotas with (ii) or without (iii) the ability to trade on such quotas. Cap-and-Trade is the popular term for tradable emission quotas; we use the single word 'Cap' when trading is not allowed. These three mechanisms are commonly used in practice to regulate various types of air, water, or land pollutants. For example, in the context of climate change, carbon taxes are used in several European Countries (Stavins 2003), as well as in Australia, India, and Japan (KPMG 2013). The European Union (E.U. Publications Office 2013), California (Barringer 2011), and China (Plumer 2013) have chosen Cap-and-Trade to limit greenhouse gas emissions by large industrial emitters[1]. In the United States, taxes and Cap-and-Trade have also been debated at the national level. The current proposal by the U.S. regulator consists in limiting carbon emissions by coal and natural-gas power plants to 1,100 and 1,000 lbs per Megawatt-hour, respectively (Shear 2013). This is an example of the Cap mechanism (i.e., emission quotas without trading).

[1] Cap-and-Trade was first used in the U.S. starting in 1995 to combat acid rains caused by sulfur-dioxide emissions from coal-fired power plants

As advocated by Plambeck and Toktay (2013), the operations management (OM) community can help mitigate climate change, for example, by extending "the knowledge base to design products, services and systems ... that require little energy" (Plambeck and Toktay 2013, p. 523). With its pragmatic systems approach, the OM community has an important role to play in greening today's production and consumption systems. Such a role would naturally include an analysis of processes, business models, and incentive systems that guarantee that the stated objectives are met. In the context of pollution regulations, while regulators define the objectives, it is important to recognize that the firms act as independent entities, and ultimately determine the environmental outcomes. Hoffman (2004) argues that "no solution to the environmental problems society faces will be solved without the involvement of business" (Hoffman 2004, p. 5), because it is the firms, not the regulator, who design and make the products, and firms typically have more information, and often more resources also, than the regulators themselves. For this reason, we focus on powerful, strategic firms that interact in noncooperative games with each other, and with a pollution-sensitive regulator. In our models, the firms have many options to comply with regulations, including abating pollution, reducing output, trading in emission allowances, paying emission taxes, investing in abatement innovations, colluding, and combining some of these options.

The contribution of this work arises from the combination of four key elements:

- We make a *methodological contribution* by constructing a rich, yet tractable, integrated pollution-production model. From a pollution point of view, the model captures (i) pollution generation as an output of production, (ii) abatement efforts by the polluting firms, (iii) the damage to society, and (iv) the enacted regulations. The model is presented in detail in the next chapter (see section 2.3). The proofs are in Appendix A.

  We use the model throughout this work to answer a series of related questions. In Chapter 2 we study the effects of the three pollution control mechanisms mentioned above on firms, consumers, and society as a whole, and discuss which mechanisms regulators should use to control pollution in a fair, effective, and practical manner. In Chapter 3, we focus on the risks of collusion under Cap-and-Trade regulation. In Chapter 4, we compare the mechanisms in terms of their effectiveness at stimulating the adoption of abatement innovations;

- An important component of this work is our focus on *strategic firms in imperfect mar-*

kets. In large, concentrated industries such as those subject to pollution regulations, firms are powerful. Previous empirical research points to the presence of market power is these industries (see, for example, U.S. EPA 2010). Wielding market power gives firms more leverage as they deal with pollution regulations. It is not uncommon for firms targeted by government-mandated regulations to issue threats to curb or even shut down operations. In our model, the firms can manipulate the output prices by controlling the supply side of the market.

In addition to controlling output, our strategic firms can abate pollution, trade (under Cap-and-Trade), pay the tax (under the Tax mechanism), invest in abatement innovations, collude, and combine some of these actions.

- In reality, competition can limit the firms' ability to exercise market power. *We allow the intensity of competition to vary on a continuum*, from no competition at all, i.e., the firms are local monopolies, to competition a la Cournot. As we show, the results are sensitive to the degree of competition.

- Finally, we introduce the *temporal dimension* in Chapters 3 and 4. As discussed in these chapters, there are few dynamic models in the literature. The timing of trading under Cap-and-Trade is a key determinant of the collusion outcome in Chapter 3. In Chapter 4, it is the ability to invest in an abatement innovation *before* production that drives our results.

The rest of the work consists of three independent chapters, each of which provides answers to the questions mentioned above. As indicated previously, these chapters rely on the same modeling framework, i.e., the integrated pollution-production model. The full-fledged n-firm model is presented and solved in Chapter 2. In Chapter 3, we limit the number of firms to two. This makes the analysis easier, but no important insights are lost. In Chapter 4, we consider n firms but ignore competition. All proofs are in the appendices. To improve navigation through the document, there is a separate appendix for each chapter.

CHAPTER 2

POLLUTION REGULATION OF FIRMS PRODUCING PARTIAL SUBSTITUTES

2.1 Introduction

Pollution regulation in the United States is a controversial affair, with every new rule giving rise to heated debate. Opponents argue that pollution controls constrain output and choke businesses, while proponents point to the adverse effects of pollution on human health, wildlife habitat, and the natural environment. The regulator needs to balance the costs and benefits of pollution reduction (Arrow et al. 1996).

Pollution is an inevitable by-product of production, and an ancient problem. Hong et al. (1996) analyze air molecules trapped in Greenland ice to track air-pollution from copper smelting over the last 5,000 years. They find evidence of soaring pollution levels 2,000 and 900 years ago, coinciding precisely with the peaks of the Roman empire and the Chinese Song dynasty– two periods of bustling economic activity (see Figure 2.1).

During the last few decades, concerns over climate change have thrown greenhouse gases (GHG) under the spotlight. Carbon dioxide (CO_2) emissions represent the bulk of man-made GHG emissions (83.7% in the U.S., 82.4% in Europe, and 94.8% in Japan).

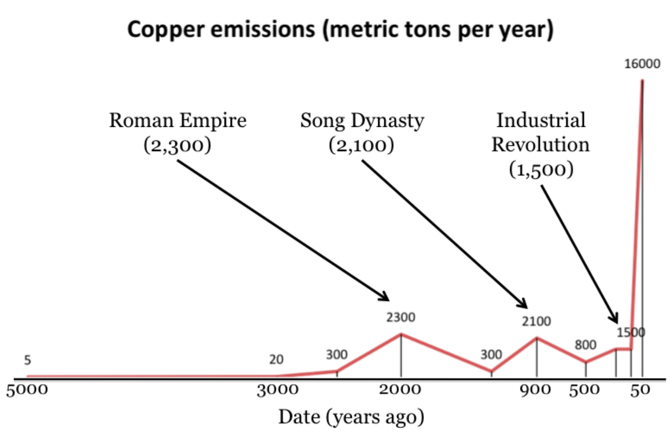

Figure 2.1. History of air pollution from copper smelting

In the U.S., business operations contribute 62% of GHG emissions, while personal vehicle use and residential buildings account for the rest (Hockstad and Cook 2012). Carbon dioxide emissions "track economic growth, slowing with recessions, but essentially rising and rising" (Matthew Arnold–World Resources Institute–in Iannuzzi 2002, Foreword). A reading of the carbon barometer for May 2013 puts carbon dioxide concentrations in the Earth's atmosphere at 399.89 ppm, a 41 percent increase since the early 1800s (Scripps 2013). Before carbon emissions became an issue, concerns crystallized around acid rains arising from sulfur dioxide discharges by fossil-fuel power plants, smog caused by particulate matter and ozone emissions around our cities, and other environmental problems originating from rapidly expanding business activities.

A firm's pollution imposes a *negative externality* on society, in that the pollution affects people, wildlife, and the natural environment outside the firm's boundaries.[1] An *unregulated* firm does not bear the full costs of its pollution, since its incentives to control or abate its pollution are not commensurate with the pollution damage it causes. Thus, regulation is inevitable to mitigate pollution in the context of negative externalities (cf. Baumol and Oates 1988, Cropper and Oates 1992). Almost inevitably, firms and industry lobby groups oppose regulation, claiming that it increases their production costs and forces them to reduce output, resulting in higher prices, lower consumer surplus, and lower social welfare. For instance, consider the regulation of airborne mercury pollution. Mercury is a deadly poison that can permanently damage the development of the brain and nervous systems in fetuses and children. In 2010, the Environmental Protection Agency (EPA) in the U.S. proposed national standards for airborne mercury pollution from coal-fired power plants. The National Association of Manufacturers, representing power generating firms, objected vehemently to this proposal, stating that "...overly burdensome and unnecessary rules...will crush economic growth and job creation" (Broder and Stolberg 2010). American Electric Power, one of the nation's largest utilities, warned that "new air quality rules could force it to 'prematurely' shut down about two dozen big coal-fired units and fire hundreds of workers" (New York Times 2011).

Polluting industries are typically large and concentrated, which raises the concern that firms operating in these markets may be in a position to exercise market power (Cropper and Oates 1992). As an example, the EPA conducted a survey of market concentration in the power generation sector in 31 states in the eastern half of the United States (U.S. EPA 2010). Power generation from fossil fuels produces large quantities of pollutants, such as

[1] We will refer to these adverse societal effects collectively as the *pollution damage*.

sulfur dioxide, nitrogen oxides, carbon dioxide, ozone, mercury, and particulate matter. In 16 states, only one or two companies controlled more than 60% of the power generation capacity. The Herfindahl-Hirschman Indices (Rhoades 1993), a commonly excepted measure of market concentration, were greater than 1,800 in 25 states, suggesting a strong likelihood that market power exists in these markets. The threat to shut down production facilities and layoff employees is more credible when firms hold some market power.

In this research, we develop analytical models to rigorously study the impact of pollution regulation on firms, consumers, and the economy when the output markets are imperfectly competitive. There are n firms in our model, each of which chooses how much to produce of a single product. The output price is a function of the firm's output, as well as of the output chosen by the other firms. In other words, the products are partial substitutes, and competition is exercised by the firms' output choices. For completeness, we allow for different degrees of product substitutability from homogeneous (i.e., perfectly substitutable) to heterogeneous goods, which enables us to vary the intensity of competition between the firms. Many products fit such a description. A used, e.g., remanufactured, product may be a substitute for the original, new product. Different generations of the same product are also partial substitutes. Even when the product is a commodity, there may be some heterogeneity between customers; for example, some customer segments may have a preference for products locally supplied. Alternative forms of a commodity, such as renewable versus fossil-fuel energy, are another example. We model the processes of pollution generation, pollution abatement, and pollution damage to society. We identify normative criteria to guide the regulator's choice of pollution control mechanisms and develop theoretical benchmarks based on these criteria. We also model and compare three widely-used pollution regulation mechanisms–Cap, Cap-and-Trade, and Taxes. With a Cap, the regulator limits the firm's total emissions or its emission rate. Under Cap-and-Trade, the regulator also sets a cap, but firms can trade emission allowances among themselves. Under Taxes, the firms pay a fee proportional to their pollution. We derive the Subgame-Perfect Nash equilibria for a series of games involving the regulator and multiple profit-maximizing firms with heterogeneous abatement costs. We study firms' choices of trade-offs between costly pollution abatement and revenue losses due to output reduction under each of these regulatory mechanisms, and analyze their ripple effects on the economy. We analyze and compare the output, pollution abatement, firms' profits, consumer surplus, and social welfare under Cap, Cap-and-Trade, and Taxes.

Our results show that firms always reduce output to comply with pollution regulations, a finding well supported by the empirical literature. Firms that hold some market power strategically lower their output to comply with pollution constraints, and at the same time command higher prices for their products. Competition limits the firms' ability to reduce output strategically, and thus mitigates the output reduction effect. Because the firms cannot reduce output as much, they have to exert more effort toward pollution abatement. Such a shift in the firms' strategies is good for social welfare. We find that when competitive forces are sufficient, the introduction of pollution constraints improves welfare because the mitigation of the pollution damage and gains in consumer surplus outweigh the losses in firm profits. Our results also confirm the equivalence between Tax and Cap-and-Trade, except for firm profits, which are always higher under Cap-and-Trade. More interesting, we find that, if the cap or tax rate are chosen appropriately, Cap-and-Trade and Tax are both fair and effective, in the sense that they can achieve any pollution limit set by the regulator, and at the same time, ensure that firms bear the exact and entire cost of their pollution, but no more. The "pure Cap" mechanism is less effective. The optimal regulatory policy depends critically on how intense the competition is. When firms are local monopolies, we show that the regulator should charge firms *less* than the full extent of their pollution damage, in other words, set a tax rate lower than the tax rate corresponding solely to the marginal pollution damage. Conversely, when competition is very intense (but firms still hold some market power), the regulator can maximize welfare by charging *more* than the firms' pollution damage. This is a direct outcome of firms' output reduction and their competitive interactions under regulation.

2.2 Literature Review

When markets are perfectly competitive, the increase in firms' marginal production costs due to regulatory constraints causes the output price to rise. As a result, demand falls. Thus, when regulations are introduced to curb pollution in perfectly competitive industries, output reduction is an expected theoretical outcome. When markets are imperfectly competitive, the producers can simultaneously influence demand and pollution by adjusting their production quantities (or setting prices) strategically. Requate (1993a) shows that pollution regulations also cause a reduction in output when firms are local monopolies (Requate 1993a). Requate (1993b) extends this result to a Cournot duopoly. The connection between pollution regulations and lower productivity and output is well supported in the empirical literature (cf. Denison 1978, GAO 1986, Gray 1987, Jorgenson and Wilcoxen 1990, Boyd and McClelland 1999, Greenstone 2002). Boyd and McClelland (1999) find a reduction in productivity between 1% and 4% at U.S. paper plants due to government-mandated environmental investments. Greenstone (2002) studies the economic impact of the Clean Air Act for the period 1972-1987. He finds that counties in the U.S. subject to stringent regulations lost 590,000 more jobs and $75 billion more in output in polluting industries than counties that experienced looser regulatory supervision. Jorgenson and Wilcoxen (1990) simulate the long-term growth of the U.S. economy and find that pollution controls are responsible for a 0.19% annual reduction in the gross national product. A study of sulfur dioxide pollution from nonferrous smelters in the U.S. found that, between 1970 and 1984, pollution abatement accounted for just 44% of the pollution reductions, while the remaining 56% reduction was achieved by output reduction (GAO 1986).

A reduction in output mechanically lowers the pollution; consequently, the resulting damage on consumers, the environment, and society as a whole, is reduced. However less production has a negative impact on consumer surplus, e.g., in the form of higher prices and fewer jobs. The net effect on social welfare is ambiguous. In economic theory, the problem of optimal pollution control is traditionally framed as one of maximizing a comprehensive measure of social welfare, which captures the benefits of production to producers and consumers, as well as the dis-utility arising from pollution (see for examples Baumol and Oates 1988, Requate 1993b, Requate 1993a, Nault 1996, Levi and Nault 2004). This social optimal defines the *first-best* standard against which various mechanisms for pollution control are evaluated.

In this paper, we develop normative criteria and a set of benchmarks to guide our discussion. In addition to the classical first-best (i.e., welfare-maximization) approach, we apply the Groves mechanism, a theoretical benchmark from the public-goods literature (Clarke 1971, Groves 1973, Groves and Loeb 1975) to our context. In the first-best approach, the firms are not strategic. The main advantage of the Groves mechanism is that it focuses solely on the externality without interfering with the market structure, competition, or the firms' incentives to maximize profits. The Groves mechanism has only rarely been used before by researchers in Operations. In particular, Anand and Mendelson (1997) develop a Groves mechanism-based transfer pricing scheme that aligns the incentives of local branches of a firm's supply chain so that they behave as a *team* and jointly maximize the firm's total profits.

Under the assumption of perfect competition and complete and perfect information, it is well known that Taxes are equivalent to Cap-and-Trade (see, for example, Weitzman 1974). If the tax rate is set equal to the marginal social damage from pollution, the first-best outcome (as defined above) can be achieved. This optimal taxation scheme is known as the Pigovian tax after economist Arthur Pigou (1932). Similarly, Cap-and-Trade can achieve first-best if the total number of allowances equals the aggregate pollution generated under first-best. The above results are highly sensitive to the assumption of perfect competition. Buchanan (1969) and Barnett (1980) study the effect of Pigovian taxes on a monopoly. Buchanan (1969) shows that Pigovian taxation of a monopoly could result in lower welfare because the tax causes the monopoly to contract output even further, and the loss in profit and consumer surplus exceeds the reduction in the pollution damage.[2] Barnett (1980) shows that the welfare-maximizing tax rate should be less than the marginal social cost of pollution, in order to counterbalance the output distortion due to the exercise of monopoly power. Requate (1993a) (cited earlier) confirms the equivalence between emission taxes and Cap-and-Trade when firms are local monopolies; however, Requate (1993b) shows that this equivalence no longer holds in an asymmetric Cournot duopoly in which firms are allowed to trade emission rights to maximize their joint profits. In Requate (1993b), there is no unambiguous ranking of Taxes and Cap-and-Trade from a welfare point of view. We see that the exercise of market power and the presence of competitive forces significantly impact the outcome of pollution control regulations. We study these interactions in a context in which strategic firms produce imperfectly substitutable goods. This allows us to isolate the effect of varying degrees of competition.

Requate (2006) surveys the literature on environmental policy under imperfect competition. Emission taxes are reviewed in some detail. Barnett's (1980) finding that the tax rate should be less than the social cost of pollution is found to be robust to several variants of the original model (e.g., models with or without abatement technology, and with linear or convex costs). Requate (2006) shows that a subsidy on output could restore the efficiency of the Pigovian tax. The main limitations of this literature is the paucity of work on Cap-and-Trade systems. Research on Cap-and-Trade under imperfect competition is limited to local monopolies or two-firm duopolies, due to the analytical difficulty of solving the trading model in more complicated industry settings. We address this gap. We specifically model the trading process among multiple competing firms when the level of competition varies. Thus, our research makes an important methodological contribution by proposing a reusable, integrated, pollution-production model that incorporates Cap-and-

[2] In Buchanan's model, this happens because the damage from pollution is linear in the output quantity of the monopoly. In this special case, the loss in pollution damage dues to output reduction is always less than the combined losses in firm profits and consumer surplus.

Trade regulation.

Environmental concerns are becoming increasingly important in business today. As suggested by Kleindorfer et al. (2005), "The modelers (the OR-based OM population) must *revisit the classical models...* to *reformulate the objective function and the set of constraints...* in the new context" [Emphasis added]. In our model, *(i)* firms are profit-maximizers (as commonly assumed in traditional supply chain research) but constrained by pollution regulations, and *(ii)* these regulatory constraints are a key driver of the equilibrium outcomes, profits, consumer surplus, and welfare.

There is a growing body of literature dedicated to the impact of pollution regulations, in particular carbon regulations, on operations. Carbon emissions markets are an artifact of requiring that pollution costs be borne by the pollution creators (cf. Pigou 1912), known in international parlance as the "polluter pays" principle (OECD 1972). Both Drake (2011) and Plambeck et al. (2012) study the impact of carbon pricing under differential stringency of regulations in different regions, when firms can choose where to locate (or relocate) their production. Production relocation can lead to *emission leakage*– a net increase in global emissions. In Drake's (2011) model, the regulator can use border tariffs as a corrective surcharge to products imported into the regulated region, which have profound but unintended consequences for clean technology choice, off-shoring, and entry. For instance, when the border tariff is equal to the carbon cost in the regulated region, Drake (2011) finds that foreign entrants and offshore plants will use cleaner technologies than local plants. Plambeck et al. (2012) study the impact of emission price variability under Cap-and-Trade on firms' optimal location, production and export quantities, emissions, and expected profits. They find that, surprisingly, emission price variability does not necessarily hurt firms' profits, and can, under certain conditions, lead to higher social welfare and lower expected emissions than a fixed-price scheme such as a carbon tax.

Under very different settings, Benjaafar et al. (2010) and Cachon (2011) show that operational leverage can enable emissions reductions at minimal cost. In Benjaafar et al. (2010), the firm, facing mandated carbon constraints, chooses the quantity and timing of production, but cannot abate pollution. Their numerical studies suggest that adjustments to the ordering policy can have a large impact on emissions with limited cost increases, and that the choice of pollution control mechanism (Carbon Cap, Carbon Tax, Cap-and-Trade with exogenously-specified carbon prices or carbon offsets) can significantly affect the firm's incentives to adopt more energy-efficient technologies. Cachon (2011) studies the problem of deciding on the number and location of retail stores to minimize total transportation costs, including associated carbon costs, incurred by a retailer and its customers, when replenishments and customer visits are regular (and exogenous). Cachon (2011) shows that his solution is robust to errors in the model parameters, and in particular, to whether carbon costs are internalized or not.

Several researchers have analyzed regulatory alternatives to Taxes and Caps, such as subsidies for pollution abatement and legal mandates requiring firms to disclose information, recycle, or dispose used products. Nault (1996) shows the equivalence of subsidies and taxes in terms of output, pollution damage, and welfare. Kalkanci et al. (2012) find that voluntary disclosure of a firm's environmental footprint leads to more learning by the firm and lower environmental impact than mandatory disclosures. Atasu et al. (2009), Subramanian et al. (2009), and Jacobs and Subramanian (2012) study *extended producer responsibility*, a mechanism wherein the manufacturer is legally responsible for collecting and treating some fraction of end-of-use products, thus supporting recycling or disposal. Plambeck and Taylor (2010) study competitive testing and whistle-blowing as a means to achieve compliance on environmental, health, and safety standards. Keskin and Plambeck (2011) study the effect of accounting rules on allocation of carbon emissions across co-products serving a domestic and an export market. They find that letting the firm choose the allocation rule, as is current practice, can contribute to higher emissions, and identify the allocation rule that leads to the lowest emissions.

Similar to Benjaafar et al. (2010), we model the firm's operational decision-making process under different pollution control mechanisms. However, unlike Benjaafar et al. (2010), the trade-off between pollution abatement and output reduction is one of the key features of our model, since firms can choose to abate pollution (albeit at a cost) in addition to reducing output. Another key difference is that the price of emission allowances in our Cap-and-Trade model is endogenous, and arises as the unique equilibrium outcome of the firms' trading game. We also model the regulator's problem of selecting a fair, effective, and practical mechanism to achieve her pollution reduction goals. Finally, we derive analytical, closed-form solutions to generate insights. Like Drake (2011) and Plambeck et al. (2012), we show that the introduction of environmental regulations has significant implications for firms' operations, but our focus is the firms' internal operations *within* the regulated region. We refer the reader to Kleindorfer et al. (2005), Corbett and Klassen (2006), Linton et al. (2007), and Souza (2012) for comprehensive reviews of the literature in sustainable operations.

To summarize, our contribution is threefold: (1) We make a methodological contribution, extending a traditional supply chain model to include environmental and regulatory constraints. Alternatively, our approach can also be seen as an extension of environmental economic models that focuses on strategic firms operating in imperfectly competitive markets; (2) We compare three widely-used pollution control mechanisms–Cap, Cap-and-Trade, and Tax–to the Groves mechanism, a theoretical benchmark from the public-goods literature; (3) Finally, we model competition on a continuum, which allows us to study how the firms' reactions under Cap, Cap-and-Trade, and Tax, and the regulator's optimal policies change with competition.

2.3 Elements of the Model

2.3.1 Modeling Pollution

Consider a firm whose production generates a harmful pollutant. We model four interrelated aspects of pollution: *(i) Pollution generation*: This relates the quantity of pollution emitted to the production quantity q as well as to the degree of pollution abatement; *(ii) Pollution abatement:* This describes how the firm can (fully or partially) abate the pollution it generates, and the costs of abatement; *(iii) Pollution damage:* This quantifies the dis-utility to society from pollution; and, finally *(iv) Pollution regulation:* This describes the mechanisms that the regulator could employ to control pollution. We discuss each of these four elements below.

2.3.1.1 Pollution Generation

Let $\widetilde{P}$ denote the total quantity of pollution emitted by the firm. Clearly, the quantity of pollutant should be an increasing function of production. We further assume that the total pollution $\widetilde{P}$ (prior to any investment in abatement) is *proportional* to the production quantity q; i.e., $\widetilde{P} = e \cdot q$ where $e \geq 0$ is the emissions rate. Several factors suggest that our linearity assumption is reasonable in the context of many industrial sectors. Pollution concentrations arise from diffusion patterns which, by the law of conservation of mass, are typically linear in the quantity of pollution released. In many industries–the power generation industry being a classical example–the output and the pollution generated are linear functions of fuel consumption, and hence, of each other. Without loss of generality, we normalize e to 1, i.e., $\widetilde{P} = q$.

2.3.1.2 Pollution Abatement

Our model of pollution abatement relies on two complementary notions: *(i)* the *abatement level*, which determines how much pollution is abated, and *(ii)* the cost of abating pollution. In our model, the firm can control the quantity of pollution it generates (albeit at a cost) by setting the abatement level, denoted by x, where $x \in [0, 1]$. The decision variable x can be interpreted as the percentage of pollution abatement chosen by the firm. In other words, $q \cdot x$ is the quantity of pollution abated. The relation between the *net* (or *residual*) pollution P, the total pollution $\widetilde{P}$, and abatement is modeled as $P = \widetilde{P} - q \cdot x = q \cdot (1 - x)$. At one extreme, when $x = 0$, the pollution is unabated (hence, $P = \widetilde{P} = q$). When $x = 1$, the pollution is completely abated and $P = 0$. Intermediate values of x correspond to partial abatement.

In our model, we assume that pollution abatement costs are *increasing* and *convex* in the quantity of pollution abated (which is $q \cdot x$). Specifically, we assume that the pollution abatement cost $C(q; x) = c \cdot (q \cdot x)^2$, where c is the abatement cost coefficient. We justify our assumption of a convex abatement cost curve on several grounds: *(i)* It is logical that the first units of pollution are easy to abate, but once the low-hanging fruits have been exploited, pollution abatement becomes increasingly difficult. *(ii)* Hartman et al. (1997) estimate the cost of pollution abatement for 7 common air pollutants, namely particulates,[3] sulfur oxides, nitrogen dioxide, carbon monoxide, hydrocarbons, lead, and other hazardous emissions, using census data from 100,000 U.S. manufacturing firms across 37 industrial sectors. They find support for quadratic abatement cost curves in several industrial sectors. *(iii)* Finally, quadratic abatement costs are commonly assumed in the extant academic literature (cf. Parry and Toman 2002, Subramanian et al. 2007). Nault (1996) and Levi and Nault (2004) assume a convex, but not necessarily quadratic, cost function.

2.3.1.3 Pollution Damage

Pollution affects human health, wildlife habitat, and the natural environment. In a widely cited study, Pope et al. (2002) found that a 10 $\mu g/m^3$ increase in fine particulate air pollution was associated with an increased risk of all-cause, cardiopulmonary, and lung cancer mortality, by 4%, 6%, and 8%, respectively. Particulates contribute to the creation of haze, increase the acidity of lakes and rivers, and alter the balance of nutrients in waters and the soil.[4] Sulfur dioxide, another common air pollutant, contributes to acid rains, which cause widespread damage to surface waters, aquatic animals, forests, crops, and buildings.

The *pollution damage function*, which we introduce next in our model, captures both present and future damage to society from emissions. Clearly, pollution damage would

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[3]Particulates are a mixture of fine solid particles and liquid droplets suspended in the air.

[4]http://www.epa.gov/air/particlepollution/health.html

be increasing in the net pollution generated (Nault 1996; Jacobs and Subramanian 2012). Furthermore, Tietenberg and Lewis (2011) suggests that "the marginal damage caused by a unit of pollution increases with the amount emitted" [p. 359]. Intuitively, while pollution is tolerable in small quantities, the damage from pollution increases with the quantity of pollution *at an increasing rate.* Also, the vast majority of epidemiological studies use either a log-linear or logistic functional form, suggesting that epidemiologists generally believe that the health impact of pollution is convex in the pollution concentration. Thus, we model the pollution damage function $D(P)$ as an increasing, *convex* function of the net total pollution, P. Specifically, we let $D(P) = d \cdot P^2$, where $d \geq 0$, the *pollution damage factor*, varies with the pollutant under consideration. A high value of d indicates a very toxic pollutant, whereas low d suggests a pollutant with moderate, albeit still harmful, impact on society. By modeling the damage as a function of the total pollution, we focus on a *global* pollutant, i.e., firm i's pollution affects consumers everywhere, not just in market i. This assumption is appropriate for many pollutants that travel over extensive areas, such as sulfur dioxide and carbon dioxide. It is also important to note that Cap-and-Trade regulation is well suited for global pollutants, and not so well for localized pollutants, because local pollutants typically result from fewer point sources, and hence result in smaller emission trading markets. Cap-and-Trade has been successfully deployed for global pollutants in large markets (e.g., the European Union, California, the Eastern United States).

2.3.1.4 Pollution Regulation

Perhaps the earliest recorded instance of pollution regulation was in London in 1272, when King Edward I banned the burning of sea-coal– a cheap, abundant but very smoky fuel.[5] Centralized control mechanisms used today include technology mandates and performance standards such as the maximum permissible emissions rate for a particular technology.[6] Often, such centralized mandates are suboptimal because *(i)* the regulator is unlikely to be fully informed about each firm's operating conditions (e.g., its abatement costs); *(ii)* efficiencies that could be achieved by tapping into firms' expertise are forgone (Tietenberg 1985); *(iii)* the regulator incurs high monitoring and information acquisition costs, particularly when the incentives of firms and the regulator diverge (cf. Iannuzzi 2002);

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[5] http://www.epa.gov/aboutepa/history/topics/perspect/london.html

[6] For example, according to the U.S. EPA Standards of Performance for Stationary Combustion Turbines (2006), a turbine of less than 50 MMBTU/h must not emit more than 8.7 pounds of nitrogen oxides per megawatt-hour.

and *(iv)* such mandates frequently invite litigation, with its related financial burdens and compliance delays (Tietenberg 1985).

To overcome these difficulties, economists have long urged the use of economic incentives, such as pollution taxes and tradable emission allowances (Stavins 1998, 2003). In this research, we analyze and compare three widely-used mechanisms: Cap, Cap-and-Trade, and Tax. Under the Cap mechanism, the regulator directly imposes a pollution limit (the 'Cap') on each firm with heavy fines as a deterrent for flouting. Firms must comply using some combination of *(a)* pollution abatement and *(b)* output reduction. Under Cap-and-Trade also, the regulator specifies a Cap, but firms can comply through some combination of *three* actions: *(a)* pollution abatement, *(b)* output reduction, and *(c)* trading in emission allowances, which effectively shifts firms' pollution constraints up or down. The premise is that Cap-and-Trade would facilitate *efficient* allocation of emission allowances via the market mechanism (Coase 1960, Dales 1968, Montgomery 1972, Schmalensee et al. 1998). Since 1995, the U.S. Acid Rain Program has included a Cap-and-Trade system for the reduction of Sulfur Dioxide (SO_2) emissions by coal-fired power plants. Cap-and-Trade is increasingly popular among environmental regulators, in particular for greenhouse gas (GHG) regulations. In 2005, the European Union launched a large-scale Cap-and-Trade system for GHG emissions–the E.U. Emissions Trading Scheme. The State of California is currently rolling out a Cap-and-Trade system for GHG as part of the Global Warming Solutions Act (Barringer 2011), and China recently announced that it will operate carbon markets to curb GHG emissions in several large cities (Forrister and Bledsoe 2013).

Under the Tax mechanism, the regulator charges each firm with a tax commensurate with its pollution. Formally, the tax is equal to $\tau \cdot [q \cdot (1-x)]$, where $q \cdot (1-x)$ is the net pollution generated by the firm and $\tau \geq 0$ is the tax rate set by the regulator, common to all firms. By increasing the tax rate, the regulator makes pollution more costly to the firm causing it to reduce its emissions. Stavins (2003) identifies ten applications of emission taxes in Europe, including to carbon monoxide (CO), carbon dioxide (CO_2), sulfur dioxide (SO_2), and nitrogen oxides (NO_x). France and Sweden tax emissions of sulfur and nitrogen oxide. Finland was the first country in the world to introduce a Carbon Tax in 1990, with Denmark, Italy, Netherlands, Norway, and Sweden following suit. In the United States, the Carbon Tax is being debated as an alternative to Cap-and-Trade.

2.3.2 Modeling Firms, the Regulator, and Their Interactions

In the model, we consider n firms and study the strategic interactions among them and a pollution-sensitive regulator. Since our research focuses on output and its interaction with pollution abatement under competition, we model competing firms with a measure of control over their output, i.e., firms with some market power. As mentioned earlier, we study the case of partial substitutes. For each horizontal market demand is characterized by the linear inverse demand curve $p_i = a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}$ where p_i is the price in market i, $a > 0,\ b > 0,\ 0 \leq \gamma \leq 1$, q_i is the quantity produced by firm i for market i, and Q_{-i} the quantity serving all the other markets (market i excluded). When $\gamma = 0$, the firms are local monopolies. The case $\gamma = 1$ is the classical Cournot oligopoly. The parameter γ captures the intensity of the competition between the firms.

Firm i has an abatement cost coefficient c_i, meaning that the total cost of pollution abatement to firm i, $C_i = c_i \cdot (q_i \cdot x_i)^2$, where x_i is the abatement level (i.e., the percentage of pollution abated), q_i is the output, and $q_i \cdot x_i$ is the quantity of pollution abated by firm i. We assume that the abatement cost coefficients c_i can take one of two values– c_l or c_h, where $0 \leq c_l \leq c_h$ without loss of generality. Abatement cost coefficients vary across pollutants, geographic regions, and industries. Even for the same pollutant and the same product, these coefficients vary across different abatement technologies, and even across abatement technologies of different vintages (Pittman 1981, Hartman et al. 1997, Swinton 1998, U.S. Census Bureau 2005, Creyts et al. 2007). We will use the subscript l to denote a *low-cost* firm; i.e., one with a *low* abatement cost coefficient c_l, and the subscript h to denote a *high-cost* firm, which has a *high* abatement cost coefficient c_h. Let m denote the number of firms with cost coefficient c_l. Thus, $n - m$ firms have a cost coefficient of c_h. The firms are otherwise identical.

2.3.3 Information Assumptions

We assume that $n,\ m,\ c_l$, and c_h are common knowledge. Thus, the regulator knows the size of the industry, is aware of the existence of two different abatement technologies, and knows the distribution of these technologies within the industry. Note that we do not assume that the regulator knows precisely the cost coefficient of each firm. This is known with certainty only by the firm itself. However, the regulator and the firms know in the aggregate the number of firms with each abatement technology. In practice, trade associations (or the technology vendors themselves) often make publicly available information about the diffusion of particular technologies without disclosing the identity of the firms that employ such technologies.

Based on scientific, historical, and political considerations, the regulator typically specifies a pollution cap for the entire region. Let S denote this cap. Given that the regulator cannot distinguish between firms based on their cost coefficients, and that the firms are otherwise identical, it is reasonable to assume that the same cap will be allocated to each firm under the Cap mechanism. In the early stages of Cap-and-Trade implementation, each firm typically receives for free an endowment of emission rights. Free allocation is called “grandfathering”. The sum of these initial endowments equals the overall cap S for the region. Most Cap-and-Trade programs make provisions in later stages for auctioning of the emission allowances, either in part or in full. In the US Acid Rain program, auctioning was limited to 3% of the total allowances. In the European Cap-and-Trade program, more than 95% of the emission allowances were grandfathered during the initial phase covering the period 2005-2008, and more than 90% for the period 2008-2012. In California, auctions are conducted on a quarterly basis, but a large number of allowances is also grandfathered for transition assistance, and in sectors that are vulnerable to external competition and emission leakage (California Air Resources Board 2010). It is important to note that if the trading market is efficient, the equilibrium outcome is independent of the allocation mechanism (whether grandfathered or auctioned off). The firms’ optimal production schedule is unaffected by the allocation mechanism (Requate 2006), although the firms’ profits are. For the same reason as the Cap mechanism, we will hereafter assume that the firms are initially given the same allocation. Let $s = S/n$ denote the pollution cap imposed on an arbitrary firm, and t be the number of emission allowances traded by the firm. Without loss of generality, $t \geq 0$ indicates that the firm is a net seller of allowances, and $t < 0$ that the firm is a net buyer. The firm’s constraint is $q \cdot (1 - x) \leq s - t$. The ‘pure Cap’ mechanism emerges as a special case of the Cap-and-Trade mechanism, with pollution limits but no trading in emission allowances (i.e., $t = 0$ for all firms). This arises if there is only one firm being regulated, or if firms are not allowed to trade with each other for regulatory or other reasons. By comparing Cap with Cap-and-Trade, we can isolate the impact of trading on firms’ production quantities, abatement levels, and profits.

Knowing $n,\ c_l,\ c_h$, and m is sufficient for the regulator to achieve any predetermined pollution goal under Tax. To facilitate comparisons, we assume that the pollution target, S, is the same under each of the regulatory mechanisms under consideration.

Firms maximize their profits. Firm i’s profit before Tax,

$$\pi_i\left(q_i, x_i \mid Q_{-i}\right) = q_i \cdot \left(a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}\right) - c_i \cdot \left(q_i \cdot x_i\right)^2$$

is the difference between its revenues and its pollution abatement costs. We assume that all other production costs, whether fixed or variable, are zero. It is straightforward to relax this assumption in our analysis; nevertheless, this assumption enables us to minimize clutter.

2.3.4 Performance Measures

The performance measures we use to evaluate the three mechanisms include the total output, the total abated pollution, firms' profits, consumer surplus, and social welfare. We augment the concept of consumer surplus (and by extension, welfare) to include environmental effects. The traditional measure of consumer surplus focuses solely on consumers' economic surplus (CES). CES is the monetary gains enjoyed by consumers from the acquisition of a good or service, and is measured as the difference between their willingness-to-pay and the price they actually pay. A pollution-sensitive regulator should be concerned not only with the welfare of consumers measured in monetary terms, but also with society's dis-utility from pollution, which is the pollution damage $D(P)$. Thus, we measure consumer surplus as $CS = CES - D(P)$. Social welfare, measured as the sum of producers' profits (Π) and the consumer surplus, automatically incorporates the effects of pollution damage as well. In this, we follow the approach adopted by Nault (1996), Jacobs and Subramanian (2012), and others: Welfare, $W = \Pi + CS = \Pi + CES - D$. Using this augmented measure of social welfare, our model helps us study the trade-offs between pollution and production for society, between the benefits of pollution abatement and economic efficiency, and between consumers' monetary utility from consumption and their dis-utility from pollution. Our model also enables comparisons of the different pollution control mechanisms along these different dimensions. Our notations are summarized in Table 2.1.

2.4 Developing Benchmarks

Pollution regulation is contentious, since it affects multiple economic actors including firms and consumers in myriad, complex ways. Thus, normative guidelines are needed to help frame regulations that balance the interests of these diverse constituencies. In this section, we develop a set of normative guidelines, propose regulatory benchmarks, and finally, examine how well these benchmarks meet our normative criteria. Benchmarks help clarify some of the challenges facing the regulator, and will also be useful to assess the performance of the Cap, Cap-and-Trade, and Tax regulatory mechanisms that we analyze later.

2.4.1 Normative Criteria

Any pollution control mechanism should meet three important criteria: *(i) effectiveness*, *(ii) fairness*, and *(iii) practicality*. *Effectiveness* is the ability to achieve a given pollution reduction goal–the primary reason for the regulation. *Fairness* is a controversial concept, given that there are multiple parties affected in different ways. We use the term "fairness" to

Table 2.1. Model notations (Chapter 2)

n	Number of regulated firms
q_i	Production quantity chosen by firm $i, q_i \geq 0$
Q	Total production quantity, $Q = \sum_{i=1}^{n} q_i$
Q_{-i}	Production of all the firms except firm i
γ	Coefficient of product substitutability
p_i	Price in firm i 's market, $p_i = a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}$
x_i	Abatement level chosen by firm $i, 0 \leq x_i \leq 1$
c_i	Abatement cost coefficient of firm i , $c_i \in \{c_l, c_h\}$
m	Number of firms with the low cost coefficient c_l
P_i	Pollution generated by firm $i, P_i = q_i \cdot (1 - x_i)$
P	Total pollution generated by the firms, $P = \sum_{i=1}^{n} P_i$
π_i	Profit of firm i
Π	Firms' joint profits, $\Pi = \sum_{i=1}^{n} \pi_i$
d	Pollution damage factor, $d \geq 0$
D	Pollution damage, $D = d \cdot P^2$
CES	Consumer economic surplus
CS	Consumer surplus, $CS = CES - D$
W	Social welfare, $W = \Pi + CS$
S	Pollution cap specified for the entire region (all firms)
s	Cap assigned to individual firms, $s \geq 0$
t_i	Number of emission allowances traded by firm i
τ	Tax rate, $\tau \geq 0$

mean that *(a)* the polluter pays for the damages caused by its pollution–the so-called "polluter pays principle"(Pigou 1912, OECD 1972), and *(b)* the polluting firm pays *only* for the damages it causes, i.e., it is not charged excessively ("unfairly"). Finally, a mechanism is *practical* if *(a)* it is easy to understand, *(b)* it can be implemented at reasonable control and administrative costs, and *(c)* the various decision-makers have the right information to make their decisions.

It should be understood that these criteria are not absolute, and in fact, often in conflict. A mechanism that is highly effective might be impractical for political reasons, or viewed as unfair by some constituencies. For a concrete example, consider that firms rather than the regulator are likely to have the most accurate information on pollution-reduction opportunities and costs. We know from previous research that for effective organizations, the decision rights structure should be aligned with the information structure (cf. Anand and Mendelson 1997), suggesting that firms should have a significant say on their pollution targets. However, given the conflicting goals of the regulator and the profit-maximizing firm, effectiveness might require that the regulator have the greater say in setting a firm's pollution targets, setting up a conflict between practicality and effectiveness of the regulation. Nevertheless, these normative criteria serve as useful guideposts to evaluate any proposed regulation, and any regulation should strive to fulfill these criteria as far as possible.

2.4.2 Optimizing Trade-offs: The First-Best as Benchmark

A direct approach through this morass of conflicting incentives is for the regulator to optimize the trade-offs between the costs and benefits of pollution reduction by maximizing social welfare. Presumably, the *first-best* solution (social welfare-maximization) would balance firms' pollution abatement costs against the benefits of pollution damage reduction to society. The problem of social welfare-maximization may be formulated as follows:

$$\begin{aligned}\max_{\mathbf{q}\geq 0,\ 0\leq \mathbf{x}\leq 1} W(\mathbf{q},\mathbf{x}) = \quad & \sum_{i=1}^{n}\left[q_i\cdot(a-b\cdot q_i-\gamma\cdot b\cdot Q_{-i})-c_i\cdot(q_i\cdot x_i)^2+\frac{b}{2}\cdot q_i^2\right] \\ & -d\cdot\left(\sum_{i=1}^{n} q_i\cdot(1-x_i)\right)^2 \end{aligned} \tag{2.1}$$

where $\mathbf{q}=(q_1,...,q_n)$ and $\mathbf{x}=(x_1,...,x_n)$ are the production quantity and abatement level vectors, $\frac{b}{2}\cdot q_i^2$ is the consumer economic surplus in market i, and the last term in (2.1) is the pollution damage $D(P)$, where the total *net* pollution $P=\sum_{i=1}^{n} q_i\cdot(1-x_i)$.

One major difficulty under the *first-best* scenario is that the regulator prescribes the quantity and abatement-level decisions for all the firms, thus also setting prices in all markets. Such a centralized, statist approach from a *pollution-focused regulator* (such as the Environmental Protection Agency) would be correctly viewed as excessive interference in the economy. Thus, the first-best (welfare-maximization) approach has limited merit as a benchmark, since it conflates pollution and output regulation. While *effective* for pollution regulation, it is neither fair nor practical. We explore an alternative approach in the next section.

2.4.3 Internalizing the Externality: The Groves Mechanism as Benchmark

We saw that the need for pollution regulation is largely driven by the negative externalities created by pollution (see discussion in Section 2.1). Thus, an alternative approach would be to force firms to internalize the costs of the pollution damage they create–ideally, without compromising much on their incentives to exercise market-power and maximize their profits. We adapt the Groves mechanism, a classic solution to the public good problem in economics (Clarke 1971, Groves 1973, Groves and Loeb 1975), to our context. In our context, the Groves mechanism consists of charging each firm with a corrective tax equal to the extra pollution damage that its production and abatement decisions inflict on society.

Let P_i denote the pollution generated by firm i, P_{-i} the pollution generated by all the firms except firm i, and P the total pollution (i.e., $P = \sum_{i=1}^{n} P_i$). We have the following relationships: $P_i = q_i \cdot (1 - x_i)$ and $P_{-i} = \sum_{j \neq i} P_j = P - P_i$. Recall that the pollution damage $D = d \cdot P^2$. The formula for the Groves tax charged to firm i, denoted G_i, is

$$G_i(\mathbf{q}, \mathbf{x}) = d \cdot P^2 - d \cdot P_{-i}^2 \tag{2.2}$$

Note that the taxes levied are simply a money transfer from the firms to the regulator, and so, do not directly affect welfare. However, the Groves tax affects firms' incentives (specifically, their choice of production quantities and abatement levels), and hence, welfare, indirectly. Firm i's problem ($i = 1, ..., n$) is to maximizes its profits *net* of the Groves tax:

$$\max_{q_i \geq 0,\ 0 \leq x_i \leq 1} \pi_i(q_i, x_i \mid Q_{-i}) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c_i \cdot (q_i \cdot x_i)^2 - G_i(\mathbf{q}, \mathbf{x}) \tag{2.3}$$

Theorem 1 shows that there is a unique Nash Equilibrium for the system of n optimization problems given by (2.3) ($i = 1, ..., n$), and also provides the solution. All proofs are in Appendix A.

Theorem 1 *A unique Nash Equilibrium exists under the Groves tax given by* (2.2). *There exists a threshold value of the pollution damage factor,* $\underline{d} = \frac{c_l c_h}{(n-m)(c_h - c_l)}$, *such that the equilibrium production quantities and abatement levels under the* Groves *mechanism are*[7]*:*

Case 1: $d \leq \underline{d}$

$$\begin{aligned} q_l^g &= q_h^g = \frac{a\left[\left((n-m)c_l + mc_h\right)d + c_l c_h\right]}{b\left(2 + (n-1)\gamma\right)\left(\left((n-m)c_l + mc_h\right)d + c_l c_h\right) + 2nc_l c_h d} \\ x_l^g &= \frac{nc_h d}{\left((n-m)c_l + mc_h\right)d + c_l c_h} \\ x_h^g &= \frac{nc_l d}{\left((n-m)c_l + mc_h\right)d + c_l c_h} \end{aligned}$$

The total emissions are $S^g = \frac{nac_l c_h}{b(2+(n-1)\gamma)(((n-m)c_l+mc_h)d+c_l c_h)+2nc_l c_h d}$.

[7] We use the following superscripts: g to denote the Groves mechanism, *fb* to denote the first-best solution (discussed below), *ct* for Cap-and-Trade, *cap* for Cap, and *tax* for Tax.

Case 2: $d > \underline{d}$

$$q_l^g = \frac{a\left(b\left(2-\gamma\right)+\frac{2(n-m)c_h d}{c_h+(n-m)d}\right)}{\begin{array}{c}\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]\left[b\left(2+\left(n-m-1\right)\gamma\right)+\frac{2(n-m)c_h d}{c_h+(n-m)d}\right]\\ -m\left(n-m\right)\gamma^2 b^2\end{array}}$$

$$q_h^g = \frac{a\left[b\left(2-\gamma\right)+2c_l\right]}{\begin{array}{c}\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]\left[b\left(2+\left(n-m-1\right)\gamma\right)+\frac{2(n-m)c_h d}{c_h+(n-m)d}\right]\\ -m\left(n-m\right)\gamma^2 b^2\end{array}}$$

$$x_l^g = 1$$

$$x_h^g = \frac{\left(n-m\right)d}{c_h+\left(n-m\right)d}$$

The total emissions are $S^g = \frac{(n-m)ac_h(b(2-\gamma)+2c_l)}{[b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)(c_h+(n-m)d)+2(n-m)c_h d]-m(n-m)\gamma^2 b^2(c_h+(n-m)d)}$.

Theorem 1 shows that the optimal pollution abatement levels x_l^{fb} and x_h^{fb} are strictly increasing, and the production quantities q_l^{fb} and q_h^{fb} strictly decreasing, in the pollution damage factor d. Thus, as might be expected, a more harmful pollutant requires a more stringent pollution abatement strategy. When $d = 0$ (i.e., the pollutant is harmless), it is socially optimal not to abate any pollution (i.e., $x_l^{fb} = x_h^{fb} = 0$) and have the firms produce $q_l^{fb} = q_h^{fb} = \frac{a}{b[2+(n-1)\gamma]}$. This is identical to the unregulated oligopoly outcome– each firm's output is $\frac{a}{b[2+(n-1)\gamma]}$. When d is very high, it is optimal to abate all the pollution; in this case, $x_l^{fb} = 1$ and even x_h^{fb} converges to 1. For even the slightest pollution damage (i.e., any $d > 0$), the Groves mechanism induces a reduction of output to *below* unregulated monopoly levels in every market, and further, we see that $x_l^g > x_h^g$ and $q_l^g \geq q_h^g$ i.e., firms with low pollution abatement costs produce more output and also abate more pollution than the high-cost firms.

We now return briefly to the Welfare maximization scenario, simply for the sake of evaluating the effectiveness of the Groves mechanism as a benchmark. Theorem 2 proves that the solution to the optimization problem (2.1) exists and is unique, and also provides the solution.

Theorem 2 *The* first-best *solution exists and is unique. The optimal production quantities and abatement levels in the* first-best *solution are:*

Case 1: $d \leq \underline{d}$

$$
\begin{aligned}
q_l^{fb} &= q_h^{fb} = \frac{a\left[\left((n-m)\,c_l + mc_h\right)d + c_l c_h\right]}{b\left(1+(n-1)\,\gamma\right)\left(\left(\left((n-m)\,c_l + mc_h\right)d + c_l c_h\right) + 2nc_l c_h d\right.} \\
x_l^{fb} &= \frac{nc_h d}{\left((n-m)\,c_l + mc_h\right)d + c_l c_h} \\
x_h^{fb} &= \frac{nc_l d}{\left((n-m)\,c_l + mc_h\right)d + c_l c_h}
\end{aligned}
$$

Case 2: $d > \underline{d}$

$$
\begin{aligned}
q_l^{fb} &= \frac{a\left(b\,(1-\gamma) + \frac{2(n-m)c_h d}{c_h + (n-m)d}\right)}{\begin{array}{c}\left[b\left(1+(m-1)\,\gamma\right) + 2c_l\right]\left[b\left(1+(n-m-1)\,\gamma\right) + \frac{2(n-m)c_h d}{c_h+(n-m)d}\right] \\ -m\,(n-m)\,\gamma^2 b^2\end{array}} \\
q_h^{fb} &= \frac{a\left[b\,(1-\gamma) + 2c_l\right]}{\begin{array}{c}\left[b\left(1+(m-1)\,\gamma\right) + 2c_l\right]\left[b\left(1+(n-m-1)\,\gamma\right) + \frac{2(n-m)c_h d}{c_h+(n-m)d}\right] \\ -m\,(n-m)\,\gamma^2 b^2\end{array}} \\
x_l^{fb} &= 1 \\
x_h^{fb} &= \frac{(n-m)\,d}{c_h + (n-m)\,d}
\end{aligned}
$$

As under the Groves mechanism, we find that the optimal pollution abatement levels x_l^{fb} and x_h^{fb} are strictly increasing, and the production quantities q_l^{fb} and q_h^{fb} strictly decreasing, in the pollution damage factor d. When $d = 0$ (i.e., the pollutant is harmless), it is socially optimal not to abate any pollution (i.e., $x_l^{fb} = x_h^{fb} = 0$) and have the firms produce $q_l^{fb} = q_h^{fb} = \frac{a}{b}$, driving prices in all markets to 0. This confirms the heavy-handed nature of the *first-best* solution. When d is very high, it is optimal to abate all the pollution; in this case, $x_l^{fb} = 1$ and even x_h^{fb} converges to 1. For even the slightest pollution damage (i.e., any $d > 0$), we see that $x_l^{fb} > x_h^{fb}$ and $q_l^{fb} \geq q_h^{fb}$ i.e., firms with low pollution abatement costs produce more output and also abate more pollution than the high-cost firms.

Proposition 1 compares the two proposed benchmarks–Welfare maximization (i.e., *first-best*) and the Groves mechanism. We find that total pollution is lower under the Groves tax than under the *first-best* solution obtained through welfare maximization. Thus, the Groves tax is a surprisingly effective mechanism for pollution regulation, even compared to the *first-best* solution.

Proposition 1 *The abatement levels under first-best and Groves are identical (i.e., $x_l^g = x_l^{fb}$ and $x_h^g = x_h^{fb}$). The total pollution is lower under the Groves mechanism than under first-best.*

Figures 2.2 and 2.3 plot the pollution and production outcomes under *first-best* and Groves as a function of the pollution damage factor d. The graphs are for $\gamma = 0$. However,

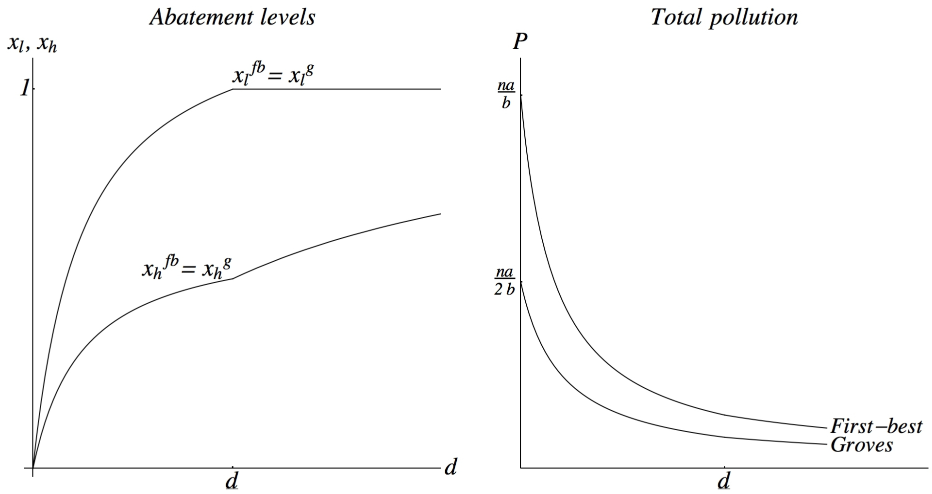

Figure 2.2. Abatement levels and Total pollution under First-best and Groves ($\gamma = 0$)

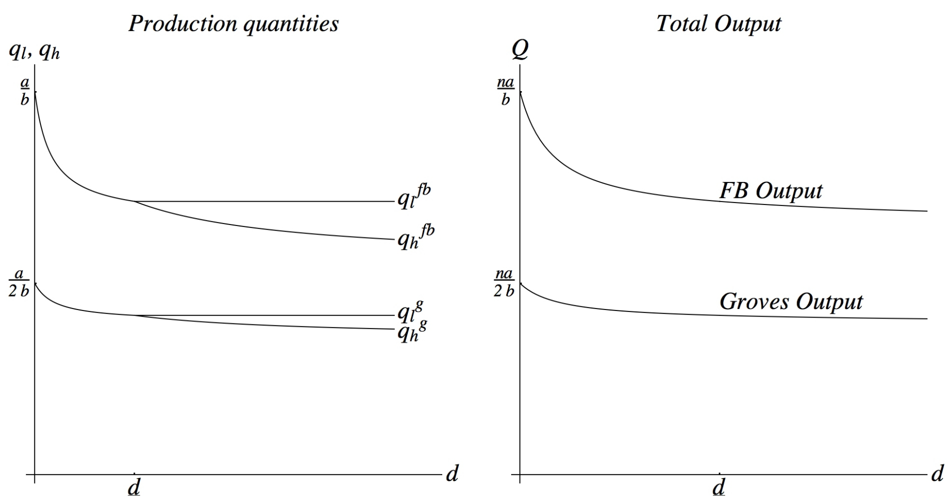

Figure 2.3. Production quantities and Total output under First-best and Groves ($\gamma = 0$)

these structural results are the same for any γ. In fact, the abatement levels do not depend on γ. Under Groves and *first-best*, the total pollution decreases as γ increases because the output decreases while the abatement levels remain constant, but the total pollution under *first-best* is always strictly greater than the pollution under Groves. One might expect the output to increase with competition. For example, it is well known that the output of a Cournot duopoly is greater than that of a monopoly. Our results differ because we are considering n markets, and not just one. The aggregate output of n monopolies is greater than the aggregate output of n oligopolistic firms producing imperfect substitutes. We see that the Groves mechanism induces firms to abate pollution exactly to *first-best* levels.

However, production quantities, both individual and aggregate, are lower under Groves than under *first-best* (see Figure 2.3). These two factors together explain why there is less pollution under Groves than under *first-best*.

The Groves mechanism meets the gold standard for *fairness*, because each firm pays for the incremental pollution damage that it inflicts on society (and no more). Unlike the *first-best* solution, the Groves mechanism also does not interfere excessively in firms' output decisions: Firms are free to maximize their profits *net* of the social cost of their pollution. In addition, we see from Proposition 1 that the Groves mechanism is surprisingly effective as well–generating less total pollution in equilibrium than even the *first-best solution*. Unfortunately, the Groves tax charged to a firm depends on the emissions of all the other firms, which are unknown to the firm when it makes its production decisions. This drawback renders the Groves mechanism impractical. Nevertheless, the Groves mechanism is suitable as a benchmark against which to measure the performance of the more practical pollution control mechanisms such as Cap, Cap-and-Trade, and Tax.

2.5 Alternative Mechanisms: Equilibrium Analysis

We saw that setting welfare-maximization as the goal of pollution regulation is neither fair nor practical, and is, besides, a clear overreach of the pollution regulator's mandate (Recall Section 2.4.2). In practice, pollution regulations such as Cap, Cap-and-Trade, and Taxes focus on pollution *targets*, usually determined by some combination of cost-benefit analyses and political expediency.

Consider first the case when the pollution constraint is slack (e.g., the cap S is set very high). The firm's unconstrained problem is to choose a production quantity $q_i \geq 0$ and an abatement level $x_i \in [0,1]$ that maximize its profit $q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c \cdot (q_i \cdot x_i)^2$. It is not difficult to see that the solution is ($x_i = 0,\ q_i = \frac{a}{b(2+(n-1)\gamma)}$)[8], and the total pollution generated is $S^u = n \cdot q_i \cdot (1 - x_i) = \frac{na}{b(2+(n-1)\gamma)}$, where the superscript u stands for *unfettered*. Thus, an *unfettered oligopoly* produces a pollution of $S^u = \frac{na}{b(2+(n-1)\gamma)}$, and any cap $\geq S^u$ is irrelevant. Another trivial case arises when $S = 0$, requiring that the firm's pollution $q_i \cdot (1 - x_i) = 0,\ \forall i$. In this case, the firm's optimal solution is ($x_i = 1, q_i = \frac{a[b(2-\gamma)+2c_{-i}]}{b^2(2-\gamma)(2+(n-1)\gamma)+2b[(2+(n-m-1)\gamma)c_l+(2+(m-1)\gamma)c_h]+4c_lc_h}$), resulting in a positive profit. Thus, the only meaningful (nontrivial) case arises in the range $S \in \left(0, \frac{na}{b(2+(n-1)\gamma)}\right)$, which will be the focus of the rest of this analysis. The firms can react to the pollution constraint in one of several ways:

[8]For details of this proof, see the proof of Theorem 3 in Appendix A.

1. *Abate pollution (as intended by the regulator)*, i.e., increase the abatement level x to bring down the pollution $q \cdot (1-x)$ to the level enforced by the regulator.

2. *Purchase emission allowances (only possible under Cap-and-Trade).* This relaxes the firm's pollution constraint, and may be less expensive than abating pollution directly.

3. *Pay emissions taxes (only possible under the Tax mechanism).* Again, this may be less expensive than abating pollution.

4. *Reduce the output.* Such a strategy mechanically brings pollution down. However, as discussed in Section 2.1, lower output (leading, presumably, to higher prices) is socially undesirable, and even more so under monopoly, as in our model.

5. *Some combination of the above options.*

In this section, we derive the *unique* (Subgame-perfect) Nash equilibrium under each type of regulation, for any *arbitrary* pollution target. We will then compare the equilibrium outcomes in Section 2.6.

2.5.1 The Cap Mechanism

Under Cap, the regulator directly assigns a cap to each firm's emissions, with a hefty fine for exceeding the cap. We assume that the penalty is sufficiently high that the firm will prefer to operate within the cap. A profit-maximizing firm i (for $i \in \{1, ..., n\}$) subject to a cap $s = S \diagup n$ solves the following problem:

$$\begin{aligned} \max_{q_i \geq 0,\ 0 \leq x_i \leq 1} \pi_i \left(q_i, x_i \mid Q_{-i}\right) &= q_i \cdot \left(a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}\right) - c_i \cdot \left(q_i \cdot x_i\right)^2 \\ \text{subject to } q_i \cdot (1 - x_i) &\leq s \end{aligned}$$

Theorem 3 gives the firm's optimal response to the Cap mechanism.

Theorem 3 *The firm's* unique *optimal strategy under the Cap mechanism is*

$$\begin{aligned} q_i^{cap} &= \frac{a\left[b\left(2-\gamma\right)+2c_{-i}\right]+2s\left[\left(2+\left(m_{-i}-1\right)\gamma\right)bc_i-m_{-i}\gamma bc_{-i}+2c_ic_{-i}\right]}{\left[b\left(2+\left(m_i-1\right)\gamma\right)+2c_i\right]\left[b\left(2+\left(m_{-i}-1\right)\gamma\right)+2c_{-i}\right]-m_im_{-i}\gamma^2b^2} \\ x_i^{cap} &= \frac{\left[b\left(2-\gamma\right)+2c_{-i}\right]\left[a-bs\left(2+\left(n-1\right)\gamma\right)\right]}{a\left[b\left(2-\gamma\right)+2c_{-i}\right]+2s\left[\left(2+\left(m_{-i}-1\right)\gamma\right)bc_i-m_{-i}\gamma bc_{-i}+2c_ic_{-i}\right]} \end{aligned}$$

where c_{-i} is the cost coefficient of the other type of firms, and m_{-i} the number of firms of the other type.

Theorem 3 shows that as s decreases (i.e., regulation becomes more stringent), the production quantities q_l^{cap} and q_h^{cap} strictly decrease while the abatement levels x_l^{cap} and x_h^{cap} strictly increase. This has several implications: *(i)* Output reduction is inevitable under the Cap mechanism. *(ii)* When the regulator lowers s, the firms simultaneously abate pollution and reduce output. Thus, the regulator achieves her pollution target partly through (desirable) pollution abatement, and partly through (undesirable) output reduction. *(iii)* Firms' reactions to the cap depend on their abatement cost coefficients. In particular, a firm with low abatement costs ($c_i = c_l$) produces a higher output and abates more pollution than a high cost firm ($c_i = c_h$), i.e., $q_l^{cap} > q_h^{cap}$ and $x_l^{cap} > x_h^{cap}$.

2.5.2 The Cap-and-Trade Mechanism

As with the Cap mechanism, the regulator assigns a cap s to each firm under Cap-and-Trade; however, firms have the additional option of trading emission allowances amongst themselves. Recall that t_i denotes the number of emission allowances sold by firm i (a negative t_i means that the firm is a net buyer of emission allowances). Clearly $t_i \leq s$, because a firm can only sell allowances up to its initial endowment. Firm i's problem is given by:

$$\begin{aligned} \max_{q_i \geq 0,\ 0 \leq x_i \leq 1,\ t_i \leq s} \pi_i \left(q_i, x_i\ , t_i \mid Q_{-i},\ r\right) &= q_i \cdot \left(a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}\right) - c_i \cdot \left(q_i \cdot x_i\right)^2 + r \cdot t_i \\ \text{subject to } q_i \cdot (1 - x_i) &\leq s - t_i \end{aligned}$$

where $\underline{t} = (t_1, t_2, ..., t_n)$ denotes the vector of trades among the firms, and r is the price of emission allowances at which the firms trade, i.e., the market clearing price. The equilibrium price r is uniquely determined by the market-clearing condition $\sum_{i=1}^{n} t_i = 0$.

Theorem 4 gives the unique Nash Equilibrium of the Cap-and-Trade game.

Theorem 4 *When all firms have the same abatement cost coefficient (i.e., $m = 0$ or $m = n$), no trading occurs ($\underline{t} = \underline{0}$), and the solution is identical to that of the Cap mechanism (Theorem 3). When $0 < m < n$, the unique Nash Equilibrium of the Cap-and-Trade game is given by:*

Case 1: $\underline{S} \leq S < S^u$

$$\begin{aligned} q_l^{ct} &= q_h^{ct} = \frac{a\left(\left(n-m\right)c_l + mc_h\right) + 2c_lc_hS}{\left(2 + \left(n-1\right)\gamma\right)b\left(\left(n-m\right)c_l + mc_h\right) + 2nc_lc_h} \\ x_l^{ct} &= c_h \frac{na - b\left(2 + \left(n-1\right)\gamma\right)S}{a\left(\left(n-m\right)c_l + mc_h\right) + 2c_lc_hS} \\ x_h^{ct} &= c_l \frac{na - b\left(2 + \left(n-1\right)\gamma\right)S}{a\left(\left(n-m\right)c_l + mc_h\right) + 2c_lc_hS} \end{aligned}$$

where $\underline{S} = \frac{(n-m)a(c_h-c_l)}{c_h[b(2+(n-1)\gamma)+2c_l]}$.

The market clearing price is $r = 2c_l c_h \frac{na-b(2+(n-1)\gamma)S}{(2+(n-1)\gamma)b((n-m)c_l+mc_h)+2nc_lc_h}$, *and each low-cost firm sells* $t_l = \frac{(n-m)(c_h-c_l)[a-b(2+(n-1)\gamma)s]}{b(2+(n-1)\gamma)((n-m)c_l+mc_h)+2nc_lc_h}$ *emission allowances to the high-cost firms.*

Case 1: $\underline{S} \le S < S^u$

$$
\begin{aligned}
q_l^{ct} &= \frac{a\left[b\left(2-\gamma\right)+2c_h\right]-2\gamma b c_h S}{\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]\left[b\left(2+\left(n-m-1\right)\gamma\right)+2c_h\right]-m\left(n-m\right)\gamma^2 b^2} \\
q_h^{ct} &= \frac{a\left[b\left(2-\gamma\right)+2c_l\right]+2c_h\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]S\diagup\left(n-m\right)}{\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]\left[b\left(2+\left(n-m-1\right)\gamma\right)+2c_h\right]-m\left(n-m\right)\gamma^2 b^2} \\
x_l^{ct} &= 1 \\
x_h^{ct} &= \frac{a\left[b\left(2-\gamma\right)+2c_l\right]-b\left[b\left(2-\gamma\right)\left(2+\left(n-1\right)\gamma\right)+2c_l\left(2+\left(n-m-1\right)\gamma\right)\right]S\diagup\left(n-m\right)}{a\left[b\left(2-\gamma\right)+2c_l\right]+2c_h\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]S\diagup\left(n-m\right)}
\end{aligned}
$$

The market clearing price is $r = 2c_h \frac{a[b(2-\gamma)+2c_l]-b[(2-\gamma)(2+(n-1)\gamma)b+2c_l(2+(n-m-1)\gamma)]S\diagup(n-m)}{[b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)+2c_h]-m(n-m)\gamma^2b^2}$, *and the low-cost firms sell all their emission allowances* ($t_l = s$) *to the high-cost firms.*

The structural results under Cap-and-Trade are similar to Cap. When the regulator lowers S, both types of firms simultaneously abate pollution and reduce output. Thus, output reduction is inevitable under Cap-and-Trade. Low-cost firms abate pollution faster. $\underline{S}$ corresponds to the cap at which low-cost firms have abated all their pollution, and further abatement is achieved solely by the high-cost firms.

2.5.3 The Tax Mechanism

We assume in our model that the regulator charges a tax proportional to the firm's emissions, i.e., firm i pays a tax $\tau \cdot q_i \cdot (1-x_i)$, where τ is the linear tax rate and $q_i \cdot (1-x_i)$ is firm i's emissions. Stavins (2003) identifies seven subcategories of environmental taxes. With the exception of fixed administrative charges such as permit fees, taxes are almost universally linear in the pollution generated. Linear taxes are simple to understand and implement, and moreover, are analytically tractable.

In choosing the tax rate τ, the regulator anticipates firms' reactions, and chooses the minimum τ to ensure that the total pollution generated by the firms is at most S. Recall that the regulator does not know which firm has which cost coefficient, but knowing m is a sufficient statistic for determining the tax rate that achieves S. Then, each firm chooses its production quantity and pollution abatement level to maximize its profits net of pollution taxes. The firms simultaneously solve

$$\max_{q_i \ge 0,\ 0 \le x_i \le 1} \pi_i\left(q_i,\ x_i \mid Q_{-i},\ \tau\right) = q_i \cdot \left(a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}\right) - c_i \cdot \left(q_i \cdot x_i\right)^2 - \tau \cdot q_i \cdot \left(1 - x_i\right)$$

We solve the two-stage game of our model using backward induction. First, we solve for the firms' optimal production quantities and abatement levels as a function of τ. Then, we plug the firms' decisions into the regulator's problem, which is to find the minimum τ such that $P \leq S$. Theorem 5 shows that there is a *unique* Subgame-Perfect Nash Equilibrium for the two-stage Tax game.

Theorem 5 *The unique Subgame-Perfect Nash Equilibrium of the Tax game is given by:*

Case 1: $\underline{S} \leq S < S^u$

The regulator chooses $\tau = 2c_l c_h \frac{na-b(2+(n-1)\gamma)S}{(2+(n-1)\gamma)b((n-m)c_l+mc_h)+2nc_l c_h}$.

The firms react with

$$\begin{aligned} q_l^{tax} &= q_h^{tax} = \frac{a\left((n-m)\,c_l+mc_h\right)+2c_l c_h S}{\left(2+(n-1)\,\gamma\right)b\left((n-m)\,c_l+mc_h\right)+2nc_l c_h} \\ x_l^{ct} &= c_h \frac{na-b\left(2+(n-1)\,\gamma\right)S}{a\left((n-m)\,c_l+mc_h\right)+2c_l c_h S} \\ x_h^{ct} &= c_l \frac{na-b\left(2+(n-1)\,\gamma\right)S}{a\left((n-m)\,c_l+mc_h\right)+2c_l c_h S} \end{aligned}$$

Case 2: $0 < S < \underline{S}$

The regulator chooses $\tau = 2c_h \frac{a[b(2-\gamma)+2c_l]-b[(2-\gamma)(2+(n-1)\gamma)b+2c_l(2+(n-m-1)\gamma)]S/(n-m)}{[b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)+2c_h]-m(n-m)^2\gamma^2 b^2}$.

The firms react with

$$\begin{aligned} q_l^{tax} &= \frac{a\left[b\left(2-\gamma\right)+2c_h\right]-2\gamma b c_h S}{\left[b\left(2+(m-1)\,\gamma\right)+2c_l\right]\left[b\left(2+(n-m-1)\,\gamma\right)+2c_h\right]-m\left(n-m\right)\gamma^2 b^2} \\ q_h^{tax} &= \frac{a\left[b\left(2-\gamma\right)+2c_l\right]+2c_h\left[b\left(2+(m-1)\,\gamma\right)+2c_l\right]S/\left(n-m\right)}{\left[b\left(2+(m-1)\,\gamma\right)+2c_l\right]\left[b\left(2+(n-m-1)\,\gamma\right)+2c_h\right]-m\left(n-m\right)\gamma^2 b^2} \\ x_l^{tax} &= 1 \\ x_h^{tax} &= \frac{a\left[b\left(2-\gamma\right)+2c_l\right]-b\left[b\left(2-\gamma\right)\left(2+(n-1)\,\gamma\right)+2c_l\left(2+(n-m-1)\,\gamma\right)\right]S/\left(n-m\right)}{a\left[b\left(2-\gamma\right)+2c_l\right]+2c_h\left[b\left(2+(m-1)\,\gamma\right)+2c_l\right]S/\left(n-m\right)} \end{aligned}$$

We see that output reduction is inevitable under emissions taxes as well. When the regulator wants to make the pollution targets more stringent, she has to raise the tax rate τ. In response, both types of firms simultaneously abate pollution and reduce output.

2.6 Analysis and Comparisons

2.6.1 Cap-and-Trade versus Cap

Clearly, the differences in outcomes under Cap-and-Trade and Cap are solely due to the ability to trade emission permits, since the models are identical in all other respects. Thus, their comparison illustrates the value of tradable emissions. The results that follow hold for any γ :

- Low-cost firms abate more pollution under Cap-and-Trade than under Cap (i.e., $x_l^{ct} > x_l^{cap}$) because they can sell emission allowances to the high-cost firms, who in turn need to abate less pollution (i.e., $x_h^{ct} < x_h^{cap}$).

- Total abatement costs across all firms are lower under Cap-and-Trade, because high-cost firms purchase emission allowances from low-cost firms instead of spending on (costly) pollution abatement.

- The opportunity to trade also drives output. *Ceteris paribus*, low-cost firms produce less under Cap-and-Trade (i.e., $q_l^{ct} < q_l^{cap}$) because this lowers their pollution; hence, they can sell more emission allowances to the high-cost firms, who can then increase their output–thus, $q_h^{ct} > q_h^{cap}$. We see that total production (across all firms) is greater under Cap-and-Trade than under Cap for any S and γ, because emissions trading exploits abatement cost heterogeneity. Thus, *the ability to trade under Cap-and-Trade mitigates the output reduction effect.*

Figures 2.4 and 2.5 illustrate these results (for $\gamma = 0$. The results are structurally the same for other values of γ).

Firms' total profits are also higher under Cap-and-Trade than Cap, because of higher output and lower pollution abatement costs. Higher output also translates into greater consumer economic surplus under Cap-and-Trade. Total pollution, and hence the pollution damage, are the same under both mechanisms. So consumer surplus (which is consumer economic surplus net of pollution damage) is also higher under Cap-and-Trade. Since both

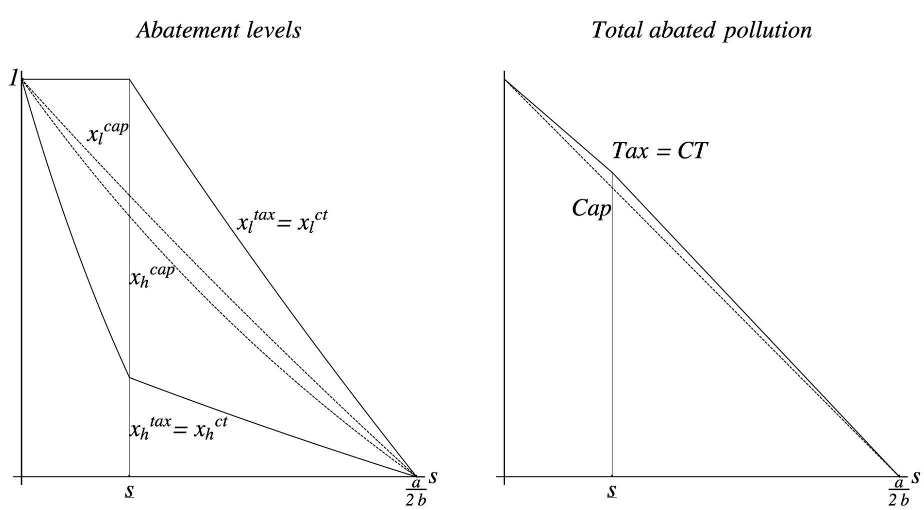

Figure 2.4. Abatement levels and Total abated pollution for Cap, Cap-and-Trade, and Tax ($\gamma = 0$)

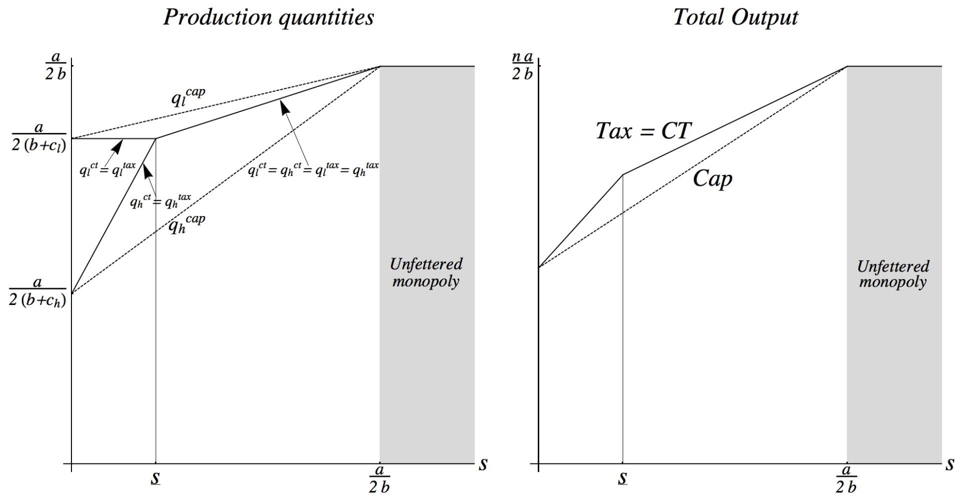

Figure 2.5. Production quantities and Total output for Cap, Cap-and-Trade, and Tax ($\gamma = 0$)

firms and consumers are better off, social welfare is also higher under Cap-and-Trade. These results are summarized in Proposition 2 below.

Proposition 2 *For any γ, firms' total output and profits, the total abated pollution, consumer economic surplus, consumer surplus, and social welfare are all higher under Cap-and-Trade than Cap.*

Proposition 2 shows that Cap-and-Trade outperforms Cap by all measures. This difference is amplified when the abatement cost c_l is small. Observe that for $c_l \longrightarrow 0$, and $S \geq \underline{S}$, the output under Cap-and-Trade is close to $\frac{na}{b[2+(n-1)\gamma]}$, the total output of n unfettered producers. In other words, the output reduction effect is almost entirely mitigated, whereas under Cap, the output reduction is still significant. The intuition is that when c_l is small, low-cost firms can produce at a level close to the unfettered scenario, and further, support most of the pollution abatement for high-cost firms, allowing them also to purchase emissions and produce at close to the level of the unfettered scenario. This observation emphasizes the relevance of Cap-and-Trade when a cheap pollution abatement technology exists. With technological innovation, the cost of pollution abatement is likely to decrease over time, suggesting that Cap-and-Trade is likely to improve all outcomes in the long run, relative to the pure Cap model. *Ceteris paribus*, Cap-and-Trade provides the highest benefits relative to Cap when the number of low-cost and high-cost firms are almost equal, ensuring adequate supply and demand of emission permits.

2.6.2 Cap-and-Trade versus Tax

Comparing Theorems 4 and 5, we see that Cap-and-Trade and Tax induce identical output and abatement levels from firms for any γ. Consequently, firms' total output, total abated pollution, consumer economic surplus, consumer surplus, and social welfare are identical under the two regimes. Firms' profits are greater under Cap-and-Trade than under Tax; the difference is accounted for by the pollution tax paid to the regulator. Proposition 3 summarizes these results.

Proposition 3 *(i) For any pollution target, and for any γ, firms' total output, the total abated pollution, consumer economic surplus, consumer surplus, and social welfare are all identical under Cap-and-Trade and Tax. (ii) The one exception is firms' profits: $\forall$ S, $\forall$ i, $\pi_i^{CT} > \pi_i^{T}$. The difference $\pi_i^{CT} - \pi_i^{T}$ is exactly equal to the emission taxes paid by firm i to the regulator, viz., $\tau \cdot s = \tau \cdot S/n$. (iii) $\forall$ S, the tax rate τ is equal to the Cap-and-Trade equilibrium (market-clearing) price r.*

For every cap S, there is a unique tax rate τ that induces firms to reduce pollution in a manner identical to Cap-and-Trade. Further, this tax rate is equal to the equilibrium (market-clearing) price r under Cap-and-Trade. The effect of either mechanism is to shift the burden of pollution abatement from the high-cost firms to the low-cost firms. Under Cap-and-Trade, low-cost firms are rewarded for bearing a higher load of the pollution reduction through the sale of surplus emission permits. Under Tax, they abate more pollution simply because abatement is cheaper than paying more emission taxes, until their marginal cost of pollution abatement equals the tax rate.

Proposition 3 shows that Taxes and Cap-and-Trade are equivalent for any γ. The general equivalence between price and quantities under perfect competition (Weitzman 1974) extends to imperfect competition. In particular, our result extends the findings in Requate (1993a), who showed such an equivalence for n local monopolies (which in our model corresponds to $\gamma = 0$).

The equivalence of the Tax and Cap-and-Trade mechanisms can be formally shown by observing that the firms' programs under each mechanism differ only by a constant. Recall that under Cap-and-Trade, firm i's objective is $\max_{q_i \geq 0,\ 0 \leq x_i \leq 1,\ t_i \leq s} \pi_i\left(q_i, x_i \mid Q_{-i}\right) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c_i \cdot (q_i \cdot x_i)^2 + r \cdot t_i$, subject to $q_i \cdot (1 - x_i) \leq s - t_i$. First, we show that the pollution constraint is binding under Cap-and-Trade. Suppose it is not; then the firm can increase t_i slightly, or decrease x_i slightly to make the constraint binding holding everything else constant. This action improves its profits, as long as $r \geq 0$. If $t_i = s$, the firm can no longer increase t_i but the pollution constraint is binding because the upper bound of the pollution constraint is $s - t_i = 0$. Since the pollution constraint is binding, we have $t_i = s - q_i \cdot (1 - x_i)$. Substituting in the objective function, the program becomes[9] $\max_{q_i \geq 0,\ 0 \leq x_i \leq 1} \pi_i (q_i, x_i | Q_{-i}) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c_i \cdot (q_i \cdot x_i)^2 - r \cdot q_i \cdot (1 - x_i) + r \cdot s$. Let $\tau = r$. This is exactly the Tax mechanism up to the constant $r \cdot s$. Under Cap-and-Trade, the price, r, at which firms trade emission allowances is determined by the market clearing condition $\sum_{i=1}^{n} t_i = 0 \iff P = S$, which means that the total pollution constraint is satisfied. Note that τ is also determined in such a way that the total pollution $P = S$.

Stiglitz (2007) argued that: "[A] tax would increase global efficiency. Of course, polluting industries like the cap-and-trade system. While it provides them an incentive not to pollute, emission allowances offset much of what they would have to pay under a tax system. Some firms can even make money off the deal." Proposition 3 shows that, in fact, *all* firms "make money off the deal"– but only because they save on the taxes paid to the regulator. The difference in the total profits across all firms, between Cap-and-Trade and Tax, is the tax payment $\tau \cdot S$. In principle, the regulator could correct this gap in profits through a lump-sum payment (or subsidy) to each firm equal to $\tau \cdot s$. In this spirit, several initiatives to make pollution taxes "revenue-neutral" have been proposed recently–to ensure that pollution taxes are used exclusively for externality correction, not for revenue generation.

2.6.3 The Effect of Competition

We found that output reduction is inevitable under all three pollution regulation mechanisms analyzed. By reducing output, firms reduce pollution; in addition, the higher prices they are able to charge partially compensates them for the lower sales volume. When firms invest in pollution abatement, there is no such compensatory dynamic. When markets are imperfect, and firms can set quantities, the firms exercise their market power to reduce demand even further.

As γ increases, the products become more and more similar, and the competition more fierce. In this section, we discuss how output and abatement efforts vary as the competition intensifies while the regulator maintains the same pollution target S. Two related observations arise: first, the output reduction effect is less pronounced when the competition intensifies. In other words, the decrease in total output relative to the unfettered level is smaller if γ is higher (for the same fixed S); secondly, the abatement levels *for both types* of firms and *for all mechanisms* are increasing in γ. This means that a more intense competition forces firms to increase their abatement efforts. The intuition behind this

[9] The constraint $t_i \leq s$ is automatically satisfied.

finding is that competition limits the firms' ability to use quantities strategically. Since the firms cannot reduce output as easily, they are compelled to abate more pollution. Thus, *from a pollution regulation standpoint, competition has a beneficial effect.*

Proposition 4 summarizes these results.

Proposition 4 *Competition mitigates the output reduction effect. The output reduction effect is less pronounced when competition increases. The abatement levels for both types of firms and for all mechanisms are increasing in the competition intensity.*

2.6.4 Welfare Considerations

Clearly, the *output reduction* effect common to all mechanisms has a negative impact on consumer economic surplus. The effect of regulation on welfare is a trade-off between lower firm profits and lower consumer economic surplus (on the one hand) and avoidance of pollution damage on the other. The magnitude of pollution damage, i.e., the social dis-utility from pollution, is driven by the pollution damage factor d. Theorem 6 shows that pollution regulations improve welfare for a very large range of parameters.

Theorem 6 *Tax and Cap-and-Trade improve welfare iff* $d > \frac{bc_lc_h(1-(n-1)\gamma)}{2nc_lc_h+(2+(n-1)\gamma)b((n-m)c_l+mc_h)}$.

Theorem 6 shows that there is a threshold value of the pollution damage factor above which both Tax and Cap-and-Trade improve welfare. Intuitively, regulation is desirable for a pollutant with a high pollution damage factor. Conversely, when d is small (i.e., less than the threshold), the pollutant's harmful effects are mild, and so regulation worsens social welfare (similar to Buchanan's (1969) finding–recall the discussion in Section 2.2) by constraining output excessively, leading to lower firm profits and lower consumer economic surplus. In this case, *laissez-faire* is the optimal policy. The shaded area of Figure 2.6 corresponds to parameter values for which regulation improves social welfare.

As the figure illustrates, it is generally the case that Cap-and-Trade and Tax regulations will improve welfare. Specifically, *regulations always improve social welfare* when $\gamma > 1/n-1$, where n is the number of regulated firms. Thus, our findings show that Buchanan's (1969) finding that regulating a single polluting monopoly with a Pigovian tax could reduce welfare, would be limited to cases where there is very little competition, and the pollutant is not too harmful. In practice, pollution regulations would improve welfare in the majority of cases. In Figure 2.6, $d_0 = \frac{bc_lc_h}{2n(c_lc_h+b\widetilde{c})}$ denotes the threshold value of the pollution damage factor above which regulations improve welfare if the firms are local monopolies, i.e., $\gamma = 0$. When c_l is small, d_0 is small also. This suggests that pollution regulations are even more

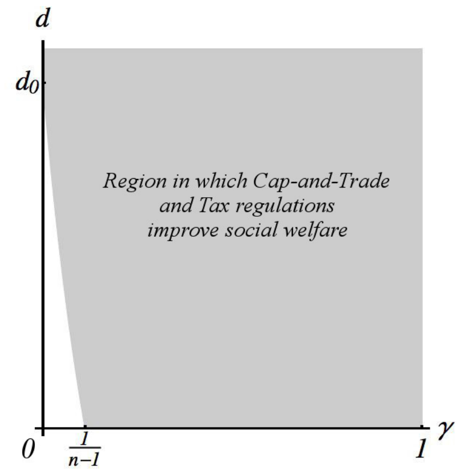

Figure 2.6. Region in which Cap-and-Trade and Tax regulations improve social welfare.

likely to improve social welfare when a low-cost abatement technology is available and operated by some firms. One could expect the cost of pollution abatement to go down over time as technologies scale up and improve, making the case for pollution regulations even more compelling.

2.6.5 Comparisons with the Groves Benchmark

We saw that the Groves mechanism is both fair and effective, but impractical (Section 2.4). In this section, we explore how well these practical mechanisms (Cap, Cap-and-Trade, and Tax) can mimic the Groves mechanism. We then compare these outcomes to welfare-maximization, with important policy consequences.

It is easy to show that for any d, the total pollution under Groves, S^g (given by Theorem 1), can be attained under Cap or Cap-and-Trade by setting the cap $S = S^g$. A total pollution of S^g can also be attained under Tax by setting the tax rate $\tau = \tau^g$, where τ^g is the tax rate that corresponds to S^g, i.e., $\tau^g = \tau\left(S^g\right)$. (The function τ is given in Theorem 5). Proposition 5 shows that, remarkably, for any γ, *both Cap-and-Trade and Tax can induce outcomes (i.e., abatement levels and output quantities at the firm level) identical to the Groves mechanism,* simply by setting $S = S^g$ and $\tau = \tau^g$, respectively. The Cap mechanism cannot.

Proposition 5 *Either Cap-and-Trade or Tax can exactly mimic the Groves mechanism by setting the cap S^g or the tax rate τ^g. The Cap mechanism cannot.*

Proposition 5 implies that the regulator can, simply by choosing the cap or tax rate judiciously under Cap-and-Trade or Tax, induce an outcome that is *(a) fair*: Every firm pays exactly for the pollution damage externalities that it imposes on society, *(b) effective*: Any pollution target (depending on the pollution damage factor d of the specific pollutant and other factors) can be achieved and, moreover, *(c) practical*: Unlike the Groves mechanism, the firm knows its initial emissions allocation (Cap) or the tax rate before it makes its production decisions. By setting $S = S^g$, the Cap mechanism can achieve the same pollution level as Groves, but the total output, the total abated pollution, consumer surplus, and welfare are all lower than Groves. Thus the Cap mechanism, unlike Cap-and-Trade, does not perform as well as Groves, illustrating the power of the trading mechanism once again.

An important policy question is whether, and under what conditions, a welfare-maximizing regulator should replicate the Groves mechanism using either Cap-and-Trade or Tax (by setting $S = S^g$ or $\tau = \tau^g$). To answer this, Theorem 7 compares S^g, the cap under the Cap-and-Trade mechanism that induces the Groves outcome, with S^*, the cap that maximizes social-welfare under Cap-and-Trade. We focus on the two polar cases: $\gamma = 0$, the case of local monopolies, and $\gamma = 1$, the Cournot scenario, because the large number of parameters in our model compounds the welfare calculations for any γ.

Theorem 7 *When* $\gamma = 0$, $S^g < S^*$ *for any* $d > 0$.
When $\gamma = 1$, $S^* < S^g$ *for any* $d > 0$.

Theorem 7 shows that the Groves mechanism is overly restrictive from the perspective of welfare-maximization when the firms are local monopolies (i.e., $\gamma = 0$), but not sufficiently strict when firms compete à la Cournot (i.e., $\gamma = 1$). Firms respond to regulatory constraints by both abating pollution and reducing output. When firms are local monopolies, output reduction raises prices, lowers consumers' economic surplus, and reduces welfare. By taxing firms for the entire pollution they produce, the Groves mechanism neglects the effect of output reduction on welfare. On the other hand, a welfare-maximizing cap should balance the social costs of pollution damage with the *full* costs of output reduction to society. Thus, S^* is higher than S^g, meaning that to maximize welfare, local monopolies should not be constrained to the full extent of the pollution damage they cause– the gains in firms' profits and consumer economic surplus from the higher cap more than compensate for any additional pollution damage. This result extends Barnett's (1980) result who finds that the optimal tax rate on a single monopoly may be less than the marginal pollution damage.

When the product is a homogeneous commodity resulting in intense competition between the firms (i.e., $\gamma = 1$), the result is reversed. To maximize welfare, the regulator must increase regulatory stringency above the level corresponding solely to the firms' pollution damage, because the reduction in pollution damage now dominates the welfare losses due to output reduction. Again, competition is good for pollution regulation. Setting the cap at S^g is suboptimal, and a strategic regulator should set a more aggressive cap; doing so allows more pollution reductions, while tapping into competitive forces that will prevent the firms from reducing output too much. In other words, the regulator can use competition to her advantage. Results for Tax are analogous, using τ^g and τ^* instead of S^g and S^*. Proposition 6 summarizes this insight.

Proposition 6 *The regulator's optimal strategy under Tax and Cap-and-Trade depends on how intense the competition is. To maximize welfare: the regulator must subsidize firms' pollution when the firms are local monopolies; the regulator must over-penalize firms' pollution when the firms compete à la Cournot.*

2.7 Concluding Remarks

Several years ago, research into the field of "supply chain management" exploded in response to the increasing importance of *interfirm* operational issues. This research integrated interfirm coordination, information and agency issues within the framework of traditional research in Operations. Similarly, sustainable operations is an increasingly important research area that expands the scope of traditional supply chain management to include environmental considerations. In this spirit, our model contributes several analytical "building-blocks"–notably of pollution generation, abatement, damage, and regulation–that can be integrated into traditional supply chain models for use by future researchers in sustainable operations.

CHAPTER 3

DOES CAP-AND-TRADE ENABLE COLLUSION?

3.1 Introduction

Emission Taxes and Cap-and-Trade are two leading approaches for pollution regulation. Proponents of Taxes have argued that Cap-and-Trade could facilitate collusion among firms via the trading mechanism, leading to suboptimal welfare outcomes. We examine this claim using a rigorous yet rich model of production and pollution under competition that allows for the possibility of collusion among firms via trading.

Beginning in the 1970s, regulators around the world have started using various mechanisms to regulate pollution from industrial sources. Centralized command-and-control mechanisms, such as technology mandates and performance standards, are a common form of regulation.[1] As discussed in Chapter 2, such centralized mandates are suboptimal, and decentralized mechanisms relying on economic incentives should be preferred. In this paper, we analyze and compare two widely used decentralized mechanisms for pollution control: Tax and Cap-and-Trade. Under the Tax mechanism, the regulator charges each firm with a tax proportional to its pollution. Under Cap-and-Trade, the regulator directly imposes a pollution limit (the "cap") on firms, but firms have the ability to trade to lower or increase their individual caps (see Chapter 2 for more details on Cap-and-Trade).

Polluting industries are typically large and concentrated. Consider the case of the U.S. power generation industry. It generates large quantities of sulfur dioxide, nitrogen oxides, ozone, mercury, particulate matter, and carbon dioxide, all of which adversely affect human health and the natural environment. In 2010, the U.S. Environmental Protection Agency

[1] An example of technology mandate is the Zero-Emission Vehicles program of the California Air Resources Board. It requires large-volume manufacturers to produce a minimum percentage of electric and hybrid vehicles, namely 12% for the period 2012-2014, and 14% for 2015 through 2017 (California Secretary of State 2012). Performance standards that limit the maximum permissible emission rate for a particular technology are also very widely used. See, for example, the proposal by the U.S. Environmental Protection Agency to limit carbon dioxide emissions by new natural gas power plants to 1,000 pounds per megawatt-hour (Shear 2013).

(EPA) released a study on market concentration in the power generation industry in 31 continental states and the District of Columbia (U.S. EPA 2010). The study found that around 5% of the generation owners control more that 40% of the overall generation capacity. Power generation is highly concentrated in many states: in Tennessee, one firm controls 100% of the generation capacity; in Delaware, Maryland, Nebraska, and North Carolina, two firms control more than 90%; in Alabama, Georgia, Illinois, Iowa, Kansas, Kentucky, Massachusetts, Michigan, Mississippi, South Carolina, and West Virginia, one or two firms control more than 60%. The study also calculated Herfindahl-Hirschman Indices (Rhoades 1993) scores in every state, and found high scores in 25 out of 31 states, suggesting that the industry is concentrated and that market power is likely to be present. Mansur (2007) found empirical evidence for the presence of market power in the Pennsylvania, New Jersey, and Maryland electricity market, the world's largest restructured electricity market.

With the exercise of market power comes the risk that powerful firms may take advantage of the flexibility offered by market mechanisms to shift the burden of pollution control to smaller firms, rivals, and potential entrants, and by so doing increase their market position even more. Heyes (2009) discusses several examples of environmental regulations that have weakened competition: for example, environmental regulations in the U.S. have put small firms at a disadvantage relative to large firms (Pashigian 1984), and have resulted in fewer small business creations (Dean et al. 2000). Ryan (2012) found that the Clean Air Act Amendments of 1990 raised the costs of entry in the U.S. cement industry by 35%.

A popular argument *against* Cap-and-Trade is that it is vulnerable to market manipulations by firms who try to bypass pollution regulations at the expense of society. Shapiro (2007) commented that "unlike cap-and-trade schemes, carbon taxes cannot be manipulated by the markets" (see also Stiglitz 2007). Market manipulations are not possible under taxes, because (i) the regulator sets the tax rate, not the firms, and (ii) the firms pay the same tax rate regardless of their market position. As a result, the firms' responses to emission taxes are independent of each other. Under Cap-and-Trade, however, the firm's trading quantity affects the permit price paid by all the firms. Strategic trading in the market for emission allowances could impact the firms' production costs, and indirectly the conditions of competition in the output market. More importantly, trading gives firms a mechanism for collusion whereby they may collectively choose a trading strategy to reach some common goal. Such a mechanism does not exist with taxes.

We define collusion as a form of market manipulation in which firms that sell in competitive markets cooperate to set prices. Through collusion, the firms seek to collectively control the output market as one monopoly. Collusive price-fixing is forbidden by antitrust laws in many countries. For example, see the 1890 Sherman Act in the U.S., and articles 101 and 102 of the Treaty on the Functioning of the European Union. In this paper, we examine the firms' incentives to collude using an integrated pollution-production model that allows for the possibility of collusion among firms via trading. We analyze several related questions: does cap-and-trade enable collusion between the regulated firms? If collusion is possible, what is its effect on firms, consumers, and society as a whole? What can the regulator do to limit the possibly negative consequences of collusion?

3.2 Literature Review

Antitrust regulations encourage competition to resist the firms' desire to monopolize, and ensure that consumers are not overcharged. A well-known collusion model is analyzed and discussed in Gibbons (1992) and Viscusi et al. (2000). In it, two Cournot duopolists with linear production technology and inverse demand function compete for output. As a result of competition, the duopolists produce more than the monopoly output. The firms can maximize their profits if each of them charges exactly half the monopoly quantity. However, such an arrangement is difficult to enforce because each firm would like to produce a little more to increase its revenues at the expense of its competitor. In other words, the collusive outcome is not a Nash equilibrium. Friedman (1971) was the first to show that a collusive Nash equilibrium can be achieved in a repeated Cournot game (i.e., two symmetric firms repeatedly play the one-period Cournot game) by the use of a trigger strategy in which collusion is played at every period until a deviation is observed; from that point on, collusion breaks down and the game forever moves to the Cournot equilibrium. Due to the inherent difficulty in enforcing collusion, there have been some empirical studies focusing on cartel stability and durability (see, for example, Grossman 2004). The empirical literature on collusion is sparse because collusion is difficult to observe. Connor (2007) analyzed hundreds of social science reports and judicial decisions from antitrust authorities published between 1770 and 2004 around the world. He found (only) 262 cartels suspected of practicing some form of price-fixing, resulting in an average surcharge of 25% compared to the price of competitive benchmarks. In this paper, we also adopt the Cournot duopoly model, but we focus on a one-period model in which trading in the market for emission allowances is the mechanism whereby collusion can take place. Our objective is not to understand cartel self-policing; it is to know if emission trading can give rise to collusion.

As discussed in the introduction, there is some empirical evidence that environmental regulations have weakened competition by increasing market concentration or deterring entry and small business creation (Heyes 2009), but very little work on collusion per se. Burtraw et al. (2009) conduct experiments to investigate whether collusion in emission allowance auctions would be more likely under discriminatory price, uniform price, or clock auctions. We do not model the emission allowance allocation process. We take the initial allocations as given, and focus on the trading process.

Requate (2006) reviews several papers studying pollution regulations (especially emission taxes) under imperfect competition, of which Requate (1993b) and Von der Fehr (1993) are closest to ours. In Requate (1993b), two asymmetric firms with linear production technologies engage in Cournot competition. The firms differ in their marginal production costs and emission rates. Production generates pollution, and the firms have no other abatement option than reducing output. Requate (1993b) compares a linear emission Tax to a Cap-and-Trade system in which firms trade to maximize their joint profits. He shows that Tax and Cap-and-Trade differ from each other; neither can implement the welfare-maximizing outcome; and no mechanism is always superior to the other. Recall that in Chapter 2, we prove the equivalence of Tax and Cap-and-Trade under more general assumptions. In this paper, we resolve this apparent contradiction.

Von der Fehr (1993) specifically studies the firms' incentives under Cap-and-Trade to use trading strategically to increase their market power (i.e., monopolize) or exclude entry. He models a symmetric Cournot duopoly (i.e., the firms have the same cost structure) in which a dominant firm is initially allocated more emission allowances than its competitor, because it is bigger. Von der Fehr shows the existence of Pareto-improving trades between the firms, and that the dominant firm can use these trading opportunities to its advantage. By trading, the dominant firm can influence the permit price to lower its own abatement costs, and improve its strategic position in the output market; this manipulation also impacts its rival's cost structure. He shows that, if the products are homogeneous, monopolization can occur (under some conditions on the firms' cost structures). However, monopolization is less likely when the products are differentiated and the cost functions exhibit dis-economies of scale. He also shows that, if the quantities are strategic substitutes, the exercise of market power in the output market causes firms to over-invest in emission rights. This is because buying more emission rights lowers the buyer's marginal cost, which allows it to be more aggressive in the output market. The rival facing a decreasing output price is forced to reduce its production quantity, which improves the profits of the first. This is an example of a Top-Dog commitment strategy in which the decision to over-invest (i.e., act tough) causes the rival to behave less aggressively. This result follows from Von der Fehr's assumption that marginal costs are decreasing in the number of emission rights.

Under cap-and-trade, ad-hoc markets are created for the exchange of emission allowances. Such markets may themselves be subject to imperfections, in particular if some firms receive a large percentage of the initial emission allowances. For example, in the initial phase of the U.S. Acid Rain Program, the first large-scale implementation of cap-and-trade system in the world, four companies were allocated 43 percent of the total emission allowances (Liski and Montero 2011). Thus, imperfect competition in the output market may interact or be compounded by imperfections in the allowance market. Hahn (1984) was the first to study the situation in which a firm exercises power in the trading market (while all the other firms are price-takers), but the output market is perfectly competitive. He shows that if the dominant firm is a buyer of permits, it will tend to buy more emission allowances in order to keep the allowance price down. If it is a seller, it will sell less to bring the price up. Thus, in equilibrium, the firms' aggregate compliance costs are higher than if the permit market was perfectly competitive. Misiolek and Elder (1989) extend the work in Hahn (1984) by studying what would happen if the dominant firm intentionally purchased more permits to exclude its rivals. They find that such exclusionary manipulations could improve or aggravate the efficiency losses due to the dominant firm's power in the permit market.

Our contribution is threefold:

- We explicitly model the trading mechanism under Cap-and-Trade, and analyze two different trading modes: in the first one, trading occurs at the time of production. This means that the production, abatement, and trading decisions are simultaneous. In the second one, the firms trade *before* production is realized. This subtle distinction in the timing of trading offers a clean and insightful comparison of the mechanisms in a manner not previously done. By comparing two variants of Cap-and-Trade to the Tax mechanism under which no collusion is possible, we are able to generate new insights into the firms' incentives to collude. Von der Fehr (1993) finds that collusion is possible under limited conditions. We show that the incentive to collude is unavoidable. Contrary to Von der Fehr (1993), we show that firms *under-invest* in trading under collusion.

- We use a richer yet tractable model, which allows for a more detailed analysis, by disentangling the firms' production, abatement, and trading decisions. In Requate (1993b), the firms can reduce pollution only by reducing output, and there is no alternative way to abate pollution. In (Requate, 2006, chapter 7), this assumption is relaxed but the model becomes intractable. In Von der Fehr (1993), emission allowances serve as a proxy for production, in the sense that firms cannot produce without emission allowances; so capturing permits from the other firm restricts its production capacity, and could push the firm out of the market. Furthermore, Von der Fehr (1993) does not explicitly model the trading process. In our paper, we allow the firms to choose their production quantities, levels of abatement, and trading quantity as three separate and independent decision variables. We distinguish and explicitly model the processes of production, pollution generation, abatement, and regulation. Our closed form solutions make it possible to compare outcomes, including output, abatement efforts, firm profits, consumer surplus, and welfare.

- Finally, we generalize the polar cases of monopoly and duopoly by considering strategic firms serving their respective markets with partially substitutable products, as in Chapter 2. In our model, varying the degree of substitutability allows us to explore the effect that different levels of competition intensity has on the firms' incentives to collude. As we show, competition intensity is a key parameter of the equilibrium outcomes.

3.3 The Model

The model in this chapter is closely related to the model in Chapter 2. We adopt the same modeling assumptions and notations as in Chapter 2, except that, for simplicity, we consider two firms instead of n. This makes the analysis easier without losing any important insights. We refer the reader to Chapter 2 for justification of our assumptions. Specifically, we adopt the same integrated pollution-production model as in Chapter 2.

The firms produce partially substitutable products. Each firm chooses a production quantity q_i for its product i. The price of product i is determined by the production quantities chosen by firm i and its competitor according to the following formula: $p_i = a - bq_i - \gamma bq_j$, where $0 \leq \gamma \leq 1$ is the coefficient of substitutability. The parameter γ determines how similar the products are, and serves as a proxy for competition intensity. When $\gamma = 0$, the products are sufficiently different that the price p_i is determined only by the quantity produced by firm i. In other words, each firm is a local monopoly in its market. The firms do not compete. At the other extreme, when $\gamma = 1$, the product is a homogeneous commodity. This means that the firms produce the same product, and competition is intense. Recall that the pollution model in Chapter 2 is characterized by four components:

1. **Pollution generation.** The pollution $\widetilde{P}_i$ generated by each firm is equal to its production quantity,[2] i.e., $\widetilde{P}_i = q_i$.

2. **Pollution abatement.** The firms can abate pollution albeit at a cost. Each firm independently determines what percentage of its pollution to abate. x_i denotes the percentage of pollution abated by firm i. If the firm abates $q_i \cdot x_i$, with $x_i \in [0, 1]$, the residual pollution is $P_i = \widetilde{P}_i - q_i \cdot x_i = q_i \cdot (1 - x_i)$. The costs of abating pollution are quadratic, convex, and increasing in the quantity of pollution abated, i.e., the costs are $c_i \cdot (q_i \cdot x_i)^2$. There are two different cost coefficients, c_i and c_j. Without loss of generality, we assume that $0 < c_i \leq c_j$. Note that when $c_i = 0$, pollution abatement is costless. The firm can effortlessly ensure that the pollution constraint is not binding. This implies that the problem coincides with the unregulated case or business-as-usual. From now on, we will let $c_l = c_i$ and $c_h = c_j$, and we will use the subscript l to denote the *low-cost* firm; i.e., the firm with a *low* abatement cost coefficient c_l, and the subscript h to denote the *high-cost* firm, which has a *high* abatement cost coefficient c_h. We assume that c_l and c_h are common knowledge.

3. **Pollution damage.** The firms' pollution causes damage. The pollution damage function D denotes the economic value of this damage to society. We assume that D is a quadratic, convex, and increasing function of the total pollution P, where $P = P_l + P_h$. Thus, $D = d \cdot P^2$, where the pollution damage factor $d \geq 0$ captures how harmful the pollutant is.

4. **Pollution regulation.** As mentioned previously, we focus on two popular mechanisms:

   (a) **The Tax mechanism**. Under the Tax mechanism, the regulator charges the firm with a fee proportional to its emissions. Under this mechanism, the tax is equal to $\tau \cdot [q \cdot (1 - x)]$ where $q \cdot (1 - x)$ is the net pollution generated by the firm and $\tau \geq 0$ is the tax rate set by the regulator, common to all firms. By increasing the tax rate, the regulator makes pollution more costly to the firm causing it to reduce its emissions. Thus, the regulator can strategically set the tax rate to achieve a particular emission reduction goal.

[2] As in Chapter 2, the emission rate is normalized to 1

(b) **The Cap-and-Trade mechanism**. Under Cap-and-Trade, the regulator specifies a limit on emissions but firms are allowed to trade with each other. At the end of each year, each firm must surrender a number of allowances equal to its actual emissions, or pay a hefty fine. Based on scientific, historical, and political considerations, the regulator typically assigns a cap S for the entire region. As argued in Chapter 2, we assume that each firm is initially allocated the same number of emission allowances, i.e., $s_l = s_h = S/2$, where s denotes an individual allocation of emission allowances. Recall that t denotes the number of emission allowances traded by the firm. Without loss of generality, $t \geq 0$ indicates that the firm is a net seller of allowances, and $t < 0$ that the firm is a net buyer. The firm's constraint is $q \cdot (1 - x) \leq s - t$. We consider two variants of Cap-and-Trade that differ only in the timing of trading. (1) The first Cap-and-Trade model is called the "single-stage" model because trading occurs simultaneously with production; (2) In the second Cap-and-Trade model, trading occurs *before* production. We call this variant the "two-stage" Cap-and-Trade model.

To allow meaningful comparisons between Tax and Cap-and-Trade, we further assume that the regulator's goals are the same: that the total pollution does not exceed an amount S.

Firms maximize their profits. Firm i's profit ($i = l$ or h) $\pi_i (q_i, x_i \mid q_j) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot q_j) - c_i \cdot (q_i \cdot x_i)^2$ is the difference between its revenues and its pollution abatement costs. The firms' joint profits or industry profits are denoted $\Pi = \pi_l + \pi_h$.

As in Chapter 2, the performance measures we use to evaluate the two mechanisms include the total output, $Q = q_l + q_h$, the total abated pollution, $q_l x_l + q_h x_h$, the firms' joint profits, Π, consumer surplus, CS, which includes the damage from pollution, $D = d \cdot P^2$, and the social welfare $W = \Pi + CS$, where Π is the profits before tax. Our notations are summarized in Table 3.1.

3.4 Equilibrium Analysis

In this section, we analyze the Tax mechanism, and the two variants of the Cap-and-Trade mechanism, and derive the *unique* (Subgame-Perfect) Nash equilibrium under each type of regulation, for any *arbitrary* pollution target, and for any γ.

Table 3.1. Model notations (Chapter 3)

q_i	Production quantity chosen by firm i, $i = l$ or h , $q_i \geq 0$
Q	Total production quantity, $Q = q_l + q_h$
γ	Competition coefficient, $0 \leq \gamma \leq 1$
p_i	Price in firm i's market, $p_i = a - b \cdot q_i - \gamma \cdot b \cdot q_j$, $a > 0$, $b > 0$
x_i	Pollution abatement level chosen by firm i, $0 \leq x_i \leq 1$
c_i	Abatement cost coefficient of firm i, $c_i \in \{c_l, c_h\}$, $0 < c_l \leq c_h$
P_i	Pollution generated by firm i, $P_i = q_i \cdot (1 - x_i)$
P	Total pollution generated by the firms, $P = \sum_{i=1}^{2} P_i$
π_i	Profit of firm i
Π	Firms' joint profit, $\Pi = \sum_{i=1}^{2} \pi_i$
d	Pollution damage factor, $d \geq 0$
D	Pollution damage, $D = d \cdot P^2$
CS	Consumer surplus $CS = CES - D$
W	Social welfare $W = \Pi + CS$
S	Total cap chosen by the regulator, $S \geq 0$
s_i	Individual firm cap
t_i	Number of emission allowances traded by firm i
τ	Tax rate, $\tau \geq 0$

3.4.1 The Tax Mechanism

As mentioned previously, under the Tax mechanism, the regulator charges a tax *proportional* to the firm's emissions, i.e., firm i pays a tax $\tau \cdot q_i \cdot (1 - x_i)$, where τ is the linear tax rate and $q_i \cdot (1 - x_i)$ is firm i's emissions. In choosing the tax rate τ, the regulator anticipates firms' reactions, and chooses the minimum τ to ensure that the total pollution generated by the firms is at most S. Then, each firm chooses its production quantity and pollution abatement level to maximize its profits net of pollution taxes. Firm i's objective is

$$\max_{q_i \geq 0,\ 0 \leq x_i \leq 1} \pi_i \left(q_i, x_i \mid q_j,\ \tau\right) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot q_j) - c_i \cdot (q_i \cdot x_i)^2 - \tau \cdot q_i \cdot (1 - x_i)$$

The regulator then chooses τ such that the total pollution, $P \leq S$. We know from chapter 2 that this game has a unique Subgame-Perfect Nash Equilibrium. Theorem 8 gives the solution to that game.

Theorem 8 *Let* $s^u = \frac{a}{b(2+\gamma)}$ *and* $\underline{s} = \frac{a(c_h - c_l)}{2c_h(b(2+\gamma)+2c_l)}$.

1. $s \geq s^u$. *The optimal tax rate is* $\tau = 0$.

The optimal production quantities and abatement levels are

$$\begin{aligned} q_l &= q_h = \frac{a}{b\,(2+\gamma)} \\ x_l &= x_h = 0 \end{aligned}$$

2. $\underline{s} \leq s < s^u$. *The optimal tax rate is* $\tau = 4c_l c_h \frac{a-b(2+\gamma)s}{(2+\gamma)b(c_l+c_h)+4c_l c_h}$.

   The firms react with

$$\begin{aligned} q_l^{tax} &= q_h^{tax} = \frac{a\,(c_l+c_h)+4c_l c_h s}{(2+\gamma)\,b\,(c_l+c_h)+4c_l c_h} \\ x_l &= 2c_h \frac{a-b\,(2+\gamma)\,s}{a\,(c_l+c_h)+4c_l c_h s} \\ x_h &= 2c_l \frac{a-b\,(2+\gamma)\,s}{a\,(c_l+c_h)+4c_l c_h s} \end{aligned}$$

3. $0 < s < \underline{s}$. *The optimal tax rate is*

$$\tau = 2c_h \frac{a\,(b\,(2-\gamma)+2c_l)-2b\left(b\left(4-\gamma^2\right)+4c_l\right)s}{4\,(b+c_l)\,(b+c_h)-\gamma^2 b^2}$$

   The optimal production quantities and abatement levels are:

$$\begin{aligned} q_l &= \frac{a\,(b\,(2-\gamma)+2c_h)-4\gamma b c_h s}{4\,(b+c_l)\,(b+c_h)-\gamma^2 b^2} \\ q_h &= \frac{a\,(b\,(2-\gamma)+2c_l)+8c_h\,(b+c_l)\,s}{4\,(b+c_l)\,(b+c_h)-\gamma^2 b^2} \\ x_l &= 1 \\ x_h &= \frac{a\left[b\,(2-\gamma)+2c_l\right]-2b\left[b\left(4-\gamma^2\right)+4c_l\right]s}{a\,(b\,(2-\gamma)+2c_l)+8c_h\,(b+c_l)\,s} \end{aligned}$$

Theorem 8 is a special case of Theorem 5 (see Chapter 2) with $n = 2$ and $m = 1$. See Figure 3.1 for a graph of the production quantities and abatement efforts as a function of s for $\gamma = 0,\ \gamma = .5$, and $\gamma = 1$.

As the regulation becomes more stringent (i.e., the cap decreases), the firms simultaneously reduce output and abate pollution. The emission tax acts as a damper on output for both firms. The *low-cost* firm abates more pollution than the *high-cost* firm (i.e., $x_l > x_h$) because it can abate more pollution for every dollar spent. When the cap is sufficiently low, the low-cost firm has abated all its pollution (i.e., $x_l = 1$). Beyond that point (i.e., $s \leq \underline{s}$), in the absence of competition (i.e., $\gamma = 0$), the *low-cost* firm will maintain its output constant. Since it has abated all its pollution, it no longer pays any taxes. To maximize its revenues, the *low-cost* firm sells all its emission allowances to the *high-cost* firm. However, when the

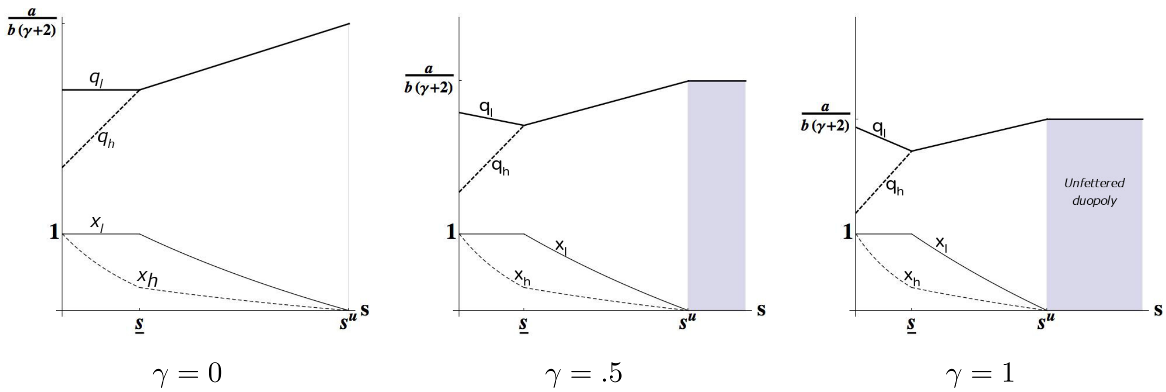

Figure 3.1. Production quantities and abatement levels under the Tax mechanism.

firms compete (i.e., $\gamma > 0$, see the center and right panes), the dynamic of the Cournot competition changes for $s \leq \underline{s}$. The *high-cost* firm reduces its output more and abates more pollution, while the *low-cost* firm *increases* its output. Overall, the total output, $Q = q_l + q_h$, decreases. The quantities are the outcome of a noncooperative game. Recall that the firms have three levers to comply with the regulation: *(i)* reduce the output, *(ii)* increase their abatement effort, or *(iii)* pay a tax. We saw in Chapter 2 that the firms prefer output reduction over pollution abatement, because reducing their production quantities allows them to increase prices. When $x_l = 1$, the low-cost firm abates all its pollution. Since its pollution abatement costs are sunk, it can credibly increase its output. The *low-cost* firm's cost advantage translates into the ability to commit credibly. Because the *low-cost* firm is more aggressive in the Cournot game, the *high-cost* firm is forced to decrease its output. The *high-cost* firm still has to balance output and abatement, and pay a tax for its residual emissions. The *low-cost* firm does not.

3.4.2 The Single-stage Cap-and-Trade Mechanism

In the single-stage Cap-and-Trade mechanism, trading, production, and abatement are simultaneous. The firm i's problem is given by:

$$\max_{q_i \geq 0,\ 0 \leq x_i \leq 1,\ t_i \leq s} \pi_i \left(q_i, x_i, t_i \mid q_j\right) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot q_j) - c_i \cdot (q_i \cdot x_i)^2 + r \cdot t_i$$

where r is the price of emission allowances at which the firms trade, i.e., the market clearing price. Firm j solves a similar problem. The market clearing condition stipulates that the demand for emission allowances equals the supply, i.e., $t_i + t_j = 0$. Rewrite $t_i = t$; the market clears if there exists a price $r \geq 0$ such that $t_j = -t$. The single-stage model correspond to the Cap-and-Trade model of Chapter 2. From Chapter 2, we know that this game has a unique Nash Equilibrium, which is given in Theorem 9.

Theorem 9 *The production quantities and abatement levels under the single-stage Cap-and-Trade are the same as the Tax mechanism. Furthermore*

1. *When $s \geq s^u$, the firms do not trade (i.e., $t = 0$).*
2. *When $\underline{s} \leq s < s^u$, the low-cost firm sells $t = \frac{(c_h - c_l)[a - b(2+\gamma)s]}{(2+\gamma)b(c_l+c_h)+4c_lc_h}$ emissions allowances at the price $r = 4c_lc_h\frac{a-b(2+\gamma)s}{(2+\gamma)b(c_l+c_h)+4c_lc_h}$ to the high-cost firm.*
3. *When $0 < s < \underline{s}$, the low-cost firm sells all its emission allowances (i.e., $t = s$) at the price $r = 2c_h\frac{a(b(2-\gamma)+2c_l)-2b\left(b\left(4-\gamma^2\right)+4c_l\right)s}{4(b+c_l)(b+c_h)-\gamma^2b^2}$ to the high-cost firm.*

Theorem 9 is a special case of Theorem 4 and Proposition 3 of Chapter 2 with $n = 2$ and $m = 1$.

The equivalence between Tax and the single-stage Cap-and-Trade mechanism are discussed in some detail in Chapter 2. In equilibrium, the marginal abatement costs of the firms are equal to each other and to the emission allowance price r, which is the shadow price of the pollution constraint. Under Tax, the firms' marginal abatement costs are also equal to each other and to the tax rate. The firms abate pollution up to the point where their marginal abatement costs (which are increasing in the abatement effort) equal the tax rate. Beyond that point, the firms' marginal abatement costs continue to increase and the firms are better off paying the tax at the fixed rate. The equilibrium tax rate is equal to the equilibrium shadow price of the pollution quantity allocated to firms under Cap-and-Trade (i.e., $r = \tau$). As expected, trading is always from the *low-cost* to the *high-cost* firm (i.e., $t \geq 0$).

This equivalence between Cap-and-Trade and Tax proves that no collusion occurs under the single-stage Cap-and-Trade mechanism. There is a unique price r at which the supply of emission allowances by the *low-cost* firm equals the demand from the *high-cost* firm. The market clearing condition deprives the firms from the opportunity to choose t strategically.

The fact that the marginal abatement costs across firms are equal guarantees that the abatement costs are minimized. Cost minimization is a key argument in favor of Tax and Cap-and-Trade.

3.4.3 The Two-stage Cap-and-Trade Model

In the previous section, we have studied a Cap-and-Trade mechanism where the market-clearing price is the driving force of the equilibrium. We found a unique price at which the volume of allowances offered by the *low-cost* firm equals the demand from the *high-cost* firm. This price is equal to the marginal abatement cost of the firms, and is the shadow price of the firms' pollution constraints. We did not impose any other condition on the equilibrium. A natural question arises: is the equilibrium in Theorems 8 and 9 Pareto-optimal? It not, does there exists another trading equilibrium that Pareto-dominates the trading equilibrium of the single-stage Cap-and-Trade? Remember from our discussion in the literature review (see Section 3.2) that Von der Fehr (1993) and Requate (1993b) use Pareto optimality as the trading objective in their Cap-and-Trade models.

A trade is Pareto-optimal if it is impossible to come up with a different trade that would make one of the firms better off without hurting the other firm. Pareto optimality is a powerful criterion because, given different options, the firms will prefer a Pareto-optimal equilibrium, provided that its gives each firm a pay-off at least equal to what they would get otherwise through a different trade. Pareto optimality requires that the firms maximize their joint profits. In other words, the firms trade to make the pie as big as possible, and then figure out a way to share the profits. The trading quantities and prices may not be unique and will typically depend on the firms' bargaining power. A powerful firm, whether buyer or seller of emission allowances, may have the ability to extract all the Pareto improvements; however, to participate in the trade, each firm will require a payoff that is at least as much as what they get under the single-stage model. Otherwise, the less powerful firm will not trade in the first stage, and instead force the duopoly to trade at the time of production. Note that the trading price being simply a transfer between firms does not affect total firm profits, consumer surplus, or welfare. With this in mind, consider the following time-line for the Cap-and-Trade mechanism:

First, the regulator chooses an aggregate cap S, and assigns a cap $s = \frac{S}{2}$ to each firm (as before).[3] Second the firms trade with each other to maximize their joint profits. Third and finally, the firms play the Cournot game. This scenario differs from the previous one in the timing of the trade. In the previous scenario (Section 3.4.2), the firms trade at the time of production. In other words, the production, competition, and trading stages are simultaneous (hence, the adjective *single-stage* to denote this trading process). In the alternative scenario (current section), the trading is decoupled from production/competition, and occurs *before*. We call this the *two-stage* Cap-and-Trade model. At the time of trading, the firms anticipate each other's reaction in the final stage of the game and focus on executing

[3] As in Chapter 2, we assume that the regulator cannot discern between the low-cost and the high-cost firm, although she knows the value of c_l and c_h. See Chapter 2 for a discussion of this assumption.

a Pareto-optimal trade. By placing trading before production, we allow the firms to use the ability to trade as a strategic lever in the Cournot game. Such a lever could be used as a means to collude. Formally, the firms solve the following problem:

1. **Trading:** choice of t

$$\max_{-s\le t\le s} \Pi = q_l\cdot(a-b\cdot q_l-\gamma\cdot b\cdot q_h)+q_h\cdot(a-b\cdot q_h-\gamma\cdot b\cdot q_l)-c_l\cdot(q_l\cdot x_l)^2-c_h\cdot(q_h\cdot x_h)^2$$

   where $q_l,\ q_h,\ x_l$, and x_h are determined by the Cournot game below, and are functions of t.

2. **Cournot competition**: choice of $q_l,\ q_h,\ x_l$, and x_h, given t

$$\begin{aligned}\max_{q_l\ge 0,\ 0\le x_l\le 1} \pi_l\left(q_l,x_l \mid t,q_h\right) &= q_l\cdot(a-b\cdot q_l-\gamma\cdot b\cdot q_h)-c_l\cdot(q_l\cdot x_l)^2\\ s.t.\ q_l\cdot(1-x_l) &\le s-t\\ \max_{q_h\ge 0,\ 0\le x_h\le 1} \pi_h\left(q_h,x_h \mid t,q_l\right) &= q_h\cdot(a-b\cdot q_h-\gamma\cdot b\cdot q_l)-c_h\cdot(q_h\cdot x_h)^2\\ s.t.\ q_h\cdot(1-x_h) &\le s+t\end{aligned}$$

We solve by backward induction. Theorem 10 shows that a Subgame-Perfect Nash Equilibrium of the two-stage Pareto-optimal Cap-and-Trade game exists and is unique. See Appendix B for the proof of this Theorem.

Theorem 10 *Assume that none of the firms shuts down. Let*

$$\begin{aligned} s_1 &= \frac{a\left(c_h-c_l\right)\left(b^2\gamma^2+4\left(1-\gamma\right)\left(b+c_l\right)\left(b+c_h\right)\right)}{c_h\left[\begin{array}{c}16\left(b+c_l\right)\left(b+c_h\right)\left(b+\left(1-\gamma\right)c_l\right)-\\ b\gamma^2\left(\left(8-\gamma^2\right)b^2+8bc_l+12bc_h+12c_lc_h\right)\end{array}\right]}\\ s_2 &= \frac{a\left[b\left(2-\gamma\right)^2\left(4+\gamma\right)+2c_h\left(8-\gamma\left(6+\gamma\right)\right)\right]}{2b\left[b\left(4-\gamma^2\right)^2+4c_h\left(4-3\gamma^2\right)\right]}\\ s_3 &= \frac{a\left(\begin{array}{c}b^2\left(2-\gamma\right)^2\left(c_l\gamma+c_h\left(4+\gamma\right)\right)+\\ 2bc_h\left(8-\gamma\left(6+\gamma\right)\right)\left(c_l+c_h\right)+16c_lc_h^2(1-\gamma)\end{array}\right)}{2bc_h\left(\begin{array}{c}b^2\left(4-\gamma^2\right)^2+2b\left(c_l\left(8-\gamma\left(4+\gamma\left(2+\gamma\right)\right)\right)+2\left(4-3\gamma\right)c_h\right)\\ +8c_lc_h\left(2-\gamma-\gamma^2\right)\end{array}\right)}\end{aligned}$$

Then $0\le s_1\le s_2\le s_3\le s^u$*, and there exists a unique* $\tilde{s}\in[s_2,s_3]$ *such that:*

Case 1: $s\ge\tilde{s}$*. The optimal trade is*

$$t=\frac{2a\left[b\left(2-\gamma\right)^2+4\left(1-\gamma\right)c_h\right]}{b\left[b\left(4-\gamma^2\right)^2+4c_h\left(4-3\gamma^2\right)\right]}-s$$

The optimal production quantities and abatement levels are

$$
\begin{aligned}
q_l &= \frac{a\left[b(2-\gamma)^2(2+\gamma)+2c_h\left[4-\gamma(2+\gamma)\right]\right]}{b\left[b\left(4-\gamma^2\right)^2+4c_h\left(4-3\gamma^2\right)\right]}\\
q_h &= \frac{a\left[b(2-\gamma)^2(2+\gamma)+8(1-\gamma)c_h\right]}{b\left[b\left(4-\gamma^2\right)^2+4c_h\left(4-3\gamma^2\right)\right]}\\
x_l &= 0\\
x_h &= \frac{b(2-\gamma)^2\gamma}{b(2-\gamma)^2(2+\gamma)+8(1-\gamma)c_h}
\end{aligned}
$$

Case 2: $s_1 < s < \tilde{s}$.

The optimal trade is

$$
t = \frac{(c_h-c_l)\left[\begin{array}{c} 2a\left(b^2\gamma^2+4(1-\gamma)(b+c_l)(b+c_h)\right)- \\ bs\left(16(b+c_l)(b+c_h)-\gamma^2\left(\left(8-\gamma^2\right)b^2+12b(c_l+c_h)+12c_lc_h\right)\right)\end{array}\right]}{\left[\begin{array}{c} 16(b+c_l)(b+c_h)\left(b(c_l+c_h)+2(1-\gamma)c_lc_h\right)- \\ b\gamma^2\left(\left(8-\gamma^2\right)b^2(c_l+c_h)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h(c_l+c_h)\right)\end{array}\right]}
$$

The optimal production quantities and abatement levels are

$$
\begin{aligned}
q_l &= \frac{\left(a(c_l+c_h)+4c_lc_hs\right)\left(\begin{array}{c} 8(b+c_l)(b+c_h)- \\ b\gamma^2\left(2(1-\gamma)b+2c_h\right)-4\gamma(b+2c_l)(b+c_h)\end{array}\right)}{\left(\begin{array}{c} 16(b+c_l)(b+c_h)\left(b(c_l+c_h)+2(1-\gamma)c_lc_h\right)- \\ b\gamma^2\left((8-\gamma)b^2(c_l+c_h)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h(c_l+c_h)\right)\end{array}\right)}\\
q_h &= \frac{\left(a(c_l+c_h)+4c_lc_hs\right)\left(\begin{array}{c} 8(b+c_l)(b+c_h)- \\ b\gamma^2\left(2(1-\gamma)b+2c_l\right)-4\gamma(b+c_l)(b+2c_h)\end{array}\right)}{\left(\begin{array}{c} 16(b+c_l)(b+c_h)\left(b(c_l+c_h)+2(1-\gamma)c_lc_h\right)- \\ b\gamma^2\left((8-\gamma)b^2(c_l+c_h)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h(c_l+c_h)\right)\end{array}\right)}\\
x_l &= 1-\frac{s-t}{q_l}\\
x_h &= 1-\frac{s+t}{q_h}
\end{aligned}
$$

Case 3: $0 < s \leq s_1$. *The optimal trade is* $t = s$. *The optimal production quantities and abatement levels are the same as the Tax mechanism.*

The assumption in Theorem 10 that none of the firms shuts down should not be overlooked. Under single-stage Cap-and-Trade, the firms never shut down (see Theorem 9). When the pollution constraints are extremely stringent, the firms continue to produce, even if they are forced to abate all their pollution. By contrast, we show in the proof of the two-stage Cap-and-Trade model that, although the *low-cost* firm never shuts down, there are values of the model parameters for which the *high-cost* firm shuts down. Selling the *high-cost* firm to the *low-cost* firm could be a form of collusion; however, we prefer to focus on the interesting case where both firms are producing.

As can be seen from Theorem 10–Case 3, the three mechanisms (Tax, single-stage Cap-and-Trade, and two-stage Cap-and-Trade) coincide when $s \leq s_1$, except for the firms' profits. The firms' profits under Tax are lower than under Cap-and-Trade by $\tau \cdot S$, which is the tax payment to the regulator. The situation $s \leq s_1$ corresponds to a very stringent pollution cap requiring the *low-cost* firm to abate all its pollution (i.e., $x_l = 1$) and, under Cap-and-Trade, to sell all its emission allowances to the *high-cost* firm. In other words, when $s \leq s_1$, the firms no longer choose t, but are forced to set $t = s$ in both the single-stage and two-stage models. The firms cannot collude, because they do not have the freedom to trade at the Pareto-optimal level. Although the firms would like to trade more, trading is limited by the supply of emission allowances.

When $s > s_1$, differences in the firms reactions suggest that collusion may be present. We investigate the potential for collusion, and its effects on output, abatement, and pollution in the next section.

3.5 Results

Our results are derived from the comparison of the firms' compliance strategies and the resulting outcomes under the single-stage and two-stage Cap-and-Trade mechanisms.

3.5.1 The Evidence for Collusion

Consider first the special case $\gamma = 0$. This corresponds to the firms being local monopolies, i.e., the firms do not compete, and have full market power in their respective markets. Note that the concept of collusion makes sense only if the firms are competing (i.e., $\gamma > 0$). It is easy to see from Theorems 9 and 10 that single-stage and two-stage Cap-and-Trade mechanisms coincide when $\gamma = 0$. This means that the single-stage Cap-and-Trade is Pareto optimal if $\gamma = 0$. When $\gamma > 0$, the firms' response under the two-stage Cap-and-Trade is different from the single-stage Cap-and-Trade mechanism if and only if $s > s_1$. In particular, this means that the single-stage Cap-and-Trade mechanism is not Pareto optimal when $\gamma > 0$ and $s > s_1$. Note that the same conclusion can be reached for the Tax mechanism.

The total output of the duopoly as a function of the cap s is plotted in Figure 3.2 for $\gamma = 0$, $\gamma = .5$, and $\gamma = 1$.

When $\gamma > 0$, the total output is less under two-stage than under single-stage Cap-and-

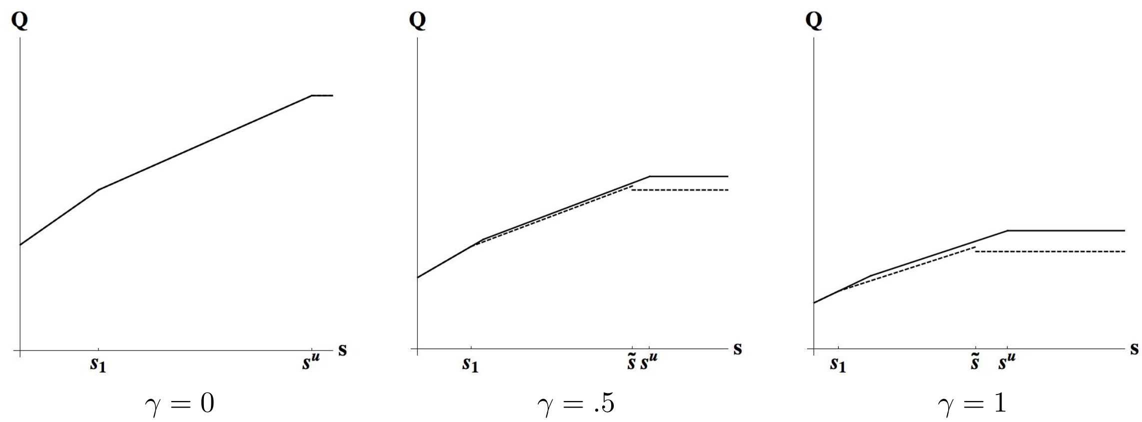

Figure 3.2. Total output under single-stage (solid line) and two-stage (dashed line) Cap-and-Trade.

Trade. As a result, the prices of both products are higher,[4] and the consumer economic surplus is reduced. In other words, the output reduction effect of pollution regulations is *amplified* in the *two-stage* Cap-and-Trade mechanism. Consider next the firms' abatement efforts. To evaluate the extent of the firms' abatement, we define the pollution abatement ratio as the ratio of the total quantity of pollution abated, $q_l x_l + q_h x_h$, to the total unabated pollution, $q_l + q_h$. Figure 3.3 compares the pollution abatement ratio under each mechanism. The firms' abatement effort under the two-stage is less than under the single-stage Cap-and-Trade when $s \leq \widetilde{s}$. In summary, under two-stage Cap-and-Trade, the firms use trading to reduce the output, which allows them to increase prices, and at the same time relax their abatement efforts. This result gives support to the claim that firms can use Cap-and-Trade to by-pass the regulations at the expense of society (Shapiro 2007). The production quantities, abatement levels and trading volume are discontinuous at $\widetilde{s}$; however, the firms' profits are not. At $\widetilde{s}$, the firms' compliance strategies shift from a regime where the pollution constraints are binding for both firms, and both firms have to abate pollution (when $s \leq \widetilde{s}$), to a regime where the *low-cost* firm abates no pollution (i.e., $x_l = 0$) and buys emission allowances from its rival who abates pollution (when $s > \widetilde{s}$). The industry profits in this case are even higher than the unregulated scenario (i.e., when $x_l = x_h = 0$). Recall that the *low-cost* firm has the upper hand in the Cournot game because of its cost advantage. By increasing output, it forces the *high-cost* firm to reduce output drastically and abate more pollution. This drives the output price up, and at the same time generates a surplus of emission allowances which allow the *low-cost* firm to produce even more than the unfettered

[4]Note that $\forall i, \quad p_i = a - bq_i - \gamma b q_j = a - b\,(1-\gamma)\,q_i - \gamma b Q$

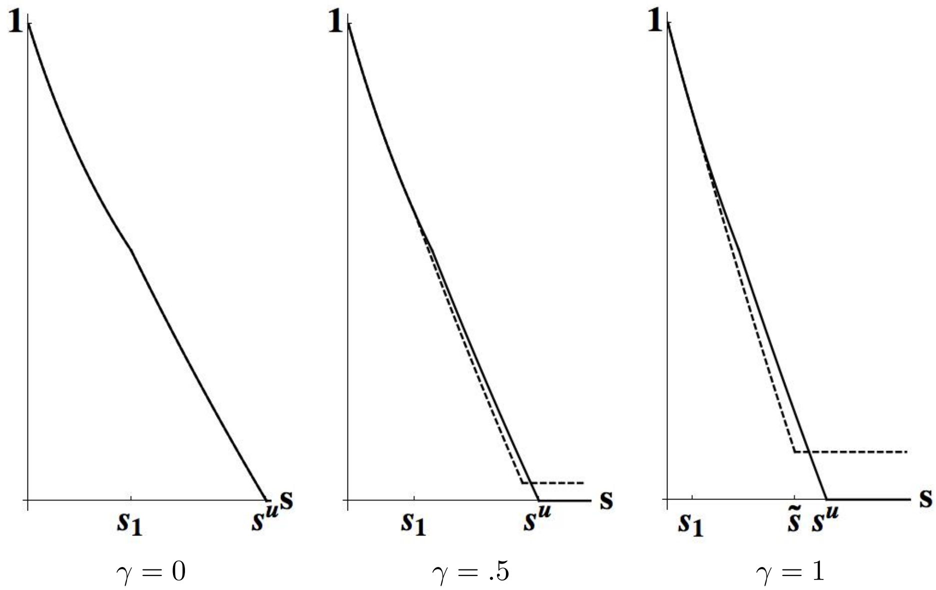

Figure 3.3. Pollution abatement ratio under single-stage (solid line) and two-stage (dashed line) Cap-and-Trade.

level. Because of its cost advantage, the *low-cost* firm can produce at a higher level of output than the *high-cost* firm. The output effect once again dominates the abatement effect. The increase in revenues outweigh the pollution abatement costs borne by the *high-cost* firm.

Note that this phenomenon occurs for large caps, in and around the unregulated region (i.e., in the vicinity of s^u). In general, firms have an incentive to over-report their emissions before Cap-and-Trade is introduced in order to get the regulator to choose a large cap, or to get more emission allowances from the regulator. Our results show that the possibility of collusion makes over-reporting even more attractive to the firms. Not only will the pollution constraints not be binding, but the firms will be able to improve their profits through trading.

When implementing Cap-and-Trade, the regulator supplies the emission allowances, but she does not have any say in the timing of the trade. It is the firms' choice. By definition, the firms' joint profits in the Pareto-optimal condition are greater than under any other trading arrangement; this means that the Pareto-optimal equilibrium is a dominant strategy. Since there exists a unique Pareto-dominant equilibrium for each $s > s_1$, we can expect collusion under Cap-and-Trade when $s > s_1$. Proposition 7 summarizes these results.

Proposition 7 *The firms have an incentive to collude under Cap-and-Trade if $\gamma > 0$ and $s > s_1$.*

An important corollary is that when $s \leq s_1$, the firms cannot collude. It follows that by setting stringent pollution reduction objectives, the regulator could preclude collusion.

3.5.2 The Effects of Collusion

Figure 3.4 compares the trading volume under both mechanisms.

In the left pane of Figure 3.4, the firms do not compete. The trading pattern is determined by supply and demand. For $s \leq \underline{s}$, the supply effect dominates. The firms would like to trade more, but trading is limited by the number of allowances available. As s increases, more allowances become available, and the trading volume increases. When $s > \underline{s}$, the demand effect dominates. The *high-cost* firm needs fewer allowances as s increases. The trading volume is decreasing in s.

When firms compete, the same dynamic is at play. The supply effect dominates for low caps; the demand effect for high caps. Because the demand effect dominates for large caps (i.e., $s \geq s_1$), the *high-cost* firm is now in the best position to commit. Collusion in the market for emission allowances hinges on the *high-cost* firm's output. The *high-cost* firm will drastically reduce its output. As a result, its pollution is also greatly reduced. If s is sufficiently large, it no longer needs to buy emission allowances, and instead becomes a seller, as evidenced by negative values of t.

Figure 3.5 shows the firms' individual decisions under collusion.

Contrary to Von der Fehr (1993), we find that trading in emission allowances is depressed under collusion, meaning that the trading volume under collusion is less than under no collusion. The intuition behind this result is that collusion makes emission allowances less desirable. This effect is mainly driven by the fact that output is reduced more under collusion.

As a result of collusion, the total output goes down. This can actually be good for society if the pollutant is very harmful, because the damage avoided is very large, and more

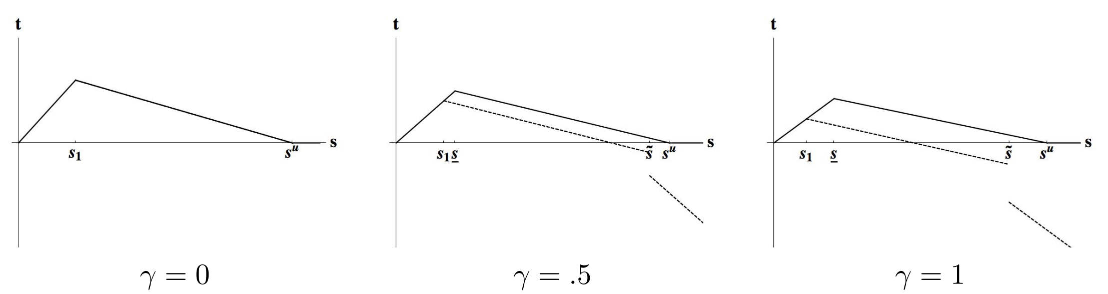

Figure 3.4. Trading volume t under the single-stage (solid line) and two-stage (dashed line) Cap-and-Trade.

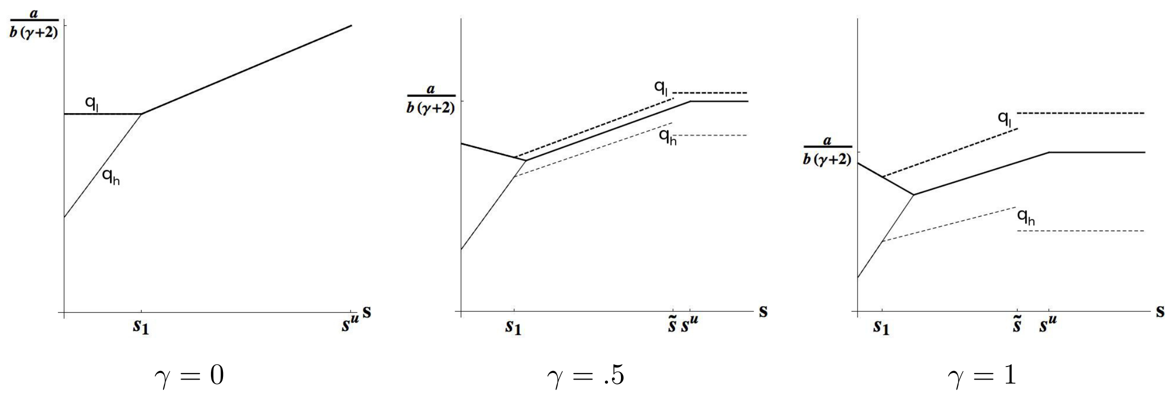

Figure 3.5. Production quantities under the single-stage (solid line) and two-stage (dashed line) Cap-and-Trade.

than compensates for losses in consumer surplus.

The firms' joint profits are plotted in Figure 3.6. Note that when $\gamma > 0$, there is a range of caps for which the pollution constraints bind for both firms (i.e., $s < \widetilde{s}$) and the firms' joint profits are higher than when regulations are absent (i.e., when $s \geq s^u$). In other words, the introduction of pollution constraints actually *improves* the firms' profits. This phenomenon is explained once again but the effect of output reduction, which not only lowers the firms' pollution, which limits the need for pollution abatement, but also increases revenues through higher prices. Note that this result does not hold for $\gamma = 0$. The industry profits when firms do not compete are strictly increasing in the cap, meaning that relaxing the cap increases industry profits. As mentioned in Chapter 2, under competition pollution regulations improve social welfare. We find that these welfare improvements are driven in part by improvements in industry profits.

Using two decades of panel data on the U.S. Portland cement industry, a highly concentrated industry, Ryan (2012) (cited earlier, see Section 3.1) found that the 1990 Amendments to the Clean Air Act were responsible for significant welfare losses, primarily due to increased industry concentration; interestingly, he also found that, consistent with our results, incumbent firms had benefited from the regulation. The increase in industry profit was driven by higher prices due to increased market concentration.

3.6 Conclusion

In this paper, we show that collusion is possible under Cap-and-Trade regulation. Collusion requires two conditions: *(i)* the presence of imperfect competition in the output markets; and *(ii)* a sufficient supply of emission allowances. Although the pollution regulator may

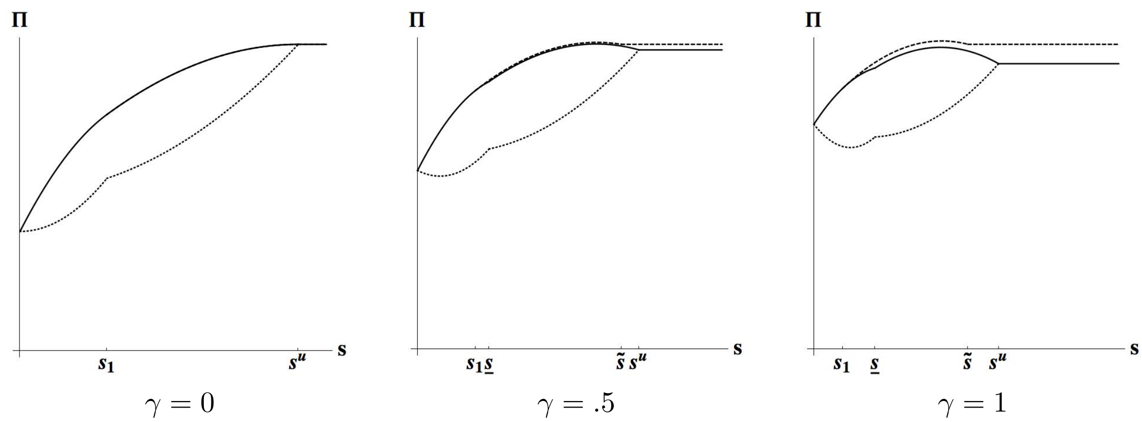

Figure 3.6. Industry profits under single-stage (solid line) and two-stage (dashed line) Cap-and-Trade. The dotted line is the industry profits under Tax.

be limited in her ability to stimulate competition, she can prevent collusion by controlling tightly the supply of emission allowances; in other words, the regulator should set ambitious pollution reduction goals under Cap-and-Trade to limit the risk of collusion.

Collusion occurs as the firms manipulate the market for emission allowances to execute Pareto-optimal trades. If there are many firms on the emission allowance exchange, collusion may be limited; however, there is no way to preclude it entirely. The effects of collusion are not all bad. The firms will reduce output more under collusion. If the pollutant is very harmful, this could improve social welfare. We show that the firms profits can be higher under pollution constraints than in the absence of regulation, because firms exercise their market power to increase prices through output reduction, which boosts their revenues. By squeezing output, the pollution constraints actually help the firms.

One of the main insights of this research is that the timing of trading is critical. One can think of several other important aspects of the trading process that require further attention, such as, for example, multiperiod trading with or without banking, the relationship between allowance auctions and trading, or the impact of trading on firms' incentives in vertical supply chains.

CHAPTER 4

INVESTMENT IN POLLUTION ABATEMENT INNOVATIONS UNDER CAP, CAP-AND-TRADE, AND TAX

4.1 Introduction

Regulations can influence technological progress (Jaffe et al. 2003) by creating incentives or obstacles to the adoption of new technologies. This is especially true in the environmental area. For example, technology mandates in pollution control can stifle innovations in abatement technologies. Under flexible, decentralized pollution control mechanisms such as Cap-and-Trade and Taxes, firms have considerable latitude in the production and abatement technologies that they choose to use. In this paper, we study how three widely used pollution control mechanisms–Cap, Cap-and-Trade, and Tax–influence investments in abatement innovations by regulated firms.

To illustrate the connection between environmental regulations and the diffusion of innovations, consider the case of climate change. The most recent assessment report of the Intergovernmental Panel on Climate Change (Alexander et al. 2013) unequivocally confirms the warming of the climate system, and reaffirms the human factor as the dominant cause of the warming. It is well-known that the human component of global warming is primarily driven by uncontrolled emissions of greenhouse gases (GHG). The International Energy Agency (Birol 2012) analyzes energy trends for the foreseeable future, and estimates that, given the known reserves and under current policies, fossil fuels will constitute 80% of the world's total primary energy demand in 2035. Such a trend will lead to an increase of 46% in carbon emissions relative to 2010 levels. Under any scenario, the agency identifies energy efficiency and carbon capture and storage (CCS) as key options for reducing man-made GHG emissions, CO_2 being the most abundant GHG, and mitigating the risks associated with climate change.

Energy efficiency designates the set of technological options for reducing the ratio of energy use per unit of output, also known as energy intensity. Common examples of energy efficient alternatives are found in building technologies for lighting (compact fluorescent lights and LEDs), insulation, and electronic energy management, as well as in motor vehicle technologies (e.g., fuel economy light trucks and cars). Energy efficiency works to reduce energy consumption at the source.

CCS consists in capturing the carbon dioxide emitted by large stationary industrial sources, e.g., coal-fired power plants, and injecting it in deep geological formations for long-term storage. The technology was developed in the oil and gas industry to facilitate oil recovery; several large-scale demonstration projects are ongoing to evaluate the feasibility of this abatement technology for power generation, and other energy intensive industries (see Global CCS Institute 2013 or CRC for Greenhouse Gas Technologies 2013). Krass et al. (2013) explains that CCS technology could also be used in the cement industry.

A 2007 McKinsey study (Creyts et al. 2007) evaluated 250 opportunities for reducing GHG emissions in the U.S., including energy efficient technologies and CCS. For each option, the potential for emission reduction was estimated, and the corresponding cost per ton of CO_2 abated was calculated. The study found that a combination of energy efficiency options could reduce emissions in buildings and transportation by 1 to 1.5 Gt[1] of CO_2 per year depending on the scenario, or between 17 and 26% of 2010 net U.S. emissions. All of these options come with a negative price tag, meaning that their adoption provides net benefits to their adopters. For example, Creyts et al. (2007) estimates that efficient lighting in buildings yields a net benefit of about $80 for every tonne of CO_2 abated. However, in spite of the fact that energy efficiency pays for itself, much of the potential for carbon abatement through improvements in energy efficiency remains unrealized (four fifth of the potential in buildings and more than half in industry according to the International Energy Agency), and public policies are required to achieve large-scale adoption (Birol 2012). It may also occur that the policy objectives of the government require abatement levels above and beyond what energy efficiency alone can provide. According to the study, this would be the case if reductions in excess of 1.5 Gt of CO_2 per year are required.

The McKinsey study cited earlier also considers CSS. It estimates that CCS could reduce emissions by about .5 Gt per year (or about 9% of 2010 emissions). An MIT study (Katzer et al. 2007) considers CCS a critical carbon abatement technology if coal use is to remain at or exceed current levels. This MIT study concludes that installing CCS at coal-fired power

[1] 1 Gt = 1 Giga tonne, or 1 billion metric ton.

plants can reduce carbon emissions by 87%. However, power plants with CCS are almost twice as expensive to build (from \$1,280 to \$2,230 per kilowatt for pulverized coal generators, see Katzer et al. 2007, p. 19). Islegen and Reichelstein (2011) study the U.S. power generation industry, and find that the break-even point of CCS investments in coal-fired power plants is at about \$30 per tonne of CO_2 (\$60 for plants relying on natural gas), a number consistent with both the McKinsey and the MIT studies. In other words, unless carbon emissions are costly to the firms, and the cost of emitting one tonne of CO_2 exceeds \$30, power generators are not likely to adopt CCS.

To control GHG emissions, Carbon Taxes and Cap-and-Trade are currently used by regulators around the world. Cap-and-Trade is growing in popularity among regulators with large-scale implementations in Europe (E.U. Publications Office 2013), California (Barringer 2011), and China (Plumer 2013). Taxes are also widely used. Australia, India, Japan (KPMG 2013), the Canadian provinces of British Columbia and Quebec, and several European countries have opted for Carbon Taxes (SBS 2013). In the U.S., the EPA recently announced its intention to lower permissible carbon emissions by new power plants to 1,000 lbs per megawatt-hour for gas-fired plants, and 1,100 lbs for coal-burning generators (Shear 2013). The most advanced coal-fired plants currently have emissions in the order of 1,800 lbs per megawatt-hour. If implemented, the new regulation would force power generators to take drastic measures to cut their emissions, and CCS would likely to become an essential technology.

The above examples illustrate that regulations can play a role in facilitating the diffusion of abatement technologies. In particular, large (and hence costly) reductions in GHG emissions would necessitate putting a price on emissions either directly through a tax, or indirectly by forcing emission reductions.

In this paper, we study how the choice of pollution control mechanisms by the regulator influences the diffusion of abatement innovations. Specifically, we analyze the firms' incentives to invest in a new abatement technology under three popular mechanisms: *(i)* a direct limit (or cap) on the firm's emission (the Cap mechanism); *(ii)* a cap on emissions with the ability to adjust the cap by trading emission allowances (the Cap-and-Trade mechanism); and *(iii)* a tax proportional to the firm's emissions (the Tax mechanism). We address the following research questions: which mechanism encourages more firms to adopt an abatement innovation? Is more adoption always better for society? Given the firms' technology adoption strategies, which mechanism maximizes social welfare?

4.2 Literature Review

Following the Schumpeterian tradition, the literature categorizes the process of technological change into three sequential stages (Jaffe et al. 2003): *(i)* the *invention* phase at which a scientific or technical breakthrough enables the creation of a new product, service, or process; *(ii)* the *innovation* phase: at this stage, the invention is packaged by a firm into a marketable merchandise. In other words, innovation occurs only after commercialization in the marketplace; and *(iii)* the *diffusion* process during which the innovation disseminates through the market to its intended users.

The invention and innovation phases are often lumped together into the generic term of R&D (for research and development). The literature identifies two main drivers of the effect of environmental regulations on the R&D process. First, by making polluting inputs more expensive, environmental regulations persuade firms to look for better, cheaper ways to produce and reduce pollution, which may result in some firms pushing the efficiency frontier. This effect is known as *induced innovation* (Jaffe et al. 2003). It is supported in several empirical studies. For example, Johnstone et al. (2010b) find evidence that public policies play a significant role in determining patent applications in renewable energies. Johnstone et al. (2010a) find that the stringency, predictability, and flexibility of the pollution control mechanisms are positively associated with the number of patent applications in air pollution abatement, wastewater effluent treatment, and solid waste management. In the second approach, called *evolutionary*, boundedly rational managers do not optimize, but instead *satisfice* (Simon 1979), i.e., make decisions that meet a set of criteria but are not guaranteed to be optimal. Due to information, time, or other resource limitations, managers sometimes overlook cost reduction opportunities that are hard to see. In their study of the pulp and paper industry, Boyd and McClelland (1999) find that input use and pollution output could be simultaneously reduced between 2% and 8% while maintaining given productive output, suggesting that managers consistently miss opportunities for productivity improvements. King and Lenox (2002) find additional evidence in a sample of U.S. manufacturing firms that report to the Toxic Release Inventory of the EPA. Regulations that force managers to reconsider their operating practices may bring such opportunities to their attention.[2] Porter and van der Linde (1995) give several examples of firms that improved their profitability after innovating to reduce their pollution to meet a new regulatory requirement. Profits

[2]The same has been said of quality management programs, which generally pay for themselves because of the costs of poor quality that are saved after managers redesign their production processes to avoid defects (Crosby 1979).

improved because the innovations provided benefits in excess of the costs of compliance.

The innovation process may be hampered by the existence of a (positive) R&D externality because knowledge spillover and imitation by competitors may prevent the innovator from capturing all the rents from innovation. Parry (1998) and Fischer et al. (2003) take into account both the R&D and environmental externalities in their comparative study of several pollution control mechanisms. Fischer et al. (2003) find that the welfare rankings of the various mechanisms they study depend on the ability to imitate the innovation, the cost of innovation, the slope and level of the marginal damage function, and the number of polluting firms, suggesting that there exists no unambiguous ranking of the mechanisms. The literature concludes that no mechanism adequately addresses both market failures (Goulder and Parry 2008).

Our focus in this paper is on the diffusion phase. The fundamental difference between R&D and diffusion models is that R&D models typically include a stochastic element (e.g., the outcome of the research process is uncertain) or features related to intellectual property such as technology patents or the existence of knowledge spillovers (Requate 2005). Diffusion has important economic significance because through it, the latent benefits of the invention are actually realized at a large scale. We distinguish three drivers of the diffusion of innovations: *(i)* social interactions; *(ii)* market forces; and *(iii)* regulatory pressures.

4.2.1 Diffusion as Social Process

Rogers (2003) conceptualizes diffusion as a social process by which new ideas spread among a population over time. Such a diffusion process involves interpersonal communication through various channels requiring high levels of social interaction. This process is captured analytically as an information transfer from innovators to potential adopters in models called *epidemic* (Kemp 1997) because the diffusion resembles the spread of a disease (Griliches 1957, Bass 1969). Alternatively, researchers have used *threshold* models in which adoption occurs after a stimulus variable exceeds a certain value (David 1969, Bonus 1973). Adopters are characterized by a distribution of values with early adopters having the highest value. The diffusion swipes the entire distribution over time as the innovation becomes more affordable giving rise to the familiar S-shaped curve. These models do not take into account the market position of the innovation in relation to the products it intends to replace, and particularly the risk of cannibalization of old products by innovative ones.

4.2.2 Market-driven Diffusion of Innovations

Market forces can have a powerful influence on the diffusion of innovations, especially when managers create new and attractive value propositions through product or service innovations. Christensen (2003) finds that a well-chosen new product development strategy can harness market forces to spur diffusion. He finds many instances of what he calls *disruptive* innovations in several industries. Such innovations reached market dominance and displaced incumbents by changing the product attributes and performance measures along which firms compete. The diffusion pattern of a disruptive innovation typically starts at the low-end of the market, a customer segment typically ignored by incumbent firms, and diffuses upward, eventually luring away mainstream customers of the initial market. Schmidt and Druehl (2008) propose an analytical framework for identifying and categorizing disruptive innovations. The point is that the extent to which managers understand their markets to create attractive value propositions seems to play a key role in the diffusion of new products or services.

4.2.3 Regulations and the Diffusion of Environmental Innovations

Regulations play a role in the adoption of new technologies, particularly when the innovations provide environmental benefits such as a reduction in harmful emissions. This is because without regulations, firms under-invest in environmentally-friendly (but costly) innovations, because they cannot appropriate all the benefits from their investments. In other words, investments in emission reductions have the characteristics of a public good. Popp (2006) studies the adoption of NOx control technology by U.S. coal-fired power plants, and finds that regulations play a dominant role in the firms' adoption decisions.

Several papers reviewed in Kemp (1997), Jaffe et al. (2002), and Newell (2009) study the impact of regulations on the diffusion of environmental innovations. The literature broadly falls into three main categories: "graphical" models (to use Kemp's (1997) terminology), game theoretic models, and empirical studies. A widely shared result is that decentralized mechanisms based on economic incentives, such as emission taxes and emission trading, promote more technology adoption than direct command-and-control.

4.2.3.1 The "Graphical" Models

In the graphical models, the firms' incentives to adopt pollution control innovations are analyzed by comparing the change in aggregate industry compliance costs before and after the innovation is adopted (Wenders 1975, Downing and White 1986, Milliman and Prince 1989). The effect of the innovation is to lower the industry's marginal abatement cost curves.

A mechanism is deemed to provide higher incentives than another if its aggregate compliance costs are lower. The regulator is assumed to be perfectly informed. In particular, she knows the firms' costs prior to innovation and the pollution damage function, and implements the optimal, i.e., welfare-maximizing, policy before the innovation is introduced. However, due a lag in perceiving the innovation or political pressures, she does not adjust the control mechanism after the innovation has diffused. Wenders (1975) and Downing and White (1986) focus on a single polluter. Milliman and Prince (1989) consider n firms, one of which innovates while the others merely adopt the innovation, and analyze the process of technological change under five mechanisms: emission taxes, emission subsidies, free and auctioned tradable emission allowances, and direct controls. The process begins with the innovation, then proceeds with diffusion from the innovator to all the other firms– they distinguish between patented and unpatented technologies–, and ends with the firms' incentives to lobby for a response from the regulator after the innovation has diffused. If the regulator responds, she will implement the optimal policy, which will require more pollution reductions. In other words, the regulator's optimal response is to ratchet. They find that direct controls provide the least incentives to promote technological change. This key finding underpins the move away from centralized command-and-control, such as technology mandates and performance standards, in environmental policy. They also find significant differences between the mechanisms based on economic incentives, with auctioned emission allowances providing the most incentives for innovation and diffusion, and emission taxes the most incentives for lobbying in favor of policy adjustment. Under tradable emission allowances, the innovation lowers the demand for allowances, reducing their price. This benefits all the firms under auctioned allowances, including the innovator and those that adopt the innovation, because all firms are buyers. Under free allowances, only the firms that do not adopt are buying. Auctioned allowances encourage diffusion while emission taxes have no impact on diffusion, unless the regulator reacts. Firms have an incentive to lobby for policy adjustments under taxes because under the optimal policy the regulator lowers the tax rate, which means that firms will pay less tax on their residual emissions. Under all the other mechanisms, ratcheting increases the firms costs in greater proportion. Critics of the graphical models argue that focusing solely on the aggregate industry costs is not sufficient to capture an individual firm's incentives (Requate 2005). Instead, researchers need to consider strategic, autonomous firms and the consumer markets they serve. In particular, the graphical models assume that all the firms adopt the innovation, and completely ignore the output market. These limitation have been addressed in several papers.

4.2.3.2 Game Theoretic Models

Requate (1995, 1998) and Requate and Unold (2003) show that the above results no longer hold when one explicitly considers the output market and strategic firms in an equilibrium context. In Requate (1995), n perfectly competitive firms can choose between two abatement technologies, a conventional one and an innovative (i.e., more efficient) technology, and both types of firms may freely enter the market (i.e., n is endogenous). Similar to Milliman and Prince (1989), the industry is initially regulated optimally, and the regulation is unchangeable. In addition to choosing an abatement technology, each firm chooses a production quantity and an emission level to minimize its costs. The main objective of the paper is to investigate how emission taxes (for any tax rate) and auctioned tradable allowances (for any aggregate limit on emissions) spur the diffusion of the abatement innovation in the long run, and what is the effect of this induced diffusion on social welfare under each mechanism. He shows that the two technologies do not coexist under tax in equilibrium, meaning that either the conventional technology or the innovation fill the entire market. When the social damage of pollution is moderate, an emission tax may lead to over-investment or under-investment in the new technology compared to the social optimum, with possibly a reduction in welfare compared to the level prior innovation. This happens when firms over-invest because too much money is spent on new abatement equipments. Under auctioned tradable allowances, Requate (1995) shows that partial adoption is possible. Since the regulation is unchangeable, this also leads to a suboptimal welfare outcome. (Remember that the regulation was initially optimal.) However, under tradable allowances, the introduction and diffusion of the innovation never lead to a reduction in welfare. A critical assumption in Requate (1995) is that the regulator does not adjust the tax rate or the number of emission allowances after the innovation is introduced. Because of political pressures or delays in the regulatory process, the regulator may be hampered in her ability to ratchet. However, since she is fully informed, she can ultimately implement the social optimum after some time. In other words, in the long run, the regulator should be allowed to adjust.

Requate (1998) also considers the output market and a fully informed regulator, but the focus is on the firms' incentives to pursue R&D activities, not on diffusion. In his model, the firms can reduce pollution by reducing output, but cannot exert effort to abate pollution. In Chapter 2, we have shown that although output reduction is an inevitable component of the firms' compliance strategies, firms will also exert effort to abate pollution even when they exercise monopoly power.

Requate and Unold (2003) consider the incentives for perfectly competitive firms to adopt an innovation in *equilibrium*. They show that, contrary to Milliman and Prince (1989), the incentives are the same under auctioned and free emission allowances in equilibrium because the initial allocation method translates into a lump sum payment (or fee) to the firms whether they adopt or not, and the effect of adoption on the allowance price does not depend on the allocation rule.

Several authors study the effect of the timing of the regulation on adoption and welfare (Petrakis and Xepapadeas 1998, Kennedy and Laplante 2000, and Requate and Unold 2003). All of these papers assume perfect competition and a perfectly-informed regulator. Petrakis and Xepapadeas (1998) study a single monopolist, and only consider emission taxes. In their three-stage model, the monopolist decides on its abatement effort before or after the regulation is enacted; output is determined in the last stage; however, the firm's technology is fixed. They show that when the regulator cannot commit ex-ante, the firm strategically increases its abatement effort to obtain a lower tax rate in the second period. However, welfare is always higher if the regulator can precommit. Kennedy and Laplante (2000) analyze technology adoption and welfare under tax and tradable emission allowances in a two-period model to determine which mechanism is time-consistent, meaning that the regulator can credibly precommit to a level of regulatory enforcement. If she cannot pre-commit, ratcheting will be required to achieve welfare optimality. They find instances where neither the tax nor the emission trading policy are time-consistent if the damage function is strictly convex.

Montero (2002) and Subramanian et al. (2007) are the only analytical papers that consider imperfect competition in a dynamic game of technology adoption. Montero (2002) compares investments in environmental R&D under emission standards (i.e., without trading) and emission trading in a Cournot duopoly. He finds that standards can offer greater incentives than emission trading because the effort of the investing firm spills over to its rival through the emission allowance market, thus allowing the rival to increase output which hurts the investing firm. Subramanian et al. (2007) focus on the compliance strategies of profit-maximizing firms under auctioned emission allowances in a three stage model. In the first stage, firms choose how much to invest to lower their emission rates. Then, firms bid for emission allowances in a sealed-bid uniform price share auction. Finally, firms produce output to serve consumers in monopolistic or oligopolistic markets. Their main finding is that changing the number of available allowances influences the abatement to a lesser extent in dirty firms (i.e., firms with high emission intensity) relative to firms relying on cleaner technology.

Drake et al. (2012) study the effects of demand uncertainty on the capacity and production decisions of a firm under emission tax or emission trading. They model emission trading as an emission tax with a stochastic rate. In their model, the firm first chooses production capacity in two technologies, clean and dirty, when demand is uncertain, and then how much to produce with each technology after demand is observed. They find that the expected firm profits under emission trading are higher than under emission tax, contrary to the popular belief that uncertainty in the emission price would hurt the firm's profitability under emission trading regulation. This is because the firm can forgo production when the emission trading price is so high as to make production unprofitable. Drake et al. (2012) also find that demand uncertainty makes the capacity decision more sensitive to changes in a subsidy rate than the production decision. They identify conditions under which an increase in the tax rate decreases the share of the clean technology in the firm's portfolio. Similarly, Krass et al. (2013) find that an increase in the tax rate encourages the firm to switch to a cleaner technology up to a certain point after which the firm reverts back to the dirty technology. Their model is deterministic and the result is driven by the affine structure (i.e., fixed and variable cost) of the firm's production and abatement costs. In Krass et al. (2013), the regulator moves first as a Stackelberg leader. They analyze the welfare properties of a policy mix, including an emission tax, a lump sum subsidy, and consumer rebates.

Plambeck et al. (2012) also find that variability in the emission trading price can improve firm profits in a model where firms choose where to locate their production facilities in or out of a regulated region, accounting for entry and competition.

Chen and Tseng (2011) study in a real options framework the timing of investment in natural gas power generation (a cleaner technology) by a coal-fired power plant subject to load obligation and various price shocks under emission taxes and emissions trading. They also find that price volatility creates value, in their case by creating profitable opportunities. They have two main findings: (1) emission trading could trigger adoption of clean technology at a lower emission price than emission taxes; (2) volatility in emission price under emission trading are likely to induce firms to adopt earlier to hedge against emission risks.

4.2.3.3 Empirical Studies

The empirical literature on the role played by regulations in the diffusion of abatement innovations is sparse due to the lack of usable data (see Jaffe et al. 2002 and Popp et al. 2009 for detailed reviews). Finding adequate measures of regulatory stringency and of the firms' innovativeness is difficult. Early studies focused on tax credits and energy-efficient standards in buildings (Jaffe and Stavins 1995, Hassett and Metcalf 1995), home appliances (Newell et al. 1999), and motor vehicles (Greene 1990) because data were available. Popp (2005) discusses the use of patent data to measure technological change in environmental models. The recent interest in Cap-and-Trade and its deployment in the U.S. and Europe provide opportunities for theory testing. Kerr and Newell (2003) find that increased regulation increased the adoption of new technology in the context of the lead phase-down at U.S. oil refineries. Johnstone et al. (2010b) find empirical evidence that public policies play a significant role in determining patent applications in renewable energies. Johnstone et al. (2010a) find that the stringency, predictability and flexibility of the pollution control mechanisms are positively associated with the number of patent applications in air pollution abatement, wastewater effluent treatment, and solid waste management, suggesting that the design characteristics of environmental regulations matter. Keohane (2007) finds that adoption decisions for sulfur dioxide scrubbers in the U.S. power generation industry was more sensitive to cost differences under Cap-and-Trade than under direct command and control, confirming the theory that incentive-based mechanisms provide more incentives for innovation. Hascic et al. (2010) find an acceleration of the rate of innovation in a selection of climate change mitigation technologies coinciding with the implementation of the Kyoto Protocol.

Rogge et al. (2011) conduct and analyze 19 case studies in the German power sector, including power generators, specialized technology providers, and project developers. They find evidence that the EU Emissions Trading System (EU ETS) strongly increased R&D efforts toward CCS, and to a lesser extent, toward research improvements in coal efficiency. They also find that the change effective in 2013 from gratis allocation of allowances to auctioning makes new coal-fired power plants less profitable but does not ultimately change the decision to invest in coal versus natural gas. This confirms the theory that whether emission allowances are auctioned off or given out for free does not change the firms' incentive to adopt (Requate and Unold 2003). Their analysis further suggests that the EU ETS, by putting a price on carbon emissions, encouraged coal plant retrofits, particularly for older plants, by making them more cost effective. Calel and Dechezleprêtre (2013) find that the EU ETS has had a strong impact on the patenting activity of regulated firms. They estimate that the EU ETS has increased the patenting of low-carbon technologies by the sampled firms by 36.2% compared to what would have happened in the absence of regulation.

Zhang et al. (2011) study the drivers of diffusion of alternative fuel vehicles in the U.S in an agent-based simulation that interestingly combines word-of-mouth (a form of social interaction), manufacturer technology choices (i.e., market forces), and government-mandated fuel efficiency standards with a penalty for noncompliance (i.e., regulatory pressures). They find that word-of-mouth has a positive impact on the adoption of electric vehicles, but a negative effect on SUVs, both hybrid and gasoline engines. Their findings also suggest that fuel efficiency standards increase the overall market share of alternative fuel vehicles, but result in higher emissions because the share of highly emitting SUVs (both gasoline and hybrid) increases sharply.

4.2.4 Our Contribution

To the best of our knowledge, our paper is the only one to analyze the firms' incentives to invest in an innovation under Cap, Cap-and-Trade, and Tax when markets are imperfect. In our model, the firms and the regulator are strategic. We do not assume that the regulator is fully informed, nor that the regulation was optimal to begin with. The equilibrium outcomes are the consequences of the strategic interactions in the dynamic game. In particular, the number of firms that adopt the innovation is endogenously determined, as are the production quantities and abatement levels that determine output, pollution, consumer surplus, and welfare. Based on the equilibrium outcomes, the regulator can decide which mechanism to use, and how stringent the aggregate cap should be.

4.3 The Model

In order to analyze the impact of regulations on the diffusion of abatement innovations, we develop a dynamic version of the model in Chapter 2 to account specifically for technological choices in pollution abatement. We adopt the same modeling assumptions and notations as in Chapter 2 and refer the reader to Section 2.3 for the justification of these assumptions.

Recall that in Chapter 2, we analyze a game in which the regulator moves first by choosing (i) a pollution reduction goal in the form of an overall cap, S, on the region's aggregate emissions, and (ii) a mechanism to achieve that goal. Within the regulated region, strategic firms play a Cournot game: they compete on quantities and sell substitutable products in n horizontal markets. The model incorporates pollution generation, pollution abatement, and the social damage ensuing from residual emissions. In addition to choosing their production quantity, the firms also determine how much of their pollution to abate. In the baseline model of Chapter 2, the firms' abatement technologies are fixed (i.e., the cost coefficients that characterize the abatement technologies are exogenous). In this paper, we endogenize the choice of abatement technology by the firms. In other words, our approach consists in including one additional step in the game after the regulator moves and before the Cournot game. The timeline of this new game is shown in Figure 4.1. The baseline game of Chapter 2 consists of steps 1, 2 and 4. In this paper, we add the third step to the game. We first review the principal features of the baseline model, which we will hereafter refer to as the *static* model, because the firms' actions are concentrated on a single step, i.e., the last stage of the model.

4.3.1 The Static Model

In the static model, we consider n symmetric, profit-maximizing firms operating within the same regulated region. For simplicity, we assume that the firms are local monopolies. In other words, the firms exercise full market power over their local markets; competition is absent. This simplification compensates for the added complexity of the game, which results from adding one more move. Each firm chooses a production quantity, q, to serve the customers in its local market. The choice of q determines the selling price p through the linear inverse-demand function $p = a - b \cdot q$, with $a,\ b > 0$. Pollution occurs as a by-product of production. We assume that firms' unabated emissions are proportional to their production quantities, and that the firms have the same emission rate, which we normalize to 1 without loss of generality. The firms can spend money to abate pollution. The cost of pollution abatement is increasing convex in the quantity of pollution abated. Let x_i denote the fraction of pollution that firm i chooses to abate. The cost of abating $q_i \cdot x_i$ units of pollution is $c_i \cdot (q_i \cdot x_i)^2$. The residual emissions of firm i after abatement are $P_i = q_i \cdot (1 - x_i)$.

The pollution control mechanisms are the same as in Chapter 2. We model Cap, Cap-and-Trade, and Tax as done previously. The key performance measures are the total output, the firms' profits, and the social welfare (see Section 2.3 for a description of these measures).

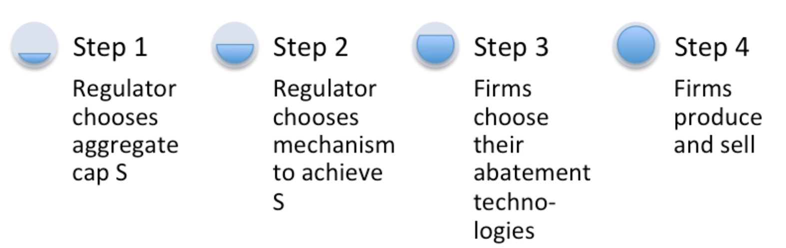

Figure 4.1. Timeline of the model with technological adoption

4.3.2 The Dynamic Model

In the static model, the firms have no control over their pollution abatement costs. Such costs are forced upon them by external factors or the firms have no time to make changes to their abatement technology after the regulation is introduced. As mentioned above, the purpose of the dynamic model is to make the abatement cost coefficients, c_i, endogenous. As in Chapter 2, we assume that there are two pollution control technologies, each of which is characterized by its cost coefficient c_i: an existing, widely-used, technology whose abatement cost coefficient is c_h, and a new, more efficient, technology whose cost coefficient is $c_l < c_h$. The pollution abatement innovation results in a lower cost coefficient for the firms that adopt it. At the beginning of the game, all n firms operate under the existing technology. We assume that the new technology is licensed by a third party. We allow each firm to choose whether or not to invest in the new technology. Let $F \geq 0$ denote the fixed investment cost if the firm invests. F captures both the acquisition cost, as well as the one-time costs associated with retrofitting the firm, changing the production processes and training the workforce. We will interchangeably use the terms fixed cost, investment cost, or switching cost to denote F. By investing F, the firm is able to change its cost coefficient to c_l. Thus, the firm faces a trade-off between the fixed cost of investing and the benefits of lower abatement costs.

Although the cap may be adjusted frequently (say, every year), the choice of a particular mechanism is a long-term decision possibly spanning several decades. For this reason, it makes sense that this decision would be made first. Let S denote the total pollution allowed, i.e., the cap. As before, we assume that, when choosing the Tax mechanism, the regulator sets the tax rate in such a manner that the total pollution is less than the cap S.

The investment model is characterized by two parameters: the cap S, which captures how stringent the regulation is, and the exogenous switching cost F. We compare the three mechanisms in terms of their ability to induce firms to invest for a given pair $\{S,\ F\}$, and study the impact of the firms' decisions on output, pollution abatement, firms' profits, and welfare. The model also allows to study the impact of a reduction in S keeping F constant, and reciprocally of a reduction in F keeping S constant. The former case corresponds to an increase in regulatory stringency, while the latter describes a situation where the regulation is stable, but the technology becomes increasingly affordable. Learning effects and economies of scale may cause F to decrease over time. The notations are summarized in Table 4.1.

4.4 Equilibrium Analysis

4.4.1 The Cap Mechanism

Under the Cap mechanism, each firm is not allowed to pollute more than $s = \frac{S}{n}$. Since s and the mechanism have been chosen, there remains two steps to the Cap game. First, each firm decides whether to adopt the abatement innovation or not. Second, the firms maximize their profits subject to the pollution constraint. The equilibrium is derived by backward induction. In the first stage of the solution procedure, each firm maximizes its profits, given its choice of c_i. Formally,

$$\max_{q_i \geq 0,\ 0 \leq x_i \leq 1} \pi_i\left(q_i,\ x_i \mid c_i\right) = q_i \cdot (a - b \cdot q_i) - c_i \cdot (q_i \cdot x_i)^2 \text{ subject to } q_i \cdot (1 - x_i) \leq s \quad (4.1)$$

In the second stage of the solution procedure, each firm decides whether to invest in the innovation or not. Specifically, firm i will invest, i.e., choose $c_i = c_l$ if and only if $\pi_i\left(q_i^*,\ x_i^* \mid c_l\right) - F > \pi_i\left(q_i^*,\ x_i^* \mid c_h\right)$, where π_i is defined by equation (4.1) and F is the fixed switching cost. It will keep $c_i = c_h$ otherwise. Theorem 11 shows that the Cap game has a unique Subgame-Perfect Nash equilibrium. All the proofs are in Appendix C.

Theorem 11 *Let* $F^{cap} = \frac{(c_h - c_l)(a - 2bs)^2}{4(b + c_l)(b + c_h)}$.
All the firms invest, i.e., $c_i = c_l\ \forall i$, *if and only if* $F < F^{cap}$. *Otherwise* $c_i = c_h\ \forall i$.

Table 4.1. Model notations (Chapter 4)

n	Number of regulated firms
q_i	Production quantity chosen by firm $i, q_i \geq 0$
p_i	Price in firm i 's market, $p_i = a - b \cdot q_i$
x_i	Abatement level chosen by firm $i, 0 \leq x_i \leq 1$
c_i	Abatement cost coefficient of firm i , $c_i \in \{c_l, c_h\}$
F	Fixed cost of adopting the abatement innovation, also called switching cost
m	Number of firms with the low cost coefficient c_l
M	Nash Equilibrium of the investment game in which m firms invest
P_i	Pollution generated by firm $i, P_i = q_i \cdot (1 - x_i)$
P	Total pollution generated by the firms, $P = \sum_{i=1}^{n} P_i$
π_i	Profit of firm i
π_i^a	Firm i's profits if it takes action $a = I$ for invest or N for do not invest
π_m^a	Firm's profits when it takes action a and m firms invest
S	Pollution cap specified for the entire region (all firms)
s	Cap assigned to individual firms, $s \geq 0$
t_i	Number of emission allowances traded by firm i
τ	Tax rate, $\tau \geq 0$

The firms' production quantities and abatement levels are

$$q_i = \frac{a + 2c_i s}{2(b + c_i)}$$
$$x_i = \frac{a - 2bs}{a + 2c_i s}$$

The equilibrium outcome under the Cap mechanism is a bang-bang, all-or-nothing outcome in which either all the firms invest in the innovation, or none of the firms invest. This result is intuitive. The firms do not invest if the switching cost is prohibitively expensive or the cap not sufficiently stringent. Because the firms' objective functions are independent of each other, the firms investment strategies are identical. See Figure 4.2 for a representation of the firms' investment strategies under the Cap mechanism.

4.4.2 The Cap-and-Trade Mechanism

The Cap-and-Trade is an extension of the Cap model in which firms are allowed to adjust their pollution constraints by trading emission allowances amongst themselves. Similar to the Cap mechanism, the game also consists of two steps. In the first step, each firm chooses whether to invest in the innovation or not. In the second step, the firms make their production, abatement, and trading decisions. Similar to Chapter 2, we assume that production, abatement, and trading happen simultaneously. Solving for the Subgame-Perfect Nash Equilibrium also involves backward induction. First, we determine the firms' production quantities, abatement levels, and trading quantities given that all the firms have

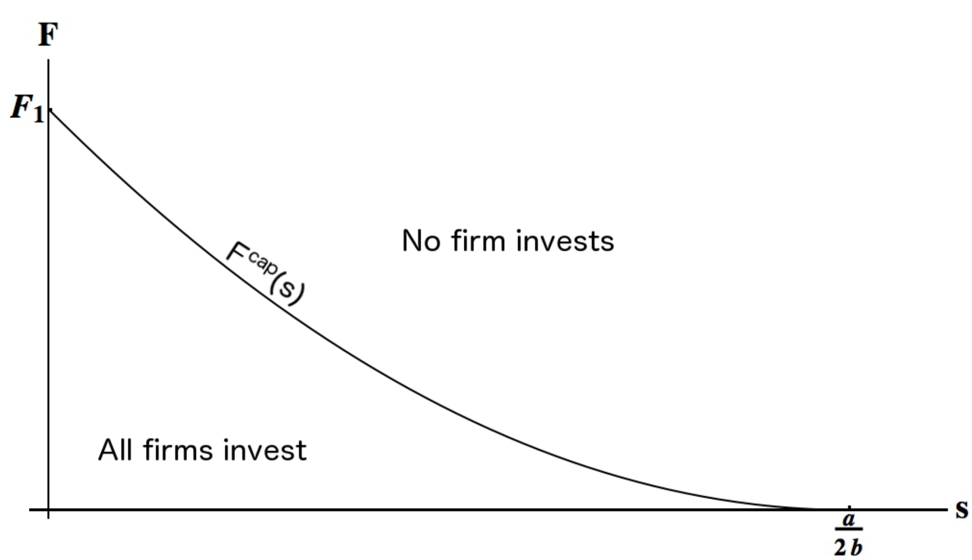

Figure 4.2. Investment equilibrium under the Cap mechanism, $F_1 = \frac{a^2(c_h - c_l)}{4(b+c_l)(b+c_h)}$

made the decision to invest or not, and the firms' decisions are common knowledge. The firms simultaneously maximize their profits given their abatement technologies,

$$\begin{aligned} \forall i, \quad \max_{q_i \geq 0,\ 0 \leq x_i \leq 1,\ t_i} \pi_i \left(q_i,\ x_i,\ t_i \mid c_i\right) &= q_i \cdot (a - b \cdot q_i) - c_i \cdot (q_i \cdot x_i)^2 + r \cdot t_i \\ \text{subject to } q_i \cdot (1 - x_i) &\leq s - t_i \text{ and } t_i \leq s \text{ and } \sum_{i=1}^{n} t_i = 0 \end{aligned}$$

Second, the firms choose their abatement technologies. The Nash equilibrium of pure strategies is the appropriate equilibrium concept since the firms' investment decisions are observed by all the firms before production is determined.

Let π_m^a denote the profits of a firm that takes action a, where $a = I$ if it invests, $a = N$ if it does not invest, and m is the number of firms that invest. We know from Chapter 2 that a firm's equilibrium response under Cap-and-Trade, and hence its profits, is uniquely determined by two parameters: (i) its own cost coefficient (i.e., whether it invests or not), and (ii) m, the number of firms with the low-cost coefficient (which is a sufficient statistic for the distribution of costs in the industry). Since each firm has the same action space $\{I,\ N\}$, there are 2^n different strategy profiles. However, the m firms that invest have the same payoff, and so do the $n - m$ firms that do not invest. Thus, there are only $n + 1$ strategy profiles to consider, depending on the value taken by $m \in \{0, 1, ..., n\}$. For each m, there are $\left(\begin{array}{c} n \\ m \end{array} \right)$ identical equilibria. We let M denote an equilibrium in which m firms invest.

$$M \text{ is a Nash equilibrium} \iff \pi_m^I > \pi_{m-1}^N \ \ (m > 0) \text{ and } \pi_m^N \geq \pi_{m+1}^I \ \ (m < n) \tag{4.2}$$

The first inequality gives the equilibrium condition for a firm that invests; the second, the equilibrium condition for a firm that does not invest. In equation (4.2), the profits if the firm invests incorporate the fixed cost F. Theorem 12 gives the unique Subgame-Perfect Nash Equilibrium of the Cap-and-Trade game.

Theorem 12 *For every F and every s, there exist n continuous, decreasing, functions of s, $\left\{F_k^{ct}\right\}_{k=1,...,n}$ such that $\forall k \in \{1, ..., n\}$,*

$$\begin{aligned} F_k^{ct}(0) &= F_1 \\ F_k^{ct}\left(\frac{a}{2b}\right) &= 0 \\ \forall s \in \left(0, \tfrac{a}{2b}\right), \quad F_{k+1}^{ct}(s) &< F_k^{ct}(s) \end{aligned}$$

Then $\forall s \in \left(0, \frac{a}{2b}\right)$, if $F_{m+1}^{ct}(s) \leq F < F_m^{ct}(s)$, then M is the unique Subgame-Perfect Nash Equilibrium of the Cap-and-Trade game.

The firms' production quantities and abatement levels are given in Theorem 4 with $\gamma = 0$ *(see Chapter 2). In particular, the firms' reactions are the same as the Cap mechanism if* $F \geq F_1^{ct}(s)$ *or* $F \leq F_n^{ct}(s)$*. In the former case,* $m = 0$*; in the latter case,* $m = n$*.*

The mathematical expressions for the series of functions $\left\{F_k^{ct}\right\}_{k=1,\ldots,n}$ is given in the proof of Theorem 12 in Appendix C. A graph of $\left\{F_k^{ct}\right\}_{k=1,\ldots,n}$ is given in Figure 4.3. When the switching cost is either very high, or very low, the Cap-and-Trade and Cap mechanisms coincide, because in this case, the firms all have the same cost coefficients, and there are no gains from trade. Trading is useless. For intermediate values of the switching cost (i.e., inside the leaf-like pattern defined by F_1^{ct} and F_n^{ct}), partial adoption of the innovation is the unique equilibrium. m firms will invest, with $0 < m < n$, abate more pollution than they would have, had they not invested, and trade the excess emission allowances to the remaining $n - m$ firms that do not invest. The price of emission allowances is sufficiently low that the firms prefer buying emission allowances rather than investing. As the cap becomes more stringent, the supply of emission allowances is depleted. At some point, it becomes profitable for one additional firm to invest. The revenue to the investing firm from the sale of emission allowances more than compensates the switching cost F. Theorem 12 extends to the case of local monopolies the result of partial adoption under Cap-and-Trade in Requate (1995). Recall that in Requate (1995), the markets are perfectly competitive.

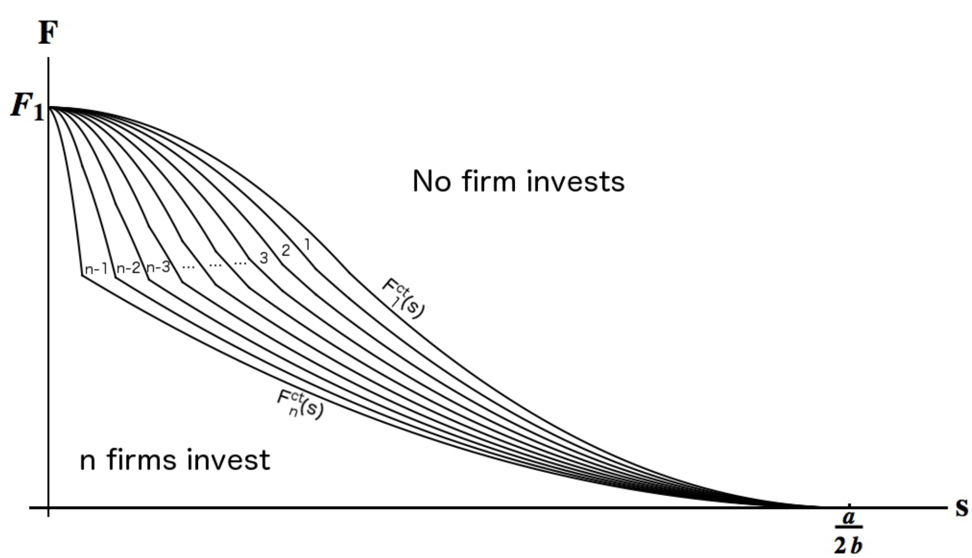

Figure 4.3. Partial adoption under the Cap-and-Trade mechanism, $F_1 = \frac{a^2(c_h - c_l)}{4(b+c_l)(b+c_h)}$

4.4.3 The Tax Mechanism

Under the Tax mechanism, every firm is assessed a fee linear in its emissions. The tax rate τ is fixed and identical for all firms. Thus, the tax paid by firm i is $\tau \cdot q_i \cdot (1 - x_i)$. The regulator anticipates the firms' reactions and chooses τ so that the aggregate pollution is less than S. As discussed in Chapter 2, knowing m is sufficient to determine such a tax rate.

The Tax game has three steps: (i) The regulator chooses τ; (ii) Each firm determines if it will adopt the abatement innovation; and (iii) The firms produce and sell their products in their respective markets. Each firm's problem during the production stage is

$$\max_{q_i \geq 0,\ 0 \leq x_i \leq 1} \pi_i \left(q_i,\ x_i \mid c_i,\ \tau\right) = q_i \cdot (a - b \cdot q_i) - c_i \cdot (q_i \cdot x_i)^2 - \tau \cdot q_i \cdot (1 - x_i)$$

When investigating the abatement innovation, each firm chooses the option with the highest payoff. The condition to adopt the innovation is

$$\pi_i \left(q_i^*,\ x_i^* \mid c_l,\ \tau\right) - F > \pi_i \left(q_i^*,\ x_i^* \mid c_h,\ \tau\right) \tag{4.3}$$

Based on condition (4.3) the cost coefficients are determined for every firm.

In choosing the tax rate, the regulator chooses the smallest τ such that the total pollution $P = \sum_{i=1}^{n} q_i \cdot (1 - x_i) \leq S$.

Theorem 13 shows that there is a unique Subgame-Perfect Nash Equilibrium for every s and every F, and gives the solution.

Theorem 13 *Let*

$$\begin{aligned}
F_0 &= \frac{a^2 c_l \left(c_h - c_l\right)}{4 c_h \left(b + c_l\right)^2} \\
F^t(s) &= \frac{c_l \left(c_h - c_l\right)\left(a - 2bs\right)^2}{4 c_h \left(b + c_l\right)^2} \\
F^T(s) &= \begin{cases} \frac{a^2 (c_h - c_l)}{4(b + c_l)(b + c_h)} - \frac{b c_h s^2}{b + c_h}, & \text{for } 0 \leq s \leq \frac{a(c_h - c_l)}{2 c_h (b + c_l)} \\ \frac{c_h (c_h - c_l)(a - 2bs)^2}{4 c_l (b + c_h)^2}, & \text{for } \frac{a(c_h - c_l)}{2 c_h (b + c_l)} \leq s \leq \frac{a}{2b} \end{cases}
\end{aligned}$$

Then $F^t(s) < F^T(s)$, $\forall s \in \left(0, \frac{a}{2b}\right)$

There exists a unique Subgame Perfect Nash Equilibrium of the Tax game.

The firms' reactions are the same as the Cap mechanism if $F \leq F^t(s)$ *or* $F \geq F^T(s)$.

All the firms invest, i.e., $c_i = c_l\ \forall i$, *if and only if* $F < F^T$. *Otherwise* $c_i = c_h\ \forall i$.

When $F^t < F < F^T$,

If $F < F_0$, the firms' reaction is

$$
\begin{aligned}
q_i &= \frac{1}{2b}\left(a - 2\sqrt{\frac{c_l c_h F}{c_h - c_l}}\right) \\
x_i &= 2b\frac{a\sqrt{c_h\left(c_h - c_l\right)F/c_l} + 2c_h F}{a^2\left(c_h - c_l\right) - 4c_l c_h F}
\end{aligned}
$$

If $F \geq F_0$, the firms' reaction is

$$
\begin{aligned}
q_i &= \frac{a}{2\left(b + c_l\right)} \\
x_i &= 1
\end{aligned}
$$

In either case, the total pollution is less than S, i.e., $P < S$.

The firms' investment equilibrium strategies under the Tax mechanism are represented in Figure 4.4.

Similar to the Cap mechanism, the equilibrium under Tax is an all-or-nothing outcome in which the firms' investment decisions coincide. There is no partial adoption like we saw under Cap-and-Trade. The firms face the same switching cost F and the same tax rate τ. In equilibrium, the incentive to invest is the same for all the firms, and they all invest when the switching cost is sufficiently low, or the tax rate high enough. There is however an interesting difference for intermediate values of F. When $F^t < F < F^T$, all the firms invest and, as a whole, abate more pollution than required, so that $P < S$. This is because the regulator has to increase the tax rate to force the firms to invest. After the firms

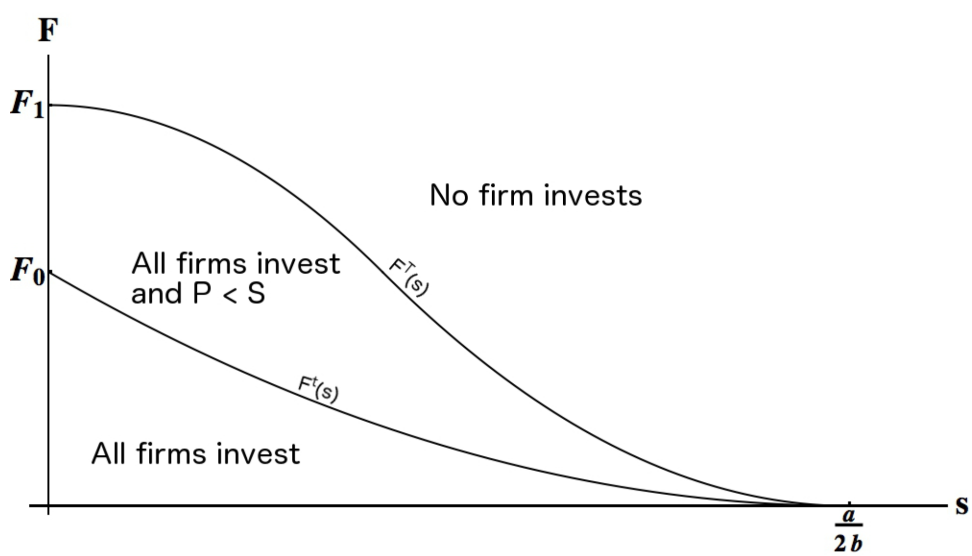

Figure 4.4. Investment equilibrium under the Tax mechanism

have invested, their abatement cost coefficient is low, and it is more profitable for them to abate pollution rather than pay a prohibitive tax. If the regulator lowers the tax rate, the firms will not invest. If the firm lowered the abatement level x to save on abatement costs and make the pollution constraint bind, it would have to pay in tax more than it saves by abating less pollution. Thus, a profit-maximizing firm will abate more pollution than required to forgo having to pay an excessive tax. Pollution abatement is the cheaper of the two compliance options. When $F < F^t$, the pollution constraint becomes binding again, i.e., $P = S$. The region between F^T and F^T is a region of *over-abatement* under the Tax mechanism.

4.5 Comparisons of the Three Mechanisms

Based on the analysis of each mechanism in equilibrium, we compare the mechanisms to each other in terms of their effectiveness at inducing investment in the abatement innovation. To illustrate the discussion, the equilibrium investment strategies under Cap, Cap-and-Trade, and Tax are represented in Figure 4.5, where the cap s, representing the level of regulatory stringency, is plotted along the x-axis and the investment cost F along the y-axis. The numbers $0,\ 1,\ 2,\ ...,\ n-2,\ n-1,\ n$ are the number of firms that invest under Cap-and-Trade. This figure is plotted for $n = 10$.

Note that the strategies coincide for $F \geq F^T$ and $F \leq F^t$. When $F \geq F^T$, the switching cost is prohibitively high, or the regulation too lax to justify investments in the

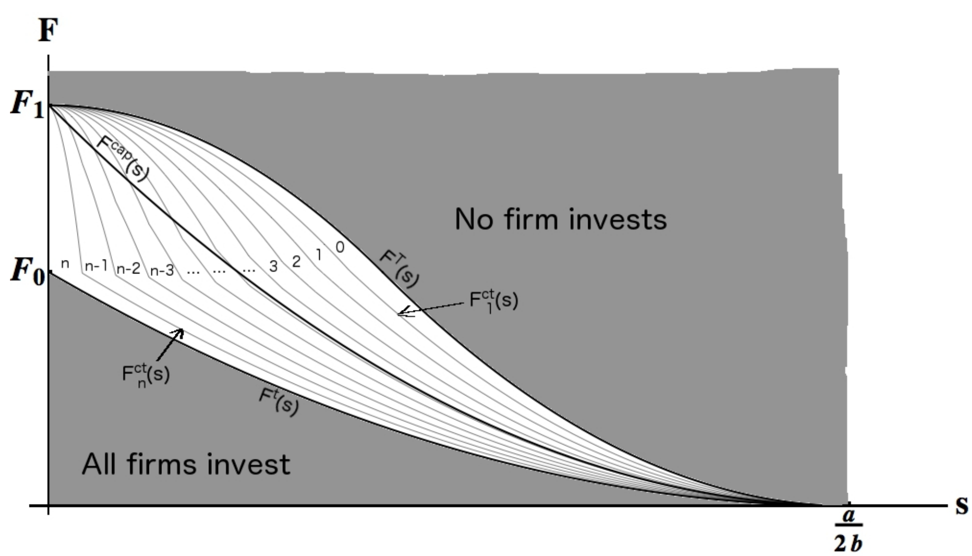

Figure 4.5. Investment equilibrium under Cap, Cap-and-Trade, and Tax

innovation. Because none of the firms invest, they all have the same cost coefficients (i.e, $c_i = c_h,\ \forall i$), and the mechanisms coincide exactly. Conversely, when $F \leq F^t$, all the firms invest (i.e, $c_i = c_l,\ \forall i$) and the mechanisms coincide. This happens because the technology is affordable and the regulation strict. We will hereafter focus on the interesting case where $F^t < F < F^T$. This region of the $\{s, F\}$ space delineates a leaf-like pattern.

Recall that F^T, F^{cap}, and F_1^{ct} are the limits of the investment strategy under Tax, Cap, and Cap-and-Trade, respectively, i.e., the lines above which no firm invests, and below which at least one firm invests. F_n^{ct} is the line below which the n^{th} firm invests under Cap-and-Trade, and F^t the lower bound of the *over-abatement* region under Tax. Theorem 14 reveals an interesting pattern.

Theorem 14 $\forall s \in \left(0,\ \frac{a}{2b}\right),\ F^t < F_n^{ct} < F^{cap} < F_1^{ct} < F^T$

Theorem 14 shows that there are more firms that invest in the abatement innovation under Tax than under any other mechanism. Specifically $F^T > F_1^{ct}$ shows that more firms invest under Tax than Cap-and-Trade, whereas $F^T > F^{cap}$ shows the superiority of Tax over Cap. We compare the mechanisms two by two in the next sections and provide some intuition for this result. To facilitate the discussion, we include Figure 4.6, which contains three graphs comparing Cap and Tax, Cap and Cap-and-Trade, and Cap-and-Trade and Tax, respectively.

4.5.1 Cap versus Tax

Because $F^{cap} < F^T$, there is more investment under Tax than Cap. Recall that the Cap mechanism focuses on attaining a predetermined pollution target, whereas the Tax mechanism charges a fee for emitting any amount of pollution. Under Cap, when the pollution goal has been met, the incentive to abate pollution ceases to function. Under Tax, however, the incentive to abate is always present because even though the firm has reduced its emissions down to the permissible level, it continues to pay a tax on its residual emissions. Thus, putting a price on residual emissions nudges the firms to continue to look for cheap abatement opportunities. Such incentives are absent under Cap. We summarize this result in proposition 8.

Proposition 8 *The Tax mechanism provides more incentives to invest in abatement innovations than the Cap mechanism, because under Tax, the firms pay a tax on residual emissions, which means that the incentives continue to take effect after the pollution target has been achieved.*

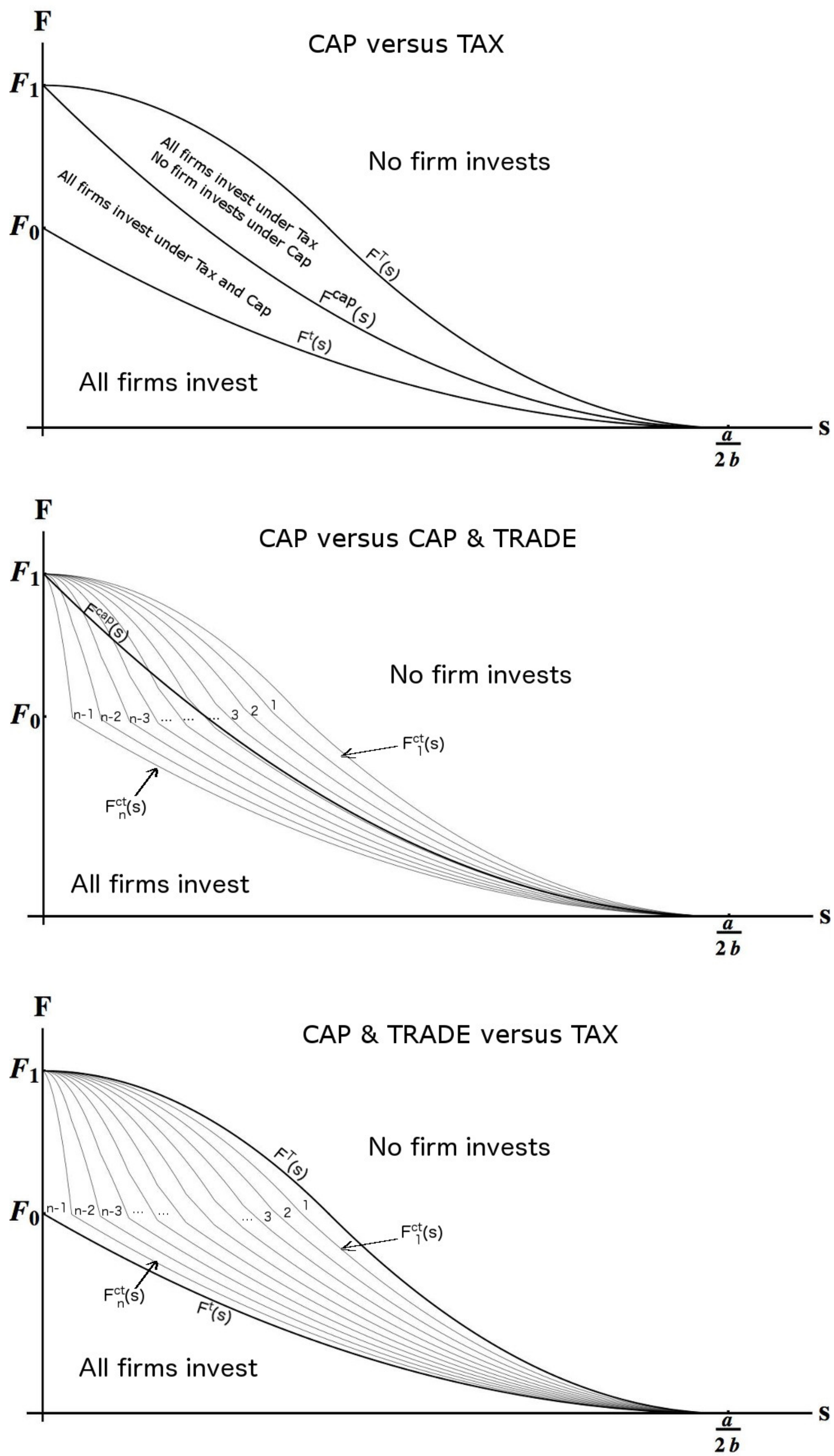

Figure 4.6. Investment equilibria. Comparisons two by two

4.5.2 Cap versus Cap-and-Trade

Comparing the Cap and Cap-and-Trade mechanisms allows us to isolate the effects of trading on the decision to invest. As we have seen, investment in the abatement innovation may be partial under Cap-and-Trade with some firms investing when others do not, whereas all the firms adopt the same strategy under Cap. As shown in Theorem 14, $F_1^{ct} > F^{cap}$ proves that some (but not all) firms invest under Cap-and-Trade *before* they invest under Cap, meaning that the firms would invest for higher values of F and s. The ability to trade creates an incentive for some firms to invest because the future gains from trade more than compensate these firms for paying the fixed investment cost F.

$F^{cap} > F_n^{ct}$ shows that some firms will defer investment under Cap-and-Trade when compared to Cap. In other words, between F^{cap} and F_n^{ct}, more firms invest under Cap than Cap-and-Trade. The intuition behind this result is that, for some firms, trading acts as a substitute for investment. Whenever a firm invests, it creates an opportunity for all the firms that have not yet invested to purchase emission allowances from that firm, rather than invest themselves. Our results show that there are values of F and s for which firms are better off trading than investing. To summarize, the effect of trading on investment is a double-edged sword: it encourages investment by some firms while at the same time discouraging investment by other firms. Our results show that in the aggregate, there could be less investment under Cap-and-Trade than under Cap.

4.5.3 Cap-and-Trade versus Tax

Because $F_1^{ct} < F^T$, there is more adoption under Tax than Cap-and-Trade. Recall that in the static model of Chapter 2, Tax and Cap-and-Trade are equivalent. In our dynamic investment model, however, Tax and Cap-and-Trade provide different incentives for investing in abatement innovations. When compared to Tax, the possibility to trade emission allowances acts as a deterrent to investment for all firms. Our first research question was to determine which mechanisms encourage more investment in abatement innovations. Our results show that the Tax mechanism emerges as the dominant mechanism in this regard. Our results also show that the ability to trade under Cap-and-Trade does not always improve the incentives to invest compared to the strict Cap mechanism.

When discussing the results of Theorem 13 (the Tax mechanism equilibrium analysis), we noted that in the *over-abatement* region defined by $F^t < F < F^T$, the firms abate more than required under the Tax mechanism, i.e., $P < S$. By contrast, $P = S$ always under Cap and Cap-and-Trade. Thus, we get an additional benefit with the Tax mechanism in the form of additional emission reductions. These results are summarized in Proposition 9.

Proposition 9 *Trading acts as a substitute for investment. By putting a price on residual emissions, the Tax mechanism generates perpetual incentives for investment and abatement, leading to more investment under Tax than any other mechanism, as well as more pollution abatement under Tax than any other mechanism.*

Proposition 9 establishes the superiority of the Tax mechanism to promote both technological investments and abatement efforts. Over-abatement causes additional reductions in the pollution damage, which is clearly good for society. However, it could significantly hurt the firms' profits to the point that the overall effect on society is negative because the firms are losing more than the welfare gains resulting from less pollution. In addition to forcing excessive pollution reductions, the Tax mechanism may also cause too much investment in the abatement innovation, i.e., investments in excess of what is optimal from a welfare point of view. If the regulation is too heavy-handed, the firms could be forced to spend a lot of money, which hurts their profitability, or to reduce output a lot, which hurts consumers. To sift through these conflicting objectives, we investigate in the next section the welfare effects of Cap, Cap-and-Trade, and Tax.

As explained from the outset, the goal of pollution regulators (such as the EPA in the U.S) is to limit pollution to a certain level, and possibly to bring this level down over time. In practice, pollution limits are the outcomes of a complex process in which scientific, economic, and political factors are weighed. In this paper, we do not know what the ideal pollution limits are. For this reason, we consider *any* pollution limit, and analyze the firms' compliance strategies taking this limit (i.e., the cap S) as given. We compare the mechanisms with each other, and not to some external benchmarks.

Recall that, consistent with the literature (Nault 1996, Jacobs and Subramanian 2012), we define welfare as the sum of the firms' profits and the consumer surplus after correction for the pollution damage, because consumers ultimately support the social costs of pollution. This measure of social welfare captures the net effects of the firms' compliance strategies on society by taking into account changes in firm profits, consumer surpluses, and the pollution damage.

Let W denote the social welfare, Π the firms' joint profits, CES the consumer economic surplus, and D the pollution damage. Consistent with previous chapters, we have

$$W = \Pi + CES - D$$

where $D = d \cdot P^2$ and P is the total *net* pollution $P = \sum_{i=1}^{n} q_i \cdot (1 - x_i)$. Formally,

$$W(\mathbf{q},\mathbf{x}) = \sum_{i=1}^{n}\left[q_i \cdot (a - b \cdot q_i) - c_i \cdot (q_i \cdot x_i)^2 + \frac{b}{2} \cdot q_i^2\right] - d \cdot \left(\sum_{i=1}^{n} q_i \cdot (1 - x_i)\right)^2$$

where $\mathbf{q} = (q_1, ..., q_n)$ and $\mathbf{x} = (x_1, ..., x_n)$ are the production quantity and abatement level vectors, $\frac{b}{2} \cdot q_i^2$ is the consumer economic surplus in market i, and the last term is the pollution damage $D(P)$.

At this point, we make two important observations:

1. In the formula for W, the firm profits Π are *before* tax, because the tax is simply a transfer between the firms and the regulator. The proceeds from the tax stay in the economy. Likewise, we consider the profits net of the investment cost F. The rationale for this choice is that the money spent by the firms that adopt the innovation also stays in the economy. From a welfare point of view, it is not lost. It is redistributed to other economic actors, e.g., the third party that licenses the innovation.

2. If the pollutant is very harmful (i.e., the pollution damage factor d is large), over-abatement under Tax will generate enough welfare gains to compensate any losses in firm and consumer surpluses. In this case, more investment under Tax will be good for society. Conversely, when the pollutant causes very little harm (i.e., d is small), regulations do not improve welfare. The regulator should actually do nothing. In the next section, we will assume that the pollutant is sufficiently harmful to warrant regulation, but not to the point that the Tax mechanism would always dominate. As we show next, the welfare rankings of the Cap, Cap-and-Trade, and Tax mechanisms in this case are not unambiguous.

4.6 Welfare Considerations: A Numerical Analysis

Our focus will be on the welfare outcomes for any pair $\{s,F\}$ within the leaf-life region identified in the previous section, because outside of this region, the mechanisms coincide. In the leaf-like region, the firms always invest under Tax, but not necessarily under Cap and Cap-and-Trade. To simplify the analysis without losing any important insight, we consider only two firms. We compare the welfare outcomes numerically. Our analysis relies on four graphs represented in Figures 4.7—4.10. The graphs correspond to different values of the fixed investment cost F. The graphs are ordered by decreasing value of F. On each of these graphs, the cap s runs along the x-axis while welfare is measured on the y-axis. The regions shaded in gray correspond to values of s and F outside the leaf-like pattern. The focus of

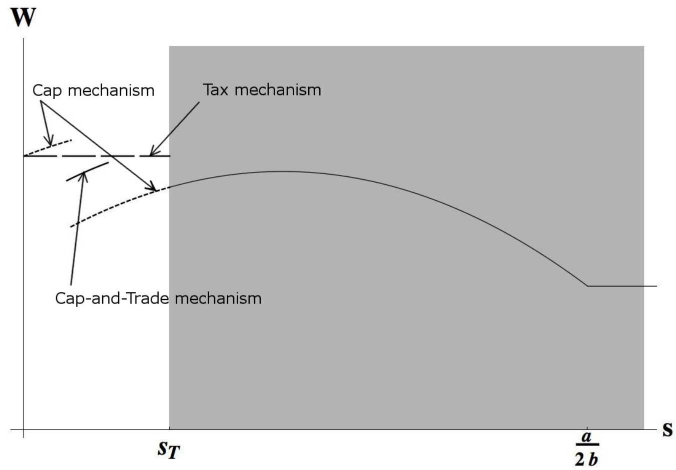

Figure 4.7. Welfare comparisons for large $F > F_0$.

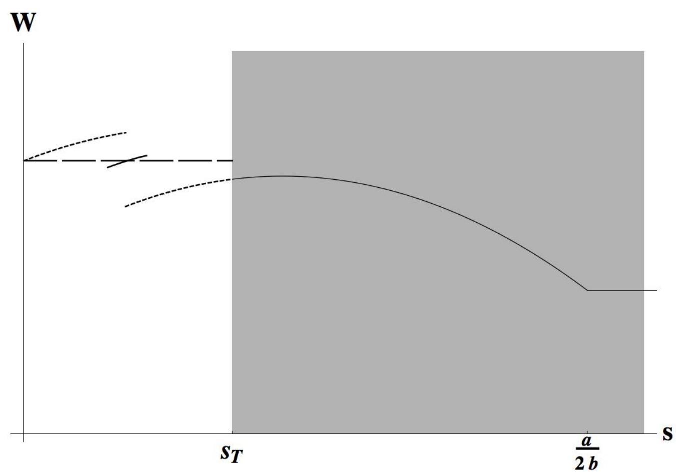

Figure 4.8. Welfare comparisons for small $F > F_0$

our discussion will be on the unshaded region delimited by s_t and s_T. Three functions are plotted on each graph: (1) the solid line corresponds to the welfare under Cap-and-Trade; (2) the dashed line the welfare under Tax; and (3) the dotted line the welfare under Cap.

The first two figures (4.7 and 4.8) correspond to $F > F_0$. Under the Tax mechanism, the firms produce a fixed quantity $Q = \frac{a}{b+c_l}$ and abate all their pollution (i.e., $x_i = 0$).

Figure 4.7 corresponds to F significantly higher than F_0 (but less than F_1). The Tax mechanism dominates Cap and Cap-and-Trade except when both firms invest under Cap. The output is greater under Tax than under Cap when the firms do not invest under Cap. Since the firms abate all their pollution under Tax, they do not pay a tax and the pollution damage is completely avoided. When the cap is sufficiently small so that the firms invests under Cap, the output is higher under Cap than Tax, and the Cap mechanism becomes

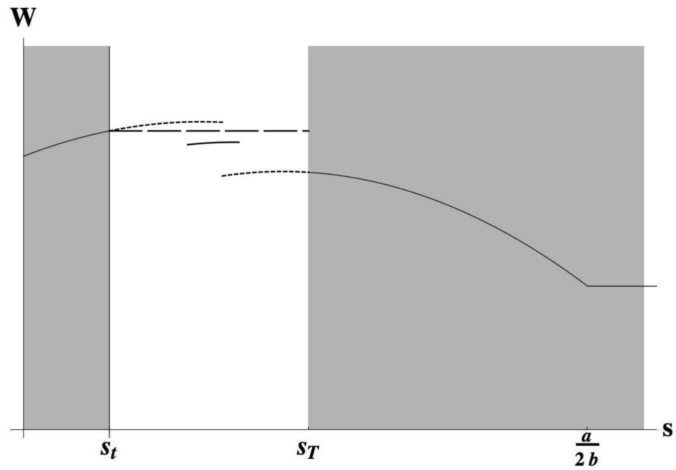

Figure 4.9. Welfare comparisons for large $F < F_0$

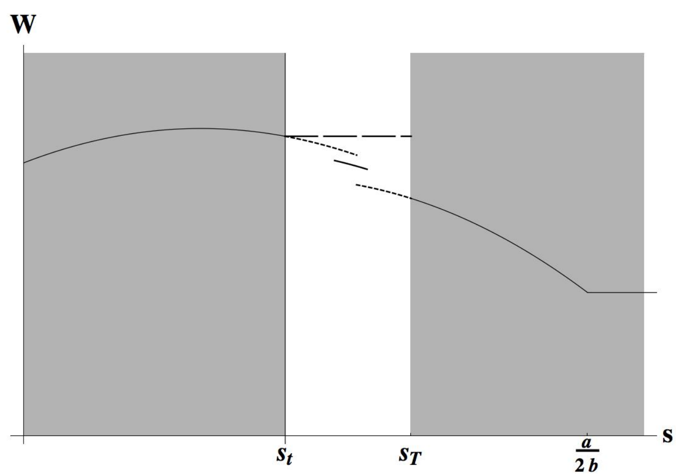

Figure 4.10. Welfare comparisons for small $F < F_0$

dominant.

Figure 4.8 corresponds to a lower value of F, although $F > F_0$ still. The pattern identified in Figure 4.7 continues to hold (i.e., Cap dominates for low caps, while Tax dominates elsewhere), except that Cap-and-Trade may become dominant for some values of s. In the figure, this is evidenced by the solid line intersecting the dashed line. Note that while the welfare under Tax is constant, as F decreases, the range of caps for which trading will occur shifts to the right. This means that the cap is less stringent: the firms' profits and the output are increasing in s. The welfare under Cap-and-Trade increases to the point that it surpasses the welfare under Tax. Next, we consider values of $F < F_0$ in Figures 4.9 and 4.10.

In this case, the firms still produce a fixed quantity under Tax (for any s), but they do not abate all their pollution. Similar to the case $F > F_0$, the Cap mechanism dominates for small s, while the Tax mechanism dominates everywhere else. For large values of $F < F_0$, the Cap-and-Trade mechanism could dominate Tax. (There is no figure corresponding to this scenario.) The main difference in this case is that, as illustrated by Figure 4.10, the Tax mechanism will dominate for all $s \in (s_t, s_T)$ when F is small.

Comparing Cap and Cap-and-Trade shows that more investment in innovations always improves welfare. The welfare jumps up when the number of firms that invest goes from 0 to 1, or from 1 to 2. Although there is always more investment under Tax, the comparisons with Cap and Cap-and-Trade are confounded by the contradictory effects of over-abatement on firm profits and consumer surplus.

4.7 Concluding Remarks

Because of environmental externalities, polluting firms lack the incentives to reduce their pollution, and markets fail to control pollution. As a result, regulations are needed. Cap-and-Trade regulation has several appealing features: (i) from a practical standpoint, it directly controls pollution; (ii) from an implementation point of view, firms have a preference for Cap-and-Trade over emission Taxes, because with Cap-and-Trade, some firms can make money, whereas under Tax, everybody pays, because everybody pollutes to some extent. Thus, firms are less likely to resist Cap-and-Trade than they would a flat, uniform tax; (iii) Cap-and-Trade is conceptually appealing because it creates ad hoc markets, i.e., the exchanges on which emission allowances are traded, to correct for the failure of traditional markets. In spite of its merits, this paper shows that the ability to trade under Cap-and-Trade deters investments in abatement innovations, because trading serves as a substitute for investing. This results in less firms investing in abatement innovation than under the Tax mechanism, and even the centralized Cap mechanism. This is a serious drawback for two main reasons: (i) Innovations make it easier for firms to abate pollution. If pollution abatement is cheap, there will be very few residual emissions, and pollution becomes a nonissue. For this reason, it is paramount that cost-efficient abatement innovations reach critical mass quickly; and (ii) the long-term benefits of abatement innovations can be huge, because they compound over extended periods of time. We show that under taxes, there is more investment than under any other mechanism. The Tax mechanism puts a direct, unavoidable price on emissions. Under taxes, the firms still pay something for their residual emissions. For this reason, the incentive to invest never goes away. Under emission quotas, the incentive disappears as soon as the quota is met. We find one additional benefit with taxation. Firms abate more pollution under taxes because investing makes it cheaper for the firm to abate pollution, than to pay a tax on emissions. Thus, the unpopular tax mechanism has several merits of its own. It is very effective at controlling pollution, encourages the adoption of cost-effective abatement innovations, and can generate additional government revenues to fund infrastructures, health care, education, and research.

APPENDIX A

CHAPTER 2 PROOFS

A.1 Proof of Theorem 1

There are four main steps to the proof:

1. We analyze the Lagrangian and derive necessary conditions on $q_i,\ x_i$ based on the Kuhn-Tucker conditions;
2. We show that the equilibrium is symmetric, meaning that firms that have the same cost coefficient adopt the same response;
3. We solve for the equilibrium (there is only one);
4. We prove that the equilibrium is a global maximum.

A.1.1 Step 1 - Analysis of the Lagrangian

Write the Lagrangian of the problem for firm i,

$\mathfrak{L} = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c_i \cdot (q_i \cdot x_i)^2 - G_i(\mathbf{q}, \mathbf{x}) + \lambda_i \cdot q_i + \mu_{i1} \cdot x_i - \mu_{i2} \cdot (x_i - 1)$.
The Kuhn-Tucker necessary conditions are

$$\begin{aligned} a - \gamma \cdot b \cdot Q_{-i} - 2q_i \cdot \left(b + c_i \cdot x_i^2\right) - 2d \cdot (1 - x_i) \cdot P + \lambda_i &= 0 \qquad \text{(A.1)} \\ 2q_i \cdot (d \cdot P - c_i \cdot q_i \cdot x_i) + \mu_{i1} - \mu_{i2} &= 0 \qquad \text{(A.2)} \end{aligned}$$

with the complementary slackness conditions $\lambda_i \cdot q_i = 0,\ \mu_{i1} \cdot x_i = 0$ and $\mu_{i2} \cdot (x_i - 1) = 0$, and the feasibility constraints $0 \le x_i \le 1$ and $q_i,\ \lambda_i,\ \mu_{i1},\ \mu_{i2} \ge 0$.

For each i, it is not possible for μ_{i1} and μ_{i2} to be simultaneously > 0.

First, we show that $\mu_{i1} = 0$. *Proof.* (By contradiction.) Suppose that $\mu_{i1} > 0$. Then $\mu_{i2} = 0$ and $x_i = 0$, and by equation $(A.2)$ $\mu_{i1} = -2dPq_i \le 0$. □

From now on, rewrite μ_{i2} simply as μ_i (the Lagrangian associated with the constraint $x_i \le 1$).

We will assume that $q_i > 0$ (which implies that $\lambda_i = 0$), and check that the solutions give positive production quantities and profits. There are two cases:

- $\mu_i > 0$. Then $x_i = 1$ and by equation $(A.1)$,

  $$q_i = \frac{a - \gamma b Q_{-i}}{2(b + c_i)}$$

  The solution holds if

  $$dP > c_i q_i$$

  (This reflects the condition $\mu_i > 0$).

- $\mu_i = 0$. Then by equation $(A.2)$, $dP = c_i q_i x_i$. After substitution in (A.1), we get

$$q_i^* = \frac{a - \gamma b Q_{-i}}{2\left(b + c_i x_i\right)}$$

The condition

$$x_i \leq 1 \iff dP \leq c_i q_i$$

A.1.2 Step 2 - Proof of Symmetry

Next, we show the following lemma.

Lemma 1 *The equilibria are symmetric, i.e., if firms i and j have the same abatement cost coefficient, $\mu_i = \mu_j$. This implies that firms with the same cost coefficient will adopt the same response.*

Proof. Consider two firms, indexed i and $j \neq i$, and suppose that $c_i = c_j = c$.

Consider the case where $\mu_i = 0$ and suppose that $\mu_j > 0$. Then, because $\mu_i = 0$, the Kuhn-Tucker conditions imply that

$$\begin{aligned} q_i &= \frac{a - \gamma b Q_{-i}}{2\left(b + c x_i\right)} \\ dP &= c q_i x_i \end{aligned} \tag{A.3}$$

and because $\mu_j > 0$, we have

$$\begin{aligned} x_j &= 1 \\ q_j &= \frac{a - \gamma b Q_{-j}}{2\left(b + c\right)} \\ dP &> c q_j \end{aligned} \tag{A.4}$$

Rewrite (A.3) as

$$2bq_i + 2cx_i q_i = a - \gamma b Q_{-i}$$

Subtract $\gamma b q_i$ from both sides of the equation: in equilibrium, we must have

$$q_i = \frac{a - \gamma b Q}{b\left(2 - \gamma\right) + 2cx_i}$$

Similarly, for firm j

$$q_j = \frac{a - \gamma b Q}{b\left(2 - \gamma\right) + 2c}$$

The conditions

$$\begin{aligned} dP = c q_i x_i &> c q_j \\ \iff \frac{\left(a - \gamma b Q\right) x_i}{b\left(2 - \gamma\right) + 2cx_i} &> \frac{a - \gamma b Q}{b\left(2 - \gamma\right) + 2c} \\ \iff x_i &> 1 \end{aligned}$$

Thus, if $\mu_i = 0$, then $\mu_j = 0$ also.

Consider now the case $\mu_i > 0$. We have just shown that we must have $\mu_j > 0$. (If not, $\mu_j = 0$ leads to $x_j > 1$.)

Thus, we have

$$\begin{aligned} x_i &= x_j = 1 \\ q_i &= q_j = \frac{a - \gamma b Q}{b(2-\gamma) + 2c} \\ \mu_i &= 2q_i (dP - cq_i) = 2q_j (dP - cq_j) = \mu_j \square \end{aligned}$$

A.1.3 Step 3 - Equilibrium Analysis

Consider the special case $m = 0$ ($c_i = c_h$ for all i). There are only two cases to consider:

- $\mu_i > 0$. In this case, $x_i = 1 \; \forall i$ and $P = 0$. This is true as long as $c_h q_i < dP = 0$, a contradiction.

- $\mu_i = 0$. Then, $dP = dnq_i (1 - x_i) = c_h q_i x_i$ which implies that

$$x_i = \frac{nd}{c_h + nd} \in [0, 1]$$

  from which we derive

$$q_i = \frac{a - \gamma b Q_{-i}}{2(b + c_h x_i)} \iff 2q_i \left(b + \frac{nc_h d}{c_h + nd} \right) + \gamma b Q_{-i} = a \tag{A.5}$$

  We can rewrite

$$q_i = \frac{a - \gamma b Q}{b(2-\gamma) + 2c_h x_i}$$

  And since all the x_i are equal, we immediately have that all the q_i are equal.

$$\begin{aligned} (A.5) \;\; &\Rightarrow \;\; 2q_i \left(b + \frac{nc_h d}{c_h + nd} \right) + (n-1)\gamma b q_i = a \\ &\Rightarrow \;\; q_i = \frac{a}{b[2 + (n-1)\gamma] + \frac{2nc_h d}{c_h + nd}} \end{aligned}$$

The symmetric case $m = n$ is obtained from the case $m = 0$ by substituting c_l for c_h.

Assume now $0 < m < n$. Knowing that the equilibria are symmetric, the n–firm problem simplifies to a 2-firm problem. There are four cases to analyze depending on the values taken by μ_l and μ_h :

1. $\mu_l > 0$ and $\mu_h > 0$. Then, $x_l = x_h = 1$. This implies that $P = 0$ and $\mu_l = -2c_l q_l^2 \leq 0$, a contradiction (similarly $\mu_h \leq 0$).

2. $\mu_l = 0$ and $\mu_h > 0$. Then, $x_h = 1$, $q_h = \frac{a-\gamma bQ_{-h}}{2(b+c_h)}$, $dP > c_h q_h$ and $q_l = \frac{a-\gamma bQ_{-l}}{2(b+c_l x_l)}$, $dP = c_l q_l x_l$. Note that

$$\begin{aligned} dP &= md q_l (1 - x_l) = c_l q_l x_l \\ \iff & x_l = \frac{md}{c_l + md} \in [0,1] \end{aligned}$$

We have the following expressions for q_l and q_h :

$$\begin{aligned} q_l &= \frac{a - \gamma bQ}{b(2-\gamma) + \frac{2mc_l d}{c_l + md}} \\ q_h &= \frac{a - \gamma Q}{b(2-\gamma) + 2c_h} \end{aligned}$$

Finally, we must have

$$\begin{aligned} dP &= c_l q_l x_l > c_h q_h \\ \iff & \frac{mc_l (a - \gamma bQ) d}{b(2-\gamma)(c_l + md) + 2mc_l d} > \frac{c_h (a - \gamma bQ)}{b(2-\gamma) + 2c_h} \\ \iff & mb(2-\gamma) c_l d + 2mc_l c_h d > b(2-\gamma) c_h (c_l + md) + 2mc_l c_h d \\ \iff & d < -\frac{c_l c_h}{m(c_h - c_l)} < 0, \text{ a contradiction} \end{aligned}$$

3. $\mu_l > 0$ and $\mu_h = 0$. Then, $x_l = 1$, $q_l = \frac{a-\gamma bQ_{-l}}{2(b+c_l)}$, $dP > c_l q_l$ and $q_h = \frac{a-\gamma bQ_{-h}}{2(b+c_h x_h)}$, $dP = c_h q_h x_h$. Note that

$$\begin{aligned} dP &= (n-m) d q_h (1 - x_h) = c_h q_h x_h \\ \iff & x_h = \frac{(n-m) d}{c_h + (n-m) d} \in [0,1] \end{aligned}$$

We have the following expressions for q_l and q_h :

$$\begin{aligned} & \begin{cases} 2(b + c_l) q_l + \gamma bQ_{-l} = a \\ 2\left(b + \frac{(n-m)c_h d}{c_h + (n-m)d}\right) q_h + \gamma bQ_{-h} = a \end{cases} \\ \iff & \begin{cases} (b[2 + (m-1)\gamma] + 2c_l) q_l + (n-m)\gamma b q_h = a \\ m\gamma b q_l + \left(b[2 + (n-m-1)\gamma] + \frac{2(n-m)c_h d}{c_h + (n-m)d}\right) q_h = a \end{cases} \end{aligned}$$

The unique solution is (using Cramer's rule)

$$\begin{aligned} q_l &= \frac{a\left[b(2-\gamma) + \frac{2(n-m)c_h d}{c_h + (n-m)d}\right]}{[b(2 + (m-1)\gamma) + 2c_l]\left[b(2 + (n-m-1)\gamma) + \frac{2(n-m)c_h d}{c_h + (n-m)d}\right] - m(n-m)\gamma^2 b^2} \\ q_h &= \frac{a[b(2-\gamma) + 2c_l]}{[b(2 + (m-1)\gamma) + 2c_l]\left[b(2 + (n-m-1)\gamma) + \frac{2(n-m)c_h d}{c_h + (n-m)d}\right] - m(n-m)\gamma^2 b^2} \end{aligned}$$

Recall that

$$
\begin{aligned}
x_l &= 1 \\
x_h &= \frac{(n-m)\,d}{c_h+(n-m)\,d}
\end{aligned}
$$

Finally, we must check that

$$
\begin{aligned}
dP &= c_h q_h x_h > c_l q_l \\
\iff & \frac{(n-m)\left[b\,(2-\gamma)+2c_l\right]c_h d}{c_h+(n-m)\,d} > c_l\left[b\,(2-\gamma)+\frac{2\,(n-m)\,c_h d}{c_h+(n-m)\,d}\right] \\
\iff & \frac{(n-m)\,b\,(2-\gamma)\,c_h d}{c_h+(n-m)\,d} > b\,(2-\gamma)\,c_l \\
\iff & (n-m)\,c_h d > c_l c_h + (n-m)\,c_l d \\
\iff & d > \frac{c_l c_h}{(n-m)\,(c_h-c_l)} \equiv \underline{d}
\end{aligned}
$$

Note that $q_l,\ q_h > 0$. Call this stationary point M_1.

4. $\mu_l = \mu_h = 0$. Then, $q_l = \frac{a-\gamma bQ_{-l}}{2(b+c_l x_l)}$, $q_h = \frac{a-\gamma bQ_{-h}}{2(b+c_h x_h)}$ and $dP = c_l q_l x_l = c_h q_h x_h$. Rewrite

$$
\begin{aligned}
q_l &= \frac{a-\gamma bQ_{-l}}{2\,(b+c_l x_l)} = \frac{a-\gamma bQ}{b\,(2-\gamma)+2c_l x_l} \\
\iff q_l &= \frac{a-\gamma bQ-2dP}{b\,(2-\gamma)}
\end{aligned}
$$

Similarly

$$
q_h = \frac{a-\gamma bQ-2dP}{b\,(2-\gamma)}
$$

Thus,

$$
q_l = q_h
$$

This implies that $c_l x_l = c_h x_h$. Thus,

$$
\begin{aligned}
dP &= dq_l\left[m\,(1-x_l)+(n-m)\,(1-x_h)\right] \\
&= dq_l\left[n-\left(m+\frac{(n-m)\,c_l}{c_h}\right)x_l\right] = c_l q_l x_l
\end{aligned}
$$

We can now solve for x_l and calculate $x_h = \frac{c_l x_l}{c_h}$ and $q_l = q_h$. We find

$$
\begin{aligned}
x_l &= \frac{nc_h}{(n-m)\,c_l+mc_h+\frac{c_l c_h}{d}} = \frac{nc_h d}{((n-m)\,c_l+mc_h)\,d+c_l c_h} \\
x_h &= \frac{nc_l d}{((n-m)\,c_l+mc_h)\,d+c_l c_h} \\
q_l &= \frac{a-\gamma bQ}{b\,(2-\gamma)+2c_l x_l} = \frac{a-n\gamma bq_l}{b\,(2-\gamma)+\frac{2nc_l c_h d}{((n-m)c_l+mc_h)d+c_l c_h}} \\
\iff q_l &= q_h = \frac{a\left[((n-m)\,c_l+mc_h)\,d+c_l c_h\right]}{b\,(2+(n-1)\,\gamma)\,(((n-m)\,c_l+mc_h)\,d+c_l c_h)+2nc_l c_h d}
\end{aligned}
$$

Clearly, $x_l,\ x_h,\ q_l$, and q_h are always positive, and $x_h \leq x_l$. The condition

$$x_l \leq 1 \iff d \leq \underline{d}$$

Call this stationary point M_2. Note that M_1 and M_2 extend to the special cases $m = 0$ and $m = n$.

A.1.4 Step 4 - Second Order Sufficient Condition

To complete the proof, we now check the second order sufficient condition, a test we must do for each firm i. The Hessian H_i for firm i is the following 2×2 matrix:

$$H_i = -2 \begin{pmatrix} b + c_i x_i^2 + d\left(1 - x_i\right)^2 & 2c_i q_i x_i - d\left[P + q_i\left(1 - x_i\right)\right] \\ 2c_i q_i x_i - d\left[P + q_i\left(1 - x_i\right)\right] & q_i^2\left(c_i + d\right) \end{pmatrix}$$

We prove that H is negative definite. The diagonal elements of H_i are negative.

We need to show that $\det\left(H_i\right) > 0$.

Let us start with stationary point M_2. At stationary point M_2, we have $dP = c_l q_l x_l = c_h q_h x_h$. Thus,

$$H_i = -2 \begin{pmatrix} b + c_i x_i^2 + d\left(1 - x_i\right)^2 & c_i q_i x_i - dq_i\left(1 - x_i\right) \\ c_i q_i x_i - dq_i\left(1 - x_i\right) & q_i^2\left(c_i + d\right) \end{pmatrix}$$

and $\det\left(H_i\right) = 4q_i^2\left(bc_i + bd + c_i d\right) > 0$.

At stationary point M_1, we have $dP = c_h q_h x_h$. Thus, $\det\left(H_h\right) > 0$. For the low-cost firms, however, we have $x_l = 1$ and $dP > c_l q_l$.

$$\begin{aligned} H_l &= -2 \begin{pmatrix} b + c_l & 2c_l q_l - dP \\ 2c_l q_l - dP & q_l^2\left(c_l + d\right) \end{pmatrix} \\ &\Rightarrow \det\left(H_l\right) = 4\left[\left(b + c_l\right)\left(c_l + d\right) q_l^2 - \left(2c_l q_l - dP\right)^2\right] \end{aligned}$$

If d is sufficiently large, $\det\left(H_l\right)$ could be negative. We use a direct approach to show that M_1 is a local maximum for low-cost firms using a method inspired from Luenberger and Ye (2008). Note that for low-cost firms, M_1 is on the boundary of the feasible set. There is only one active constraint at $M_1 : g\left(q_l, x_l\right) = x_l - 1$. $\bigtriangledown g = \begin{pmatrix} 0 & 1 \end{pmatrix}^T \neq 0,$ where the first coordinate is the production quantity q_l, and the second the abatement level x_l. Thus, M_1 is regular.

We will show that for any feasible move away from M_1, the value of the objective function decreases. The gradient of the objective function at M_1 is

$$\bigtriangledown \pi_l|_{M_1} = \begin{pmatrix} 0 & 2q_l\left(dP - c_l q_l\right) \end{pmatrix}^T$$

A non-zero feasible direction, $d = \begin{pmatrix} d_1 & d_2 \end{pmatrix}^T$, is defined by $(\bigtriangledown g)^T \cdot d \leq 0$. This condition is equivalent to

$$d_2 \leq 0$$

Note that if d is a feasible direction, $d \cdot \bigtriangledown \pi_l|_{M_1} = 2q_l \cdot (dP - c_l q_l) \cdot d_2 \leq 0$, because at M_1, $dP > c_l q_l$. $\square$

A.2 Proof of Theorem 2

Write the Lagrangian

$\mathfrak{L} = \sum_{i=1}^{n} q_i \cdot \left[a - \left(\frac{b}{2} + c_i \cdot x_i^2\right) \cdot q_i - \gamma \cdot b \cdot Q_{-i}\right] - d \cdot P^2 + \sum_{i=1}^{n} \left[\lambda_i \cdot q_i + \mu_{i1} \cdot x_i - \mu_{i2} \cdot (x_i - 1)\right]$, where λ_i, μ_{i1} and μ_{i2} are Lagrange multipliers. The Kuhn-Tucker necessary (first order) conditions are, for all i:

$$a - \gamma \cdot b \cdot Q_{-i} - q_i \cdot \left(b + 2c_i \cdot x_i^2\right) - 2d \cdot (1 - x_i) \cdot P + \lambda_i = 0 \tag{A.6}$$

$$2q_i \cdot (d \cdot P - c_i \cdot q_i \cdot x_i) + \mu_{i1} - \mu_{i2} = 0 \tag{A.7}$$

(and the usual complementary slackness conditions and feasibility constraints).

(A.7) is exactly (A.2) (*Groves* mechanism) and (A.6) is identical to (A.1) except for the coefficient of q_i which is $-\left(b + 2c_i x_i^2\right)$ instead of $-2\left(b + c_i x_i^2\right)$. The proof is very similar to Theorem 1, and we only highlight the main points. Please, refer to the proof of Theorem 1 above for details.

Conditions similar to Theorem 1 are derived from the Kuhn-Tucker conditions (step 1), and the equilibrium is also symmetric (step 2).

- $dP \leq c_i q_i$ (this corresponds to $\mu_i = 0$) : Then,

$$\begin{aligned} dP &= c_i q_i x_i \\ q_i &= \frac{a - \gamma b Q_{-i}}{b + 2c_i x_i} \end{aligned}$$

- $dP > c_i q_i$ (this corresponds to $\mu_i > 0$) : Then,

$$\begin{aligned} x_i &= 1 \\ q_i &= \frac{a - \gamma b Q_{-i}}{b + 2c_i} \end{aligned}$$

When analyzing the equilibrium (step 3), the cases ($\mu_l > 0;\ \mu_h > 0$) and ($\mu_l = 0;\ \mu_h > 0$) lead to contradictions. The following cases lead to the unique equilibrium:

1. $\mu_l > 0$ and $\mu_h = 0$. Then, $x_l = 1,\ q_l = \frac{a-\gamma b Q_{-l}}{b+2c_l},\ dP > c_l q_l$ and $q_h = \frac{a-\gamma b Q_{-h}}{b+2c_h x_h},\ dP = c_h q_h x_h$. Note that

$$\begin{aligned} dP &= (n-m)\, d q_h (1-x_h) = c_h q_h x_h \\ \Longleftrightarrow \quad & x_h = \frac{(n-m)\, d}{c_h + (n-m)\, d} \in [0,1] \end{aligned}$$

We have the following expressions for q_l and q_h :

$$\begin{aligned} &\begin{cases} (b+2c_l)\, q_l + \gamma b Q_{-l} = a \\ \left(b + \frac{2(n-m)c_h d}{c_h+(n-m)d}\right) q_h + \gamma b Q_{-h} = a \end{cases} \\ \Longleftrightarrow \quad &\begin{cases} (b\,[1+(m-1)\,\gamma] + 2c_l)\, q_l + (n-m)\,\gamma b q_h = a \\ m\gamma b q_l + \left(b\,[1+(n-m-1)\,\gamma] + \frac{2(n-m)c_h d}{c_h+(n-m)d}\right) q_h = a \end{cases} \end{aligned}$$

The unique solution is (using Cramer's rule)

$$\begin{aligned} q_l &= \frac{a\left[b\,(1-\gamma) + \frac{2(n-m)c_h d}{c_h+(n-m)d}\right]}{\begin{array}{c}[b\,[1+(m-1)\,\gamma] + 2c_l]\left[b\,[1+(n-m-1)\,\gamma] + \frac{2(n-m)c_h d}{c_h+(n-m)d}\right] \\ -m\,(n-m)\,\gamma^2 b^2\end{array}} \\ q_h &= \frac{a\,[b\,(1-\gamma) + 2c_l]}{\begin{array}{c}[b\,[1+(m-1)\,\gamma] + 2c_l]\left[b\,[1+(n-m-1)\,\gamma] + \frac{2(n-m)c_h d}{c_h+(n-m)d}\right] \\ -m\,(n-m)\,\gamma^2 b^2\end{array}} \end{aligned}$$

Recall that

$$\begin{aligned} x_l &= 1 \\ x_h &= \frac{(n-m)\, d}{c_h + (n-m)\, d} \end{aligned}$$

Finally, we must check that

$$\begin{aligned} dP &= c_h q_h x_h > c_l q_l \\ \Longleftrightarrow \quad & \frac{(n-m)\,[b\,(1-\gamma) + 2c_l]\, c_h d}{c_h + (n-m)\, d} > c_l \left[b\,(1-\gamma) + \frac{2\,(n-m)\, c_h d}{c_h + (n-m)\, d}\right] \\ \Longleftrightarrow \quad & \frac{(n-m)\, b\,(1-\gamma)\, c_h d}{c_h + (n-m)\, d} > b\,(1-\gamma)\, c_l \\ \Longleftrightarrow \quad & (n-m)\, c_h d > c_l c_h + (n-m)\, c_l d \\ \Longleftrightarrow \quad & d > \frac{c_l c_h}{(n-m)\,(c_h - c_l)} \equiv \underline{d} \end{aligned}$$

Note that $q_l,\ q_h > 0$. Call this stationary point M_1.

2. $\mu_l = \mu_h = 0$. Then, $q_l = \frac{a-\gamma bQ_{-l}}{b+2c_l x_l}$, $q_h = \frac{a-\gamma bQ_{-h}}{b+2c_h x_h}$ and $dP = c_l q_l x_l = c_h q_h x_h$. Rewrite

$$\begin{aligned} q_l &= \frac{a-\gamma bQ_{-l}}{b+2c_l x_l} = \frac{a-\gamma bQ}{b(1-\gamma)+2c_l x_l} \\ \Longleftrightarrow q_l &= \frac{a-\gamma bQ-2dP}{b(1-\gamma)} \end{aligned}$$

Similarly

$$q_h = \frac{a-\gamma bQ-2dP}{b(1-\gamma)}$$

Thus,

$$q_l = q_h$$

This implies that $c_l x_l = c_h x_h$. Thus,

$$\begin{aligned} dP &= dq_l \left[m(1-x_l)+(n-m)(1-x_h)\right] \\ &= dq_l \left[n - \left(m + \frac{(n-m)c_l}{c_h}\right) x_l\right] = c_l q_l x_l \end{aligned}$$

We can now solve for x_l and calculate $x_h = \frac{c_l x_l}{c_h}$ and $q_l = q_h$. We find

$$\begin{aligned} x_l &= \frac{nc_h}{(n-m)c_l + mc_h + \frac{c_l c_h}{d}} = \frac{nc_h d}{((n-m)c_l+mc_h)d + c_l c_h} \\ x_h &= \frac{nc_l d}{((n-m)c_l+mc_h)d + c_l c_h} \\ q_l &= \frac{a-\gamma bQ}{b(1-\gamma)+2c_l x_l} = \frac{a-n\gamma bq_l}{b(1-\gamma)+\frac{2nc_l c_h d}{((n-m)c_l+mc_h)d+c_l c_h}} \\ \Longleftrightarrow q_l &= q_h = \frac{a\left[((n-m)c_l+mc_h)d+c_l c_h\right]}{b(1+(n-1)\gamma)\left(((n-m)c_l+mc_h)d+c_l c_h\right)+2nc_l c_h d} \end{aligned}$$

Clearly, $x_l,\ x_h,\ q_l$, and q_h are always positive, and $x_h \leq x_l$. The condition

$$x_l \leq 1 \iff d \leq \underline{d}$$

Call this stationary point M_2. Note that M_1 and M_2 extend to the special cases $m = 0$ and $m = n$. □

A.3 Proof of Theorem 3

Suppose that the pollution constraint is binding. Then, $s = S/n = q_i \cdot (1-x_i)$. Rewrite $q_i \cdot x_i = q_i - s$ and substitute in the objective function. We have

$$\max_{q_i \geq 0} q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c_i \cdot (q_i - s)^2$$

The objective function for firm i having to choose q_i is strictly concave. The first order condition is necessary and sufficient. We get the unique solution:

$$q_i = \frac{a - \gamma b Q_{-i} + 2c_i s}{2(b + c_i)}$$

Rewrite this equation

$$q_i = \frac{a - \gamma b Q + 2c_i s}{b(2 - \gamma) + 2c_i} \tag{A.8}$$

So in equilibrium, the firm's production quantity is uniquely determined by its cost coefficient, and the equilibrium is symmetric.

Note that $(A.8)$ implies that in equilibrium $q_i > 0 \ \forall i$, because $a - \gamma Q > 0$. If it were not the case, then the price of product i,

$$p_i = a - bq_i - \gamma b Q_{-i} = a - \gamma b Q - b(1 - \gamma) q_i, \ \forall i$$

and there exists at least one market k for which $q_k > 0$ leading to $p_k < 0$.

q_l and q_h solve the following equations:

$$\begin{cases} 2(b + c_l) q_l + \gamma b Q_{-l} = a + 2c_l s \\ 2(b + c_h) q_h + \gamma b Q_{-h} = a + 2c_h s \end{cases}$$

$$\Longleftrightarrow \begin{cases} [b(2 + (m-1)\gamma) + 2c_l] q_l + (n - m)\gamma b q_h = a + 2c_l s \\ m\gamma b q_l + [b(2 + (n - m - 1)\gamma) + 2c_h] q_h = a + 2c_h s \end{cases}$$

This implies (using Cramer's rule)

$$\begin{aligned} q_l &= \frac{a[b(2-\gamma) + 2c_h] + 2s[(2 + (n-m-1)\gamma) bc_l - (n-m)\gamma bc_h + 2c_l c_h]}{[b(2 + (m-1)\gamma) + 2c_l][b(2 + (n-m-1)\gamma) + 2c_h] - m(n-m)\gamma^2 b^2} \\ q_h &= \frac{a[b(2-\gamma) + 2c_l] + 2s[(2 + (m-1)\gamma) bc_h - m\gamma bc_l + 2c_l c_h]}{[b(2 + (m-1)\gamma) + 2c_l][b(2 + (n-m-1)\gamma) + 2c_h] - m(n-m)\gamma^2 b^2} \end{aligned}$$

from which we derive

$$\begin{aligned} x_l &= \frac{q_l - s}{q_l} \\ &= \frac{[b(2-\gamma) + 2c_h][a - bs(2 + (n-1)\gamma)]}{a[b(2-\gamma) + 2c_h] + 2s[(2 + (n-m-1)\gamma) bc_l - (n-m)\gamma bc_h + 2c_l c_h]} \\ x_h &= \frac{q_h - s}{q_h} \\ &= \frac{[b(2-\gamma) + 2c_l][a - bs(2 + (n-1)\gamma)]}{a[b(2-\gamma) + 2c_l] + 2s[(2 + (m-1)\gamma) bc_h - m\gamma bc_l + 2c_l c_h]} \end{aligned}$$

The conditions

$$0 \le x_i \le 1, \ \forall i \iff 0 \le s \le \frac{a}{b(2 + (n-1)\gamma)}$$

Suppose now that the pollution constraint is not binding. It is clear that the firm will choose $x_i = 0$. The objective function is strictly concave. The unique solution satisfies

$$q_i = \frac{a - \gamma b Q_{-i}}{2b} \iff q_i = \frac{a - \gamma b Q}{b(2 - \gamma)}$$

Thus, in equilibrium, the firms' quantities are equal.

We get

$$\begin{aligned} b\,(2-\gamma)\,q_i &= a-\gamma bQ = a-n\gamma bq_i \\ \iff q_i &= \frac{a}{b\,(2+(n-1)\,\gamma)} > 0 \end{aligned}$$

The condition

$$q_i \cdot (1-x_i) < s \iff s > \frac{a}{b\,(2+(n-1)\,\gamma)} \ \square$$

A.4 Proof of Theorem 4

The firms' objective is, $\forall i$

$$\max_{q_i \geq 0,\ 0 \leq x_i \leq 1,\ t_i \leq s} \quad \pi_i\,(q_i, x_i|Q_{-i}) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c_i \cdot (q_i \cdot x_i)^2 + r \cdot t_i$$

$$\text{subject to} \begin{cases} q_i \cdot (1-x_i) \leq s - t_i & \text{(pollution constraint)} \\ \sum_{i=1}^{n} t_i = 0 & \text{(market clearing condition)} \end{cases}$$

This is a nonlinear constrained optimization problem.

Note that the conditions $q_i \geq 0,\ x_i \leq 1$ and the pollution constraint jointly guarantee that $t_i \leq s$.

There are five main steps to the proof:

1. We analyze the Lagrangian and derive necessary conditions on $q_i,\ x_i,\ t_i$, and the market clearing price r based on the Kuhn-Tucker conditions;
2. We show that the equilibria are symmetric;
3. We solve for the equilibrium (there is only one);
4. We show that firms always produce a positive quantity;
5. We prove that the equilibrium is a global maximum.

A.4.1 Step 1 - Analysis of the Lagrangian

Write the Lagrangian

$\mathfrak{L} = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c_i \cdot (q_i \cdot x_i)^2 + r \cdot t_i + \lambda_i \cdot q_i + \mu_{i1} \cdot x_i - \mu_{i2} \cdot (x_i - 1) - \nu_i \cdot (q_i \cdot (1-x_i) - s + t_i)$. The Kuhn-Tucker necessary conditions are:

$$
\begin{aligned}
a - \gamma b Q_{-i} - 2q_i \cdot \left(b + c_i \cdot x_i^2\right) + \lambda_i - \nu_i \cdot (1 - x_i) &= 0 \qquad \text{(A.9)}\\
q_i \cdot (\nu_i - 2c_i \cdot q_i \cdot x_i) + \mu_{i1} - \mu_{i2} &= 0 \qquad \text{(A.10)}\\
\nu_i &= r \qquad \text{(A.11)}\\
\lambda_i \cdot q_i &= 0\\
\mu_{i1} \cdot x_i &= 0\\
\mu_{i2} \cdot (x_i - 1) &= 0\\
\nu_i \cdot (q_i \cdot (1 - x_i) - s + t_i) &= 0\\
q_i,\ \lambda_i,\ \mu_{i1},\ \mu_{i2},\ \nu_i &\geq 0\\
0 \leq x_i &\leq 1\\
q_i \cdot (1 - x_i) &\leq s - t_i
\end{aligned}
$$

Necessary conditions are $\nu_i = r,\ \forall i$ and also (as in Theorem 1) $\mu_{i1} = 0$. Rewrite μ_{i2} as μ_i.

Suppose that $\lambda_i > 0$. Then $q_i = 0$ and $Q_{-i} = Q$.

When $\nu_i = 0$, equation (A.9) implies that

$$a - \gamma b Q = -\lambda_i \qquad \text{(A.12)}$$

Consider the output price in market j for which $q_j > 0$ (there exists at least one such market):

$$
\begin{aligned}
p_j &= a - b q_j - \gamma b Q_{-j}\\
&= a - \gamma b Q - b\,(1 - \gamma)\, q_j
\end{aligned}
$$

Equation $(A.12)$ implies that

$$p_j = -\lambda_i - b\,(1 - \gamma)\, q_j < 0,\ \text{a contradiction}$$

Thus, $\nu_i > 0$, which means that

$$q_i\,(1 - x_i) = s - t_i = 0 \iff t_i = s$$

As expected, the firm sells all its emission allowances, and makes a profit

$$\pi_i = rs$$

Suppose for now that $\lambda_i = 0,\ \forall i$.

- If $\nu_i = 0$, then $r = 0$ and

$$\begin{aligned} (A.10) &\Rightarrow \mu_i = x_i = 0 \\ (A.9) &\Rightarrow q_i = \frac{a - \gamma b Q_{-i}}{2b} \\ t_i &\leq s - q_i \text{ (pollution constraint)} \end{aligned}$$

- If $\nu_i > 0$, then

$$q_i (1 - x_i) = s - t_i \tag{A.13}$$

There are two subcases to consider:

1. $\mu_i > 0$: Then

$$x_i = 1$$

$$\begin{aligned} (A.13) &\Rightarrow t_i = s \\ (A.9) &\Rightarrow q_i = \frac{a - \gamma b Q_{-i}}{2(b + c_i)} \\ \mu_i &> 0 \Rightarrow r > 2c_i q_i \end{aligned}$$

2. $\mu_i = 0$: Then

$$\begin{aligned} (A.10) &\Rightarrow r = 2c_i q_i x_i \Rightarrow x_i = \frac{r}{2c_i q_i} \geq 0 \\ (A.13) &\Rightarrow t_i = s - q_i (1 - x_i) \end{aligned}$$

Rewrite $(A.9)$

$$\begin{aligned} & a - \gamma b Q_{-i} - 2bq_i - 2c_i q_i x_i^2 - r + r x_i = 0 \\ \iff & a - \gamma b Q_{-i} - r - 2bq_i + x_i (r - 2c_i q_i x_i) = 0 \\ \iff & q_i = \frac{a - \gamma b Q_{-i} - r}{2b} \end{aligned}$$

The conditions

$$x_i \leq 1 \iff r \leq 2c_i q_i$$

A.4.2 Step 2 - Proof of Symmetry

Next, we show that only symmetric equilibria are possible, i.e., equilibria in which if firms i and j have the same abatement cost coefficient, $\mu_i = \mu_j$. This implies that firms with the same cost coefficient will adopt the same response.

Proof. If $r = 0$, then $\mu_i = \mu_j = 0$. If $r > 0$, then write $r = 2dP$ and use Lemma 1 (cf. proof of Theorem 1). □

A.4.3 Step 3 - Equilibrium Analysis

Knowing that the equilibrium is symmetric, the n–firm problem simplifies to a 2-firm problem. If the firms all have the same abatement cost coefficient (i.e., $m = 0$ or $m = n$), we know that in equilibrium

$$t_i = t_j \ \forall i \neq j$$

This implies that

$$t_i = S/n = s, \ \forall i$$

In other words, the firms do not trade. They maintain their initial allowances. The solution is given by Theorem 3 with $c_i = c_h$ if $m = 0$, and $c_i = c_l$ if $m = n$. The corresponding profits are positive, and therefore, the firms would produce strictly positive quantities (i.e., $\lambda_i = 0, \ \forall i$).

Suppose now that $0 < m < n$. We know that, in equilibrium, firms with the same abatement cost coefficient will have the same posttrading cap (i.e., if $c_i = c_j$, then $t_i = t_j$). Let s_l and s_h denote the equilibrium (posttrading) cap of the low-cost and high-cost firms, respectively. Since all firms have an initial cap s, when a low-cost firm sells $s - s_l$ emission allowances, a high-cost firm buys $s_h - s$ allowances. The market clearing condition stipulates that

$$m(s - s_l) + (n - m)(s - s_h) = 0 \iff s - s_h = -\frac{m}{n-m}(s - s_l)$$

We can rewrite the n-firm problem as a *two*-firm problem:

$$\begin{array}{ll} \max\limits_{q_l \geq 0, \ 0 \leq x_l \leq 1, \ q_l(1-x_l) \leq s-t} & q_l \cdot (a - b \cdot q_l - \gamma \cdot b \cdot Q_{-l}) - c_l \cdot (q_l \cdot x_l)^2 + r \cdot t \\ \max\limits_{q_h \geq 0, \ 0 \leq x_h \leq 1, \ q_h(1-x_h) \leq s + \frac{m}{n-m} t} & q_h \cdot (a - b \cdot q_h - \gamma \cdot b \cdot Q_{-h}) - c_h \cdot (q_h \cdot x_h)^2 - \frac{m}{n-m} r \cdot t \end{array}$$

where $t = t_l = s - s_l$ and $t_h = s - s_h = -\frac{m}{n-m} t_l$. The first order necessary conditions are given by equations $(A.9 - A.11)$. We need to consider the following five cases:

1. $r = 0$. Then $\forall i$

$$\begin{aligned} x_i &= 0 \\ q_i &= \frac{a - \gamma b Q_{-i}}{2b} \\ t_i &\leq s - q_i \end{aligned}$$

The production quantities must solve

$$q_i = \frac{a - \gamma b Q}{b(2 - \gamma)}$$

Thus, in equilibrium,

$$q_l = q_h \equiv q$$

and

$$b\,(2-\gamma)\,q = a - n\gamma bq \iff q = \frac{a}{b\,(2+(n-1)\,\gamma)}$$

We get

$$\begin{aligned} t &\leq s - q \text{ (for firm } l\text{)} \\ -\frac{m}{n-m}t &\leq s - q \text{ (for firm } h\text{)} \end{aligned}$$

This is equivalent to

$$\frac{n-m}{m}\left(\frac{a}{b\,(2+(n-1)\,\gamma)} - s\right) \leq t \leq s - \frac{a}{b\,(2+(n-1)\,\gamma)}$$

A necessary condition is that $LHS \leq RHS$ which implies

$$\begin{aligned} \frac{n-m}{m}\left(\frac{a}{b\,(2+(n-1)\,\gamma)} - s\right) &\leq s - \frac{a}{b\,(2+(n-1)\,\gamma)} \\ \iff s &\geq \frac{a}{b\,(2+(n-1)\,\gamma)} \end{aligned}$$

Note that since $r = 0$, the trading volume t has no impact on the firms' objective functions. Assume that in this case, the firms will choose the smallest feasible $|t|$. Since $t = 0$ is feasible, then

$$t^* = 0$$

2. $r > 0,\ \mu_l > 0,\ \mu_h > 0$. Then,

$$\begin{aligned} x_l &= x_h = 1 \\ t &= s = -\frac{m}{n-m}s \Rightarrow s = 0 \end{aligned}$$

When $s = 0$, the firms have nothing to trade, and the solution coincides with the pure Cap mechanism with $s = 0$ (see Theorem 3).

3. $r > 0,\ \mu_l > 0,\ \mu_h = 0$. Then

$$\begin{aligned} x_l &= 1 \\ t &= s \\ q_l &= \frac{a - \gamma b Q_{-l}}{2(b + c_l)} \\ r &> 2c_l q_l \end{aligned}$$

and

$$\begin{aligned} r &= 2c_h q_h x_h \\ t &= \frac{n-m}{m}\left[q_h(1-x_h) - s\right] && \text{(A.14)} \\ q_h &= \frac{a - \gamma b Q_{-h} - r}{2b} && \text{(A.15)} \\ r &\leq 2c_h q_h \end{aligned}$$

Since $t = s$,

$$\begin{aligned} (A.14) \Rightarrow \frac{ms}{n-m} + s &= q_h - q_h x_h \\ \Longleftrightarrow q_h x_h &= q_h - \frac{ns}{n-m} \end{aligned}$$

This in turn implies that

$$r = 2c_h q_h x_h = 2c_h q_h - \frac{2nc_h s}{n-m}$$

We can substitute this expression of r in $(A.15)$ and solve for q_l and q_h :

$$\begin{cases} \left[b(2 + (m-1)\gamma) + 2c_l\right] q_l + (n-m)\gamma b q_h = a \\ m\gamma b q_l + \left[b(2 + (n-m-1)\gamma) + 2c_h\right] q_h = a + \frac{2nc_h s}{n-m} \end{cases}$$

From which we derive (using Cramer's rule)

$$\begin{aligned} q_l &= \frac{a\left[b(2-\gamma) + 2c_h\right] - 2\gamma b c_h S}{\begin{array}{c}\left[b(2+(m-1)\gamma) + 2c_l\right]\left[b(2+(n-m-1)\gamma) + 2c_h\right] \\ -m(n-m)\gamma^2 b^2\end{array}} \\ q_h &= \frac{(n-m)a\left[b(2-\gamma) + 2c_l\right] + 2c_h\left[b(2+(m-1)\gamma) + 2c_l\right] S}{(n-m)\left[\begin{array}{c}\left[b(2+(m-1)\gamma) + 2c_l\right]\left[b(2+(n-m-1)\gamma) + 2c_h\right] \\ -m(n-m)\gamma^2 b^2\end{array}\right]} \end{aligned}$$

It follows that

$$\begin{aligned} r &= 2c_h\left[q_h - \frac{S}{n-m}\right] \leq 2c_h q_h \\ x_h &= 1 - \frac{S}{(n-m)q_h} \\ &= \frac{\begin{array}{c}(n-m)a\left[b(2-\gamma) + 2c_l\right] - \\ bS\left[b(2-\gamma)(2+(n-1)\gamma) + 2c_l(2+(n-m-1)\gamma)\right]\end{array}}{(n-m)a\left[b(2-\gamma) + 2c_l\right] + 2c_h S\left[b(2+(m-1)\gamma) + 2c_l\right]} \end{aligned}$$

The condition

$$\begin{aligned} r > 2c_l q_l \Rightarrow 2c_h q_h - \frac{2c_h S}{n-m} &> 2c_l q_l \\ \iff S &< \frac{(n-m)\,a\,(c_h - c_l)}{c_h\,[b\,(2+(n-1)\,\gamma) + 2c_l]} \equiv \underline{S} \end{aligned}$$

The condition

$$x_h \leq 1 \iff S \geq 0$$

Finally, the condition

$$\begin{aligned} x_h \geq 0 \iff (n-m)\,q_h &\geq S \\ \iff S &\leq \frac{(n-m)\,a\,[b\,(2-\gamma)+2c_l]}{b\,[b\,(2-\gamma)\,(2+(n-1)\,\gamma) + 2c_l\,(2+(n-m-1)\,\gamma)]} \end{aligned}$$

This condition also guarantees that $r \geq 0$. In the equation above, the $RHS \geq \underline{S}$, so that the solution holds as long as

$$0 \leq S < \underline{S}$$

In this case,

$$\begin{aligned} t^* &= s \\ r &= 2c_h\left[q_h - \frac{S}{n-m}\right] \\ &= 2c_h \frac{\begin{array}{c} a\,[b\,(2-\gamma)+2c_l] - \\ b\,[(2-\gamma)\,(2+(n-1)\,\gamma)\,b + 2c_l\,(2+(n-m-1)\,\gamma)]\,S\,/\,(n-m) \end{array}}{\begin{array}{c} [b\,(2+(m-1)\,\gamma)+2c_l]\,[b\,(2+(n-m-1)\,\gamma)+2c_h] - \\ m\,(n-m)\,\gamma^2 b^2 \end{array}} \end{aligned}$$

Call this stationary point M_1.

4. $r > 0,\ \mu_l = 0,\ \mu_h > 0$. Then,

$$\begin{aligned} r &= 2c_l q_l x_l \\ t &= s - q_l\,(1-x_l) \\ q_l &= \frac{a - \gamma b Q_{-l} - r}{2b} \\ r &\leq 2c_l q_l \end{aligned} \tag{A.16}$$

and

$$\begin{aligned} x_h &= 1 \\ t &= -\frac{n-m}{m} s \\ q_h &= \frac{a - \gamma b Q_{-h}}{2\,(b+c_h)} \\ r &> 2c_h q_h \end{aligned}$$

Since

$$t = s - q_l (1 - x_l) = -\frac{n-m}{m} s$$

We have

$$q_l x_l = q_l - \frac{ns}{m}$$

This in turn implies that

$$r = 2c_l q_l x_l = 2c_l q_l - \frac{2nc_l s}{m}$$

We can substitute this expression of r in $(A.16)$ and solve for q_l and q_h :

$$\begin{cases} [b(2+(m-1)\gamma)+2c_l]\, q_l + (n-m)\gamma b q_h = a + \frac{2nc_l s}{m} \\ m\gamma b q_l + [b(2+(n-m-1)\gamma)+2c_h]\, q_h = a \end{cases}$$

From which we derive (using Cramer's rule)

$$\begin{aligned} q_l &= \frac{ma[b(2-\gamma)+2c_h] + 2c_l S[b(2+(n-m-1)\gamma)+2c_h]}{\begin{array}{c} m[b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)+2c_h] \\ -m(n-m)\gamma^2 b^2 \end{array}} \\ q_h &= \frac{a[b(2-\gamma)+2c_l] - 2\gamma b c_l S}{\begin{array}{c} [b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)+2c_h] \\ -m(n-m)\gamma^2 b^2 \end{array}} \end{aligned}$$

It follows that

$$\begin{aligned} r &= 2c_l\left(q_l - \frac{S}{m}\right) \leq 2c_l q_l \\ x_l &= 1 - \frac{S}{mq_l} \\ &= \frac{ma[b(2-\gamma)+2c_h] - bS[b(2-\gamma)(2+(n-1)\gamma)+2c_h(2+(m-1)\gamma)]}{ma[b(2-\gamma)+2c_h]+2c_l S[b(2+(n-m-1)\gamma)+2c_h]} \end{aligned}$$

The condition

$$\begin{aligned} r &> 2c_h q_h \Rightarrow c_l q_l - \frac{c_l S}{m} > c_h q_h \\ \Rightarrow S &< -\frac{mab(2-\gamma)(c_h-c_l)}{2c_l\left[\begin{array}{c} b^2(2-\gamma)(2+(n-1)\gamma)+ \\ bc_l(2+(n-m-1)\gamma)+bc_h(4+(m-2)\gamma)+2c_l c_h \end{array}\right]} \\ &< 0, \text{ a contradiction} \end{aligned}$$

5. $r > 0,\ \mu_l = \mu_h = 0$. Then,

$$\begin{aligned} r &= 2c_l q_l x_l = 2c_h q_h x_h \\ t &= s - q_l\left(1 - x_l\right) = \frac{n-m}{m}\left[q_h\left(1 - x_h\right) - s\right] && \text{(A.17)} \\ q_l &= \frac{a - \gamma b Q_{-l} - r}{2b} && \text{(A.18)} \\ q_h &= \frac{a - \gamma b Q_{-h} - r}{2b} \\ r &\leq 2c_l q_l \text{ and } r \leq 2c_h q_h \end{aligned}$$

Rewrite equation $(A.18)$

$$2bq_l = a - \gamma b Q_{-l} - r$$

Subtract $\gamma b q_l$ from both sides

$$\begin{aligned} b\left(2-\gamma\right) q_l &= a - \gamma b Q - r \\ \Longleftrightarrow q_l &= \frac{a - \gamma b Q - r}{b\left(2-\gamma\right)} \end{aligned}$$

Likewise

$$q_h = \frac{a - \gamma b Q - r}{b\left(2-\gamma\right)} = q_l \tag{A.19}$$

This means that

$$\begin{aligned} Q &= nq_l = nq_h \\ c_l x_l &= c_h x_h \end{aligned}$$

We can solve for q_l and x_l by rearranging $(A.17)$ and $(A.19)$

$$\begin{aligned} &q_l x_l = \tfrac{c_h}{\widetilde{c}}\left(q_l - s\right) \\ &r = 2c_l q_l x_l = a - b\left(2 + \left(n-1\right)\gamma\right) q_l \end{aligned}$$

From which we derive

$$\begin{aligned} \frac{2c_l c_h}{\widetilde{c}}\left(q_l - s\right) &= a - b\left(2 + \left(n-1\right)\gamma\right) q_l \\ \Longleftrightarrow q_l &= q_h = \frac{a\left(\left(n-m\right)c_l + mc_h\right) + 2nc_l c_h s}{\left(2 + \left(n-1\right)\gamma\right) b\left(\left(n-m\right)c_l + mc_h\right) + 2nc_l c_h} \end{aligned}$$

$$\begin{aligned} x_l &= \frac{c_h}{\widetilde{c}}\left(1 - \frac{s}{q_l}\right) = c_h \frac{na - b\left(2 + \left(n-1\right)\gamma\right) S}{a\left(\left(n-m\right)c_l + mc_h\right) + 2nc_l c_h s} \\ x_h &= c_l \frac{na - b\left(2 + \left(n-1\right)\gamma\right) S}{a\left(\left(n-m\right)c_l + mc_h\right) + 2nc_l c_h s} \end{aligned}$$

x_l and x_h will be positive if

$$S \leq \frac{na}{b\left(2+(n-1)\gamma\right)}$$

We have $x_h \leq x_l$ and

$$x_l \leq 1 \iff S \geq \frac{(n-m)\,a\,(c_h-c_l)}{c_h\left[b\left(2+(n-1)\gamma\right)+2c_l\right]} = \underline{S}$$

In this case,

$$\begin{aligned} t^* &= \frac{(n-m)\,(c_h-c_l)\left[a-b\left(2+(n-1)\gamma\right)s\right]}{b\left(2+(n-1)\gamma\right)\left((n-m)\,c_l+mc_h\right)+2nc_lc_h} \\ r &= 2c_lq_lx_l = a-b\left(2+(n-1)\gamma\right)q_l \\ &= \frac{2c_lc_h\left[na-b\left(2+(n-1)\gamma\right)S\right]}{\left(2+(n-1)\gamma\right)b\left((n-m)\,c_l+mc_h\right)+2nc_lc_h} \end{aligned}$$

Call this stationary point M_2.

A.4.4 Step 4 - Proof that $q_i > 0,\ \forall i$

Under Cap-and-Trade, a firm can produce nothing and sell all its emission allowances on the emission trading market. We show that such a strategy is always dominated. Assume that $n > 2$.

The firm's optimal profits are

$$\begin{aligned} \pi_i^* &= q_i^*\left(a-bq_i^*-\gamma bQ_{-i}^*\right)-c_i\left(q_i^*x_i^*\right)^2+rt_i^* \\ &= q_i^*\left[a-\left(b+c_ix_i^{*2}\right)q_i^*-\gamma bQ_{-i}^*\right]+rt_i^* \end{aligned}$$

In equilibrium, the pollution constraint is always binding. Thus,

$$t_i^* = s-q_i^*\left(1-x_i^*\right)$$

and

$$\begin{aligned} \pi_i^* &= q_i^*\left[a-\left(b+c_ix_i^{*2}\right)q_i^*-\gamma bQ_{-i}^*-r\left(1-x_i^*\right)\right]+rs \\ &= \left(b+c_ix_i^{*2}\right)q_i^{*2}+rs \text{ (by the first order condition w.r.t. } q_i) \end{aligned}$$

If a firm produces nothing, it will make a profit $\pi_i = r_{n-1}\cdot s$, where r_{n-1} denotes the price for emission allowances when there are $n-1$ firms producing. By not producing, the firm reduces the demand for allowances, which drives the price down.

This is formally verified by

$$\frac{\partial r_n}{\partial n} > 0$$

Thus, $r_{n-1} \leq r_n$. By producing $q_i^* > 0$, the firm will earn

$$\left(b+c_ix_i^{*2}\right)q_i^{*2}+r_n\cdot s \geq r_{n-1}\cdot s$$

A.4.5 Step 5 - Second Order Sufficient Condition

We complete the proof by checking the second order condition at our two stationary points, a test we must do for each individual firm. We need only show the optimality in terms of $(q,\ x)$ because the objective function is separable in (q, x) and t, and linear in t. For an arbitrary firm i, the Hessian H is the following 2×2 matrix.

$$H = \begin{pmatrix} -2\left(b + c_i x_i^2\right) & r - 4c_i q_i x_i \\ r - 4c_i q_i x_i & -2c_i q_i^2 \end{pmatrix}$$

Note that the diagonal elements are strictly negative. A strictly positive determinant establishes that the Hessian is negative definite, and proves strict concavity.

At stationary point M_2, $r = 2c_l q_l x_l = 2c_h q_h x_h$. Hessian becomes

$$-2 \begin{pmatrix} b + c_i x_i^2 & c_i q_i x_i \\ c_i q_i x_i & c_i q_i^2 \end{pmatrix}$$

Its determinant is $4bc_i q_i^2 > 0$.

Similarly, at stationary point M_1, the Hessian is negative definite for the high-cost firms (because it is still true that $r = 2c_h q_h x_h$). For the low-cost firms, however, the Hessian becomes

$$\begin{pmatrix} -2\left(b + c_l\right) & r - 4c_l q_l \\ r - 4c_l q_l & -2c_l q_l^2 \end{pmatrix}$$

Its determinant is $4\left(b + c_l\right) c_l q_l^2 - \left(r - 4c_l q_l\right)^2$ could be negative for r sufficiently large.

We use a direct approach to show that M_1 is a local maximum for low-cost firms using a method inspired from Luenberger and Ye (2008). Note that for low-cost firms, M_1 is on the boundary of the feasible. The active constraints at M_1 are $g_1\left(q_l, x_l, t_l\right) = x_l - 1$ and $g_2\left(q_l, x_l, t_l\right) = q_l\left(1 - x_l\right) - s + t_l$.

$\bigtriangledown g_1 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T$ and $\bigtriangledown g_2 = \begin{pmatrix} 0 & -q_l & 1 \end{pmatrix}^T \neq 0$, where the first coordinate is the production quantity q_l, the second the abatement level x_l, and the third the trading volume t_l. $\bigtriangledown g_1$ and $\bigtriangledown g_2$ are linearly independent. Thus, M_1 is regular.

We will show that for any feasible move away from M_1, the value of the objective function decreases. The gradient of the objective function at M_1 is

$$\bigtriangledown \pi_i|_{M_1} = \begin{pmatrix} 0 & q_l\left(r - 2c_l q_l\right) & r \end{pmatrix}^T$$

A non-zero feasible direction, $d = \begin{pmatrix} d_1 & d_2 & d_3 \end{pmatrix}^T$ is defined by $\left(\bigtriangledown g_1\right)^T \cdot d \leq 0$ and $\left(\bigtriangledown g_2\right)^T \cdot d \leq 0$. These conditions are equivalent to

$$d_2 \leq 0 \text{ (condition for } g_1\text{)}$$

and

$$-d_2 \cdot q_i + d_3 \leq 0 \Rightarrow d_3 \leq d_2 \cdot q_i \leq 0 \text{ (condition for } g_2\text{)}$$

Note that if d is a feasible direction, $\bigtriangledown\pi_i|_{M_1} \cdot d = q_l \cdot (r - 2c_i q_i) \cdot d_2 + r \cdot d_3 \leq 0$, because at M_1, $r > 2c_i q_i$. $\square$

A.5 Proof of Theorem 5

There are three main steps to the proof:

1. Using the backward induction procedure, we show that the firms' Nash Equilibrium is symmetric, and solve it given the tax rate τ;
2. We solve for the Subgame-Perfect Nash equilibrium (there is only one), by finding the minimum τ such that the total pollution $P \leq S$;
3. We prove that the equilibrium is a global maximum.

A.5.1 Step 1 - The Firms' Nash Equilibrium Given τ

The firms' objective is, $\forall i$

$$\max_{q_i \geq 0, 0 \leq x_i \leq 1} \pi_i^t (q_i, x_i | Q_{-i}) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c_i \cdot (q_i \cdot x_i)^2 - \tau \cdot q_i \cdot (1 - x_i)$$

Write the Lagrangian

$\mathfrak{L} = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot Q_{-i}) - c_i \cdot (q_i \cdot x_i)^2 - \tau \cdot q_i \cdot (1 - x_i) + \lambda_i \cdot q_i + \mu_{i1} \cdot x_i - \mu_{i2} \cdot (x_i - 1)$. The Kuhn-Tucker necessary conditions are:

$$a - \gamma \cdot b \cdot Q_{-i} - 2q_i \cdot \left(b + c_i \cdot x_i^2\right) - \tau \cdot (1 - x_i) + \lambda_i = 0 \tag{A.20}$$

$$q_i \cdot (\tau - 2c_i \cdot q_i \cdot x_i) + \mu_{i1} - \mu_{i2} = 0 \tag{A.21}$$

(and the usual complementary slackness conditions and feasibility constraints).

First, we show that $\mu_{i1} = 0$. *Proof.* (By contradiction.) Suppose that $\mu_{i1} > 0$. Then, $\mu_{i2} = 0$ and $x_i = 0$, and by equation $(A.21)$ $\mu_{i1} = -\tau q_i \leq 0$. $\square$

We next show that $\lambda_i = 0$. *Proof.* If $\lambda_i > 0$ ($i.e.$, $q_i = 0$), the firm makes a profit of 0. If $\lambda_i = 0$, it is easy to verify that, if q_i and x_i satisfy $(A.20)$ and $(A.21)$, $\pi_i^t = \left(b + c_i x_i^2\right) q_i^2 > 0$. $\square$

There are only two cases to consider:

- $\mu_{i2} > 0$. Then $x_i = 1$ and by equation (A.20)

$$\begin{aligned} q_i &= \frac{a - \gamma b Q_{-i}}{2(b + c_i)} \\ \Longleftrightarrow \ q_i &= \frac{a - \gamma b Q}{b\,(2-\gamma) + 2c_i} \end{aligned}$$

  Note that $q_i > 0$, because $a - \gamma Q > 0$. If it were not the case, then the price of product i,

$$p_i = a - bq_i - \gamma b Q_{-i} = a - \gamma b Q - b\,(1-\gamma)\,q_i, \ \forall i$$

  and there exists at least one market k for which $q_k > 0 \Rightarrow p_k < 0$.

  The solution holds if

$$\mu_{i2} > 0 \iff \tau > 2c_i q_i$$

- $\mu_{i2} = 0$. Then, by equation (A.21), $\tau = 2c_i q_i x_i$. After substitution in (A.20), we get

$$\begin{aligned} q_i &= \frac{a - \tau - \gamma b Q_{-i}}{2b} \\ \Longleftrightarrow \ q_i &= \frac{a - \tau - \gamma b Q}{b\,(2-\gamma)} \\ \Longleftrightarrow \ q_i &= \frac{a - \gamma b Q}{b\,(2-\gamma) + 2c_i x_i} > 0 \end{aligned}$$

  and $x_i = \frac{\tau}{2c_i q_i}$. The condition

$$x_i \leq 1 \Rightarrow \tau \leq 2c_i q_i$$

In particular, we have shown that in equilibrium $q_i > 0, \ \forall i$.

Lemma 1 (cf. proof of Theorem 1) with $\tau = 2dP$ shows that the equilibrium must be symmetric, i.e., firms with the same cost coefficients adopt the same responses. Because of symmetry, we only need to consider the following 4 cases to solve the general n-firm case:

1. $\tau > \max\{2c_l q_l, \ 2c_h q_h\}$. Then, $x_l = x_h = 1$ and

$$\begin{aligned} &\left\{ \begin{array}{l} q_l = \frac{a - \gamma b Q_{-l}}{2(b+c_l)} \\ q_h = \frac{a - \gamma b Q_{-h}}{2(b+c_h)} \end{array} \right. \\ \Longleftrightarrow \ &\left\{ \begin{array}{l} \left[b\,(2 + (m-1)\,\gamma) + 2c_l\right] q_l + (n-m)\,\gamma b q_h = a \\ m\gamma b q_l + \left[b\,(2 + (n-m-1)\,\gamma) + 2c_h\right] q_h = a \end{array} \right. \end{aligned}$$

   From which we derive

$$\begin{aligned} q_l &= \frac{a\left[b\,(2-\gamma) + 2c_h\right]}{\left[b\,(2+(m-1)\,\gamma) + 2c_l\right]\left[b\,(2+(n-m-1)\,\gamma) + 2c_h\right] - m\,(n-m)\,\gamma^2 b^2} \\ q_h &= \frac{a\left[b\,(2-\gamma) + 2c_l\right]}{\left[b\,(2+(m-1)\,\gamma) + 2c_l\right]\left[b\,(2+(n-m-1)\,\gamma) + 2c_h\right] - m\,(n-m)\,\gamma^2 b^2} \end{aligned}$$

This is feasible as long as $\tau > \max\{2c_l q_l, 2c_h q_h\}$

$$\Rightarrow \tau > \frac{2ac_h\left[b\left(2-\gamma\right)+2c_l\right]}{\begin{array}{c}\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]\left[b\left(2+\left(n-m-1\right)\gamma\right)+2c_h\right]\\ -m\left(n-m\right)\gamma^2 b^2\end{array}} \equiv \tau_1$$

Call this stationary point M_1.

2. $2c_h q_h < \tau \leq 2c_l q_l$. Then $x_h = 1,\ x_l = \frac{\tau}{2c_l q_l}$ and

$$\begin{cases} q_l = \frac{a-\tau-\gamma b Q_{-l}}{2b} \\ q_h = \frac{a-\gamma b Q_{-h}}{2(b+c_h)} \end{cases}$$
$$\iff \begin{cases} b\left(2+\left(m-1\right)\gamma\right)q_l + \left(n-m\right)\gamma b q_h = a-\tau \\ m\gamma b q_l + \left[b\left(2+\left(n-m-1\right)\gamma\right)+2c_h\right]q_h = a \end{cases}$$

From which we derive:

$$\begin{aligned} q_l &= \frac{a\left[b\left(2-\gamma\right)+2c_h\right]-\tau\left[b\left(2+\left(n-m-1\right)\gamma\right)+2c_h\right]}{b\left[\left(2-\gamma\right)\left(2+\left(n-1\right)\gamma\right)b+2c_h\left(2+\left(m-1\right)\gamma\right)\right]} \\ q_h &= \frac{a\left(2-\gamma\right)+m\gamma\tau}{\left(2-\gamma\right)\left(2+\left(n-1\right)\gamma\right)b+2c_h\left(2+\left(m-1\right)\gamma\right)} \end{aligned}$$

We must have

$$\begin{aligned} & 2c_h q_h < \tau \leq 2c_l q_l \\ \iff & \frac{2ac_h}{\left(2+\left(n-1\right)\gamma\right)b+2c_h} < \tau \\ \leq & \frac{2ac_l\left(b\left(2-\gamma\right)+2c_h\right)}{\begin{array}{c}\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]\left[b\left(2+\left(n-m-1\right)\gamma\right)+2c_h\right]\\ -m\left(n-m\right)\gamma^2 b^2\end{array}} \end{aligned} \tag{A.22}$$

This solution is not feasible because the LHS of $(A.22)$ is greater than the RHS.

3. $2c_l q_l < \tau \leq 2c_h q_h$. Then $x_l = 1,\ x_h = \frac{\tau}{2c_h q_h}$ and

$$\begin{cases} q_l = \frac{a-\gamma b Q_{-l}}{2(b+c_l)} \\ q_h = \frac{a-\tau-\gamma b Q_{-h}}{2b} \end{cases}$$
$$\iff \begin{cases} \left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]q_l + \left(n-m\right)\gamma b q_h = a \\ m\gamma b q_l + b\left(2+\left(n-m-1\right)\gamma\right)q_h = a-\tau \end{cases}$$

From which we derive:

$$\begin{aligned} q_l &= \frac{a\left(2-\gamma\right)+\left(n-m\right)\gamma\tau}{\left(2-\gamma\right)\left(2+\left(n-1\right)\gamma\right)b+2c_l\left(2+\left(n-m-1\right)\gamma\right)} \\ q_h &= \frac{a\left[b\left(2-\gamma\right)+2c_l\right]-\tau\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]}{b\left[\left(2-\gamma\right)\left(2+\left(n-1\right)\gamma\right)b+2c_l\left(2+\left(n-m-1\right)\gamma\right)\right]} \\ x_h &= \frac{\tau}{2c_h q_h} = \frac{b\left[\left(2-\gamma\right)\left(2+\left(n-1\right)\gamma\right)b+2c_l\left(2+\left(n-m-1\right)\gamma\right)\right]\tau}{2c_h\left[a\left[b\left(2-\gamma\right)+2c_l\right]-\tau\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]\right]} \end{aligned}$$

We must have

$$\begin{aligned} & 2c_l q_l < \tau \le 2c_h q_h \\ \iff \quad & \tau_2 \equiv \frac{2ac_l}{(2+(n-1)\gamma)b + 2c_l} < \tau \le \tau_1 \end{aligned}$$

It is true that

$$\tau_2 < \tau_1$$

Call this stationary point M_2.

4. $\tau \le \min\{2c_l q_l,\ 2c_h q_h\}$. Then

$$\begin{aligned} \tau &= 2c_l q_l x_l = 2c_h q_h x_h \\ q_l &= \frac{a - \tau - \gamma b Q_{-l}}{2b} \quad \text{(A.23)} \\ q_h &= \frac{a - \tau - \gamma b Q_{-h}}{2b} \quad \text{(A.24)} \end{aligned}$$

Rewrite $(A.23)$

$$2bq_l = a - \tau - \gamma b Q_{-l}$$

and subtract $-\gamma b q_l$ from both sides:

$$b(2-\gamma) q_l = a - \tau - \gamma b Q$$

Do the same with $(A.24)$. This implies that

$$q_h = \frac{a - \tau - \gamma b Q}{b(2-\gamma)} = q_l$$

Thus

$$Q = nq_l$$

We can now solve for q_l and calculate x_l and x_h. We find

$$\begin{aligned} q_l &= q_h = \frac{a-\tau}{b(2+(n-1)\gamma)} \\ x_l &= \frac{\tau}{2c_l q_l} = \frac{b(2+(n-1)\gamma)\tau}{2c_l(a-\tau)} \\ x_h &= \frac{\tau}{2c_h q_h} = \frac{b(2+(n-1)\gamma)\tau}{2c_h(a-\tau)} \end{aligned}$$

Clearly, $x_l \ge x_h \ge 0$. The condition

$$x_l \le 1 \iff \tau \le \frac{2ac_l}{b(2+(n-1)\gamma) + 2c_l} \equiv \tau_2$$

Call this stationary point M_3.

A.5.2 Step 2 - The Subgame-Perfect Nash Equilibrium

Next, we solve the regulator's problem which is to choose the smallest τ such that $P \leq S$. We need to do this for the intervals of τ for which each of our 3 stationary points is defined. We start with M_3 :

1. $(M_3):\ 0 \leq \tau \leq \tau_2 = \frac{2ac_l}{(2+(n-1)\gamma)b+2c_l}$

   Given the firms' reaction functions, the total pollution generated is

$$\begin{aligned}
P(\tau) &= \sum_{i=1}^{n} \left[q_i(\tau)(1-x_i(\tau))\right] \\
&= mq_l(1-x_l) + (n-m)q_h(1-x_h) \\
&= q_l\left[n - mx_l - (n-m)x_h\right] \\
&= nq_l - mq_lx_l - (n-m)q_hx_h \\
&= \frac{n(a-\tau)}{b(2+(n-1)\gamma)} - \frac{m\tau}{2c_l} - \frac{(n-m)\tau}{2c_h}
\end{aligned}$$

   P is linear decreasing in τ. Thus, the smallest tax rate τ such that $P(\tau) \leq S$ is

$$\begin{aligned}
& \frac{n(a-\tau)}{b(2+(n-1)\gamma)} - \frac{m\tau}{2c_l} - \frac{(n-m)\tau}{2c_h} = S \\
\iff \quad & \tau = \frac{2c_lc_h\left[na - b(2+(n-1)\gamma)S\right]}{(2+(n-1)\gamma)b((n-m)c_l + mc_h) + 2nc_lc_h} \equiv \tau_3^*
\end{aligned}$$

   The condition

$$\begin{aligned}
0 &\leq \tau_3^* \leq \tau_2 \\
\iff \frac{(n-m)a(c_h-c_l)}{c_h\left[b(2+(n-1)\gamma)+2c_l\right]} &\leq S \leq \frac{na}{b(2+(n-1)\gamma)}
\end{aligned}$$

   Recall that $\underline{S} = \frac{(n-m)a(c_h-c_l)}{c_h[b(2+(n-1)\gamma)+2c_l]}$ and $S^u = \frac{na}{b(2+(n-1)\gamma)}$.

   We have just shown that when $\underline{S} \leq S \leq S^u$

$$\begin{aligned}
q_l^* &= q_h^* = \frac{a-\tau_3^*}{b(2+(n-1)\gamma)} = \frac{a\widetilde{c} + 2c_lc_hS/n}{(2+(n-1)\gamma)b\widetilde{c} + 2c_lc_h} \\
x_l^* &= \frac{\tau_3^*}{2c_lq_l^*} = \frac{c_h\left[na - b(2+(n-1)\gamma)S\right]}{a((n-m)c_l + mc_h) + 2c_lc_hS} \\
x_h^* &= \frac{c_lx_l^*}{c_h} = \frac{c_l\left[na - b(2+(n-1)\gamma)S\right]}{a((n-m)c_l + mc_h) + 2c_lc_hS}
\end{aligned}$$

   The firm's profits are

$$\pi_i = q_i\left[a - \gamma bQ_{-i} - \tau(1-x_i) - \left(b + c_ix_i^2\right)q_i\right]$$

By equation $(A.20)$

$$a-\gamma bQ_{-i}-\tau\left(1-x_i\right)=2q_i\left(b+c_i\cdot x_i^2\right)$$

Thus,

$$\pi_i=\left(b+c_ix_i^2\right)q_i^2=bq_i^2+\frac{\tau^2}{4c_i}$$

and after algebraic simplifications

$$\pi_i^*=\frac{\begin{array}{c}a^2\left[b\left(\left(n-m\right)c_l+mc_h\right)^2+c_ic_{-i}^2\right]+\\ 2abc_ic_{-i}\left[2\left(\left(n-m\right)c_l+mc_h\right)-\left(2+\left(n-1\right)\gamma\right)c_{-i}\right]S\diagup n+\\ bc_ic_{-i}^2\left[b\left(2+\left(n-1\right)\gamma\right)^2+4c_i\right]\left(S\diagup n\right)^2\end{array}}{\left[\left(2+\left(n-1\right)\gamma\right)b\left(\left(n-m\right)c_l+mc_h\right)+2c_ic_{-i}\right]^2}$$

When $\tau=0$,

$$P\left(0\right)=\frac{na}{b\left(2+\left(n-1\right)\gamma\right)}=S^u$$

This means that the pollution constraint is slack if $S\geq S^u$.

2. $\left(M_2\right):\ \tau_2<\tau\leq\tau_1=\frac{2ac_h\left[b(2-\gamma)+2c_l\right]}{\left[b(2+(m-1)\gamma)+2c_l\right]\left[b(2+(n-m-1)\gamma)+2c_h\right]-m(n-m)\gamma^2b^2}$

The low-cost firms do not pollute (i.e., $x_l=1$). The total pollution generated comes only from the high-cost firms:

$$\begin{aligned}P\left(\tau\right)&=\left(n-m\right)q_h\left(1-x_h\right)\\&=\left(n-m\right)\left[q_h-q_hx_h\right]\\&=\left(n-m\right)\left[q_h-\frac{\tau}{2c_h}\right]\\&=\left(n-m\right)\left[\frac{a\left[b\left(2-\gamma\right)+2c_l\right]-\tau\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]}{b\left[\left(2-\gamma\right)\left(2+\left(n-1\right)\gamma\right)b+2c_l\left(2+\left(n-m-1\right)\gamma\right)\right]}-\frac{\tau}{2c_h}\right]\end{aligned}$$

It is strictly decreasing in τ. Thus, the smallest tax rate such that $P\leq S$ is

$$\begin{aligned}&\left(n-m\right)\left[\frac{a\left[b\left(2-\gamma\right)+2c_l\right]-\tau\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]}{b\left[\left(2-\gamma\right)\left(2+\left(n-1\right)\gamma\right)b+2c_l\left(2+\left(n-m-1\right)\gamma\right)\right]}-\frac{\tau}{2c_h}\right]=S\\ \Longleftrightarrow\tau&=2c_h\frac{\begin{array}{c}a\left[b\left(2-\gamma\right)+2c_l\right]-\\ b\left[\left(2-\gamma\right)\left(2+\left(n-1\right)\gamma\right)b+2c_l\left(2+\left(n-m-1\right)\gamma\right)\right]S\diagup\left(n-m\right)\end{array}}{\begin{array}{c}\left[b\left(2+\left(m-1\right)\gamma\right)+2c_l\right]\left[b\left(2+\left(n-m-1\right)\gamma\right)+2c_h\right]-\\ m\left(n-m\right)^2\gamma^2b^2\end{array}}\\&\equiv\tau_2^*\end{aligned}$$

The condition

$$\begin{aligned}\tau_2&<\tau_2^*\leq\tau_1\\ \Longleftrightarrow 0&\leq S<\underline{S}\end{aligned}$$

In this case,

$$
\begin{aligned}
q_l &= \frac{a(2-\gamma)+(n-m)\gamma\tau_2^*}{(2-\gamma)(2+(n-1)\gamma)b+2c_l(2+(n-m-1)\gamma)} \\
&= \frac{a[b(2-\gamma)+2c_h]-2\gamma b c_h S}{\begin{array}{c}[b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)+2c_h] \\ -m(n-m)\gamma^2 b^2\end{array}} \\
q_h &= \frac{a[b(2-\gamma)+2c_l]-\tau[b(2+(m-1)\gamma)+2c_l]}{b[(2-\gamma)(2+(n-1)\gamma)b+2c_l(2+(n-m-1)\gamma)]} \\
&= \frac{a[b(2-\gamma)+2c_l]-2c_h[b(2+(m-1)\gamma)+2c_l]S/(n-m)}{\begin{array}{c}[b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)+2c_h] \\ -m(n-m)\gamma^2 b^2\end{array}} \\
x_l &= 1 \\
x_h &= \frac{\tau_2^*}{2c_h q_h^*} \\
&= \frac{\begin{array}{c}a[b(2-\gamma)+2c_l]- \\ b[b(2-\gamma)(2+(n-1)\gamma)+2c_l(2+(n-m-1)\gamma)]S/(n-m)\end{array}}{a[b(2-\gamma)+2c_l]+2c_h[b(2+(n-m-1)\gamma)+2c_l]S/(n-m)}
\end{aligned}
$$

The firms' profits are

$$
\begin{aligned}
\pi_l^* &= (b+c_l)q_l^{*2} \\
&= \frac{(b+c_l)[a[b(2-\gamma)+2c_h]-2\gamma b c_h S]^2}{\left[\begin{array}{c}[b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)+2c_h] \\ -m(n-m)^2\gamma^2 b^2\end{array}\right]^2} \\
\pi_h^* &= b q_h^{*2}+\frac{\tau_2^{*2}}{4c_h} \\
&= \frac{1}{\left[\begin{array}{c}[b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)+2c_h] \\ -m(n-m)^2\gamma^2 b^2\end{array}\right]^2}\times \\
&\quad \left[\begin{array}{c}b[a[b(2-\gamma)+2c_l]-2c_h[b(2+(m-1)\gamma)+2c_l]S/(n-m)]^2 \\ +c_h\left[\begin{array}{c}a[b(2-\gamma)+2c_l]- \\ b[(2-\gamma)(2+(n-1)\gamma)b+2c_l(2+(n-m-1)\gamma)]S/(n-m)\end{array}\right]^2\end{array}\right]
\end{aligned}
$$

3. $(M_1):\ \tau>\tau_1=\frac{2ac_h[b(2-\gamma)+2c_l]}{[b(2+(m-1)\gamma)+2c_l][b(2+(n-m-1)\gamma)+2c_h]-m(n-m)\gamma^2b^2}$

   In this case, the total pollution generated is $P(\tau)=0$. This is true for any S. The minimum τ that satisfies the pollution constraint is simply $\tau_1^*=\tau_1$. We show that this tax rate is higher than τ_2^* and τ_3^* so that it is dominated.

   τ_2^* is decreasing in S, whereas τ_1^* is independent of S, and $\tau_2^*(0)=\tau_1^*$.

   Similarly, τ_3^* is also decreasing in S, and at $S=\frac{(n-m)a(c_h-c_l)}{c_h[b(2+(n-1)\gamma)+2c_l]}$, we have $\tau_3^*=\tau_2^*=\frac{ac_l}{b(2+(n-1)\gamma)+2c_l}$.

A.5.3 Step 3 - Second Order Sufficient Condition

We complete the proof by checking the second order sufficient condition at stationary points M_2 and M_3, a test we must do for each individual firm. In general, the Hessian for firm i is

$$\begin{pmatrix} -2\left(b + c_i x_i^2\right) & \tau - 4c_i q_i x_i \\ \tau - 4c_i q_i x_i & -2c_i q_i^2 \end{pmatrix}$$

Note that the diagonal elements are strictly negative. A strictly positive determinant established that the Hessian is negative definite, which proves strict concavity.

At stationary point M_3, $\tau = 2c_l q_l x_l = 2c_h q_h x_h$. The Hessian becomes

$$-2 \begin{pmatrix} b + c_i x_i^2 & c_i q_i x_i \\ c_i q_i x_i & c_i q_i^2 \end{pmatrix}$$

Its determinant is $4bc_i q_i^2 > 0$.

Similarly, at stationary point M_2, the Hessian is negative definite for the high-cost firms (because it is still true that $\tau = 2c_h q_h x_h$). For the low-cost firms, however, the Hessian becomes

$$\begin{pmatrix} -2\left(b + c_l\right) & \tau - 4c_l q_l \\ \tau - 4c_l q_l & -2c_l q_l^2 \end{pmatrix}$$

Its determinant is $4\left(b + c_l\right) c_l q_l^2 - \left(\tau - 4c_l q_l\right)^2$ could be negative for τ sufficiently large.

We use a direct approach (similar to the proofs of Theorems 1 and 4). Note that for low-cost firms, M_2 is on the boundary of the feasible set. The gradient of the only binding constraint, i.e., $g\left(q_i, x_i\right) = x_i - 1$, is $\begin{pmatrix} 0 & 1 \end{pmatrix}^T \neq 0$. Thus, M_2 is regular. We will show that for any feasible move away from M_2, the value of the objective function decreases. The gradient of the objective function at M_2 is

$$\triangledown \pi_i\left(M_2\right) = \begin{pmatrix} 0 & q_i \cdot \left(\tau - 2c_l q_l\right) \end{pmatrix}^T$$

where the first coordinate is the production quantity q_i, and the second the abatement level x_i. A non-zero feasible direction, $d = \begin{pmatrix} d_1 & d_2 \end{pmatrix}^T$, is defined by $\left(\triangledown g\right)^T \cdot d \leq 0$. This condition is equivalent to $d_2 \leq 0$. If d is a feasible direction, $\triangledown \pi_i\left(M_2\right)^T \cdot d = q_i \cdot \left(\tau - 2c_l q_l\right) \cdot d_2 \leq 0$, because at M_1, $\tau > 2c_l q_l$. $\square$

A.6 Proof of Theorem 6

The social welfare W is continuous in S because W is a continuous function of q_i and x_i, which are themselves continuous in S.

For $S \geq S^u$, W is the constant

$$W^u = \frac{na^2\left(3b - 2nd\right)}{2b^2\left(2 + \left(n-1\right)\gamma\right)^2}$$

We study the conditions under which W is decreasing to the left of S^u. This means that by lowering the cap below S^u, W increases above W^u.

$$W(S) = \Pi(S) + CES(S) - D(S)$$

Recall that under the Tax mechanism, the tax payment is a transfer from the firms to the regulator, and as such does not impact welfare. Similarly, under Cap-and-Trade, money is transferred between the firms as they trade emission allowances, but these transfers do not impact the firms' joint profits, and welfare is no affected. Thus, in calculating welfare, we only need to consider the profits before tax.

CES is given by

$$CES = \sum_{i=1}^{n} \int_0^{q_i} \left[d_i(q) - p_i(q_i)\right] dq$$

where

$$d_i(q) = a - \gamma b Q_{-i} - bq$$

is the demand curve for product i, and

$$p_i(q) = a - \gamma b Q_{-i} - bq_i$$

is the price charged for product i.

$$CES = \sum_{i=1}^{n} \int_0^{q_i} b(q_i - q)\, dq = \frac{b}{2} \sum_{i=1}^{n} q_i^2$$

Finally, since the pollution constraints are binding,

$$D(S) = dS^2$$

Thus,

$$\begin{aligned} W(S) \quad = \quad & mq_l(a - \gamma b Q_{-l} - bq_l) - mc_l(q_l x_l)^2 \\ & + (n-m) q_h (a - \gamma b Q_{-h} - bq_h) - (n-m) c_h (q_h x_h)^2 \\ & + \frac{b}{2}\left(mq_l^2 + (n-m) q_h^2\right) \\ & - dS^2 \end{aligned}$$

On the interval $[\underline{S},\ S^u]$,

$$\begin{aligned} W(S) \quad = \quad & \frac{a\left(3b\widetilde{c} + 2c_l c_h\right)\left(na\widetilde{c} + 4c_l c_h S\right)}{2\left(2c_l c_h + (2 + (n-1)\gamma) b\widetilde{c}\right)^2} \\ & - \left(d + \frac{bc_l c_h\left(2\left(1 + 2(n-1)\gamma\right)c_l c_h + (2 + (n-1)\gamma)^2 b\widetilde{c}\right)}{n\left(2c_l c_h + (2 + (n-1)\gamma) b\widetilde{c}\right)^2}\right) S^2 \end{aligned}$$

where $\widetilde{c} = \frac{(n-m)c_l + mc_h}{n}$.

The condition is

$$
\begin{aligned}
& W'(S^u) < 0 \\
\iff \quad & \frac{2a\left[n\left(2c_lc_h + (2+(n-1)\gamma)\, b\widetilde{c}\right)d - bc_lc_h\right]}{b\left(2+(n-1)\gamma\right)\left(2c_lc_h + (2+(n-1)\gamma)\, b\widetilde{c}\right)} > 0 \\
\iff \quad & d > \frac{bc_lc_h\left(1-(n-1)\gamma\right)}{n\left(2c_lc_h + (2+(n-1)\gamma)\, b\widetilde{c}\right)} \equiv f(\gamma)
\end{aligned}
$$

Cap-and-Trade and Tax regulations improve welfare iff

$$d > f(\gamma)$$

Note that this will always be the case if $\gamma > \frac{1}{n-1}$. In the special monopoly case, i.e., $\gamma = 0$, the condition is

$$d > d_0 = \frac{bc_lc_h}{2n\left(c_lc_h + b\widetilde{c}\right)} \square$$

A.7 Proof of Proposition 5

By Theorems 4 and 5, we have, for $i \in \{l, h\}$ and for all s,

$$
\begin{aligned}
q_i^{tax} &= q_i^{ct} \\
x_i^{tax} &= x_i^{ct}
\end{aligned}
$$

Next, we calculate $q_i^{ct}(S^g)$ and $x_i^{ct}(S^g)$. In other words, we calculate the firms' decision variables when the cap is set equal to the total pollution under Groves.

Assume $d \leq \underline{d}$. Then,

$$S^g = \frac{nac_lc_h}{b\left(2+(n-1)\gamma\right)\left(n\widetilde{c}d + c_lc_h\right) + 2nc_lc_hd}$$

Note that

$$d \leq \underline{d} \iff S^g \geq \underline{S}$$

Thus,

$$
\begin{aligned}
q_l^{ct}(S^g) &= q_h^{ct}(S^g) = \frac{a\widetilde{c} + 2c_lc_hS^g/n}{\left(2+(n-1)\gamma\right)b\widetilde{c} + 2c_lc_h} \\
&= \frac{a\left(n\widetilde{c}d + c_lc_h\right)}{b\left(2+(n-1)\gamma\right)\left(n\widetilde{c}d + c_lc_h\right) + 2nc_lc_hd} \\
&= q_l^g = q_h^g
\end{aligned}
$$

where $\widetilde{c} = \frac{(n-m)c_l + mc_h}{n}$ and

$$
\begin{aligned}
x_l^{ct}(S^g) &= \frac{c_h\left[a - b\left(2+(n-m-1)\gamma\right)S^g/n\right]}{a\widetilde{c} + 2c_lc_hP^g/n} \\
&= \frac{nc_hd}{n\widetilde{c}d + c_lc_h} = x_l^g
\end{aligned}
$$

$$x_h^{ct}(S^g) = \frac{c_l x_l}{c_h} = \frac{n c_l d}{n\tilde{c}d + c_l c_h} = x_h^g$$

Assume $d > \underline{d}$. Then,

$$S^g = \frac{(n-m)\, a c_h \left(b(2-\gamma) + 2c_l\right)}{\begin{array}{c}\left[b\left(2+(m-1)\gamma\right)+2c_l\right]\left[b\left(2+(n-m-1)\gamma\right)+\frac{2(n-m)c_h d}{c_h+(n-m)d}\right]\\ -m(n-m)\gamma^2 b^2\end{array}}$$

Note that

$$d > \underline{d} \iff S^g < \underline{S}$$

Thus,

$$\begin{aligned}
q_l^{ct}(S^g) &= \frac{a\left[b(2-\gamma)+2c_h\right] - 2\gamma b c_h S^g}{\begin{array}{c}\left[b\left(2+(m-1)\gamma\right)+2c_l\right]\left[b\left(2+(n-m-1)\gamma\right)+2c_h\right]\\ -m(n-m)\gamma^2 b^2\end{array}}\\
&= \frac{a\left(b(2-\gamma)+\frac{2(n-m)c_h d}{c_h+(n-m)d}\right)}{\begin{array}{c}\left[b\left(2+(m-1)\gamma\right)+2c_l\right]\left[b\left(2+(n-m-1)\gamma\right)+\frac{2(n-m)c_h d}{c_h+(n-m)d}\right]\\ -m(n-m)\gamma^2 b^2\end{array}}\\
&= q_l^g
\end{aligned}$$

$$\begin{aligned}
q_h^{ct}(S^g) &= \frac{a\left[b(2-\gamma)+2c_l\right] + 2c_h\left[b\left(2+(m-1)\gamma\right)+2c_l\right]S^g/(n-m)}{\begin{array}{c}\left[b\left(2+(m-1)\gamma\right)+2c_l\right]\left[b\left(2+(n-m-1)\gamma\right)+2c_h\right]\\ -m(n-m)\gamma^2 b^2\end{array}}\\
&= \frac{a\left[b(2-\gamma)+2c_l\right]}{\begin{array}{c}\left[b\left(2+(m-1)\gamma\right)+2c_l\right]\left[b\left(2+(n-m-1)\gamma\right)+\frac{2(n-m)c_h d}{c_h+(n-m)d}\right]\\ -m(n-m)\gamma^2 b^2\end{array}}\\
&= q_h^g
\end{aligned}$$

and

$$x_l^{ct}(S^g) = 1 = x_l^g$$

$$\begin{aligned}
x_h^{ct}(S^g) &= \frac{a\left[b(2-\gamma)+2c_l\right] - b\left[b(2-\gamma)\left(2+(n-m-1)\gamma\right)S^g/(n-m)\right]}{a\left[b(2-\gamma)+2c_l\right] + 2c_h\left[b\left(2+(m-1)\gamma\right)+2c_l\right]S^g/(n-m)}\\
&= \frac{(n-m)\,d}{c_h+(n-m)\,d} = x_h^g
\end{aligned}$$

Hence, when $S = S^g$, all outcomes, except the firms' profits, are identical under Groves, Tax, and Cap-and-Trade (the expressions as functions of q_i and x_i are the same). From Proposition 2, we know that Cap-and-Trade dominates Cap for all outcomes and for any cap S. In particular, when $S = S^g$, Cap does worse than Cap-and-Trade, which mimics Groves. $\square$

A.8 Proof of Theorem 7

The social welfare W is continuous in S because W is a continuous function of q_i and x_i, which are themselves continuous in S.

More precisely, $W(S)$ is concave, quadratic on $[0,\ \underline{S}]$ and on $[\underline{S},\ S^u]$, and constant after that, because W is a quadratic function of $\{q_i\}_{1\leq i\leq n}$ and $\{q_i x_i\}_{1\leq i\leq n}$, all of which are linear in S.

From Proposition 5, we know that $W^{ct}(S^g) = W^g$. Recall that $\underline{S} = \frac{(n-m)a(c_h-c_l)}{c_h(b(2+(n-1)\gamma)+2c_l)}$.

(a) Assume that $\gamma = 0$.

On each interval (i.e., $[0,\ \underline{S}]$ and $[\underline{S},\ S^u]$), W is a parabola whose summit is at $-B/2A$, where A is the coefficient of S^2 and B the coefficient of S.

There are two cases:

1. $d \leq \underline{d}$: Then $S^g = \frac{nac_lc_h}{2[bc_lc_h+n(b\widetilde{c}+c_lc_h)d]}$, and $\underline{S} \leq S^g \leq S^u$, where $\widetilde{c} = \frac{(n-m)c_l+mc_h}{n}$.

   On $[0,\ \underline{S}]$,

$$\begin{aligned} W^{ct}(S) &= W^{tax}(S) \\ &= \frac{ma^2(3b+2c_l)}{8(b+c_l)^2} + \frac{(n-m)a^2(3b+2c_h)}{8(b+c_h)^2} \\ &\quad + \frac{c_h(n-m)a(3b+2c_h)S}{2(n-m)(b+c_h)^2} \\ &\quad - \left(d + \frac{bc_h(2b+c_h)}{2(n-m)(b+c_h)^2}\right)S^2 \end{aligned}$$

   The summit is at

$$\frac{(n-m)ac_h(3b+2c_h)}{2(n-m)d(b+c_h)^2 + bc_h(2b+c_h)} > \underline{S}$$

   In other words, W^{ct} is increasing on $[0,\ \underline{S}]$, which implies that $S^* > \underline{S}$.

   On $[\underline{S},\ S^u]$,

$$\begin{aligned} W^{ct}(S) &= W^{tax}(S) \\ &= \frac{na^2\widetilde{c} + 4c_lc_hS(a-bS/n)}{4(b\widetilde{c}+c_lc_h)} \\ &\quad + \frac{nb(a\widetilde{c}+2c_lc_hS/n)^2}{8(b\widetilde{c}+c_lc_h)^2} \\ &\quad - dS^2 \end{aligned}$$

The summit is at

$$\begin{aligned}\frac{nac_lc_h\left(3b\widetilde{c}+2c_lc_h\right)}{2\left[2nd\left(b\widetilde{c}+c_lc_h\right)^2+bc_lc_h\left(2b\widetilde{c}+c_lc_h\right)\right]} &> \\ \frac{nac_lc_h}{2\left[bc_lc_h+n\left(b\widetilde{c}+c_lc_h\right)d\right]} &= S^g\end{aligned}$$

2. $d > \underline{d}$: Then $S^g = \frac{(n-m)ac_h}{2[bc_h+(n-m)(b+c_h)d]} < \underline{S}$.

   On $[0,\ \underline{S}]$,

$$\begin{aligned}W^{ct}(S) &= W^{tax}(S) \\ &= \frac{ma^2(3b+2c_l)}{8(b+c_l)^2}+\frac{(n-m)a^2(3b+2c_h)}{8(b+c_h)^2} \\ &\quad +\frac{c_h(n-m)a(3b+2c_h)S}{2(n-m)(b+c_h)^2} \\ &\quad -\left(d+\frac{bc_h(2b+c_h)}{2(n-m)(b+c_h)^2}\right)S^2\end{aligned}$$

   The summit is at

$$\begin{aligned}\frac{(n-m)ac_h(3b+2c_h)}{2(n-m)d(b+c_h)^2+bc_h(2b+c_h)} &> \\ \frac{(n-m)ac_h}{2\left[bc_h+(n-m)(b+c_h)d\right]} &= S^g\end{aligned}$$

(b) Assume that $\gamma = 1$.

There are also two cases:

1. $d \leq \underline{d}$: Then $S^g = \frac{nac_lc_h}{(n+1)b(c_lc_h+n\widetilde{c}d)+2nc_lc_hd}$, and $\underline{S} \leq S^g \leq S^u$.

   On $[\underline{S},\ S^u]$,

$$\begin{aligned}W^{ct}(S) &= \frac{a\left(3b\widetilde{c}+2c_lc_h\right)\left(na\widetilde{c}+4c_lc_hS\right)}{2\left(2c_lc_h+(n+1)b\widetilde{c}\right)^2} \\ &\quad -\left(d+\frac{bc_lc_h\left(2(2n-1)c_lc_h+(n+1)^2b\widetilde{c}\right)}{n\left(2c_lc_h+(n+1)b\widetilde{c}\right)^2}\right)S^2\end{aligned}$$

   The summit is at

$$\begin{aligned}&\frac{nac_lc_h\left(3b\widetilde{c}+2c_lc_h\right)}{n\left(2c_lc_h+(n+1)b\widetilde{c}\right)^2d+bc_lc_h\left(2(2n-1)c_lc_h+(n+1)^2b\widetilde{c}\right)} \\ < \ &\frac{nac_lc_h}{2\left[bc_lc_h+n\left(b\widetilde{c}+c_lc_h\right)d\right]} = S^g,\ \text{for } n \geq 2.\end{aligned}$$

   In other words, W is strictly decreasing on $[S^g,\ S^u]$, which implies that $S^* < S^g$.

2. $d > \underline{d}$:

Then $S^g = \frac{(n-m)ac_h(b+2c_l)}{((m+1)b+2c_l)((n-m+1)b(c_h+(n-m)d)+2(n-m)c_hd)-m(n-m)b^2(c_h+(n-m)d)} < \underline{S}$.

On $[\underline{S},\ S^u]$, the summit is at

$$\frac{nac_lc_h\left(3b\widetilde{c}+2c_lc_h\right)}{n\left(2c_lc_h+(n+1)\,b\widetilde{c}\right)^2 d+bc_lc_h\left(2\left(2n-1\right)c_lc_h+(n+1)^2\,b\widetilde{c}\right)} < \underline{S}$$

In other words, W^{ct} is strictly decreasing on $[\underline{S},\ S^u]$, which implies that $S^* < \underline{S}$.

On $[0,\ \underline{S}]$,

$$W^{ct}\left(S\right)=-dS^2+\frac{1}{2\left[(n+1)\,b^2+2b\left((n-m+1)\,c_l+(m+1)\,c_h\right)+4c_lc_h\right]^2}\times$$
$$\left[\begin{array}{c} a^2\left[2m\left(c_h-c_l\right)\left(5b^2+6b\left(c_l+c_h\right)+4c_lc_h\right)+n\left(3b+2c_h\right)\left(b+2c_l\right)^2\right] \\ +4ac_h\left[\left(3b+2c_h\right)\left(b+2c_l\right)^2-4mb^2\left(c_h-c_l\right)\right]S- \\ \frac{2bc_h}{n-m}\left[\begin{array}{c} (n+1)^2\,b^3+2b^2\left(2\left(n+1\right)\left(n-m+1\right)c_l-\left(n\left(m-2\right)+4m+1\right)c_h\right) \\ +4bc_l\left(\left(n-m+1\right)^2 c_l+\left(n\left(m+4\right)-m\left(m+6\right)-2\right)c_h\right) \\ +8\left(2n-2m-1\right)c_l^2c_h \end{array}\right]S^2 \end{array}\right]$$

The summit is at

$$\frac{\left(n-m\right)ac_h\left(\left(3b+2c_h\right)\left(b+2c_l\right)^2-4mb^2\left(c_h-c_l\right)\right)}{\begin{array}{c} bc_h\left(\begin{array}{c} (n+1)^2\,b^3+2b^2(2\left(n+1\right)\left(n-m+1\right)c_l-\left(n\left(m-2\right)+4m+1\right)c_h \\ +4bc_l\left(\left(n-m+1\right)^2 c_l+\left(n\left(m+4\right)-m\left(m+6\right)-2\right)c_h\right) \\ +8\left(2n-2m-1\right)c_l^2c_h \end{array}\right) \\ +\left(n-m\right)\left[(n+1)\,b^2+2b\left((n-m+1)\,c_l+(m+1)\,c_h\right)+4c_lc_h\right]^2 d \end{array}}$$
$$< \frac{\left(n-m\right)ac_h\left(b+2c_l\right)}{\left(\begin{array}{c} \left((m+1)\,b+2c_l\right)\left((n-m+1)\,b\left(c_h+(n-m)\,d\right)+2\left(n-m\right)c_hd\right) \\ -m\left(n-m\right)b^2\left(c_h+\left(n-m\right)d\right) \end{array}\right)} = S^g\ \square$$

APPENDIX B

CHAPTER 3 PROOFS

B.1 Proof of Theorem 10

We assume that, if several trades are feasible and give the same payoff, the firms prefer the smallest trading volume, *i.e.*, $t = \arg\min\{|t|, \forall t \text{ feasible}\}$. In particular, if $t = 0$ if feasible, the firms will choose not to trade.

In the *two-stage* model, the firms first trade emission allowances with each other to maximize their joint profits, then produce and compete in the market. The solution is derived by backward induction, starting from the production/competition stage, and working backwards to the trading stage.

There are six main steps to the proof:

1. We analyze the Lagrangian and derive necessary conditions on q_i, x_i, and t_i based on the Kuhn-Tucker conditions of the Cournot game;
2. We solve the Cournot game (choice of q_i and x_i);
3. For each Cournot Nash Equilibrium in step 2, we solve the trading game (choice of t_i) and derive the Subgame-Perfect Nash Equilibrium;
4. For s given, we derive the Pareto-dominant Subgame-Perfect Nash Equilibrim.
5. We investigate conditions under which one of the firms may shut down (i.e., $q_i = 0$ for some i);
6. We prove that the objective function is strictly concave.

B.1.1 Step 1 - Analysis of the Lagrangian

In the first stage, the firms' problem is to solve the following joint-maximization problem (Cournot game):

$$\max_{q_l>0,\ 0\le x_l\le 1} \pi_l\left(q_l, x_l|t, q_h\right) = q_l \cdot (a - b \cdot q_l - \gamma \cdot b \cdot q_h) - c_l \cdot (q_l \cdot x_l)^2$$
$$\text{subject to } q_l \cdot (1 - x_l) \le s - t \text{ and } t \le s$$

$$\max_{q_h>0,\ 0\le x_h\le 1} \pi_h\left(q_h, x_h|t, q_l\right) = q_h \cdot (a - b \cdot q_h - \gamma \cdot b \cdot q_l) - c_h \cdot (q_h \cdot x_h)^2$$
$$\text{subject to } q_h \cdot (1 - x_h) \le s + t \text{ and } t \ge -s$$

Consider an arbitrary firm i. Write the Lagrangian

$$\mathfrak{L} = q_i\left(a - bq_i - \gamma bq_j\right) - c_i\left(q_i x_i\right)^2 + \lambda_i q_i + \mu_{i1} x_i - \mu_{i2}\left(x_i - 1\right) - \nu_i\left[q_i\left(1 - x_i\right) - s + t_i\right]$$

where λ_i, μ_{i1}, μ_{i2}, and ν_i are Lagrange multipliers. The Kuhn-Tucker necessary conditions are:

$$a - \gamma b q_j - 2q_i \left(b + c_i x_i^2\right) + \lambda_i - \nu_i \left(1 - x_i\right) = 0 \tag{B.1}$$

$$q_i \left(\nu_i - 2c_i q_i x_i\right) + \mu_{i1} - \mu_{i2} = 0 \tag{B.2}$$

with the complementary slackness conditions $\lambda_i q_i = 0$, $\mu_{i1} x_i = 0$, $\mu_{i2}\left(x_i - 1\right) = 0$, and $\nu_i \left[q_i \left(1 - x_i\right) - s + t_i\right] = 0$, and the feasibility constraints $0 \leq x_i \leq 1$, and $q_i,\ \lambda_i,\ \mu_{i1},\ \mu_{i2},\ \nu_i \geq 0$.

First, we show that $\mu_{i1} = 0$. *Proof.* (By contradiction.) Suppose that $\mu_{i1} > 0$. Then, $\mu_{i2} = 0$ and $x_i = 0$, and by equation $(B.2)$, $\mu_{i1} = -\nu_i q_i \leq 0$. ■

Assume for now that $\lambda_i = 0$. (We analyze the case $\lambda_i > 0$, *i.e.*, $q_i = 0$ later.)

Rewrite $\mu_{i2} = \mu_i$. There are 2 Lagrange multipliers leading to the following cases:

- $\mu_i = \nu_i = 0$ (The pollution constraint is slack):

  $$(B.2) \Rightarrow x_i = 0 \text{ (assume for now that } q_i > 0.)$$

  By equation $(B.1)$,

  $$q_i = \frac{a - \gamma b q_j}{2b}$$

  We also have

  $$q_i \leq s - t_i$$

- $\mu_i = 0,\ \nu_i > 0$ (The pollution constraint binds):

  $$\nu_i > 0 \Rightarrow q_i \left(1 - x_i\right) = s - t_i \iff q_i x_i = q_i - \left(s - t_i\right) \tag{B.3}$$

  $$(B.2) \Rightarrow \nu_i = 2c_i q_i x_i \tag{B.4}$$

  Note that this case is feasible only if $q_i > 0$ and $x_i > 0$.

  Combining $(B.3)$ and $(B.4)$ into $(B.1)$, we get

  $$q_i = \frac{a - \gamma b q_j + 2c_i \left(s - t_i\right)}{2\left(b + c_i\right)}$$

  Then

  $$x_i = 1 - \frac{s - t_i}{q_i} \leq 1$$

  provided that

  $$q_i > s - t_i \text{ (this condition guarantees that } x_i > 0)$$

- $\mu_i > 0$. Then $x_i = 1$ and by equation $(B.1)$, $q_i = \frac{a-\gamma b q_j}{2(b+c_i)}$. We must have $\nu_i > 0$, otherwise by $(B.2)$, $\mu_i = -2c_i q_i^2 \leq 0$.

$$\nu_i > 0 \Rightarrow q_i\left(1 - x_i\right) = 0 = s - t_i \Rightarrow t_i = s$$

  This is a special case of the previous situation where the pollution constraint binds.

To summarize, we have the following two cases:

The pollution constraint is slack:

$$q_i \leq s - t_i : \; q_i = \frac{a - \gamma b q_j}{2b}, \; q_i \geq 0, \; x_i = 0$$

The pollution constraint binds:

$$q_i > s - t_i : \; q_i = \frac{a - \gamma b q_j + 2c_i\left(s - t_i\right)}{2\left(b + c_i\right)}, \; q_i \geq 0, \; x_i = 1 - \frac{s - t_i}{q_i}, \; t_i \leq s$$

B.1.2 Step 2 - Cournot Nash Equilibrium

Next, we solve the Cournot game, given t. Four production equilibria are feasible:

1. **The pollution constraints are slack for both firms.**

   The firms solve the following system of equations:

$$\begin{cases} 2bq_l + \gamma b q_h = a \\ \gamma b q_l + 2bq_h = a \end{cases}$$

   From which we derive

$$q_l = q_h = \frac{a}{b\left(2 + \gamma\right)}$$

   We also have

$$x_l = x_h = 0$$

   The feasibility conditions are

$$\frac{a}{b\left(2 + \gamma\right)} - s \leq t \leq s - \frac{a}{b\left(2 + \gamma\right)}$$

   In particular, we must have $s \geq \frac{a}{b(2+\gamma)} \equiv s^u$. Call this stationary point E_1.

   Let Π denote the firms' joint profits. The firms' joint profits under E_1 are

$$\Pi_1 = \frac{2a^2}{b\left(2 + \gamma\right)^2}$$

   In the second stage (step 3 - Sub-game perfect Nash equilibrium), the firms choose t to maximize their joint profits. Since the profits are independent of t, the firms will choose the smallest feasible $\left|t\right|$.

$$t_1^* = 0$$

2. **The pollution constraint is slack for firm l but binds for firm h.**

The firms must solve

$$\begin{cases} 2bq_l + \gamma b q_h = a \\ \gamma b q_l + 2\left(b + c_h\right) q_h = a + 2c_h \left(s + t\right) \end{cases}$$

The solution is

$$\begin{aligned}
q_l &= \frac{a\left[b\left(2-\gamma\right) + 2c_h\right] - 2\gamma b c_h \left(s+t\right)}{b\left[b(4-\gamma^2) + 4c_h\right]} \\
q_h &= \frac{a(2-\gamma) + 4c_h\left(s+t\right)}{b(4-\gamma^2) + 4c_h} \\
x_l &= 0 \\
x_h &= \frac{\left(2-\gamma\right)\left[a - b\left(\gamma+2\right)\left(s+t\right)\right]}{a\left(2-\gamma\right) + 4c_h\left(s+t\right)} \\
\Pi_2 &= \frac{1}{b\left[b\left(4-\gamma^2\right) + 4c_h\right]^2} \times \\
& \left[\begin{array}{c} a^2\left[2b^2\left(2-\gamma\right)^2 + bc_h\left(6-\gamma\right)\left(2-\gamma\right) + 4c_h^2\right] \\ +4abc_h\left[b\left(2-\gamma\right)^2 + 4c_h\left(1-\gamma\right)\right]\left(s+t\right) \\ -b^2c_h\left[b\left(4-\gamma^2\right)^2 + 4c_h\left(4-3\gamma^2\right)\right]\left(s+t\right)^2 \end{array}\right]
\end{aligned}$$

Call this stationary point E_2. Feasibility conditions are

$$\begin{aligned}
q_l &\leq s - t \iff t \leq \frac{\left(b\left(2-\gamma\right) + 2c_h\right)\left(b\left(2+\gamma\right)s - a\right)}{b\left(2-\gamma\right)\left(b\left(2+\gamma\right) + 2c_h\right)} \equiv f\left(s\right) \\
q_h &> s + t \iff t < \frac{a}{b\left(\gamma+2\right)} - s \\
t &\geq -s
\end{aligned}$$

f intersects the line $-s$ at $s_f = \frac{a(b(2-\gamma)+2c_h)}{2b(b(4-\gamma^2)+4c_h)}$ and the line $\frac{a}{b(\gamma+2)} - s$ at $s = s^u$. (See Figure B.1).

In the second stage (step 3 - Subgame-Perfect Nash Equilibrium), the firms choose t to maximize Π_2.

Π_2 is quadratic and concave in $s+t$; thus, there is a unique maximum

$$t_2^* = \begin{cases} f\left(s\right) & \text{if } s_f \leq s < s_2 \\ \frac{2a\left[b(2-\gamma)^2 + 4(1-\gamma)c_h\right]}{b\left[b(4-\gamma^2)^2 + 4c_h(4-3\gamma^2)\right]} - s & \text{if } s \geq s_2 \end{cases}$$

where

$$s_2 = \frac{a\left[b\left(2-\gamma\right)^2\left(4+\gamma\right) + 2c_h\left(8 - \gamma\left(6+\gamma\right)\right)\right]}{2b\left[b\left(4-\gamma^2\right)^2 + 4c_h\left(4-3\gamma^2\right)\right]}$$

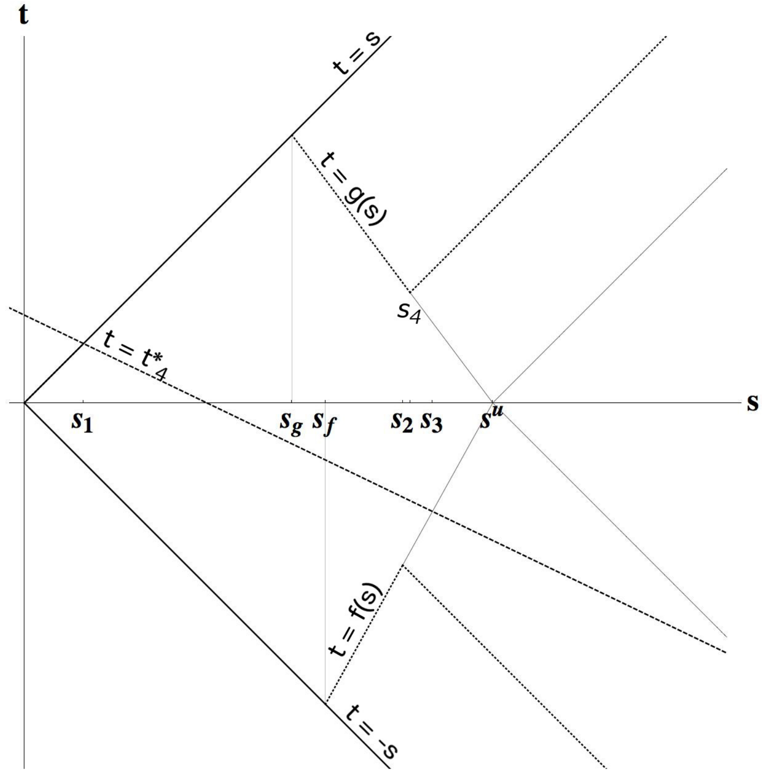

Figure B.1. Representation of the feasible region in the $\{s, t\}$ space.

is the cap at which

$$f(s) = \frac{2a\left[b(2-\gamma)^2 + 4(1-\gamma)c_h\right]}{b\left[b(4-\gamma^2)^2 + 4c_h(4-3\gamma^2)\right]} - s$$

When $s \geq s_2$, the optimal production quantities and abatement levels are:

$$\begin{aligned}
q_l^* &= \frac{a\left[b(2-\gamma)^2(2+\gamma) + 2c_h[4-\gamma(2+\gamma)]\right]}{b\left[b(4-\gamma^2)^2 + 4c_h(4-3\gamma^2)\right]} \\
q_h^* &= \frac{a\left[b(2-\gamma)^2(2+\gamma) + 8(1-\gamma)c_h\right]}{b\left[b(4-\gamma^2)^2 + 4c_h(4-3\gamma^2)\right]} \\
x_l^* &= 0 \\
x_h^* &= \frac{b(2-\gamma)^2\gamma}{b(2-\gamma)^2(2+\gamma) + 8(1-\gamma)c_h} \\
\Pi_2^* &= \frac{a^2\left[2b(2-\gamma)^2 + c_h[8-\gamma(8-\gamma)]\right]}{b\left[b(4-\gamma^2)^2 + 4c_h(4-3\gamma^2)\right]}
\end{aligned}$$

(Step 4) It is straightforward to verify that $\Pi_2^* \geq \Pi_1^*$ so that E_1 is dominated by E_2. We will later show that E_2 is dominated when $s < s_2$ and $t^* = f(s)$, so we can ignore that case.

3. **The pollution constraint is binding for firm l but slack for firm h.**

The firms must solve

$$\begin{cases} 2(b+c_l)q_l + \gamma b q_h = a + 2c_l(s-t) \\ \gamma b q_l + 2bq_h = a \end{cases}$$

The solution is

$$\begin{aligned}
q_l &= \frac{a(2-\gamma)+4c_l(s-t)}{b(4-\gamma^2)+4c_l} \\
q_h &= \frac{a\left[b(2-\gamma)+2c_l\right]-2\gamma b c_l(s-t)}{b\left[b(4-\gamma^2)+4c_l\right]} \\
x_l &= \frac{(2-\gamma)\left[a-b(\gamma+2)(s-t)\right]}{a(2-\gamma)+4c_l(s-t)} \\
x_h &= 0 \\
\Pi_3 &= \frac{1}{b\left[b(4-\gamma^2)+4c_l\right]^2} \times \\
&\left[\begin{array}{c} a^2\left[2b^2(2-\gamma)^2+bc_l(6-\gamma)(2-\gamma)+4c_l^2\right] \\ +4abc_l\left[b(2-\gamma)^2+4c_l(1-\gamma)\right](s-t) \\ -b^2c_l\left[b\left(4-\gamma^2\right)^2+4c_l\left(4-3\gamma^2\right)\right](s-t)^2 \end{array}\right]
\end{aligned}$$

Call this stationary point E_3. Feasibility conditions are

$$\begin{aligned}
q_l &> s-t \iff t > s - \frac{a}{b(2+\gamma)} \\
t &\leq s \\
q_h &\leq s+t \iff t \geq \frac{(b(2-\gamma)+2c_l)(a-b(2+\gamma)s)}{b(2-\gamma)(b(2+\gamma)+2c_l)} \equiv g(s)
\end{aligned}$$

g intersects the line s at $s_g = \frac{a(b(2-\gamma)+2c_l)}{2b(b(4-\gamma^2)+4c_l)}$ and the line $s - \frac{a}{b(\gamma+2)}$ at $s = s^u$. (See Figure B.1.)

In the second stage (step 3 - Sub-game perfect Nash equilibrium), the firms choose t to maximize Π_3.

Π_3 is quadratic and concave in $s-t$; thus,

$$t_3^* = \begin{cases} g(s) & \text{if } s_g \leq s \leq s_4 \\ s - \frac{2a\left[b(2-\gamma)^2+4(1-\gamma)c_l\right]}{b\left[b(4-\gamma^2)^2+4c_l(4-3\gamma^2)\right]} & \text{if } s \geq s_4 \end{cases}$$

where $s_4 = \frac{a\left[b(2-\gamma)^2(4+\gamma)+2c_l(8-\gamma(6+\gamma))\right]}{2b\left[b(4-\gamma^2)^2+4c_l(4-3\gamma^2)\right]}$ is the cap at which $g(s) = s - \frac{2a\left[b(2-\gamma)^2+4(1-\gamma)c_l\right]}{b\left[b(4-\gamma^2)^2+4c_l(4-3\gamma^2)\right]}$.

The optimal production quantities and abatement levels are:

$$
\begin{aligned}
q_l^* &= \frac{a\left[b\left(2-\gamma\right)^2\left(2+\gamma\right)+8\left(1-\gamma\right)c_l\right]}{b\left[b\left(4-\gamma^2\right)^2+4c_l\left(4-3\gamma^2\right)\right]} \\
q_h^* &= \frac{a\left[b\left(2-\gamma\right)^2\left(2+\gamma\right)+2c_l\left[4-\gamma\left(2+\gamma\right)\right]\right]}{b\left[b\left(4-\gamma^2\right)^2+4c_l\left(4-3\gamma^2\right)\right]} \\
x_l^* &= \frac{b\left(2-\gamma\right)^2\gamma}{b\left(2-\gamma\right)^2\left(2+\gamma\right)+8\left(1-\gamma\right)c_l} \\
x_h^* &= 0 \\
\Pi_3^* &= \frac{a^2\left[2b\left(2-\gamma\right)^2+c_l\left[8-\gamma\left(8-\gamma\right)\right]\right]}{b\left[b\left(4-\gamma^2\right)^2+4c_l\left(4-3\gamma^2\right)\right]}
\end{aligned}
$$

(Step 4) It is straightforward to verify that $\Pi_2^* \geq \Pi_3^*$, meaning that E_3 is dominated by E_2. We will later show that E_3 is also dominated when $s < s_4$ and $t^* = g(s)$, so we can ignore that case. In other words, E_3 is dominated.

4. **The final case is for both pollution constraints to bind.**

The firms must solve

$$
\begin{cases} 2\left(b+c_l\right)q_l+\gamma b q_h = a+2c_l\left(s-t\right) \\ \gamma b q_l+2\left(b+c_h\right)q_h = a+2c_h\left(s+t\right) \end{cases}
$$

From which we derive

$$
\begin{aligned}
q_l &= \frac{a\left[b\left(2-\gamma\right)+2c_h\right]-2\gamma b c_h\left(s+t\right)+4c_l\left(b+c_h\right)\left(s-t\right)}{4\left(b+c_l\right)\left(b+c_h\right)-\gamma^2 b^2} \\
q_h &= \frac{a\left[b\left(2-\gamma\right)+2c_l\right]-2\gamma b c_l\left(s-t\right)+4c_h\left(b+c_l\right)\left(s+t\right)}{4\left(b+c_l\right)\left(b+c_h\right)-\gamma^2 b^2}
\end{aligned}
$$

The feasibility conditions

$$
\begin{aligned}
q_l &> s-t \iff t > f(s) \\
q_h &> s+t \iff t < g(s)
\end{aligned}
$$

A necessary condition is that $s < s^u$. (This is because f and g are linear functions of s that intersect at s^u.)

$$
\begin{aligned}
q_l &= \frac{a\left(b+2c_h\right)-2\gamma b c_h\left(s+t\right)+4c_l\left(b+c_h\right)\left(s-t\right)}{4\left(b+c_l\right)\left(b+c_h\right)-\gamma^2 b^2} \\
q_h &= \frac{a\left(b+2c_l\right)-2\gamma b c_l\left(s-t\right)+4c_h\left(b+c_l\right)\left(s+t\right)}{4\left(b+c_l\right)\left(b+c_h\right)-\gamma^2 b^2} \\
x_l &= 1-\frac{s-t}{q_l} \\
x_h &= 1-\frac{s+t}{q_h}
\end{aligned}
$$

The expression for the joint profits is complicated.

$$\Pi_4 = -\frac{1}{\left(4\left(b+c_l\right)\left(b+c_h\right)-\gamma^2 b^2\right)^2}\times$$
$$\left[\begin{array}{c} \left[a\left(b\left(2-\gamma\right)+2c_h\right)+2b\left(2c_l\left(s-t\right)-c_h\gamma\left(s+t\right)\right)+4c_lc_h\left(s-t\right)\right]\times \\ \left[\begin{array}{c} a\left(b+2c_l\right)\left(b\gamma-2\left(b+c_h\right)\right)+ \\ 2b\left(b\left(c_h\gamma\left(s+t\right)+c_l\left(2-\gamma^2\right)\left(s-t\right)\right)+2c_lc_h\left(\left(1+\gamma\right)s-\left(1-\gamma\right)t\right)\right) \end{array}\right] \\ +\left[a\left(b\left(2-\gamma\right)+2c_l\right)-2\left(b\left(c_l\gamma\left(s-t\right)-2c_h\left(s+t\right)\right)+2c_lc_h\left(s+t\right)\right)\right]\times \\ \left[\begin{array}{c} a\left(b+2c_h\right)\left(b\gamma-2\left(b+c_l\right)\right)+ \\ 2b\left(2c_lc_h\left(s-t+\gamma\left(s+t\right)\right)+b\left(c_l\gamma\left(s-t\right)+c_h\left(2-\gamma^2\right)\left(s+t\right)\right)\right) \end{array}\right] \\ +c_h\left[\left(b\left(2-\gamma\right)+2c_l\right)\left(a-b\left(2+\gamma\right)s\right)-b\left(2-\gamma\right)t\left(b\left(2+\gamma\right)+2c_l\right)\right]^2 \\ +c_l\left[\left(b\left(2-\gamma\right)+2c_h\right)\left(a-b\left(2+\gamma\right)s\right)+b\left(2-\gamma\right)t\left(b\left(2+\gamma\right)+2c_h\right)\right]^2 \end{array}\right]$$

Call this stationary point E_4. Π_4 is quadratic and concave in t.

The unique maximum is at

$$t_4^* = \frac{\left(c_h-c_l\right)\left[\begin{array}{c} 2a\left(b^2\gamma^2+4\left(1-\gamma\right)\left(b+c_l\right)\left(b+c_h\right)\right) \\ -bs\left(16\left(b+c_l\right)\left(b+c_h\right)-\gamma^2\left(\left(8-\gamma^2\right)b^2+12b(c_l+c_h)+12c_lc_h\right)\right) \end{array}\right]}{\left[\begin{array}{c} 16\left(b+c_l\right)\left(b+c_h\right)\left(b\left(c_l+c_h\right)+2\left(1-\gamma\right)c_lc_h\right) \\ -b\gamma^2\left(\left(8-\gamma^2\right)b^2\left(c_l+c_h\right)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h\left(c_l+c_h\right)\right) \end{array}\right]}$$

t^* is a linear, decreasing function of s. We need to check that it is feasible.

Define

$$\begin{array}{lll} s_1 & \text{as the point at which } t_4^* \text{ intersects} & t = s \\ s_3 & \text{as the point at which } t_4^* \text{ intersects} & t = f\left(s\right) \end{array}$$

We have

$$s_1 = \frac{a\left(c_h-c_l\right)\left(b^2\gamma^2+4\left(1-\gamma\right)\left(b+c_l\right)\left(b+c_h\right)\right)}{c_h\left[\begin{array}{c} 16\left(b+c_l\right)\left(b+c_h\right)\left(b+\left(1-\gamma\right)c_l\right)- \\ b\gamma^2\left(\left(8-\gamma^2\right)b^2+8bc_l+12bc_h+12c_lc_h\right) \end{array}\right]}$$

$$s_3 = \frac{a\left(b^2\left(2-\gamma\right)^2\left(c_l\gamma+c_h\left(4+\gamma\right)\right)+2bc_h\left(8-\gamma\left(6+\gamma\right)\right)\left(c_l+c_h\right)+16c_lc_h^2(1-\gamma)\right)}{2bc_h\left(\begin{array}{c} b^2\left(4-\gamma^2\right)^2+ \\ 2b\left(c_l\left(8-\gamma\left(4+\gamma\left(2+\gamma\right)\right)\right)+2\left(4-3\gamma\right)c_h\right)+8c_lc_h\left(2-\gamma-\gamma^2\right) \end{array}\right)}$$

By construction, we have

$$0 \le s_1 \le s_3 \le s^u$$

(see Figure B.1)

$$t_4^* \text{ is feasible } \iff s_1 \le s \le s_3$$

The optimal production quantities and abatement levels are

$$
\begin{aligned}
q_l^* &= \frac{\left(a\left(c_l+c_h\right)+4c_lc_hs\right)\left(\begin{array}{c} 8\left(b+c_l\right)\left(b+c_h\right)- \\ b\gamma^2\left(2\left(1-\gamma\right)b+2c_h\right)-4\gamma\left(b+2c_l\right)\left(b+c_h\right)\end{array}\right)}{\left[\begin{array}{c} 16\left(b+c_l\right)\left(b+c_h\right)\left(b\left(c_l+c_h\right)+2\left(1-\gamma\right)c_lc_h\right) \\ -b\gamma^2\left(\left(8-\gamma^2\right)b^2\left(c_l+c_h\right)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h\left(c_l+c_h\right)\right)\end{array}\right]} \\
q_h^* &= \frac{\left(a\left(c_l+c_h\right)+4c_lc_hs\right)\left(\begin{array}{c} 8\left(b+c_l\right)\left(b+c_h\right)- \\ b\gamma^2\left(2\left(1-\gamma\right)b+2c_l\right)-4\gamma\left(b+c_l\right)\left(b+2c_h\right)\end{array}\right)}{\left[\begin{array}{c} 16\left(b+c_l\right)\left(b+c_h\right)\left(b\left(c_l+c_h\right)+2\left(1-\gamma\right)c_lc_h\right) \\ -b\gamma^2\left(\left(8-\gamma^2\right)b^2\left(c_l+c_h\right)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h\left(c_l+c_h\right)\right)\end{array}\right]} \\
x_l^* &= 1-\frac{s-t_4^*}{q_i^*} \\
x_h^* &= 1-\frac{s+t_4^*}{q_j^*}
\end{aligned}
$$

The firms' joint profits are

$$
\Pi_4^{t<s} = \tag{B.5}
$$

$$
\frac{\left[\begin{array}{c} a\left(8\left(1-\gamma\right)\left(b+c_l\right)\left(b+c_h\right)+b\gamma^2\left(2b+\left(c_l+c_h\right)\right)\right)\left(a\left(c_l+c_h\right)+8c_lc_hs\right) \\ -4bc_lc_hs^2\left(16\left(b+c_l\right)\left(b+c_h\right)-\gamma^2\left(\left(8-\gamma^2\right)b^2+12b\left(c_l+c_h\right)+16c_lc_h\right)\right)\end{array}\right]}{\left[\begin{array}{c} 16\left(b+c_l\right)\left(b+c_h\right)\left(b\left(c_l+c_h\right)+2\left(1-\gamma\right)c_lc_h\right) \\ -b\gamma^2\left(\left(8-\gamma^2\right)b^2\left(c_l+c_h\right)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h\left(c_l+c_h\right)\right)\end{array}\right]}
$$

When $0 \leq s \leq s_1$, t_4^* hits a boundary of the feasible set.

$$
\begin{aligned}
t_4^* &= s \\
q_l^* &= \frac{a\left(b\left(2-\gamma\right)+2c_h\right)-4\gamma bc_hs}{4\left(b+c_l\right)\left(b+c_h\right)-\gamma^2b^2} \\
q_h^* &= \frac{a\left(b\left(2-\gamma\right)+2c_l\right)+8c_h\left(b+c_l\right)s}{4\left(b+c_l\right)\left(b+c_h\right)-\gamma^2b^2} \\
x_l^* &= 1 \\
x_h^* &= 1-\frac{2s}{q_j^*} \\
\Pi_4^{t=s} &= \frac{1}{\left(4\left(b+c_l\right)\left(b+c_h\right)-\gamma^2b^2\right)^2}\times \qquad \text{(B.6)} \\
&\left[\begin{array}{c} a^2\left[b^2\gamma^2\left(2b+c_l+c_h\right)+4\left(b+c_l\right)\left(b+c_h\right)\left(2b\left(1-\gamma\right)+c_l+c_h\right)\right] \\ +8ac_hs\left(b+c_l\right)\left(4\left(b+c_l\right)\left(b+c_h\right)-b\gamma\left(\left(4-\gamma\right)b+4c_h\right)\right) \\ -4bc_hs^2\left[b^3\gamma^4-4b\gamma^2\left(b+c_l\right)\left(2b+3c_h\right)+16\left(b+c_l\right)^2\left(b+c_h\right)\right]\end{array}\right]
\end{aligned}
$$

When $s_3 \leq s \leq s^u$, t_4^* hits another boundary of the feasible set.

$$
t_4^* = f\left(s\right)
$$

(Step 4) We now show that this solution is dominated. The firms' production and abatement decisions are continuous along the boundary $t = f(s)$ and so are the joint profits. We have

$$s_3 \geq s_2$$

which implies that the joint profits under E_4 along $t = f(s)$ (when t^* hits the boundary of the feasible set) are less than the interior solution of E_2. In other words, E_2 dominates E_4 when $s \geq s_3$. We have the following result:

$$E_2 \text{ is the global maximum for } s \geq s_3 \tag{B.7}$$

A similar reasoning is used to prove that E_4 dominates E_2 when $s \leq s_2$ (in this case, E_2 sits on the boundary and E_4 is the interior point solution), and that E_4 dominates E_3 when $s \leq s_4$ (in this case, E_3 sits on the boundary and E_4 is the interior point solution). Since $s_2 \leq s_3$, we have the following result:

$$E_4 \text{ is the global maximum for } s \leq s_2 \tag{B.8}$$

Combining results $(B.7)$ and $(B.8)$ and the fact that Π_2^* is independent of s and Π_4^* is strictly concave in s, we have the following result:

There exists a unique $\widetilde{s} \in [s_2, s_3]$ such that E_4 is the unique global maximum for $s \in (s_1, \widetilde{s})$ and E_2 is the unique global maximum for $s \geq \widetilde{s}$ $\square$.

B.1.3 Step 5a - Can the High-cost Firm Shut Down, i.e., $q_h = 0$?

When $q_h = 0$, firm l is a monopolist. The firms jointly maximize

$$\begin{array}{l} \max\limits_{q_l \geq 0,\ 0 \leq x_l \leq 1} \Pi^m (q_l, x_l) = q_l \cdot (a - b \cdot q_l) - c_l \cdot (q_l \cdot x_l)^2 \\ \text{subject to } q_l \cdot (1 - x_l) \leq s - t \text{ and } t \geq -s \end{array}$$

Using the same notations as previously, the first order conditions are:

$$a - 2q_l \left(b + c_l x_l^2\right) - \nu_l (1 - x_l) = 0 \tag{B.9}$$

$$q_l (\nu_l - 2c_l q_l x_l) - \mu_l = 0 \tag{B.10}$$

- Suppose $\nu_l = 0$ (slack pollution constraint). Equation $(B.10)$ implies that $\mu_l = 0$ and $q_l x_l = 0$. Equation $(B.9)$ implies that

$$q_l = \frac{a}{2b}$$

Since $q_l > 0$, $x_l = 0$. We have the following conditions

$$-s \le t \le s - \frac{a}{2b}$$

A necessary condition is that $s \ge \frac{a}{4b}$. The optimal joint profits are

$$\Pi^{m*} = \frac{a^2}{4b}$$

- Suppose $\nu_l > 0$ (binding pollution constraint). Then, $q_l\,(1 - x_l) = s - t$.

  - If $\mu_l = 0$, equations $(B.9)$ and $(B.10)$ imply

$$\begin{aligned} \nu_l &= 2c_l q_l x_l \\ q_l &= \frac{a + 2c_l\,(s-t)}{2\,(b + c_l)} \\ x_l &= \frac{a - 2b\,(s-t)}{a + 2c_l\,(s-t)} \end{aligned}$$

provided that

$$\begin{aligned} s - \frac{a}{2b} &< t \le s \\ t &\ge -s \end{aligned}$$

The firms' joint profits are

$$\Pi^m = \frac{a^2 + 4ac_l\,(s-t) - 4bc_l\,(s-t)^2}{4\,(b + c_l)}$$

If $s \ge \frac{a}{4b}$, to maximize their joint profits, the firms will choose

$$t^* = s - \frac{a}{2b}$$

and

$$\Pi^{m*} = \frac{a^2}{4b}$$

If $s < \frac{a}{4b}$, then $t^* = -s$ and the joint profits are

$$\Pi^m = \frac{a^2 + 8ac_l s - 16bc_l s^2}{4\,(b + c_l)}$$

Π^m is continuous at $s = \frac{a}{4b}$.

– $\mu_i > 0$ is a special case of the previous case with $t = s = 0$, leading to a joint profit $= \frac{a^2}{4(b+c_l)}$.

We conclude by comparing the firms' joint profits when $q_h = 0$ and when $q_h > 0$. Note that $s_1 \leq \frac{a}{4b} \leq s_2$. This implies that $\frac{a}{4b} \leq \widetilde{s}$.

- When $s > \widetilde{s}$, if firm h shuts down, the firms' joint profits is

$$\Pi^m = \frac{a^2}{4b}$$

If the firms compete, they make a joint profit

$$\Pi_2 = \frac{a^2 \left[2b\,(2-\gamma)^2 + c_h\,[8 - \gamma\,(8-\gamma)]\right]}{b\left[b\,(4-\gamma^2)^2 + 4c_h\,(4-3\gamma^2)\right]}$$

Thus firm h shuts down if and only if

$$\frac{a^2}{4b} > \Pi_2 \tag{B.11}$$

The condition $(B.11)$ is equivalent to

$$b\gamma^4 - 16\,(b+c_h)\,\gamma^2 + 32\,(b+c_h)\,\gamma - 16\,(b+c_h) > 0$$

The LHS has a unique root $\underline{\gamma}$ between 0 and 1. Define $\alpha_h = \frac{c_h}{b}$.

$$\underline{\gamma} = 2\sqrt{1+\alpha_h}\left[\sqrt{1+\sqrt{\frac{1}{1+\alpha_h}}} - 1\right]$$

$\underline{\gamma}$ ranges from $2\left(\sqrt{2}-1\right) \approx .83$ to 1.

$$\text{The condition } (B.11) \iff \gamma > \underline{\gamma}$$

- When $\frac{a}{4b} < s < \widetilde{s}$, the firms still make a profit $\Pi^m = \frac{a^2}{4b}$ if firm h shuts down. However, when they compete, the firms' joint profits is given by equation $(B.6)$.

$$\begin{aligned}\Pi_4^{t<s} \quad = \quad & \frac{1}{\left(4\,(b+c_l)\,(b+c_h) - \gamma^2 b^2\right)^2} \times \\ & \left[\begin{array}{c} a^2\left[b^2\gamma^2\,(2b+c_l+c_h) + 4\,(b+c_l)\,(b+c_h)\,(2b\,(1-\gamma)+c_l+c_h)\right] \\ +8ac_h s\,(b+c_l)\,(4\,(b+c_l)\,(b+c_h) - b\gamma\,((4-\gamma)\,b + 4c_h)) \\ -4bc_h s^2\left[b^3\gamma^4 - 4b\gamma^2\,(b+c_l)\,(2b+3c_h) + 16\,(b+c_l)\,(b+c_h)\,(b+c_l)\right] \end{array}\right]\end{aligned}$$

Thus, firm h shuts down if and only if

$$\frac{a^2}{4b} > \Pi_4^{t<s} \tag{B.12}$$

This condition is complicated.

Since $\Pi_4^{t<s}$ is strictly decreasing on $\left[\frac{a}{4b}, \widetilde{s}\right]$, we can derive the following sufficient conditions:

1. If $\gamma \leq \underline{\gamma}$, then $\Pi_4^{t<s} > \Pi_2 \geq \frac{a^2}{4b}$, meaning that firm h will **not** shut down.
2. There is a threshold value of γ above which firm h will shut down. Specifically, firm h will shut down if
$$\Pi_4^{t<s}\left(\frac{a}{4b}\right) < \frac{a^2}{4b}$$
This condition is equivalent to
$$\begin{aligned}&b^2\gamma^4 - 16\left(b^2 + bc_l + bc_h + c_l c_h\right)\gamma^2 + \\ &32\left(b^2 + bc_l + bc_h + c_l c_h\right)\gamma - 16\left(b^2 + bc_l + bc_h + c_l c_h\right) > 0\end{aligned}$$
This equation in γ has a unique root $\overline{\gamma}$ between 0 and 1. Define $\alpha_l = \frac{c_l}{b}$, and recall that $\alpha_h = \frac{c_h}{b}$.
$$\overline{\gamma} = \frac{\beta}{b}\left[\sqrt{\frac{1 + \alpha_l + \alpha_h + \alpha_l\alpha_h + \sqrt{(1+\alpha_l)(1+\alpha_h)}}{(1+\alpha_l)(1+\alpha_h)}} - 1\right]$$
where $\beta = 2\sqrt{(b+c_l)(b+c_h)}$.
$$\text{The condition } (B.12) \iff \gamma > \overline{\gamma}$$
Note that $\underline{\gamma} \leq \overline{\gamma} \leq 1$.

We provide a graphical representation of condition $(B.12)$ in Figure B.2. Our results show that if γ is sufficiently high, firm h shuts down. There are parameter values for which collusion could lead to the extreme situation in which q_h is forced out (*i.e.*, the *low-cost* firm becomes a monopoly).

- When $s_1 \leq s \leq \frac{a}{4b}$, if firm h shuts down, the firms' joint profits is
$$\Pi^m = \frac{a^2 + 8ac_l s - 16bc_l s^2}{4(b+c_l)}$$
If the firms compete, they still make a joint profit
$$\Pi_4^{t<s} \text{ (see equation B.6)}$$
Thus, firm h shuts down if and only if
$$\frac{a^2 + 8ac_l s - 16bc_l s^2}{4(b+c_l)} > \Pi_4^{t<s} \tag{B.13}$$
This condition is also complicated, but the result is the same as before. For γ sufficiently high, firm h shuts down. A graphical representation is given in Figure B.2.

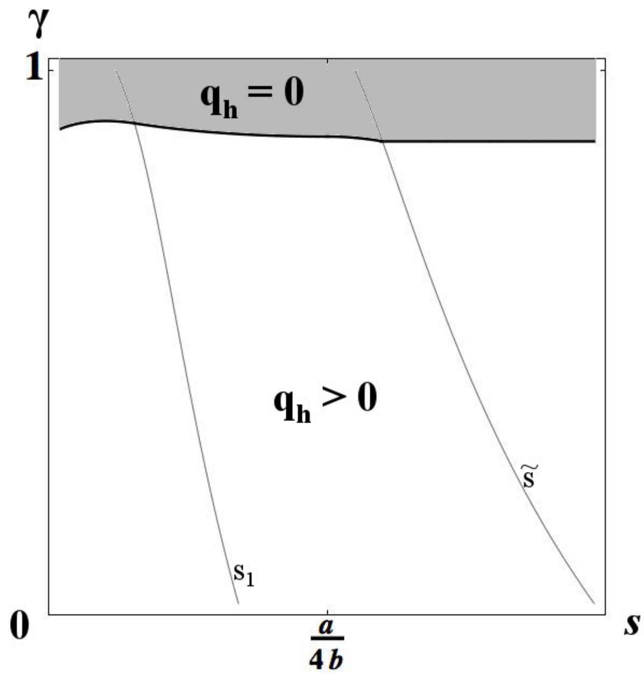

Figure B.2. Does the high-cost firm shut down?

- When $0 \leq s \leq s_1$, if firm h shuts down, the firms' joint profits is still

$$\Pi^m = \frac{a^2 + 8ac_l s - 16bc_l s^2}{4(b + c_l)}$$

However, when they compete, the firms' joint profits is given by equation $(B.6)$.

$$\begin{aligned} \Pi_4^{t=s} &= \frac{1}{(4(b+c_l)(b+c_h) - \gamma^2 b^2)^2} \times \\ & \left[\begin{array}{c} a^2 \left[b^2\gamma^2 (2b + c_l + c_h) + 4(b+c_l)(b+c_h)(2b(1-\gamma) + c_l + c_h) \right] \\ +8ac_h s (b + c_l)(4(b+c_l)(b+c_h) - b\gamma((4-\gamma)b + 4c_h)) \\ -4bc_h s^2 \left[b^3\gamma^4 - 4b\gamma^2 (b+c_l)(2b+3c_h) + 16(b+c_l)^2 (b+c_h) \right] \end{array} \right] \end{aligned}$$

Thus, firm h shuts down if and only if

$$\frac{a^2 + 8ac_l s - 16bc_l s^2}{4(b + c_l)} > \Pi_4^{t=s} \tag{B.14}$$

Similar to the previous case, when γ is large, condition $(B.14)$ will be satisfied. See Figure B.2. □

B.1.4 Step 5b - Proof That the Low-cost Firm Never Shuts Down

The case $q_l = 0$ is the symmetric of $q_h = 0$. The proof is obtained by swapping l and h. When comparing firm l's profits if $q_l = 0$ to its profits if $q_l > 0$, note that the conditions are the same as the case $q_h = 0$ when $s > \frac{a}{4b}$. In other words, we have a tie: if one of the firms is to shut down, it does not matter which one. When $s \leq \frac{a}{4b}$, however, note that for any s,

$$\frac{a^2 + 8ac_l s - 16bc_l s^2}{4(b + c_l)} \geq \frac{a^2 + 8ac_h s - 16bc_h s^2}{4(b + c_h)}$$

with equality only when $s = \frac{a}{4b}$. This means that the *high-cost* firm will shut down before the *low-cost* firm. Using a continuity argument at $s = \frac{a}{4b}$, we conclude that the *low-cost* firm never shuts down. $\square$

B.1.5 Step 6 - Second Order Necessary Conditions

We now check the second order conditions. We need to do this for the Cournot game only, because we have already established that the joint-profit maximization problems are quadratic concave in t.

For an arbitrary firm i, the Hessian H_i of the Cournot game is the following 2×2 matrix.

$$H_i = \begin{pmatrix} -2\left(b + c_i x_i^2\right) & -4c_i q_i x_i + v_i \\ -4c_i q_i x_i + v_i & -2c_i q_i^2 \end{pmatrix}$$

Note that the diagonal elements are negative. If $\det(H_i) > 0$, then the Hessian is negative definite, which implies concavity of the objective function.

At stationary point E_1, we have

$$H_i = -2\begin{pmatrix} b & 0 \\ 0 & c_i q_i^2 \end{pmatrix}$$

whose determinant is $\det(H_i) = 4bc_i q_i^2 > 0$.

At stationary point E_2, we have $\nu_l = 0$ and $x_l = 0$, which implies that

$$H_l = -2\begin{pmatrix} b & 0 \\ 0 & c_l q_l^2 \end{pmatrix}$$

and $\det(H_l) = 4bc_l q_l^2 > 0$.

We also have $\nu_h = 2c_h q_h x_h$, which implies that

$$H_h = -2\begin{pmatrix} b + c_h x_h^2 & c_h q_h x_h \\ c_h q_h x_h & c_h q_h^2 \end{pmatrix}$$

and $\det(H_h) = 4bc_h q_h^2 > 0$.

E_3 is off the equilibrium path.

At stationary point E_4, $\nu_i = 2c_i q_i x_i \ \forall i$ which implies that

$$H_i = -2\begin{pmatrix} b + c_i x_i^2 & c_i q_i x_i \\ c_i q_i x_i & c_i q_i^2 \end{pmatrix}$$

and $\det(H_i) = 4bc_i q_i^2 > 0$. $\square$

APPENDIX C

CHAPTER 4 PROOFS

C.1 Notations and Assumptions

π_i^a denotes the profit of firm i if it takes action a, where $a = I$ if the firm invests, and $a = N$ if the firm does not invest.

π_m^a denotes the profit of a firm that takes action a, where $a = I$ or N, when m firms invest.

Let

$$\begin{aligned} F_0 &= \frac{a^2 c_l (c_h - c_l)}{4c_h (b + c_l)^2} \\ F_1 &= \frac{a^2 (c_h - c_l)}{4 (b + c_l)(b + c_h)} \end{aligned}$$

Without loss of generality, we will assume that if the firm is indifferent between investing and not investing, it will not invest.

C.2 Proof of Theorem 11

Consider any firm i. The firm's profit are given by Theorem 3 of Chapter 2.

$$\begin{aligned} \pi_i^N &= \frac{a^2}{4(b + c_h)} + \frac{c_h (a - bs) s}{(b + c_h)} \\ \pi_i^I &= \frac{a^2}{4(b + c_l)} + \frac{c_l (a - bs) s}{(b + c_l)} - F \end{aligned}$$

Firm i will invest if and only if

$$\pi_i^I > \pi_i^N \iff F < \frac{(c_h - c_l)(a - 2bs)^2}{4(b + c_l)(b + c_h)} \equiv F^{cap}(s) \ \square$$

C.3 Proof of Theorem 12

We solve by backward induction.

The solution to the last stage is given by Theorem 4 of Chapter 2. Then, we solve the n-firm investment game.

$$M \text{ is a Nash equilibrium} \iff \pi_m^I > \pi_{m-1}^N \ (m > 0) \text{ and } \pi_m^N \geq \pi_{m+1}^I \ (m < n)$$

The first inequality specifies the equilibrium condition for a firm that invests; the second the equilibrium condition for a firm that does not invest. Let

$$\begin{aligned} c_m &= \frac{(n - m) c_l + m c_h}{n} \\ s_m &= \frac{(n - m) a (c_h - c_l)}{2n c_h (b + c_l)} \\ s &\in \left(0, \frac{a}{2b}\right) \end{aligned}$$

We first consider the two special cases $m = 0$ and $m = n$.

- When $m = 0$, none of the firms invest. This will be an equilibrium iff

$$\pi_0^N \geq \pi_1^I \iff \begin{cases} \frac{a^2+4c_h s(a-bs)}{4(b+c_h)} \geq \frac{a^2}{4(b+c_l)} + \frac{c_h s[(n-1)a-2nbs]}{(n-1)(b+c_h)} - F, \text{ if } s < s_1 \\ \frac{a^2+4c_h s(a-bs)}{4(b+c_h)} \geq \frac{a^2\left(bc_1^2+c_l c_h^2\right)+4c_l c_h s(a-bs)[b(2c_1-c_h)+c_l c_h]}{4(bc_1+c_l c_h)^2} - F, \text{ otherwise} \end{cases}$$

$$\iff F > F_1^{ct}(s) \equiv \begin{cases} F_1 - \frac{(n+1)bc_h s^2}{(n-1)(b+c_h)}, \text{ if } s < s_1 \\ \frac{a^2\left(bc_1^2+c_l c_h^2\right)+4c_l c_h s(a-bs)[b(2c_1-c_h)+c_l c_h]}{4(bc_1+c_l c_h)^2} - \frac{a^2+4c_h s(a-bs)}{4(b+c_h)}, \text{ otherwise} \end{cases}$$

- When $m = n$, all the firms invest. This will be an equilibrium iff

$$\pi_n^I > \pi_{n-1}^N \iff \begin{cases} \frac{a^2+4c_l s(a-bs)}{4(b+c_l)} - F > \frac{a^2}{4(b+c_h)} + \frac{c_h s[a+n(n-2)bs]}{b+c_h}, \text{ if } s < s_{n-1} \\ \frac{a^2+4c_l s(a-bs)}{4(b+c_l)} - F > \frac{a^2\left(bc_{n-1}^2+c_l^2 c_h\right)+4c_l c_h s(a-bs)[b(2c_{n-1}-c_l)+c_l c_h]}{4(bc_{n-1}+c_l c_h)^2}, \text{ otherwise} \end{cases}$$

$$\iff F < F_n^{ct}(s) \equiv \begin{cases} F_1 + \frac{c_l s(a-bs)}{b+c_l} - \frac{c_h s[a+n(n-2)bs]}{b+c_h}, \text{ if } s < s_{n-1} \\ \frac{a^2+4c_l s(a-bs)}{4(b+c_l)} - \frac{a^2\left(bc_{n-1}^2+c_l^2 c_h\right)+4c_l c_h s(a-bs)[b(2c_{n-1}-c_l)+c_l c_h]}{4(bc_{n-1}+c_l c_h)^2}, \text{ otherwise} \end{cases}$$

Now, consider the general case $0 < m < n$.

As we will shortly show, for any fixed s, the equilibrium condition for a firm that invests, i.e., $\pi_m^I > \pi_{m-1}^N$, translates into the condition $F < g(m, s)$, where g is some function of m and s.

Observe that the equilibrium condition for a firm that does not invest, $\pi_m^N \geq \pi_{m+1}^I$, can be written as $F \geq g(m+1, s)$. In other words,

$$M \text{ is a Nash equilibrium} \iff g(m+1, s) \leq F < g(m, s)$$

We will show that g is strictly decreasing in m and that $g(1, s) = F_1^{ct}(s)$ and $g(n, s) = F_n^{ct}(s)$. This means that there exists a unique equilibrium for every F and s. The set of functions $\{g(m, s),\ m = 1, ..., n\}$ partition the $\{s, F\}$ space in $m+1$ regions, one for each of the Nash equilibria.

From Theorem 4 of Chapter 2, we have

$$\pi_m^I = \begin{cases} \frac{a^2}{4(b+c_l)} + \frac{c_h s[(n-m)a-2nbs]}{(n-m)(b+c_h)} - F & \text{if } s < s_m \\ \frac{a^2\left(bc_m^2+c_l c_h^2\right)+4c_l c_h s(a-bs)[b(2c_m-c_h)+c_l c_h]}{4(bc_m+c_l c_h)^2} - F & \text{otherwise} \end{cases}$$

and

$$\pi_{m-1}^N = \begin{cases} \frac{a^2}{4(b+c_h)} + \frac{c_h s\left[(n-m+1)^2 a-n(n-2m+2)bs\right]}{(n-m+1)^2(b+c_h)} & \text{if } s < s_{m-1} \\ \frac{a^2\left(bc_{m-1}^2+c_l^2 c_h\right)+4c_l c_h s(a-bs)[b(2c_{m-1}-c_l)+c_l c_h]}{4(bc_{m-1}+c_l c_h)^2} & \text{otherwise} \end{cases}$$

Since $s_m < s_{m-1}$, the equilibrium conditions for a firm that invests are

$$F < \begin{cases} g_1(m,s), & \text{if } s < s_m \\ g_2(m,s), & \text{if } s_m \le s < s_{m-1} \\ g_3(m,s), & \text{otherwise} \end{cases}$$

where

$$\begin{aligned}
g_1(m,s) &= F_1 - \frac{bc_h s^2}{b+c_h}\left[\frac{n(n+2)}{(n-m+1)^2} + \frac{2n}{(n-m)(n-m+1)^2}\right] \\
g_2(m,s) &= \frac{a^2\left(bc_m^2 + c_l c_h^2\right) + 4c_l c_h s(a-bs)\left[b(2c_m - c_h) + c_l c_h\right]}{4(bc_m + c_l c_h)^2} \\
&\quad -\frac{a(a+4c_h s)}{4(b+c_h)} + \frac{n(n-2m+2)bc_h s^2}{(n-m+1)^2(b+c_h)} \\
g_3(m,s) &= \frac{a^2\left(bc_m^2 + c_l c_h^2\right) + 4c_l c_h s(a-bs)\left[b(2c_m - c_h) + c_l c_h\right]}{4(bc_m + c_l c_h)^2} \\
&\quad -\frac{a^2\left(bc_{m-1}^2 + c_l^2 c_h\right) + 4c_l c_h s(a-bs)\left[b(2c_{m-1} - c_l) + c_l c_h\right]}{4(bc_{m-1} + c_l c_h)^2}
\end{aligned}$$

Note that

$$\begin{aligned}
& g_1(m,s_m) = g_2(m,s_m) \\
= \; & \frac{a^2(c_h - c_l)}{4c_h(b+c_l)^2(b+c_h)} \\
& \times\left[b\frac{[2m(n-m+1)-n]c_h + (n-m)[(n-m)(n+2)+2]c_l}{n(n-m+1)^2} + c_l c_h\right] \\
& g_2(m,s_{m-1}) = g_3(m,s_{m-1}) \\
= \; & \frac{a^2 c_l(c_h - c_l)\left[\frac{bc_h(b+c_l)(c_h-c_l)}{n(bc_m+c_l c_h)^2} + \frac{2[c_l(b+c_h)-mc_h(b+c_l)]}{m(bc_m+c_l c_h)} + n + 2 - \frac{2}{m}\right]}{4nc_h(b+c_l)^2}
\end{aligned}$$

Define $g(m,s)$ as the concatenation of g_1, g_2, and g_3. The above establishes the continuity of g with respect to s.

It is easy to show that g_1 is strictly decreasing in m.[1] Next, we show that g_3 is also strictly decreasing in m.

Let

$$\begin{aligned}
G_i(m) &= \frac{bc_m^2 + c_l c_h c_i}{(bc_m + c_l c_h)^2}, \; i \in \{l,h\} \\
H_i(m) &= \frac{b(2c_m - c_i) + c_l c_h}{(bc_m + c_l c_h)^2}, \; i \in \{l,h\}
\end{aligned}$$

We have

$$g_3(m,s) = \frac{a^2}{4}G_h(m) + c_l c_h s(a-bs)H_h(m) - \left(\frac{a^2}{4}G_l(m) + c_l c_h s(a-bs)H_l(m)\right)$$

[1] $\frac{\partial g_1}{\partial m} = -\frac{2nbc_h s^2}{(b+c_h)(n-m+1)^3}\left[n + 2 + \frac{3(n-m)+1}{(n-m)^2}\right] < 0$

and

$$\begin{aligned}\frac{\partial G_i(m)}{\partial m} &= \frac{2bc_lc_h(c_h-c_l)(c_m-c_i)}{n(bc_m+c_lc_h)^3}\\ \frac{\partial H_i(m)}{\partial m} &= \frac{2b^2(c_h-c_l)(c_i-c_m)}{n(bc_m+c_lc_h)^3}\end{aligned}$$

Thus,

$$\begin{aligned}\frac{a^2}{4}\frac{\partial G_h(m)}{\partial m}+c_lc_hs(a-bs)\frac{\partial H_h(m)}{\partial m} &= -\frac{bc_lc_h(c_h-c_l)(c_h-c_m)(a-2bs)^2}{2n(bc_m+c_lc_h)^3}<0\\ \frac{a^2}{4}\frac{\partial G_l(m)}{\partial m}+c_lc_hs(a-bs)\frac{\partial H_l(m)}{\partial m} &= -\frac{bc_lc_h(c_h-c_l)(c_l-c_m)(a-2bs)^2}{2n(bc_m+c_lc_h)^3}>0\end{aligned}$$

and

$$\frac{\partial g_3(m,s)}{\partial m}<0$$

Finally, we need to show that $g_2(m,s)>g_3(m+1,s)$ for $s\in[s_m,s_{m-1}]$ and that $g_1(m,s)>g_2(m+1,s)$ for $s\in[s_{m+1},s_m]$. Observe that

$$\begin{aligned}\frac{\partial g_2(m,s)}{\partial s} &= \frac{c_lc_h(a-2bs)[b(2c_m-c_h)+c_lc_h]}{(bc_m+c_lc_h)^2}-\frac{c_h\left(a-2\frac{n(n-2m+2)}{(n-m+1)^2}bs\right)}{b+c_h}\\ \frac{\partial g_3(m,s)}{\partial s} &= c_lc_h(a-2bs)\left[\frac{b(2c_m-c_h)+c_lc_h}{(bc_m+c_lc_h)^2}-\frac{b(2c_{m-1}-c_l)+c_lc_h}{(bc_{m-1}+c_lc_h)^2}\right]\end{aligned}$$

From which we derive

$$\frac{\partial g_2(m,s)}{\partial s}-\frac{\partial g_3(m,s)}{\partial s}=\frac{c_lc_h(a-2bs)[b(2c_{m-1}-c_l)+c_lc_h]}{(bc_{m-1}+c_lc_h)^2}-\frac{c_h\left(a-2\frac{n(n-2m+2)}{(n-m+1)^2}bs\right)}{b+c_h}$$

Note that $n(n-2m+2)\leq(n-m+1)^2$. Thus, for $s\geq 0$,

$$\frac{\partial g_2(m,s)}{\partial s}-\frac{\partial g_3(m,s)}{\partial s}\leq c_h(a-2bs)\left[\frac{c_l[b(2c_{m-1}-c_l)+c_lc_h]}{(bc_{m-1}+c_lc_h)^2}-\frac{1}{b+c_h}\right]\leq 0$$

with equality only when $m=1$.

When $m=1$, $g_3(m+1,s)<g_3(m,s)=g_2(m,s)$ for any $s\in\left[0,\frac{a}{2b}\right)$.

When $m>1$, $g_3(m+1,s)<g_3(m,s)<g_2(m,s)$ because $g_3(m,s_{m-1})=g_2(m,s_{m-1})$ by continuity of g, and g_2 decreases faster than g_3 on $[s_m,s_{m-1}]$.

Next, we show that g_2 is also strictly decreasing in m.

$$\frac{\partial g_2(m,s)}{\partial m} = -\frac{bc_l c_h (c_h - c_m)(a-2bs)^2}{2n(bc_m + c_l c_h)^3} - \frac{2n(m-1)bc_h s^2}{(n-m+1)^2(b+c_h)} < 0$$

Thus, $g_2(m+1,s) < g_2(m,s)$. We conclude by showing that $g_1(m,s) \geq g_2(m,s)$ for $s \in [s_{m+1}, s_m]$. By continuity of g, we have $g_2(m,s_m) = g_1(m,s_m)$.

$$\begin{aligned} g_1(m,s) - g_2(m,s) &= -\frac{b}{(bc_m + c_l c_h)^2} \times \\ & \left[\begin{array}{c} \frac{(b+c_l)c_h^2\left[2m^2 b(c_h-c_l)+n(n+m)c_l(b+c_h)\right]}{(n-m)n(b+c_h)} s^2 \\ -\frac{ac_h(c_h-c_l)\left[m^2 b(c_h-c_l)+n^2 c_l(b+c_h)\right]}{n^2(b+c_h)} s \\ +\frac{(n-m)^2 a^2 c_l (c_h-c_l)^2}{4n^2(b+c_l)} \end{array} \right] \end{aligned}$$

$g_1 - g_2$ is in inverted parabola whose summit is

$$s^* = \frac{(n-m)a(c_h-c_l)\left[m^2 b(c_h-c_l) + n^2 c_l(b+c_h)\right]}{2nc_h(b+c_l)\left[2m^2 b(c_h-c_l) + n(n+m)c_l(b+c_h)\right]} < s_m$$

It has one real root at s_m and another one at some $s < 0$. Thus, $g_1 - g_2 \geq 0$ on $[s_{m+1}, s_m]$.

The verification that $g(1,s) = F_1^{ct}(s)$ and $g(n,s) = F_n^{ct}(s)$ is straightforward. Note in particular that $g_2(1,s) = g_3(1,s)$ and that $s_n = 0$.

The series of functions F_k^{ct} is the series $g(k,s)$. □

C.4 Proof of Theorem 13

We solve by backward induction.

From the proof of Theorem 5 of Chapter 2, we have firm $i's$ reaction function given τ and c_i as:

$$\begin{cases} q_i^* = \frac{a-\tau}{2b}, & x_i^* = \frac{b\tau}{c_i(a-\tau)}, \quad \text{for } 0 \leq \tau \leq \frac{ac_i}{b+c_i} \\ q_i^* = \frac{a}{2(b+c_i)}, & x_i^* = 1, \quad \text{otherwise} \end{cases}$$

This gives the solution to the third stage.

In the second stage,

$$\text{Firm } i \text{ will invest} \iff \pi_i^I > \pi_i^N$$

Note that the objective functions of firm i and $j \neq i$ are independent of each other. Since $c_l < c_h$, we have $\frac{ac_l}{b+c_l} < \frac{ac_h}{b+c_h}$, and there are three cases to consider:

1. $0 \leq \tau \leq \frac{ac_l}{b+c_l}$: In this case,

$$\begin{aligned} \pi_i^I &= \frac{(a-\tau)^2}{4b} + \frac{\tau^2}{4c_l} - F \\ \pi_i^N &= \frac{(a-\tau)^2}{4b} + \frac{\tau^2}{4c_h} \end{aligned}$$

$$\pi_i^I > \pi_i^N \iff F < \frac{(c_h - c_l)\tau^2}{4c_l c_h} \equiv F_l(\tau)$$

2. $\frac{ac_l}{b+c_l} < \tau \leq \frac{ac_h}{b+c_h}$: In this case,

$$\begin{aligned} \pi_i^I &= \frac{a^2}{4(b+c_l)} - F \\ \pi_i^N &= \frac{(a-\tau)^2}{4b} + \frac{\tau^2}{4c_h} \end{aligned}$$

$$\pi_i^I > \pi_i^N \iff F < \frac{a^2}{4(b+c_l)} - \frac{(a-\tau)^2}{4b} - \frac{\tau^2}{4c_h} \equiv F_h(\tau)$$

3. $\tau > \frac{ac_h}{b+c_h}$: In this case,

$$\begin{aligned} \pi_i^I &= \frac{a^2}{4(b+c_l)} - F \\ \pi_i^N &= \frac{a^2}{4(b+c_h)} \end{aligned}$$

$$\pi_i^I > \pi_i^N \iff F < F_1$$

Define

$$f(\tau) = \begin{cases} F_l(\tau) & \text{for } 0 \leq \tau \leq \frac{ac_l}{b+c_l} \\ F_h(\tau) & \text{for } \frac{ac_l}{b+c_l} < \tau \leq \frac{ac_h}{b+c_h} \\ F_1 & \text{otherwise} \end{cases}$$

Let $F_0 = \frac{a^2 c_l (c_h - c_l)}{4c_h (b+c_l)^2}$, and recall that $F_1 = \frac{a^2(c_h-c_l)}{4(b+c_l)(b+c_h)}$. f is continuous differentiable, increasing and positive on $[0,+\infty)$. It is useful to consider its inverse f^{-1}. It is easy to show that

$$f^{-1}(F) = \begin{cases} f_1^{-1}(F) = 2\sqrt{\frac{c_l c_h F}{c_h - c_l}}, & \text{for } 0 \leq F \leq F_0 \\ f_2^{-1}(F) = \frac{ac_h}{b+c_h} - 2\sqrt{\frac{bc_h}{b+c_h}(F_1 - F)}, & \text{for } F_0 < F \leq F_1 \end{cases}$$

See Figure C.1 for a plot of the function f^{-1}.

$$\begin{aligned} \text{The firm invests} &\iff F < f(\tau) \\ &\iff \tau > f^{-1}(F) \end{aligned}$$

We conclude with the regulator's problem to find the minimum τ such that the pollution generated by the firms is less than S.

Suppose that $F \in [0, F_0]$. In that case, $f^{-1} = f_1^{-1}$ and $0 \leq f_1^{-1}(F) \leq \frac{ac_l}{b+c_l}$.

If $\tau > f_1^{-1}(F)$, the firms invest (*i.e.*, $c_i = c_l$). We need to consider the following two sub-cases:

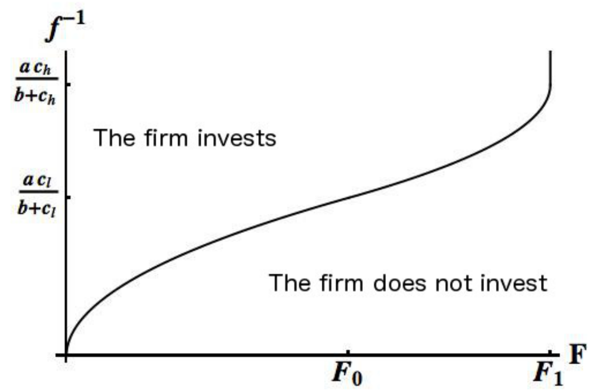

Figure C.1. Graph of f^{-1}

1. $\tau > \frac{ac_l}{b+c_l}$. The firms' reaction function is $q_i = \frac{a}{2(b+c_l)}$ and $x_i = 1$. The pollution generated is 0 (the firms abate all their pollution), and the minimum tax rate that satisfies the pollution constraint is simply $\tau^* = \frac{ac_l}{b+c_l}$.

2. $f_1^{-1}(F) < \tau \leq \frac{ac_l}{b+c_l}$. The firms' reaction function is $q_i = \frac{a-\tau}{2b}$ and $x_i = \frac{b\tau}{c_l(a-\tau)}$. The pollution generated is $P(\tau) = n \cdot \frac{ac_l-(b+c_l)\tau}{2bc_l}$, which is strictly decreasing in τ. The minimum τ such that $P(\tau) \leq S$ is

$$P(\tau^*) = S \iff \tau^* = \frac{c_l(a-2bs)}{b+c_l}, \text{ if } \tau^* > f_1^{-1}(F)$$

and $\tau^* = f_1^{-1}(F)$, otherwise. The condition

$$\tau^* > f_1^{-1}(F) \iff F < \frac{c_l(c_h-c_l)(a-2bs)^2}{4c_h(b+c_l)^2} \equiv F^t(s)$$

When $F < F^t$, $\tau^* = \frac{c_l(a-2bs)}{b+c_l}$ and the firm reacts with $q_i = \frac{a+2c_ls}{2(b+c_l)}$ and $x_i = \frac{a-2bs}{a+2c_ls}$. When $F^t \leq F \leq F_0$, $\tau^* = f_1^{-1}(F) = 2\sqrt{\frac{c_lc_hF}{c_h-c_l}}$ and the firm reacts with $q_i = \frac{1}{2b}\left(a-2\sqrt{\frac{c_lc_hF}{c_h-c_l}}\right)$ and $x_i = 2b\frac{a\sqrt{c_h(c_h-c_l)F/c_l}+2c_hF}{a^2(c_h-c_l)-4c_lc_hF}$. Note that in this case, $P(\tau^*) = \frac{1}{b}\left(a-2(b+c_l)\sqrt{\frac{c_hF}{c_l(c_h-c_l)}}\right)$, which is decreasing in F. When $F > F^t$, $P(\tau^*) < P(F^t) = S$. **The pollution constraint is slack.**

If $\tau \leq f_1^{-1}(F)$, the firm does not invest, and since $\tau \leq \frac{ac_l}{b+c_l} < \frac{ac_h}{b+c_h}$, the firm's reaction function is $q_i = \frac{a-\tau}{2b}$ and $x_i = \frac{b\tau}{c_h(a-\tau)}$. The pollution generated is $P(\tau) = n \cdot \frac{ac_h-(b+c_h)\tau}{2bc_h}$. The minimum τ such that $P(\tau) \leq S$ is $\tau^* = \frac{c_h(a-2bs)}{b+c_h}$.

The condition

$$\tau^* \leq f_1^{-1}(F) \iff F \geq \frac{c_h(c_h - c_l)(a-2bs)^2}{4c_l(b+c_h)^2} \equiv F^{T1}(s)$$

The firms react with $q_i = \frac{a+2c_h s}{2(b+c_h)}$ and $x_i = \frac{a-2bs}{a+2c_h s}$.

Note that for $F \in [0, F_0]$,

$$\begin{aligned} \frac{c_l(a-2bs)}{b+c_l} &\leq \min\left\{\frac{c_h(a-2bs)}{b+c_h}, \frac{ac_l}{b+c_l}\right\} \\ f_1^{-1}(F) &\leq \frac{ac_l}{b+c_l} \end{aligned}$$

This means that the regulator will choose $\tau^* = \frac{c_l(a-2bs)}{b+c_l}$ when $F < F^t$.

When $F^t \leq F \leq F_0$, the regulator will choose $\tau^* = \min\left\{f_1^{-1}(F), \frac{c_h(a-2bs)}{b+c_h}\right\}$.

In other words, (as we have just established), she will choose

- $\tau^* = f_1^{-1}(F) \iff F^t \leq F \leq \min\{F^{T1}, F_0\}$;
- $\tau^* = \frac{c_h(a-2bs)}{b+c_h} \iff F^{T1} \leq F \leq F_0$.

Suppose now that $F \in (F_0, F_1]$. In that case, $f^{-1} = f_2^{-1}$ and $\frac{ac_l}{b+c_l} < f_2^{-1}(F) \leq \frac{ac_h}{b+c_h}$.

If $\tau > f_2^{-1}(F)$, the firms invest, and since $\tau > \frac{ac_l}{b+c_l}$, the firms' reaction function is $q_i = \frac{a}{2(b+c_l)}$ and $x_i = 1$.

The pollution generated is 0 (the firms abate all their pollution), and the minimum tax rate that satisfies the pollution constraint is simply $\tau^* = f_2^{-1}(F) = \frac{ac_h}{b+c_h} - 2\sqrt{\frac{bc_h}{b+c_h}(F_1 - F)}$.

If $\tau \leq f_2^{-1}(F)$ the firms do not invest, and since $\tau \leq \frac{ac_h}{b+c_h}$, the firms' reaction function is $q_i = \frac{a-\tau}{2b}$ and $x_i = \frac{b\tau}{c_h(a-\tau)}$.

The pollution generated is $P(\tau) = n \cdot \frac{ac_h - (b+c_h)\tau}{2bc_h}$, and $\tau^* = \frac{c_h(a-2bs)}{b+c_h}$.

The condition

$$\tau^* \leq f_2^{-1}(F) \iff F \geq \frac{a^2(c_h - c_l)}{4(b+c_l)(b+c_h)} - \frac{bc_h s^2}{b+c_h} \equiv F^{T2}(s)$$ [2]

Thus, the regulator will choose

$$\tau^* = \begin{cases} f_2^{-1}(F) & \text{if } F_0 < F \leq F^{T2} \\ \frac{c_h(a-2bs)}{b+c_h} & \text{if } F > \max\{F_0, F^{T2}\} \end{cases}$$

When the regulator chooses $\tau^* = f_2^{-1}(F)$, the firms respond with $q_i = \frac{a}{2(b+c_l)}$ and $x_i = 1$.

[2] F_{T2} is greater than F_t because the former is always greater than F_0 while the latter is always smaller than F_0.

When the regulator chooses $\tau^* = \frac{c_h(a-2bs)}{b+c_h}$, the firms respond with $q_i = \frac{a+2c_h s}{2(b+c_h)}$ and $x_i = \frac{a-2bs}{a+2c_h s}$.

Finally, since $F \in (F_0, F_1]$, then $s \in \left[0, \frac{a(c_h-c_l)}{2c_h(b+c_l)}\right)$.

Let

$$F^T(s) = \begin{cases} F^{T2}(s) & \text{for } 0 \leq s \leq \frac{a(c_h-c_l)}{2c_h(b+c_l)} \\ F^{T1}(s) & \text{for } \frac{a(c_h-c_l)}{2c_h(b+c_l)} \leq s \leq \frac{a}{2b} \end{cases}$$

It is straightforward to show that F^T is continuous, decreasing, and positive. In particular, we have shown that the firm invests if and only if $F < F^T$, and that when $F^t < F < F^T$, the pollution constraint is slack. $\square$

C.5 Proof of Theorem 14

We need to show that $\forall\ s \in \left(0, \frac{a}{2b}\right)$

$$F^t(s) < F_n^{ct}(s) < F^{cap}(s) < F_1^{ct}(s) < F^T(s)$$

We begin by showing that $\forall\ s \in \left(0, \frac{a}{2b}\right)$

$$F^t(s) < F_n^{ct}(s)$$

Recall that

$$F^t(s) = \frac{c_l(c_h-c_l)(a-2bs)^2}{4c_h(b+c_l)^2}$$

On the interval $(0, s_{n-1})$ where $s_{n-1} = \frac{a(c_h-c_l)}{2nc_h(b+c_l)}$

$$F_n^{ct}(s) = \frac{a^2}{4(b+c_l)(b+c_h)} + \frac{c_l s(a-bs)}{b+c_l} - \frac{c_h s[a+n(n-2)bs]}{b+c_h}$$

We will show that $F_n^{ct} - F^t$ is strictly decreasing for $s \in (0, s_{n-1})$, which implies that

$$\begin{gathered}\left(F_n^{ct} - F^t\right)(s) > \left(F_n^{ct} - F^t\right)(s_{n-1}) = \\ \frac{a^2 b c_l (c_h-c_l)^2 [b(c_l + 2(n-1)c_h) + (2n-1)c_l c_h]}{4n^2 c_h^3 (b+c_l)^4} > 0\end{gathered}$$

$$\begin{aligned}\left(F_n^{ct} - F^t\right)'(s) &= -\frac{b}{c_h(b+c_l)^2(b+c_h)} \times \\ &\quad \left[ab(c_h-c_l)^2 + 2s\left[(n-1)^2 c_l c_h(2bc_h + c_l c_h) + b^2\left(c_l(2c_h-c_l) + n(n-2)c_h^2\right)\right]\right] \\ &< 0\end{aligned}$$

On the interval $\left[s_{n-1}, \frac{a}{2b}\right)$

$$F_n^{ct}(s) = \frac{c_l(c_h - c_l)\left[n^2 c_l c_h + b\left(c_l + \left(n^2-1\right)c_h\right)\right](a-2bs)^2}{4(b+c_l)\left[b(c_l+(n-1)c_h)+nc_lc_h\right]^2}$$

Likewise, we show that $F_n^{ct} - F^t$ is strictly decreasing for $s \in \left(s_{n-1}, \frac{a}{2b}\right)$, which implies that

$$\left(F_n^{ct} - F^t\right)(s) > \left(F_n^{ct} - F^t\right)\left(\frac{a}{2b}\right) = 0$$

$$\left(F_n^{ct} - F^t\right)'(s) = -\frac{b^2 c_l (c_h - c_l)^2 \left[b(c_l + 2(n-1)c_h) + (2n-1)c_lc_h\right](a-2bs)}{c_h(b+c_l)^2\left[b(c_l+(n-1)c_h)+nc_lc_h\right]^2} < 0$$

Next, we show that $\forall\ s \in \left(0, \frac{a}{2b}\right)$

$$F_n^{ct}(s) < F^{cap}(s)$$

Recall that

$$F^{cap}(s) = \frac{(c_h - c_l)(a-2bs)^2}{4(b+c_l)(b+c_h)}$$

On the interval $(0, s_{n-1})$

$$F_n^{ct}(s) = \frac{a^2}{4(b+c_l)(b+c_h)} + \frac{c_l s(a-bs)}{b+c_l} - \frac{c_h s\left[a + n(n-2)bs\right]}{b+c_h}$$

We have

$$F^{cap}(s) - F_n^{ct}(s) = \frac{(n-1)^2 bc_h s^2}{b+c_h} > 0$$

On the interval $\left(s_{n-1}, \frac{a}{2b}\right)$

$$F_n^{ct}(s) = \frac{c_l(c_h - c_l)\left[n^2 c_l c_h + b\left(c_l + \left(n^2-1\right)c_h\right)\right](a-2bs)^2}{4(b+c_l)\left[b(c_l+(n-1)c_h)+nc_lc_h\right]^2}$$

We have

$$F^{cap}(s) - F_n^{ct}(s) = \frac{(n-1)^2 bc_h(c_h-c_l)^2(a-2bs)^2}{4(b+c_h)\left[b(c_l+(n-1)c_h)+nc_lc_h\right]^2} > 0$$

Next, we show that $\forall\ s \in \left(0, \frac{a}{2b}\right)$

$$F^{cap}(s) < F_1^{ct}(s)$$

On the interval $(0, s_1)$ where $s_1 = \frac{(n-1)a(c_h-c_l)}{2nc_h(b+c_l)}$

$$F_1^{ct}(s) = \frac{a^2(c_h-c_l)}{4(b+c_l)(b+c_h)} - \frac{(n+1)bc_hs^2}{(n-1)(b+c_h)}$$

We have

$$\begin{aligned} F_1^{ct}(s) - F^{cap}(s) = & \frac{bs}{(n-1)(b+c_l)(b+c_h)} \times \\ & \left[(n-1)a(c_h-c_l) - \left[(n+1)c_lc_h + b\left(c_l + n(2c_h - c_l)\right)\right]s\right] \end{aligned}$$

$$
\begin{aligned}
F_1^{ct}(s) - F^{cap}(s) &> 0 \\
\Longleftrightarrow s &< \frac{(n-1)\,a\,(c_h - c_l)}{(n+1)\,c_l c_h + b\,(c_l + n\,(2c_h - c_l))}
\end{aligned}
$$

This is true because

$$
s_1 = \frac{(n-1)\,a\,(c_h - c_l)}{2nc_h\,(b + c_l)} < \frac{(n-1)\,a\,(c_h - c_l)}{(n+1)\,c_l c_h + b\,(c_l + n\,(2c_h - c_l))}
$$

On the interval $\left(s_1, \frac{a}{2b}\right)$

$$
F_1^{ct}(s) = \frac{c_h\,(c_h - c_l)\left[n^2 c_l c_h + b\left(\left(n^2 - 1\right)c_l + c_h\right)\right](a - 2bs)^2}{4\,(b + c_h)\,[b\,((n-1)\,c_l + c_h) + nc_l c_h]^2}
$$

We have

$$
F_1^{ct}(s) - F^{cap}(s) = \frac{(n-1)^2\,bc_l\,(c_h - c_l)\,(a - 2bs)^2}{4\,(b + c_l)\,[b\,((n-1)\,c_l + c_h) + nc_l c_h]^2} > 0
$$

We conclude with the proof that $\forall\ s \in \left(0, \frac{a}{2b}\right)$

$$
F_1^{ct}(s) < F^T(s)
$$

There are three cases:

1. On the interval $(0, s_1)$

$$
\begin{aligned}
F_1^{ct}(s) &= \frac{a^2\,(c_h - c_l)}{4\,(b + c_l)\,(b + c_h)} - \frac{(n+1)\,bc_h s^2}{(n-1)\,(b + c_h)} \\
F^T(s) &= \frac{a^2\,(c_h - c_l)}{4\,(b + c_l)\,(b + c_h)} - \frac{bc_h s^2}{b + c_h}
\end{aligned}
$$

   It follows immediately that

$$
F_1^{ct}(s) < F^T(s)
$$

2. On the interval (s_1, s_{n-1})

$$
\begin{aligned}
F_1^{ct}(s) &= \frac{c_h\,(c_h - c_l)\left[n^2 c_l c_h + b\left(\left(n^2 - 1\right)c_l + c_h\right)\right](a - 2bs)^2}{4\,(b + c_h)\,[b\,((n-1)\,c_l + c_h) + nc_l c_h]^2} \\
F^T(s) &= \frac{a^2\,(c_h - c_l)}{4\,(b + c_l)\,(b + c_h)} - \frac{bc_h s^2}{b + c_h}
\end{aligned}
$$

F^T is strictly concave[3], while F_1^{ct} is strictly convex[4]. Thus, $F^T - F_1^{ct}$ is strictly concave, which implies that on the interval (s_1, s_{n-1})

$$\left(F^T - F_1^{ct}\right)(s) \geq \min\left\{\left(F^T - F_1^{ct}\right)(s_1),\ \left(F^T - F_1^{ct}\right)\left(\frac{a\,(c_h - c_l)}{2c_h\,(b + c_l)}\right)\right\}$$

$$\Rightarrow \quad \left(F^T - F_1^{ct}\right)(s) \geq \min\left\{\begin{array}{c} \frac{(n-1)a^2 b (c_h - c_l)^2}{2n^2 c_h (b+c_h)(b+c_l)^2}, \\ \frac{a^2 b c_l (c_h - c_l)^2 [b(2(n-1)c_l + c_h) + (2n-1)c_l c_h]}{4c_h (b+c_l)^2 [b((n-1)c_l + c_h) + n c_l c_h]^2} \end{array}\right\} > 0$$

3. On the interval $\left(s_{n-1}, \frac{a}{2b}\right)$

$$\begin{aligned} F_1^{ct}(s) &= \frac{c_h\,(c_h - c_l)\left[n^2 c_l c_h + b\left(\left(n^2 - 1\right)c_l + c_h\right)\right](a - 2bs)^2}{4\,(b + c_h)\left[b\left((n-1)\,c_l + c_h\right) + n c_l c_h\right]^2} \\ F^T(s) &= \frac{c_h\,(c_h - c_l)\,(a - 2bs)^2}{4c_l\,(b + c_h)^2} \end{aligned}$$

The condition

$$\begin{aligned} & F_1^{ct}(s) < F^T(s) \iff \\ & b\,(c_h - c_l)\left[(2n-1)\,c_l c_h + 2\,(n-1)\,b c_l + b c_h\right] > 0 \end{aligned}$$

which is always true. $\square$

[3] $F^{T\prime\prime}(s) = -\frac{2bc_h}{b+c_h} < 0$

[4] $F_1^{ct\prime\prime}(s) = \frac{2b^2 c_h (c_h - c_l)\left[b\left(\left(n^2-1\right)c_l + c_h\right) + n^2 c_l c_h\right]}{(b+c_h)[b((n-1)c_l + c_h) + n c_l c_h]^2} > 0$

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