On Sphere Detection for OFDM Based MIMO Systems

Md. Shamser Alam

Master's student in Department of Telecommunication, Blekinge Institute of Technology

Karlskrona, Sweden

msashimul@gmail.com

Abstract— **In this paper, a OFDM transceiver system including a Depth-first Stack based Sequential decoding solution that is based on Schnorr-Eucner by Maximum likelihood method and the SNR loss compensation method due to the insertion of cyclic prefix in the OFDM transceiver system to solve the inter symbol interference and inter carrier interference has been implemented. The analysis and algorithms are general in nature.**

Keywords: Sphere detection, OFDM transceiver system, SNR loss, MIMO systems, Cyclic prefix.

I. INTRODUCTION

Wireless communications has emerged as one of the largest and most rapidly growing sectors of the global telecommunications industry driven by the demand for increasingly sophisticated connectivity at anytime and at anywhere .Communication by using Multiple Input Multiple Output (MIMO) antenna architectures promises to play a key role in fuelling this tremendous growth which is one of the most significant technological developments of the last decade. OFDM is a multi carrier technique which mitigates inter symbol interference. The key transmission technique for future wireless communication is the combination of MIMO and OFDM. Particularly, signal detection has been the subject of intensive study at the receiver end of the MIMO channel. The Sphere Decoder (SD) [1] is one of the most important and industrially relevant algorithms to emerge from these efforts. Optimal detection of signals or the Maximum Likelihood (ML) transmitted over MIMO channels is well-known to be an NP-complete problem. Sphere decoding (SD) came into view as a challenging method to exhibit the optimum ML solution for the MIMO decoding problem which reformulates the impractical exhaustive search over all possible vectors that is transmitted into an efficient depth-first tree search.

Existing Sphere Detection algorithms show two major weaknesses: First, the value chosen for the search radius parameter is highly sensitive for the performance of most current proposals. Secondly, the complexity coefficient can become very large when the SNR [1] is low or when the problem dimension is high, e.g., at the high spectral efficiencies required to support higher communication rates, although its time complexity is polynomial in the average case. Another problem is OFDM transceiver system is SNR loss due to the addition of cyclic prefix.

In this paper, a simulation of the OFDM transceiver system [2] including the Sphere Detection is accomplished and the SNR loss due to the insertion of cyclic prefix in the OFDM transceiver system also compensated for different QAM modulations. For that purpose, a generic codeword Depth-first Stack-based sequential decoding Sphere Detector which is based respectively on both the Fincke-Pohst and Schnorr-Euchner enumerations was implemented to obtain the optimal solution.

The first of my contributions introduces the sensitivity of the sphere decoder's performance to its radius parameter. In section II, overview of the physical layer of the LTE system where as the system model is described in section III. Section IV details the basic concepts and fundamentals of mathematical implementation and the description of Sphere Decoding algorithm. In section V simulation results are stated and in section VI conclusions are given.

II. LTE PHYSICAL LAYER

The Fig.1 demonstrates the LTE design of the transmitter and receiver on the basis of physical layer information and the structure which is depicted below is based on OFDM system.
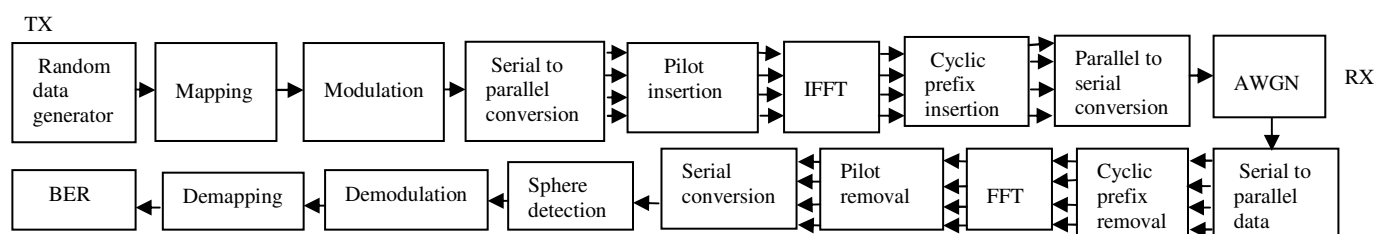

Figure 1. Full Block structure for the UMTS-LTE Transceiver

The modeling of the data digital or analog in the communication system is done by a random binary source generator with equi-probable bits.

$$D = [d0, d1, d2, d3, d4\dots dn] \text{ and } di = \{0, 1\}$$

The number of generated digital data is paralleled and is mapped into the complex symbol data block by modulation techniques and subcarriers are also attached to data. Mapping schemes improve the spectral efficiency and increase the bit rate. The data rate and the efficiency are developed by the constellation. Pilot insertion will increase the spectrum signal length which is not the integer multiple of total length of signal. Length is adjusted by the time band limits or the frequency band limits. Pilot tones used in my design, are transmitted together with the data symbols which make the design of the UMTS-LTE transceiver more robust against the fading effect of the channel. IFFT generates OFDM as it converts a N number of frequency domain complex symbol into time domain. Wireless communication in multi-path faces many problems while the transmission is going on. Interference symbol and inter carrier interference are common over the time varying frequency selective channels. Cyclic prefix reduces the inter symbol interference which is similar to the last part of the transmitted OFDM symbol. Cyclic prefix must be adapted to delay spread related to signal. There are some conditions in the length must be same or longer than the channel impulse response length to get rid of interference. To face inter symbol interference a guard time interval is added to each symbol which is cyclic prefix and is put before OFDM symbol. OFDM transmitted symbol does not affect in the actual transmitted data of every next symbol. Receiver side has some additional operations with respect to the transmitter side. The signal received is in the form of convolution of the multi-path channel impulse response h(t) and the transmitted signal s(t).This is inverse process to the one at transmitter side. The received signal is comprised of convolution of the multi-path channel impulse response h(t) and the transmitted signal s(t).

III. SYSTEM MODEL

Consider the linear MIMO system as shown in Fig.2 to communicate over the channel.
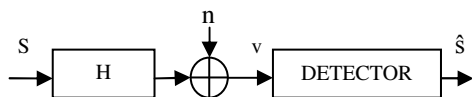

Figure 2. MIMO Communication System Diagram

The simplified linear MIMO communication system diagram is showing the discrete time signals transmitted symbols vector $S \in X^M$, channel matrix $H \in R^{NXM}$, additive noise vector $n \in R^N$, received vector $v \in R^N$ and detected symbol vector $\hat{S} \in R^M$. Finite alphabet $X = \{x_2,\dots x_B\}$ of size B indicates the transmitted symbols to get the goal. The B possible transmitted symbol vectors, $S \in X^M$ are selected based on the available data. As a result an optimal detector should returns as $\hat{S} = S_*$, given the observed signal vector v and its posterior probability of having been sent as:

$$s_* \triangleq \text{argmax}_{s \in X^M} P(S \text{ was sent } | v \text{ is observed }) \quad (1)$$

Equation (1) is called as the Maximum A posterior Probability (MAP) detection rule. Suppose the symbol vectors $S \in X^M$ are equi-probable that P (S was sent) is constant then the optimal MAP detection rule can be:

$$s_* = \text{argmax}_{s \in X^M} P(v \text{ is observed } | S \text{ was sent }) \quad (2)$$

Maximum Likelihood (ML) detector always returns an optimal solution to satisfy (2). The additive white Gaussian noise n is considered to express the ML detection problem as the minimization of the squared Euclidean distance metric in order to meet a vector v over an M-dimensional finite discrete search set as:

$$s_* = \text{argmin}_{s \in X^M} |v - HS|^2 \quad (3)$$

The optimization variables S and $|v - HS|^2$ as the objective function are measured from the optimization literature. The wireless communication problems examples can be explained by defining the channel matrix H, the ML detection of lattice coded signals, QAM-modulated signals transmitted over MIMO at fading channels and frequency selective fading channels, as well as multi-user channels.In efficient wireless communication the propagation of the radio frequencies is carried out through communication channels. There are a few suggestions for detection orderings which take the channel matrix, H and the vector of received signals, y into account [10], [11]. However, the detection ordering can only be based on channel complexity. An advantage is that the channel matrix pre-processing has to be done once as dealing with the constant deterministic channel or the channel matrix remains constant for many realization of y. There are certain channels as the Additive White Gaussian Noise channel model (AWGN) and the frequency-selective channel model. The (AWGN) channel model investigates the effects of real channels on the performance of communications systems. It is simple and comes from the impairment with this wireless model and the white noise. This noise is characterized by a random signal of certain spectral density and it is obtained by independent random samples from a Gaussian distributions. AWGN does not include any fading, dispersion or interference and a mathematical mode to represent the effect of thermal noise. It consists of a simple wireless channel and is used widely.

IV. SPHERE DETECTION

The input and output relationship of the MIMO channel is given below:

$$y = HS + v \quad (4)$$

where $S \in X^m$ is the finite set of transmitted vector symbols , $y \in F^n$ is the received signal vector, $H \in F^{nxm}$ is the channel matrix and $v \in F^n$ is the additive white Gaussian noise.

Here F is the set of real or complex numbers, i.e. $F \in \{R,C\}$,according to the context. Detection of the vector symbols transmitted over the system channel model according to (4) is based upon y and H. A finite set of linearly modulated symbols transmitted over a known linear channel subject to Gaussian noise is modeled on the basis of (4). That is, a detector is defined by some (possibly random) map.

$$\Phi: F^n F^{nxm} \rightarrow X^m \quad (5)$$

where S=Φ (y, H) and F is R or C. Computation of Φ relates to the implementation of the detector. Naturally, the exact interpretation how the detectors are beneficial in this system is debatable. The possibility that the minimum probability of error provided by the receiver in case of transmitted messages $S \in X^m$, is the maximum-likelihood (ML) receiver, expressed as:

$$\hat{s}_{ML} = argmin_{S \in S^m} ||v - HS||^2 \qquad (6)$$

The detector and receiver are always are interchangeable and referred to same thing. The number of symbols m is large enough and results are computationally difficult.

Sphere decoding which was introduced originally by Finke and Pohst ,enumerates all lattice points in a sphere centered at a given vector. This detection technique was first applied to the ML detection problem which gained main stream recognition with a later series of papers [7], [8].The principle of the Sphere Detection algorithm is to find the closest lattice point to the received signal within a sphere of radius. Hence, it is possible to reduce the computational complexity, by restricting the search area. The choice of is very crucial to the speed of the algorithm where as in practice, it can be adjusted according to the noise (and eventually the fading) variance. The sphere decoder is developed on two stages. Firstly a pre processing stage computes the QR factorization of the channel matrix, H and after this a search stage finds the estimate, $\hat{s}_{ML}$ This detection algorithm has also been observed under many different communications scenarios. Focusing on the multiple antenna channel, with focus on the CDMA scenario and to generate soft information required by concatenated coding schemes are the examples of different communication scenarios. The sphere decoding algorithm can be illustrated as a tree search procedure by a pruning criteria to reduce the search. It is also notable that there is no possibility that the Sphere decoding estimation does not belong to the set of leaf nodes visited by the algorithm (assuming there are some leaf nodes visited). This decoding algorithm [9] performs a depth-first search over the tree by visiting child nodes before sibling nodes. If the distance exceeds a certain radius d, it falls outside the sphere and is automatically pruned along with its children and siblings (if the latter are enumerated). The radius is updated as the distance to that leaf, if a leaf inside the sphere of radius d is reached. The sphere decoding algorithm also [1] depends on the initial search radius. There will be too many lattice points in the sphere if the initial search radius is too large where as there will be no points in the sphere if the radius is too small. A MIMO channel detector which produces a set of symbols S $\in X^m$ given a set of signals v $\in F^N$ observed at the output of the communication channel, is typically modeled as a linear system H $\in F^{nxm}$ combined with an additive noise vector n $\in F^n$. I surmise that $M \leq N$ and that H is of full rank M, i.e., there are at least as many observations as symbols to be detected. In the tree search analogy, the sequence of symbol decisions, {$\hat{s}_{m-k+1}$ ....$\hat{s}_m$ } which corresponds to a node of the search tree at the kth level, starts counting from the root of the tree which by default is at the 0 level. The nodes ordering before and during the tree search is important for the algorithm. With appropriate ordering, SD can improve detection performance significantly and provide the number of nodes required to search or the required number of multiplications to achieve maximum likelihood detection performance.

A. *Sphere Decoding Fundamentals*

The sphere decoding is based on the enumeration of points in the search set which are found within the sphere of some radius centred at a target such as the received signal point. The Fincke-Pohst (F-P) and Schnorr-Euchner (S-E) techniques are the two computationally efficient means of realizing this enumeration [9] and the foundation of most existing sphere decoders are formed by these. The F-P and S-E enumerations and all SDs are the QR-factorization of the channel matrix: N by $M \leq N$ matrix H with linearly independent columns factorization can be shown in factors as:

$$H= Q \begin{bmatrix} R \\ 0 \end{bmatrix} \qquad (7)$$

Q is NxN and orthogonal, R is MxM, upper invertible and triangular, and 0 is an (N-M)xM matrix of zeros. As the objective function is invariant under orthogonal transformation, minimization problem can be written as

$$argmin_{s \in x^M} |v\text{-}Hs|^2 = argmin_{s \in x^M} | \ Q^T v\text{-} \begin{bmatrix} R \\ 0 \end{bmatrix} S|^2 \qquad (8)$$

$$= argmin_{s \in x^M} \ | \tilde{v} - RS|^2 \qquad (9)$$

Where $\tilde{v}= \begin{bmatrix} Q^T v \end{bmatrix}$, the lower limit 1 and the upper limit M extract the first M elements of the orthogonally transformed target. The factored matrix of the upper triangular structure then enables the decoder to decompose the equivalent objective function (9) recursively as:

$$|\tilde{v} - RS|^2 = d^2(\tilde{v}_M, r_{MM}S_M)$$
$$+ \left|(\tilde{v} - r_M S_M)_{\backslash M} - R_{\backslash MM}S_1^{M-1}\right|^2 \qquad (10)$$

$$= d^2(\tilde{v}_M, r_{MM}S_M) + |\tilde{y}(S_M) - R_{\backslash MM}S_1^{M-1}|^2 \qquad (11)$$

$$= \sum_{D=M-1}^{O} d^2(\tilde{y}(S_{D+1}^M)_D, r_{DD}S_D) \qquad (12)$$

Where the squared Euclidean distance metric is $d^2(.)$ where as $(\tilde{v} - r_M S_M)_{\backslash M}$ indicates the tall sub matrix comprised of all columns but M^{th} and $R_{\backslash MM}$ indicates the square sub matrix formed by removing MM^{th} row and column and I have

$$\begin{cases} \tilde{y}(S_{D+1}^M) = \quad \tilde{y}(\Phi) = \tilde{v}, \qquad D = M \\ \\ (\ \tilde{y}(S_{D+2}^M) - r_{D+1}S_{D+1})_{\backslash D+1} \quad D = M - 1, \dots \dots \dots \dots .0 \end{cases} \qquad (13)$$

a set of L= M-D constraint values are applied to optimization variables $s_{D+1}\dots\dots\dots\dots,s_M$ and parameterize the residual target. Here $\tilde{y}$ shows that it resides in the same orthogonally transformed space as $\tilde{v}$. In this way the QR factorization provides the means of evaluating objective

function efficiently. Many shared terms are contained in (12) summation for the decomposition. There are the values of the objective function for all B^{M-1} as in the first term of (11) which are involved in the search set satisfying $ş_M = s_M$. So associate the constraint $ş_M = s_M$ with this term. The (13) summation lends itself naturally to a weighted representation B-ary tree. It is shown in Fig. 3 for the case where $M = B = 2$ and $X = \{-1,1\}$. Each of the terms in the diagram in the summations of (12) associated with a constraint as well as with a branch. Then each node encapsulates a set of constraints $ş_{D+1}^M = s_{D+1}^M$ and these have been applied in a way that specified by the branches traversed along its path from the root node. A residual target can also be associated with each node by computing (13). I observe that the variables must be constrained in order from $ş_M$ to $ş_1$ because of the QR factorization.
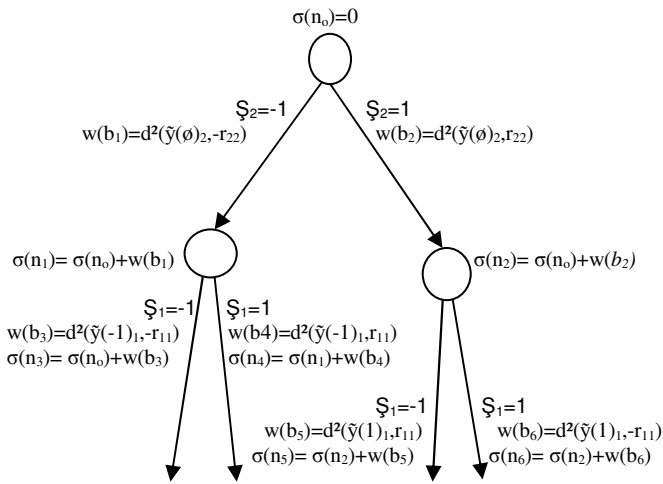

Figure 3. A weighted B-ary tree with M = 2 and X = {1; 1}

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The above tree is explained by the problem parameters v, H, and X. A few properties of the tree are important to study sphere decoding algorithms.

a) Nodes are distributed over M +1 level, as the numbering from the root level node n at level 0 to the leaf nodes at M and no leaf nodes are at levels 0 through M-1.

b) Branches at levels L and L + 1 (L = 0,..........,M-1) are linked with variable $ş_D$, where I let D = M- L.

c) The B-ary, tree with each non leaf node is the parent of exactly B child. The each child branch corresponds to one of the B values in X in a way that the associated variable can take. There are B^L nodes at level L, and each is associated with a set of L constraints $ş_{D+1}^M = s_{D+1}^M$. Particularly, each leaf node is associated with a full vector of constraints $ş = s$, where $s \in X^M$ corresponds to a point in the search set.

d) This weighted tree; non-negative weights $w(b_j)$ and $ϭ(n_k)$ are related with the branches and nodes, respectively by the proper assigning to the root node n_0 the weight 0.

e) Nodes at level L to L+1 have branches to assign weights $d^2(ỹ(s_{D+1}^M)_D, r_{DD}s_D)$,so that the constraints $ş_{D+1}^M = s_{D+1}^M$ are

related with the parent node at level L (Property c), and $ş_D = s_D$ is the constraint associating to the branch.

f) The summation of branch weights along the path from the root provides each node weight or as the sum of weights of its parent node and the connecting branch.

g) The node weight are not decreasing following any path from the root to a leaf node.

h) The summation (13) is equal to the leaf nodes weights such that these values of objective function evaluated at each of the point in the search set.

The properties c and h depict that the ML solution which is specified by the point in search set associated with the smallest leaf node weight in the tree of Fig. 3. An exponential number of leaf nodes are considered, and in the similar way a comparable number of non-leaf nodes whose weights must all be computed in order to determine those of the leaf nodes.

B. The Fincke-Pohst and Schnorr-Euchner enumerations

The smallest weight leaf node starting from the root is searched by a sphere decoder. It should start at the root and only can express itself further by computing the weights of connected branches and nodes, because of the recursive definition of the node weights. The clever pruning process of the tree, makes it able to declare an ML solution which is based on the intermediate node weights after the computation of a polynomial number of weights in the average case [3]. This is done by the property g. Considering the geometric structure, the leaf node weight corresponds to the squared Euclidean distance from a point in the search set to the target. Points located within the sphere of radius C centred at the target can be enumerated by exploring from the root along all the branches such that node n_k is encountered as $ϭ(nk) > C^2$. Descendents of node n_k have weights at least as $ϭ(nk)$ because of property g. Therefore the point linked with the leaf nodes must lie outside of the search sphere. Then reduction of the time in computing the tree based search is done by pruning at node n_k. It means that the weights of branches and nodes which are descendants of node n_k, need not to be computed. By traversing the tree in depth-first procedure, from left to right, until all nodes having weights not greater than C^2, are discovered. It returns a list of leaf nodes that relate to points located within the search sphere. This shows the behaviour of the Fincke-Pohst (F-P) enumeration with respect to the tree in Fig. 3. The implementation can be found in works such as [6], [3]. A characteristic of the F-P strategy is that a search radius must be specified. Remember if C is too large, many node weights will have to be computed and a large number of leaf nodes may be returned. If it is too small, no leaf nodes will be found and the decoder must then be restarted with a larger search radius. These factors impact the overall computation time negatively and so one of the main weaknesses of the F-P decoder is the sensitivity of its performance to the choice of C. Particularly the distance of the Babai point [6], is a point in the search set and it is assured to find one leaf node. Values of C and the F-P enumeration are used in the first decoder referred as the FPB. Specifically, the Schnorr-Euchner (S-E) enumeration adds a certain refinement to the F-P approach. The tree is traversed in depth first from left to right in the F-P

strategy, in a way as the children of a node are considered in order of increasing $S_i \in X$, where i is the level of the parent node and recall that each of its children is linked with applying the additional constraint $ş_i = s_i$. The strategy S-E also shows that the traversing of the tree in depth- first rules instead of considering child nodes from left to right explores in increasing order in weights according to the computation of connecting branch weights.

The S-E enumeration discovers eligible leaf nodes more quickly than that of the F-P enumeration [6]. In case if there were the only refinement, the S-E enumeration would still have to compute the same number of branch and node weights as the F-P strategy. But it is observed that as a leaf node n_l is discovered, the search radius can be adaptively reduced to C = √σ(n_l). Actually after having discovered a point in the search set, I am only interested in locating those points which are even closer to the target than that point. The decoder based on the S-E enumeration has been the current state-of-the art by adaptively adjusting the search radius.

V. SIMULATION RESULT

The designed OFDM transceiver including Sphere Detection is considered operating with different QAM modulation. The full block diagram for the implemented system channel model is shown in Fig.1. With the transceiver structure in Fig.1 the different parts of the system are easily configurable and adaptable for parameters changes.

With a small loss of transmission energy using the concept of a cyclic prefix the inter symbol interference (ISI) and inter carrier interference (ICI) within an OFDM symbol can be avoided. The insertion of a silent guard period between successive OFDM symbols avoiding ISI in a dispersive environment does not avoid the loss of the subcarrier orthogonality where as with the introduction of a cyclic prefix, this problem is solved. This cyclic prefix both defends the orthogonality of the subcarriers and protects ISI between successive OFDM symbols. Therefore, equalization at the receiver is very simple which motivates the use of OFDM in wireless systems. First, to minimize the loss of SNR the length of the cyclic prefix should be chosen to be a small fraction of the OFDM symbol length. The length of the OFDM symbol or equivalently, the number of subcarriers are directly related to the size of the cyclic prefix .The disadvantage of the cyclic prefix insertion is that there is a reduction in the Signal to Noise Ratio due to a lower efficiency by duplicating the symbol. The SNR loss is

$$SNR_{loss} = l0log10(\frac{S}{S + Tcp}) \qquad (14)$$

Where S is the length of transmitted OFDM fft symbol and T_{cp} is the length of cyclic prefix. For larger Eb/N0 values the gap between the theoretical and simulated curve increases significantly for higher modulations due to SNR loss, as the remaining equalization error becomes more significant. A loss of SNR due to the Cyclic Prefix insertion directly influence the achievable bit error rate (BER) regarding Eb/N0.

In the simulator, 20 packets of data were passed consecutively through the system channel model in which

each time code word bits were passed through sphere detector. Each packet contains 5200*m bits of data where m depends on modulation scheme. To analyze the performance and the results of the designed system channel model that is depicted in Fig.1, 20 dB of EbNo is used for theoretical , simulated and simulated with SNR loss compensation curves. The length of cyclic prefix is 16 where as the length of fft symbol is 64 having 52 subcarriers for each QAM modulation technique. The performance of the designed system depicted in Fig.4 to Fig.6 is evaluated by AWGN channel model for different QAM modulation including sphere detection. In Fig.4, for QPSK modulation both simulated and simulated with SNR loss compensation curves are plotted on the theoretical curve as the SNR and compensated SNR loss are same. The best performance is obtained comparing the result with the theoretical curve. One observable fact is that after 10 dB EbNo, the bit error rate is same for the remaining EbNo. After passing the data through the several SNR, the obtained bit error rate is within the range of 0.09 to 0.0003.In the output Fig.5, for 16-QAM modulation it is clearly noticeable that the SNR loss gap between theoretical and the simulated curve increases comparatively higher than that of previous execution. According to the equation 14, the simulated SNR loss is compensated. In the simulated with SNR loss compensated curve, a better performance is obtained than that of simulated curve. The bit error rate ranges with in 0.2 to 0.0004.A significant SNR loss gap is obtained for the 64-QAM modulation technique comparing with the theoretical output. The bit error rate of the simulated curve is with in the range of 0.3 to 0.006.
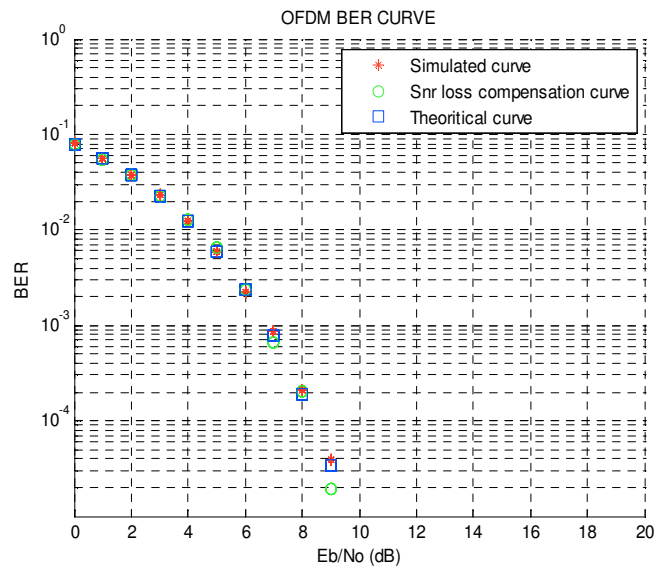

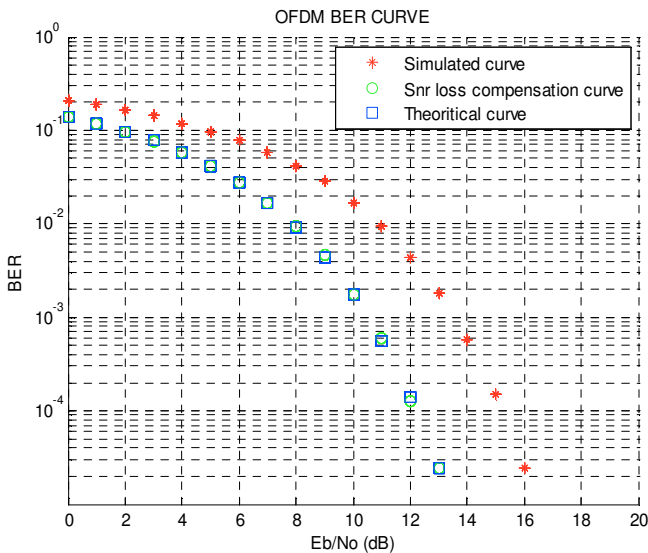Figure 4. Performance analysis using QPSK modulation.

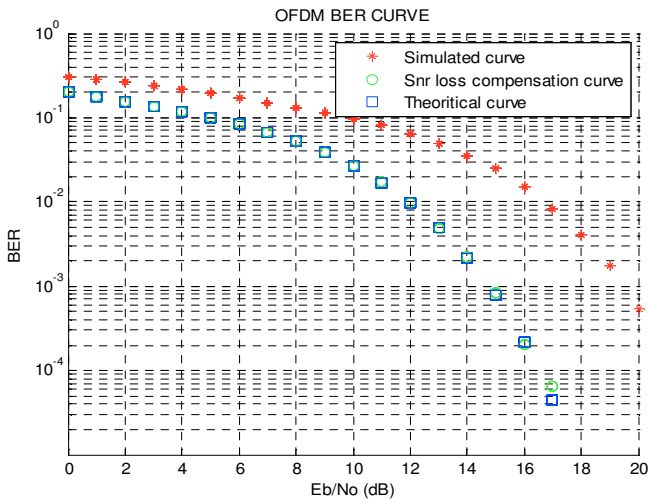Figure 5. Performance analysis using 16-QAM modulation.

Figure 6. Performance analysis using 64-QAM modulation.

VI. CONCLUSION

OFDM is shown to be an adequate modulation technique for the future generation systems. Hence LTE that is based on OFDM, is expected to be deployed by many mobile operators in the near future. Detection ordering and search radius make a statement that a large class of sphere decoder algorithms is included in that purpose. This is proven in case of detection ordering that the dependency on y in sphere decoding is a lot. Maximum eigen value of $H^H H$ will converge to some finite non random limit as m approaches to infinity in the case of channel matrix. Suboptimal implementation with low complexity is future research as the exponential complexity is present in the optimal sphere decoder. Sphere decoder has complexity not only in the worst case but in a probabilistic and every sense as well. This approach is given in [5] as the fixed complexity sphere decoder with low error probability at high SNR and a sub exponential complexity of $O(|S|^{\sqrt{m}})$.

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BIOGRAPHY

Md. Shamser Alam completed Master's in Electrical Engineering on Telecommunication from Blekinge Institute of Technology, Sweden and received BSC in computer science and Engineering from Chittagong University Of Engineering and Technology, Bangladesh. His interest of study is in signal processing and in wireless communication systems.