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Twelve quick tips for designing sound dynamical models for bioprocesses

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Because of the inherent complexity of bioprocesses, mathematical models are more and more used for process design, control and optimization etc... These models are generally 2 based on a set of biochemical reactions. Model equations are then derived from mass balance, 3 coupled to empirical kinetics. Biological models are nonlinear and represent processes, which by essence are dynamic and adaptive. The temptation to embed most of the biology 5 is high, with the risk that calibration would not be significant anymore. The most important 6 task for a modeler is thus to ensure a balance between model complexity and ease of use. Since a model should be tailored to the objectives which will depend on applications and 8 environment, a universal model representing any possible situation is probably not the best g option. 10

Here are twelve tips to develop your own bioprocess model. For more details on bioprocess ¹¹ modelling, the readers could refer to [1]. More tips concerning computational aspects can ¹² be found in [2,3]. ¹³

Tip 1: Define your objective and the application context

Years of high school learning about how to set-up mechanistic models based on the funda-15 mental F = m.a relationship of mechanics, or on the Ohm law have corrupted our minds. 16 It took centuries to identify the corpus of laws supporting today's physical models. Fig 17 1 recalls that, previously, there used to be some "less accurate" predictive models that 18 have been forgotten. At present, models in these fields, even if empirical, are excellent 19 approximations and -at least for those we studied at school- always ended-up in rather 20 simple, often linear and mathematically tractable models. The complexity of biological 21 systems requires a more open viewpoint, where different models of the same process can 22 be useful and complementary. Therefore, before writing equations, one must first clearly 23 define the model objective. The model can be designed for numerous reasons, among 24 which prediction of future evolution, understanding of the process behaviour, estimation 25 of unmeasured variables or fluxes, operator training, detection and diagnosis of failures, 26 optimization and control. 27

Tip 2: Adapt your modelling framework with your objective, ²⁸ your knowledge and your data set ²⁹

When developing a model, it is crucial to keep in mind the objectives of the model and 30 the framework for its application. A model targeting the understanding of some metabolic 31 processes inherently requires the user to embark on the details of the cell metabolism [5, 6]. 32 Predicting the impact of meteorology on outdoor microalgal processes means that light 33 and temperature must be included somewhere in the model. A model for on-line control 34 can be more straightforward (often because it will benefit from on-line information on 35 process state). So, keeping in mind the model objective, one has to choose which variables 36 to include, but also the type of model: deterministic versus stochastic, homogeneous versus 37 heterogeneous (in terms of space or phenotype). The available data set or data that can 38 be provided by the experimental set-up will also constrain the choice of model complexity. 39



Fig. 1. Medieval theory of the canon ball trajectory, from Walther Hermann Ryff (1547) [4]. The canon ball trajectory was an assemblage of circular arcs and segments. Models in physics are now excellent approximations, but they have sometimes been improved during century-long periods. In biology, we are still at the dawn of model development.

Parameters should be calibrated at some point, or at least reasonably determined from ⁴⁰ the experimental information. Model complexity can first be measured by the number of ⁴¹ state variables (variables with dynamics) together with the number of parameters and stay ⁴² compatible with the objectives and data. ⁴³

Tip 3: Take care with dimensions, intensive and extensive 44 properties 45

This tip seems very basic, but, in our opinion, it is worth emphasising. The dimension of the model equation should be checked. Particular care should be taken between intensive and extensive variables [7]. This is particularly true when dealing with a metabolic model. A metabolite concentration could be expressed per unit of culture volume or intracellular volume. The concentration dynamics should then include the dilution by the reactor feeding 50 or by cellular growth, respectively. Moreover, the kinetics of intracellular reactions should ⁵¹ depend on intracellular concentrations, not culture concentrations. In several studies, it ⁵² remains unclear. ⁵³

Tip 4: Do not assume gas concentrations equilibrate with 54 atmosphere

Assuming gas concentrations equilibrate with the atmosphere is a common mistake. If we measure the dissolved CO₂ concentration in a glass of water in equilibrium with the atmosphere, it will be proportional to P_{CO_2} , the CO₂ partial pressure at the interface (*i.e.* 58 in the air): $[CO_2] = K_h P_{CO_2}$ where K_h is Henry's constant at the considered temperature and salinity. At steady state, there is no more gas exchange between the atmosphere and liquid phase.

If algae are developing in the glass, the CO_2 concentration will be lower, because the algae permanently consume it. As a consequence, there is a permanent flux of CO_2 from air to water, with a flow rate

$$Q_{CO_2} = K_L a (CO_2 - K_h P_{CO_2})$$

which will balance the consumption of CO_2 by the algae. Now the concentration of CO_2 is lower than $K_h P_{CO_2}$, its natural equilibrium value without algae.

Tip 5: Check the mathematical soundness of your model

A mathematical analysis of your model may help to detect potential errors, limitations and ⁶⁵ drawbacks in model design, and to better apprehend the process. Whenever possible, one ⁶⁶ should check mass conservations, check the boundedness of the variables (in particular their ⁶⁷ positivity), and study the asymptotic behaviour of the model. This last point could be, ⁶⁸ for some models, particularly challenging. It is essential to keep in mind that nonlinear ⁶⁹ dynamical models are complex mathematical objects with potentially weird behaviours, ⁷⁰

including limit cycles, chaos or abrupt change in behaviours after bifurcation when one 71 of the model parameters has been slightly modified [8]. Mathematicians spend months 72 trying to understand and prove the behaviour of systems of low dimension, e.g. with "only" 73 three state variables. The mathematical complexity is breath-taking when considering 74 standard bioprocess models. Often, the properties of these models are hardly suspected, 75 and Pandora's box stays closed. Even the number of equilibria that can be produced is 76 rarely discussed. Adding new features or including more realism into a model extends the 77 risk of unexpected model behaviours. 78

The objective is to determine whether the trajectories of your system converge towards an equilibrium (a global equilibrium, or different equilibria depending on the initial conditions), if they present sustained oscillations (limit cycle) or even show a chaotic behaviour. These properties should be in line with the behaviour of your bioprocess, otherwise the model should be revised.

Tip 6: Be aware of structural identifiability

Most of the parameters in physical modelling have a clear meaning and can be directly⁸⁵ measured on the process. Also, physical models are often linear. The theory of linear⁸⁶ systems and their identification has received much attention, indirect identification of a⁸⁷ tenth of parameters can be accurately carried out by modern algorithms [9,10]. For the⁸⁸⁸ biological systems, which are in turn nonlinear and described by rough approximations,⁸⁹⁹ more modesty is required.⁹⁰⁰

Theoretical identifiability of the parameters is a complex mathematical property [11], ⁹¹ which is often characterized by cryptic (but accurate) mathematical formulations. In a ⁹² nutshell, this theoretical mathematical property states that a parameter value can be ⁹³ uniquely determined by (nonlinear) combinations of measurements and their derivatives ⁹⁴ (with respect to time) at any order. More simply, a unique set of parameters can produce a ⁹⁵ given model output. With non-linear models, it is possible that two sets of parameters can ⁹⁶ produce exactly the same results. To illustrate the non-identifiability pathology, we present ⁹⁷

in Table 1 two illustrative astonishing examples for trivial models.

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The first example is unfortunately not so rare. It consists in representing an inhibition 99 kinetics (from substrate S) with a product of Monod and a hyperbolic inhibition term. A 100 numerical example is given in Table 1 (Example 1), where two parameter sets produce 101 exactly the same values. Parameters here are only locally structurally identifiable. 102

The second example in Table 1 uses a trivial logistic equation (x is the biomass) modified to deal with mortality rate (which is obviously a very bad idea). Here, an infinity of parameters provide the same biomass dynamics, they are structurally not identifiable.

These two examples also demonstrate that it is useless to attribute a biological meaning $_{106}$ to a non-identifiable parameter. In the first case, what was, in turn, the inhibition constant: $_{107}$ K_i or K_s ? In the second example, is K the carrying capacity of the medium? $_{108}$

Perhaps more problematic when using an automatic algorithm for parameter identification, non-identifiable parameters will kill any approach. Especially if it is a global approach, 110 any optimisation algorithm will oscillate between several of the possible solutions, or average 111 them, and often will never converge. 112

In general, assessing identifiability for complex dynamical models is very challenging. ¹¹³ This is a reason why modellers must refrain from embedding too many processes into a ¹¹⁴ model, and privilege lower complexity models when only a limited set of measurements is ¹¹⁵ available for validation. ¹¹⁶

Tip 7: Double check numerical implementation

If your model has been implemented only once, then it probably contains at least three ¹¹⁸ mistakes. We know this is not true for you, but it is for most of the people. So if the model ¹¹⁹ was right, after a rapid change in one of the equations for testing the effect of one factor, it ¹²⁰ would become wrong because eventually the test is not removed. There are strict coding ¹²¹ rules and use of validation tests [12], but they are rarely respected for model development ¹²² because the model implementation is generally not carried out by computer scientists. Also, ¹²³ the way models are implemented can highly differ, and some computer languages may be ¹²⁴

	Parameter	Parameter	False claim	Function
	set $\#1$	set $\#2$	parameter meaning	
Example 1:	substrate upt	ake with inhibition		$\phi(S) = \bar{\mu} \frac{S}{S+K_s} \frac{K_i}{S+K_i}$
Numerical values	$\bar{\mu} = 2,$ $K_i = 1,$ $K_s = 2.$	$\bar{\mu} = 1,$ $K_i = 2,$ $K_s = 1.$	max. growth rate inhibition constant affinity constant	$\phi(S) = 2\frac{S}{(S+1)(S+2)}$
General case	$ar{\mu},\ K_i,\ K_s.$	$ \begin{array}{l} \bar{\mu} \frac{K_i}{K_s}, \\ K_s, \\ K_i. \end{array} $	max. growth rate inhibition constant affinity constant	$\phi(S) = \bar{\mu} \frac{S}{S+K_s} \frac{K_i}{S+K_i}$
Example 2:	logistic grow	th with mortality	$\dot{x} = \bar{\mu}(1 - \frac{x}{K})x - Rx$	
Numerical values	$ar{\mu} = 2, \ K = 1, \ R = 1.$	$\bar{\mu} = 3, K = 1.5, R = 2.$	max. growth rate carrying capacity mortality rate	$\dot{x} = (1 - 2x)x$
General case	$ar{\mu},\ K,\ R.$	$ar{\mu} + heta, \ K rac{ar{\mu} + heta}{ar{\mu}}, \ R + heta.$	max. growth rate carrying capacity mortality rate	$\dot{x} = \bar{\mu}(1 - \frac{x}{K})x - Rx$

Table 1. Analysis	of two simple	examples with	identifiability issues.
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In Example 1, two different parameter sets produce the same value of the function $\phi(S)$. In Example 2, an infinite number of parameter sets can produce the same dynamics \dot{x} for an arbitrary value of θ . The parameters meaning (as often claimed) does then not make any sense.

more difficult to cross check. Excel[®] is an excellent tool for displaying data and for simple ¹²⁵ computations, but it is not an appropriate tool for simulating complex models since it is ¹²⁶ almost impossible to cross-check implementation. Some graphical languages also have these ¹²⁷ drawbacks when a connection to a wrong node can corrupt the result while being almost ¹²⁸ impossible to detect. ¹²⁹

One way of reducing the risk of error is a double implementation, with two different ¹³⁰ computer programmers and two different languages. This has been the case for the models ¹³¹ used in wastewater treatment, ADM1 for anaerobic digestion [13] and ASM1 for activated ¹³² sludge [14]. The first comparison between different implementations revealed to be quite ¹³³ quaint. Also, simple case studies must help to check simple theoretical properties (positivity ¹³⁴ of variables, mass conservation, etc...) that must be respected. ¹³⁵

Tip 8: Pay attention to practical identifiability

The cost criterion to be optimised (typically the sum of squared errors) is generally non-137 convex, and many local minima perturb parameter identification. In practice, it is often 138 not possible to get an accurate estimate of parameters from the data sets. The most 139 efficient algorithms are generally limited to three parameters to be determined per measured 140 quantity (assuming a reasonable sampling over time). The weird consequence is that fitting 141 a model to a set of data is generally possible, but that does not mean that the estimated 142 parameters are reasonable. Whenever a parameter has a clear meaning, the validity of the 143 identified value must always be checked, and bounds can be added during the identification 144 process. Multiple algorithm initialisations are also strongly recommended. Collecting 145 informative data is also key for practical identifiability, which means data corresponding 146 to high sensitivities of the model outputs with respect to parameter variations (cf. Fisher 147 information matrix [9]). As a matter of illustration, it is not possible to estimate a parameter 148 related to growth inhibition if substrate concentration is always too low to trigger inhibition. 149

Finally, a literature review is an essential resource for parameter values, in particular ¹⁵⁰ for algorithm initialisation. Nonetheless, exotic chimaera can appear when picking up ¹⁵¹

Tip 9: apply the "divide and conquer" strategy to identify 153 your parameters

Do not try to get all your parameters at once, through a never converging optimization 155 algorithm and rather identify subsets of parameters. In many cases, after simple algebraic 156 manipulations some parts of the model can lead to relationships between some measured 157 quantities and eventually provide some combinations of the parameters. For example, the 158 pseudo-stoichiometry can often be identified independently of the reaction rates after some 159 straightforward transformations [15]. Some working modes do considerably simplify the 160 model, and are often an opportunity to extract such relationships. For example, during a 161 phase when nutrients are nonlimiting, the Michaelis-Menten kinetics can be replaced by 162 constants. Similarly, if different equilibria can be observed for various inputs, they would 163 probably lead to very interesting relationships between some of the model parameters [16]. 164

Tip 10: determine parameter and model uncertainties

Assessing measurement uncertainty propagation is of utmost importance to assess model 166 accuracy. This first means that the experimental data must be associated to the variance of 167 their measurement error. There are different strategies to compute not only the parameter 168 values but also their confidence intervals. This is straightforward when parameters are 169 deduced from linear relationship, but is can also be estimated in a more complex case 170 thanks to the covariance matrix of parametric errors [9]. The strong scientific added value 171 is that the simulation scheme will predict not only outputs but also the confidence intervals 172 derived from the covariance matrix of prediction errors, or from Monte Carlo simulations. 173

Tip 11: Validate the model with data not used for identification

When observing the vast diversity in bioprocess models, only a few of them have been 176 appropriately validated. First, because it is not possible to validate a model, a model 177 can only be discarded when it is not compliant with experimental records [17]. However, 178 assuming a relaxed use of the "validation" term, it would mean that the model has been 179 proven accurate for a large variety of cases. In particular for cases significantly different 180 from the learning data set (data that has been used for the calibration). This ideal situation 181 is very difficult to meet in practice, and most of the time the validation datasets only differ 182 by some initial conditions, or by a single different forcing variable. If the model has enough 183 parameters, it can probably fit a calibration dataset nicely with only a few points. However, 184 it will exhibit abysmal performances for cross-validation. For larger calibration data sets, 185 the fit will probably less successfully highlight the quality of the model, but prediction 186 capacity might be highly enhanced. The plot will not look that nice, but the model will 187 definitely be more powerful and relevant. 188

Claiming that the model is valid is, therefore, an act of faith, and a very weak scientific ¹⁸⁹ assertion. As running experiments takes time and is money consuming, the number of ¹⁹⁰ experiments is by essence limited. As consequence, it becomes clear that the conditions for ¹⁹¹ which the model has been validated must be clearly stated. Knowing the "model validation ¹⁹² domain" will in itself be precious for future model use. Also, providing data sets for which ¹⁹³ the model did not do its job is intrinsically useful, although rarely done. ¹⁹⁴

Often, the question is instead to choose the best model among a few candidates. A more 195 complex model, with more parameters, will mechanically better fit the data. However, that 196 does not mean it is more correct, it just means it is more flexible. The Akaike criterion [18] 197 is a good option to compare the performance of two models of different levels of complexity. 198 However, the only real criterion to assess the predictive power of a model, and therefore 199 to compare model performances is cross-validation, assessing the model with data which 200 were not used for calibration (and data whose dynamics are significantly different from 201 the calibration data set). Additionally, the candidate models can even be used to find the 202 experimental conditions that will allow to differentiate them better [17]. 203

Finally, models can include the effects of different factors which often have been studied ²⁰⁴ separately. The models then gather these effects classically by multiplying the different ²⁰⁵ terms or using Liebig's law of minimum. Validation experiments could be the last chance ²⁰⁶ to test possible interactions between these factors and find the best way to combine their ²⁰⁷ effects in the model. ²⁰⁸

Tip 12: Share codes, tips, tools, and model limitations

More and more journals require this, and it is to be welcomed. Providing your model - with ²¹⁰ all the files necessary to reproduce your simulations (including parameter values, initial ²¹¹ conditions etc...) - will favour its dissemination within the scientific community. Your ²¹² model would thus be further validated with new data sets. Additionally, it promotes error ²¹³ checking, helps the reader if some model details in the manuscript are unclear, and removes ²¹⁴ any suspicion of fraud. ²¹⁵

More generally, what makes the success and the efficiency of a model, is not limited to 216 the biology it embeds and to the realism of its predictions. A model is inexorably associated 217 with a set of tools to calibrate it, estimate which are the most sensitive parameters, optimise 218 a criterion, determine the input which maximizes productivity etc... The associated toolbox 219 to make the model applicable and efficient is probably at least as necessary as the model 220 itself. Great models can have complex structures or behaviours, which eventually make 221 their use more tricky. For example, the outstanding Geider model [19] is in turn rather 222 challenging to calibrate, and specific methods dedicated to its calibration are needed [20]. 223 Even simpler models, such as the Hinshelwood model [21] for temperature, advantageously 224 predicts a mortality rate [22], but calibrating this model often turns into a nightmare [23]. 225 Keeping two different modelling approaches can significantly help in this case, by using 226 the toolbox of one of the models to manage the other one. Typically, using a temperature 227 response model from [24] as a gauging device makes the calibration of Hinshelwood's model 228

much less painful. Providing all these kinds of information on your model should promote 229 its adoption by the community. 230

Conclusion

Modelling in biology is a question of choices and trade-offs. The striking difference between 232 two different modellers is often the choice in model complexity. Extensive tests, using 233 cross-validation datasets or based on Akaike criteria may reveal that one model has a 234 better prediction capability than the other, but in other circumstances, it might be the 235 opposite. Our culture has contributed to hatch the illusion of a unique and universal model 236 behind nature. However, even if this idea were right, we are far from having discovered it. 237 Also, always trying to run after such universal representation of nature, inexorably leads to 238 models whose complexities do not match the available measurements and our capability 230 to validate the model. So, why should we keep a unique model? Why not use a series 240 of models of increasing complexity? Surrogate models consist of a simplified version of 241 a simulator, which is easier to handle mathematically, resulting in more straightforward 242 use for optimisation or control. The surrogate model can be derived and calibrated from 243 the most complex model, but the opposite is also true. A simplified model, with limited 244 accuracy, can provide bounds for a more detailed model. Also, a complicated model can 245 be simplified into different sub-models depending on the environment and the limiting 246 factor (nutrients, light or temperature). Working with a set of coherent models should 247 not necessarily increase difficulty, it creates a consistent framework that can prove to be 248 very useful for different purposes, from model calibration and process optimisation, up to 249 advanced control. 250

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