On the Social (Sub)Optimality of Divisionalization under Product Differentiation*

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Abstract

We revisit the interplay between differentiation and divisionalization in a duopoly version of Ziss (1998). We model divisionalization as a discrete problem to prove that (i) firms may choose not to become multidivisional; and (ii) there may arise asymmetric outcomes in mixed strategies, due to the existence of multiple symmetric equilibria. If industry-wide divisionalization is the unique equilibrium, it can be socially efficient provided goods are *almost* perfect substitutes. Even small degrees of differentiation may suffice to make industry-wide divisionalization socially desirable because of the prevalence of consumers' taste for variety over the replication of fixed costs.

Keywords: divisionalization; product differentiation; duopoly

JEL Codes: L13; L22; L41

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1 Introduction

The early literature on multidivisional firms investigates monopoly models where the subject matter is the use of divisionalization as a strategic entry barrier (Schwartz and Thompson, 1986; Veendorp, 1991). A subsequent stream of literature translates the idea into oligopoly games to show that divisionalization may be an instrument to acquire a dominant position, or, replicate Stackelberg leadership[1] (Corchón, 1991) but, since all firms share this incentive, the outcome is definitely procompetitive and may even drive the industry towards perfect competition if divisionalization is costless (Polasky, 1992).[2] Baye *et al.* (1996a,b) add up a fixed cost per-division to show that the resulting equilibrium, with the same finite number of divisions per-firm, can be socially efficient if the industry is a duopoly.[3] This literature relies on product homogeneity and linear variable costs.

To the best of our knowledge, the only exception contemplating product differentiation in the representative consumer approach used in the bulk of the aforementioned literature is Ziss (1998), where each firm's product is an imperfect substitute of the rivals' products, in such a way that intra-firm (resp., inter-firm) competition takes place in differentiated (resp., homogeneous) products. Leaving aside the issue of social efficiency, Ziss (1998) treats the number of divisions as a continuous variable and illustrates the effect of product differentiation on the number of divisions at equilibrium. Ziss (1998) assumes products to be differentiated across firms but not across

[1] Of course, this has a lot to do with strategic delegation (Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987). The relationship between divisionalization and delegation is explicitly considered in Ziss (1999), Bárcena-Ruiz and Espinosa (1999) and González-Maestre (2000). For a reconstruction of the debate, see Lambertini (2017).

[2] However, going multidivisional may facilitate implicit collusion (Dargaud and Jacques, 2015).

[3] Corchón and González-Maestre (2000) extend the analysis to the case of concave demand functions, to prove that either perfect competition or a natural oligopoly may obtain at equilibrium as the cost of divisionalization shrinks. See also González-Maestre (2001), where an analogous result emerges from a discrete choice model of spatial differentiation.

divisions and demonstrates the existence of a pure-strategy equilibrium with symmetric multidivisional firms, even in the limit case in which divisionalization is costless, for any degree of product differentiation. This finding has a twofold implication: (i) the result outlined in Polasky (1992) emerges only when the product is homogeneous also across firms and not only across the divisions belonging to the same firm; and (ii) the perfectly competitive outcome remains out of reach even when divisionalization costs are nil, provided products are not perfect substitutes. Moreover, Ziss (1998) also shows that the peak of the optimal per-firm number of divisions is sensitive to both industry structure and product differentiation.

This approach, as well as the bulk of the aforementioned literature, focusses itself on the existence and characterization of optimal divisionalization, under the (somewhat implicit) assumption that firm will divisionalise themselves. By doing so, it overlooks the possibility for a firm facing a rival which goes multidivisional to respond by not divisionalising itself. This is the route we take here, to produce the following results.

- Accounting for the integer problem, the decision on divisionalization gives rise to a discrete-strategy game in which firms maintain the option not to do it, and indeed don't divisionalise themselves if the cost of divisionalization is too high and product substitutability is low enough. It is worth stressing that an analogous result has emerged in the aforementioned debate. In particular, González-Maestre (2000, Props. 1(i) and 3(i), pp. 327-28) shows that single-division firms will appear at equilibrium in a homogeneous good duopoly where divisionalization and strategic delegation *à la* Vickers (1985) coexist. This is due to the fact that if the number of firms is small, then every firm obtains a relatively high fraction of industry profits; consequently, the possibility of becoming multidivisional is not appealing, as the incentive to increase market share is more than offset by the damage created by the increase in the intensity of market competition. Moreover, González-Maestre (2001, Prop. 1(ii), p. 1303) uses a spatial model *à la* Salop (1979) to find, amongst other things, that if the number of firms is sufficiently small as compared to the elasticity of the transportation cost function borne by consumers, then firms will remain single-division entities. Additionally, Corchón, (1991, Example 1, p. 2) using a hyperbolic demand function for a homogeneous good, proves that the number of divisions is undetermined in duopoly (which admits the case of single-division firms).

- There exists an intermediate range of values of fixed costs and product substitutability in which multiple equilibria arise, including, with a positive probability, asymmetric equilibria.
- The alignment of private and social preferences towards industry-wide divisionalization occurs only for sufficiently low levels of fixed costs and product differentiation.

To keep the model manageable, we confine our attention to the duopoly case, although we also extend the analysis to the scenario in which, should firms speculate about becoming multidivisional, they could examine the possibility of activating a finite generic number of divisions. The interesting feature emerging from this slight generalisation consists in the fact that the choice between a single division or more than two does not give rise to a prisoners' dilemma.

The remainder of the paper is organized as follows. The model is laid out in section 2. The solution of the game is outlined in section 3. Section 4 deals with the welfare analysis. the extension to a generic number of divisions is assessed in section 5. Section 6 contains some concluding remarks.

2 Setup

The model looks the same as in Ziss (1998), except for the number of firms and the explicit consideration of the integer problem concerning the number of divisions. We consider two firms operating in a Cournot market for differentiated goods produced at a common and constant marginal cost which can be normalised to zero for the sake of simplicity and without further loss of generality. Each firm offers a homogeneous good, which is an imperfect substitute of the variety supplied by the rival. Firm i has $d_i \geq 1$ divisions (or franchisees), each one controlled by a manager competing with all others (including those operating the remaining $d_i - 1$ divisions of firm i). The inverse demand function for division h belonging to firm i is $p_{ih} = a - q_{ih} - \sum_{\ell \neq h} q_{i\ell} - s \sum_{m=1}^{d_j} q_{jm}$, where $a > 0$ is the common choke price and $s \in (0, 1]$ measures the degree of substitutability between any two varieties offered by different firms. The cost of building up each division is $F > 0$. Hence, the profit function of firm i if it has d_i divisions is $\pi_i = \sum_{\ell=1}^{d_i} p_{i\ell} q_{i\ell} - d_i F$. Since every firm's divisions sell the same homogeneous good, $p_{i\ell} = p_i$ for all $\ell = 1, 2, ...d_i$, and therefore the profit function of firm i can be written as $\pi_i = p_i \sum_{\ell=1}^{d_i} q_{i\ell} - d_i F$.

Unlike the extant literature on multidivisional firms which we have summarised in the introduction, our purpose consists in modelling the firms' divisionalization choices as a game in discrete strategies, which, by construction, encompasses the possibility of asymmetric outcomes where firms are endowed with different numbers of divisions. This choice is motivated by the intent of showing that the scenario in which firms decide not to become multidivisional entities may indeed be an equilibrium one, and the simplest way of achieving this result consists in supposing that initially both firms have a single division and then ask themselves whether to add a second one or not. Accordingly, we have to examine three cases: (i) both firms are multidivisional entities; (ii) only one is multidivisional; (iii) neither one is.

The game has a two-stage structure. Initially, both firms have a single division each. In the first stage, firms simultaneously and noncooperatively decide whether to build up a second division or not; in the second stage, market competition takes place in the output space under complete, symmetric and imperfect information, given the number of divisions chosen at the previous stage. The solution concept is subgame perfection by backward induction, with the choices of firms at the first stage being public domain before setting output levels.

3 Solving the game

To clarify the reason why we shall focus upon the binary choice between one and two divisions, we may summarise the general setup in Ziss (1998) by supposing, initially, that both firms are multidivisional organizations, with $d_i \geq 1$ divisions each. The profit function of the generic division $h = 1, 2, ...d_i$ of firm $i = 1, 2$ is

$$\pi_{ih} = \left(a - q_{ih} - \sum_{\ell \neq h} q_{i\ell} - s \sum_{m=1}^{d_j} q_{jm} \right) q_{ih} - F \qquad (1)$$

and the profit function of firm i is $\Pi_i = \sum_{h=1}^{d_i} \pi_{ih}$. Firms' owners choose their respective number of divisions to maximise their firms' profits in the first stage, and managers choose outputs to maximise the profits of their individual divisions in the second stage. Agents' choices are noncooperative and simultaneous in both stages. The Cournot-Nash output of every single division is

$$q_{ih}^{CN} = \frac{a \left[1 + d_j (1 - s)\right]}{1 + d_j + d_i \left[1 + d_j (1 - s^2)\right]} \qquad (2)$$

so that the relevant objective function of the owner of firm i at the first stage is

$$\Pi_i^{CN} = \frac{d_i a^2 \left[1 - d_j (1 - s)\right]^2}{\left[1 + d_j + d_i \left(1 + d_j (1 - s^2)\right)\right]^2} - d_i F \qquad (3)$$

Given the functional form of (3), the resulting first order condition $\partial \Pi_i^{CN} / \partial d_i = 0$, in addition to making its analytical solution cumbersome, does not lend itself to an intuitive interpretation, even under the symmetry condition $d_i = d_j = d$:

$$\frac{a^2 \left[1 - d^2 (1 - s^2)\right]}{\left[1 + d (1 - s)\right] \left[1 + d (1 + s)\right]^3} - F = 0 \qquad (4)$$

Clearly, the quartic equation in (4) could be solved w.r.t. d, and its solution studied numerically, as in Ziss (1998). This procedure, however, involves treating d first as a continuous variable (which allows one to write (4)), and then accounting for the fact that d is indeed an integer.

Our alternative approach consists in modelling the divisionalization choice as a discrete problem, in which firms must initially decide whether to build up a second division or not. Following the backward induction procedure, we first have to solve the three relevant subgames determined by the firms' decisions at the first stage. In case (i), both firms are multidivisional organizations; the profit function of the generic division $h = 1, 2$ of firm $i = 1, 2$ is

$$\pi_{ih} = [a - q_{ih} - q_{i\ell} - s(q_{j1} + q_{j2})] q_{ih} - F \tag{5}$$

and straightforward calculations deliver the Cournot-Nash equilibrium levels of outputs and parent firm's profits:

$$q_{MM}^{CN} = \frac{a}{3+2s} \, ; \, \pi_{MM}^{CN} = \frac{2\left[a^2 - F(3+2s)^2\right]}{(3+2s)^2} \tag{6}$$

where subscript MM indicates that both firms are multidivisional, and

$$\pi_{MM}^{CN} > 0 \forall F \in (0, F_{MM}) \, , \, F_{MM} \equiv \frac{a^2}{(3+2s)^2} \tag{7}$$

In case (ii), firm i has two divisions while firm j has a single one. Accordingly, the relevant profit functions at the market stage are

$$\pi_{ih} = (a - q_{ih} - q_{i\ell} - sq_j) q_{ih} - F \tag{8}$$

$$\pi_j = [a - q_j - s(q_{ih} + q_{i\ell})] q_j - F \tag{9}$$

Solving for the asymmetric Cournot-Nash equilibrium, one obtains

$$q_{i1}^{CN} = q_{i2}^{CN} = q_{MS}^{CN} = \frac{a(2-s)}{2(3-s^2)} \, ; \, q_j^{CN} = q_{SM}^{CN} = \frac{a(3-2s)}{2(3-s^2)} \tag{10}$$

with $q_{SM}^{CN} \in \left(q_{MS}^{CN}, 2q_{MS}^{CN}\right)$, and

$$\begin{aligned} \pi_i^{CN} = \pi_{MS}^{CN} &= \frac{a^2 (2-s)^2 - 4F (3-s^2)^2}{2 (3-s^2)^2} \\ \pi_j^{CN} = \pi_{SM}^{CN} &= \frac{a^2 (3-2s)^2 - F (3-s^2)^2}{4 (3-s^2)^2} \end{aligned} \tag{11}$$

where subscripts MS and SM identify the multidivisional firm and the single-division firm, respectively, and

$$\pi_{MS}^{CN} > 0 \forall F \in (0, F_{MS}) \,,\; F_{MS} \equiv \frac{a^2 (2-s)^2}{(3-s^2)^2} \tag{12}$$

$$\pi_{SM}^{CN} > 0 \forall F \in (0, F_{SM}) \,,\; F_{SM} \equiv \frac{a^2 (3-2s)^2}{(3-s^2)^2} \tag{13}$$

Then, case (iii) depicts the Cournot duopoly made up by single-division firms, whose equilibrium is familiar from Singh and Vives (1984):

$$q_{SS}^{CN} = \frac{a}{2+s} \,;\; \pi_{SS}^{CN} = \left(q_{SS}^{CN}\right)^2 - F = \frac{a^2 - F (2+s)^2}{(2+s)^2} \tag{14}$$

with

$$\pi_{SS}^{CN} > 0 \forall F \in (0, F_{SS}) \,,\; F_{SS} \equiv \frac{a^2}{(2+s)^2} \tag{15}$$

Tedious but trivial calculations are needed to establish the following:

Lemma 1 $F_{SS} \geq F_{SM} > F_{MS} \geq F_{MM}$ *for all* $s \in (0,1]$.

This Lemma implies that the condition $F \in (0, F_{MM})$ is necessary and sufficient for equilibrium profits to be strictly positive in any possible subgame (or, for any shape taken by the internal organization of firms). In order to leave aside trivial outcomes, this is the range in which we are going to investigate the first stage of the game, taking place in discrete strategies and described by Matrix 1.

		2	
		S	M
1	S	π_{SS}^{CN} ; π_{SS}^{CN}	π_{SM}^{CN} ; π_{MS}^{CN}
	M	π_{MS}^{CN} ; π_{SM}^{CN}	π_{MM}^{CN} ; π_{MM}^{CN}

Matrix 1

In order to determine the equilibrium outcome(s) of the stage game in Matrix 1, one has to assess the sign of $\pi_{SS}^{CN} - \pi_{MS}^{CN}$ and $\pi_{SM}^{CN} - \pi_{MM}^{CN}$, as well as $\pi_{SS}^{CN} - \pi_{MM}^{CN}$, which determines the Paretian properties of symmetric equilibria. This exercise yields

$$\pi_{SS}^{CN} \geq \pi_{MS}^{CN} \,\forall\, F \geq \max\left\{0, F_{MS}^{SS}\right\}, F_{MS}^{SS} \equiv \frac{a^2\left[s^2\left(4-s^2\right)-2\right]}{2\left(2+s\right)^2\left(3-s^2\right)^2} \tag{16}$$

with $F_{MS}^{SS} > 0$ for all $s \in \left(\sqrt{2-\sqrt{2}}, 1\right]$;

$$\pi_{SM}^{CN} \geq \pi_{MM}^{CN} \,\forall\, F \geq \max\left\{0, F_{MM}^{SM}\right\}, F_{MM}^{SM} \equiv \frac{a^2\left[8s^2\left(3-s^2\right)-9\right]}{4\left(3+2s\right)^2\left(3-s^2\right)^2} \tag{17}$$

with $F_{MM}^{SM} > 0$ for all $s \in \left(\sqrt{3\left(2-\sqrt{2}\right)/2}, 1\right]$. Instead, $\pi_{SS}^{CN} > \pi_{MM}^{CN}$ always. This property has relevant consequence, which we shall dwell upon below.

Moreover, on the basis of (7) and (16-17), we may formulate the following:

Lemma 2 $F_{MM} > \max\left\{0, F_{MM}^{SM}\right\} \geq \max\left\{0, F_{MS}^{SS}\right\}$ *for all* $s \in (0, 1]$.

For any given value of the choke price a, the critical levels of F appearing in Lemma 2 look like the curves in Figure 1.

Recalling that we must confine our attention to $F \in (0, F_{MM})$, the map can be used to characterise the equilibrium outcomes of the first stage of the game, by referring to the areas $I - III$ identified by the curves:

- in region I, $F \in \left(0, F_{MS}^{SS}\right)$; therefore, $\pi_{SS}^{CN} < \pi_{MS}^{CN}$ and $\pi_{SM}^{CN} < \pi_{MM}^{CN}$. Here, playing strategy M is convenient irrespective of the rival's behaviour, which, in the game-theoretical jargon, entails that the game is solvable in dominant strategies

- in region II, $F \in \left(F_{MS}^{SS}, F_{MM}^{SM}\right)$; therefore, $\pi_{SS}^{CN} > \pi_{MS}^{CN}$ and $\pi_{SM}^{CN} < \pi_{MM}^{CN}$

- in region III, $F \in \left(F_{MM}^{SM}, F_{MM}\right)$; therefore, $\pi_{SS}^{CN} > \pi_{MS}^{CN}$ and $\pi_{SM}^{CN} > \pi_{MM}^{CN}$

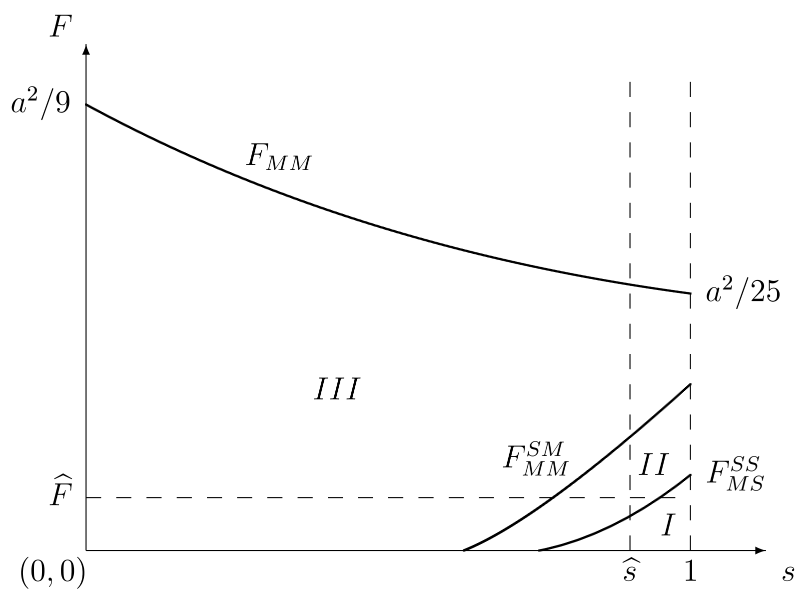

Figure 1 The partition of the (s, F) space emerging from Matrix 1

Accordingly, we are in a position to claim

Proposition 1 *For all* $F \in \left(0, F_{MS}^{SS}\right)$, *i.e., in region* I, (M, M) *is the unique equilibrium at the intersection of dominant strategies; in region* II, *i.e., for* *all $F \in \left(F_{MS}^{SS}, F_{MM}^{SM}\right)$, the first stage portrays a coordination game with two symmetric equilibria in pure strategies, (M,M) and (S,S); in region III, i.e., for all $F \in \left(F_{MM}^{SM}, F_{MM}\right)$, (S,S) is the unique equilibrium at the intersection of dominant strategies.*

The above Proposition implies the following:

Corollary 1 *If (M,M) is the unique equilibrium at the first stage (region I), then the 2×2 game describing owners' decisions about divisionalization is a prisoners' dilemma. If instead (S,S) is the unique equilibrium, it is also Pareto-efficient for firms.*

The fact that $\pi_{SS}^{CN} > \pi_{MM}^{CN}$ is implicit in the nature of the delegation contract instructing each manager to compete against any of the rival's divisions as well as the other division inside the same firm, all of this in a setting where differentiation inside the same firm is altogether absent by assumption. Hence, the effect of the instructions given to any manager implies a sort of cannibalization effect amplified by perfect substitutability.

The arising of multiple equilibria in region II means, of course, that for all $F \in \left(F_{MS}^{SS}, F_{MM}^{SM}\right)$, the mixed-strategy equilibrium becomes relevant too, and its outcome may be asymmetric (see the Appendix). Intuitively, Proposition 1 says that the number of divisions at the industry equilibrium decreases in the size of F. This reflects the essence of the problem faced by firms when deciding whether to go multidivisional or not: each firm has to balance the business-stealing effect exerted upon the rival, on one side, against the cannibalization effect taking place internally, together with the duplication of the fixed cost, on the other. This is the source of the prisoners' dilemma situation emerging in region I, when profit incentives drive industry-wide divisionalization but cannibalization and competition, combined with the duplication of fixed costs, make the equilibrium inefficient for players. The same argument explains the multiplicity of equilibria arising in region II, as soon as the additional fixed cost required for setting up the second division slightly decreases below F_{MM}^{SM}: at that point, firms know that becoming multidivisional is rational only if both do so, because the incremental gross profits created by a second division may not justify the fixed cost involved in its creation (recall that the product is homogeneous across a firm's divisions).

Along the orthogonal dashed lines, one can reconstruct the same considerations under a slightly different light. Moving upwards along the vertical line at $\widehat{s}$, one can replicate the results in Proposition 1, while for any $\widehat{F} \in \left(0, F_{MS}^{SS}\big|_{s=1}\right)$ it can be seen that the progressive decrease of product differentiation (as $s \to 1$) drives firms towards (M, M), illustrating the seemingly counterintuitive idea that increasing product substitutability favours divisionalization. To explain this result, take the opposite viewpoint, and observe that any $s \in \left(0, \sqrt{2-\sqrt{2}}\right]$ prevents divisionalization. This is quite obvious in the limit case $s \to 0$, where $\pi_{MM}^{CN} = \pi_{SS}^{CN}$ because firms are independent monopolists in two unrelated markets and therefore divisionalization is pointless. As s increases, the negative externality exerted by firm j's variety along the demand function faced by each division of i increases as well, creating the necessary pressure to go multidivisional as differentiation has decreased enough, which happens as soon as $s > \sqrt{2-\sqrt{2}}$.

4 Welfare analysis

Since Baye *et al.* (1996a), we are accustomed with the idea that profit incentives will induce a socially efficient level of divisionalization in duopoly, as in this case the increase in industry output - which, by definition, is procompetitive - may more than offset the replication of fixed costs (Baye *et al.*, 1996a). If the number of firms is higher than two, the second effect prevails and hinders welfare, much the same way as in entry models (see, e.g., Mankiw and Whinston, 1986). However, in Baye *et al.* (1996a) the good is homogeneous, and we are about to see that imperfect product substitutability makes a difference. The relevant welfare levels can be easily calculated substituting equilibrium outputs appearing in (6), (10) and (14) in the definition of the welfare function $SW = U - TC$ in which

$$U = a\left(\sum_{\ell=1}^{d_i} q_{i\ell} + \sum_{m=1}^{d_j} q_{jm}\right) - \frac{1}{2}\left[\sum_{\ell=1}^{d_i} q_{i\ell}^2 + \sum_{m=1}^{d_j} q_{jm}^2 + 2\left(\sum_{h=1}^{d_i} q_{ih} \sum_{\ell \neq h} q_{i\ell}\right.\right.$$

$$\left.\left.\sum_{h=1}^{d_j} q_{jh} \sum_{m \neq h} q_{jm} + s\left(\sum_{\ell=1}^{d_i} q_{i\ell}\right)\left(\sum_{m=1}^{d_j} q_{jm}\right)\right)\right] \tag{18}$$

is the utility function of the representative consumer, and $TC = (d_i + d_j) F$ is the total cost function of the industry. This procedure delivers

$$\begin{aligned} SW_{MM} &= \frac{4a^2(2+s)}{(3+2s)^2} - 4F \\ SW_{MS} = SW_{SM} &= \frac{a^2[59 - 4s(11 + s(1-s))]}{8(3-s^2)^2} - 3F \\ SW_{SS} &= \frac{a^2(3+s)}{(2+s)^2} - 2F \end{aligned} \tag{19}$$

with $SW_{SS} > 0$ for all $F \in \left(0, \widetilde{F}_{SS}\right)$, $\widetilde{F}_{SS} \equiv a^2(3+s) / \left[2(2+s)^2\right]$, and

$$\begin{aligned} SW_{SM} = SW_{MS} > SW_{SS} \forall F \in \left(0, \widetilde{F}_{SM}^{SS}\right), \widetilde{F}_{SM}^{SS} &\equiv \frac{a^2[3(15-8s) - 4s^2(5-2s)]}{8(3+2s)^2(3-s^2)^2} \\ SW_{MM} > SW_{SM} = SW_{MS} \forall F \in \left(0, \widetilde{F}_{MM}^{SM}\right), \widetilde{F}_{MM}^{SM} &\equiv \frac{a^2[4(5-3s) - s^2(5-4s)]}{8(2+s)^2(3-s^2)^2} \end{aligned} \tag{20}$$

with the intuitive ranking $\widetilde{F}_{SS} > \widetilde{F}_{SM}^{SS} \geq \widetilde{F}_{MM}^{SM} \geq 0$ for all $s \in (0, 1]$, entailing that having both firms multidivisional is socially efficient if and only if the fixed cost is lower than the lowest threshold, i.e., $F \in \left(0, \widetilde{F}_{MM}^{SM}\right)$. Moreover, $\widetilde{F}_{SM}^{SS}$ and $\widetilde{F}_{MM}^{SM}$ are decreasing and convex in s, unlike those describing firms' incentives in Matrix 1, the reason being that preference for variety boosts consumer surplus and expands the space for a socially efficient divisionalization, *coeteris paribus*. Thus far, the welfare analysis obviously implies that social preferences are monotone in F, as usual. However, while $\widetilde{F}_{SS} > F_{MM}$ and $\widetilde{F}_{MM}^{SM} \geq F_{MM}^{SM}$ for all $s \in (0, 1]$, F_{MM}^{SM} and F_{SM}^{SS} cross $\widetilde{F}_{MM}^{SM}$ at $s \cong 0.933$

and $s \cong 0.976$, respectively. This fact, which is represented in Figure 2 (in which $\widetilde{F}_{SS}$ does not appear), identifies regions $\{$A, B, C, D, E, F, G$\}$, whose inspection reveals the following:

Proposition 2 *Private and social preferences about divisionalization (or product proliferation) are reciprocally aligned only in regions A and G. In G, the unique equilibrium at the upstream stage is* (S, S), *which is privately and socially Pareto-efficient. In A, the unique equilibrium is* (M, M), *which is socially efficient (although inefficient for firms).*

In particular, the conflict between private and social preferences is solved if the fixed cost and product differentiation are both very low, in which case industry-wide divisionalization is socially welcome. Leaving aside region G, this alignment breaks up as soon as consumers' taste for variety becomes relevant enough. To grasp the reason, one may think of s being arbitrarily close to zero: in such a case, products are (almost) completely unrelated and firms operate as (almost) independent monopolists on two different markets; hence, resorting to divisions whose mandate is to act independently is not a sound idea. In order to acquire a significant role and give rise to an equilibrium featuring bilateral divisionalization (or franchising), products must be (or become) very close, if not necessarily perfect, substitutes.

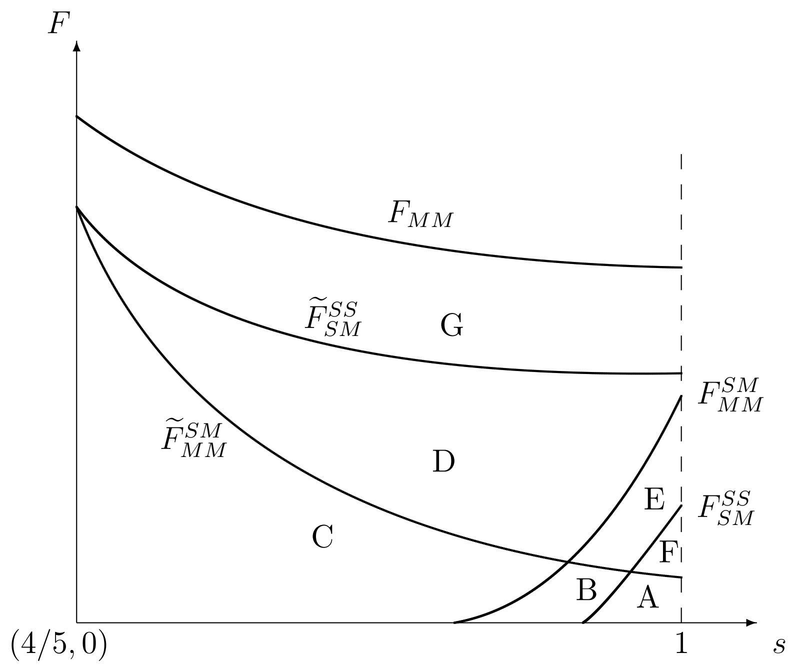

Figure 2 Private vs social preferences concerning divisionalization

We may now take a quick look at the remaining regions. In B and C, a public authority would like both firms to build up a second division, but this may happen only in mixed strategies (in region B), or cannot happen because (S, S) is the unique equilibrium (in region C). These two regions portray all situations in which the taste for variety and the resulting consumer surplus are strong enough to more than offset the replication of fixed costs and make the divisionalization of both firms socially desirable, but firms' incentives do not yield this outcome (whose attainment would require subsidising firms through a tax - possibly lump-sum - levied on consumers' income). In regions D, E and F it would be socially efficient to have a single divisionalised firm but this may happen probabilistically only in E, while in D and F firms end up ‘undersupplying’ and ‘oversupplying’ divisionalization, respectively.

5 Extension

To complement the foregoing analysis, we also illustrate the scenario in which firms operate either a single division or $d \geq 2$ divisions each, as this perspective permits one to single out an interesting property of the model.

In the case of industry-wide divisionalization, straightforward calculations deliver the Cournot-Nash equilibrium levels of outputs and parent firm's profits:

$$q_{MM}^{CN} = \frac{a}{1+d(1+s)} \, ; \, \pi_{MM}^{CN} = \frac{d\left[a^2 - F(1+d(1+s))^2\right]}{(1+d(1+s))^2} \tag{21}$$

with

$$\pi_{MM}^{CN} > 0 \forall F \in (0, F_{MM}) \, , \, F_{MM} \equiv \frac{a^2}{(1+d(1+s))^2} > 0 \tag{22}$$

The critical level F_{MM} in (22) implies that, if d were arbitrarily large, the choice of becoming multidivisional would become unfeasible, and the whole game we are investigating would be inadmissible, since $\lim_{d \to \infty} F_{MM} = 0$, for any finite value of the choke price a.

If firm i has d divisions while firm j has a single one, one obtains

$$q_{i1}^{CN} = q_{i2}^{CN} = q_{MS}^{CN} = \frac{a(2-s)}{2+d(2-s^2)} \, ; \, q_j^{CN} = q_{SM}^{CN} = \frac{a[1+d(1-s)]}{2+d(2-s^2)} \tag{23}$$

with $q_{SM}^{CN} \in \left(q_{MS}^{CN}, 2q_{MS}^{CN}\right)$, and

$$\begin{aligned} \pi_i^{CN} = \pi_{MS}^{CN} &= \frac{d\left[a^2(2-s)^2 - F(2+(2-s^2))^2\right]}{[2+d(2-s^2)]^2} \\ \pi_j^{CN} = \pi_{SM}^{CN} &= \frac{a^2[1+d(1-s)]^2 - F[2+(2-s^2)]^2}{[2+d(2-s^2)]^2} \end{aligned} \tag{24}$$

Moreover,

$$\pi_{MS}^{CN} > 0 \forall F \in (0, F_{MS}) \, , \, F_{MS} \equiv \frac{a^2(2-s)^2}{[2+d(2-s^2)]^2} \tag{25}$$

$$\pi_{SM}^{CN} > 0 \forall F \in (0, F_{SM}), \; F_{SM} \equiv \frac{a^2 [1 + d(1-s)]^2}{[2 + d(2 - s^2)]^2} \tag{26}$$

Obviously, the residual case in which both firms are single-division entities coincides with case (iii) in section 3.

The critical thresholds of the fixed cost F can be calculated anew, to find that the equivalent of Lemma 1 holds qualitatively unmodified, and

$$F_{MM}^{SS} = \frac{a^2 \left[1 - d(1+s)^2\right]}{(2+s)^2 [1 + d(1+s)]^2} < 0 \forall d \geq 2 \text{ and } s \in (0,1] \tag{27}$$

in which F_{MM}^{SS} is the critical level of the fixed cost beyond which $\pi_{SS}^{CN} > \pi_{MM}^{CN}$, and therefore define the *prisoner's dilemma condition*; moreover,

$$F_{MM}^{SM} = \frac{a^2 \left[1 + d - d^2(1 - 2s^2) + d^3 (1 - s^2)^2\right]}{[1 + d(1+s)]^2 [2 + d(2 - s^2)]^2} \tag{28}$$

$$F_{MS}^{SS} = \frac{a^2 \left[4 - d(2 - s^2)^2\right]}{(2+s)^2 [2 + d(2 - s^2)]^2} \tag{29}$$

While $F_{MM}^{SM} > 0$ provided d is finite and substitutability is sufficiently high, $F_{MS}^{SS} > 0$ if and only if $d \in \{1, 2, 3\}$. This is the consequence of the fact that $F_{MS}^{SS} \leq 0$ for all $d > 4/(2 - s^2)^2$, which increases in s and is equal to four at $s = 1$, i.e., under perfect substitutability.

This finding implies that, for any $d \geq 4$, region I in Figure 1 disappears, and therefore the prisoners' dilemma with (M, M) as an inefficient equilibrium of Matrix 1 cannot arise. Hence, there remain regions II and III, with the same equilibrium properties illustrated in Proposition 1. In turn, this entails that the alignment between private and social preferences only obtains in region G in Figure 2, in which (S, S) is the unique equilibrium both privately and socially efficient.[4] The conclusion emerging from this additional exercise can be summarised in the following remark, which indeed reflects the social inefficiency of replicating sunk costs:

[4] The details of calculations related to the welfare analysis with d divisions are omitted for the sake of brevity, as the relevant expressions of the critical thresholds of F pertaining to the welfare assessment keep the properties and signs illustrated in section 4.

Remark 1 *If firms contemplate the possibility of having $d \geq 4$ divisions each, the 2×2 upstream game is never a prisoners' dilemma, and this prevents the alignment of social and private preferences onto multidivisionalization.*

6 Concluding remarks

Relying on a duopolistic version of the model used by Ziss (1998), with divisionalization choices modelled in discrete strategies, we have shown that firms may decide not to become multidivisional, or may give rise to asymmetric outcomes in mixed strategies, due to the arising of multiple equilibria. As for the scenario in which the unique equilibrium involves industry-wide divisionalization, this is also socially efficient in a limited range of product differentiation (indeed, very close to perfect substitutability). This result qualifies a well known conclusion dating back to Baye *et al.* (1996a), obtained under the assumption of product homogeneity. In particular, a small degree of product differentiation may suffice to jeopardise the alignment between private and social preferences.

The parameter region wherein the choice concerning divisionalization is a prisoners' dilemma vanishes as soon as firms assess the possibility of setting up a symmetric number of divisions larger than two. This also implies that the alignment between private and social incentives may only arise in correspondence of the equilibrium outcome in which firms remain single-division entities.

Appendix

In order to characterise the Nash equilibrium in mixed strategies relevant for region II, we may assume that firm 1 (resp., 2) attaches probability $\mathfrak{p} \in [0,1]$ (resp., $\mathfrak{q} \in [0,1]$) to strategy S. Firm 1 must choose $\mathfrak{p}$ so as to make firm 2 indifferent between S and M, and the problem of firm 2 in choosing $\mathfrak{q}$ is analogous. The relevant expected profits for the two firms, calculated along columns or rows, are the following:

$$\begin{aligned} E\pi_2\left(S\right) &= \mathfrak{p}\pi_{SS}^{CN} + \left(1-\mathfrak{p}\right)\pi_{SM}^{CN} \\ E\pi_2\left(M\right) &= \mathfrak{p}\pi_{MS}^{CN} + \left(1-\mathfrak{p}\right)\pi_{MM}^{CN} \end{aligned} \tag{A1}$$

$$\begin{aligned} E\pi_1\left(S\right) &= \mathfrak{q}\pi_{SS}^{CN} + \left(1-\mathfrak{q}\right)\pi_{SM}^{CN} \\ E\pi_1\left(M\right) &= \mathfrak{q}\pi_{MS}^{CN} + \left(1-\mathfrak{q}\right)\pi_{MM}^{CN} \end{aligned} \tag{A2}$$

Solving the system $E\pi_i\left(S\right) = E\pi_i\left(M\right)$, $i = 1, 2$, we obtain

$$\mathfrak{p}^* = \mathfrak{q}^* = \frac{\pi_{MM}^{CN} - \pi_{SM}^{CN}}{\pi_{MM}^{CN} - \pi_{SM}^{CN} + \pi_{SS}^{CN} - \pi_{MS}^{CN}} \tag{A3}$$

which is positive and lower than one in region II.

The probabilities of observing each of the four possible outcomes are

$$\begin{aligned} P\left(S,S\right) &= \mathfrak{p}^*\mathfrak{q}^* = \left(\mathfrak{p}^*\right)^2 = \left(\mathfrak{q}^*\right)^2 \\ P\left(M,M\right) &= \left(1-\mathfrak{p}^*\right)\left(1-\mathfrak{q}^*\right) = \left(1-\mathfrak{p}^*\right)^2 = \left(1-\mathfrak{q}^*\right)^2 \\ P\left(M,S\right) = P\left(S,M\right) &= \left(1-\mathfrak{p}^*\right)\mathfrak{q}^* = \left(1-\mathfrak{q}^*\right)^2\mathfrak{p}^* \end{aligned} \tag{A4}$$

The last step consists in assessing the probability for firms to play one of the two Nash equilibria in pure strategies, $P\left(\mathcal{N}\right) = P\left(S,S\right) + P\left(M,M\right)$ against the probability of making a mistake, $P\left(\mathcal{M}\right) = P\left(M,S\right) + P\left(S,M\right)$, with

$$P\left(\mathcal{N}\right) - P\left(\mathcal{M}\right) = \frac{\left(\pi_{MM}^{CN} - \pi_{SM}^{CN} - \pi_{SS}^{CN} + \pi_{MS}^{CN}\right)^2}{\left(\pi_{MM}^{CN} - \pi_{SM}^{CN} + \pi_{SS}^{CN} - \pi_{MS}^{CN}\right)^2} > 0. \tag{A5}$$

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