NINTH-GRADE STUDENTS WITH DISABILITIES: A CONCRETE-REPRESENTATIONAL-ABSTRACT + WRITING STRATEGY FOR SOLVING RATE OF CHANGE PROBLEMS

by

Kaitlin A. Bundock

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STATEMENT OF DISSERTATION APPROVAL

The dissertation of **Kaitlin A. Bundock**

has been approved by the following supervisory committee members:

Leanne S. Hawken	, Chair	**10/5/2015** Date Approved
Sharlene A. Kiuhara	, Chair	**10/5/2015** Date Approved
Robert E. O'Neill	, Member	**10/5/2015** Date Approved
Breda V. O'Keeffe	, Member	**10/5/2015** Date Approved
Margarita B. Cummings	, Member	**10/5/2015** Date Approved

and by **Robert E. O'Neill** , Chair/Dean of the Department/College/School of **Special Education**

and by David B. Kieda, Dean of The Graduate School.

ABSTRACT

The U.S. finishes in the bottom fifth of industrialized nations in math achievement, based on the Program for International Student Assessment (PISA) scores. The National Assessment of Educational Progress (NAEP) classifies almost 10% of U.S. students as low achieving, and students with disabilities score particularly poorly on such assessments. Experts describe U.S. students as lacking conceptual understanding and requiring remedial instruction in math. When implemented across multiple grade and ability levels, math instruction incorporating a concrete-representational-abstract (CRA) sequence has increased math achievement. Writing To Learn Math (WTLM) is a strategy proven through research to improve students' conceptual understanding through writing. CRA and WTLM have similar cognitive foundations, yet no studies have evaluated a combination of CRA and WTLM. Combining CRA and WTLM has the potential to address the challenges of adjusting to the national Common Core standards and assessments, which include improving conceptual understanding and writing across all content areas. This unique combination of interventions could offer promising results for effective curriculum development and remedial instruction.

This study included three ninth-grade students from a suburban school who are below state proficiency levels in math, and employed a single-subject across-participants design to investigate the following research questions: (1) What is the effect of implementing a concrete-representational-abstract (CRA) instructional sequence incorporating writing to learn math strategies on students with disabilities' proficiency in solving rate of change

problems, and (2) Do students with disabilities find WTLM math and a CRA instructional sequence to be socially acceptable?

Results indicated that the CRA + Writing intervention may be effective in improving students' with disabilities understanding of rate of change. All 3 students improved their scores on the math items of the rate of change probes, and maintained these improvements on maintenance assessments administered between 1 and 7 weeks following the completion of the intervention. Two of the 3 students also displayed moderate improvements in their scores on the writing items of the rate of change probes. The findings of this study provide multiple implications for both research and practice, as well as several directions for future research.

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CHAPTER 1

REVIEW OF THE LITERATURE

Despite governmental reforms geared towards improving the educational outcomes of students in public education in the United States, such as the No Child Left Behind Act and the Individuals with Disabilities Education Act, evidence indicates low performance in mathematics nationwide (Maccini et al., 2007). The U.S. ranks 26th out of the 34 most industrialized nations in math achievement, according to data obtained from a test administered by the Organization for Economic Cooperation and Development, the Program for International Student Assessment (PISA) (Ryan, 2013). Based on the 2013 results of the National Assessment of Educational Progress (NAEP), 39% of eighth-graders are proficient in mathematics (National Center for Education Statistics [NCES], 2013), and only 26% of students in 12th-grade were identified as proficient on the 2011 NAEP (Schmidt & Burroughs, 2013). Additionally, 21% of eighth-graders are below even a basic level of proficiency (NCES, 2013). There are also severe discrepancies in math achievement between specific subpopulations in the U.S., including students with disabilities, English Language Learners (ELLs), students from low socioeconomic backgrounds, and students of color (Bryant, 2005; Butler et al., 2003; Jitendra & Star, 2011; Maccini, Mulcahy, & Wilson, 2007; Schmidt & Burroughs, 2013).

Students with disabilities are among the lowest achieving subpopulations in the U.S., with only 8% of eighth-graders with disabilities at or above proficiency and 65% below even a basic level of math understanding (NCES, 2013). Approximately 25% of students with disabilities can count with one to one correspondence to 10, and only 4-8% of this population can perform basic math computations (addition, subtraction, multiplication, and division) to solve applied problems (Saunders, Bethune, Spooner, & Browder, 2013). About 5-8% of students in the United States have learning disabilities in math (Bryant, 2005; Fuchs et al., 2008; Jitendra & Star, 2011; Judge & Watson, 2011). Students identified with a learning disability in mathematics consistently have low rates of achievement, with 95% ranking in the lowest 25th percentile on standardized assessments throughout all grade levels (Gersten et al., 2012; Judge & Watson, 2011). Recent research demonstrates, however, that students with moderate and even severe disabilities can learn grade level content when instructed on basic numeracy (Saunders, Bethune, Spooner, & Browder, 2013).

Two main contributors to lower math performance have been isolated: 1) inadequate remedial instruction for students who are behind, and 2) a disconnect between the conceptual and algorithmic components of mathematics that results from algorithmic components being taught in isolation instead of as part of a complete conceptual package (Bryant, 2005). Research has demonstrated inadequate special education instruction in mathematics across all settings (Hammrich, 2001). In self-contained and resource settings for students with mild to moderate disabilities, there is a strong emphasis on basic skills, remediation, and fact drilling (Hord & Bouck, 2012). These approaches leave little time and resources to dedicate to even intermediate-level mathematics instruction, and often leave out conceptual connections that may help struggling learners gain a better grasp on the concepts (Bryant, 2005; Hammrich, 2001).

The results of a recent metaanalysis of mathematics instruction for students with disabilities indicate that the following mathematics instructional components result in significant positive effects: explicit instruction, heuristics, visual representations, student verbalizations of their problem-solving processes, teachers providing specific and ongoing feedback, and cross-age tutoring (Gersten, Chard et al., 2009). The only two strategies for which significant positive effects were not present were peer-assisted learning within a class and student feedback with goal setting (Gersten, Chard et al., 2009). Gersten, Chard et al. (2009) indicate that the two strategies with both practical and statistical significance from their study are explicit instruction and the use of heuristics. Additionally, the Institute for Education Sciences recommends that educators follow eight practices when intervening to improve the mathematics performance of students who struggle: universally screen students in mathematics; focus on whole numbers in grades K-5 and rational numbers in grades 4-8; provide explicit and systematic instruction that includes modeling, verbalization, guided practice, review, and feedback; provide instruction on solving word problems; include visual representations of mathematics; include approximately 10 minutes of instruction on basic fact retrieval per intervention session; frequently monitor the progress of students receiving interventions; and incorporate motivational strategies for students receiving interventions (Gersten, Beckmann et al., 2009). Based on these research recommendations, it is clear that educators must focus on providing explicit instruction, developing students' conceptual understanding of mathematics through the use of visual representations, and also provide frequent opportunities for students to build fluency with basic mathematics skills. These goals may seem difficult to achieve, based on limitations of time and resources often found in schools.

Mathematics education and interventions must be approached with the same sense of urgency as reading is approached to effectively address students' learning difficulties

(Fuchs et al., 2008). Proficiency in mathematics is essential for success in not only academics, but also daily living (Hodge, Riccomini, Buford, & Herbst, 2006). Math problem-solving is a necessary prerequisite skill for students to perform well in other courses, such as chemistry and physics. Math is also vital for individuals to know how to use public transportation, shop for food and other essential items, and manage personal finances. While the National Council for Teaching Mathematics (NCTM) has emphasized changes in the teaching of mathematics to provide more of a balance between algorithmic, skills-based instruction and applied inquiry-based learning, there is a lack of research on mathematics instruction and intervention for struggling learners (Bryant, 2005). As our world becomes more technologically advanced, students will need to have stronger applied mathematical problem-solving skills and technological literacy in order to access information and opportunities (Fuchs et al., 2008). For example, mathematical problem-solving skills are vital for success in many careers, including careers in science, finance, and construction. Students who have proficient mathematical problem-solving skills have more options available to them following high school than students who lack proficiency in math problem-solving skills.

It is important to address low math achievement because math knowledge is a gateway to not only higher education, but also career success and economic rights (National Mathematics Advisory Panel (NMAP), 2008; Rasmussen et al., 2011). Recently, many states have increased high school mathematics requirements, based on the need for students to achieve college and career readiness by the time they graduate, as well as ties between higher mathematics achievement and college success (Rasmussen et al., 2011). Underperformance in mathematics results in lower confidence, less equity in employment, and fewer opportunities in society (Bell & Norwood, 2010; Fullerton, 1995; Fuchs et al., 2008). Each of these implications is exacerbated for groups of students who consistently perform at disproportionately lower levels in mathematics, including students with disabilities and from diverse backgrounds (Bryant, 2005; Butler et al., 2003; Maccini et al., 2007). With a significant proportion of workers expected to retire from science, technology, and engineering fields within the upcoming three decades, it is essential that this inadequacy be addressed in order for the U.S. to maintain its current economic standing (NMAP, 2008). It is especially important to address math achievement, and find efficient and effective methods that can be used to raise the math achievement of secondary students with disabilities in the current climate of public education in the U.S.

The Emergence of the Common Core State Standards

The National Governors Association (NGA) and Council of Chief State School Officers (CCSSO) established the Common Core State Standards Initiative (CCSSI) in 2009 to create academic expectations that would not only advance educational achievement in the U.S., but also decrease variability in indicators of academic proficiency across states (Dingman, Teuscher, Newton, & Kasmer, 2013). A final report of the standards, created by state representatives, content area experts, and educators, was released in 2010. As of March 2013, 45 states, four territories, the District of Columbia, and the U.S. Department of Defense Education Activity have adopted the Common Core Standards (Dingman et al., 2013). The broad adoption of these standards dramatically shifts how math education is approached in the U.S., creating a need for new research aligned with the key elements of the Common Core Standards.

Key Changes Made in the Common Core

The Common Core State Standards for Mathematics (CCSSM) change how math is taught in four main ways: (a) the grade level at which math content is taught; (b) the number of grade levels certain topics span; (c) shifts in emphasis on certain math topics; and (d) the addition of Standards for Mathematical Practice (SMP), which emphasize problem-solving across the standards (Dingman et al., 2013; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2012). The shift in embedding math reasoning throughout all standards and grade levels, as well as the push for assessments to evaluate deeper levels of student understanding, will provide challenges for teachers (Alberti, 2013; Dingman et al., 2013; Rothman, 2012b). Research is needed to help determine effective ways to incorporate the SMP and prepare students to demonstrate the knowledge required on new assessments. Due to greater emphasis on certain math concepts in the CCSSM, it is important that researchers determine effective methods of teaching high priority topics.

Changes in Concepts Taught at Each Grade Level

In order to be ready to master higher level algebra by the time students complete high school, students must be exposed to math concepts that are critical foundations for algebra at earlier grades. The CCSSM attempt to fulfill this need by changing the grade level at which certain math concepts are taught. When compared with previous state standards, the CCSSM move foundational concepts, including basic computation facts and operations with fractions to earlier grade levels. One of the biggest changes occurs with addition and subtraction of fractions, which are moved to earlier grades for 40 out of 42 states whose standards were compared to the CCSSM (Dingman et al., 2013; National Governors

Association Center for Best Practices & Council of Chief State School Officers, 2012). Shifting essential concepts to lower grades may provide students with a stronger foundation for algebra, as it allows teachers to provide more in depth instruction on concepts, and builds in more time for students to practice and master these skills (Dingman et al., 2013; Schmidt & Borroughs, 2013).

Changes in the Number of Grade Levels Topics Span

A second change that is made with implementation of the CCSSM is the number of grade levels certain math topics range. This change is primarily influenced by the goal of reducing the number of standards taught at each grade level in order to provide more in depth focus on the math content included within each standard (Schmidt & Houang, 2012). When compared with previous state standards, the CCSSM increase the number of grade levels in which addition, subtraction, multiplication, and division of whole numbers are addressed by an average of two grade levels (Dingman et al., 2013; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2012). The number of grade levels at which some concepts are taught are also decreased. One of the main concepts receiving decreased grade level coverage is addition and subtraction of fractions, for which the majority of states allotted at least three grade levels. The CCSSM reduce coverage of addition and subtraction of fractions to two grade levels (Dingman et al., 2013; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2012). In sum, the changes regarding the number of grade levels topics span made in the CCSSM aim to increase focus on important math concepts in order to provide a more solid foundation for higher level math.

Shifts in Emphasis on Certain Math Concepts

The third main change identified with the switch to CCSSM is a change in emphasis on specific math concepts (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2012). Dingman et al. (2013) discuss this change occurring in two main ways. First, when compared to some states' standards, the CCSSM decreases the overall emphasis on specific math concepts such as patterning and measurement. Additionally, the CCSSM moves the emphasis on some concepts, such as statistics and measurement, to higher grade levels. The second way changes in emphasis occur is through increasing emphasis on certain topics, both overall as well as within specific grade levels. Algebra is the main math concept that receives additional emphasis under the CCSSM, which is consistent with the National Mathematics Advisory Panel's call for students to master algebra by the end of high school (NMAP, 2008). Rate of change, or slope, is one algebraic concept that is heavily emphasized in the CCSSM. Rate of change is first introduced in seventh-grade, and spans all five of the high school domain areas in the CCSSM, which include algebra, functions, modeling, geometry, and statistics and probability (National Governors Association Center for Best Practice & Council of Chief State School Officers, 2012b). Specific math concepts, including working with mathematical properties and relationships between operations, both of which are foundations for algebra, also have increased emphasis under the CCSSM (Dingman et al., 2013).

The Standards for Mathematical Practice (SMP)

The final, and perhaps most noticeable change from previous state standards and the CCSSM is the addition of the SMP, which emphasize approaches towards thinking mathematically across all grade levels (National Governors Association Center for Best

Practices & Council of Chief State School Officers, 2012; Schifter & Granofsky, 2012). Prior to the CCSSM, concepts captured within the SMP, such as reasoning and problem-solving, were isolated to specific grade levels. The incorporation of eight standards for mathematical practice therefore represents a big shift in math standards (Dingman et al., 2013). The SMP outline the following expectations for how students interact with mathematics:

> Make sense of problems and persevere in solving them….Reason abstractly and quantitatively….Construct viable arguments and critique the reasoning of others…. Model with mathematics…. Use appropriate tools strategically…. Attend to precision….Look for and make use of structure…. Look for and express regularity in repeated reasoning. (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2012, Standards for Mathematical Practice, paras. 3-10)

Each of the SMP focuses on students developing and being able to express conceptual understanding of math, as well as higher level mathematical thinking. New assessments aligned to the CCSSM relate directly to the SMP, as portions of these exams require students to solve open-ended problems and explain their reasoning (Dingman, et al., 2013; Hakuta, Santos, & Fang, 2013; NMAP, 2008; Rothman, 2012b). By embedding the SMP within specific content standards, teachers may be able to help their students build new ways of thinking mathematically that can help students successfully approach unique and complex problems.

Common Core and U.S. Math Achievement

Early research conducted on the CCSSM indicates that the changes likely place the U.S. on the right track to advance mathematics education. When compared with the standards of eight top scoring nations on the Third International Mathematics and Science Study (TIMSS), the CCSSM ranked well on the three criteria identified as being indicators of math achievement (Schmidt & Houang, 2012). The top eight scoring countries on the 1995

TIMSS included Singpore, Japan, Korea, Hong Kong, Flemish Belgium, and the Czech Republic (NMAP, 2008). Curriculum analysis conducted following the TIMSS indicates that criteria separating the top scoring nations from low scoring nations are focus, rigor, and coherence (Schmidt & Houang, 2012). Focus refers to the number of standards taught at each grade level. Nations with a greater degree of focus in their curriculum (i.e., fewer topics taught per grade level) tended to score higher on the TIMSS than nations with less focus (Schmidt & Houang, 2012). Nations with curricula that had a higher degree of rigor, referring to high expectations and high-level content, also scored better on the TIMSS. Coherence is indicated by standards that advance sequentially and hierarchically between grade levels in a logical way. Nations with a greater degree of coherence also scored better on the TIMSS than those with less coherence (Schmidt & Houang, 2012).

The development of the CCSSM took into account the characteristics of rigor, coherence, and focus, which are reflected in each of the four main changes previously discussed (Schifter & Granofsky, 2012; Schmidt & Houang, 2012). When compared to the standards from the top eight scoring nations on the TIMSS, the CCSSM have a high degree of similarity, which increases the likelihood of test score improvement (Schmidt & Houang, 2012). Despite this promising early research, educators will likely face difficulties switching over to the CCSSM (NCTM, 2013; Rothman, 2012a; Schifter & Granofsky, 2012; Schmidt & Burroughs, 2013).

Common Core and Instructional Practice

While initial evidence indicates that the Common Core State Standards set U.S. math education on the right course (Rothman, 2012a; Schmidt & Houang, 2012), teachers will need assistance with teaching to these standards, especially in regards to preparing students to explain their math reasoning on assessments (Dingman et al., 2013; Hakuta, Santos & Fang, 2013; NMAP, 2008; Rothman, 2012b). First, educators may struggle with the transition to assessments that are aligned with the changes in the CCSSM (Alberti, 2013; NCTM, 2013). Additionally, educators will likely struggle with determining how to effectively integrate the SMP into their everyday teaching practices (Alberti, 2013; Schmidt & Burroughs, 2013). The CCSSM reflect a higher degree of rigor than found in past standards (Dingman et al., 2013). Assessments will be accordingly aligned to the increased rigor of the CCSSM. In order to advance student understanding to meet the new demands of the CCSSM, educators will need to focus on improving students' conceptual understanding, procedural skill and fluency, and applied knowledge (Alberti, 2013). This may present challenges if educators are not familiar with many of the changes in grade level content and emphasis made in the CCSSM (Dingman et al., 2013; Schmidt & Burroughs, 2013).

It is essential that researchers and those responsible for preparing teachers invest in developing effective strategies for teaching content with increased emphasis, such as algebra. Interventions that emphasize connections between concrete, representational, and abstract mathematical concepts may be effective in improving the algebraic computation skills of students with disabilities (Bryant, 2005; Shayer & Adhami, 2007), which is particularly important because the CCSSM emphasizes proficient understanding of algebra for all students (Witzel, 2005). It is equally important that energy be invested in developing effective and efficient methods to teach math content moved to lower grades, as well as math content that will receive attention in fewer grade levels. The shorter window for students to master an understanding of these concepts before they have to apply them to new problems and higher level mathematics necessitates that conceptual understanding be prioritized (Dingman et al., 2013).

The SMP will also provide a challenge for educators, as integrating these practices into everyday teaching may prove difficult for those who are less familiar with teaching the conceptual elements that underlie math procedures. Teachers must also be provided with training and support to understand the practices that are not as traditionally tied to mathematics content, such as the ability to construct viable arguments and critique the reasoning of others, which students are expected to demonstrate through their responses to open ended items on assessments aligned to the CCSSM (Dingman et al., 2013; Marzano, 2012). Researchers should focus on finding efficient, effective, and socially acceptable methods to implement the SMP in conjunction with math content. This is perhaps most important with math content that has been prioritized by the CCSSM, such as algebra, because the SMP could provide an impetus for increased conceptual understanding of algebra concepts. Greater conceptual understanding of algebra concepts is important for students to be able to answer new CCSSM assessment questions probing for deeper and more complex understanding of content. Strategies that provide students with conceptual understanding of math content have the potential to assist students with this transition. The following section will detail one such strategy, the Concrete-Representational-Abstract (CRA) instructional sequence, and the current research base supporting the use of the CRA instructional sequence with students with disabilities.

CRA Instructional Sequence in Mathematics Interventions

One point of controversy within mathematics education relates to whether inquiry-based instruction should be prioritized over explicit instruction (Cole & Washburn-Moses, 2010). Inquiry-based instruction typically occurs in general education settings, and involves students approaching novel problems to generate ideas for how to solve the problems prior to being introduced to algorithms or being taught how to approach the problems explicitly, as is typical with explicit instruction (Jitendra, 2013). The goals of inquiry-based instruction include generating mathematical discussions, as well as building connections between real life and math concepts (Jitendra, 2013). While some research indicates the effectiveness of inquiry-based instruction in mathematics, research conducted with students with disabilities indicates that explicit instruction is more effective than inquiry-based methods (Gersten, Chard et al., 2009; Jitendra, 2013). Students with disabilities may struggle with inquiry-based instruction due to their unique struggles with working and long-term memory, metacognition, processing speed, and difficulties with math computation and procedural skills (Jitendra, 2013; Shin & Bryant, 2015). One method to achieve the goals of inquiry-based instruction while using explicit instruction is to teach mathematics using concrete and pictorial representations.

Researchers have demonstrated through cognitive learning theories that people learn via a progression of concrete depictions to pictorial representations to abstract expressions of any given concept (Bell & Bell, 1985; Bruner, 1973). The CRA instructional sequence is based on this cognitive foundation, providing students with a structure to build conceptual understanding of even highly advanced math content. The concrete phase involves physical depictions of math concepts, the pictorial representation phase incorporates drawing diagrams and pictures to represent math concepts, and the abstract phase displays math concepts using numbers and formulas. For example, when CRA is used to teach students how to solve algebraic equations, the concrete phase may involve the use of differently sized tiles to represent variables and numbers, the representational phase may involve drawing diagrams to depict these tiles, and the abstract phase involves writing out and solving algebraic equations with variables, numerals, and math symbols (as typically taught). The

CRA instructional sequence has been studied with students with and without disabilities in grades 2 through 9, and has been used to effectively teach concepts including basic math facts such as addition, subtraction, and multiplication, as well as fractions, word problems, and algebra (Butler et al., 2003; Flores, 2010; Manl, Miller, & Kennedy, 2012; Miller & Kaffar, 2011; Morin & Miller, 1998; Strickland & Maccini, 2012; Witzel, 2005; Witzel et al., 2003). In each of these studies, students' math scores improved significantly, and these improvements were maintained and applied to novel problems even 6 weeks following the interventions (Flores, 2010; Strickland & Maccini, 2012), suggesting the promise of the CRA approach.

The CRA Instructional Sequence With Elementary Level Content

Researchers have demonstrated the potential effectiveness of the CRA instructional sequence in elementary schools in improving students' understanding and use of a variety of mathematics concepts. The majority of studies implemented at the elementary school level have been with students with disabilities or with students who have difficulty with math concepts (Flores, 2009; Flores, 2010; Flores, Hinton, & Schweck, 2014; Flores, Hinton, & Strozier, 2014; Mancl, Miller, & Kennedy, 2012; Miller & Kaffar, 2001). Miller and Kaffar (2011) implemented a study during a summer program for students with math difficulties to determine the effectiveness of the CRA instructional sequence on improving the ability of 24 second-grade students to solve problems involving addition with regrouping. After receiving 16 CRA sequenced lessons, the students in the experimental group scored higher on curriculum-based measures and pre- and posttests than those in the comparison group (Miller & Kaffar, 2011). Students in the experimental group specifically scored higher than students in the comparison group on both computation and fluency measures, and performed similarly to the comparison group on word problems (Miller & Kaffar, 2011). The generalizability of Miller and Kaffar's study was limited because it involved a sample of convenience. While there were no significant differences in math scores between the groups prior to the intervention, both groups of students were selected from students enrolled in a summer program for students with math difficulties (Miller & Kaffar, 2011).

A study by Flores (2009) indicated that the CRA instructional sequence may be an effective approach for improving third-grade Latino students' fluency in computing subtraction problems with regrouping in the tens place. Flores implemented 10 CRA sequenced lessons with 6 low performing students, 4 of whom qualified for special education services. The first three lessons implemented focused on the concrete phase of instruction, the next three focused on the representational phase, the seventh lesson included instruction in a mnemonic device, and the last three lessons focused on the abstract phase. Through her study, Flores demonstrated a functional relation between the CRA instructional sequence and the number of correct digits by students scored on 2-minute probes incorporating problems that required subtraction with regrouping (Flores, 2009). Flores developed the probes used for this study, based on Beck, Conrad, and Anderson's *Basic Skill Builders* (1999) (as cited in Flores, 2010). All 6 participants showed substantial improvements in their computation skills (Flores, 2009) following participation in 10 CRA sequenced lessons. One limitation of Flores' study is the small group of students, so the results cannot be generalized to the broader population. More research is needed with students whose skills vary to provide more evidence for the effectiveness of the CRA instructional sequence.

Flores (2010) replicated and extended her research on the use of the CRA instructional sequence to improve students' abilities to solve subtraction problems involving regrouping in the tens and hundreds places. In this study, Flores implemented the CRA instructional sequence using a multiple-probe across-participants design with 6 third-grade participants, 5 of whom were Hispanic and 1 of whom was African American. All 6 students were identified as low performing in mathematics, but none had been referred for special education evaluation. Flores (2010) used the same materials and followed the same instructional and assessment procedures as done in her 2009 study, with the addition of subtraction problems involving regrouping in the tens and hundreds place, rather than just in the tens place. All 6 participants again showed significant improvements in their scores on the subtraction probes, and maintained their improvements 6 weeks following intervention. A functional relation between CRA instruction and subtraction with regrouping was demonstrated across all students (Flores, 2010). Flores indicated that additional research should be conducted on CRA for subtraction with regrouping to determine the generality of the intervention, as well as across concepts and grades to determine the efficacy of the instructional procedure as an intervention.

Mancl, Miller, and Kennedy (2012) also implemented a study to assess the effectiveness of the CRA instructional sequence for solving problems involving subtraction with regrouping. Their study used a multiple-probe across-participants design with 5 fourth- and fifth-grade students with disabilities. The researchers evaluated the effectiveness of the CRA instructional sequence and explicit instruction using students' scores on researcher developed probes. Results indicated that all 5 students were able to achieve at least 80% mastery of each of the 11 lessons delivered as part of the intervention. Mancl et al. (2012) established a functional relation between the CRA instructional sequence and students' scores on probes involving subtraction with regrouping. The Mancl et al. (2012) study was limited because it was implemented only with students receiving services in a resource classroom.

Flores, Hinton, and Strozier (2014) evaluated the CRA instructional sequence, with the addition of the Strategic Instruction Model (SIM) on the mathematics performance of three low-performing third-grade students. This study involved a multiple-probe across-behaviors design, in which each student participated in baseline and intervention phases for three mathematical skills: subtraction with regrouping in the ones place, subtraction with regrouping in the ones and tens places, one-digit multiplication with regrouping, and two-digit multiplication with regrouping. Students progressed between the phases after they reached the criterion of 30 correct digits on a progress monitoring probe for each skill. For each mathematical skill taught, the instructional sequence was the same as previous Flores' studies (2009, 2010), with the addition of the incorporation of SIM, which involved explicit instruction with an emphasis on procedural knowledge. The instructional sequence involved three concrete lessons, three representational lessons, a mnemonic device taught in the seventh lesson, and the last three lessons at the abstract phase. Two of the participants demonstrated a functional relation between the CRA-SIM intervention and their math performance for three of the phases, and 1 of the participants demonstrated a functional relation between the CRA-SIM intervention and his math performance for four of the phases. The authors recommended that additional research be conducted on CRA-SIM to replicate these effects and to extend CRA-SIM to teaching higher level skills.

Flores, Hinton, and Schweck (2014) implemented a study to evaluate whether there was a functional relation between CRA-SIM and the computation performance of students with disabilities on solving multiplication problems that involved regrouping. In this study, Flores and colleagues used a multiple-probe design with 4 fourth- and fifth-grade students, all of whom were identified as having a Specific Learning Disability (SLD). The instructional sequence was similar to the previous studies implemented by Flores (2009), Flores (2010), and Flores, Hinton, and Strozier (2014), with the additional incorporation of SIM. Additionally, the concepts taught in the Flores, Hinton, and Schweck (2014) study involved teaching two digit multiplication problems that required regrouping. The results of this study provided evidence of a functional relation between the use of CRA-SIM and the performance of students with SLD on solving two-digit multiplication problems involving regrouping. All of the students maintained their scores in maintenance probes administered 1-4 weeks following instruction, and performed at levels higher than baseline on generalization probes (which involved problems with a three-digit multiplicand and two-digit multiplier) administered 2 weeks following maintenance. One interesting finding to note is that students did not show an immediate improvement in scores during intervention. However, Flores and colleagues pointed out that fluency in computation is developed over time, rather than immediately. The researchers recommended that future research be conducted on the CRA instructional sequence to demonstrate its efficacy with other populations of students, as well as additional math concepts.

CRA Instructional Sequence with Secondary Level Content

The CRA instructional sequence may also be effective for teaching both remedial and advanced concepts to secondary students with and without disabilities who struggle with math concepts (Butler et al., 2003; Morin & Miller, 1998; Strickland & Maccini, 2012; Witzel, 2005; Witzel, Mercer, & Miller, 2003). Morin and Miller (1998) demonstrated that the CRA instructional sequence can be an effective approach for teaching multiplication facts and associated word problems to seventh-grade students with moderate to severe disabilities. In their study, they implemented the CRA instructional sequence for 21 lessons with 3 students identified for special education services under the category of mental retardation (MR) using a single-subject design. Students were assessed prior to and following the intervention, as well as daily during the baseline and intervention phases (Morin & Miller, 1998). During the baseline phase of the study, a special education teacher provided instruction using traditional third-grade materials. Morin and Miller used four measures in this study: the baseline probes and pre- and posttests from Multiplication Facts 0 to 81, as well as researcher developed daily lesson sheets. They found that each student improved substantially based on a comparison of the participants' pre- and posttest scores. One student improved his/her ability to solve multiplication computation and word problems by 20%, another by 40%, and the 3^{rd} by 70%. Morin and Miller (1998) were able to establish functional relation between the CRA instructional sequence and students' scores on multiplication and word problem tasks. This study demonstrated that students with more significant academic needs can benefit from the CRA instructional sequence (Morin & Miller, 1998). Morin and Miller discussed two main limitations. First, the authors pointed out that the results of this study were not generalizable to a broader population since their study was implemented with only 3 seventh-graders. Additionally, Morin and Miller did not compare the CRA instructional sequence to other math interventions or strategies.

CRA Instructional Sequence and Fraction Concepts

Butler et al. (2003) conducted a study with students with mild to moderate disabilities in grades 6-8 to determine if there were differences in the pre- and posttest scores for students exposed to the CRA instructional sequence or Representational Abstract (RA) mathematics instruction pertaining to fraction concepts. Twenty-four students were exposed to 10 lessons involving the RA sequence, while 26 students were exposed to 10 lessons involving the CRA instructional sequence. The posttests of an additional 65 students in general education mathematics classrooms were compared to determine what mathematical concepts "typical" same age peers were able to master (Butler et al., 2003). The measures used by Butler et al. included pre- and posttests composed of three subtests from the Brigance Comprehensive Inventory of Basic Skills-Revised (CIBS-R) (Brigance, 1999) (as cited in Butler et al., 2003) and two additional researcher designed subtests. The students in the CRA instructional sequence group were exposed to concrete depictions and manipulative devices in the first three lessons, followed by another three lessons utilizing representational drawings, and the last four lessons focused on working with the mathematical concepts in an abstract algorithm (Butler et al., 2003). A paired samples *t* test indicated that students in both the RA and CRA instructional sequence groups improved on all outcome measures and a MANCOVA indicated that the students in the CRA instructional sequence group had significantly higher posttest scores when compared with the RA group. A MANOVA comparing both the RA and CRA instructional sequence groups to the group of typical peers found that the students in the intervention groups were able to master approximately the same amount of content at the same level as their peers in general education (Butler et al., 2003). The results of this study indicated that the CRA instructional sequence is effective in improving the mathematic performance of middle school students with disabilities. Butler et al. indicated that some of the main limitations of this study included the short time frame of implementation, a lack of follow-up data, and that the results of the intervention could not be generalized to students with disabilities other than learning disabilities.

CRA Instructional Sequence and Algebra

Witzel (2005) conducted a study comparing the pre- and posttest scores of sixth- and seventh-grade students with and without learning disabilities receiving traditional abstract mathematics instruction with those receiving a CRA instructional sequence in mathematics to determine the effects of the CRA instructional sequence on students' ability to solve linear algebraic functions. Witzel developed the assessments used for this intervention and had them reviewed by experts to ensure they were appropriate and aligned with the content involved in the intervention. The intervention focused on teaching 5 math skills in 19 lessons (Witzel, 2005). For the 108 students in the CRA instructional sequence group, the 1st day of instruction for each concept focused on concrete depictions and models, the 2nd day incorporated pictorial representations, and the 3rd and 4th days connected these concepts to abstract algorithms. The 123 students in the control group received traditional abstract instruction for all class sessions. Results indicated that both groups showed significant improvement between pre- and posttests. Students in the CRA instructional sequence group outperformed the students in the abstract group on the posttest, as demonstrated by a statistically significant difference between scores per instructional group (Witzel, 2005). This indicates that the CRA instructional sequence was successful in improving the students' ability to solve algebra problems. The results of this study also indicated that the CRA instructional sequence was effective in improving the performance of students with low, medium, and high mathematical abilities. One of the limitations of this study was that, based on the high standard deviations of the results for both the RA and CRA groups, neither strategy had an immediate effect for all students. This finding could be due to students being at different levels of readiness for algebra. Currently, algebra performance is typically predicted by arithmetic skill. However, due to the complexities of algebra, as well as the many different thought processes required for algebra, basing students' readiness for algebra solely on their arithmetic skill may not be accurate. Witzel recommended that additional research be conducted to evaluate student variables that could affect performance in algebra.

Witzel, Mercer, and Miller (2003) conducted a similar study comparing the pre- and posttest outcomes of sixth- and seventh-grade students between groups receiving traditional abstract instruction and a CRA instructional sequence. All of the students who participated in the Witzel et al. study were either low performers in math or students receiving special education services. Witzel et al. defined low-performing math students as those who were described by their teachers as having below average performance in math class, scored below the 50th percentile in math on statewide achievement tests, and were from low socio-economic backgrounds. Each group of students participated in 19 lessons designed to instruct students how to simplify and solve algebraic equations with variables on both sides of the equal sign. The experimental group received a CRA sequence of instruction, while the control group received only abstract instruction. Witzel and his colleagues again developed the pre- and post assessments used in this study based on the curriculum and had experts review the assessments (Witzel et al., 2003). This study followed the same design as the Witzel (2005) study and demonstrated larger, statistically significant differences between pre- and posttest scores for both groups. The 34 students in the CRA instructional sequence group outperformed the 34 students in the traditional abstract condition and had fewer computation errors on the posttest, although the differences between the CRA and abstract groups were not statistically significant on the posttest. The CRA instructional sequence group also outperformed the traditional abstract group on a follow-up test 3 weeks after the conclusion of the intervention (Witzel et al., 2003). The results of this study indicated that students exposed to the CRA instructional sequence improved their mathematics performance at a greater rate than those exposed to traditional abstract instruction. Witzel et al. pointed out two main limitations of this study. First, the assessment used was not externally validated, and may have been too difficult, based on teacher feedback. Additionally, the 19 lessons implemented in this study followed a sequence of instruction typically found in algebra textbooks. While the sequence of instruction has been found to affect students' performance, little scientific research has assessed the sequence of algebra instruction found in most textbooks. Therefore, a poor sequence of instruction could contribute to students' scoring low in math.

The CRA instructional sequence has also been effective in teaching eighth- and ninth-grade students with learning disabilities more advanced algebra content, specifically how to multiply linear algebraic expressions within the context of area problems (Strickland & Maccini, 2012). In their multiple-probe design across participants study, Strickland and Maccini (2012) integrated the concrete, representational, and abstract phases of instruction, rather than isolating each phase into different lessons. All of the assessments used in this study were designed by the researchers and reviewed by experts. The domain probes were designed to assess all objectives in the instructional unit. The lesson probes were developed to assess objectives of individual lessons, and were administered at the end of each lesson. A final measure, a transfer probe, was developed to assess students' performance on novel problems, and included three tasks that synthesized the concepts taught during the intervention. Strickland and Maccini assessed students' performance on the three domain probes administered prior to the intervention, following the intervention, and 3 to 6 weeks after the intervention. In addition, Strickland and Maccini also assessed students at the end of each lesson using lesson probes. A final transfer probe was also administered to determine how readily students were able to transfer the knowledge learned during the intervention to unique prompts. The 3 participants in their study showed immediate improvements following the three 40-minute intervention lessons, and 2 of the 3 correctly solved over 95% of items on a maintenance measure administered 3 and 6 weeks following the intervention (Strickland & Maccini, 2012). Additionally, 1 student scored 83% on an assessment item administered to assess the degree to which students were able to transfer the strategies learned in the intervention to novel problems (Strickland & Maccini, 2012). The other two students scored 67% and 50% on the transfer item, which still reflects a promising degree of transfer (Strickland & Maccini, 2012). All 3 of the students also found the intervention to be socially acceptable, based on their responses on Likert scale questions and open-ended questions on a questionnaire developed by the researchers (Strickland & Maccini, 2012). All of the students indicated that the CRA instructional sequence was helpful, the intervention was enjoyable and fun, and they would recommend it to other students (Strickland & Maccini, 2012). The authors suggested that using an integrated CRA instruction sequence could be one method of differentiation used in classes in which general and special education students receive instruction in an inclusive setting (Strickland & Maccini, 2012). The authors discussed a few key limitations of their study. First, because the study included only three participants who received individualized instruction, more research is needed to establish external validity. Additionally, the study was limited in that it addressed only one math concept and was conducted with individual students as opposed to a group setting (Strickland & Maccini, 2012).

Summary of CRA Instructional Sequence Research

The preceding studies demonstrated that the CRA instructional sequence may be effective in teaching students with and without disabilities who struggle with math concepts (Butler et al., 2003; Flores, 2009; Flores, 2010; Flores, Hinton, & Schweck, 2014; Flores, Hinton, & Strozier, 2014; Mancl, Miller, & Kennedy, 2012; Miller & Kaffar, 2011; Morin & Miller, 1998; Strickland & Maccini, 2012; Witzel, 2005; Witzel, Mercer, & Miller, 2003). In particular, the CRA instructional sequence has been a promising practice for teaching high level content, such as algebra (Strickland & Maccini, 2012; Witzel, 2005; Witzel, Mercer, & Miller, 2003). The successes of the CRA instructional sequence in teaching algebra concepts to students with disabilities demonstrated promising future directions for this strategy, as the CCSSM prioritize algebra readiness for all students (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2012). The next section will detail how the CRA instructional sequence could specifically align with the CCSSM.

CRA Instructional Sequence and Common Core

The research conducted thus far on the CRA instructional sequence indicated that it may be a promising strategy to help students with disabilities adapt to the rigorous CCSSM (Flores, Hinton, & Schweck, 2014; Flores, Hinton, & Strozier, 2014). The primary challenges students will face in adapting to the CCSSM are solving complex, novel problems on assessments aligned with the CCSSM, and also demonstrating more conceptual understanding. Two of the main areas of difficulty students face in mathematics currently include overreliance on formulas, as well as a lack of conceptual understanding (Alberti, 2013; NCTM, 2013; NMAP, 2008; Shifter & Granofsky, 2012). The CRA instructional sequence has the potential to address these areas of need because it provides a strategy that students can use independently on assessments, even if they forget a formula, because the CRA instructional sequence is rooted in conceptual understanding of math content (Flores, Hinton, & Schweck, 2014; Flores, Hinton, & Strozier, 2014; Mancl, Miller, & Kennedy, 2012; Miller & Kaffer, 2011; Strickland & Maccini, 2012; Strom, 2012). The CRA instructional sequence also incorporates multiple representations and provides students with a method to model with mathematics, reason abstractly and quantitatively, and look for and make use of structure, all of which are reflected in the SMP (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2012; Strickland & Maccini, 2012). Additional research should be conducted to determine the effectiveness of the CRA instructional sequence in the context of CCSSM implementation. Currently, there is evidence that students struggle with transitioning between the representational and abstract phases in the CRA sequence of instruction (Strickland & Maccini, 2012), demonstrating a need for research to determine if there are strategies that can be combined with the CRA instructional sequence to improve students' ability to transition between these phases. One strategy that has the potential to assist students with developing this skill is writing to learn mathematics, which will be detailed in the following section.

Writing as a Vehicle for Learning

The concept of writing to learn (WTL) developed from cognitive theories demonstrating that writing is a unique linguistic practice that provides individuals with an opportunity for inner dialogue, allowing them to engage in self-reflection, further explore and understand the world, and solve problems (Emig, 1977; Klein & Yu, 2013; Lindemann, 1987; Vygotzsky, 1962). Additionally, writing is both a process and a product (Vygotsky, 1962). Writing can both be used to process and learn material, and serve as a permanent product through which to assess students' understanding. As evidenced by several metaanalyses of writing to learn studies, writing may be a promising intervention for improving content area knowledge ranging from psychology, literature, social studies, science, to math (Bangert-Drowns, Hurley, & Wilkinson, 2004; Graham, McKeown, Kiuhara, & Harris, 2012; Klein & Yu, 2013). The success of WTL strategies has been evidenced from prekindergarten to higher education levels, and is likely due to the unique features of writing that promote intensive and varied avenues for learners to engage more intimately with the material (Emig, 1977). Two main reasons have been proposed for why writing is an effective strategy for learning. First, writing about concepts learned is a unique form of communication that allows students to access a higher level of thinking. Additionally, writing is a form of self-discovery and reflection that helps increase student understanding (Lindemann, 1987; Vygotzky, 1962).

Writing is an effective mechanism to help students learn because it provides a structure to communicate information. The act of communicating via writing is different than talking because writing requires that the writer provide any necessary context or background essential for understanding, whereas a speaker could rely on visual and social cues (Emig, 1977; Hebert, Gillespie, & Graham, 2013; Lindemann, 1987; Vygotsky, 1962). This means that writers must provide maximal detail (i.e., precise word choice, clear elaboration of ideas through the use of examples, etc.) to clearly communicate the context to their intended audience. Writing aims to communicate understanding of a subject to someone, and a writer only has words to convey his/her meaning (Emig, 1977; Lindemann, 1987). In order to convey their understanding, writers have to necessarily detach themselves from the actual content or situation to be able to represent it in their own words (Vygotsky, 1962). This level of abstraction necessitates a higher level of understanding regarding the concepts being written about (see Gillespie, Graham, Kiuhara, & Hebert, 2014).

Writing also helps students learn because it is a form of self-discovery and reflection. The process of writing involves moving fluidly between thought and word, which requires constant reflection. Throughout this process, students are able to connect ideas to one another, which allows for further development of relationships between concepts (Emig, 1977; Vygotsky, 1962). Writing helps people understand the world and themselves. Especially within specific career fields that have particular jargon and vocabulary, writing can help students master unique vocabulary and discourse. This mastery provides them with increased opportunities to access careers and college (Lindemann, 1987). As a product, writing also allows the learner to demonstrate and communicate his or her understanding of concepts. Therefore, writing is a productive process that helps students learn, and results in a product that is useful for students to communicate their understanding.

Writing to Learn Research

Writing to learn has been studied extensively for over 30 years, and metaanalyses of WTL studies demonstrate overall positive effects (Bangert-Drowns et al., 2004; Graham & Hebert, 2011). In a review of 50 WTL studies, Bangert-Drowns et al. (2004) found that WTL had small but positive effects on school achievement (average weighted effect size for WTL studies overall was 0.17 standard deviations), and Graham and Hebert (2011) reported that WTL studies improved reading comprehension in their review of 95 studies (average weighted effect sizes for each research question of the study were 0.22, 0.35, and 0.37). Bangert-Drowns et al. found that writing involving reflection on current knowledge, confusion, and the learning process was more effective than personal writing. The two metaanalyses had slightly different findings regarding for which school level WTL was most effective. Bangert-Drowns et al. found that WTL had significant, but lower, effects in grades 6-8 than elementary or high school levels, whereas Graham and Hebert found that the highest effect sizes were at the middle school level, followed by high school. Based on these conflicting results, studies incorporating WTL activities especially at the secondary level, are necessary to further determine which types of writing are most effective, as well as for which subjects writing to learn provides the most benefit (Klein & Yu, 2013).

Writing to Learn Mathematics

Educational theorists began to consider the potential benefits of using writing to improve math understanding in the early 1980s, in part due to the emerging success of writing as a vehicle to learn reading and other content areas, as well as to address frustrations with the inadequacy of basic skills instruction in U.S. public education (Bell & Bell, 1985). Writing may particularly assist students in becoming more proficient in math due to parallels between the cognitive functions of writing and problem-solving (Bell & Bell, 1985; Lindemann, 1987; Wallas, 1926). Both writing and math require students to engage with multiple representations of concepts; writing requires transferring between spoken, written, and read information, while math necessitates connecting symbolic, numeric, and linguistic representations. Due to the high level of abstraction found in both math and writing, combining these content areas may provide students with additional methods to engage with the material more fluently (Bell & Bell, 1985). Whereas specific approaches to writing to learn math (WTLM) vary, all typically involve students writing about their experiences with math. For example, students may write in math journals to discuss their feelings towards math, if they are improving, and/or what they find confusing. Students could also write to plan out how to approach solving a problem, to reflect on how they solved a problem, or to construct an argument in defense of their answer. Studies have suggested rubric criteria with which to assess WTLM, including the use of appropriate mathematical language, use of specific examples, the inclusion of mathematical representations as support for ideas, and the use of appropriate mathematical notation (Stonewater, 2002). Additionally, it is recommended that distinctions be made in the quality of mathematical writing based on whether the writing summarizes (i.e., reports steps completed to solve the problem), or presents dialogue about mathematics (i.e., poses critical questions, analyzes, and makes connections between math concepts; Craig, 2011).

The Connection Between Writing and Problem-Solving

The specific processes in math problem-solving and writing fall in the planning stages undertaken when composing a piece of writing or approaching and solving a problem (Lindemann, 1987). The root of these cognitive similarities lies in how humans approach all problem-solving attempts, which was originally outlined by Wallas in *The Art of Thought* (1926) (as cited in Bell & Bell, 1985). Wallas identified four steps involved in solving any problems: preparation, incubation, illumination, and verification. Preparation involves defining the problem at hand, incubation describes the process of reflecting on the problem and considering multiple ways of approaching it, illumination refers to the stage at which a strategy is employed to solve the problem, and verification is the final stage of the process in which the solution is tested (Bell & Bell, 1985; Wallas, 1926). This process takes the specific form in the math context of determining what information a math problem is asking for, setting up the problem and considering how to solve it, engaging in the calculations to solve the problem, and checking the answer. For writing, the process is seen in the stages of selecting a topic, determining what to say about the topic and outlining the written response, writing a draft, and revising the written product (Bell & Bell, 1985). Using WTLM can therefore be successful for skills in both content areas, as "their underlying processes are so very similar, practice in one area can reinforce competency in both by strengthening the student's critical thinking ability" (Bell & Bell, 1985, p. 213).

Research on Writing to Learn Mathematics

Researchers have investigated WTLM with a variety of grade levels and math content, both to gain insight into students' understanding of math concepts (Craig, 2011; Gopen & Smith, 1990; Stonewater, 2002; Waywood, 1994), as well as to measure changes in math performance due to writing (Albert, 2000; Bell & Bell, 1985; Evans, 1984; Kostos & Shin, 2010; Porter & Masingila, 2000). The majority of studies showed positive results relating to students' math assessment scores, confidence and feelings towards math, and abilities to express mathematical thinking through writing (Akkus & Hand, 2011; Albert, 2000; Bell & Bell, 1985; Craig, 2011; Evans, 1984; Gopen & Smith, 1990; Kosko & Norton, 2012; Kostos & Shin, 2010; Kroll & Halaby, 1997; Miller & England, 1989; Porter & Masingila, 2002; Stonewater, 2002; Waywood, 1994). In addition to the studies on WTLM that have been published, numerous research articles discussed the theoretical concepts of WTLM and provided support to educators attempting to implement it in their classrooms (Baxter, Woodward, Olson, & Robyns, 2002; Bosse & Faulconer, 2008; Burns, 2004; Burns & Silby, 2001; Countryman, 1993; Flores & Brittain, 2003; Goldsby & Cozza, 2002; McIntosh & Draper, 2001; Miller, 1991; O'Connel et al., 2005; Rider-Bertrand, 2012; Ryan, Filero, Cheland, & Zambo, 1996). To date, all of the studies on WTLM have been implemented with students in general education settings. In light of the promising research base on WTLM, additional research is needed to explore the wide-ranging potential benefits of WTLM, including its potential use in interventions with students with disabilities.

Research on Using Writing to Gain Insight Into Student Understanding

Much of the research on WTLM pertains to analyzing students' writing to gain insight regarding how students think about various math concepts. Waywood (1994) implemented a qualitative study to analyze how questioning operated in students' mathematical writing. Waywood used an exegetical method of analysis to determine the extent to which questioning in math journals can be used to create learner profiles. Exegetical analysis "aims to understand the meaning of a text by a sensitive reading and analysis of the inter-relatedness of the parts to the whole" (Waywood, 1994, p. 327). Waywood evaluated written math responses of 3 10th-grade students in an all-girls' high school that implemented writing in mathematics as part of its curriculum across all grade levels. Through interviews with students and evaluation of their writing in math, Waywood determined that students who had a higher level of sophistication in writing, as evidenced by the use of specific math vocabulary and accurate descriptions of concepts, also scored higher in math. Waywood also found that writing in math was favored positively by students, about which he concluded, "journals, as a learning tool, address the needs of the person to reflect on and integrate their experience of schooling in mathematics" (Waywood, 1994, p. 339). Waywood also analyzed one student's writing in ninth-grade and then in 11th-grade, to get a sense of how journal writing, questioning, and mathematical thinking changed over the course of 3 years. Waywood found this particular student's questioning grew in complexity over time, which provided additional evidence for the correlation between writing and the development of mathematical thinking. Waywood's study demonstrated that writing can also provide a valuable method to witness students' cognitive processes in solving math problems and therefore gain more information about students' math understanding. Although Waywood's study provided promising results for the power of WTLM, it lacked experimental control, as many factors other than journal writing could have contributed to the improvements in students' questioning and math scores on assessments. Additionally, Waywood's study was implemented with a specific population of students in general education, so it did not provide insight regarding whether math journals could provide similar benefits to students with disabilities.

Writing to Learn in Calculus

Researchers have used writing to develop further understanding of students' thinking in calculus (Craig, 2011; Gopen & Smith, 1990; Kosko & Norton, 2012; Porter & Masingila, 2000; Stonewater, 2002). Kosko and Norton (2012) conducted a study to determine if preservice teachers could engage in written communication with high school calculus students that would demonstrate the high school students' understanding of the SMP found in the new CCSSM. Kosko and Norton paired 27 preservice mathematics teachers with one precalculus student each. The preservice teachers and the students wrote six letters to one another about discrete mathematics. Kosko and Norton found, through external review of the letters, that the majority of the preservice teachers were able to get students to use representation (i.e., diagrams) or communication (i.e., written descriptions)- based SMP, but rarely both (Kosko & Norton, 2012). While representation and communication are both viewed as parts of the problem-solving process, typically students are most comfortable with one or the other. Kosko and Norton concluded from their study that math teachers should encourage students to use both math representation and written communication, as "using both processes in conjunction . . . may help students make connections between two processes that appear to be often separated" (Kosko & Norton, 2012, p. 347). Additionally, Kosko and Norton identified letter writing as a potentially effective strategy to link written and representational math expression. Kosko and Norton's study was limited, as it was a case study implemented only with general education students, and did not involve the assessment of math achievement.

Craig (2011), Gopen and Smith (1990), and Stonewater (2002) conducted studies with college-level calculus students to determine criteria that indicate higher levels of mathematical understanding in students' writing. In each of these studies, the authors concluded that WTLM improves students' math problem-solving ability, as indicated by students' written responses and test performance (Craig, 2011; Gopen & Smith, 1990; Stonewater, 2002). Additionally, each author found that students who used correct and specific math terminology and vocabulary, relevant examples to illustrate concepts, and multiple modes of representation to convey their ideas performed better than students whose responses did not include these traits (Craig, 2011; Gopen & Smith, 1990; Stonewater, 2002). These findings are helpful in establishing key criteria to use when evaluating students' writing in WTLM.

The research on WTLM discussed thus far (Craig, 2011; Gopen & Smith, 1990; Kosko & Norton, 2012; Stonewater, 2002; Waywood, 1994) demonstrated that writing may be an effective mechanism to gain insight into students' understanding, which may provide educators with valuable information to use when planning and implementing instruction. Although the previous studies did not explicitly evaluate the changes in students' math performance as a result of writing, several studies have assessed changes in students' math outcomes, which will be discussed in the following section.

Research Measuring Change in Math Performance Related to Writing Tasks

While the majority of studies on WTLM have focused on higher level math content generally taught at the high school or postsecondary level, the few studies conducted with elementary school students to teach basic math concepts have provided initial evidence of the effectiveness of WTLM (Evans, 1984; Kostos & Shin, 2010). Kostos and Shin (2010) conducted a mixed methodology action research project with 16 students in second grade to determine if math journals improved students' ability to communicate mathematically. Students were assessed prior to and following a 5-week unit on problem-solving and patterns using a math assessment obtained from a state's department of education (Kostos & Shin, 2010). The math journals, which were written in approximately three times per week, were also assessed using the *Saxon Math Teacher Rubric for Scoring Performance Tasks* (Larson, 2008; as cited in Kostos & Shin). This rubric evaluated students' responses for processes and strategies expressed, for level of knowledge and skills understanding, and for communication and representation (Kostos & Shin, 2010). As additional sources of data, Kostos and Shin interviewed the students and completed teacher-researcher reflection journals. Kostos and Shin found that the majority of students improved their math thinking, math communication, and increased their use of math vocabulary. Additionally, math journals were found to be an effective assessment tool that provided teachers with valuable insight regarding students' understanding of math concepts (Kostos & Shin, 2010). Although Kostos and Shin conducted this study with students who were not receiving special education services, this study suggested that even very young students, who may have rudimentary writing skills, may benefit from WTLM. This study was limited, in that it was conducted only with one classroom of students and one teacher-researcher. Additionally, the results did not connect mathematical communication to general math computation, so it is unknown the extent to which writing helped students improve their mathematical computation (Kostos & Shin, 2010).

Evans (1984) conducted a quasi-experimental group study, using her fifth-grade classroom composed of 22 general education students as a treatment group and another fifth-grade classroom composed of 6 gifted students and 17 general education students as a comparison group, to determine if writing improved students' understanding of multiplication and geometry concepts. Evans used three main types of writing in her study, including prompts in which students had to explain how to solve problems, writing about the definitions of math vocabulary, and writing to explain and correct errors made. Evans found that even though the pretest scores of her students were lower than those of the control group, her students had higher scores than the control group on the teacher-designed end of unit tests for both multiplication and geometry. Statistical analyses were not run for the differences in pretest scores between groups, the change in scores from pre- to posttest within each group, or the differences in posttest scores between groups, so it is unknown whether these differences were statistically significant. While lacking experimental control and methodological rigor, Evans' study suggested that WTLM may help average fifth-grade students make significant gains in foundational concepts.

Writing to Learn in Secondary Math

WTLM researchers at the secondary level have focused on general problem-solving and math abilities (Albert, 2000; Bell & Bell, 1985; Waywood, 1994), algebra (Akkus & Hand, 2011; Miller & England, 1989), precalculus (Kosko & Norton, 2012), and calculus (Craig, 2011; Gopen & Smith, 1990; Porter & Masingila, 2002; Stonewater, 2002). Bell and Bell (1985) conducted a pilot study incorporating a group design with ninth-grade students in general math to determine if WTLM improves students' math proficiency. The experimental group of 18 students in this study received traditional math instruction with the addition of a structured expository writing component. The writing incorporated into the intervention involved students writing about the math processes used to solve problems, as well as writing other problems and problem contexts (Bell & Bell, 1985). The 20 students in the comparison group received traditional math methods. Results indicated that the experimental group scored significantly higher than the comparison group on the posttest following the 4-week WTLM intervention. Through their pilot study, Bell and Bell demonstrated that WTLM can improve students' problem-solving, likely because "exposition allows them to become more aware of their thinking processes and more conscious of the choices they are making as they carry out the computation and analysis involved in solving math problems" (Bell & Bell, 1985, p. 220). Bell and Bell's study was limited in that it was conducted with a small group of students in general education math, so their results cannot be generalized to students with disabilities or broader populations.

Albert (2000) conducted an interpretative case study with seventh-grade students to explore the evolution of thought processes during a 14-week study. Albert implemented an instructional strategy involving group discussions and individual writing in math, as well as group evaluation of solutions. Albert's study included 7 seventh-grade students in a treatment group and three comparison groups. Albert analyzed students' completion of math problems and writing samples. Additionally, she asked the students questions about their attitudes towards math during semistructured group interviews. The interviews and writing samples involved students' solving math problems, describing math problems in their own words, and explaining their reasoning. Albert found that the instructional strategies implemented for the intervention, which included WTLM, improved students' understanding of problem-solving and performance on problems, as well as their attitudes towards math, at higher levels than the control groups. Writing allowed students to reflect on their understanding and thought processes in a structured manner. This practice allowed students to develop a deeper level of understanding of fundamental math concepts rather than relying on formulas (Albert, 2000). Albert's study suggested that WTLM may improve students' math understanding, but is limited because it was implemented with only a limited selection of general education students. Albert's study also did not include analysis of students' scores or responses prior to the implementation of the intervention, so it is unknown to what extent the instructional methods employed were responsible for improvements in math understanding.

Writing to Learn Versus Discussion-Based Activities in Calculus

WTLM researchers have primarily focused on calculus in higher education settings, and their studies have usually involved evaluating students' written responses to determine criteria that indicate level of mathematical understanding (Craig, 2011; Gopen & Smith, 1990; Stonewater, 2002). One exception is a study conducted by Porter and Masingila (2000) to determine whether students gained similar benefit from writing as they did from discussion. Porter and Masingila's study incorporated a qualitative group design, in which they analyzed and compared students' errors and responses on four course exams based on their membership in two separate sections of an introductory university calculus course. The same instructor taught each section of the course, but one section received WTLM activities, whereas the comparison section received identical instruction without writing assignments. The experimental (WTLM) group was composed of 15 students, whereas the comparison group was composed of 18 students. Both sections focused on the same concepts and procedures, but the section receiving WTLM were given in and out of class writing assignments focused on explaining course ideas, discussing the relationship between course concepts, and writing their thoughts about concepts and procedures of the course. Over the span of one semester, Porter and Masingila analyzed and categorized the responses and errors students made on each exam, and compared the two groups to determine if there was a correlation between WTLM and performance on exams. Porter and Masingila found that there were not significant differences between the class section that used WTLM practices and the section that used discussion based methods. They concluded that the real benefit that has been found in WTLM studies may have more to do with getting students to deeply and critically engage in the math material at a level that allows them to communicate their thinking with others (Porter & Masingila, 2000). This study's conflicting evidence necessitates additional research be conducted on WTLM principles.

Writing to Learn Mathematics and Common Core

Further research is needed on WTLM in light of the implementation of the CCSSM and SMP because WTLM may have unique features that provide assistance in transitioning to the common core standards, as well as addressing the challenges faced in mathematics in the U.S. currently. First, the assessments aligned with the new CCSSM require that students be able to express their reasoning for how they solved problems, as well as justify their solutions in writing (Bosse & Faulconer, 2008; Kostos & Shin, 2010). These assessments will likely be difficult for the majority of U.S. students, due to having historically demonstrated specific challenges with conceptual understanding and problem-solving (Alberti, 2013; NCTM, 2013; NMAP, 2008; Shifter & Granofsky, 2012). Students who are overly reliant on formulas often lack conceptual understanding, which makes it very difficult to describe the processes used in solving problems, or to justify why they know their answer is correct (Strickland & Maccini, 2012; Strom, 2012). WTLM could provide a solution to these struggles, as it explicitly provides a structure for students to explain their understanding conceptually (Bosse & Faulconer, 2008; Kostos & Shin, 2010).

WTLM may also provide teachers with a strategy to facilitate efficient implementation of the SMP in their teaching. Teachers and students view WTLM favorably (Akkus & Hand, 2010; Miller & England, 1989) and writing has been demonstrated to tie directly to the SMP (Kosko & Norton, 2012). Teachers may struggle with finding ways to implement the SMP in their teaching naturally and consistently (Schifter & Granofsky, 2012). WTLM ties directly to the SMP expectations of constructing viable arguments and critiquing the reasoning of others, as well as attending to precision (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2012). Additional research should be conducted to determine if WTLM can be implemented in an effective and efficient manner to integrate the SMP into the teaching of mathematics at multiple grade and content levels. Research is also needed to determine if WTLM can be implemented successfully with students with disabilities, especially since WTLM has never been studied with this population. Implementing WTL in conjunction with math instructional strategies geared towards improving conceptual understanding may provide an efficient and effective process to improve students' overall math achievement, in particular with complex algebra concepts that are traditionally taught with primarily abstract methods, such as rate of change. The following section will discuss the importance of rate of change and provide an overview of the research conducted thus far on rate of change.

Rate of Change and Students With Disabilities

While more resources and attention have been dedicated to math education research recently, there is a severe lack of research on effective instructional and remedial strategies for teaching students at the secondary level, especially strategies specific to low-performing groups, such as students with disabilities (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Graham & Hebert, 2011; Saunders, Bethune, Spooner, & Browder, 2013; Strickland & Maccini, 2012; Witzel, Mercer, & Miller, 2003). Early intervention, while important, is not adequate to address the challenges of teaching older students advanced math content. Algebraic proficiency is increasingly vital for students to graduate from high school, attend higher education, and access jobs ranging from manufacturing to science, technology, and engineering career fields (Bell & Norwood, 2010; Fuchs et al., 2008; Fullerton, 1995; NMAP, 2008; Schmidt & Burroughs, 2013). Given the current trend for students in lower grades in the U.S. to perform at higher levels of relative proficiency than older students (NCES, 2013; Schmidt & Burroughs, 2013) it is essential that strategies for teaching algebra content to older students be developed and researched to improve student achievement. This need is amplified with implementation of the CCSSM, which will likely pose difficulties for students with disabilities because they prioritize conceptual understanding and are more rigorous (Dingman et al., 2013; Saunders et al., 2013). While students with disabilities struggle in many areas of math taught in the CCSSM, rate of change is particularly important because it is an algebraic concept that is emphasized heavily in the secondary core of the CCSSM (National Governors Association Center for Best Practice & Council of Chief State School Officers, 2012b).

Importance of Rate of Change

Rate of change is the concept of a unit per unit change in quantities (Bezuidenhout, 1998), and is described by a ratio comparing two different, numeric measurable quantities (Herbert & Pierce, 2012). For example, rate of change is often computed for problems that involve finding the average miles per hour given a set of speeds and times. Rate of change is often referred to as slope in the context of graphing linear equations, and is usually taught as the change in y over the change in x. Rate of change is a key area to target for secondary level intervention, as it is a concept that spans all five of the high school domain areas in the CCSSM, which include algebra, functions, modeling, geometry, and statistics and probability (National Governors Association Center for Best Practice & Council of Chief State School Officers, 2012b). Rate of change is a concept that is first introduced in algebra as slope with linear equations, but continues and is extended in higher level mathematics in functions, limits, and derivatives (Teuscher & Reys, 2012). Rate of change is a concept that is found in calculus, engineering, and physics. Therefore, students with disabilities who struggle with rate of change may be impacted in negative ways in other content areas, such as science. Additionally, rate of change is particularly important for students with disabilities to understand because it is a concept that is important for everyday life and personal choices. Knowledge of rate of change can help individuals with functional skills, such as accurately reading a map, calculating interest on a loan, or selecting a phone plan (Hebert & Pierce, 2012). Recent surveys reveal that over half of the adults in the U.S. lack this understanding (Herbert & Pierce, 2011; NMAP, 2008; Piper, Marchand-Martella, & Martella, 2010). This poses serious risks for individuals' economic independence as well as for U.S. global economic standing (NMAP, 2008).

Research on Rate of Change

Rate of change is an area of mathematics that poses many difficulties for students, both with and without disabilities (Adjiage & Pluvinage, 2007; Herbert & Pierce, 2012). Only 12% of eighth-grade students were able to solve problems comparing rates of change on the NAEP in 1996 (Jitendra et al., 2009). Despite the importance and wide applicability of rate of change, there is little research on this topic. The majority of research conducted thus far on rate of change has focused on analyzing students' understanding of the concepts of rate of change, slope, and steepness (Hattikudor et al., 2012; Herbert & Pierce, 2007; Herbert & Pierce, 2012; Stump, 2001; Teuscher & Reys, 2012; Teuscher, Reys, Evitts, & Heinz, 2010; Wilhelm & Confrey, 2003). Two quasi-experimental studies have compared students' understanding of rate of change concepts based on students' participation in single content or integrated pathway curricula (Teuscher et al., 2010; Teuscher & Reys, 2012). Just two experimental studies have assessed the effects of instructional strategies on students' performance on rate of change assessments (Kramarski & Mevarech, 2003; Kramarski, 2004). To date, no studies on rate of change have been implemented with students with disabilities. The difficulties students with disabilities may have with rate of change problems, and the effects of interventions targeting students' with disabilities understanding of rate of change, are therefore currently unknown.

Rate of Change Research at the College Level

Researchers studying rate of change concepts have primarily focused on college level students and their understanding of rate of change concepts in the context of advanced math content, such as calculus or physics (Bezuidenhout, 1998; Christensen & Thompson, 2012; Orton, 1984; Van Dyke & White, 2004). These researchers have discovered that even advanced level students struggle with the concept of rate of change, often due to conceptual misunderstandings or a lack of conceptual knowledge (Bezuidenhout, 1998; Christensen & Thompson, 2012; Orton, 1984; Van Dyke & White, 2004). The fact that advanced level students struggle with rate of change is important to note, as it provides evidence of the need for more effective methods of instruction for rate of change concepts at the high school level.

Rate of Change Research in the Context of Precalculus

Conceptual knowledge of rate of change is a foundational skill needed for calculus. Based on this connection, many researchers have assessed students' understanding of rate of change concepts in precalculus, in part to guide teachers in how to better prepare students for calculus and teach concepts in calculus to reduce confusion (Herbert & Pierce, 2007; Herbert & Pierce, 2012; Stump, 2001; Teuscher et al., 2010; Teuscher & Reys, 2012). Herbert and Pierce (2012) conducted a qualitative study with 20 10th-grade Australian students to assess their understanding of rate of change. Herbert and Pierce interviewed the students using computer simulations of real-world contexts involving rate, such as when a window shade partially covering a square or an abnormally shaped window is raised or lowered. An additional real-world context involved comparing the rates of two figures that were walking. Students were shown a diagram depicting the real-world context, as well as a graph and table, and asked to describe how the rate was changing. Students' responses ranged from those that revealed very little understanding of mathematical concepts of rate (i.e., students described rate referring to quality as opposed to numeric value), elementary understanding (i.e., rate associated with numeric value), and more nuanced conceptual understanding (i.e., rate as a relationship between quantities; Herbert & Pierce, 2012). Based on their research, Herbert and Pierce discovered four critical aspects of rate of change that may address students' gaps of knowledge about the concept. These four aspects include the following: (a) rate as a relationship between changes in two quantities, (b) rate as a relationship between changes in two quantities that may vary, (c) rate as a numerical relationship between changes in two quantities that may vary, and (d) rate as a numerical relationship between changes in two quantities that may vary and is applicable to any context (Herbert & Pierce, 2012). Herbert and Pierce suggested that these critical aspects be used to evaluate existing approaches to teaching rate of change. One limitation of this study was that student level variables (i.e., current math class, gender, etc.) that may have affected students' understanding of rate of change concepts were not taken into account. Additionally, the study did not make any attempts to change students' understanding of rate of change, but was instead conducted to gain information regarding students' understanding of rate of change.

Herbert and Pierce (2007) conducted a similar qualitative study analyzing students' use of gestures to explain their understanding of rate of change. Herbert and Pierce (2007) video-recorded interviews with 25 10^{th}-grade students who were given the task of describing the rate of change in the real-world context of a window shade, as described in Herbert and Pierce's (2012) study. Students' responses were viewed several times, and their use of specific gestures was coded. The results of this analysis indicated that students had a good understanding of the basic concept of rate of change, but were not always able to put this understanding into words (Herbert & Pierce, 2007). Herbert and Pierce (2007) found that gestures helped students describe their reasoning, but that most students were unable to link graphic and numeric representations to rate of change. Additionally, Herbert and Pierce (2007) found that very few students had an understanding of variable rate of change, which was depicted by an abnormally shaped window. Herbert and Pierce (2007) suggested that teachers incorporate the use of gestures in their teaching to help students understand more of the conceptual aspects of rate of change, and also to be able to gain more information about students' understanding. This study was limited in that it only assessed average students' understanding of rate of change, and did not evaluate the influence of student-level variables or instruction to change understanding.

Stump (2001) interviewed 22 precalculus students in high school to analyze their understanding of slope as a measure of rate of change. In her qualitative study, Stump asked students a series of questions about problems in multiple contexts, which included pictures, models, or graphs. The problems included contexts of steepness and ski ramps, percent grade of ramps, a bicycle experiment measuring wheel rotations, the cost of tickets for a junior class dance, growth rates related to height, and descriptions of slope (Stump, 2001). Students' audio-recorded answers were evaluated and coded. Stump's analysis of the students' responses revealed that students struggle with connecting the concept of slope to steepness, appropriately using the slope of a line to measure rate of change in real-world contexts, graphic representations of slope, understanding slope as a ratio, and proportional reasoning related to rate of change. These findings demonstrate that students had several gaps in their knowledge of the concepts of, and relationships between, slope, rate of change, and steepness (Stump, 2001). Stump's study was limited, in that it did not assess why these gaps of understanding might be occurring. Future research should focus on determining ways to address students' conceptual misunderstandings of slope, rate of change, and steepness.

Influence of Curricula Pathways on Students' Understanding
of Rate of Change

Teuscher et al. (2010) conducted a quasi-experimental study to determine if the curricula path students had been involved in throughout high school influenced their understanding of rate of change concepts at the beginning of an AP calculus course. In their study, 134 students that had been involved in traditional, single-topic curricula (i.e., wherein students' typical course route is algebra, geometry, algebra II, precalculus), as well as 57 students who had been involved in integrated curricula (i.e., secondary I, secondary II, etc.), were given a piecewise function task adapted from a released AP Calculus exam item (Teuscher et al., 2010). A piecewise function is characterized by its multiple segments, each with a different rate of change. Students' open-ended responses were evaluated using a rubric developed and reviewed by mathematicians and math educators. Teuscher et al. found that all of the students had confusion and misconceptions about the concepts of rate of change, slope, and steepness, in particular with understanding the meaning and importance of the sign (positive or negative) of slope. Curricular path of the students did not impact students' understanding of these concepts (Teuscher et al., 2010). Limitations of this study include unequal group sizes, a lack of statistical analysis, and a lack of experimental control. Future research should focus on more rigorous methodology to determine the effects of different curricular paths on students' understanding of rate of change concepts.

Teuscher and Reys (2012) conducted a similar quasi-experimental study to compare students who had participated in an integrated curricula approach and those who had participated in a traditional, single-topic curricula approach on their performance on calculus readiness topics, rate of change items, open-ended rate of change items, and errors committed on rate of change items. Teuscher and Reys' study included 4 10^{th}-grade students, 28 11th-grade students, and 161 12th-grade students at the beginning of an Advanced Placement Calculus course. Of the participants, 136 had participated in a traditional curricula approach, while 57 had participated in an integrated curricula approach. Students were given assessments involving a Piecewise Function Task and a Filling the Tank Task, which were both open-ended, and their responses were evaluated and compared (Teuscher & Reys, 2012). The Filling a Tank Task provided students with a context of filling a tank of water with a hose. The rate of change in the Filling a Tank Task was variable because the hose was turned off and on and involved different water pressures. Results indicated that there were no significant differences in performance between groups ($F = 3.54$, $p = 0.063$), which suggests that students from different curricula approaches performed similarly on the tasks (Teuscher & Reys, 2012). By analyzing students' responses and errors, Teuscher and Reys concluded that students had difficulty calculating, explaining, representing, and interpreting nonconstant rates of change. Additionally, less than half (47%) of all students involved in the study answered the rate of change items on the assessment correctly (Teuscher & Reys, 2012). Teuscher and Reys' study was limited, as it did not have experimental control, did not have a control group, and also included unequal group sizes. Additional research should focus on determining effective methods to approach students' misunderstandings of rate of change concepts.

Rate of Change Research in the Context of Algebra and Pre-Algebra

Researchers have also assessed students' understanding of rate of change concepts in algebra and pre-algebra (Hattikudor et al., 2012; Wilhelm & Confrey, 2003), as well as studied the effects of instructional approaches on students' performance on rate of change assessments (Kramarski, 2004; Kramarski & Mevarech, 2003). Wilhelm and Confrey (2003) interviewed four algebra students (grade not specified) in a qualitative study assessing students' abilities to relate rate of change concepts in the context of motion to the context of money. In their study, Wilhelm and Confrey presented students with computer simulations of rate of change depicted in a bank account, as well as person's speed measured by a motion detector. After having the students calculate and describe the rate of change in each of these contexts, Wilhelm and Confrey asked the students to represent the rate of change concept shown in the motion context using the computer simulation program for the money context. One of the students was able to correctly solve the rate of change problems in both the motion and money contexts, but was unable to accurately describe how to depict the motion context using the money context (Wilhelm & Confrey, 2003). Another student was able to accurately find and explain rate of change in the context of motion, but not money. This participant was unable to demonstrate how the motion context could be modeled using the computer simulation program for the money context (Wilhelm & Confrey, 2003). The remaining 2 students each demonstrated incomplete understanding of rate of change in singular contexts; 1 student struggled with the money context, while the other struggled with both the motion and the money context. However, both of these students were able to describe how the concept of rate of change was similar between the money and motion context, and model the motion context using the computer simulation program for money (Wilhelm & Confrey, 2003). Wilhelm and Confrey concluded that rate of change is a complex concept that many students struggle with, due to a lack of conceptual understanding of rate of change in real-world contexts. One of the limitations of this study was that participants were selected to be interviewed primarily due to their good attendance records and on-task behavior during the unit on linear equations (Wilhelm & Confrey, 2003). This participant selection methodology likely prevented Wilhelm and Confrey from obtaining a sample with a range of math skills. Additional research should focus on assessing the rate of change understanding of low performing students and students with disabilities.

Students' understanding of rate of change and the y-intercept. Hattikudor et al. (2012) conducted a quasi-experimental study evaluating sixth-, seventh-, and eighth-grade students' understanding of slope and y-intercept, based on their responses on two graph construction items on a written assessment. Hattikudor et al. were interested in comparing the performance of sixth-, seventh-, and eighth-grade students to be able to look at students' intuitions regarding slope and y-intercept prior to receiving instruction on these concepts (sixth-grade), as well as to analyze how instruction alters students' performance (seventh- and eighth-grade). Participants included 59 sixth-graders, 65 seventh-graders, and 56 eighth-graders (Hattikudor et al., 2012). Hattikudor et al. found that there was a significant difference based on grade level, as well as based on whether students were provided with quantitative or qualitative (nonnumerical) information for the graph, and regarding whether students were able to better graph slope or the y-intercept. These results indicate that as students got older, they had more correct answers, that students were better able to solve problems involving quantitative as opposed to qualitative information, and that students had more difficulty graphing the y-intercept than they did slope. One of the main limitations of this study was that the assessment may not have been sensitive enough to provide accurate analysis as it only included two assessment items. Hattikudor et al. suggested that future research be conducted to determine better ways to instruct students in graphing, with a focus on the y-intercept and understanding qualitative contexts.

Research on instructional methods and rate of change performance. Only two studies have involved manipulation of variables to evaluate the effects of different strategies on students' performance on rate of change concepts (Kramarski, 2004; Kramarski &

Mevarech, 2003). Kramarski and Mevarech (2003) conducted a pre/post group study with 384 eighth-grade students to evaluate the effects of four instructional methods on students' performance on linear graph concepts, including rate of change. Primarily, Kramarski and Mevarech sought to improve students' understanding of linear graphs during a 2-week (10 lesson) unit of instruction. The unit included the concepts of slope, y-intercept, and rate of change in both quantitative and qualitative contexts, as well as graphing algebraic equations (Kramarski & Mevarech, 2003). The instructional context prior to implementation of the experimental conditions was the same across all classrooms, and consisted of heterogeneous math classes (by ability) that met five times per week. For the linear graphing unit, each class used the same textbook and supplementary materials to learn strategies of graph interpretation (tables, graphs, verbal explanations, formulas, procedural steps; Kramarski & Mevarech, 2003). The four conditions included in the study were distributed equally across 12 randomly selected classrooms in four schools that were demographically similar (Kramarski & Mevarech, 2003). The four conditions included cooperative learning combined with strategy instruction, individualized learning combined with strategy instruction, cooperative learning without strategy instruction, and individualized learning without strategy instruction (Kramarski & Mevarech, 2003). The strategy instruction conditions involved students being trained and prompted to use a strategy involving comprehension questions. Students were provided with an acronym for these comprehension questions, DATA, which stood for "*D*escribe the x-axis and the y-axis; *A*ddress the units and the ranges of each axis; *T*ell the *T*rend(s) of the graph or parts of the graph; and *A*nalyze specific points" (Kramarski & Mevarech, 2003, p. 286). The cooperative learning condition involved students working in groups of four, which included 1 high achieving student, 1 low achieving student, and 2 average students. In these groups, students took turns leading the group in solving a problem. Each student read a problem out loud and tried to solve it. He or she explained it to his/her group, and if there was not a consensus among the group members regarding the answer, the students discussed the problem to come to an agreement (Kramarski & Mevarech). In the condition that consisted of both strategy instruction and cooperative learning, the DATA acronym was used to guide problem-solving and discussion. In the individual strategy instruction, students wrote their responses to the DATA prompts. The individual condition that did not include strategy instruction or cooperative learning served as a control group, in which students worked on problems individually without the DATA prompts.

Kramarski and Mevarech (2003) used a 36-item assessment to analyze students' graph interpretation and construction. Kramarski and Mevarech, using ANOVAs, determined that there was a significant difference in posttest scores based on condition. The cooperative learning combined with strategy instruction condition was the highest performing condition, followed by the individual condition combined with strategy instruction. The cooperative learning and individual groups were the lowest performing and significant differences were not found between the two nonstrategy instruction groups (Kramarski & Mevarech, 2003). Kramarski and Mevarech's findings demonstrated that students' performance on linear equation tasks may improve with the incorporation of cooperative learning and strategy instruction. However, Kramarski and Mevarech cautioned that methods for cooperative learning and strategy instruction vary, so the results are not generalizable to strategies that were not included in their research. Additional research should be conducted to evaluate other methods incorporating similar components as those researched by Kramarski and Mevarech.

Kramarski (2004) implemented a similar pre/post mixed methods group study to gain further information about characteristics that distinguished cooperative learning with strategy instruction from cooperative learning without strategy instruction. In this study, Kramarski compared two conditions: strategy instruction combined with cooperative learning, and cooperative learning alone. The two conditions were distributed evenly across six randomly selected eighth-grade classrooms with a total of 196 students (Kramarski, 2004). The researcher used the same DATA metacognitive strategy, as well as the same cooperative learning structures, described in the Kramarski and Mevarech (2003) study. Using the same 36-item graphing assessment as in the Kramarski and Mevarech (2003) study, Kramarski evaluated students' scores on that math assessment prior to and following a 2-week (10 lesson) unit on linear graphing, and also analyzed mathematical discourse that occurred among the groups. The researcher observed 24 small groups, which were randomly selected and evenly distributed across the six participating classrooms. Within each observation, Kramarski evaluated students' metacognitive behaviors, based on criteria that included the overall group interaction level, and individual behaviors. The individual behaviors evaluated included being off-task, working individually, providing/receiving technical help, providing/receiving the final answer with no elaboration, and providing/receiving elaborated explanations.

The results of ANOVAs indicated that both groups performed significantly better on the posttests, and that the condition involving both strategy instruction and cooperative learning outperformed the condition that included cooperative learning alone (Kramarski, 2004). Observation results indicated that the students involved in the condition with strategy instruction and cooperative learning had higher levels of group interaction, while students in the cooperative learning alone condition had higher rates of individual behaviors. Students in the condition with strategy instruction and cooperative learning also provided/received more elaborated explanations than the students in the cooperative learning alone condition, who provided/received technical help without elaboration more frequently (Kramarski, 2004). The results of the ANOVAs and observation data, when considered together, indicate that students' performance on linear graphing tasks may benefit from the incorporation of strategy instruction in a cooperative learning context. The main limitation of Kramarski's (2004) study is similar to that of Kramarski and Mevarech's (2003) study, in that its results are not generalizable to all strategy instruction or cooperative learning structures since there is variability in these methods. While the authors described this as an explicit limitation of their study, it is important to note that this limitation applies more broadly to the field's use of the terms "strategy instruction" and "cooperative learning" to refer to a wide range of interventions. Kramarski (2004) suggested that additional research be conducted to determine if incorporating instructional practices to help students model and visualize graphing concepts helps reduce errors with graph construction.

Summary of the Research on Rate of Change

Research conducted thus far on rate of change demonstrates that rate of change concepts, including slope and steepness, are areas of difficulty for students from sixth-grade through college (Bezuidenhout, 1998; Christensen & Thompson, 2012; Orton, 1984; Hattikudor et al., 2012; Herbert & Pierce, 2007; Herbert & Pierce, 2012; Stump, 2001; Teuscher, Reys, Evitts, & Heinz, 2010; Teuscher & Reys, 2012; Van Dyke & White, 2004; Wilhelm & Confrey, 2003). Additionally, integrated versus traditional curricula approaches do not minimize students' difficulties with the concepts (Teuscher et al., 2010; Teuscher & Reys, 2012). In particular, students struggle with relating and differentiating the concepts of rate of change, slope, and steepness from one another (Stump, 2001; Teuscher et al., 2010;

Teuscher & Reys, 2012; Wilhelm & Confrey, 2003), and with understanding the meaning and importance of the sign of slope (Teuscher et al., 2010; Teuscher & Reys, 2012). Students also struggle with interpreting and representing qualitative graphing contexts (Hattikudor et al., 2012), as well as nonlinear rate of change contexts (Herbert & Pierce, 2007; Teuscher & Reys, 2012). Research conducted on the influence of strategy instruction and cooperative learning demonstrated that methods incorporating structured reflection on linear graphing problems may improve students' performance on interpreting and constructing graphs, including rate of change problems (Kramarski & Mevarech, 2003; Kramarski, 2004). An approach incorporating writing to learn and a CRA instructional sequence is consistent with the research recommendations on rate of change, which call for further evaluation of additional strategies involving representation and strategy instruction. Given the absence of research on rate of change with students with disabilities, as well as the lack of research applying CRA to rate of change problems, evaluating the effects of an intervention comprised of writing to learn strategies and a CRA instructional sequence on the math performance of students with disabilities would provide a significant contribution to the research literature.

Rationale for the Proposed Study

The implementation of the CCSSM has the potential to advance math education in the U.S. and correct some of the current rates of failure for students with disabilities due to its increase in focus, coherence, and rigor, as well as its emphasis on conceptual understanding. However, the CCSSM will be difficult for both teachers and students due to its increases in rigor, as well as the requirement for students to be able to explain and justify their reasoning on assessments, both of which necessitate conceptual understanding and writing skills. Overall, WTLM has been shown to be effective in improving students' performance in math, as well as their ability to express their mathematical thinking in writing. The CRA instructional sequence has been shown to be effective in improving students' with disabilities performance in mathematics, specifically with rate of change concepts, because it builds students conceptual understanding. The CRA instructional sequence teaches students how to think about math concepts strategically, rather than relying on easily forgotten formulas. Therefore, the purpose of this study is to determine whether an intervention involving the CRA instructional sequence and WTLM principles improves students' with disabilities proficiency in solving rate of change problems.

Implementing the CRA instructional sequence with WTLM elements may help facilitate a deeper level of math understanding and extend students' learning beyond manipulating mathematical symbols and formulas. Both the CRA instructional sequence and WTLM approaches are premised on cognitive foundations that pinpoint the importance of multiple representations and methods of expression in gaining conceptual understanding of math concepts. While the CRA instructional sequence focuses on providing students with a method to break down and understand math concepts, WTLM helps students internalize and express their understanding of these concepts. Therefore, an intervention that incorporates both the CRA instructional sequence and WTLM could provide students with disabilities with the skills necessary to meet the rigorous demands of the CCSSM and assessments. Achieving higher levels of conceptual understanding of foundational math topics has the potential to unlock access to many college and career opportunities currently inaccessible to students with disabilities. It is therefore important to assess the effectiveness of a combination of these methods in teaching math concepts to students with disabilities in math to determine their potential instructional and/or remedial use.

Research Questions

1. What is the effect of implementing a concrete-representational-abstract (CRA) instructional sequence incorporating writing to learn math strategies on students' with disabilities proficiency in solving rate of change problems?
2. Do students with disabilities find WTLM math and a CRA instructional sequence to be socially acceptable?

CHAPTER 2

METHODS

The purpose of this study is to determine if a functional relationship exists between the use of a rate of change intervention incorporating the CRA instructional sequence and WTLM and students' accuracy on a rate of change assessment. A multiple-probe design across participants was used for this study, as this design allows for the examination of the effects of the intervention on students' scores on math and writing measures.

Research Questions

1. What is the effect of implementing a concrete-representational-abstract (CRA) instructional sequence incorporating writing to learn math strategies on students with disabilities' proficiency in solving rate of change problems?
2. Do students with disabilities find WTLM and a CRA instructional sequence to be socially acceptable?

Setting

The study was conducted in a large, suburban public high school in the intermountain west of the United States. There were approximately 33,714 students enrolled in this district, 4% (1,436) of whom were classified as English Language Learners, and 11%

(3,873) as students with Individualized Education Plans (IEPs; National Center for Education Statistics [NCES], 2014). The student body at the high school consisted of approximately 1,828 students, 90% of whom identify as White, non-Hispanic, 5% as Hispanic, and 4% as Asian/Pacific Islander. Students identifying as American Indian/ Alaska Native, Black, or of two or more races accounted for less than 2% of the school population collectively (NCES, 2014). Approximately 16% of the students attending this school were eligible for free or reduced meals (NCES, 2004).

The school site was scheduled on a trimester calendar, in which students attended five 66-minute classes per day for 12 weeks. Students receiving special education services identified as needing intensive supports in mathematics received their primary mathematics instruction in either a resource setting taught by special educator, with class sizes ranging from 6-10 students, or inclusive general education classrooms that were typically cotaught by a general education and a special educator. The majority of the students who received this level of special education supports for mathematics qualified for special education services under the categories of specific learning disability, other health impairment, emotional/behavior disturbance, or visual impairment.

Participants

Selection Criteria and Procedures

Participants were selected using a three-stage screening process. For the first gate, students were eligible for inclusion in this study if they were identified to receive special education services, and were recommended by their special education math teacher as being in need of additional instructional support for rate of change concepts. Students qualifying for special education services under the categories of Other Health Impairment, Autism, Specific Learning Disability, and Emotional and Behavior Disturbance were considered as potential participants.

Second, to be eligible for participation in the proposed study, participants had to score at or below the 25th percentile on the Basic Concepts component of the *KeyMath-3* (Connolly, 2007) assessment and be able to write a complete sentence. Students whose scores on the Basic Concepts components of the *KeyMath-3* assessment were above the 25th percentile were not eligible to participate in this study, based on these students likely not being in need of the targeted instruction included in the intervention.

Third, students completed one rate of change assessment developed by the researcher. Students who earned less than 70% of the available points on the assessment were eligible for inclusion in the study. A score of 70% on the rate of change assessment would indicate that the student was able to graph a function with variable rate of change, as well as find the average rate of change, with minimal errors. Students with scores of 70% or higher were not likely to be in need of this intervention, because the intervention specifically targeted finding the average rate of change. Students scoring 70% or above on the researcher-developed rate of change assessment were also unlikely to show significant growth due to entering the study with a relatively high level of rate of change knowledge and were therefore excluded from the study.

Selected Participants

Four participants met the criteria for selection, including parental consent and student assent. All 4 students were in ninth-grade, received special education services for math, and were enrolled in Secondary 1 math courses aligned with the CCSSM. Each student received their math education in a general education setting (3 students were in a cotaught class, and 1 student was in a general education class that was not cotaught), and was enrolled in a special education study skills class. The special education study skills class was taught by a special education teacher, and provided students with an opportunity to work on homework, meet with teachers, or access individual assistance as needed for any of their classes. Students participated in baseline, intervention, and maintenance sessions during their study skills class.

Sara

Sara was a 14-year-old ninth-grade Caucasian female receiving special education services under the classification of Other Health Impairment. Sara received special education services for Language Arts and Math. During the first and second trimesters of the academic year, Sara was in a cotaught Secondary 1 Math class. She did not have a math class during third trimester. Sara scored 71 points on the Basic Concepts section of *KeyMath-3*, which equates to a grade equivalent of approximately 2.9 and the 3rd percentile. Sara earned 20 points on the Spontaneous Writing portion of the *TOWL-4*, which equates to a composite index of 100 (average), and the 50th percentile. On the rate of change assessment, Sara received 4 points (14%) on the math items and 9 points (25%) on the writing items. Sara stopped coming to school at the end of January, and chose to withdraw from school in March and complete her high-school coursework online. As a result, Sara did not complete the final lesson of the intervention, the final intervention assessment, maintenance assessments, or the social validity questionnaire.

Toby

Toby was a 14-year-old ninth-grade Caucasian male who received special education services under the classification of Specific Learning Disability. In addition to receiving special education services for math, Toby also received services for Language Arts. Toby was in the same cotaught Secondary 1 Math class as Sara for the first and second trimesters of the school year. He did not have math during the third trimester. On the Basic Concepts component of the *KeyMath-3* assessment, Toby scored 77 points, which is a grade equivalent of approximately 4.4, and equates to the 6th percentile. On the *TOWL-4*, Toby scored 19 points on the Spontaneous Writing portion, which equates to the 45th percentile and reflects a composite score of 98 (average). On the rate of change assessment, Toby did not score points on any of the math or writing items.

Jason

Jason was a 15-year-old ninth-grade Hispanic male receiving special education services under the classification of Autism. Jason received special education services for Math and Language Arts. Jason was in the same cotaught Secondary 1 Math class as Sara and Toby during the first trimester. He did not have math during the second trimester. During the third trimester, Jason was in a cotaught Secondary 1 Math class taught by the same special and general educators that he had during first trimester. During the second and third trimesters, Jason did not have a study skills class. As a result, he participated in intervention and maintenance sessions during an elective class (e.g., basketball, school store) each trimester. Jason scored 71 points on the Basic Concepts portion of the *KeyMath-3* assessment, which equates to the 3rd percentile and approximately a 3.3 grade equivalent. On the Spontaneous Writing portion of the *TOWL-4* assessment, Jason scored 19 points, which equates to the 45th percentile and a composite index of 98 (average). Jason did not earn any points on the math items of the rate of change assessment. He earned 2 points (5%) on the writing items of the rate of change assessment.

Abigail

Abigail was a 15-year-old ninth-grade Caucasian female who received services under the classification of Specific Learning Disability. She received services only for math. During the first trimester, Abigail was in an inclusive Secondary 1 Math class that was not cotaught. She did not have math during the second trimester. During the third trimester, she was in the same cotaught Secondary 1 Math class as Jason. On the Basic Concepts portion of the *KeyMath-3* assessment, Abigail earned 80 points, which equates to the 9th percentile and an approximate grade equivalent of 4.5. On the Spontaneous Writing portion of the *TOWL-4*, Abigail scored 29 points, placing her in the 96% percentile with a composite score of greater than 130 (very superior). On the rate of change assessment, Abigail earned 4 points (14%) on the math items and 5 points (14%) on the writing items.

Measures

Screening Measures

Math Screening Measure

KeyMath-3 is a norm-referenced diagnostic assessment that is used to measure students' knowledge of mathematical concepts and skills (Connolly, 2007). The assessment is appropriate for use with students aged 4 years and 6 months to 21 years functioning at grade levels K-12. Statistical analyses have demonstrated *KeyMath-3* to have adequate internal-consistency reliability, alternate-form reliability, and test-retest reliability (Connolly, 2007).

KeyMath-3 has also been assessed for content and construct validity (Connolly, 2007). The *KeyMath-3* assessment has a high degree of reliability and validity for students with and without disabilities, making it appropriate to use for participant identification.

The test includes 10 subtests covering basic concepts, operations, and applications (Connolly, 2007). The subtests include numeration, algebra, geometry, measurement, data analysis and probability, mental computation and estimation, addition and subtraction, multiplication and division, as well as foundations of problem-solving and applied problem-solving (Connolly, 2007, p. 3). The items on the *KeyMath-3* assessment are aligned with the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (NCTM, 2000; as cited in Connolly, 2007, p. 1), as well as the state standards for the state in which this study was conducted. For the purposes of participant identification in this study, students completed the subtests that make up the Basic Concepts component of the *KeyMath-3* assessment. These subtests include numeration, algebra, geometry, measurement, and data analysis and probability (Connolly, 2007). Scores on the *KeyMath-3* assessment are reported as raw scores that are then converted into scaled scores. Percentile ranks for each grade level or age equivalent are then determined based on the scaled scores. For this study, students' scores are reported as scaled scores and percentile ranks. Participant eligibility was determined based on the students' percentile rank.

Writing Screening Measure

TOWL-4 is a norm-referenced test of written language used to identify students in need of writing supports, identify students' strengths and needs in writing, and as a measurement tool in writing research (Hammill & Larsen, 2009). The *TOWL-4* is appropriate for use with students aged 9 years old through 17 years and 11 months old

(Hammill & Larsen, 2009). Four different measures of reliability have been evaluated for the *TOWL-4*, all of which demonstrate that the *TOWL-4* has high reliability. The *TOWL-4* has been assessed for internal consistency, alternate-form reliability, test-retest reliability, and reliability of scoring procedures. The *TOWL-4* has also been assessed for content-description validity, criterion-prediction validity, and construct-identification validity (Hammill & Larsen, 2009). The results of these analyses demonstrate that the *TOWL-4* is a reliable and valid measure of writing that can be used to effectively assess students' writing performance.

There are two test forms of equal difficulty of the *TOWL-4*, which allows for students' writing to be assessed with the *TOWL-4* twice. The three aspects of writing (conventional, linguistic, and conceptual) included in the *TOWL-4* are represented in two assessment formats (Hammill & Larsen, 2009). The Contrived writing format primarily measures students' spelling, punctuation, and word usage, whereas the Spontaneous writing format measures students' functional writing ability through their construction of passages (Hammill & Larsen, 2009). There are 7 subtests for both the Contrived and the Spontaneous test formats, which include vocabulary, spelling, punctuation, logical sentences, sentence combining, contextual conventions, and story composition (Hammill & Larsen, 2009, p. 5). For the purposes of gathering information on their initial writing abilities, students completed only the Spontaneous writing component of the assessment. The Spontaneous writing component includes two areas of evaluation, one for contextual conventions and one for story composition (Hammill & Larsen, 2009). The scores for each of the two areas of evaluation are reported as raw scores, which are then converted into scaled scores that are matched with percentile ranks based on grade level or age equivalency. For this study, students' scaled scores and percentile ranks are reported. Students were eligible to participate in this study if they could write at least one complete sentence.

Dependent Variable Measures

I created a pool of 20 assessments to measure growth in solving rate of change problems for use during the baseline and intervention phases of the study (see Appendix A for full set of assessments). I developed the assessments based on tasks and tests found through the Mathematics Assessment Resource Service (MARS; Mathematics Assessment Resource Service, University of Nottingham, 2007-2012). The MARS has compiled and drafted various tasks and tests in line with the Common Core State Standards for Mathematics (CCSSM), based on proposed assessments for the CCSSM (Mathematics Assessment Resource Service, University of Nottingham, 2007-2012).

The format of the MARS assessments and tasks primarily consists of applied problems in which students respond to multiple questions regarding a specific mathematical context that could be encountered in everyday life (Mathematics Assessment Resource Service, University of Nottingham, 2007-2012). Included within these questions are prompts for students to justify and/or explain the processes used in calculating the answers. I modeled the rate of change assessments on these assessments to ensure that the measures were consistent with the demands of the upcoming CCSSM assessments.

A math expert reviewed the assessments to ensure they were of equal difficulty, sensitive to change, and appropriate for use with ninth-grade students with disabilities. The assessments were revised based on her feedback. Revisions were made to constrain the complexity of math tasks, in part to increase the likelihood that the different forms of the assessment were equal difficulty, and that the assessment tested concepts taught in the intervention, rather than assessing students' basic math skills. Revisions included limiting the number of digits of values in problems, eliminating decimals, and using basic fractions. Specifically, the values provided in problems were no longer than two digits, although answers could include three digit values. Similarly, none of the values provided in the problems included decimals, but answers could include decimals. Unit fractions (i.e., ¼, ½, 3/5, etc.) were used to reduce errors made due to difficulties students had with fraction concepts. Expert review of researcher-designed assessments has been used to demonstrate validity of assessments in similar studies, largely due to the lack of research validated math progress monitoring measures for use with older students (Strickland & Maccini, 2012; Witzel, 2005; Witzel, Mercer, & Miller, 2003). The assessments covered the objectives of the intervention (which pertain to being able to solve rate of change problems using equations, graphs, or tables) and were of equal difficulty to allow for sensitivity to change in student performance.

There were two components of the rate of change assessment, an applied math computation component and a written constructed response component. The applied math computation component of the rate of change assessment was worth a total of 28 points and involved students completing five problems. Two problems involved students finding the slope of a line from a graph. On each assessment, one of these problems involved a line going through the origin (0,0) of a graph, and one that did not go through the origin. Additionally, one of the graphed lines had a negative slope and one had a positive slope. The order of which of these problems went first was randomized, and an equal number of lines that went through the origin were positive and negative. The next two problems asked students to find the missing value (one missing x and one missing y per assessment) when provided with the slope and a context. For each assessment, one of these questions included a fraction and one included a y-intercept. The order of these problems was also randomized.

The final math item involved answering multiple questions about a variable rate of change contextual problem. For example, students were presented with the following problem:

> A motorcyclist leaves Salt Lake City at 10:00 am and travels for an hour at 30 miles per hour, then for an hour at 60 miles per hour. The motorcyclist stops for lunch for 30 minutes, then travels for 3 hours at 45 miles per hour.

Students were then directed to make a table and graph depicting the times and distances traveled at each stage of the journey, calculate the average speed for the whole journey, find how far from Salt Lake City the motorcyclist had traveled by 2:30 pm, and determine when the motorcyclist was approximately 110 miles from Salt Lake City.

Each assessment followed the same format, consisting of students finding the slope of a graphed line for the first two problems, determining the missing value of two constant rate of change problems, and for the variable rate of change problem, completing a table and graph and calculating the average rate of change. Students received 2 points for finding the slope of a line from a graph for each of two problems, 2 points for finding the missing y value in a missing value problem, and 2 points for finding the missing x value in a missing value problem. For the variable rate of change problem, students received 2 points for graphing each phase of the problem correctly, for a total of 8 points. Students also received 2 points for correctly inputting the values for each phase in a table, for an additional 8 points. Students received 2 points for finding the average rate of change. See Table 1 for an example of how the applied math computation component of each assessment was scored.

The second component of the rate of change assessment was the written constructed response component, which measured students' ability to express mathematical reasoning and explanations through writing. Scores from this component served as a secondary dependent variable for this study. The written constructed response component was also reviewed by a math expert and revised based on feedback. The written constructed response component consisted of 4 open-ended prompts worth 9 points each. Two written constructed response items on each assessment required students to provide explanations for their answers, and two items required students to provide justifications for their answers. Students responded to the following two questions based on one of the constant rate missing value problems: (1) "Describe in writing the process you used to solve the problem above"; (2) "Explain how you know your answer makes sense. Provide an example to support your reasoning." For the variable rate of change problems similar to the one referenced above, students responded to the following prompts and questions: (1) "Explain the steps you used to determine the average speed for the whole journey"; and (2) "How do you know your answer is correct?" Each of the students' four written constructed responses on each assessment were evaluated based on whether they were factually correct (3 pts), included appropriate and specific mathematical language (3 pts; e.g., added, divided, rate of change, etc.), and either used appropriate labels and specific quantities (3 pts; for explanation type questions) or data and/or warrants to support their justifications (3 pts; for justification type questions). For each of the criteria, students received 0 points for an answer that was not proficient, 1 point for an answer that showed some emerging skills, 2 points for an answer that was approaching proficiency, or 3 points for an answer that was proficient (see Tables 2 and 3 for more detailed scoring rubrics). A total of 9 points could be earned for each of the 4 written constructed response prompts, totaling to 36 points per assessment. See Table 2 for an example of how explanatory written constructed response items were scored, and Table 3 for an example of how justification written constructed response items were scored.

Each student received two overall scores on each assessment, each of which were reported as points and percentage of points earned. For the applied mathematics computation components, students' scores were summarized as a percentage calculated by dividing the number of points earned on the math prompts by the number of points possible for the math prompts. The total points earned for the applied math computation component on each assessment were graphed and used as the primary dependent variable in this study. For the written constructed response component, students' scores were also summarized as a percentage, calculated by dividing the number of points earned for the written responses by the total number of points possible for the written responses. The total points earned for the written responses on each assessment were graphed as well. Throughout the course of the study, I administered 12-15 rate of change assessments to each student. When administering the assessments, I read the questions out loud to the student to prevent the risk of reading difficulties influencing scores on the rate of change assessments. Administration of each assessment took approximately 10-30 minutes.

Social Validity Questionnaires

To evaluate the second research question, whether students with disabilities found the rate of change intervention to be socially acceptable, participating students completed a social validity questionnaire, which was modified from the Children's Intervention Rating Profile (CIRP; Arra & Bahr, 2005; see Appendix B). The CIRP has been used in previous studies to evaluate the social validity of math interventions (Arra & Bahr, 2005). The social validity questionnaire assessed the appropriateness of the intervention, as well as its perceived effectiveness.

The questionnaire included six items to be rated from 1-6 on a Likert scale (1 indicating strongly disagree, 6 indicating strongly agree). The items were as follows: (1) using

POD✓, cubes and diagrams is a helpful way to teach math; (2) using POD✓, cubes and diagrams to teach math is too hard; (3) using POD✓, cubes and diagrams to teach math may be hard for other students; (4) there are better ways to teach math to students than using POD✓, cubes and diagrams, (5) using POD✓, cubes and diagrams is a good way to teach math to other students; (6) I like using POD✓, cubes and diagrams to learn math; and (7) I think that using POD✓, cubes and diagrams to teach math will help students do better in school. Students' social validity questionnaire responses were measured using the questionnaires as permanent products. The mean, median, and range of responses were reported per item as well as for the questionnaire overall. Each of these values were reported per student and for the overall sample of students participating in the intervention.

Design

A multiple-probe design across participants was used in this study to determine if a functional relationship existed between the use of a rate of change intervention incorporating the CRA instructional sequence and WTLM and students' total points of correct math and writing responses on a rate of change assessment. A multiple-probe design was the most appropriate to use for this study because it was expected that the baseline data would likely remain stable and not increase without exposure to instruction or intervention on solving rate of change problems. Using a multiple-probe design helps prevent complications that arise within multiple-baseline studies related to extended baseline or intervention phases, such as diminishing intervention effects due to participant boredom with content, or concerns with delaying a potentially helpful intervention (Johnston, 2011). Additionally, as this intervention pertained to academic knowledge/ skill acquisition, a reversal design would not have been appropriate because it was unlikely that a return to baseline phase would result in any substantial decrement in skills (Johnston, 2011). A multiple-probe design allows for a functional relation to be established between the rate of change intervention and students' total points of correct math and written responses because the design can demonstrate behavior change at least three different points in time (Johnston, 2011).

This study began with each of the 4 participants completing a rate of change assessment. Then, baseline assessments were administered at least two times per week with Sara until the data were stable, and for at least 3 points. Baseline assessments were also administered at least two times per week with Toby until Sara began intervention. Upon the intervention phase starting for Sara, Toby and Jason each completed an additional baseline rate of change assessment. Intervention assessments were administered for each participant in the session immediately following the completion of Lessons 2, 4, 6, and 7. Baseline assessment administration for Toby and Jason aligned with intervention assessment administration for Sara, until each participant began intervention. Decisions about phase changes were made based on students' response to intervention corresponding with the primary dependent variable, students' total points of correct math responses on a rate of change assessment. A response to intervention was defined as a score on an intervention assessment that was at least 10% above the student's median baseline score. Once Sara demonstrated a response to intervention, and Toby's baseline data were stable, he began intervention. This pattern of assessment was repeated for Jason and Abigail. Abigail was added as a study participant later than the other 3 students. Her baseline data did not begin until each of the other participants had begun intervention. The intervention phase for each participant lasted until they completed the seventh lesson and fourth intervention assessment. At that point, each participant moved into the maintenance phase, which consisted of completing one maintenance assessment per week for up to 6 weeks.

Procedures

Participant Selection

As the researcher, I met with the main school contact person, a special education math teacher, to discuss the teacher's recommendations for students whom she believed were a good fit for the rate of change intervention. I sent home consent letters to the parents of the students referred to the intervention by the teacher. Parents had a 2-week period to contact me with any questions and return the signed consent forms. One week after the consent letters were sent home, I made follow-up phone calls to parents who had not returned the consent letters. In the follow-up phone calls, I reminded the parent of the study, and asked if they needed any additional information about the study and the study procedures. I communicated to the parents that the study was completely voluntary. Students whose parents provided informed consent were then given the opportunity to assent to participate in the study. During the assent process, I provided each participant with an assent letter, explained the purpose and procedures of the study, as well as potential risk and benefit to the student. I conducted the assent process individually with each student. Students who assented to participate were assigned a pseudonym to maintain confidentiality. They were then given the *KeyMath-3* and *TOWL-4* writing assessments, as well as one of the researcher-developed rate of change assessments. I administered each of these assessments in a quiet, private setting. The *KeyMath-3* assessment required a 60-90 minute block of time, and the *TOWL-4* assessment required a 30-minute block of time. The rate of change assessment required approximately 10-30 minutes. To minimize any potential frustration, students completed only one assessment during each assessment session.

Assessments were administered during a block of time determined to be the least intrusive to the student's schedule (e.g., during a study skills class period, before or after school, or during a nonacademic content class such as Physical Education). I scored and evaluated each student's assessments to determine which students met the inclusion criteria for the study.

Rate of Change Intervention Implementation

File Reviews

After the participant enrollment process, I conducted file reviews of the students who assented (and parents consented) to participate in the study to gather demographic information. To complete the file reviews, I reviewed the information in the students' school and special education files to determine information regarding students' levels of performance (as indicated by their Individualized Education Plans), assessment scores (on the Utah CRT as well as norm-referenced and standardized measures), and disability category. While collecting demographic information, I started the baseline phase of the study.

Baseline

Prior to administering any baseline assessments, I randomized the set of 20 rate of change assessments. Each participant received the same order of assessments across baseline and intervention phases. To control for any potential variability between forms, each student took the same form of the assessment on the same day. This meant that some forms of the assessment were skipped to allow for alignment between participants. If a participant was absent on the day for which an assessment was scheduled, the student completed the scheduled assessment form as soon as they returned. If the student was absent for more than one week, then the next form in the set was administered.

During the baseline phase, I administered two rate of change assessments per week for Sara and initially for Toby. Once Sara began intervention, I administered baseline rate of change assessments for Toby and Jason on the same days Sara was given intervention assessments. Due to Abigail beginning the baseline phase later than the other participants, she completed at least two baseline assessments per week. During the baseline phase, students participated in the traditional instruction provided in their math classes, and completed baseline assessments in a one-on-one setting with me. Each of the selected participants received math instruction in inclusive math classes in the general education setting, each of which were cotaught by general and special education teachers. Abigail did not have a math class during second trimester, when her baseline phase began. Traditional instruction in the math class was aligned with the CCSSM and district curriculum map. The concepts of rate of change and variable rate of change were covered in the school's math curriculum during the first trimester, which is also when all participants had a math class. The special education teacher provided supplementary support and accommodations for each of the students as needed. The accommodations that were most often employed included reductions in the number of problems students were required to complete, extended time, and individual assistance with concepts in study skills classes or after school. The primary teaching method the special and general education teachers employed in their classroom was explicit instruction, with some lessons incorporating hands-on activities.

Rate of Change Intervention

Once the baseline data were stable, as indicated by the last 3-5 assessments of the phase having within 10% variability of the mean of the baseline (O'Neill, McDonnell, Billingsley, & Jenson, 2011), the intervention phase began for Sara. The intervention phase for each participant continued until each of the seven intervention lessons were completed. The intervention phase of the study consisted of students participating in lessons delivered by me in a one on one setting. Students were pulled out of their special education study skills class or elective classes (if they did not have a study skills class) to participate in the intervention phase of the study. Due to the multiple-probe design, each student started the intervention phase at different points in time. Each student was exposed to the same seven core intervention lessons, in the same order, to reduce the risk of conflicting variables.

Some students also completed review lessons, based on their scores on the math portion of the assessments during intervention. Review lesson 1 was administered if a student did not demonstrate an increase of at least 10% above the median of their baseline scores by the second intervention assessment. Review lesson 1 was administered between lessons 4 and 5. Review lesson 2 was administered if a student demonstrated either a decrease or a plateau in scores on their third intervention assessment. Review Lesson 2 was administered between Lessons 6 and 7.

Content of intervention lessons. The content of the lessons included in this intervention addressed two standards from the Secondary I core of the CCSSM (National Governors Association Center for Best Practice, & Council of Chief State School Officers, 2012b). These standards include F.IF.4, which focuses on students' knowledge of key components of graphs and tables related to quantities, and F.IF.6, which consists of students' ability to find the average rate of change of a function using a variety of methods (National Governors Association Center for Best Practice, & Council of Chief State School Officers, 2012b). Appendix D includes a more detailed description of the intervention objectives. The lessons for this intervention were organized into two main units, each of which targeted one of the aforementioned standards. Please refer to Appendix E for a full unit plan of the intervention lessons as well as Appendix F for the individual detailed lesson plans.

The first two lessons covered standard F.IF.4, and focused on the foundations of graphing and basic knowledge of linear equations (National Governors Association Center for Best Practice, & Council of Chief State School Officers, 2012b). In lesson 1, I focused on students' ability to identify the slope and y-intercept of linear equations, identify and match graphed linear equations with their contexts and tables, as well as review the different types of slope (positive, negative, zero and undefined). In lesson 2, I taught students how to graph one- and two-step equations when provided with contextual linear equation problems. In lesson 3, I taught students to find missing values in constant rate of change problems. Lesson 3 also provided students with more opportunities to practice skills and concepts covered in the first two lessons.

The second unit of the intervention addressed calculating the average rate of change of a function with a variable rate of change, which is covered in standard F.IF.6 (National Governors Association Center for Best Practice, & Council of Chief State School Officers, 2012b). Lesson 4, the first lesson in this unit, connected the concepts on constant and average rate of change by having students find the constant rate of change from contextual linear equation problems. Lessons 5, 6, and 7 each covered the same objective and involved students finding the average rate of change of a function with a variable rate of change, as well as depicting the problem using a graph and table. In each lesson, particular emphasis was given to students' explaining how and why the rate changes throughout the problem. The primary difference between lessons 5, 6, and 7 was the level of difficulty. In lesson 5, students worked with problems involving two or three rate phases. For example, a problem with two rate phases could consist of a description of a motorcycle traveling for 3 hours at 30 miles per hour, then 2 hours at 40 miles per hour. In lesson 6, students worked with problems involving three or four rate phases. In lesson 7, students worked with problems involving four rate phases. Lessons 6 and 7 also incorporated the student providing more detailed explanations of how and why the rate changed throughout the problem.

In addition to the core seven lessons, there were two review lessons. The first review lesson was administered between lessons 2 and 3, based on whether students had shown at least a 10% increase in scores during intervention compared to the median of their baseline assessments. The objectives addressed in the first review lesson included identification of the slope of a linear equation from a graphed line, as well as solving and graphing one and two step linear equations when provided with a context. Both of these objectives align with standard F.IF.4. The second review lesson occurred between lessons 6 and 7, and was administered if students either showed a decrease or plateau in scores on their third intervention assessment. The second review lesson focused on identifying the slope of a linear equation from a graph, solving one and two step equations when provided with a context (standard F.IF.4), and finding the average rate of change of a function with a variable rate of change (standard F.IF.6).

Instructional delivery. The rate of change intervention consisted of 7 core lessons on rate of change and 2 review lessons that were administered based on individual students' needs. Instruction was delivered at least 3 of the 5 days each week during a 20-40 minute block of time, based on student availability. I worked with students during their study skills class. The priority of the study skills class was for students to complete other course work, get help with content with which they were struggling, or make up tests. Therefore, I was limited in the amount of time I worked with students each week to accommodate their needs to complete other coursework, homework, or meet with teachers during the study skills class. Each lesson (with the exception of lesson 1) began with an explicit review of material covered in previous lessons. After the introductory review, each lesson followed the format of explicit teacher modeling and explanation of the lesson's objectives, guided practice in which the student practiced completing tasks/problems pertaining to the lesson's objectives, and individual practice in which the student individually completed tasks/problems related to the lesson's objectives. For example, in a lesson focusing on teaching the student how to classify slope, the direct instruction component of the lesson consisted of me explaining the definitions and differences between positive, negative, zero, and undefined slope, using concrete objects and diagrams. While I explained these concepts, the student filled in a graphic organizer for vocabulary. After the guided practice, the student and I completed two activities. The first activity involved the student pointing to graphs to correctly identify positive, negative, zero, and undefined slope. Second, the student completed a matching activity that involved correctly matching a graphed equation, completed table, written equation, and contextual problem for a total of four problems. The representations of each problem (graph, equation, table, context) were shown on cards that the student placed together to form a problem with all four of its representations. I provided additional explanations, corrective feedback, and praise as appropriate throughout the activities. The independent practice entailed the student independently answering 2 questions. For each question, the student was provided with one representation of a problem (graph, equation, table, context) and was asked to complete the missing representations. In addition, the student also classified the slope in each of the problems as positive, negative, zero, or undefined. The final step in each lesson involved the student completing an exit slip, which consisted of one or two problems aligned with the lesson objectives for that lesson. The student completed the exit slip problems independently, which allowed me to gather data on an ongoing basis to help tailor each lesson to the specific needs of each student. I went over the exit slip with the student at the beginning of the following lesson. I reviewed or retaught concepts the student was struggling with, and provided praise and encouragement for concepts for which the student demonstrated growth.

Each lesson incorporated the CRA instructional sequence. An integrated approach to CRA was used for this study, rather than isolating the concrete, representational, and abstract levels of the sequence to specific lessons. The CRA integration strategy, which has been used in previous studies focused on improving the math skills of students in eighth- and ninth-grades, provides a more flexible way to incorporate concrete and representational depictions of secondary math concepts (Strickland & Maccini, 2012). I taught the lesson concepts using concrete manipulatives (i.e., blocks), representations (i.e., pictures/diagrams) and abstract depictions of the concepts (i.e., formulas). During the guided practice, I assisted the student in using each of these methods (C-R-A) to gain an understanding of the lesson concepts. During the individual practice, the student was allowed to independently solve problems related to that lesson's material using any or all of the CRA methods. For example, if the lesson involved finding the missing value in a linear equation, the independent practice would also involve the student finding the missing value in a linear equation. The concrete manipulatives used for this study consisted of interlocking centimeter cubes, which could be stacked and lined up to represent various rate of change concepts. The representational strategies used included drawings of the cubes, diagrams of the stages in rate of change problems, as well as supplementary diagrams used to address gaps in students' understanding of foundational concepts (e.g., fraction bars to help understand what fractional rates meant conceptually).

In addition to the CRA instructional sequence, I discussed the lesson content with the student, focusing on encouraging the student to explain his/her reasoning. To include WTLM principles, the guided practice and individual practice each included at least one writing prompt asking the student to explain how he/she solved the problem/completed the task, and why he/she knows his/her answer is correct. I taught the student a specific writing strategy, the POD✓, which stands for Propose, Outline, Defend. The ✓ component of the POD✓ prompted students to check their work using four main steps: re-read the problem, check to make sure the problem is set up correctly, check calculations, and check for any common mistakes. The POD✓ strategy was taught alongside the math concepts, and a POD✓ graphic organizer was completed throughout the majority of the problems in each lesson.

The POD✓ math writing heuristic consisted of three main components that prompted students to write explanations of their math problem-solving processes. The first step of the POD✓ asked students to *Propose* the problem. In this step, students wrote down what they are asked in the math problem at hand, and listed the information that they were provided with in the problem. The second step prompted students to *Outline* how they would solve the problem. In this step, students wrote down steps for how they would solve the problem. The final step of the POD✓ math writing heuristic had students *Describe* and *Defend* their answer. In this final step, students first used words to describe the process used to solve the problem, and then explained how they knew their answer made sense, using pictures or an example for support. I provided the student with a POD✓ graphic organizer (See Appendix C) to assist him/her in completing the writing prompts. Through the use of the POD✓ writing heuristic, I aimed to help students improve their abilities to write mathematical explanations and justifications, as well as work through math problems. The explicit reviews at the beginning of each lesson incorporated memorization strategies to help students remember the components of the POD✓ strategy and how to use them to work through problems. Students were asked to complete a POD✓ graphic organizer for at least one of the problems on each exit slip as well, which allowed me to evaluate students' use of the strategy and their mathematical writing.

Behavior management system. I explicitly defined and taught each student the behavior I expected students to display during each intervention or assessment session. The three behavioral expectations were the following: (a) Be respectful; (b) Follow directions; and (c) Be a persistent problem solver. Be respectful was defined as keeping hands, feet and other objects to yourself, as well as using appropriate language free of put-down or derogatory comments. Follow directions was defined as the student doing what they were told to do the first time they were told to do so. Be a persistent problem solver was defined as the student trying their best on each problem, and attempting all problems in each lesson or assessment. Students' appropriate behavior was reinforced using a token economy system. Students were able to earn a maximum of 2 points per each of the three expectations each day. Students could cash in their points every two weeks for small tangible or consumable items (e.g., granola bars, pencils, etc.).

Intervention data collection. I administered rate of change assessments during the sessions immediately following the completion of Lessons 2, 4, 6, and 7 for each student. Intervention assessment administration followed the same procedures previously described for baseline assessment administration. On days the assessments were administered, students were pulled out of their study skills or elective class as they typically were for intervention instruction, but students completed assessments rather than participating in intervention lessons. I administered the assessments, reading the prompts aloud in a quiet and controlled environment. I then scanned and emailed the completed assessments to the graduate student designated to score assessments. The graduate student scored the math and writing portions of each assessment and emailed completed scoring rubrics back to me. I then graphed the total points students earned for both math and writing sections on each assessment to have an ongoing record of how students were responding to the intervention. While the rate of change assessments included points for math and writing, these two components of the assessments were scored and graphed independently to allow me to evaluate the effects of the intervention on students' writing and math performance. Social Validity data were collected for each participant following completion of the final intervention assessment. Each student completed the CIRP during the block of time they typically were pulled out of their math class to participate in the intervention lessons and assessments. To provide descriptive information, the amount of time per intervention session, as well as per lesson was recorded for each student. This information was further analyzed to determine the average lesson length per participant and across participants, as well as the average session length per participant and across participants.

Maintenance

Each student completed one maintenance rate of change assessment per week between 1 and 7 weeks following the completion of the fourth intervention assessment. Rate of change assessment administration during maintenance followed the same administration procedures described for intervention assessment administration. Students' percent of correct math and writing responses were evaluated and graphed for maintenance. If a student's maintenance scores on the math portion of the rate of change assessments fell below the median score of their intervention math scores by the third maintenance assessment, students completed a Booster Lesson that reviewed key concepts of the intervention. Booster Lessons were completed following the same procedures described above for intervention lessons.

Data analysis

Data were analyzed throughout the study on an ongoing basis to allow me to make adjustments to the intervention as necessary. Data analysis that occurred throughout the study provided me with information regarding when to make phase changes. Decisions regarding phase changes were made based on progress of the primary dependent variable, students' total points on the math items of the rate of change assessment. Once the intervention was completed, I analyzed all of the participant data. Graphs of the data were visually inspected to determine the effects of the intervention on the students' total points for correct math and writing responses on the rate of change assessments. The results of the social validity questionnaires were calculated and reported for each participant (using pseudonyms), as well as for each of the items included within the questionnaire.

Treatment Integrity Procedures

Fidelity of lesson implementation. I collected treatment integrity data for 100% of the lessons administered by completing a checklist of key components of lesson procedures, which were determined individually for each lesson. The key components included on each checklist were assessed as either present or not present within the lesson, and I completed each checklist as I administered each lesson. The minimum acceptable percentage of fidelity of implementation was 95%. This minimum percentage is consistent with the fidelity reported in the CRA research base, which ranges from averaging between 98-100% per study (Butler et al., 2003; Flores, 2009; Mancl, Miller, & Kennedy, 2012; Miller & Kaffar, 2001; Morin & Miller, 1998; Strickland & Maccini, 2012).

Interrater agreement on fidelity of lesson implementation. An undergraduate student was selected to complete interobserver agreement (IOA) on fidelity of lesson implementation for at least 30% of all lessons administered. Prior to completing IOA on any of the lessons, I trained the undergraduate student using recordings of sample lessons. The undergraduate student and I each completed fidelity checklists for three lessons, which we then compared for agreement. Agreement during training was 100%. Lessons for which IOA was assessed were then randomly selected and evenly distributed across participants and lessons. To complete IOA, the undergraduate student listened to audio recordings of selected lessons. I audio recorded all lessons administered with each student, and was blind to which lessons were selected for IOA on fidelity of lesson implementation. The undergraduate student completed the corresponding lesson fidelity checklist for each selected lesson. After IOA was assessed for each of the selected lessons, the undergraduate student provided the completed lesson fidelity checklists to me, and I calculated the overall percentage of agreement. I calculated the overall percentage of agreement by dividing the total number of agreements by the total number of agreements plus disagreements.

Fidelity of assessment administration. To evaluate the fidelity of assessment administration for baseline, intervention, and maintenance assessments, I completed assessment fidelity checklists for 100% of the administered assessments as I administered each assessment. The assessment fidelity checklist included procedural elements of assessment administration, including reading scripted directions before administering each assessment, reading each question aloud to each student, and providing only appropriate clarifications or prompts in response to student questions (see Table 4).

Interrater agreement on fidelity of assessment administration. A second undergraduate student was selected to complete IOA on fidelity of assessment administration. Prior to completing IOA on any assessments, I trained the undergraduate student using recordings of sample assessment administration sessions. The undergraduate student and I independently completed assessment fidelity checklists, then compared our agreement. Agreement on the fidelity of the practice assessments was 100%. I then randomly selected 30% of the assessments for IOA. The assessments were randomly selected and evenly distributed across phases and participants. I audio recorded 100% of the assessments administered to each participant in each phase. The undergraduate student listened to audio recordings of each of the selected assessments and completed an assessment fidelity checklist for each one. The undergraduate student provided the completed assessment fidelity checklists to me, and I calculated agreement by dividing the total number of agreements by the total number of agreements plus disagreements.

Assessment Integrity Procedures

Assessment scoring procedures. To prevent the risk of my bias influencing assessment scoring, a graduate student scored 100% of the baseline, intervention, and maintenance assessments. I trained the graduate student in scoring procedures, using the previously described rubrics and sample assessments. The graduate student and I scored four assessments together, to provide opportunities to discuss any disagreements in scoring. The graduate student and I then independently scored another four assessments, and then I evaluated our agreement by dividing number of agreements by number of agreements plus disagreements. The graduate student and I continued this process until at least 95% agreement was achieved for scoring the math component of the assessments, and at least 90% agreement was achieved for scoring the writing components of the assessments. By the end of one 3-hour training session, agreement on scores on the math component of the assessment was 98% and agreement on scores on the writing component of the assessment was 90%.

After each assessment was administered, I scanned and emailed the assessment to the graduate student. The graduate student used the previously described rubrics to score the math and writing components of each assessment, and returned the completed rubrics to me, and I then recorded and graphed the scores.

Interrater agreement on scoring of assessments. To provide interrater agreement on the scoring or assessments, I scored at least 50% of the assessments administered. I randomly selected assessments for which to evaluate IOA, and evenly distributed these assessments across participants and phases. After scoring each of the selected assessments, I compared my scores with those of the graduate student and calculated agreement by dividing the total number of agreements by the total number of agreements plus disagreements. The minimum acceptable percentage of agreements in scoring was 90% for the math components of the assessments. If agreement percentages fell below 90%, I provided a refresher training session on scoring procedures. Within the CRA research base, the range of interrater agreement in scoring of dependent measures is 93-100% (Butler et al., 2003; Flores, 2009; Mancl, Miller, & Kennedy, 2012; Miller & Kaffar, 2001; Morin & Miller, 1998; Strickland & Maccini, 2012). In this study, interrater agreement was calculated separately for the applied math computation and the written constructed response components of the rate of change assessments. Within the WTLM research base, no intervention studies have reported interrater reliability procedures or results. Conducting interrater agreement on written responses is valuable because writing is more subjective, and therefore could be more prone to lower percentages of agreement. While agreement is typically high for CRA studies (Butler et al., 2003; Flores, 2009; Mancl, Miller, & Kennedy, 2012; Miller & Kaffar, 2001; Morin & Miller, 1998; Strickland & Maccini, 2012), it is expected that agreement on the writing portion of the rate of change assessments used in this study might be slightly lower, due to writing being more subjective than math.

Table 1. Scoring Rubric for Applied Math Computation Problems on Rate of Change Assessments

Problem/Task	Not Proficient	Emerging	Proficient
M.1: Find the slope of the line from a graph. The student must correctly identify the correct numerical value of the slope, as well as the correct sign (+ or -) of the slope.	0	1- flipped slope (x/y instead of y/x) 1- correct value with incorrect sign	2
M.2: Find the slope of the line from a graph. The student must correctly identify the correct numerical value of the slope, as well as the correct sign (+ or -) of the slope.	0	1- flipped slope (x/y instead of y/x) 1- correct value with incorrect sign	2
M.3: Answer the missing value question: The student must determine the correct x or y value for the missing value question (ex: How many more cars would she need to wash to earn $50 total?; how much money would she have total if she washed 5 more cars?)	0	1- elements of correct process (ex: table with most phases correct), but incorrect final answer	2
M.4: Answer the missing value question: The student must determine the correct x or y value for the missing value question (ex: How many more cars would she need to wash to earn $50 total?; how much money would she have total if she washed 5 more cars?)	0	1- elements of correct process (ex: table with most phases correct), but incorrect final answer	2
M.5: Label Graph: Student must correctly label the x and y-axis with what each represents in the problem (ex: time on the x-axis and distance on the y-axis) (1 pt).	0	0.5 for only x or only y correct	1
M.5: Number Graph: Student must correctly number the x and y axes *Must be consistent on each axis (i.e. by 5's; not by 2's for half the axis, then by 5's), and appropriate for the values in the problem (i.e, go up high enough).	0	0.5 for only x or only y correct	1
M.5: Input values in the table: *The x values must be in the correct order in relationship to each other & the problem, but do not need to correspond to the y values (same criteria applies to the y values). *The student may receive points for the correct values if they provide a range for x or y values, if the end point of the range matches the correct value.			
M.5: Input the correct x value for the first phase in the problem	0		1
M.5: Input the correct x value for the second phase in the problem	0		1
M.5: Input the correct x value for the third phase in the problem	0		1
M.5: Input the correct x value for the fourth phase in the problem	0		1

Table 1. (Continued)

Problem/Task	Not Proficient	Emerging	Proficient
M.5: Input the correct y value for the first phase in the problem	0		1
M.5: Input the correct y value for the second phase in the problem	0		1
M.5: Input the correct y value for the third phase in the problem	0		1
M.5: Input the correct y value for the fourth phase in the problem	0		1
M.5: Graph each phase in the graph: *If axes are numbered incorrectly, student may receive points for graphing if they correctly graph the values shown on their table (if their table values are correct).			
M.5: Graph 1 coordinate point indicating the first phase	0		1
M.5: Graph 1 coordinate point indicating the second phase	0		1
M.5: Graph 1 coordinate point indicating the third phase	0		1
M.5: Graph 1 coordinate point indicating the fourth phase	0		1
M.5: Connect (0,0) to the point indicating the first phase	0		1
M.5: Connect the points indicating the first and second phases	0		1
M.5: Connect the points indicating the second and third phases	0		1
M.5: Connect the points indicating the third and fourth phases	0		1
M.6: Calculate the average rate of change: Find the correct value for average rate of change.*1.5 points if answer is incorrect due to rounding; work must be shown	0	1- correct process used, but calculation error made	2
Total Score:		**Points**	**Percentage**
		/28	

Table 2. Scoring Rubric for Explanation Constructed Responses on Rate of Change Assessments

Writing Prompts:	W.1 Describe in writing the process you used to solve the problem above. W.3 Explain the steps you used to determine….			
Type of Writing Prompt	Explanation			
Description of Criteria	*Not Proficient (0)*	*Emerging (1)*	*Approaching (2)*	*Proficient (3)*
Accuracy: Provides an accurate and appropriate description of mathematical procedures/ process used. The process described is an effective method to determine the correct answer (i.e. the process described is mathematically sound, ex: two times two equals four).	No attempt; or, attempt made demonstrates lack of understanding of how to solve the problem (ex: I found the distances and times; I did the math)	Student provides a description of an accurate process (incomplete or complete), but includes the incorrect answer (ex: I divided 26 by 2 and the result was 12), OR provides an incomplete (didn't include all steps) explanation with the correct answer (ex: I added 14 + 14 = 28).	Student provides accurate, but general explanation, and provides the correct answer. A general explanation means the explanation is incomplete (i.e., missing 1 or more steps). Ex: I set up the equation and solved it; I used the table.	Student accurately describes all steps involved in finding the correct answer (complete process) and provides correct answer. Ex: I set up the equation and solved it by dividing 14 by ½.
Math Vocab: Uses appropriate and specific mathematical language, including precise vocabulary.	No attempt; or, attempt made does not include mathematical language (ex: I thought about the numbers).	Student uses 1 precise math vocabulary word. (ex: add, subtract, divide, average, etc.)	Student uses 2 precise math vocabulary words (ex: add, subtract, divide, average, etc.)	Student provides explanation with at least 3 precise math vocabulary words (ex: add, subtract, divide, average, etc.)
Labels & Quantities: Student's explanation includes appropriate labels and specific reference to quantities.	No attempt; or, attempt made does not provide labels, units, or quantities. Or, attempt made includes labels or units, but references specific, but any inaccurate quantities (ex: "I multiple 3 and 6", but 3 and 6 are not in the problem).	Refers to specific and accurate quantities, but lacks reference to labels or units (ex: I divided 14 by 1/2).	Includes labels or units, but doesn't refer to specific quantities (ex: I added the miles and then the hours).	Includes all correct labels or units, and refers to specific and accurate quantities (ex: I divided 14 by ½ to determine how many slices of bread per 14 tablespoons of peanut butter).

Notes: 1) Student must include at least one sentence in writing in response to these questions (operations & symbols must be in writing, but numbers can be written as numerals). They can use math computation as support, but must have more than just math computation work shown to receive any points on any category. **2)** No points may be awarded in any category for verbatim repeats of the problem (ex: student restates the problem on M.3, but does not provide any additional information).

Table 3. Scoring Rubric for Explanation Constructed Responses on Rate of Change Assessments

Writing Prompts:	W.2 Explain how you know your answer makes sense. Provide an example to support your reasoning. W.4 How do you know your answer is correct? Provide an example to support your reasoning.			
Type of Writing Prompt	Justification			
Description of Criteria	*Not Proficient (0)*	*Emerging (1)*	*Approaching (2)*	*Proficient (3)*
Accuracy: Provides an accurate and appropriate justification that is mathematically sound (ex: I know my answer is correct because Jerry will use twice as many slices as tablespoons of peanut butter if he has ½ tablespoon per slice).	No attempt; or, attempt made demonstrates lack of understanding of why the problem is correct, or if it is correct (ex: I know my answer is correct because I did the math)	Student provides a justification that lacks specific reference to the problem (ex: because I added and divided correctly) OR student provides an accurate justification but provides the incorrect answer.	Student provides correct answer and justification, but does not include enough detail to confirm student understanding (ex: I know my method works because I am using a table)	Provides correct answer and accurate and appropriate justification with detailed explanation (ex: I know my method works because the table helps me keep track of my work; the graph, story problem, and table match)
Math Vocab: Uses appropriate and specific mathematical language, including precise vocabulary.	No attempt; or attempt does not include mathematical language (ex: I am not sure, but the graph is right)	Student uses 1 precise math vocabulary word. (ex: add, subtract, divide, average, etc.)	Student uses 2 precise math vocabulary words (ex: add, subtract, divide, average, etc.)	Student provides explanation with at least 3 precise math vocabulary words (ex: add, subtract, divide, average, etc.)
Support: Provides data/warrants/reasoning to support their justification.	No attempt; or, attempt made does not provide specific details or examples to support their answer; or, attempt made includes an example that does not support their reasoning/answer (I'm not sure about my method; I just know it is right)	Describes partial justification, or provides the answer in the context of the problem (ex: In four years the lake decreases 1 ft. So it would take 48 years for the lake to dry up); OR student provides partial accurate justification with incorrect answer (ex: I know my answer is right because half of 12 is 5).	Provides correct answer and describes complete justification, including specific operations and numbers (ex: 20 divided into groups of 4 is 5. Thus in 25 minutes the plane will descend 20 ft.)	Provides correct answer and complete justification. Uses specific examples from the problem or logical reasoning in defense of the answer (ex: dividing the total distances traveled by the total amount of time passed includes what occurred at each phase. Average rate of change tells generally how long it would take to get from A to B

Notes: 1) Student must include at least one sentence in writing in response to these questions (operations & symbols must be in writing, but numbers can be written as numerals). They can use math computation as support, but must have more than just math computation work shown to receive any points on any category. **2)** No points may be awarded in any category for verbatim repeats of the problem (ex: student restates the problem on M.3, but does not provide any additional information).

Table 4. Assessment Fidelity Checklist for Rate of Change Assessments

Date of Rate of Change Probe administration: ______________ Time Started: _________ Time Ended: ________	# of Components Apparent	Total # of Components	Fidelity Score (%)
Name of Participant: ______________________	Name of Assessor: __________________		
Component	Apparent (yes)	Not Apparent (no)	Not applicable (na)
Assessor reads the directions for the rate of change probe out loud.			
Assessor provides student with a calculator & explains that they may use the calculator on any of the test questions if they would like to.			
Assessor reads each of the prompts in the rate of change probe out loud (check that each of the prompts listed below is read out loud):			
Reads out loud: M.1. What is the slope of the line below?			
Reads out loud: M.2. What is the slope of the line below?			
Reads out loud: M.3. (problems vary).			
Reads out loud: W.1. Describe in writing the process you used to solve the problem above.			
Reads out loud: W.2. Explain how you know your answer makes sense. Provide an example to support your reasoning.			
Reads out loud: M.4. (problems vary).			
Reads out loud: M.5. (problems vary).			
Reads out loud: M.6. (problems vary).			
Reads out loud: W.3. Explain the steps you used. . .			
Reads out loud: W.4. How do you know your answer is correct?			
Responses/Clarifications to Student Questions			
Assessor provides clarifications that do not give away the answer or how to find it. *Examples of appropriate clarifications* include- re-reading the prompt, simplifying vocabulary in the prompt, or prompting the student to think back to the lessons. *Inappropriate clarifications* include any explanation of the steps that students need to complete in the problem, explanations of what slope or rate of change is, or providing example written responses.			*NA if the student does not ask any questions*

CHAPTER 3

RESULTS

Fidelity Results

Lesson Fidelity

As the intervention instructor, I helped ensure fidelity of intervention implementation by completing a checklist of essential lesson components. As I taught each lesson, I indicated whether each component of the lesson occurred or did not occur. I followed these procedures for 100% of the lessons administered. For all lessons and participants, I was able to mark 100% of essential lesson components as occurring.

In addition to completing fidelity checklists, I also audio-recorded each lesson. To ensure that I implemented the intervention as intended, a research assistant who was also a student at the university completed interrater reliability checks for lesson fidelity. The research assistant listened to 48% (12 out of 25 total lessons) of the lesson recordings and completed the fidelity checklists. The research assistant first randomly selected the lessons for which to complete interrater reliability checks. This procedure was followed to help prevent biases from influencing the selection of lessons for interrater reliability. The research assistant ensured that selected lessons were evenly distributed across participants. For each lesson, the research assistant checked off each component as he heard them on the recording, indicating whether the component occurred or did not occur. Agreement was calculated by dividing the total number of agreements by the total number of agreements plus disagreements. Interrater reliability ranged from 98% to 100% agreement, with an average agreement of 99.71%.

I also self-monitored assessment fidelity for 100% of the assessments I administered. As I administered each assessment, I checked off components of an assessment fidelity checklist as being present, not present, or not applicable (e.g., if students did not ask any questions, the item regarding providing only appropriate responses to student questions would not be applicable). The assessment fidelity checklist included procedural elements of assessment administration, including reading scripted directions before administering each assessment, reading each question aloud to each student, and providing only appropriate clarifications or prompts in response to student questions, if applicable (see Table 4). All assessments were administered with 100% fidelity across all participants. A second research assistant who was also a student at the university completed interrater reliability for assessment fidelity. Interrater reliability was completed for 43.75% (18 out of 41 total assessments) of assessments administered, and was randomly selected across phases and participants. There was 100% agreement on the assessments for which interrater reliability was assessed.

Interrater Reliability

To help reduce potential bias in scoring, a third research assistant (RA) who was a graduate student at the university completed 100% of the assessment scoring. The (RA) was blind to the purpose and phases of the study. Prior to the beginning of the study, I trained the research assistant in scoring procedures. The RA and I scored four practice assessments together, to provide opportunities to discuss any disagreements in scoring. The research assistant and I then independently scored another set of practice assessments I evaluated our agreement on these assessments by dividing number of agreements by number of agreements plus disagreements. The research assistant and I continued this process until at least 95% agreement was achieved for scoring the students' total points of correct math responses, and 90% agreement was achieved for scoring the students' total points of correct writing responses. By the end of one 3-hour training session, average agreement for students' total points of correct math responses on the rate of change assessment was 98% (range 91-100%) and average agreement for students' total points of correct writing responses was 90% (range 75-100%). I completed interrater reliability checks by scoring 51% (21 of 41 total assessments) of the assessments, randomly selected and evenly distributed across participants and phases, which exceeds recommended professional standards of 20-30% (McDonnell & Heathfield, 2011). For students' total points of correct math responses on the rate of change assessments, agreement ranged from between 82.6% to 100%, with an average of 96.8%. For the students' total points of correct writing responses on the rate of change assessments, agreement ranged from between 25% to 100%, with an average of 87.9%.

Effects of the Intervention

The primary dependent variable for this study was students' total points of correct math scores on rate of change assessments (math scores). The secondary dependent variable was students' total points of correct writing responses on rate of change assessments (writing scores). Descriptive data for each dependent variable and each phase (baseline, intervention, and maintenance) are presented in Tables 5 and 6. Visual analysis of the graphed data was conducted within phases, across phases, and across participants. Visual analysis within phases and participants included an evaluation of trend, variability, level, immediacy of change, and magnitude (White & O'Neill, 2011). To determine if a positive, negative, or no trend existed within each phase of the data, ordinary least square (OLS) trend lines were calculated for each phase for each participant. A positive slope indicated increases in math scores, a slope of 0 indicated no change in math scores, and a negative slope indicated a decrease in math scores. Across participants, visual analysis was conducted to evaluate the consistency of data patterns. Data were also compared vertically across participants. Figure 1 presents the graphs of students' total points of correct math responses on rate of change assessments and Figure 2 presents the graphs of students' total points of correct writing responses on rate of change assessments.

Math Scores on Rate of Change Assessments

Baseline

Descriptive results. Each participant completed between four and seven baseline assessments. As described in Chapter 2, Abigail began the baseline phase later than Toby and Jason. During baseline, students' math scores ranged from 0-7 points, with an average mean of 2.18 points and average median of 2 points (refer to Table 5 for individual participant data).

Visual analysis results. Visual analyses of students' math scores on baseline rate of change assessments were first conducted within each phase for each participant. Toby's scores were at a very low level, as he scored 0 on all baseline assessments. Jason's scores also reflected a low level, with a mean of 1 points and range of 0-2 points. Abigail's baseline scores were at a slightly higher level, with a mean of 4.43 points and range of 3-7 points. Regarding trend and variability, Toby's baseline data had a slope of 0, as he had math scores of 0 on all baseline assessments and his scores remained stable throughout baseline. Jason's math scores on the baseline assessments showed a decreasing trend with low variability with a range of 2 points (slope = - 0.006), and Abigail's baseline assessments showed a slightly greater decreasing trend (slope = - 0.027) but higher variability.

Across participants, baseline math scores were fairly consistent. Toby and Jason had similar results in terms of trend, variability and levels. Abigail also had similar results for trend, but showed higher variability (range of 3-7 points) and higher levels (mean of 4.43 points). Vertical analysis across participants showed that Toby and Jason had similar scores at the same point in time when both were in baseline. It was not possible to conduct a vertical analysis that included Abigail when looking solely at the baseline phase, as Abigail began baseline later than Toby and Jason.

Intervention

Descriptive results. During intervention, students' math scores ranged from 2-18 points, with an average mean of 6.71 points and average median of 4.5 points (see Table 5). Consistent with the procedures described in Chapter 2, each participant began intervention after displaying low and stable points of correct math responses on baseline rate of change assessments, and after the previous participant demonstrated a response to intervention (i.e., at least a 10% increase in math scores on intervention rate of change assessments compared to the median of math scores on baseline rate of change assessments). Based on the preestablished phase change criteria, Abigail was scheduled to begin intervention after Jason's math scores on intervention rate of change assessments were at least 10% above the median of his math scores on baseline rate of change assessments. Jason scored approximately 7% above the median of his baseline math scores on his first intervention assessment, but had a decrease of 1 point on his second intervention assessment. The decision was made to begin Abigail's intervention phase at that point, due to evidence of a decreasing trend in her baseline math scores and concerns regarding the time left in the school year.

Each student completed four assessments during the intervention phase, and received instruction on lessons 1-7. Toby, Jason, and Abigail all received instruction on at least one review lesson. Review lesson 1 was administered after lesson 4 if a student had not demonstrated an improvement of at least 10% above the median of their baseline math scores by the second intervention assessment. Review lesson 2 was administered after lesson 6 if a student had a decrease or plateau in math scores on their third intervention assessment.

Toby received instruction on Review Lesson 2, due to a decrease in math scores on his third intervention assessment. Jason received instruction on Review Lesson 1 because his math scores had improved by less than 10% over his median baseline score by the second intervention assessment. Abigail received instruction on Review Lesson 1 because she had demonstrated an improvement of less than 10% above the median of baseline math scores by the second intervention assessment. She also received instruction on Review Lesson 2, due to a decrease in math scores on her third intervention assessment. Table 7 summarizes the treatment dose, an indicator of treatment intensity (Codding & Lane, 2015) per participant. On average, students received instruction for 849.3 minutes over 34.3 sessions. Sessions were conducted on average of 2.31 times per week.

Visual analysis results. Within the intervention phase, increasing trends were visible for all three students' math scores. Jason demonstrated the highest trend (slope =0.24) in math scores, followed by Abigail (slope =0.11). Toby's intervention math scores had a more modest, but still increasing trend (slope =0.09). As anticipated, the level of intervention math scores for each participant remained relatively low for the first three intervention assessments. Toby had the lowest level of math scores during intervention (mean= 5 points), Jason had slightly higher math scores during intervention (mean=6.75 points), and Abigail had the highest math scores during intervention (mean=8.37 points). All 3 participants had fairly low variability in intervention math scores, with the exception of large increases between the third and fourth intervention assessments. Toby's scores ranged from 2 points to 11 points, Jason's intervention math scores ranged from 2 points to 17 points, and Abigail's intervention math scores ranged from 3.5 points to 18 points.

All 3 participants showed an immediacy of effect between baseline and intervention. Both Toby and Jason received 3 points on the first intervention assessments, compared with scores of 0 on their final baseline assessments. Abigail's first intervention score was 6 points, while her final baseline score was 3 points. Abigail's scores showed consistent variability during baseline her math score on the first intervention assessment was above the median of her baseline scores, which was 4 points. The magnitude of change in math scores between baseline and intervention was determined by comparing means of each phase (see Table 5). Jason had the largest increase in math scores (5.75 points), followed by Toby (5 points), and Abigail (3.94 points). These increases in means for each participant reflect relatively small changes in magnitude, as math scores on the rate of change assessments were out of 28 total points. However, when the median baseline scores are compared with students' highest intervention math scores, improvements ranged between 11 and 16 points.

The patterns of data across participants during intervention were fairly consistent. First, all 3 participants had increasing trends during intervention. This is in contrast to trends in baseline, which were decreasing for all 3 participants. Each participant had the greatest increase in math scores between the third and fourth intervention assessments, which were administered after lessons 6 and 7, respectively. Vertical analysis across participants shows that students for whom the independent variable (the CRA + W intervention) was not manipulated did not have improvements in their baseline scores, while at the same point in time students for whom the independent variable was manipulated did have improvements in their intervention scores. This vertical analysis provides additional evidence that the CRA + W intervention may have influenced students' improvements in scores during intervention.

Maintenance

Descriptive results. Students completed between one and six maintenance assessments 1 to 7 weeks following the end of the intervention. During maintenance, students' math scores ranged from 1-26 points with an average mean of 11.7 points and average median of 11 points. Booster lessons were administered during the maintenance phase if students' maintenance math scores fell below the median of their intervention scores. Toby received instruction on a Booster Lesson because his third maintenance math score fell below the median of his intervention scores.

Visual analysis results. There were mixed results regarding trend within the maintenance phase. Toby's math scores on maintenance assessments reflected an increasing trend (slope = 0.1831). Jason's math scores on maintenance assessments demonstrated a decreasing trend (slope = -0.2634). It was not possible to calculate slope for Abigail's maintenance phase, due to her completion of only one maintenance assessment. There was not time for Abigail to complete additional maintenance assessments because the school year ended. All 3 students maintained higher levels of scores during maintenance when compared with both baseline and intervention. Toby's mean score during maintenance was 4 points, Jason's mean score during maintenance was 17.16 points, and Abigail's score on her only maintenance assessment was 26 points. Variability during maintenance was comparable to variability during intervention. During intervention, Toby's scores had a range of 9 points and Jason's scores had a range of 15 points. During maintenance, Toby's scores had less variability, with a range of 7 points, and Jason's range remained at 15 points.

Jason and Abigail had immediate increases in scores between the intervention and maintenance phases. Toby showed an immediate decrease in scores on his first maintenance assessment, receiving 1 point. Jason demonstrated an increase in scores on his first maintenance assessment, receiving 22 points. On her maintenance assessment, Abigail received 26 points, which is also an immediate increase in scores compared to her final intervention assessment. The magnitude of change in math scores (see Table 5 for a comparison of means) between intervention and maintenance was large for Jason and Abigail. Toby's mean maintenance score (4 points) was 1 point lower than his mean intervention score (5 points). Across participants, the patterns of math score data between intervention and maintenance were not very consistent. Toby's maintenance math scores started low and gradually increased, whereas Jason's maintenance math scores started high and gradually decreased.

Writing Responses on Rate of Change Assessments

Baseline

Descriptive results. Students' baseline writing scores ranged from 0-13 points, with an average mean of 3 points and average median of 1.5 points. Individual participant data are reported in Table 6.

Visual analysis results. Regarding trend, Toby's baseline data had a slope of 0, as he had writing scores of 0 on all baseline assessments. Jason and Abigail displayed decreasing trends during baseline. The slope of Jason's writing scores on baseline assessments was mostly flat, but slightly decreasing (slope = -0.004). The slope of Abigail's writing scores on baseline assessments was also slightly decreasing (slope = -0.041). All 3 participants had relatively low levels of writing scores during baseline. Out of 36 points total, the mean of Jason's writing scores was 1 point and the mean of Abigail's writing scores was 4.43 points (see Table 6 for a comparison of means). Toby's scores, all 0 points, remained stable throughout baseline. Jason's scores had low variability (ranged between 1 and 2 points). Abigail's writing scores were highly variable, ranging from 2 to 13 points.

Across participants, baseline data were fairly consistent. Toby and Jason had similar results in terms of trend, variability and levels. Abigail also had similar results for trend, but showed higher variability and higher levels. Vertical analysis across participants showed that Toby and Jason had similar scores at the same point in time when both were in baseline. It was not possible to conduct a vertical analysis that included Abigail when looking solely at the baseline phase, as Abigail began baseline later than Toby and Jason.

Intervention

Descriptive results. Students' intervention writing scores ranged from 0-11 points, with an average mean of 3.2 points and average median of 3 points. Data for individual participants are reported in Table 6.

Visual analysis results. Within the intervention phase, all 3 students had increasing trends for their writing scores. Abigail had the highest slope (slope = 0.0944), followed by Jason (slope = 0.0306). Toby showed evidence of a slightly increasing trend (slope = 0.0105). Levels of writing scores during intervention remained low (see Table 6 for intervention means). Toby and Jason each had mean writing scores during intervention of

1.75 points, and Abigail had a mean writing score during intervention of 6.25 points. Jason and Toby had low variability in writing scores during intervention, with Toby's writing scores ranging between 0 and 4 points, and Jason's ranging from 0 to 3 points. Abigail's writing scores during intervention were more variable, ranging between 3 and 11 points.

Toby was the only participant to show a clear immediate increase in scores between baseline and intervention, with his first writing score of 3 points, compared to his writing scores of 0 points on all baseline assessments. Jason's first intervention writing score was 2 points, compared to his final baseline writing score of 1 point. Abigail's first intervention writing score was 3 points. While this was above her final baseline writing score of 2 points, it was below the median of her baseline writing scores, which was 5 points. In terms of magnitude, there were very slight improvements in level of performance for each participant between baseline and intervention, ranging between increases of 0.75 points to 1.8 points (see Table 6 for a comparison of means).

Writing scores across participants during intervention were consistent between Toby and Jason, but not Abigail. Toby and Jason both showed low levels with some variability. Abigail's writing scores steadily increased throughout intervention. Vertical analysis across participants and phases provides mixed results. Jason had stable and low scores in baseline at the same point in time that Toby had increases in intervention scores. However, Abigail showed increases in baseline scores at the same point in time that Jason had increases in intervention scores. Additionally, while Toby and Jason were in intervention and Abigail remained in baseline, all 3 students had decreases in scores at the same point in time. Therefore, the visual analysis of data between and within participants does not conclusively demonstrate that improvements made between baseline and intervention can be attributed to the intervention.

Maintenance

Descriptive results. Students' maintenance writing scores ranged from between 0-18 points, with an average mean of 4.4 points and average median of 4 points.

Visual analysis results. Regarding trend of writing scores within the maintenance phase, Toby had a decreasing trend (slope = -0.0269), Jason had a slightly increasing trend (slope = 0.0088), and it was not possible to calculate slope for Abigail's maintenance phase, as she only completed one maintenance assessment. Levels of participants' writing scores during maintenance were low relative to the number of points available on the assessments for Jason and Toby, but higher for Abigail. The mean of Toby's maintenance writing scores was 3 points and the mean of Jason's maintenance writing scores was 4.5 points. Abigail's writing score on the one maintenance assessment she completed was relatively high (18 points). Compared to intervention, Toby's writing scores were less variable during maintenance, ranging from 0 to 4 points. Jason's writing scores on maintenance assessments were more variable than they were during intervention, ranging from 1 to 7 points.

Immediate increases in writing scores between the intervention and maintenance phases were only visible for Abigail. Toby and Jason both had decreases in writing scores on their first maintenance assessment. Regarding magnitude, all 3 students also showed an increase in writing scores between baseline and maintenance, as well as intervention and maintenance phases (see Table 6 for a comparison of means between phases). Between baseline and maintenance phases, participants' writing scores improved by between 3 and 13.57 points, and between intervention and maintenance phases, participants' writing scores improved by between 1.25 and 11.75 points. Patterns of data during the maintenance phase were consistent for Toby and Jason. Both Toby and Jason had initial decreases in writing scores on their first maintenance assessments, variable writing scores that were above baseline and intervention levels on the second through fifth maintenance assessments, and a decrease in writing scores on their final maintenance assessments. Abigail's writing scores on her only maintenance assessment did not reflect same pattern as Toby and Jason's scores, as her writing scores increased instead of decreasing.

Social Validity Results

Table 8 reports the results on the modified Children's Intervention Rating Profile (Arra & Bahr, 2005). Overall, students felt positively toward the intervention. Each of the 7 items was rated on a 1-6 Likert-type scale, with 1 indicating strongly disagree and 6 indicating strongly agree. On average, the students indicated that using POD✓, cubes and diagrams was a helpful way to teach math (median = 5), that using POD✓, cubes and diagrams to teach math was a good way to teach math to other students (median = 4), that they liked using POD✓, cubes and diagrams to learn math (median = 4), and that they felt that using POD✓, cubes and diagrams to teach math would help students do better in school (median = 5). However, students also indicated that using POD✓, cubes and diagrams to teach math is too hard (median = 3), that using POD✓, cubes and diagrams to teach math may be hard for other students (median = 5), and that they felt there were better ways to teach math to students other than using POD✓, cubes and diagrams (median = 4).

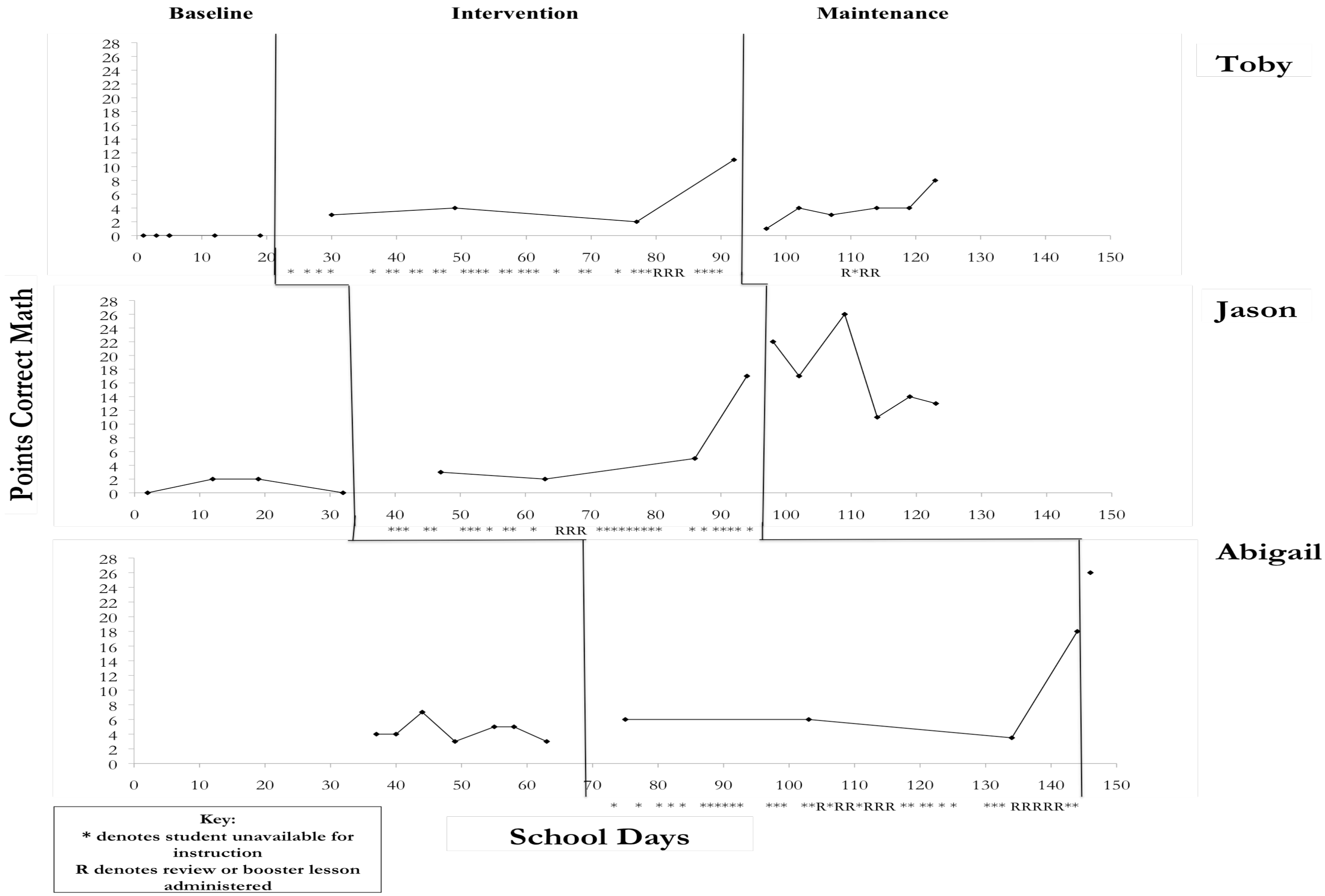


Figure 1. Students' Total Points of Correct Math Responses on Rate of Change Assessments

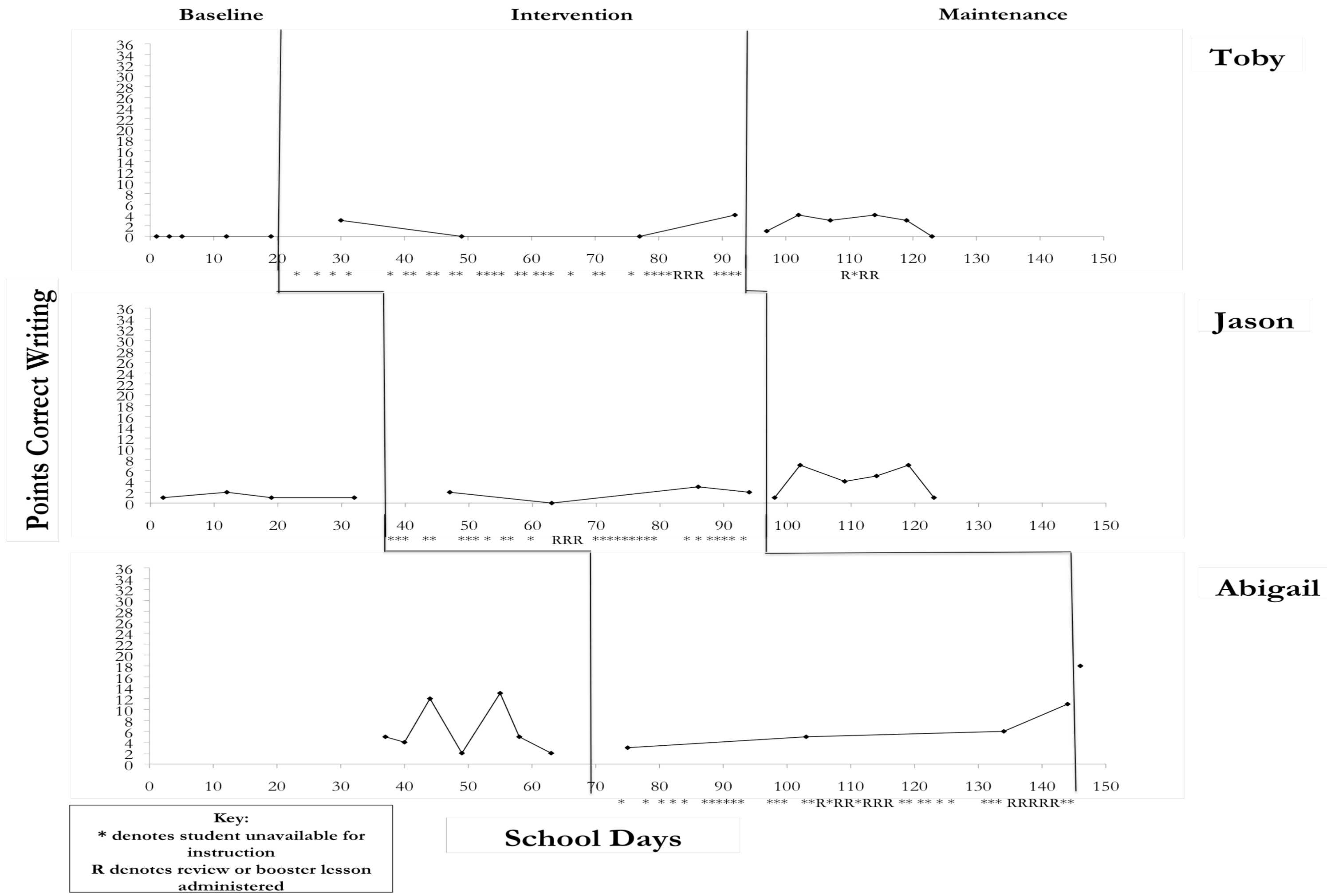


Figure 2. Students' Points of Correct Writing Responses on Rate of Change Assessments

Table 5. Descriptive Data For Students' Total Points of Correct Math Responses on Rate of Change Assessments

	Baseline			Intervention			Maintenance		
	Mean	Median	Range	Mean	Median	Range	Mean	Median	Range
Toby	0.00	0.00	0.00	5.00	3.50	2.00-11.00	4.00	4.00	1.00-8.00
Jason	1.00	1.00	0.00-2.00	6.75	4.00	2.00-17.00	17.16	15.50	11.00-26.00
Abigail	4.43	4.00	3.00-7.00	8.37	6.00	3.50-18.00	26.00	26.00	---
Total	2.18	2.00	0.00-7.00	6.71	4.50	2.00-18.00	11.77	11.00	1.00-26.00

Table 6. Descriptive Data For Students' Total Points of Correct Writing Responses on Rate of Change Assessments

	Baseline			Intervention			Maintenance		
	Mean	Median	Range	Mean	Median	Range	Mean	Median	Range
Toby	0.00	0.00	0.00	1.75	1.50	0.00-4.00	3.00	3.00	0.00-4.00
Jason	1.00	1.00	1.00-2.00	1.75	2.00	0.00-3.00	4.50	4.50	1.00-7.00
Abigail	4.40	5.00	2.00-13.00	6.25	5.50	3.00-11.00	18.00	18.00	---
Total	3.00	1.50	0.00-13.00	3.20	3.00	0.00-11.00	4.40	4.00	0.00-18.00

Table 7. Treatment Intensity Per Participant

	Lesson 1		Lesson 2		Lesson 3		Lesson 4		Lesson 5		Lesson 6		Lesson 7		Review 1		Review 2		Total	
	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D
Toby	73	2	118	4	119	5	106	4	126	7	118	6	107	3	NA	NA	88	3	855	34
Jason	52	2	99	3	138	5	92	3	107	4	93	3	53	2	90	3	NA	NA	724	25
Abigail	73	2	103	3	148	8	122	6	139	6	85	5	85	2	115	7	99	5	969	44
Average	66	2	107	3.3	135	6	107	4.3	124	5.6	98.6	4.6	81.6	2.3	102.5	5	93.5	4	849.3	34.3

Note: M indicates minutes and D indicates Days.

Table 8. Social Validity Results Per Question

Survey Questions	Responses			
	Toby	Jason	Abigail	Median
1. Using POD✓, cubes and diagrams is a helpful way to teach math	5	3	5	5
2. Using POD✓, cubes and diagrams to teach math is a good way to teach math to other students	4	3	6	4
3. I like using POD✓, cubes and diagrams to learn math	4	4	5	4
4. I think that using POD✓, cubes and diagrams to teach math will help students do better in school	5	4	5	5
5. Using POD✓, cubes and diagrams to teach math is too hard	3	4	2	3
6. Using POD✓, cubes and diagrams to teach math may be hard for other students	4	5	5	5
7. There are better ways to teach math to students than using POD✓, cubes and diagrams	4	5	3	4

Note: Likert scale range of 1-6, with 1 indicating strongly disagree and 6 indicating strongly agree.

CHAPTER 4

DISCUSSION

Summary of Results

The purpose of this study was to determine if a functional relationship exists between the use of a rate of change intervention incorporating the CRA instructional sequence and WTLM and students' accuracy on a rate of change assessment. The primary dependent variable was students' total points of correct math scores on rate of change assessments (math scores), and the secondary dependent variable was students' total points of correct writing scores on rate of change assessments (writing scores). Students' math and writing scores on the rate of change assessments in this study provided valuable information regarding concepts students struggled with related to rate of change and writing, as well as concepts for which students made significant improvements.

Evidence of a Functional Relation Between the CRA + W Intervention and Math Scores

The first research question asked what the effect of implementing a concrete-representational-abstract (CRA) instructional sequence incorporating writing to learn math strategies was on students with disabilities' proficiency in solving rate of change problems. Toby, Jason, and Abigail had increasing trends in math scores during intervention, had higher levels of math scores during intervention than baseline, and did not have substantial variability in math scores during intervention.

Typically, one sign of effective interventions is immediacy of effects aligned with the start of the intervention. Only Toby demonstrated an immediate and substantial improvement in math scores between baseline and intervention. Jason and Abigail demonstrated more gradual improvements. However, since this intervention involved acquisition of an academic skill, it was expected that improvement would be gradual as opposed to immediate. Previous research on CRA has also noted a lack of immediacy of effects (Flores, Hinton, & Schweck, 2014; Witzel, 2005). A study conducted by Flores, Hinton, and Schweck (2014) on the use of CRA to improve students' multiplication with regrouping also found a delayed effect. However, the authors indicated that this delay in improvement was balanced by the development of fluency and generalization (Flores, Hinton, & Schweck, 2014). The results of this study are similar, as the participants were able to maintain higher rates of performance compared to baseline even 6 weeks following the intervention. The concepts covered throughout the seven intervention lessons also built upon one another. The concepts covered in lessons 1-4 were foundational to those covered in lessons 5-7. Therefore, it was anticipated that students would improve gradually because the lessons covered concepts that gradually increased in complexity.

An additional consideration is that the assessments used to evaluate student progress throughout the intervention assessed all of the intervention objectives. When students took the first and second intervention assessments, they had only been exposed to approximately half of the intervention objectives. The objectives covered in Lessons 5-7 accounted for approximately 64% of the points available on the rate of change assessments. As anticipated, Toby, Jason, and Abigail showed the biggest increases in math scores on the fourth intervention assessment, which occurred after students had had exposure to all lesson concepts, as well as multiple opportunities to practice them.

All 3 students had increases in magnitude of math scores between baseline and intervention. Toby's mean math scores improved by 5 points between baseline and intervention, Jason's mean math scores improved by 5.75 points between baseline and intervention, and Abigail's mean math scores improved by 3.94 points between baseline and intervention. The overall level of performance for each participant during intervention was relatively low compared to the total number of points available on the math portion of the assessment (26 points). However, the assessment was constructed to be challenging, in part because it covered a broad range of skills. It was therefore difficult for students to achieve 26 points.

All 3 participants had fairly consistent patterns of data during intervention for math scores related to trend, level, variability, immediacy of effects, and magnitude. Abigail had the highest amount of variability in her baseline and intervention data, and also had the highest scores on average during baseline, which could have contributed to the variability. A student who had some proficiency with the skills assessed would be expected to have more fluctuation in scores versus a student who scored 0 points on the assessments. Consistency of data patterns across participants provides evidence that changes in students' math scores can be attributed at least in part to the CRA+W intervention. Additionally, based on vertical analysis across participants, students who remained in baseline did not show improvements in math scores at the same point in time that students in intervention showed improvements in math scores. This is an important finding, as it provides evidence that the intervention may have influenced improvements in math scores.

All 3 students maintained higher mean math scores during maintenance when compared with baseline and intervention, although maintenance math scores tended to be lower than the math scores on the final intervention assessment. The average mean across participants during baseline was 2.18 points, compared with 6.71 points during intervention and 11.77 points during maintenance (see Table 5 for individual participant means). A slight decrease in maintenance scores was expected, as the maintenance assessments occurred between 1 and 7 weeks following the end of the intervention, and it is documented that students with disabilities often struggle with long-term memory and retaining skills learned (Shin & Bryant, 2015). In this study, students' average scores were actually higher during maintenance than in intervention. This could be because the average intervention scores were influenced by early intervention assessments, for which students scored lower. The maintenance phase, in contrast, occurred after students had been exposed to and practiced all skills covered in the rate of change assessments. Therefore, student may have continued to improve throughout the maintenance phase. While these results are mixed, the trends in the data are promising for further development of both the CRA + W intervention and math and writing assessments.

Difficult Math Concepts for Students With Disabilities

Consistent with previous research on students' understanding of rate of change, the students in this study had difficulty understanding the concept and importance of the sign of slope (Stump, 2001; Teuscher, Reys, Evitts, & Heinz, 2010), as well as depicting variable rate of change (Herbert & Pierce, 2007; Teuscher & Reys, 2012). These findings are of particular interest, as they demonstrate areas of difficulty that students with and without disabilities may have in common, since previous research on rate of change has only included students without disabilities.

There were also concepts the students in this study struggled with that have not been documented in research on rate of change. The students struggled with constant rate of change problems in which the rate was expressed as a fraction. This difficulty may have been influenced by previous findings that show that students' with disabilities struggle with fractions (Fuchs et al., 2013). Students also struggled with skills related to number sense, including counting the slope of a graphed line and determining the scale to use for graphs. These findings are consistent with research indicating that students with disabilities have particular difficulty with number sense (Shin & Bryant, 2015). One common error that most of the students made was starting their count at 1 versus 0. Determining the scale to use for the graphs was also challenging for students. This skill involved students consistently numbering the graph to include the full range of x and y values. All four students were unfamiliar with how to set up the scale of the graph, despite having been exposed to linear equations and graphing prior to the intervention and in earlier grades. This area of difficulty may indicate that more time be spent instructing students in how to correctly set up graphs based on the specific values in a problem.

Areas of Improvement for Students' Math Scores

While the results of this study reveal several areas of difficulty in solving rate of change problems as well as students' use of writing to explain or justify their answers, they also demonstrate several areas in which students improved. While the students initially struggled with identifying and understanding the sign of slope, as well as finding the slope of a graphed line, they made significant improvements with these skills. This may have been due to the use of concrete manipulatives to help build students' understanding of the sign of slope, as well as the frequent review that occurred of each of these concepts. Review of finding slope of a line from a graph was incorporated into the beginning of most of the lessons, as students demonstrated this as an area of need. A second area of improvement was in solving constant rate of change problems. During baseline, Toby and Jason did not solve any constant rate of change problems correctly, and Abigail solved problems with approximately 33% accuracy. Following intervention, each student improved in this skill area by an average increase of 40%. Third, Jason and Abigail demonstrated significant improvements in representing variable rate of change problems in tables and graphs, as well as calculating the average rate of change for these problems. Toby demonstrated moderate improvements in this area. While Jason followed the correct process to calculate the average rate of change for these problems, he often made calculation errors that affected whether he found the correct average rate of change. Toby did not improve in his ability to graph variable rate of change problems, but did improve in his ability to depict these problems in a table. These results indicate that direct instruction incorporating CRA + W may help students specifically develop the skills of identifying the slope of a graphed line, solving constant rate of change problems, and representing variable rate of change problems using tables or graphs.

Evidence of a Functional Relation Between the CRA + W and Writing Scores

A secondary dependent variable evaluated in this study was students' writing scores. All 3 participants had increasing trends in writing scores during intervention with low variability, but the level of each participant's scores remained very low. The writing portion of the rate of change assessment was very rigorous. In order to achieve a perfect score of 36 points on the writing portion of the assessments, students had to provide a thorough, mathematically correct answer that included at least three specific math vocabulary words, as well as one concrete example to support their explanation or justification on each of the three writing prompts. Toby and Jason had moderate increases in their writing scores between baseline and intervention, and maintained these improvements up to 6 weeks following the intervention. Only Toby demonstrated a clear immediate increase in writing scores between baseline and intervention. Jason and Abigail did have small increases in mean scores across phases. One interesting finding is that students continued to improve in their writing scores during maintenance, despite having only slight improvements between baseline and intervention.

There were less consistent data patterns for writing scores than for math scores. Toby and Jason had similar patterns, which included some variability, low levels, and very small positive trends in intervention writing scores. Abigail had less variability, higher levels, and a stronger positive trend in writing scores during intervention. These data indicate that the intervention may result in larger gains when implemented with students who have higher writing scores on the rate of change assessments initially. Additionally, Abigail also had the highest math scores during baseline, and the patterns of her writing scores were closely aligned with those of her math scores. Abigail's results are similar to those observed by Waywood in his 1994 study of 10^{th}-grade students' writing in math, in which he found that students who used specific math vocabulary and accurate descriptions of concepts also had higher math scores. Overall, due to the inconsistency in writing scores, it is difficult to conclude that a functional relationship exists between the CRA + W intervention on students' writing scores on rate of change assessments.

Difficult Writing Concepts for Students With Disabilities

The particular difficulties students experienced with the writing on the rate of change assessments were consistent with previous WTLM research and rate of change. Similar to studies on the rate of change performance of students without disabilities, students had difficulty using examples and specific information to support their explanations and justifications of their problem-solving processes (Herbert & Pierce, 2007). All 3 participants struggled with explaining how they knew their answers were correct, especially for variable rate of change problems (Teuscher & Reys, 2012), and had the smallest increases in scores on the category of the rubric related to providing support for justifications of their answers. These findings indicate that students with disabilities struggle with expressing their understanding of rate of change in writing in addition to struggling with understanding rate of change conceptually. One reason students may have struggled with expressing their mathematical ideas in writing is due to having limited knowledge of how to plan, organize and compose a paragraph (Santangelo, 2014), especially for the purpose of explaining mathematical concepts. While this is a common area of difficulty, teaching students strategies to use for planning, revising, and editing their writing is one of the most effective methods to help students improve their writing (Graham & Perin, 2007). Additionally, providing data, warrants, and support are often skills learned through explicit writing instruction. Secondary teachers, especially those who teach science or math courses, often do not spend much time explicitly teaching writing, typically do not teach writing using research-based strategies, and view themselves as being unprepared to teach writing (Kiuhara, Graham, & Hawken, 2009). Students were not receiving explicit writing instruction during their math classes, and it is unknown whether they were receiving explicit writing instruction during their language arts classes. It is possible that students may have increased their writing scores more significantly had they been provided with more research-based explicit writing instruction as part of their primary classroom instruction.

Areas of Improvement for Students' Writing Scores

An area of improvement in students' writing scores was in the students' use of specific math vocabulary in their written explanations and justifications. During baseline, students often left the writing questions blank, or included very limited information. All 3 participants' writing scores increased the most on the category of the rubric related to the use of specific math vocabulary. Students may have made the most improvements in their writing scores on this area of the rubric because including specific math vocabulary words in written explanations or justifications is a lower level skill than providing warrants and reasoning to support the correctness of their answer. This finding demonstrates that whereas the results of the intervention on writing performance were inconclusive, students improved their use of specific math vocabulary during the intervention. It is interesting that students continued to improve in their use of specific math vocabulary during maintenance. It is possible that students continued to improve as a result of opportunities to practice using the terminology during instruction.

Social Validity

Students rated the social acceptability of the CRA +W intervention favorably. The medians of each item of the CIRP indicated that students overall felt that the intervention was a helpful way to learn math, was a good method to use with other students, that they liked the intervention, and that they felt it would help other students do better in school. These results are promising, as they demonstrate that students viewed the intervention positively, and felt that it might help them not only in math, but also more broadly in school. Students also indicated that they felt the intervention was hard, might be difficult for other students, and that they felt there were better ways to teach math to students other than the intervention. It is difficult to determine whether the high ratings of agreement (medians of 3-5) on the difficulty of the intervention were influenced by the difficulty of the math content, as opposed to the strategies being used to learn the math content. Although students demonstrated proficiency in using the POD✓ and representations of the problems, the lessons, especially those that addressed the objectives of variable and average rate of change, were challenging for the students.

Contribution to the Research

While the results of this study are inconclusive, they are promising for future research in the area of interventions targeting higher level math skills conducted with students with disabilities at the secondary level. In particular, the results of this study provide insight into areas in need of further curriculum and assessment development. Further research is needed to show clear functional effects and extend the research base on CRA + W.

This is an innovative study that applies CRA to the math concept of rate of change. As such, this study contributes to the research base on the use of CRA with students with disabilities in secondary math contexts (Strickland & Maccini, 2012; Witzel, 2005; Witzel, Mercer & Miller, 2003). This study also adds to the research base on an integrated approach to CRA, in which concrete, representational, and abstract depictions of the concepts are used more fluidly than found with traditional CRA (Strickland & Maccini, 2012). The results of this study demonstrate that students' math scores on the rate of change assessments improved following the CRA+W intervention. These results are therefore consistent with those of other CRA studies conducted with middle and high school students (Butler et al., 2003; Strickland & Maccini, 2012; Witzel, 2005; Witzel et al., 2003). Additionally, this study adds to the research base by including maintenance assessments up to 7 weeks following the end of the intervention. Many of the previous studies on CRA did not include maintenance data, and those that did typically assessed maintenance for 4 weeks or less after the conclusion of the intervention (Flores, 2009; Flores, Hinton, & Schweck, 2014; Witzel et al., 2003). One exception to this is the study conducted by Strickland and Maccini (2012), in which maintenance data were collected 3 to 6 weeks following the intervention. While additional research is needed to determine whether students' learning generalizes to other areas, maintenance data are important to help determine the extent to which students' retain the information taught during the intervention.

It is unknown the degree to which the WTLM strategies, as opposed to CRA, influenced the students' improvements in solving rate of change problems. The primary conclusion that can be drawn is that students' performance on rate of change problems improved after receiving the CRA + W intervention. It is possible that writing contributed to students' improvements in math performance, as this finding has been indicated by previous research on WTLM (Albert, 2000; Bell & Bell, 1985; Kostos & Shin, 2010), but additional research is necessary to draw any conclusions in this area.

While the results of this study are inconclusive regarding the influence of students' writing on learning rate of change concepts, they provide justification for further investigating the role writing may play as a learning activity in math. Very few studies have been conducted to evaluate the extent to which WTLM effects specific constructs of writing in math (i.e., specific mathematical vocabulary, the use of examples to support reasoning, accurate explanations of concepts, etc.). In their study of second-grade students' writing in math, Kostos and Shin (2010) used a rubric to score students' writing based on the expression of processes and strategies, level of knowledge and skills for understanding, and communication and representation of math concepts. They found that students' improved in their use of specific math vocabulary, as well as their ability to communicate about strategies and processes.

The results of the current study extend those of Kostos and Shin (2010) by providing information about the influence of writing on students' use of specific math vocabulary, as well as their explanations and justifications of math processes, in the context of ninth-grade students with disabilities. Consistent with the findings of Kostos and Shin (2010), the students in this study improved most in regards to the number of math vocabulary words used throughout the intervention. More research is needed in this area to determine the extent to which WTLM activities may help students more accurately express their understanding of math concepts, especially in light of CCSSM aligned assessments that include questions requiring students to explain and justify their reasoning.

Implications for Practice

There are four main implications for practice that stem from this study. First, the results of this study support the use of instructional strategies and interventions that build both conceptual and procedural knowledge. Several researchers indicate the importance of teaching conceptual and procedural skills in mathematics (Flores, Hinton, & Schweck, 2014; Flores, Hinton, & Strozier, 2014; Fuchs et al., 2012; Gersten et al., 2009; Strickland & Maccini, 2012). Students with disabilities in particular may struggle with developing both conceptual and procedural skills, and should be supported with specific strategies (Fuchs et al., 2012). The intervention addressed in this study incorporated the POD✓ strategy to help students learn both procedural and conceptual skills. Procedural skills were addressed through the outline and describe steps in the POD✓, while conceptual skills were addressed through the defend step. CRA served as a mechanism for students to represent rate of change concepts visually through using centimeter cubes and diagrams, which may have improved their conceptual understanding. The improvements students made in their math scores on the rate of change assessments add to the research base supporting the use of CRA to help students build both conceptual and procedural knowledge (Flores, Hinton, & Schweck, 2014; Flores, Hinton, & Strozier, 2014; Strickland & Maccini, 2012). Specifically, the results of this study demonstrate the potential benefits of using CRA to teach higher level math concepts.

Second, the results support current recommendations regarding effective strategies for teaching math to students with disabilities in secondary contexts, including the use of visual representations, concrete manipulatives, explicit instruction, and the incorporation of basic skills practice (Gersten et al., 2009). The results of this study showed students' math scores improved following the CRA+W intervention. This finding may support the recommendation that visual representations be incorporated in secondary math instruction to help students gain conceptual understanding (Gersten et al., 2009), and indicate that CRA + W may be one effective strategy to implement this recommendation. Researchers also recommend that concrete manipulatives be used sparingly, primarily to expedite students' making connections to algorithms (Gersten et al., 2009). An integrated CRA approach may provide a way to more flexibly and expeditiously use concrete manipulatives in the sequence of instruction (Strickland & Maccini, 2012), as lessons incorporate the different representations (concrete, visual, algorithmic) as needed. In this study, students showed a preference for using visual representations and diagrams, although initial exposure to concrete manipulatives was provided when each concept was introduced. The results of this study, in combination with research recommendations, indicate that teachers should consider using integrated CRA to teach secondary math concepts.

The results of this study are also consistent with research recommending the use of explicit, systematic instruction (Flores, Hinton, & Schweck, 2014; Flores, Hinton, & Strozier, 2014; Gersten et al., 2009; Strickland & Maccini, 2012), as well as the incorporation of a variety of instructional methods (Gersten et al., 2009; Mulcahy et al., 2014). Explicit instruction includes "providing models of proficient problem-solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review" (Gersten et al., 2009, p. 21). Additionally, Mulcahy et al. (2014) recommend that a variety of methods be included in math interventions, including mnemonics, manipulatives, and real-world contexts. The CRA + W intervention addresses all of these features. The results of this study provide some evidence that explicit instruction can incorporate opportunities for students to engage deeply with math content, through depicting mathematical information using concrete and visual representations, writing about problem-solving processes, and through the use of mnemonic strategies such as the POD✓.

The results of this study, including the findings regarding the challenges students had with basic skills (i.e., graphing, number sense, fractions) throughout the intervention, provide evidence in support of research recommending the incorporation of basic skills into grade level content interventions. Gersten et al. (2009) recommended that at least 10 minutes of each intervention session incorporate building fluency and retrieval of arithmetic facts. While this was not done explicitly with the CRA + W intervention, students did practice fluency and retrieval of arithmetic facts throughout the intervention. For example, when students created diagrams to depict constant rate of change problems, they practiced multiplication facts because they had to count up by a certain number to complete the diagram. The improvements students made on solving constant rate of change problems may have been related to gaining more practice with arithmetic facts.

In light of the results of this study, as well as the priority the Common Core State Standards for Mathematics (CCSSM) practice standards place on students being able to describe and justify their reasoning, it is recommended that math teachers build in opportunities to embed writing activities in their instruction. Specifically, students should write to make sense of problems and explain their reasoning. The POD✓ strategy in this intervention provided students with such a structure for writing in math. Consistent with theories about the role writing plays in learning, the POD✓ strategy in this intervention provided students with a structure to constantly reflect on their thinking while solving math problems, with the intent that students would build connections between concepts (Emig, 1977; Vygotsky, 1962). While the results of this study are inconclusive regarding the influence of the writing component of the intervention on improvements in math or writing scores, practitioners are encouraged to provide students with structured opportunities for writing strategically in mathematics.

In addition to supporting current research recommendations regarding effective strategies for teaching mathematics to students with disabilities, as well as incorporating writing in mathematics, the results of this study suggest the importance of effectively preparing preservice special education math teachers. In particular, preservice special education teachers should be provided with training on how to use each of the recommendations suggested by Gersten et al. (2009) in their training or certification programs. Additionally, because many students with disabilities receive their math instruction in inclusive settings, it is important that preservice general education teachers also be exposed to strategies that are effective for teaching students with disabilities mathematics (DeSimone & Parmar, 2006).

In addition to effectively preparing preservice special education math teachers, current special education math teachers should also be supported in their endeavors to incorporate research-based recommendations for teaching math to students with disabilities, as well as for incorporating writing in the teaching of mathematics. Professional development opportunities should be provided to current special and general education math teachers to help them learn practical strategies and interventions, such as CRA and the POD✓ strategy. Additionally, professional development in mathematics should model use of the strategies being recommended, and involve active learning strategies to be most effective (DeSimone, Porter, Garet, Yoon, & Birman, 2002).

One final implication for practice relates to the amount of time allocated for interventions at the secondary level. Secondary settings often do not have very much flexibility within students' daily schedules. Students are required to take core classes, and with the emphasis on high stakes testing, elective classes are often reduced. In this study, students participated in the intervention during a study skills class. However, they often needed class time to make up work missed due to absences, or get extra help from teachers. Gersten et al. (2009) found that of the studies incorporating math fact fluency, those with significant results typically involved intervention sessions ranging from 15-40 minutes in length that met three times per week for 12-18 weeks. While these studies were conducted in elementary school contexts, it is likely that effective interventions in secondary settings would require as much or more time, given what we know about how students learn, as well as the particular complexity of secondary math content. While more research needs to be conducted in this area, one implication for practice that can be drawn at this time is the need for more flexibility in students' schedules at the secondary level to allow them to receive targeted interventions, without falling behind in core content classes.

Limitations

While the results of this study are promising, there are several limitations that should be taken into account when interpreting the results. First, it should be noted that this intervention was conducted with three students at one high school. Therefore, the results cannot be generalized to other contexts, and more research is needed to improve external validity. Second, the intervention also did not evaluate the degree to which the skills students learned transferred to success in their math classes. Additional research should be conducted to evaluate the extent to which increased proficiency in solving rate of change problems influenced students' performance in other content areas, such as math and science.

The time required for the intervention was difficult to incorporate into students' schedules. At the school where this study was implemented, students had five classes each day. The participants in the study each had a study skills class, in addition to their other classes. While the study skills class did not have any assignments students were required to complete, the students did have between three to four other core content classes that involved many assignments and challenging content. Students often needed the time in the study skills class to get help on assignments, finish exams, and complete homework or make-up assignments. As a result, the length of intervention sessions varied greatly, and typically occurred between two and three times per week. On average, the intervention required 34.3 sessions, and the average length per session was 22.02 minutes (see Table 7). The range of days required for the intervention was 26-44 days, and the range of the length of intervention sessions was 7-72 minutes. It is possible that differences in the length of each intervention session, as well as the frequency of sessions per week, may have influenced students' results. Treatment intensity is a relatively new area of research in the context of special education interventions, but there is emerging evidence that treatment intensity may have an effect on the effectiveness of an intervention (Codding & Lane, 2015). It is possible that an increase in treatment intensity for the CRA+W intervention may have resulted in larger improvements in math and writing scores between baseline and intervention.

An additional limitation regarding student schedules was that students did not have math every trimester, and had math during different points of the study. All 3 participants had math during the first trimester, which is also when rate of change and variable rate of change were covered in the school's math curriculum. Students varied in terms of when they had their math class in relationship to the phases of the study, which introduces a confounding variable. Toby had math during all of the baseline phase and most of intervention phase but not during the maintenance phase. Jason had math during the baseline and maintenance phases, but not during the intervention phase. Abigail did not have math during the baseline phase or the first third of the intervention phase, but did have math during the remainder of the intervention and maintenance phases. Based on the differences in when students were enrolled in a math class compared with the phases of this study, the extent to which instruction in their math class may have influenced their math scores during baseline, intervention, or maintenance phases is unknown. Future research should focus on implementing the CRA+W intervention in schools in which students take a full year of math to help eliminate the potential of this confounding variable.

There is currently a lack of research validated curriculum-based assessments, particularly for secondary level content. This study included review of assessments by a math expert, as well as pilot testing of the assessments to help reduce variability in difficulty between the assessments, and ensure that the assessment items were clear. However, without specific research on the measures' validity and reliability, it is unknown whether variability in student math scores is due to the intervention, student factors, or flaws with the assessments themselves. Future research should incorporate the use of valid and reliable assessments to evaluate the results of the CRA + W intervention.

Directions for Future Research

Research on CRA + W

Research on CRA in secondary settings with secondary math concepts is still emerging, but is promising (Strickland & Maccini, 2012; Witzel, 2005; Witzel, et al., 2003). Research has demonstrated that CRA is effective in improving students' performance on solving linear algebraic equations, simplifying and solving algebraic equations, and multiplying linear algebraic expressions in area problems (Strickland & Maccini, 2012; Witzel, 2005; Witzel et al., 2003). Future research should focus on replicating the results of this study, as well as investigating the effects of CRA + W on more advanced secondary math concepts, including solving systems of equations, geometry concepts, and statistics. Additionally, future research should investigate whether CRA + W is an effective intervention approach for helping secondary students gain fluency with math concepts foundational for success in higher level mathematics, such as fractions, ratios, and proportions. In particular, future research should investigate whether CRA + W is an effective intervention approach for helping students build understanding of prerequisite skills needed to learn rate of change, such as numeracy, graphing, as well as modeling and solving algebraic equations. More research is also needed to evaluate the social validity of secondary math interventions, and to determine if the degree of difficulty of the math content addressed in secondary math interventions influences students' views of the interventions.

This study incorporated the use of the CRA + W intervention to address two main objectives. Lessons 1-4 of the intervention addressed constant rate of change, while lessons 5-7 addressed variable and average rate of change. One reason why students did not show immediate intervention effects may be due to the inclusion of both objectives on the rate of change assessments, especially since the variable rate of change objective accounted for the majority of points available on the assessments. It is possible that splitting the intervention into two separate interventions, one for constant rate of change and one for variable rate of change, may result in more immediate and significant effects. Future research should evaluate the effects of a CRA + W intervention on constant rate of change, separate from variable rate of change. Assessments specific to each objective may be more sensitive to changes in student performance.

Most of the research on CRA has been implemented in small group or individual settings. The results of this study indicate that students with disabilities may have similar difficulties related to rate of change concepts compared with students without disabilities. Future research should evaluate whether CRA + W is an effective instructional approach in inclusive secondary math classes. In particular, an integrated CRA sequence may be effective in inclusive secondary math classes because it provides opportunities to differentiate content through the use of concrete manipulatives, visual representations, and algorithms (Flores, Hinton, & Schweck, 2014). Future research should therefore evaluate the use of CRA + W in inclusive settings, and its outcomes for both students with and without disabilities.

As the principal investigator, I implemented instruction and administered assessments in this study. This is consistent with the many of the other research conducted on CRA (Flores, 2009; Flores, Hinton, & Schweck, 2014; Flores, Hinton, & Strozier, 2014; Strickland & Maccini, 2012). It is important that teachers be trained to effectively implement and evaluate the effects of academic interventions, to ensure sustainability of these practices. Future research should focus on training both special and general educators to use the CRA + W intervention, evaluation of teacher implementation of the CRA + W intervention, and evaluation of student results in response to the intervention.

Research on Math Measures

One of the main limitations within CRA research is the lack of research validated math measures designed to address specific math skills (Flores, Hinton, & Strozier, 2012). This is an area needed to not only improve the strength of CRA research, but also the ability of teachers to progress monitor and make data-based decisions to guide the instructional programming for their students. In particular, there is a need for research validated progress-monitoring measures for higher level math concepts, including algebra, geometry, trigonometry, and statistics. Future research should focus on developing and evaluating progress-monitoring measures for these specific math skills. Research should also focus on the use of CRA in the context of school-wide Response to Intervention (RtI) systems (Strickland & Maccini, 2012), but the application of CRA in RtI will be limited until there are effective progress monitoring measures developed, as one of the key features of RtI is data-based decision making.

Conclusion

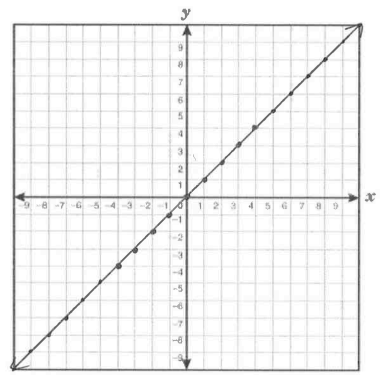
Success in mathematics, particularly algebra, is important for access to postsecondary education and employment (Bell & Norwood, 2010; Fullerton, 1995; Fuchs et al., 2008). Students with disabilities are often not successful in mathematics, particularly at the secondary level (Bryant, 2005; Butler et al., 2003; Maccini, Mulcahy, & Wilson, 2007). Additionally, the recent adoption of the CCSSM by the majority of states increases the rigor and focus on conceptual understanding in mathematics (Dingman et al., 2013). This study adds to the emerging research base on strategies to effectively improve students' with disabilities procedural and conceptual understanding and performance in mathematics. This study evaluated the effects of a CRA + W intervention on students' with disabilities total points of correct math responses on rate of change assessments. The results indicate that the CRA + W intervention may be effective in improving students' with disabilities understanding of rate of change. Further research is needed to determine the effectiveness of the CRA + W intervention on improving student performance in writing, as well as other secondary math concepts.

APPENDIX A

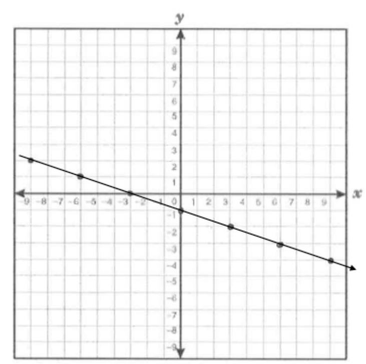
RATE OF CHANGE ASSESSMENTS

A.

M.1. What is the slope of the line below? ______



M. 2. What is the slope of the line below? _______



Solve the problems below. Show all of your work.

M.3 It takes Artemis 4 minutes to do each math problem on her homework. She has already worked for 10 minutes. She has 12 problems left. At this rate, what is the total amount of time it will take Artemis to finish all of her homework?

____

W.1. Describe in writing the ***process*** you used to solve the problem above.

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

M.4 Jerry likes to eat peanut butter on toast. He uses a ½ tablespoon of peanut butter on each slice of toast. If a jar of peanut butter has 16 tablespoons of peanut butter in it, how many slices of toast with peanut butter can Jerry have per jar?

____

M.5. A motorcycle leaves Salt Lake City at 10:00 am and travels for an hour at 30 miles per hour, then for an hour at 60 miles per hour. The motorcyclist stops for lunch for 2 hours, then travels for 3 hours at 45 miles per hour.
Make a table and graph showing the times and distances traveled at each stage of the journey. Show all of your work, and number and label the graph.

Table:

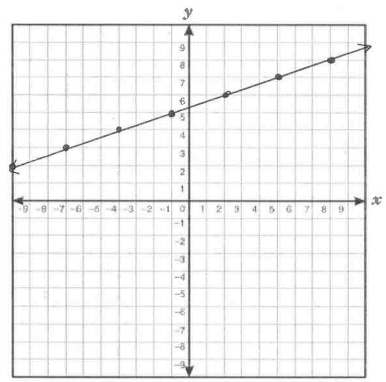
M.6. What is the average speed for the whole journey?______

W.3. ***Explain*** the steps you used to determine the average speed for the whole journey.

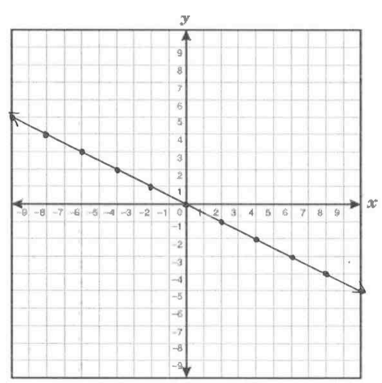
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

B.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 The water level of a lake decreases by ¼ foot per year during a drought. How many years of drought will it take for all 16 feet of the water in the lake to dry up? ______

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Joshua is in a pie-eating contest. He eats 2 pies per minute. He has already eaten 3 pies and there are 4 minutes left. At this rate, how many pies will he eat altogether during the contest?

____

M.5. A car left Provo at 11 am and traveled for two hours at 45 miles per hour, then for an hour at 50 miles per hour. The car stopped to go to a national park for one hour, then traveled for two hours at 55 miles per hour.

Make a table and graph showing the times and distances traveled at each stage of the journey. Show all of your work, and number and label the graph.

Table:

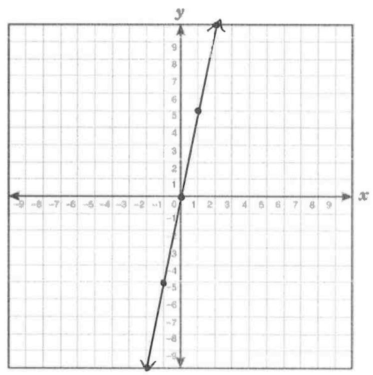
M.6. What is the average speed for the whole journey?______

W.3. ***Explain*** the steps you used to determine the average speed for the whole journey.

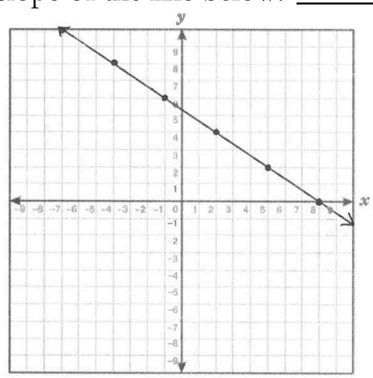
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

C.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Max runs at a rate of 4 miles per hour. He has already run for 8 miles today. He plans to run for 2 more hours today. At this rate, how many total miles will he run today? _________

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Jerry is painting fences. He paints 2/5 fences per hour. At this rate, how many hours will it take Jerry to paint 10 fences?_______

M.5. Jeff is taking a road trip. He drives for 3 hours at a rate of 60 miles per hour, then for 2 hours at a rate of 40 miles per hour. Jeff stops to eat lunch for 1 hour, then drives for 1 hour at a rate of 30 miles per hour.

Make a table and graph showing the times and distances traveled at each stage of the journey. Show all of your work, and number and label the graph.

Table:

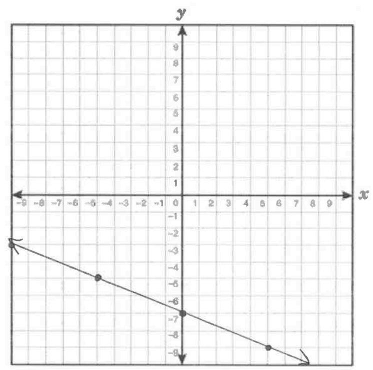
M.6. What is the average speed for the whole journey?______

W.3. ***Explain*** the steps you used to determine the average speed for the whole journey.

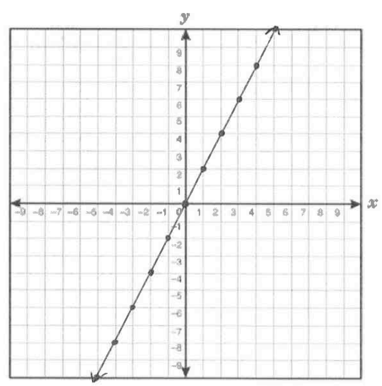
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

D.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Sandra donates 1/5 of every dollar she earns to the food bank. At this rate, how much money does she need to earn to be able to donate $12? _________

W.1. Describe in writing the ***process*** you used to solve the problem above.

___

___

___

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

___

___

___

M.4 Tina earns $5 for every car she washes. She already earned $15 this week. She plans to wash 7 more cars this week. At this rate, how much total money will she earn washing cars this week? _________

M.5. A family owns a yogurt company. They produce yogurt in one-cup containers. One worker, Abby, is responsible for filling the tubs of yogurt. She comes to work at 9 am and fills containers at a rate of 12 containers per hour for three hours. For one hour, she fills containers at a rate of 10 containers per hour. She then takes a one hour lunch break. When she comes back from lunch, she fills containers at a rate of 20 containers per hour for two hours.

Make a table and graph showing the times and containers filled at each stage of the Abby's workday.
Show all of your work, and number and label the graph.

Table:

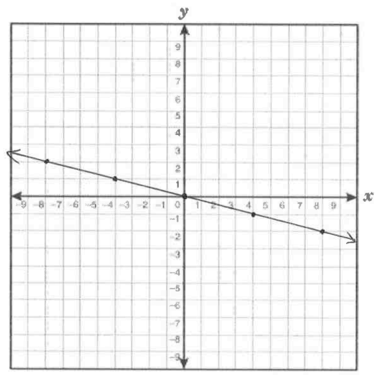
M.6. What is the average rate Abby fills containers at for the workday?______

W.3. ***Explain*** the steps you used to determine the average rate for the workday.

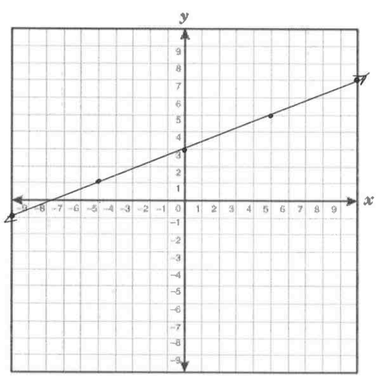
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

E.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Jeremiah runs at a pace of 3 miles per hour. Jeremiah has already run 6 miles this week. Jeremiah plans to run for 4 more hours this week. At this rate, how many total miles will Jeremiah run this week? _________

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Joey earns tips at a rate of ½ dollars per root beer float he makes. At this rate, how many floats does he need to make to earn $12 in tips? ________

M.5. Joe is filling an Olympic size swimming pool with a hose that doesn't have good water pressure. Joe fills the pool for three hours at a rate of 60 gallons per hour, then for two hours at a rate of 30 gallons per hour. Joe shuts off the hose for an hour to see if he can make the water pressure return to normal. He then fills the pool at a rate of 40 gallons per hour for four hours.

Make a table and graph showing the times and gallons at each stage of the filling process.
Show all of your work, and number and label the graph.

Table:

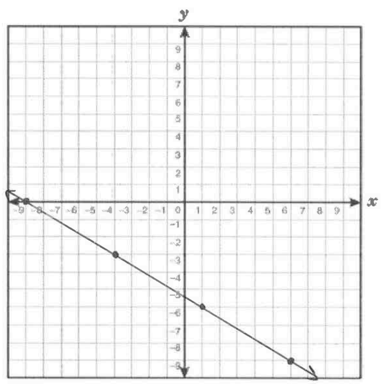
M.6. What is the average rate of the filling process?______

W.3. ***Explain*** the steps you used to determine the average gallons per hour.

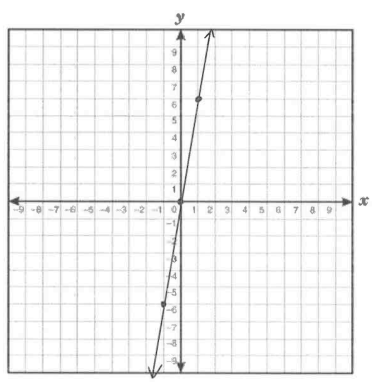
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

F.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Carrie works at a coffee shop. She earns tips at a rate of ¾ dollars per drink she makes. At this rate, how many drinks would she need to make to earn $6 in tips? _________

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Andrew can mow 2 lawns per hour. He has already mowed 4 lawns this week. He plans to mow lawns for 5 more hours this week. At this rate, how many total lawns will Andrew mow this week? _______

M.5. A car leaves Moab at 7 am and travels for three hours at 70 miles per hour, then for an hour at 50 miles per hour. The car stops for two hours, then continues for four hours at 75 miles per hour.

Make a table and graph showing the times and distances traveled at each stage of the journey. Show all of your work, and number and label the graph.

Table:

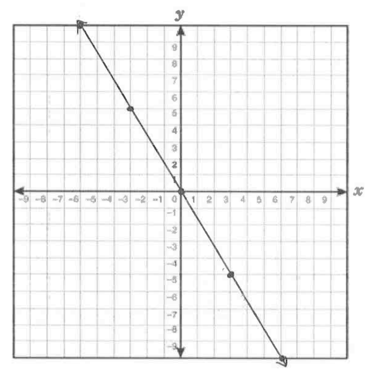
M.6. What is the average speed for the whole journey?_______

W.3. ***Explain*** the steps you used to determine the average speed for the whole journey.

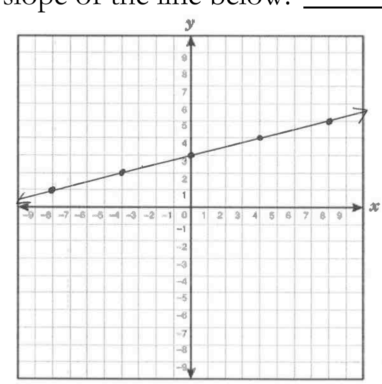
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

G.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Andrea makes soap to sell at the farmers market. She sells each bar of soap for $5. She has already made $20 today. If she sells 7 more bars of soap today at this price, how much total money will she have made today?_________

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Sarah has a credit card debt of $40. She pays off the credit card at a rate of 4/5 dollars per week. At this rate, how many weeks will it take her to pay off the full $40 she owes on her credit card? ______

M.5. Tina has a job picking berries. She gets to work at 7 am and picks berries for 2 hours at a rate of 6 baskets per hour. She then picks berries at a rate of 8 baskets per hour for 3 hours. She takes a lunch break for 1 hour. Then she picks berries at a rate of 5 baskets per hour for 1 hour.

Make a table and graph showing the times and number of berry baskets picked for each stage of Tina's workday. Show all of your work, and number and label the graph.

Table:

M.6. What is the average rate of berry picking for Tina's workday? ______

W.3. ***Explain*** the steps you used to determine the average number of baskets picked for the workday.

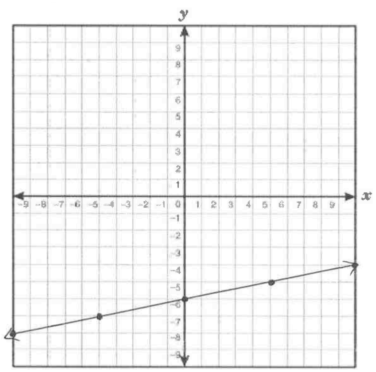
____
____
____

W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

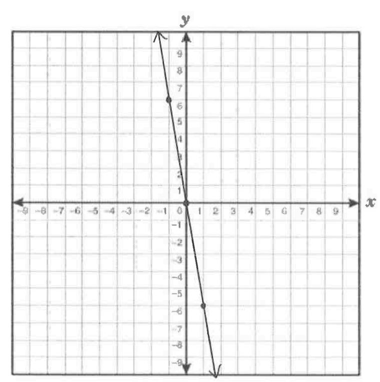
____
____
____

H.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 A tree grows ¼ inch per year. At this rate, how many years will it take for the tree to be 12 inches tall? _________

W.1. Describe in writing the ***process*** you used to solve the problem above.

___

___

___

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

___

___

___

M.4 Harriet earns $8 per hour for baby-sitting. She has already earned $16 this week for baby-sitting. She plans to baby-sit for 6 more hours this week. How much total money will she earn for baby-sitting this week? _______

M.5. A bicyclist left Ogden at 8 AM and traveled at a rate of 20 miles per hour for 3 hours, then took a break for 1 hour. The bicyclist then traveled for 2 hours at a rate of 30 miles per hour, and then 1 hour at a rate of 35 miles per hour.

Make a table and graph showing the times and distances traveled at each stage of the bike ride. Show all of your work, and number and label the graph.

Table:

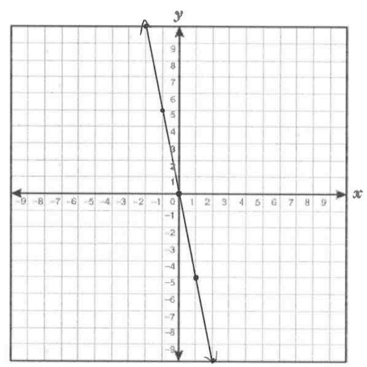
M.6. What is the average speed for the whole bike ride?_______

W.3. ***Explain*** the steps you used to determine the average speed for the whole bike ride.

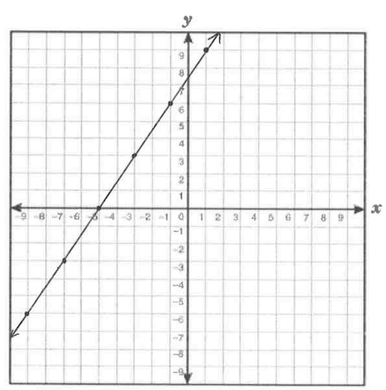
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

I.

M.1. What is the slope of the line below? ______



M. 2. What is the slope of t he line below? _______



Solve the problems below. Show all of your work.

M.3 Sam earns $3 for every dog she walks. She already earned $9 this week. She plans to walk 8 more dogs this week. At this rate, how much total money would she earn this week? __________

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Jordy reads at a rate of 2/3 pages per minute. At this rate, how many minutes will it take her to read 18 pages?_________

M.5. Steven wants to take a hot bath, but doesn't have a very good water heater. Steven fills his bathtub for 8 minutes at a rate of 2 gallons per minute, and then turns off the water for 5 minutes to let it get hot. He turns the water back on and fills the bathtub at a rate of 3 gallons per minute for 5 minutes, and then at a rate of 2 gallons per minute for 4 minutes.

Make a table and graph showing the times and gallons at each stage of filling process. Show all of your work, and number and label the graph.

Table:

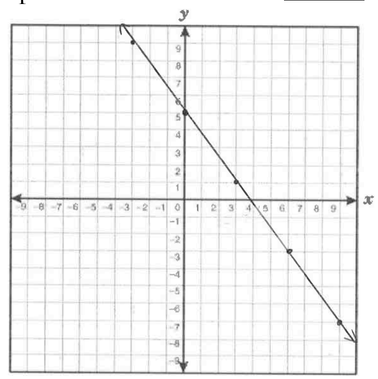
M.6. What is the average rate for the filling process?_______

W.3. ***Explain*** the steps you used to determine the average gallons per minute.

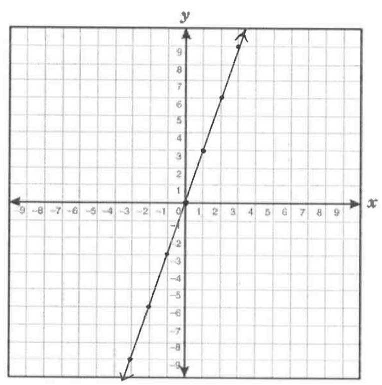
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

J.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 A tree grows at a rate of 1/3 inches per month. At this rate, how many months will it take for the tree to be 12 inches tall?

_____

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 James can bike at a rate of 6 miles per hour. James has already biked 12 miles of a trail. At this rate, James needs to bike for 4 more hours to finish the trail. How many total miles long is the trail?_________

M.5. Mike works at a juice bottling factory. He arrives at work at 10 am and fills bottles of juice at a rate of 80 bottles per hour for the first 3 hours of his shift. He then takes a 1 hour lunch break. When he returns from break, he fills bottles at a rate of 60 bottles per hour for 2 hours, and then at a rate of 90 bottles per hour for 1 hour.

Make a table and graph showing the times and bottles filled at each stage of Mike's workday. Show all of your work, and number and label the graph.

Table:

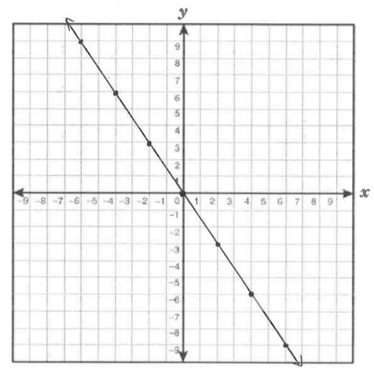
M.6. What is the average rate Mike fills bottles at for the workday?_______

W.3. ***Explain*** the steps you used to determine the average rate for the workday.

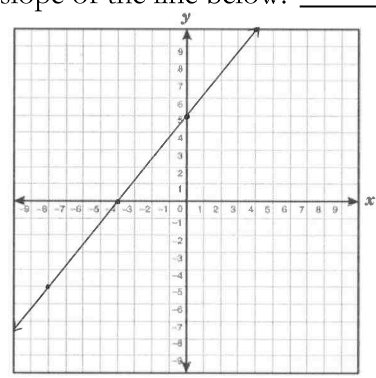
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

K.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Samantha can run at a rate of 6 miles per hour. She has already run 3 miles this week. She plans to run for 3 more hours this week. At this rate, how many total miles will she run this week?_________

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

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____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 The water level in a dog's water dish decreases by ½ inch per hour on a hot day. How many hours will it take for the dog's dish to be empty if it is 6 inches deep at the start of the day?_______

M.5. Heidi invests $60 in a savings account. Heidi earns $2 per month in interest for the first 6 months, no interest for the next 4 months, $3 per month in interest for the following 3 months, and $5 per month in interest for 9 months.

Make a table and graph showing the times and amount of interest earned at each stage of the account.
Show all of your work, and number and label the graph.

Table:

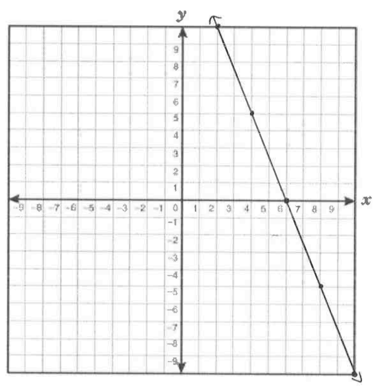
M.6. What is the average rate of interest earned during the whole time period described?_____

W.3. ***Explain*** the steps you used to determine the average amount of interest earned.

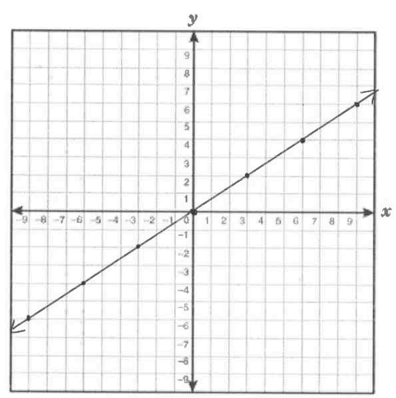
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

L.

M.1. What is the slope of the line below? ______



M. 2. What is the slope of the line below? _______



Solve the problems below. Show all of your work.

M.3 A drone descends at a rate of 4/5 feet per minute when preparing for landing. How many minutes will it take a drone to descend a total of 20 feet? _______

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Mark can skateboard at a pace of 9 miles per hour. He has already skateboarded for 5 miles this week. He plans to skateboard for 2 more hours this week. At this rate, how many total miles will he skateboard this week?__________

M.5. Jami is filling a swimming pool with water. She fills the pool for 3 hours at a rate of 30 gallons per hour. She then turns off the water when she goes to lunch for 1 hour. When she gets back, she fills the pool for 4 hours at a rate of 50 gallons per hour, and then for 1 hour at a rate of 40 gallons per hour.

Make a table and graph showing the times and gallons at each stage of filling process. Show all of your work, and number and label the graph.

Table:

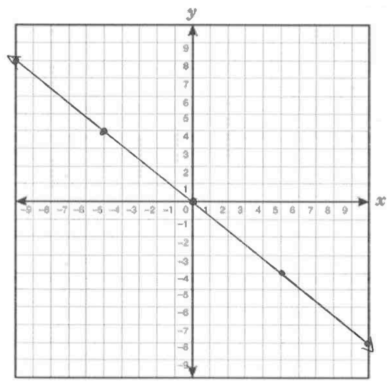
M.6. What is the average rate for the filling process?_______

W.3. ***Explain*** the steps you used to determine the average gallons per hour.

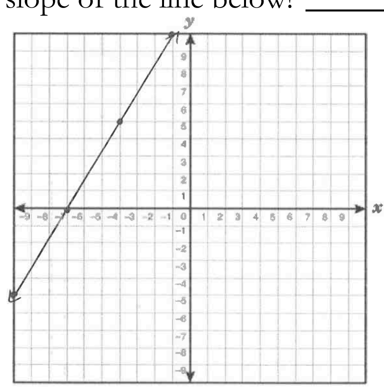
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

M.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Dale earns $2 for every dog he walks. He already earned $8 this week. He plans to walk 6 more dogs this week. At this rate, how much total money will he earn this week? _________

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. **Explain** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 John is trying to save money for a vacation. If he saves ¼ of every dollar he earns, how much money will he have to earn to save a total of $24? ______

M.5. Amy works at a toy block factory. Amy is responsible for filling boxes with blocks. She comes to work at 8 am and fills boxes at a rate of 7 boxes per hour for 4 hours. She then takes an hour lunch break. When she comes back to work she fills boxes at a rate of 9 boxes per hour for 3 hours and then 6 boxes per hour for 1 hour.

Make a table and graph showing the times and boxes filled at each stage of Amy's workday. Show all of your work, and number and label the graph.

Table:

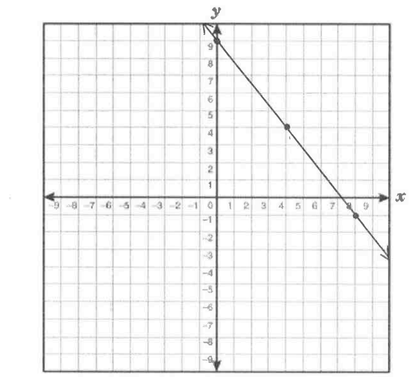
M.6. What is the average rate Amy fills boxes at for the workday?______

W.3. ***Explain*** the steps you used to determine the average rate for the workday.

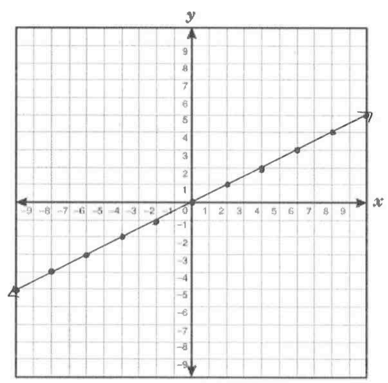
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

N.

M.1. What is the slope of the line below? ______



M. 2. What is the slope of the line below? _______



Solve the problems below. Show all of your work.

M.3 Danny is making trail mix. ¾ of the mix is peanuts. Danny has 6 cups of peanuts. At this rate, how many total cups of trail mix can Danny make?

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Joe spends $3 to buy a snack every day. He has already spent $6 on snacks this week. Joe plans to buy 4 more $3 snacks this week. At this rate, how much total money will Joe spend on snacks this week?

____

M.5. Sandy earns $5 per month in interest on her savings account for the first 12 months, then $0 interest per month for the next 3 months, $8 per month in interest for the next 9 months, and $10 per month in interest for the next 12 months.

Make a table and graph showing the times and amount of interest earned at each stage of the account. Show all of your work, and number and label the graph.

Table:

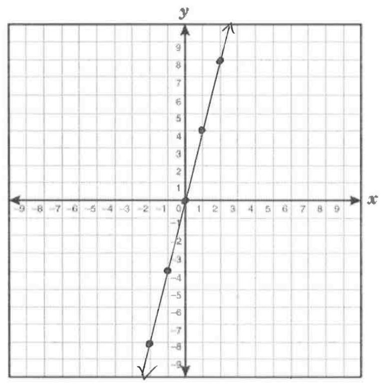
M.6. What is the average rate of interest earned during the whole time period described?______

W.3. ***Explain*** the steps you used to determine the average amount of interest earned.

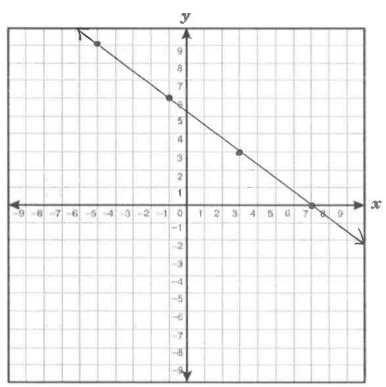
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

O.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Melinda swims 3 laps per minute. She has already swum 20 laps today. She plans to swim for 40 more minutes today. At this rate, how many total laps will she swim today?_________

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 During the spring, the snow in the mountains melts at a rate of 2/3 inches per day. How many days will it take for 36 inches of snow to melt?________

M.5. Henry is filling his horse's water tank using an unreliable hose. Henry fills the tank for 30 minutes at a rate of 3 gallons per minute. He then takes a 15-minute break to feed his animals. When he returns, he fills the tank for 15 minutes at a rate of 5 gallons per minute, and then for 10 minutes at a rate of 1 gallon per minute.

Make a table and graph showing the times and gallons at each stage of filling process. Show all of your work, and number and label the graph.

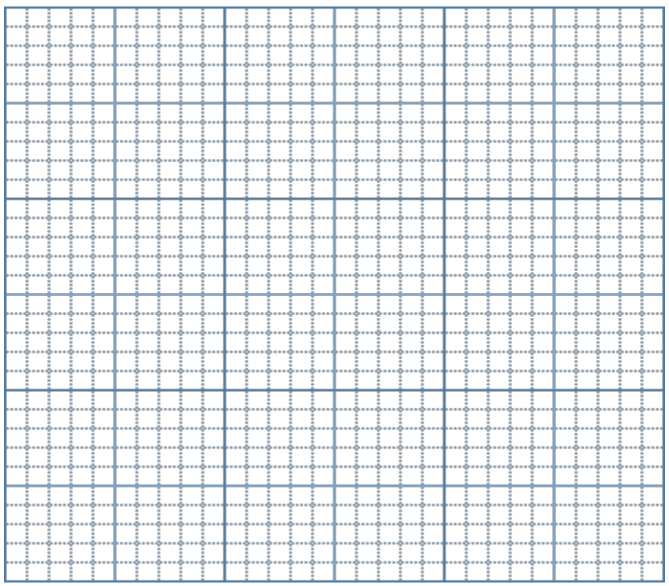


Table:

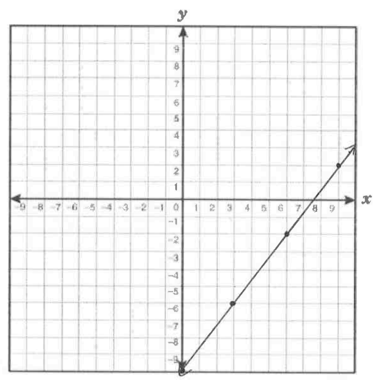
M.6. What is the average rate for the filling process?_______

W.3. ***Explain*** the steps you used to determine the average gallons per minute.

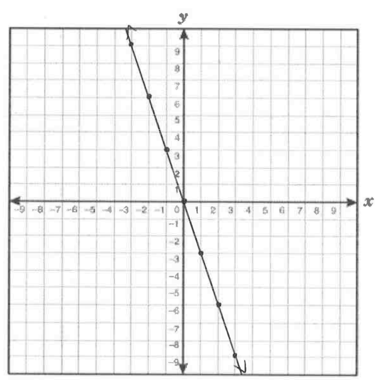
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

P.

M.1. What is the slope of the line below? ______



M. 2. What is the slope of the line below? _______



Solve the problems below. Show all of your work.

M.3 Jacky is filling a fish tank. She fills the fish tank at a rate of ¾ gallons per minute. She needs the tank to be 9 gallons full. At this rate, how many minutes will she need to fill the tank for?

____

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Mark earns $2 in tips for every sandwich he makes at work. He has already made $8 today. Mark plans on making 10 more sandwiches today. At this rate, how much total money in tips will Mark earn today? __________

M.5. Andy makes bricks at a brick factory. He starts work at 7 am. He makes bricks at a rate of 15 bricks per hour for the first 2 hours, then takes a break for 1 hour. He then makes bricks at a rate of 30 bricks per hour for 1 hour, and then at a rate of 20 bricks per hour for 3 hours.

Make a table and graph showing the times and bricks made at each stage of Andy's workday. Show all of your work, and number and label the graph.

Table:

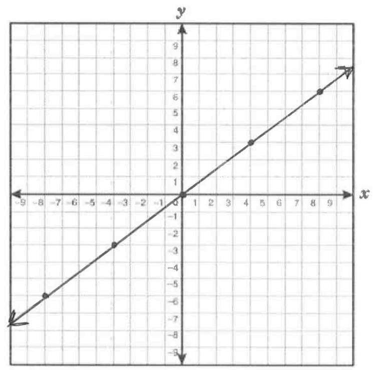
M.6. What is the average rate Any makes bricks at per hour for the workday?______

W.3. ***Explain*** the steps you used to determine the average rate for the workday.

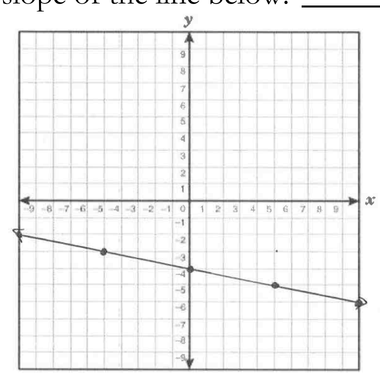
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

Q.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Jack runs at a pace of 4 miles per hour. He has already run 6 miles this week. He plans to run for 4 more hours this week. At this pace, how many total miles will he run this week?_________

W.1. Describe in writing the ***process*** you used to solve the problem above.

___

___

___

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

___

___

___

M.4 Melanie is making cookies. She uses 4/5 cups of flour per each batch. She plans on using 8 cups of flour. At this rate, how many batches of cookies can she make?______

M.5. Jerry has a loan that charges $5 per month in interest for the first 12 months, no interest while Jerry is in school for 9 months, $1 per month for the 12 months following his graduation from school, and $2 per month for the next 18 months.

Make a table and graph showing the times and amount of interest charged at each stage of the loan.
Show all of your work, and number and label the graph.

Table:

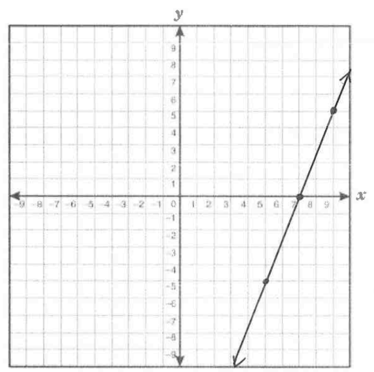
M.6. What is the average rate of interest charged during the whole time period described?______

W.3. ***Explain*** the steps you used to determine the average amount of interest charged.

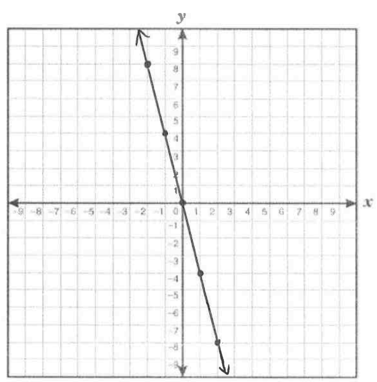
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

R.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 James is an artist. He completes paintings at a rate of 1/5 per hour. At this rate, how many hours will it take him to complete 10 paintings? _______

W.1. Describe in writing the ***process*** you used to solve the problem above.

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

M.4 Andrea earns $4 for every newspaper she delivers. She already earned $36 delivering papers this week. Andrea plans to deliver 20 more papers this week. At this rate, how much total money would she earn this week?_________

M.5. Shawna is making a homemade shark tank. She starts filling the tank at 7 am and fills it at rate of 24 gallons per hour for 2 hours. She then stops filling the tank for 1 hour to make sure there are no leaks. She then starts filling the tank at a rate of 36 gallons per hour for 3 hours, and then at a rate of 12 gallons per hour for 1 hour.

Make a table and graph showing the times and gallons at each stage of filling process. Show all of your work, and number and label the graph.

Table:

M.6. What is the average rate for the filling process?______

W.3. ***Explain*** the steps you used to determine the average gallons per hour.

____

____

____

W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

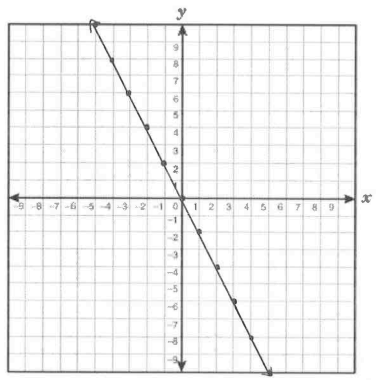
____

____

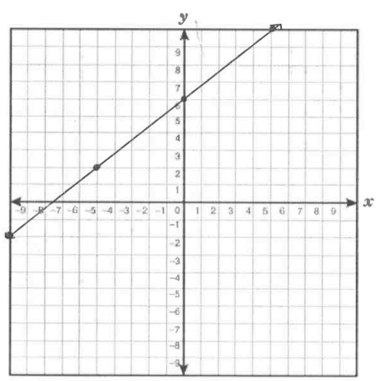
____

S.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 Susan works for 9 hours each day. She has already worked for 18 hours this week. She is scheduled to work for 5 more days this week. At this rate, how many total hours will Susan work this week? ______

W.1. Describe in writing the *process* you used to solve the problem above.

W.2. *Explain* how you know your answer makes sense. Provide an example to support your reasoning.

M.4 A bathtub drains at a rate of 1/3 inches per minute. How many minutes will it take to drain the bathtub if it is 12 inches full?_______

M.5. Sam gets a credit card that charges $1 per month in interest for the first 2 months, then no interest for the next 10 months. The card company then charges $3 per month in interest for the next 6 months, and $4 per month in interest for the next 12 months.

Make a table and graph showing the times and amount of interest charged at each stage of the credit card account.
Show all of your work, and number and label the graph.

Table:

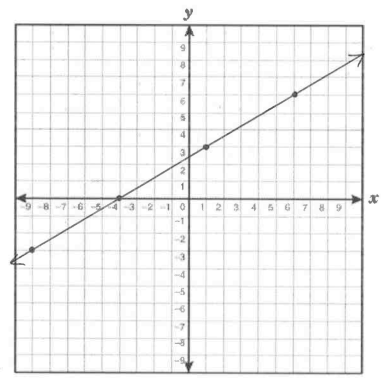
M.6. What is the average rate of interest charged during the whole time period described?_______

W.3. ***Explain*** the steps you used to determine the average amount of interest charged.

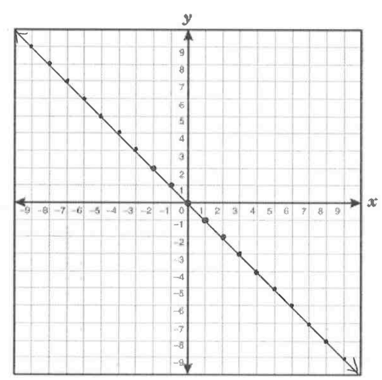
W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

T.

M.1. What is the slope of the line below? _____



M. 2. What is the slope of the line below? ______



Solve the problems below. Show all of your work.

M.3 It rains 2/5 inches per month in New Mexico. At this rate, how many months will it take to rain a total of 4 inches?

____

W.1. Describe in writing the ***process*** you used to solve the problem above.

____

____

____

W.2. ***Explain*** how you know your answer makes sense. Provide an example to support your reasoning.

____

____

____

M.4 Some hikers climb down a mountain at a rate of 5 miles per hour. They have already climbed down 3 miles. They have 2 more hours left of their hike. At this rate, how many total miles is the hike down the mountain? _________

M.5. Nancy invests \$20 in a savings account. Nancy earns \$4 per month in interest for the first 8 months, then no interest for the next 4 months. After that, Nancy earns \$6 per month in interest for the next 12 months, and then \$8 per month in interest for the next 12 months.

Make a table and graph showing the times and amount of interest earned at each stage of the account.
Show all of your work, and number and label the graph.

Table:

M.6. What is the average rate of interest earned during the whole time period described?______

W.3. ***Explain*** the steps you used to determine the average amount of interest earned.

____
____
____

W.4. ***How*** do you know your answer is correct? Provide an example to support your reasoning.

____
____
____

APPENDIX B

STUDENT'S INTERVENTION RATING PROFILE

Student's Intervention Rating Profile

Directions: Please answer the following questions using a scale ranging from 1 (*Strongly Disagree)* to 6 *(Strongly Agree)*.

1. Using POD✓, cubes and diagrams is a helpful way to teach math.

1	2	3	4	5	6
Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

2. Using POD✓, cubes and diagrams to teach math is too hard.

1	2	3	4	5	6
Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

3. Using POD✓, cubes and diagrams to teach math may be hard for other students.

1	2	3	4	5	6
Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

4. There are better ways to teach math to students than using POD✓, cubes and diagrams.

1	2	3	4	5	6
Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

5. Using POD✓, cubes and diagrams to teach math is a good way to teach math to other students.

1	2	3	4	5	6
Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

6. I like using POD✓, cubes and diagrams to learn math.

1	2	3	4	5	6
Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

7. I think that using POD✓, cubes and diagrams to teach math will help students do better in school.

1	2	3	4	5	6
Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

APPENDIX C

POD✓ GRAPHIC ORGANIZE

The POD ✓ Strategy For Writing To Learn Mathematics

P	**Propose** the problem	1. What are you asked?	2. Outline information given:
O	**Outline** how you will solve the problem	3. **Outline** the steps To Solve the Problem:	
D	**Describe & Defend** your answer	4. **Describe** in writing the process you used to solve the problem.	5. **Defend:** Explain how you know your answer makes sense. Provide an example for support.
✓	I re-read the problem ____ I set up the problem correctly____ I checked my calculations ____ I didn't make any common mistakes ____		

APPENDIX D

UNIT PLAN FLOWCHART

Teaching Rate of Change Using CRA & WTLM

1 Lesson	2 Lesson	3 Lesson	4 Lesson
Big Idea: Linear equations express a relationship between two variables. This relationship can be depicted on a graph and table in addition to the equation. The slope of a linear equation can be described as positive, negative, undefined or 0. ▪ Highlights: rule of 4; relationships between variables; types of slope.	Big Idea: Linear equations have specific features that are used to refer to different parts of what they represent. ▪ Highlights: x and y axis, locations of positive and negative values on a graph, linear equation, relationship between variables, x and y intercepts.	Big Idea: Rate of change describes the relationship between two variables, as one increases or decreases. Rate of change can be used to find missing values in a pattern. ▪ Highlights: stacking cubes to find the rate of change and missing values with sequentially stacked cubes (ex: 1,2,34).	Big Idea: Rate of change can be used to find missing values in a pattern, even when you only have two points that you know. ▪ Highlights: connect to slope formula, connect key features of stacked cubes to key features on graph, table & equation.
▪ Introduce POD✓ strategy & graphic organizer & fill in together.	▪ Complete the POD✓ graphic organizer with guidance.	▪ Complete the POD✓ graphic organizer with guidance.	▪ Individually complete the P and O parts of the graphic organizer; complete D & ✓ with guidance.
Need to make sure students have foundational understanding.	Connect concept of change between towers to concept of rate of change/ slope in LP 3.	Increase difficulty of stacking cube problems by finding missing values; builds to next lesson w/ more towers missing.	Provides foundation to connect to formula to find slope.
Review key features of graphed linear equations. Introduce rule of 4 (graph, equation, table, context). Match graphed equations w/their table, equation &/or context. Introduce types of slope (+, -, 0, undefined). Students classify the slope of the already graphed equations.	Review key features of graphed linear equations (graphing points, x and y intercepts). Provide contexts of linear equations & model them with stacking cubes. Highlight x and y intercepts & slope within the contexts.	Compare linear & exponential equations. Define rate of change (comp. w/slope). Model stacked cubes & how to find slope. Model how to find missing values using stacking cubes (ex: have 1,2,3, need 7). Make up contexts for problems, solve problems using rule of 4.	Review how to represent & solve real life problems using linear equations. Explain that sometimes we only have 2 towers to use to solve problems, and that we can use this information to find the slope. Use cubes and/or diagrams to solve at least 3 problems.
Common Mistakes: mixing up x and y; errors when graphing or counting slope from a graphed line; confusion around undefined slope	Common Mistakes: confusion around undefined slope (teach as infinite steepnees)	Common Mistakes: Difficulty with the 0^{th} tower, trouble with multiplicative vs. additive relationships.	Common Mistakes: miscounting number of towers between those provided; calculation errors with subtraction & division.
Accommodations: large grids, pre-labeled or numbered grids, rulers, prompts & corrective feedback for errors	Accommodations: frayer models for vocab, labeled grids for +, -, etc.	Accommodations: *give the 0^{th} tower for all students (don't start with the 1^{st} tower and have them fine the 0^{th}).	Accommodations: graph paper, prompts to draw out the work/write out more than 3 pairs in the table to track the pattern; multiplication charts.

RL 1 Review Lesson (If Needed)	5 Lesson	6 Lesson	RL 2 Review Lesson (If Needed)
Big Idea: Review foundational concepts of linear equations and slope. ▪ Highlights: slope from graphed lines, equations and tables; practice solving and graphing 1 & 2 step equations from contexts.	Big Idea: Some functions have multiple parts that can have different rates of change. Average RoC ▪ Highlights: variable rate of change, real life examples of when this occurs; average rate of change.	Big Idea: Some functions have multiple parts that can have different rates of change. Average RoC Describe how & why rate changes. ▪ Highlights: variable rate of change w/ problems that have 3-4 phases.	Big Idea: Review of all objectives covered during intervention prior to final lesson.
Review components of the POD✓ strategy. Student fills in POD✓ for linear equation problems.	POD✓ filled in independently for one to two problems.	POD✓ graphic organizer filled in for 3 problems.	Have student provide components of the POD✓ from memory. Student fills in POD✓ for linear equation problems and variable rate of change problems.
Reviews foundational linear equation concepts to ensure student is ready for variable rate of change problems.	Introduces concept of variable rate of change; next lesson builds conceptual understanding of how & why rate changes	Increases rigor of variable rate of change problems by increasing to 4 phases; interpreting rate of change.	Reviews key concepts from all lessons to prepare student for level of independence required for Lesson 7.
Review the definition of slope & different ways to classify slope. Practice identifying slope from a graph. Discuss common mistakes and how to prevent them. Solve real life problems, when provided with context, using diagrams, tables, equations, and graphs. Focus on explaining why answers make sense, and on identifying & correcting errors.	Provide student w/contextual example of variable RoC. Ask for ideas of how to find the average RoC. Go over ideas, demo how to solve using diagrams & stacking cubes. Work w/different contexts of variable RoC probes (fill a tank, distance/rate/time, mixed bag, make your own). Contexts include 2-3 phases.	Review concepts from previous lesson. Model how to solve, stop & jot after each demo problem for student to describe how and why rate is changing. Work with different contexts of variable RoC, with focus on describing how & why rate is changing. Review key concepts. Contexts include 3-4 phases.	Review the ways slope can be classified. Practice identifying slope from a graph, and have student describe how to do so. Review the difference between constant and variable rate of change. Work on solving problems with constant rate of change, focusing on why the answer makes sense. Work on solving problems with variable rate of change, focusing on how & why rate changes.
Common Mistakes: Counting errors when finding slope from a graph, mixing up y-intercept & slope.	Common Mistakes: Difficulty w/concept of average; calculation & process errors	Common Mistakes: Difficulty understanding how rate changes; process or calculation errors; errors w/finding average.	Common Mistakes: Mixing up slope & y-int. for constant RoC, mixing up x and y variables for variable RoC
Accommodations: provide additional practice and modeling for counting, mnemonic strategies for slope & y-int.	Accommodations: provide steps/more prompting for students in need of it. Have diff. levels of difficulty of problems. (ex: probs w/ 2 or 3 phases)	Accommodations: use stacking cubes & diagrams to show how rate changes; different levels of difficulty of problems; prompts/steps provided	Accommodations: Review what variables stand for; diagrams to show rate change; problems with less phases

7	Lesson		Lesson		Lesson		Lesson
Big Idea: Some functions have multiple parts that can have different rates of change. Average RoC Describe how & why rate changes. ▪ Highlights: variable rate of change w/ problems that have 4 phases.							
Fill in POD✓ graphic organizer for each task problem.							
This is the last lesson in the unit; focus on conceptual understanding & justifications for how rate is changing.							
Review concepts from LP 6 Talk about how & why rate changes; interpret rate of change in 1 contextual problem. Student works on problem tasks (longer, applied problems)- 1 with guidance, 1 individually. Contexts include 4 phases.							
Common Mistakes: understanding how rate changes; process or calculation errors; errors w/finding average; frustration w/ prob. lengths.							
Accommodations: Use stacking cubes & diagrams to show how rate changes; different levels of difficulty of problems; prompts/steps provided.							

APPENDIX E

LESSON PLANS

Lesson Plans

Subject: CRA &WTLM Intervention Grade level: Secondary 1 Lesson: 1 Name:
Date:

Core Standard:	**Instructional Objective**:	**Content (concepts, information, skills, new vocab, etc.)**:	**Instructional Materials Needed**:
(F.IF.4) SWBAT correctly identify and interpret key features of graphs and tables in terms of the quantities (key features include: intercepts; intervals where function is increasing, decreasing, positive, negative; relative maximums and minimums; symmetries; end behavior; and periodicity).	SWBAT identify, locate and graph coordinate pairs. SWBAT identify and locate y intercepts. SWBAT identify and graph linear equations using completed tables (ex: students will match completed tables of linear equations with their graphs; students will graph linear equations from points provided on tables). SWBAT classify the slope of a linear equation as positive or negative from a graph.	X-axis Y-axis Coordinate pair Coordinate plane Y-intercept Slope Positive Negative Undefined Zero	Graph paper (coordinate grid pre-made) Colored pencils POD graphic organizers (4 copies) Ruler Unifix cubes/ base 10 blocks Individual dry erase boards Slope, equation, table, context cards for sorting/matching Lesson 1 packet (2 copies) Lesson 1 exit slip

Outline of Lesson:

1. Today we are going to review some important graphing reminders. Graphing is an important skill for the upcoming lessons in our unit together, so I need to make sure that you understand how to do it. Why else might it be important to know the foundations of graphing?

2. Another important concept we are going to review is slope. Have you heard of slope before? We are going to talk about the different ways we can describe slope, and how we can find out what the slope of a line is today.

3. Something else we are going to be doing in all of the lessons with me is working on writing explanations for our math problems, using this graphic organizer (refer to POD ✓graphic organizer).
4. Today we are going to fill this out together as we go through the lesson.

5. Pass out the POD✓ graphic organizers and lesson 1 packets.

6-10. Let's review some of the vocabulary that is important for graphing points. For each of the terms listed above, I will say a definition, have the student repeat the definition, have the student write down a definition in their own words on a guided notes sheet in their lesson 1 packet, and have the student draw an example of the term on the guided notes sheet in their lesson 1 packet.

Coordinate plane
x-axis
y-axis
Coordinate pair
y-intercept

11. So, another new word for today's lesson is slope. When have you heard that word before, in school or outside of school? Good. The definition of slope that we are going to use today is that slope describes how steep a straight line is. Write down a definition of slope in your own words on the second page of your lesson 1 packet.

12. We are going to describe the slope of lines today. There are four main words we are going to use to describe slope; positive, negative, undefined, and zero.

13. The first way we can describe slope is "positive". Let's look at one of your stacking cubes problems from yesterday. Stack cubes to represent a problem with positive slope. In this problem, how do our stacks of blocks change from tower to tower? Student responds: they increase by 2. Good. So we are adding on to the towers as we move to the right. This shows us a positive slope because the towers increase as we add towers.
14. The definition of a positive slope is that the line increases as we move to the right. If we think about this with our x and y axis, the definition of positive slope is that the y values increase as the x values increase.
15. Write down a definition for positive slope on your lesson 2 packet.

16. Draw an example of a line with positive slope on your lesson 2 packet.
17. The next way we can describe slope is "negative". Make a prediction/hypothesis about what you think negative slope might mean. Show what a line with negative slope would look like using the stacking cubes. In this stacking cube problem, how do our stacks change as we add towers? Do they increase or decrease. Response: decrease. Good. How many were they decreasing by?

Response: (varies). Good. So this shows negative slope because the towers decrease as we add towers.

18. The definition of negative slope is that the line decreases as we move to the right. If we think about this on an x y axis (coordinate plane), the definition of negative slope is that the y values decrease as we move to the right.
19. Write down a definition of negative slope.
20. Draw an example of a line with negative slope.
21. The next way we can describe slope is as having "zero slope". I am going to show you a stacking cubes problem that shows zero slope. Show three stacks of cubes that have the same number of cubes in each stack. Do the stacks change from tower to tower? Response: No. Good. This means there is zero change, so we would describe this slope as zero slope.
22. The formal definition of zero slope is that zero slope refers to lines that have the same y value for all x values. Lines with zero slope are also called horizontal lines. You can remember horizontal because it is like the horizon.
23. Write a definition for zero slope.
24. Draw an example of a line with zero slope.

25. There is one last type of slope. I want you to watch me build this stack tower. I start stacking cubes taller and taller. I am only going to have one tower. Can I describe what the change is between towers if I only have one? Students respond. I cannot describe the change between towers if I only have one tower because I don't know what the other towers look like. This means that my slope is "undefined".
26. The definition of undefined slope is that lines with the same x value for all y values have undefined slope. Lines with undefined slope are also called vertical lines. You can think of undefined slope as infinite slope- the slope is so steep that we can't actually tell what it is.
27. Write down a definition of undefined slope
28. Draw an example of a line with undefined slope.

29. For the next part of the lesson, we are going to identify slope from an equation. What is the equation (y=3x +2)? Good. Slope is the number that is in front of x. The formula y=mx + b is the formula for a linear equation, which we are going to talk more about in lesson 2. If slope is always the number in front of x, what letter stands for slope? Good.

30. We are going to also identify slope from graphed lines. Slope can be described as the rise over the run. Let's look at this graphed line. So, for this graphed line, you first identify two coordinate pairs. Then, you count how many you move up to the second point from the first point, and how many over you move from the first point to the second point. You write this as y/x.

31. Explain the card sorting game. This card game is based on what is called the rule of 4. The rule of 4 refers to having 4 different representations for linear equations- the equation, its graph, a table, and context that describes the relationship. You have a set of cards that is all mixed up. You will need to match the cards so that you have four cards that go together. In each of these sets of four, you will have a table, equation, graphed equation, and context that need to describe the same thing. After you have determined which cards go together, classify the slope of each line in your stack as positive, negative, zero, or undefined.

32. We are going to fill out a POD✓ as we do the first problem in the card sort. P- propose the problem: the question we are answering is: "How do you know that the four representations match?".
33. What information are you given?
34. O- outline the steps you will use to solve the problem
35. Model how to match the cards.
36. D- Describe and defend you answer; use words to describe the process you used to solve the problem.
37. Explain how you know your answer makes sense. Provide pictures or an example for support.

38-41. Check: Let's check our work. We want to make sure that we re-read the problem, set up the problem correctly, check our calculations, and make sure we didn't make any common mistakes.

42-43. Prompt the student to complete the card sort. I will provide corrective feedback and assistance as needed.

44-45. Prompt the student to complete lesson 2 practice problems, which involve making concrete depictions of positive and negative slope, then drawing those representations on their paper, completing a table, and graphing them. I will provide corrective feedback and assistance as needed.

46-49. Awesome job! Let's review. Draw an example of what positive slope looks like. Now show me what negative slope looks like. Now show me what undefined slope looks like. And show me what zero slope looks like. Great job! Tomorrow we will work more with linear equations so we can learn more about why being able to identify slope is useful.

50. I want you to fill out this exit ticket independently. The exit ticket has five problems. For three problems, the student will draw an example of the type of slope described. For two problems, the student will identify the slope of a graphed line.

Adaptations/Modifications: Large grid graphs Rulers for helping students keep track of their place on the graphs. Rest of adaptations/modifications will be determined based on specific students' IEPs.	**Reinforcement Procedures**: Verbal praise. Students will earn points on daily cards for following directs, being respectful, and being persistent problem solvers. I will tally the point and tell them how many they earned at the end of each day. Students will cash in points on assessment days.	**Assessments** **Pre**: Scores on baseline probes; student's responses to checks for understanding in the rationale of the lesson. **During**: Responses to checks for understanding; I will observe their work during guided practice & provide corrective feedback as well. **Post**: Exit slip	**Follow-up Activities**: Spiral in prompts for any common errors I see based on exit tickets in lesson #1 (ex: when I graph this point, I remember that negative numbers mean I move down if it is a y-coordinate).

Subject: CRA &WTLM Intervention Grade level: Secondary 1 Lesson: 2

Name: Date:

Core Standard:	Instructional Objective:	Content (concepts, information, skills, new vocab, etc.):	Instructional Materials Needed:
(F.IF.4) SWBAT correctly identify and interpret key features of graphs and tables in terms of the quantities (key features include: intercepts; intervals where function is increasing, decreasing, positive, negative; relative maximums and minimums; symmetries; end behavior; and periodicity).	SWBAT graph one and two-step equations when provided with a context.		Graph paper (coordinate grid pre-made) Guided notes POD graphic organizer (4 copies) Frayer models Unifix cubes/ base 10 blocks Individual dry erase boards Lesson 2 packet (2 copies) Lesson 2 exit slip

Lesson Outline:

1. Today we are going to learn about representing real life situations with linear equations. Once we know how to use linear equations to represent different situations, we can use math to help us solve real life problems. Yesterday we learned about the features of linear equations (like y-intercept, x-intercept, etc.). We also learned about different ways we can describe slope.

2-5. Let's review. What is a linear equation? Good. What is the x intercept? What is the y intercept? What is slope?

6. Hand student the review sheet.

7-11. We also want to review the different ways we can describe slope. Draw an example of a line with positive slope. Draw an example of a line with negative slope. Draw an example of a line with undefined slope. Look at the graphed line. What is the slope of that line?

12. Review what POD ✓ stands for.

13. Hand the student the lesson 2 packet.

14. Now that we have reviewed some information about graphing and slope, let's talk about modeling real world contexts with

equations. The first thing we need to know before we can model real world contexts with equations is the formula for a linear equation, $y=mx+b$. In this equation, m= slope and b=y-intercept. Y and x are variables that stand for different values in the problem.

15. Read through the example problem. Amy is trying to save money so she can buy a gift for her sister. She saves ¾ of the amount of money she earns from mowing lawns. How much money would she need to earn to be able to save $30?

16. POD ✓: P-propose the problem; what are you asked?
17. POD✓: What information are you given?
18. In this problem, x stands for the amount of money Amy earns. Y stands for money because it is the amount that changes based on how much money Amy earns. M is the slope, which shows the amount of money saved per the amount of money earned. We know that we want to find out how much money she needs to earn to be able to save a total of $30.
19. So, in this problem we are trying to find x, the amount of money earned.
20. Write an equation showing the example problem. $\$30=3/4x$
21. How can we show what this means using the blocks we have? I will discuss suggestions made by the student. Then, I will show them how to model the context with the stacking cubes.
22. POD✓: O- outline the steps you will use to solve the problem. Write an equation, fill in the table, graph the equation, plug in $30 for y.

23. Now let's fill in a table for the Amy problem. I will model how to determine what x and y stand for, then how to fill in the table for the values in the problem, connecting to the stacking cubes.

24. Next, let's graph the information in the Amy problem. I will model how to label and number the x and y axes, then how to graph using the information we have in the table.

25. Now we are going to figure out the answer to the question in the problem- how much money does Amy need to earn to be able to save $30?

26. POD✓: D- Describe and defend you answer; use words to describe the process you used to solve the problem.
27: POD✓: Explain how you know your answer makes sense. Provide pictures or an example for support.

28-31. Check: Let's check our work. We want to make sure that we re-read the problem, set up the problem correctly, check our calculations, and make sure we didn't make any common mistakes.

32. Now we are going to do one more problem together. This is practice problem #1. For every book Joe reads, he earns $2. He starts with $10 in his piggy bank. How much total money will Joe earn from reading books if he reads 5 more books?

33. POD✓: Propose the problem: What are you asked? How much total money will Joe earn from reading books if he reads 5 more books?
34. POD✓: Propose the problem: What information are you given? X=books, y= money, m=2, plugging in 5 for x.
35. POD ✓: O- outline the steps you will use to solve the problem. Model it with cubes, Write an equation, fill in the table, graph it, then plug in 5 for x and solve the equation.
36. Model the problem using cubes.
37. Let's write an equation for it. I will provide corrective feedback & listen to the student's suggestions.

38. Now let's fill in a table for the problem. What variable will the x stand for? What variable will the y stand for? What values should we put in for x? What values should we put in for y? I will assist the student with completing the table.

39. The next thing we have to do is graph the problem. What are you going to label your x axis as? What are you going to label your y axis as? What is the biggest number we need to go up to for the y-axis? So, should we count by 1s, 2s, 5s, 10s, ….? What is the biggest number we need to go up to for the x-axis? So, should we count by 1s, 2s, 5s, 10s. . . .? I will assist the student & provide corrective feedback as they set up their graph.

Now, let's graph the coordinate pairs from your table. I will provide corrective feedback and/or assistance as needed.

40. POD✓: D- Describe and defend you answer; use words to describe the process you used to solve the problem.
41. POD✓: Explain how you know your answer makes sense. Provide pictures or an example for support.

41-45. Check: Let's check our work. We want to make sure that we re-read the problem, set up the problem correctly, check our calculations, and make sure we didn't make any common mistakes.

46-47. Nice job so far. Now I want you to work on practice problem #2 on your own. Let me know if you need help with any of

the steps. I will provide corrective feedback, answer questions, and assist as needed. 48-50. Awesome job with the real world problems! Let's review. What is the formula for a linear equation? What does the m stand for? What does the b stand for? Good. Tomorrow we will learn/ review the Rule of 4. 51. I want you to fill out this exit ticket independently. The exit ticket has 1 problem similar to what we worked on today so you can test yourself to see what you learned.			
Adaptations/Modifications: Graph paper for drawing stacking cubes problems. Unifix cubes instead of base 10 blocks for students with any motor difficulties. Rest of adaptations/modifications will be determined based on specific students' IEPs.	**Reinforcement Procedures**: Verbal praise for correct responses & on-task behavior. Students will earn points on daily cards for following directs, being respectful, and being persistent problem solvers. I will tally the point and tell them how many they earned at the end of each day. Students will cash in points on assessment days.	**Assessments** **Pre**: Scores on baseline probes; student's responses to checks for understanding in the rationale of the lesson. Student's scores on exit slip from lesson 1. **During**: Responses to checks for understanding; I will observe their work during guided practice & provide corrective feedback as well. **Post**: Exit slip	**Follow-up Activities**: Spiral in prompts for any common errors I see based on exit tickets in lesson #2 (ex: have student describe the slope in the stacking cube problems they complete in lesson 2).

Subject: CRA &WTLM Intervention Grade level: Secondary 1 Lesson: 3

Name: Date:

Core Standard:	**Instructional Objective**:	**Content (concepts, information, skills, new vocab, etc.)**:	**Instructional Materials Needed**:
(F.IF.6) SWBAT calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval	a. SWBAT represent linear and exponential equations using stacking cubes (w/sequentially stacked cubes ex: 1,2,3,4). SWBAT identify the rate of change using stacking cubes/ Cuisenaire rods or pictures of stacking cubes/ Cuisenaire rods. b. SWBAT find missing values in a pattern represented by stacking cubes or Cuisenaire rods, or pictures of stacking cubes/ Cuisenaire rods (ex: how many cubes would be in the 8th tower?).	Rate of change	Graph paper (coordinate grid pre-made) Guided notes POD ✓ graphic organizer (4 copies) Frayer model Unifix cubes/ base 10 blocks Individual dry erase boards Cuisenaire rods Lesson 3 packet (2 copies) Lesson 3 exit slip

Lesson Outline:

1. In the two lessons we have had so far, we have talked about real life contexts that we can use linear equations to describe, features of linear equations, and how to describe slope. Today we are going to deepen our understanding of linear equations by working with patterns.

2. Hand student the review sheet.
3. Review the steps for the POD✓ by having the student fill in what the steps are on the review sheet.
4. I will provide the student with corrective feedback and assistance as necessary as they work on the POD ✓ review.

5. Review how to represent and solve real life problems using linear equations by prompting the student to complete the problem on the review sheet: Drew is running a race. He runs at a pace of 5 miles per hour. He has already run 10 miles. How much longer will it take Drew to run the whole race if the race is 20 miles long total?

6. I will provide the student with corrective feedback and assistance as necessary as they work on the review problem.

7. Hand the student the lesson 3 packet.
8-10. Show a pattern on blocks that go in a series like this (1,3,5,7). Demonstrate how to model this problem using stacking cubes, and drawing out the stacking cubes. Think aloud: I notice that the stacks of cubes get bigger in each tower. I wonder if they change by the same amount each time. Count to show that they do. Each tower increases by two cubes. My first tower has 1 cube in it. So, if I have 1 + … I wonder what I can add to make it so I am always going to get the correct number of cubes in a tower. For the second tower, I would have to add 2, for the third tower I would have to add 4, for the fourth tower, I would have to add 6. Each tower increases by 2. So I wonder if I add 2p + 1 if that will make it so that I always have the correct number in a tower. P stands for the number of the tower in my sequence. So the second tower would be 2 (1) + 1 = 3. Show for the other towers too.

11. With the rule of 4, we want to show this problem multiple ways. Let's make a table to show the number of tower and how many cubes it has.

12. Let's also graph the pattern. (Model how to graph the pattern we have, connecting with the equation). Where does my line cross the y axis? And what is that called? Good.

13. We can tell that the slope of this pattern is 2 because each tower increases by 2.
14. We can also tell that the y-intercept of the pattern is 1, because the line crosses the y-axis at 1.
15. When we look at the cubes, the graph, and the table, how else can we describe the slope and the y-intercept?

16. We can describe this equation as having a constant of 2, because it always increases by 2. This is a linear equation. There is also another kind of equation called an exponential equation. Do you know what an exponent is? Here is an example of an exponential equation y= 2 ^(x +1). Let's figure out what this pattern would look like. I would fill in the chart and show the student how to do so by plugging in values to the equation. Let's compare this equation to our linear equation. They look very similar when they are in their equation form, but what can we tell is different between them?
17. The student and I will discuss differences between the two equations. Exponential equations change by a factor of 2, instead of a constant of 2. So, this is like we are multiplying instead of adding. For the lessons that we work on together, we will just be dealing with linear equations, but you might come across exponential equations in class or on tests.

18. Now let's look at a problem that is a real world linear equation. Lisa is collecting coins. She has 2 coins in her collection to start

(before she starts collecting any coins). She plans on adding coins at a rate of 3/5 coins per day. Let's draw towers of stacked cubes to show how many coins Lisa would have collected in 25 more days. Draw a tower for each week. Good. How many coins are in Lisa's first tower? Response: 2. Good.

19. What does x stand for in this problem?
20. What does y stand for in this problem?

21. POD ✓: P- propose the problem; what are you asked? (This is a missing y problem- we need to find the number of coins after 6 weeks).
22. POD✓: P- What information are you given? 3/5 coins per day (m), starting with 2 coins (b). We need to plug in 25 days (x).
23. POD ✓: O- outline the steps you will use to solve the problem: write an equation, fill in a table, graph the equation. Plug in x=25 to check my work.

24. Write an equation: By how much do her towers change every 5 days? Response: 3. So, what rule could we use to show how many coins Lisa has after however many weeks if she sticks with collecting 3 coins every 5 days? Students work, then show T = 3/5p + 2. Good.

25. Fill in the table: Let's complete a table for the Lisa problem.
26. Now let's make a graph for the Lisa problem.
27. State the answer.

28. POD ✓: D- Describe & defend you answer. Use words to describe the process you used to solve the problem.
29. POD✓: D- describe & defend your answer. Explain how you know your answer makes sense. Provide pictures or an example for support.

30-33: Check: I re-read the problem, I set up the problem correctly, I checked my calculations, I didn't make any common mistakes.

34-35: Now I want you to work on practice problem #2 on your own. Let me know if you need help with any of the steps. I will provide corrective feedback, answer questions, and assist as needed.

36. Work on practice problem 1 with the student. For this problem, the student will roll a di to first determine the number of cubes in the first tower (y-intercept). The first problem will be a missing x problem.

37. They will roll a second di determine the number that each tower will change by (slope).
38. I will help the student make up a context for the values they rolled.
39. What does x stand for in this problem?
40. What does y stand for in this problem?

41. POD ✓: P- propose the problem; what are you asked?
42. POD✓: P-What information are you given?
43. POD ✓: O- outline the steps you will use to solve the problem: write an equation, fill in a table, graph the equation.

44. Write an equation: By how much do the towers change? So, what would our equation be?
45. Fill in the table: Let's complete a table for the problem.
46. Now let's make a graph for the problem.
47. State the answer.

48. POD ✓ : D-describe & defend your answer. Use words to describe the process you used to solve the problem.
49. POD✓: D-describe & defend your answer. Explain how you know your answer makes sense. Provide pictures or an example for support.
50-53: Check: I re-read the problem, I set up the problem correctly, I checked my calculations, I didn't make any common mistakes.

54. I will prompt the student to work on the second practice problem independently. The second practice follows the same sequence as the first practice problem, but is a missing y problem.
55. I will provide the student with corrective feedback and assistance as necessary.
56. Review key concepts that the student has struggled with throughout this lesson and/or the previous two lessons.

57. Awesome job! I want you to fill out this exit ticket independently. The exit ticket will consist of 1 rate of change problem for which the student will have to identify the slope and y-intercept, write an equation, fill in the table, and complete a graph.

Adaptations/Modifications:	**Reinforcement Procedures**:	**Assessments**	**Follow-up Activities**:
Graph paper for drawing stacking cubes problems. Unifix cubes instead of base 10 blocks for students with any motor difficulties. Rest of adaptations/modifications will be determined based on specific students' IEPs.	Verbal praise for correct responses & on-task behavior. Students will earn points on daily cards for following directs, being respectful, and being persistent problem solvers. I will tally the point and tell them how many they earned at the end of each day. Students will cash in points on assessment days.	**Pre**: Scores on baseline probes; student's responses to checks for understanding in the rationale of the lesson. Student's scores on exit slip from lesson 2. **During**: Responses to checks for understanding; I will observe their work during guided practice & provide corrective feedback as well. **Post**: Exit slip	Spiral in prompts for any common errors I see based on exit tickets in lesson 3.

Subject: CRA &WTLM Intervention Grade level: Secondary 1 Lesson: 4

Name: Date:

Core Standard:	Instructional Objective:	Content (concepts, information, skills, new vocab, etc.):	Instructional Materials Needed:
(F.IF.6) SWBAT calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval	a. SWBAT represent linear and exponential equations using stacking cubes (w/ pairs of towers w/missing towers- ex: 3rd & 12th towers). b. SWBAT identify the rate of change using stacking cube/ Cuisenaire rod representations, or pictures of stacking cubes/Cuisenaire rods. *Stacking cube problems that involve two towers of cubes that have a certain number of towers separating them; i.e. the 6th and 12th towers.* c. SWBAT express the rate of change determined from stacking cube/ Cuiseniare rods representations (or pictures) symbolically (ex: write an equation to express the pattern & identify slope in the equation). ← Connect to slope formula	Slope formula	Graph paper (coordinate grid pre-made) Guided notes POD graphic organizer (4 copies) Frayer model Unifix cubes/ base 10 blocks Cuisenaire rods Individual dry erase boards Lesson 4 packet (2 copies) Lesson 4 exit slip

Lesson Outline:

1. Sometimes in rate of change problems you will not have all of the towers. We saw some examples of this yesterday. It is very common that you often will only have two of the towers, and they might be separated by a lot of towers in between them (ex: on tests, etc.). We are going to practice solving this type of problem today. This is one of the most important skills as we move on to solving rate of change problems and real world context function problems.

2. Hand the student the review sheet.
3. Review the steps for the POD✓ by having the student fill in what the steps are on the review sheet.
4. I will provide the student with corrective feedback and assistance as necessary as they work on the POD ✓ review.

5. Review how to represent and solve real life problems using linear equations by prompting the student to complete the problem on the review sheet: Sara drinks water at a rate of 4/5 ounces per hour. Her water bottle holds 20 ounces of water. At this rate, how many hours will it take for Sara to drink all of her water bottle?

6. I will provide the student with corrective feedback and assistance as necessary as they work on the review problem.
7. Hand the student the lesson 4 packet.
8. Review the use of cubes and diagrams to show linear equations, tables, and graphs (reference review problem).

9. Today we are going to work with towers that are bigger, and each our series is only going to show two towers that are in the series, but there will be missing towers in between the ones you are provided with.

10. Complete the POD✓ with the student for example problem 1: P- propose the problem; what are you asked?
11. Complete the POD✓ with the student for example problem 1: P-propose the problem; what information are you given?
12. Complete the POD✓ with the student for example problem 1: O-outline the steps you will use to solve the problem.
13. Fill in the steps for how to find slope from two points on the guided notes portion of the packet.

14. Model how to find the slope from two known points, using concrete manipulatives OR representations (using ex. problem 1).
15. Model how to complete a table with only two known points, using concrete manipulatives OR representations (ex. problem 1).
16. Model how to complete a graph with only two known points (example problem 1).
17. Model how to write an equation with only two known points (example problem 1).

18. Complete the POD✓ with the student for example problem 1; D- describe & defend you answer; use words to describe the process you used to find the answer. ?
19. Explain how you know your answer makes sense. Provide pictures or an example for support.
20. Check: prompt the student to fill in the four check steps.
21. Check: complete the first check step with the student; re-read the problem.
22. Check: complete the second check step with the student; set up the problem correctly.
23. Check: complete the third check step with the student; check calculations.
24. Check: complete the fourth check step with the student; check for common mistakes.

25. Work on practice problem 1 together; POD✓: P- propose the problem; what are you asked?

26. Work on practice problem 1 together; POD✓: P-propose the problem; what information are you given?
27. Work on practice problem 1 together: POD✓: O-outline the steps you will use to solve the problem.
28-31. Work on practice problem 1 together: find the rate of change from two known points, complete a table, graph the problem, write an equation.
32. Complete POD✓ with the student: D-describe & defend you answer; Use words to describe the process you used to solve the problem.
33. Complete POD✓ with the student: D-describe & defined your answer: explain how you know your answer makes sense. Provide pictures or an example for support.
34. Prompt the student to check their work.
35. Provide corrective feedback and assistance as necessary.
36. Prompt the student to write down one way they checked their work.
37. Provide corrective feedback and assistance as necessary.
38. Prompt the student to work on practice problem 2 independently.
39. Provide corrective feedback and assistance as needed while student works on problem 2 independently.
40. Write the slope formula on the board and talk about why/how what we worked on with the towers connects to the slope formula.
41. Awesome job! I want you to fill out this exit ticket independently. The exit ticket has one stacking cube problem. You need to find the rate of change for the problem, complete a table, complete a graph, and write an equation.

Adaptations/Modifications:	**Reinforcement Procedures**:	**Assessments**	**Follow-up Activities**:
Graph paper for drawing stacking cubes problems. Unifix cubes instead of base 10 blocks for students with any motor difficulties. Rest of adaptations/modifications will be determined based on specific students' IEPs.	Verbal praise for correct responses & on-task behavior. Students will earn points on daily cards for following directs, being respectful, and being persistent problem solvers. I will tally the point and tell them how many they earned at the end of each day. Students will cash in points on assessment days.	**Pre**: Scores on baseline probes; student's responses to checks for understanding in the rationale of the lesson. Student's scores on exit slip from lesson 3. **During**: Responses to checks for understanding; I will observe their work during guided practice & provide corrective feedback as well. **Post**: Exit slip	Spiral in prompts for any common errors I see based on exit tickets in lesson 4.

Subject: CRA &WTLM Intervention Grade level: Secondary 1 Lesson: 5

Name: Date:

Core Standard:	**Instructional Objective**:	**Content (concepts, information, skills, new vocab, etc.)**:	**Instructional Materials Needed**:
(F.IF.6) SWBAT calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval	a. SWBAT find the average rate of change of a function with variable rate of change. b. SWBAT interpret how and why the rate changes throughout the problem.	Variable rate of change Average rate of change	Graph paper (coordinate grid pre-made) Guided notes POD graphic organizer (4 copies) Frayer model Unifix cubes/ base 10 blocks Individual dry erase boards Lesson 5 packet (2 copies) Lesson 5 exit slip

Lesson Outline:

1. We have been working with problems involving problems that have one constant rate of change. We are going to start working with problems that have multiple parts that can each have different rates of change. Many circumstances in life are like this, and can be described as having a variable, as opposed to constant, rate of change. Variable rate of change problems are important to know how to work with, because they are common in every day life (examples: car trips, interest, etc.).

2. Hand student review sheet.
3. Review the steps for the POD✓ by having the student fill in what the steps are on the review sheet.
4. I will provide the student with corrective feedback and assistance as necessary as they work on the POD ✓ review.
5. Let's review how to find the slope, or rate of change, when we only know two points by working on this review problem: Alexis ran a 20 mile race. She ran 8 miles in the first 2 hours, and finished all 20 miles in 5 hours. How many miles per hour did she run, assuming that she ran at a constant rate?
6. I will provide the student with corrective feedback and assistance as necessary as they work on the review problem.

7. Hand lesson 5 packet to the student.
8. What does constant rate of change mean? (review definition)
9. Prompt student to write down definition of constant rate of change on their lesson 5 packet.
10. What do you think a variable rate of change means? Provide definition.
11. Prompt student to write down definition of variable rate of change on their lesson 5 packet.

12. Now we are going to work on a variable rate of change problem. (Read problem out loud): Joe left SLC at 9 am and drove for 3 hours at a rate of 60 miles per hour. He then stopped for lunch for 1 hour. After lunch, he drove for 2 hours at a rate of 45 miles per hour. On average, how many miles per hour did Joe drive during his trip?

13. Complete POD✓ with student: P-propose the problem. What are you asked?
14. Complete POD✓ with student: P-propose the problem. What information are you given?
15. Complete POD✓ with student: O-outline the steps to solve the problem.

16. Fill in steps on guided notes in lesson 5 packet for the main five steps of finding the average rate of change (just the main steps, not the sub-components of each of the steps).
17. Model an example variable rate of change problem using concrete manipulatives.
18. Explain to the student that we aren't going to write a linear equation for variable rate of change problems, because linear equations depict problems that have a constant rate of change.
19. Model how to find the average rate of change of a variable rate of change problem using manipulatives.
20. Fill in steps on guided notes in lesson 5 packet on how to fill in a table for a variable rate of change problem.
21. Model how to fill in a table for a variable rate of change problem
22. Fill in the steps on the guided notes in lesson 5 packet on how to complete a graph for a variable rate of change problem.
23. Model how to complete a graph for a variable rate of change problem.
24. Fill in the steps on the guided notes in the lesson 5 packet on how to find the average rate of change of a variable rate of change problem.
25. Model how to find the average rate of change of a variable rate of change problem.

26. Complete POD✓ with the student: D- describe & defend you answer; use words to describe the process you used to solve the problem.

27. Complete POD✓ with the student: D- describe & defend your answer. Explain how you know your answer makes sense. Provide pictures or an example for support.

28. Check: review what the four check steps are, and have the student fill them in on their packet.
29. Check: complete the first check step with the student: re-read the problem.
30. Check: complete the second check step with the student: set up the problem correctly.
31. Check: complete the third check step with the student: check calculations.
32. Check: complete the fourth check step with the student: check for any common mistakes.

33. Read practice problem 1 out loud: Mary works at a fruit canning factory. She arrives at work at 8 am and fills cans at a rate of 8 cans per hour for the first 4 hours. She then takes a lunch break for 1 hour. After lunch, she fills cans at a rate of 6 cans per hour for 3 hours. On average, how many cans does Mary fill per hour during her work day?

34. Prompt student to complete POD✓- P- propose the problem (parts 1 and 2).
35. Provide the student with assistance and corrective feedback as needed.
36. Prompt student to complete POD✓- O-outline the steps to solve the problem.
37. Provide the student with assistance and corrective feedback as needed.

38-40. Work on practice problem 1 together: complete the table, graph the problem, and find the average rate of change.

41. Prompt the student to complete POD✓-D- describe & defend your answer; use words to describe the process you used to solve the problem/
42. Provide the student with assistance and corrective feedback as needed.
43. Prompt the student to complete POD✓ D- describe & defend your answer; Explain how you know your answer makes sense. Provide pictures or an example for support.
44. Provide the student with assistance and corrective feedback as needed.

45. Prompt the student to complete the first check step: re-read the problem.
46. Prompt the student to complete the second check step: set up the problem correctly.
47. Prompt the student to complete the third check step: check calculations.

<table>
<tr><td colspan="4">48. Prompt the student to complete the fourth check step: check for common mistakes.
49. Provide corrective feedback and assistance as needed on the check steps.

50. Prompt student to work on practice problem 2 independently.
51. Provide corrective feedback & assistance as needed while student works on practice problem 2.

52. Review the difference between constant and variable rate of change.

53. Prompt student to fill out exit slip independently.</td></tr>
<tr><td>Adaptations/Modifications:
Graph paper.
Unifix cubes instead of base 10 blocks for students with any motor difficulties.
Problems can be adapted based on difficulty also (ex: 2 pieces in the function instead of 3). Adaptations /modifications will be determined based on specific students' IEPs</td><td>Reinforcement Procedures:
Verbal praise for correct responses & on-task behavior.
Students will earn points on daily cards for following directs, being respectful, and being persistent problem solvers. I will tally the point and tell them how many they earned at the end of each day. Students will cash in points on assessment days.</td><td>Assessments
Pre: Scores on baseline probes; student's responses to checks for understanding in the rationale of the lesson. Student's scores on exit slip from lesson 4.
During: Responses to checks for understanding; I will observe their work during guided practice & provide corrective feedback as well.
Post: Exit slip</td><td>Follow-up Activities:
Spiral in prompts for any common errors I see based on exit ticket in lesson 5.</td></tr>
</table>

Subject: CRA &WTLM Intervention Grade level: Secondary 1 Lesson: 6
Name: Date:

Core Standard:	**Instructional Objective**: a. SWBAT	**Content (concepts, information, skills, new vocab, etc.)**:	**Instructional Materials Needed**:
(F.IF.6) SWBAT calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval	find the average rate of change of a function w/ a variable rate of change. b. SWBAT interpret how and why the rate changes throughout the problem. Variable rate of change problems with 3-4 phases.		Graph paper (coordinate grid pre-made) Guided notes POD graphic organizer (4 copies) Frayer model Unifix cubes/ base 10 blocks Individual dry erase boards Lesson 6 packet (2 copies) Lesson 6 exit slip

Lesson Outline:

1. Today we are going to work with finding the average rate of change of a function with a variable rate of change. We will also focus on how to describe how and why the rate changes throughout the problem. It is important to be able to describe this, because doing so will help us understand what is happening in the problem.

2. Hand the student the review sheet.
3. Let's review some of the information we have talked about so far. Write each of the answers for what we talk about on your review sheet. First, What is the difference between constant rate of change and variable rate of change?
4. Sketch an example of variable rate of change on the graph.
5. What does POD✓ stand for?
6. What can you do to check your work?

7. Now, complete the review problem: Marcus runs at a rate of 2/3 miles per hour. At this rate, how many hours would it take him to run 4 miles?

8. I will provide corrective feedback and assistance as needed for the review problems.

9. Hand the student the lesson 6 packet.
10. Work on example problem 1 together; read the problem out loud to the student: Joe just got a credit card. For the first 3 months, he is charged $3 in interest per month. For the next 6 months, he isn't charged any interest per month ($0). For the next 2 months, he is charged $5 in interest per month. For the following month, he is charged $2 per month. On average, how much interest is Joe charged each month?

11. Prompt the student to use POD✓ to start solving the problem by filling in the P portion of the POD✓.
12. Provide corrective feedback and assistance as necessary.

13. Prompt the student to fill in the O portion of the POD✓, focusing on the steps that we covered for solving variable rate of change problems in lesson 5.
14. Provide corrective feedback and assistance as necessary.

15. Work with the student to complete a table for the problem.
16. Work with the student to graph the problem.
17. Work with the student to find the average rate of change for the problem.
18. Model a description of how and why the rate changes during the problem. Let's talk about how and why the rate is changing over time in this problem. The student and I will talk about each phase of the problem and describe the rate in terms of steepness of the line (rate of change), as well as what is happening overall between the phases.
19. I will prompt the student to write down an explanation of how and why the rate is changing over time in the problem on their lesson 6 packet.
20. Prompt the student to complete the D portion of the POD✓.
21. Provide corrective feedback and assistance as necessary.
22. Prompt the student to complete the ✓ portion of the POD✓
23. Provide corrective feedback and assistance as necessary.
24. Work on practice problem 1 together: read the problem out loud to the student: Sam is filling a pool with water. She fills the pool for 2 hours at a rate of 15 gallons per hour. Then, she fills the pool for 3 hours at a rate of 30 gallons per hour. She takes a 1 hour lunch break, then fills the pool for 4 hours at a rate of 20 gallons per hour. What is the average amount of gallons per hour Sam fills the pool with during her workday?

25. Prompt the student to complete both portions of the P section of the POD✓.
26. Provide corrective feedback and assistance as needed.
27. Prompt the student to complete the O portion of the POD✓.
28. Provide corrective feedback and assistance as needed.
29. Work on practice problem 1 together: complete the table.
30. Work on practice problem 1 together: graph the problem.
31. Work on practice problem 1 together: find the average rate of change.
32. Prompt the student to complete the D section of the POD✓: How do you know your answer is correct?
33. Complete D section of the POD✓ together: How and why does the rate change throughout the problem?
34. Prompt the student to complete the ✓ portion of the POD✓.
35. Prompt student to work on practice problem 2 independently.
36. Provide corrective feedback & assistance as needed while student works on practice problem 2.
37. Review: How do you find the average rate of change in a variable rate of change problem?
38. Awesome job! I want you to fill out this exit ticket independently. The exit ticket has one problem for the student to complete. The problem will involve a context similar to what we discussed during the lesson. The student will have to find the average rate of change, find a missing value, and explain how and why the rate is changing. The student will have to complete the rule of 4 by filling in missing components.

Adaptations/Modifications:	**Reinforcement Procedures**:	**Assessments**	**Follow-up Activities**:
Graph paper Unifix cubes instead of base 10 blocks for students with any motor difficulties. Can adapt/modify based on difficulty of problems as well. Adaptations/modifications will be determined based on specific students' IEPs.	Verbal praise for correct responses & on-task behavior. Students will earn points on daily cards for following directs, being respectful, and being persistent problem solvers. I will tally the point and tell them how many they earned at the end of each day. Students will cash in points on assessment days.	**Pre**: Scores on baseline probes; student's responses to checks for understanding in the rationale of the lesson. Student's scores on exit slip from lesson 5. **During**: Responses to checks for understanding; I will observe their work during guided practice & provide corrective feedback as well. **Post**: Exit slip	Emphasize how to find the average rate of change of a variable rate of change function; focus on 4 phases

Subject: CRA &WTLM Intervention Grade level: Secondary 1 Lesson: 7
Name: Date:

Core Standard:	**Instructional Objective**: a. SWBAT	**Content (concepts, information, skills, new vocab, etc.)**:	**Instructional Materials Needed**:
(F.IF.6) SWBAT calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval	find the average rate of change of a function with variable rate of change. b. SWBAT interpret how and why the rate changes throughout the problem. *problems with 4 phase changes		Graph paper (coordinate grid pre-made) Guided notes POD graphic organizer (4 copies) Frayer model Unifix cubes/ base 10 blocks Individual dry erase boards Lesson 7 packet (2 copies) Lesson 7 exit slip

Lesson Outline:

1. Today we will practice finding the average rate of change for variable rate of change problems. This is the third day we are covering this material, so the problems today will be a little harder, but I am also looking for you to really master these concepts.

2. Hand student review sheet.
3. Before we work on the variable rate of change problems, we are going to review some things we have been working on. Write the answers to each of these questions on your review sheet. First, explain the steps you use to find the average rate of change.
4. Provide corrective feedback and assistance as needed.

5. What does POD✓ stand for?
6. Provide corrective feedback and assistance as needed.
7. What can you do to check your work?
8. Provide corrective feedback and assistance as needed.

9. Now let's review a constant rate of change problem. Read problem out loud to student. Cesar wants to save $77 from working this summer. He has already saved $21. He plans to save $7 per week. How much total money will he save if he saves money for 8 more weeks?
10. Provide corrective feedback and assistance as needed.
11. Hand the lesson 7 packet to the student.
12. Prompt the student to solve a variable rate of change problem. Solving the problem involves finding the average rate of change for a variable rate of change problem, inputting the values in the problem into a table, and graphing the problem.
13. Ask the student to present an argument to justify why their answer is correct.
14. Provide corrective feedback for any components of the problem the student got incorrect (if needed).

15-17. Complete a practice problem with the student to review how to fill in the table, complete the graph, and find the average rate of change for the variable rate of change problem.

18. Prompt the student to work on practice problem 1 independently. The practice problem involves the student solving a variable rate of change problem that involves 4 phases. For the problem, the student must complete a table and graph, as well as find the average rate of change. The student will also fill in the POD✓ graphic organizer as they solve the problem.

19. Provide corrective feedback and assistance as needed while student works on practice problem 1.
20. Prompt the student to justify how they know their answer is correct.
21. Review the process and tips used to find the average rate of change of a variable rate of change problem.

22. Awesome job! I want you to fill out this exit ticket independently. The exit ticket has one problem for the student to complete. The problem will involve a context similar to what we discussed during the lesson. The student will have to find the average rate of change, fill in the table and graph, and explain how and why the rate is changing.

Adaptations/Modifications:	**Reinforcement Procedures**:	**Assessments**	**Follow-up Activities**:
Graph paper for drawing stacking cubes problems. Unifix cubes instead of base 10 blocks for students with any motor difficulties. Can adapt/modify based on	Verbal praise for correct responses & on-task behavior. Students will earn points on daily cards for following directs, being respectful, and being persistent problem solvers. I will tally the	**Pre**: Scores on baseline probes; student's responses to checks for understanding in the rationale of the lesson. Student's scores on exit slip from lesson 6.	Tell the student that this is our last day together learning material as a group, and that the next class I will have them fill out some information for

difficulty of problems as well. Rest of adaptations/modifications will be determined based on specific students' IEPs.	point and tell them how many they earned at the end of each day. Students will cash in points on assessment days.	**During**: Responses to checks for understanding; I will observe their work during guided practice & provide corrective feedback as well. **Post**: Exit slip	me about what they thought about this unit and our time together.

Subject: CRA & WTLM Intervention Grade level: Secondary 1 Review lesson 1

Core Standard:	**Instructional Objective**:	**Content (concepts, information, skills, new vocab, etc.)**:	**Instructional Materials Needed**:
(F.IF.4) SWBAT correctly identify and interpret key features of graphs and tables in terms of the quantities (key features include: intercepts; intervals where function is increasing, decreasing, positive, negative; relative maximums and minimums; symmetries; end behavior; and periodicity).	SWBAT identify the slope of a linear equation from a graph. SWBAT solve and graph one and two step equations when provided with a context.	Slope Positive Negative Undefined Zero Y-intercept	Graph paper (coordinate grid pre-made) POD graphic organizers (4 copies) Ruler Unifix cubes/ base 10 blocks Individual dry erase boards Booster Lesson 1 packet (2 copies) Booster Lesson 1 exit slip

Outline of Lesson:

1. Today we are going to review some of the concepts we have been working on. We have gone over a lot so far, but I want to make sure we take time to review since all of the information we have covered is important to remember.
2. Hand student the booster lesson 1 packet.
3. One thing we have been using and that we will use during this lesson is POD✓. What does POD✓ stand for?
4. One of the first concepts we worked with was slope. How can we define slope?
5. What are the four ways we can classify slope?
6. Let's practice identifying slope from a graph. We will work on the first problem together.
7. What are some common mistakes you might make on finding slope from a graph?
8. How can you prevent those mistakes?
9. Now I want you to work on the next 3 problems on your own.
10. I will provide corrective feedback and assistance as needed.
11. Nice job. The other big skill we have been working on is solving real life problems by modeling them using linear equations. Let's work on one of these problems. Read problem out loud to student: Jackson spends ½ of the money he earns. How much money would he earn if he spent $32?
12. Complete POD✓ with the student: P-propose the problem; What are you asked?

13. Complete POD✓ with the student: P-propose the problem; What information are you given?
14. Complete POD✓ with the student: O-outline the steps you will use to solve the problem.
15. Help the student write an equation for the problem.
16. Help the student complete a table for the problem.
17. Help the student complete the graph for the problem.
18. Help the student find the answer to the problem.
19. Complete POD✓ with the student: D-describe & defend your answer: Use words to describe the process you used to solve the problem.
20. Complete POD✓ with the student: D-describe & defend your answer: Explain how you know your answer makes sense. Provide pictures or an example for support.
21. Ask student what the check steps are.
22. Complete check step 1 with the student: re-read the problem.
23. Complete check step 2 with the student: set up the problem correctly.
24. Complete check step 3 with the student: check calculations.
25. Complete check step 4 with the student: check for common mistakes.
26. Prompt student to work on practice problem 2 on their own.
27. Provide student with corrective feedback and assistance as needed.
28. Complete practice problem #3 with the student. Read the problem out loud. Jeremiah is filling a fish tank at a rate of 2/3 gallons per minute. The tank is already 3 gallons full. For how many more minutes will Jeremiah need to fill the tank for it to be 15 gallons full total?
29. Complete POD✓ with the student: P-propose the problem; What are you asked?
30. Complete POD✓ with the student: P-propose the problem; What information are you given?
31. Complete POD✓ with the student: O-outline the steps you will use to solve the problem.
32. Help the student write an equation for the problem.
33. Help the student complete a table for the problem.
34. Help the student complete the graph for the problem.
35. Help the student find the answer to the problem.
36. Complete POD✓ with the student: D-describe & defend your answer: Use words to describe the process you used to solve the problem.
37. Complete POD✓ with the student: D-describe & defend your answer: Explain how you know your answer makes sense.

Provide pictures or an example for support.
38. Ask student what the check steps are.
39. Complete check step 1 with the student: re-read the problem.
40. Complete check step 2 with the student: set up the problem correctly.
41. Complete check step 3 with the student: check calculations.
42. Complete check step 4 with the student: check for common mistakes.
43. Prompt student to work on practice problem #4 on their own.
44. Provide student with corrective feedback and assistance as needed.
45. Review: What are some common mistakes you have seen with finding slope from a graph?
46. How can you prevent some of these mistakes?
47. What are some common mistakes you have seen with solving linear equation context problems?
48. How can you prevent some of these mistakes?
49. Prompt student to work on exit slip. The exit slip consists of 2 slope from graphed line problems, and 2 contextual linear equation problems for the student to solve (write an equation, complete a table, graph, and find the answer).

Adaptations/Modifications:	**Reinforcement Procedures**:	**Assessments**	**Follow-up Activities**:
Large grid graphs Rulers for helping students keep track of their place on the graphs. Rest of adaptations/modifications will be determined based on specific students' IEPs.	Verbal praise. Students will earn points on daily cards for following directs, being respectful, and being persistent problem solvers. I will tally the point and tell them how many they earned at the end of each day. Students will cash in points on assessment days.	**Pre**: Scores on intervention probes; student's responses to checks for understanding in the rationale of the lesson; scores on exit slips from lessons 1-4. **During**: Responses to checks for understanding; I will observe their work during guided practice & provide corrective feedback as well. **Post**: Exit slip	Spiral in prompts for any common errors I see based on exit tickets in lessons #1-4 (ex: when I graph this point, I remember that negative numbers mean I move down if it is a y-coordinate), or on intervention assessments.

Subject: CRA & WTLM Intervention   Grade level: Secondary 1   Review lesson 2

Core Standard:	**Instructional Objective**:	**Content (concepts, information, skills, new vocab, etc.)**:	**Instructional Materials Needed**:
(F.IF.4) SWBAT correctly identify and interpret key features of graphs and tables in terms of the quantities (key features include: intercepts; intervals where function is increasing, decreasing, positive, negative; relative maximums and minimums; symmetries; end behavior; and periodicity). (F.IF.6) SWBAT calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval	SWBAT identify the slope of a linear equation from a graph. SWBAT solve and graph one and two step equations when provided with a context. SWBAT find the average rate of change of a function w/ a variable rate of change.	Slope Positive Negative Undefined Zero Y-intercept Constant rate of change Variable rate of change	Graph paper (coordinate grid pre-made) POD graphic organizers (4 copies) Ruler Unifix cubes/ base 10 blocks Individual dry erase boards Booster Lesson 2 packet (2 copies) Booster Lesson 2 exit slip

Outline of Lesson:

1. Today we are going to review some of the concepts we have been working on. We have gone over a lot so far, but I want to make sure we take time to review since all of the information we have covered is important to remember.
2. Hand student the booster lesson 2 packet.
3. One thing we have been using and that we will use during this lesson is POD✓. What does POD✓ stand for?
4. One of the first concepts we worked with was slope. What are the four ways we can classify slope?
5. Let's practice identifying slope from a graph. Describe how you would find the slope of the line on this graph.
6. Provide assistance and corrective feedback as needed.
7. Prompt student to find the slope of a line on a second graph.
8. I will provide corrective feedback and assistance as needed.
9. Nice job. Another concept we have been working with is constant versus variable rate of change. Can you describe the difference

between constant and variable rate of change?
10. I will provide corrective feedback and assistance as needed.
11. Let's work on a problem with a constant rate of change. Read problem out loud to student: Sara is filling a fish tank at a rate of ¾ gallons per minute. The tank is already 3 gallons full. At this rate, for how many more minutes does Sara need to fill the tank for it to be 6 gallons full total?
12. Complete POD✓ with the student: P-propose the problem; What are you asked?
13. Complete POD✓ with the student: P-propose the problem; What information are you given?
14. Complete POD✓ with the student: O-outline the steps you will use to solve the problem.
15. Help the student write an equation for the problem.
16. Help the student complete a table for the problem.
17. Help the student complete the graph for the problem.
18. Help the student find the answer to the problem.
19. Complete POD✓ with the student: D-describe & defend your answer: Use words to describe the process you used to solve the problem.
20. Complete POD✓ with the student: D-describe & defend your answer: Explain how you know your answer makes sense. Provide pictures or an example for support.
21. Ask student what the check steps are.
22. Complete check step 1 with the student: re-read the problem.
23. Complete check step 2 with the student: set up the problem correctly.
24. Complete check step 3 with the student: check calculations.
25. Complete check step 4 with the student: check for common mistakes.
26. Prompt student to work on practice problem #2 on their own.
27. Provide student with corrective feedback and assistance as needed.
28. Now let's work on a variable rate of change problem. Read the problem out loud to the student: Andrew has a credit card that charges $1 per month in interest the first 3 months. He is then charged $0 in interest per month for the next 3 months, and $2 in interest per month for the following 6 months. For the next 3 months, he is charged $3 in interest. On average, how much interest is Andrew charged each month?
29. Complete POD✓ with the student: P-propose the problem; What are you asked?
30. Complete POD✓ with the student: P-propose the problem; What information are you given?
31. Complete POD✓ with the student: O-outline the steps you will use to solve the problem.

32. Help the student complete a table for the problem.
33. Help the student complete the graph for the problem.
34. Help the student find the answer to the problem.
35. Complete POD✓ with the student: D-describe & defend your answer: Use words to describe the process you used to solve the problem.
36. Complete POD✓ with the student: D-describe & defend your answer: Explain how you know your answer makes sense. Provide pictures or an example for support.
37. Ask student what the check steps are.
38. Complete check step 1 with the student: re-read the problem.
39. Complete check step 2 with the student: set up the problem correctly.
40. Complete check step 3 with the student: check calculations.
41. Complete check step 4 with the student: check for common mistakes.
42. Review: What are some common mistakes you have seen with solving variable rate of change problems?
43. How can you prevent some of these mistakes?
44. Prompt student to work on exit slip. The exit slip consists of 1 slope from graphed line problem, 1 contextual linear equation problem for the student to solve (write an equation, complete a table, graph, and find the answer), and 1 variable rate of change problem for the student to solve (complete a table, graph, and find the average rate of change).

Adaptations/Modifications:	**Reinforcement Procedures**:	**Assessments**	**Follow-up Activities**:
Large grid graphs Rulers for helping students keep track of their place on the graphs. Rest of adaptations/modifications will be determined based on specific students' IEPs.	Verbal praise. Students will earn points on daily cards for following directs, being respectful, and being persistent problem solvers. I will tally the point and tell them how many they earned at the end of each day. Students will cash in points on assessment days.	**Pre**: Scores on intervention probes; student's responses to checks for understanding in the rationale of the lesson; scores on exit slips from lessons 1-6. **During**: Responses to checks for understanding; I will observe their work during guided practice & provide corrective feedback as well. **Post**: Exit slip	Spiral in prompts for any common errors I see based on exit tickets in lessons #1-6 (ex: when I graph this point, I remember that negative numbers mean I move down if it is a y-coordinate), or on intervention assessments.

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