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## STUDY OF THE DAILY IRREGULARITY ON SPECIFIC ROUTES, SERVICING THE PASSENGER STOPS IN RUSE BULGARIA

**Summary.** For this study, an analysis of a system of transport nodes of the railway and bus service transportation was carried out. A principal scheme of a transport node in typical cities for Bulgaria and a sample flow chart are presented. The categories of basic requirements for transport nodes in cities are also defined. On the basis of a railway system and bus transport nodes in the city of Ruse, Bulgaria, research on the interaction of the system with the mass urban passenger transport has been conducted. As a result, a mathematical model is proposed for the average daily inbound flows of passengers to a specific passenger stop, and a specific route has been modelled to be used as a server. An assessment of the daily irregularity on this route has been performed and a system has been modelled as a system of mass service for assessing the capacity and organization of work. The incoming passenger flows and the speed of passenger service have also been defined as a non-stationary Poisson flow. Under the conditions of non-stationarity flows, the basic values of the system parameters have been calculated and an application has been created in the MatLab platform. For a model of the system of mass service, the system of the differential-algebraic equations Kolmogorov/Erlangen for stochastic processes has been selected.

### 1. INTRODUCTION

The analysis of most of the transport nodes in the cities in Bulgaria shows that the conditions for transferring passengers through them do not meet a number of requirements such as the following:

- transport flow through the node is not rationalized;
- the flows of vehicles and passengers cross each other;
- the passengers are not protected from bad weather conditions;
- car parks have insufficient capacity;
- the optimal technological requirements for the operation of passenger transport are not met;
- the information technology for passengers is not sufficient, etc.

In many cities around the world, there are well-developed and optimized transport systems for different types of passenger transport.

An essential element of the European Union's policy is to focus all efforts on achieving optimal links between road, rail, air and water transport (by sea and inland waterways) [19].

In [20], it is stated that the optimal location and design of modal shift nodes for potential users involved in multimodal transfers in an urban and metropolitan context is one of the most important issues in transport system design and planning. Several algorithms have been developed to deal with this issue. The main idea was to locate such nodes in areas of adequate size, well connected to the road network and public transit to minimize the time required for the transfer, taking into

account the variation of transport demand in the various stages of the decision-making process (generation, distribution, modal choice and transfer route). The aim of the proposed research is to analyse the attractiveness towards passenger modal shift nodes in relation to their potential territorial role and the optional services they can offer, which are relevant in vast urban areas.

In [6], the authors present a library of classes implemented in Python, which could be used for computer simulations of public transfer nodes. The proposed software allows researchers to change technological parameters during simulation procedures and makes possible automatization of simulation experiments in the field of passenger transportation. Two cognitively different environments are distinguished in [9], which are called network space and space on the stage. The network space consists of the public transport network. Stage space consists of the environment at the nodes of the public transport system, through which passengers enter and leave the system and in which they change the means of transport. The work explores the properties of the two types of spaces and their interaction. A modelling capability is shown for the following: graphics can be used for network space, graphs can be used for scene space and we propose a novel model based on cognitive schemata and partial orders.

A number of authors are addressing these issues in their research [10-13, 22].

Despite the existing scientific results, the specificity of each of the transport links related to infrastructure, passenger flows, geographical location and development over the years necessitates a survey of each node and/or node system for planning its efficient operation [7].

## **2. GENERAL CHARACTERISTIC OF MULTIMODAL TRANSPORT NODES IN THE CITIES OF BULGARIA**

Urban transport nodes are part of the territory of a given settlement, where passenger transport is provided between different modes of passenger transportation and private transportation (Fig. 1).

Types of transportation from the National Transport Scheme are road, rail, air, rarely river and sea, Ordinances 2 and 33 [20, 21].

District and municipal schemes are implemented by bus transportation.

In Bulgaria, as a mass urban transport, the bus and trolleybus transportation are mainly developed in 10 cities: Sofia, Burgas, Varna, Ruse, Stara Zagora, Pleven, Haskovo, Sliven, Vratsa and Pazardzhik. Only the capital of the country Sofia is serviced by two more types of transportation: the tram and metro transportation. From and to public transport stops, people walk. Data on the speed of movement of people on foot are presented in [23].

Taxi service is well developed in the country, including a surplus of capacity to date.

After 1990, the use of automobiles in the country significantly increased, especially the use of private transport for transportation in the cities. Today shared travel ("car sharing") is quite common, whereby for a more comfortable journey, citizens create a group through social networks and travel together with one vehicle owned by one of them in a given direction. In the Republic of Bulgaria, its application is now growing, mainly among young people when travelling in big cities or between them.

The functional objective of each transport node in a given location is to provide maximum comfort and safety for the passengers in its territory, as well as to ensure maximum attractiveness of public transportation. This can be achieved by applying a system approach to:

- determining and analysing the structure of the transport system and its subsystems;
- determining the boundaries of the transport system to separate the object from the external environment and differentiate its internal and external connections;
- analysing the interconnected elements;
- building the system structure;
- establishing the functions of the system and its subsystems;
- coordinating the system's objectives with each of its subsystems, etc.

One of the main tasks for proper transport node planning is the construction of a flow chart that takes maximum account of the structure of connections in the transport node. When constructing the

Flow Diagram; it is necessary to consider the operation of each separate transport system in a given node and then the structure and their interconnection.

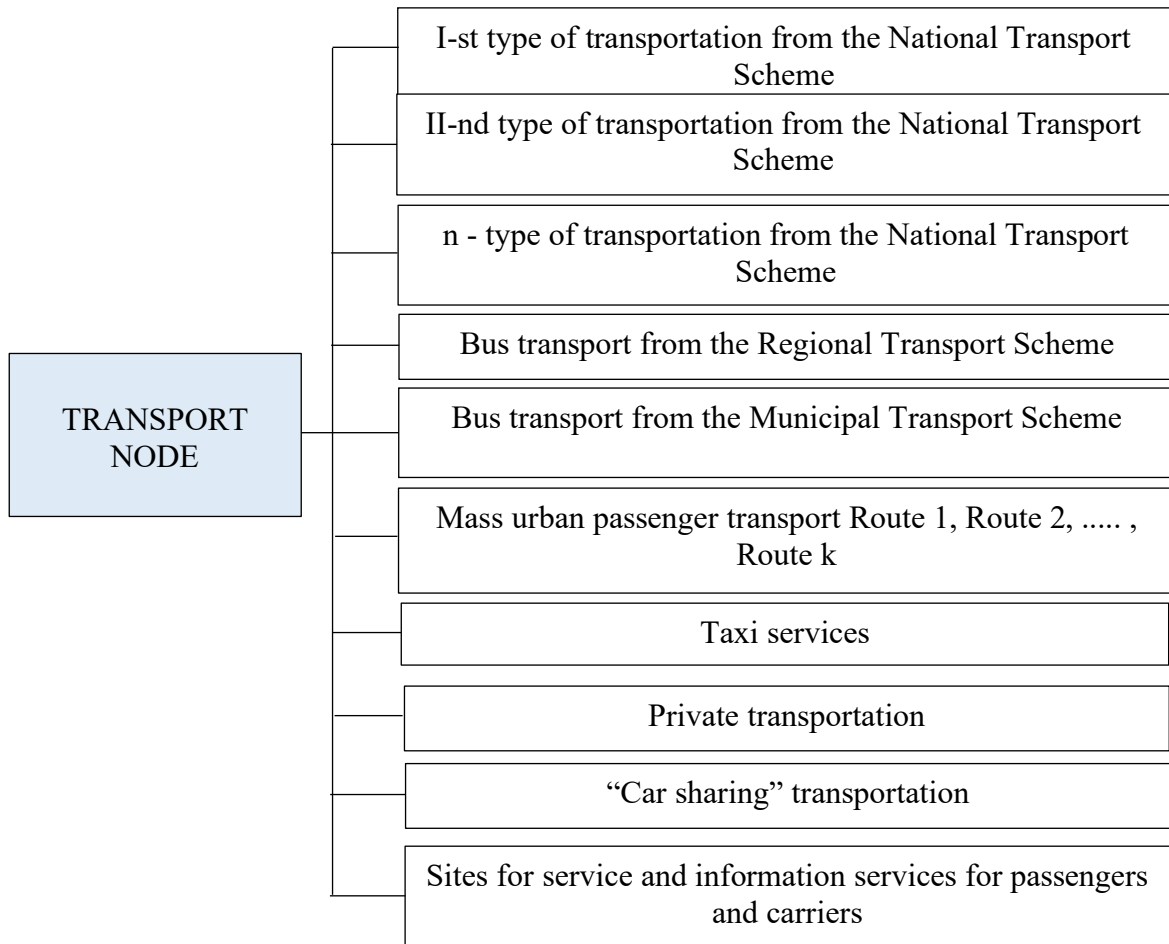


Fig. 1. A basic scheme of a transport node in a populated area

A sub layer of transport node is a transport node element in which not only communication functions are realized but also the accumulation of passengers with different travel purposes.

The connections in the transport node are the provision of communication functions of the node.

Of the flow chart shown in Fig. 2, the arrows pointing upward indicate the passengers arriving at the transport node and those pointing down indicate the departure from the node.

The model allows us to estimate the time of passenger flow in different schedules to assess the carrying capacity of each element of the transport node to ensure the normal transport of the passengers passing through the transport node with a minimum number of delays.

The main purpose of the transport nodes is to ensure that the passengers can rely on the appropriate quality and regulatory requirements and that the public transport system is their best choice for transportation.

The main requirements for urban transport nodes can be divided into three main categories:

- transportation;
- staff for realization of the main functions of the transport node; and
- technology (requirements to ensure efficient operation of the passenger transport system in the transport node).

The transport requirements are related to the following:

- optimization of the time for switching or changing the mode of transport: ensuring the interoperability of each of the elements of the transport node; lack of conflict points in pedestrian flows; and providing different technical means for faster and safer passenger movement;

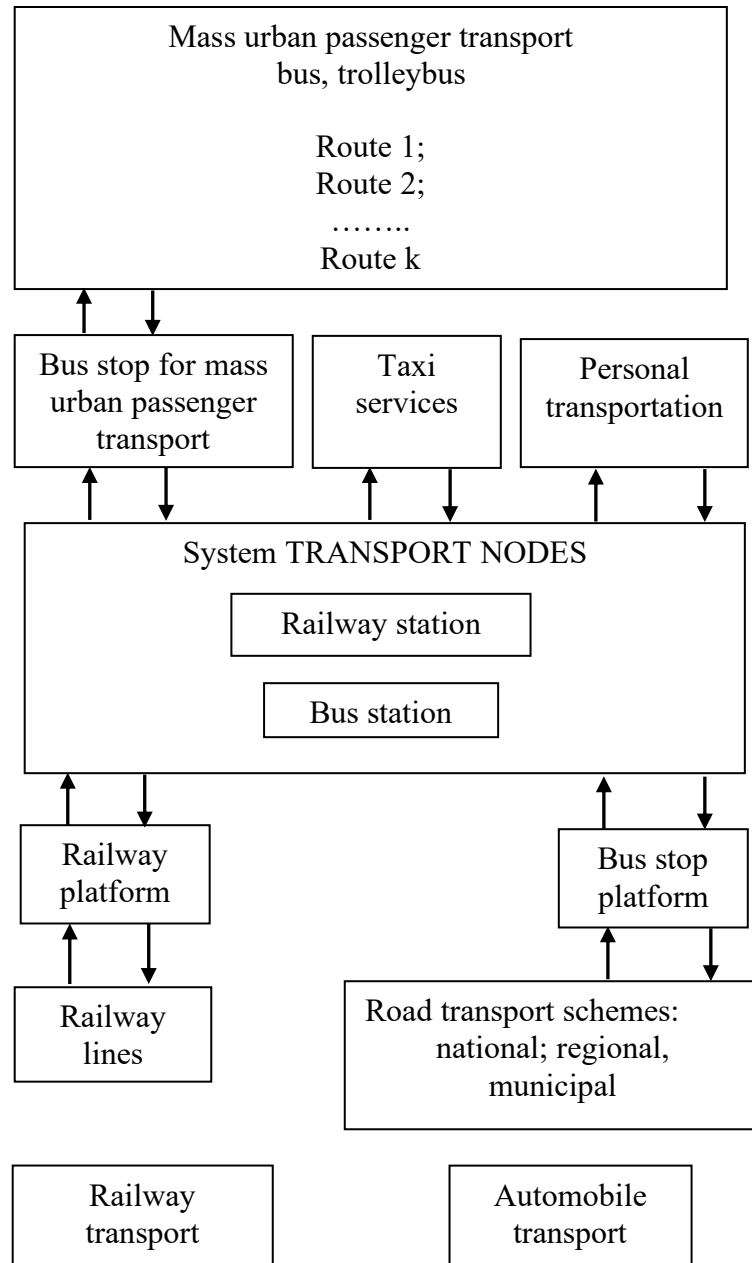


Fig. 2. Sample flow diagram of the transport node system consisting of a railway and bus station. Arrows pointing upward  $\uparrow$  indicate the arrival of passengers at the transport node and those pointing down  $\downarrow$  indicate the departure from the node

- confidence and comfort of passengers switching or changing the mode of transport: providing protection against bad weather conditions when moving between the main sections of the transport node; providing an information system with corresponding signboards and information for comfortable passenger navigation on the territory of the transport node; avoiding the intersection of transport and pedestrian flows to ensure the safety of passengers and the smooth movement of transport within the territory of the transport node; providing convenient pedestrianized

connections that are close to the transport node and are personalized for pedestrians; and provision of conditions for the mobility of disabled people;

- development of the transport network that is part of the transport node in terms of improvement of the carrying capacity and increasing the speed of movement of the flows; and
- provision of parking spaces for personal vehicles and taxi stops for taxis.

The technological requirements include the coordination of different types of transport by using modern methods of dispatching; implementation of navigation and computerized traffic management systems for public transport; the establishment of a single timetable for all modes of transport interacting in the transport nodes; optimization of transport traffic time in the vehicle (increase of the carriage capacity, creation of conditions for priority movement of the public urban passenger transport, provision of conditions for manoeuvring of the transport vehicles on the territory of the transport node and when entering and leaving it; and avoiding intersection of transport flows of public and private transport; placing technological devices on the territory of the transport node (separation of the platforms for embarking and disembarking passengers, technical and technological premises for ensuring the operation of different modes of transport, etc.).

The sites ensuring the realization of the main functions of the transport node are service sites; parking and taxi stands; and administrative and commercial sites.

### **3. ANALYSIS OF THE SYSTEM OF TRANSPORT NODES WITH MASS URBAN PASSENGER TRANSPORT (MUPT)**

Ruse is the largest Bulgarian port city on the Danube River and is the fifth largest city in Bulgaria. It is an administrative centre of Ruse region and the municipality of Ruse, as well as an economic, transport, cultural and educational centre of regional and national importance. The population of the town of Ruse is 142 902 (according to NSI data as of 31.12.2018).

The European roads E70 and E85 pass through the city of Ruse, as well as the following corridors: Main - Rhine - Danube; Helsinki - Kiev / Moscow - Odessa / Chisinau - Bucharest - Ruse - Stara Zagora – Dedeagac, the White Sea.

The transport facilities on the territory of the city of Ruse are the Central Railway Station; the Railway Yard Station; the Ruse Freight Train Station; the Ruse-North Freight Train Station; the Ruse-West Freight Train Station; the South Bus Terminal; the East Bus Terminal; Port of West; Port of East; the Ro-Ro Terminal; and Central Passenger Quay - Ruse. There are 26 operating lines in the city, including 8 trolleybus and 18 bus lines.

There are two systems of transport nodes in town Ruse: railway transport and bus transport, also including urban trolleybus transport and river passenger transport (which is not sufficiently developed):

- Central Railway Station and South Bus Terminal;
- Railway yard station and East Bus Terminal.

The study is based on the example of the node transport system at the Railway Yard Station and the East Bus Terminal in Ruse.

At this station 19 trains arrived, Fig. 3.

The total bus routes are 15, all of which are from the district and municipal transport circuits.

The distribution of the number of arriving and departing buses from East Bus Terminal, Ruse, during the day shift period is shown in Fig. 4, a variant of workdays that are busier than usual.

On tracking the logic for volume of passenger flows from the flow chart (Fig. 2), we can accept that the largest passenger traffic is directed to the stops of the mass passenger public transport, mainly from the pedestrian access zone near the respective stops. This is why we have studied the service of the passengers from the system of the two transport nodes – Railway Yard Station - Ruse and East Bus Terminal – Ruse from the mass urban passenger transport system.

Through the mass urban passenger transport system run trolley and bus routes.

To describe the operation mode of a particular route passing through a passenger stop as a mass service, it is necessary to know the characteristics of the incoming flow of orders to the particular

route considered as a stochastic process, the service intensity, the maximum allowable waiting time at the stop (queue line), as well as the number and type of service units (routes passing through the line), [2, 3].

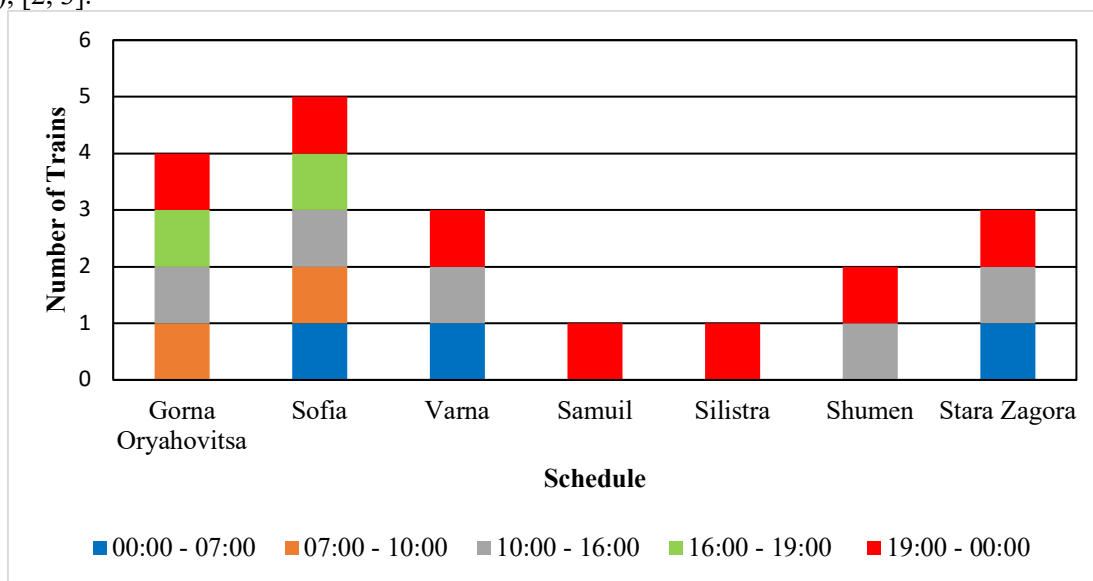


Fig. 3. The distribution of the number of arrivals from the Railway Yard Station - Ruse in the periods of the daily 24 hours

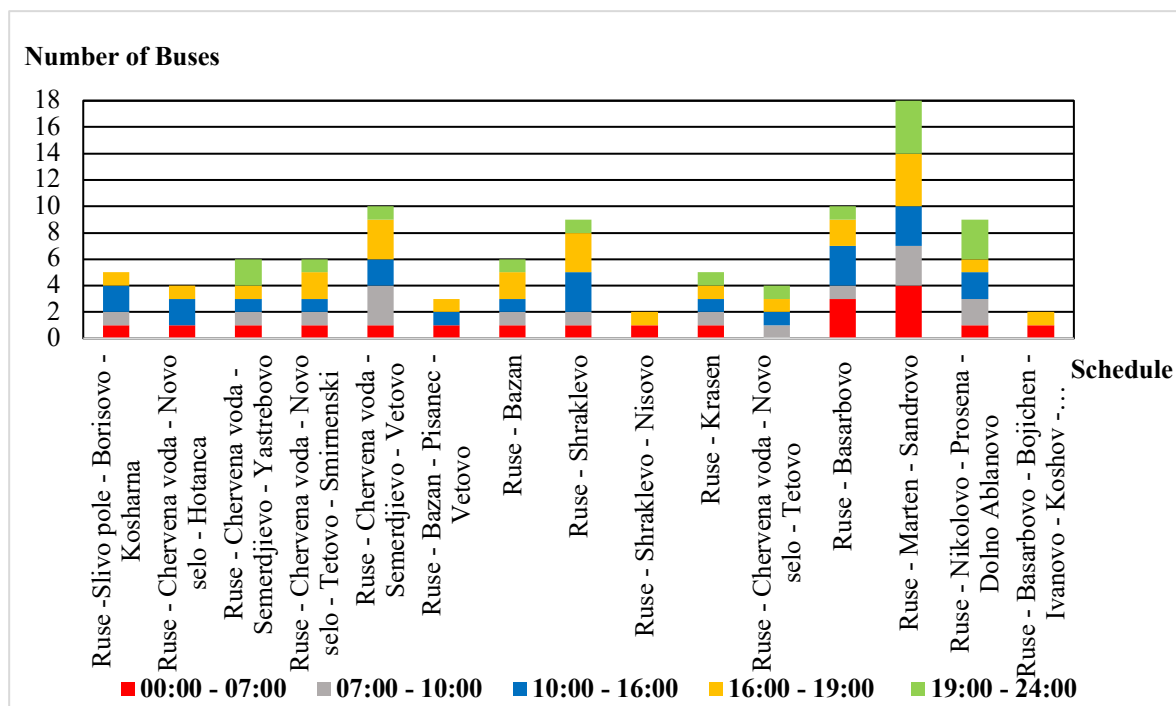


Fig. 4. Distribution of the number of arriving buses from the East Bus Terminal- Ruse on work days

For the incoming flow of cargo, the following prerequisites can be made:

- The flow is normal;
- The flow is without consequences (Poisson flow), [4].

- The flow is stationary / non-stationary - for sufficiently long periods of time - 1 month, 6 months, 1 year, etc.; it is possible to assume that there is a stationary incoming flow i.e. the probability of occurrence of a certain number of cars in a given, sufficiently long interval depends only on the length of that interval. Generally, at random intervals, the flow  $\lambda$  is non-stationary  $\lambda =$

$\lambda(t)$ . This non-stationary trend stands out well for 1 business day (24 hours). The incoming and outgoing flow is considered and modelled [5, 6, 14]. Depending on the measurements and observations made, an approximating function of the incoming flow density is determined. The same applies to service speed (outgoing flow).

The incoming flow  $\lambda(t)$  of travellers (requests) selecting a particular line at the given passenger stop is considered the sum of three independent flows  $\lambda(t) = \lambda_1(t) + \lambda_2(t) + \lambda_3(t)$ , where  $\lambda_1(t)$  is the incoming flow of passengers arriving from the railway station waiting for the specific line from the given stop.  $\lambda_2(t)$  and  $\lambda_3(t)$ , respectively, denote flows of passengers arriving from the bus terminal and passengers living in proximity to the bus stop wishing to use a particular bus line, respectively. Due to the fact that every traveller wants a specific line (which means that they cannot be serviced by a random line), a single-channel system with a server for the specific line should be considered. To describe the overall operation of the stop, it is necessary to map multiple independent single-channel mass service systems with servers for the respective routes from that stop, as well as the respective incoming flows to these routes. This paper discusses the work of only one particular line and the flow of requests to it.

The intensity of service  $\mu(t)$  can be inferred from the daily schedule of the respective route and from the capacity of the servicing unit (capacity of buses/trolley buses that are serving this route). It is also known that at a time when the server is busy, the order received is waiting for a certain time  $T_w$  to be served (waiting for the respective bus/trolley bus) and then it is rejected (the passenger is looking for another alternative way for travel). To investigate the system's operation, it is necessary to find the probability that the system will have the number of passengers  $k$  in the moment  $t$  for the particular server (route), i.e.

$$P_k(t) = ?, \quad k = \overline{0, \infty}, \quad t \in [0, T], \quad n = 1, \tag{1}$$

where a full day  $T = 0$  of 24 hours is taken as a unit of time  $T$ . The beginning of the working day  $t = 0$  coincides with the astronomical start of the day (0 hours and 0 minutes), the end of the working day - with the end of the astronomical day  $t = T = 24$  (24 hours and 0 minutes).

For the model of the system of mass service, the following can be summed up: for the mass service system with  $n = 1$  service channels, a non-stationary flow of requests with density  $\lambda(t)$  arrives. Request service time is of random quantity with an indicative distribution and service rate  $\mu(t)$ , also unstationary over time. The number  $n$  of serving channels is one. An order that arrives at a time when the channel is busy, waits for a time  $T_w$  and then leaves the system as unusable. The reciprocal value of the waiting time is  $\nu = 1/T_w$ , which is a constant over time.

Thus, the system is of the type  $(M / M / n)$  in a non-stationary mode. To describe a system of this type, the following set of differential equations of Kolmogorov (Erlang-Kolmogorov) [8, 14] is in effect:

$$\begin{aligned} \frac{dP_0(t)}{dt} &= -\lambda(t)P_0(t) + \mu(t)P_1(t) \\ &\dots\dots\dots \\ \frac{dP_k(t)}{dt} &= \lambda(t)P_{k-1}(t) - (\lambda(t) + k\mu(t))P_k(t) + \mu(t)(k+1)P_{k+1}(t) \\ &\dots\dots\dots \\ \frac{dP_n(t)}{dt} &= \lambda(t)P_{n-1}(t) - (\lambda(t) + n\mu(t))P_n(t) + (n\mu(t) + \nu)P_{n+1}(t) \\ &\dots\dots\dots \\ \frac{dP_{n+s}(t)}{dt} &= \lambda(t)P_{n+s-1}(t) - (\lambda(t) + n\mu(t) + s\nu)P_{n+s}(t) + (n\mu(t) + (s+1)\nu)P_{n+s+1}(t) \end{aligned} \tag{2}$$

The number of passengers in the queue when the server is busy is marked by  $s$ . The system of differential equations is infinite and for the numerical solution, it is necessary to take a finite number of equations. The number of equations is such that for a given number  $N$ , the sum  $\sum_{i=N+1}^{\infty} P_i(t)$  must not exceed the given number  $\varepsilon$ . In this case, a system of  $N = 502$  differential equations are solved as an error from the "cutting" and it has to be less than  $\varepsilon = 10^{-8}$ . After

selecting a finite number of equations, the number of probabilities  $P_k(t)$  is one more than the number of equations. This requires the introduction of the extra algebraic normality equation  $\sum_{i=0}^N P_i(t) = 1$ . The computational features associated with system (2) after "cutting" and introducing an algebraic equation are as follows:

- the system is large;
- the system is differential-algebraic; and
- the system is of the "stiff system" type.

This can be summarized as a system of stiff differential–algebraic equations with a large dimension. To overcome these difficulties, special numerical methods have been developed. A program in Matlab was developed for solving the system (2) by means of built-in "solvers" *ode 15 s*, embodying the method of Gir. When entering  $\lambda(t), \mu(t), N$ , the application returns the numerical solution of  $P_k(t)$ . When manually done, the precision is greater than the default for the solver - from  $10^{-6}$  up  $10^{-9}$  for absolute error and from  $10^{-3}$  to  $10^{-5}$  for relative error. The initial state of the system  $P_k(t_0)$  is unknown. It is known that these types of processes are persistent and after a long period of time, they enter into a regular mode of operation. Therefore, a random state may be taken arbitrarily. Integrating the system into the interval needs to be done not one time, but a sufficient number of times, and after each integration, the values at the end of the period become initial values for the next integration. In this way, the probabilistic functions  $P_k(t)$  begin to bend to their regular values. After multiple integration, it was found that only after 5-6 periods, the functions  $P_k(t)$  entered into regular mode (remain the same for two adjacent periods). Accuracy is also increased here, with integration being done for 20 periods, as the difference of all  $P_k(t)$  in the last and penultimate integration is less than  $10^{-8}$  for each  $t$ .

The incoming flow  $\lambda(t)$  is the sum of three independent flows:

$$\lambda(t) = \lambda_1(t) + \lambda_2(t) + \lambda_3(t) \quad (3)$$

where  $\lambda_1(t)$ - the incoming flow of passengers arriving from the railway station;  $\lambda_2(t)$ - the incoming flow of passengers arriving from the bus station;  $\lambda_3(t)$ - the flow of passengers who have not arrived from the railway station and the bus station but wish to use the specific line of the bus stop reviewed.

One good way to model incoming flows is to have a large enough historical database, to approximate by the smallest square method or by some other method, like the minimal method. Here, the flow  $\lambda_1(t)$  is modelled and analysed in the absence of sufficient historical data, but the approximate average of the total number of passengers  $A_1$ , arriving from the railway station and waiting for the particular line from the analysed bus stop, is known. It can be assumed that the proportion  $p_0, p_1, p_2, \dots, p_k$  of this number of passengers in specific time ranges is proportional to the number of trains arriving in the corresponding interval. For modelling  $\lambda_1(t)$ , it is advisable to select a relatively simple function with a periodicity (24 hours). The integral amount of  $\lambda_1(t)$  for a specific daytime interval must match the number of requests for that interval. One 24-hour day is divided into time intervals  $[t_i, t_{i+1}]$  ( $t_0$  corresponds to 0 hours and 0 minutes), for which the average number of passengers is known. One such distribution of  $A_1 = 450$  is given in Table 1.

For a model of  $\lambda_1(t)$ , the five-member Fourier line with a 24-hour periodicity was chosen [1]:

$$\lambda_1(t) = a_0 + a_1 \cos\left(\frac{2\pi t}{24}\right) + b_1 \sin\left(\frac{2\pi t}{24}\right) + a_2 \cos\left(\frac{4\pi t}{24}\right) + b_2 \sin\left(\frac{4\pi t}{24}\right) \quad (4)$$

The coefficients  $a_0, a_1, b_1, a_2, b_2$ , selected so that the integral amount  $\lambda_1(t)$  for specific interval of the day  $[t_i, t_{i+1}]$ , match the number of orders  $p_i A_i$  for the interval:

$$\int_{t_i}^{t_{i+1}} \lambda_1(t) dt = p_i A_i, \quad i = \overline{0, 5}. \quad (5)$$

Conditions (5) lead to the following system of linear algebraic equations in relation to coefficients  $a_0, a_1, b_1, a_2, b_2$ :

$$\begin{aligned} a_0 \int_{t_i}^{t_{i+1}} dt + a_1 \int_{t_i}^{t_{i+1}} \cos\left(\frac{2\pi t}{24}\right) dt + b_1 \int_{t_i}^{t_{i+1}} \sin\left(\frac{2\pi t}{24}\right) dt + a_2 \int_{t_i}^{t_{i+1}} \cos\left(\frac{4\pi t}{24}\right) dt + \\ + b_2 \int_{t_i}^{t_{i+1}} \sin\left(\frac{4\pi t}{24}\right) dt = p_i A_i, \quad i = \overline{0, 4}. \end{aligned} \quad (6)$$



The coefficients of (4) after the solution of system (6) are  $a_0=18,75$ ;  $a_1=3,58$ ;  $b_1=-7,45$ ;  $a_2=8,98$ ;  $b_2=-8,15$ .

Table 1

Average number of requests for the scheduled 24-hour intervals for trains

Interval of daytime $[t_i, t_{i+1}]$ with beginning and end $t_i^{-th}$ and $t_{i+1}^{-th}$ hour	Part of $p_i$ of incoming flow for the time interval of $[t_i, t_{i+1}]$ , proportional to arriving trains	Average number of requests $p_i A_1$ for the time interval of $[t_i, t_{i+1}]$
[0,7]	3/19	71.05
[7,10]	2/19	47.36
[10,16]	5/19	118.42
[16,19]	2/19	47.36
[19,24]	7/19	165.78
Total	19	450

Like in Table 1, Table 2 gives the incoming flow portion  $p_i$  for a time period  $[t_i, t_{i+1}]$  proportional to buses arriving, the average number  $p_i A_2$  of time interval requests for a period of  $[t_i, t_{i+1}]$ , with  $A_2 = 630$ .

Table 2

Average number of requests for the specified intervals on buses

Interval of daytime $[t_i, t_{i+1}]$ with beginning and end $t_i^{-th}$ and $t_{i+1}^{-th}$ час	Part of $p_i$ of incoming flow for the time interval of $[t_i, t_{i+1}]$ , proportional to arriving trains	Average number of requests $p_i A_1$ for the time interval of $[t_i, t_{i+1}]$
[0,7]	0/98	0
[7,10]	24/98	154.28
[10,16]	23/98	147.85
[16,19]	31/98	199.28
[19,24]	20/98	128.57
Total	98	630

In the 24-hour interval between  $0^{00}$  and  $7^{00}$ h, there are no buses; this means that the incoming flow  $\lambda_2(t)$  in this interval is zero. In this case, the interval between  $7^{00}$  and  $24^{00}$ h is considered. At this time, the flow  $\lambda_2(t)$  will be interpolated with a third-degree polynomial, and the flow is zero during the rest of the day, i.e.:

$$\tilde{\lambda}_2(t) = \begin{cases} 0, & 0 \leq t < 7 \\ a_3 t^3 + a_2 t^2 + a_1 t + a_0, & 7 \leq t < 24 \end{cases} \quad (7)$$

Conditions for the incoming flow volumes are placed in the middle of the time intervals, the value of  $\tilde{\lambda}_2(t)$  being  $p_i A_2$ :

$[t_i, t_{i+1}] / 2$ , the value of  $\tilde{\lambda}_2(t)$  to be  $p_i A_2$ :

$$\tilde{\lambda}_2((t_i, t_{i+1}) / 2) = p_i A_1, \quad i = \overline{1, 4}. \quad (8)$$

The conditions of (8) are conditions for constructing a Third-Grade Lagrangian polynomial, which has the form:

$$\tilde{\lambda}_2(t) = \frac{\tilde{\lambda}_2(t_1^*)(t-t_2^*)(t-t_3^*)(t-t_4^*)}{(t_1^*-t_2^*)(t_1^*-t_3^*)(t_1^*-t_4^*)} + \frac{\tilde{\lambda}_2(t_2^*)(t-t_1^*)(t-t_3^*)(t-t_4^*)}{(t_2^*-t_1^*)(t_2^*-t_3^*)(t_2^*-t_4^*)} + \frac{\tilde{\lambda}_2(t_3^*)(t-t_1^*)(t-t_2^*)(t-t_4^*)}{(t_3^*-t_1^*)(t_3^*-t_2^*)(t_3^*-t_4^*)} + \frac{\tilde{\lambda}_2(t_4^*)(t-t_1^*)(t-t_2^*)(t-t_3^*)}{(t_4^*-t_1^*)(t_4^*-t_2^*)(t_4^*-t_3^*)} \quad t \in [7, 24]. \quad (9)$$

where  $t_i^* = (t_i + t_{i+1}) / 2$  is the middle of the interval  $[t_i, t_{i+1}]$ . After replacing and simplifying (9), we obtain:

$$\tilde{\lambda}_2(t) = \begin{cases} 0, & 0 \leq t < 7 \\ -0.3733t^3 + 15.9873t^2 - 213.8481t + 1046.1607, & 7 \leq t < 24. \end{cases} \quad (10)$$

Built like that the Lagrange polynomial better reflects the dynamics of change at different intervals of the day, but the integral amount of volume  $\int_{t_1}^{t_5} \tilde{\lambda}_2(t)dt$  is not equal to  $A_2$  ; it requires the selection of a normalization coefficient  $\tilde{C}$  , with:

$$\tilde{C} \int_{t_1}^{t_5} \tilde{\lambda}_2(t)dt = A_2. \quad (11)$$

Then, the inbound flow after normalization takes the form of:

$$\lambda_2(t) = \tilde{C} \tilde{\lambda}_2(t). \quad (12)$$

In this case,  $\tilde{C} = 0.17854$ .

We finally have for  $\lambda_2(t)$ :

$$\lambda_2(t) = \begin{cases} 0, & 0 \leq t < 7 \\ 0.17854(-0.3733t^3 + 15.9873t^2 - 213.8481t + 1046.1607), & 7 \leq t < 24. \end{cases} \quad (13)$$

To determine the incoming flow  $\lambda_3(t)$ , we do the same thing as with  $\lambda_1(t)$ , by choosing the same model and method of calculating the coefficients in the model. The necessary values for determining the coefficients in the model are given in Table 3, with  $A_3 = 480$ .

Table 3

Average number of requests for scheduled passenger intervals from the area

Interval of daytime $[t_i, t_{i+1}]$ with beginning and end $t_i^{-th}$ and $t_{i+1}^{-th}$ hour	Part of $p_i$ incoming flow for the time interval of $[t_i, t_{i+1}]$ , proportional to arriving trains	Average number of requests $p_i A_1$ for the time interval of $[t_i, t_{i+1}]$
[0,7]	0.2	96
[7,10]	0.14	67.2
[10,16]	0.24	115.2
[16,19]	0.13	62.4
[19,24]	0.12	57.6

Incoming flow coefficients:

$$\lambda_3(t) = a_0 + a_1 \cos\left(\frac{2\pi t}{24}\right) + b_1 \sin\left(\frac{2\pi t}{24}\right) + a_2 \cos\left(\frac{4\pi t}{24}\right) + b_2 \sin\left(\frac{4\pi t}{24}\right). \quad (14)$$

are  $a_0 = 22,45$ ;  $a_1 = -8,44$ ;  $b_1 = 0,41$ ;  $a_2 = -6,08$ ;  $b_2 = -1,08$ .

The density graphs of the three types of flows  $\lambda_1(t), \lambda_2(t), \lambda_3(t)$  are given in Fig. 5, as well as the cumulative flow  $\lambda(t) = \lambda_1(t) + \lambda_2(t) + \lambda_3(t)$ .

An approximated speed of service is given in Fig. 6. The chosen model is not very accurate, but it reflects the behaviour of the speed (service density).

When modelling the service intensity ( $t$ ), it is necessary to know the schedule and capacity of the vehicles on the route. In this case, a route is observed, and it is shown that it is served by one type of transport vehicle with a maximum service capacity of 30 passengers. The maximum number of passengers who can be serviced for 24 hours is  $M = 2940$ . The number of vehicles and their maximum capacity for a given daytime interval are given in Table 4.

The servicing happens in moments  $t_j^*$  of time in which up to 30 passengers are serviced at a time. More precisely, for a time interval  $\Delta t$  , theoretically,  $\Delta t \rightarrow 0$  , but in practice, this interval is never 0 and it is negligible in comparison to the 24-hour day. More precise modelling takes place with the so-called " Dirac delta function -  $\delta$  " on the right-hand side of (2). The presence of  $\delta$  functions in the right-hand part leads to serious computational difficulties for system (2). Standard numerical methods are not suitable; therefore, more specific numerical methods are constructed [16, 17].

To avoid this moment, the speed of service  $\mu(t)$  is modelled in such a way that for each environment of subinterval  $[t_i, t_{i+1}]$ , the values are approximated in the smallest square (may be approximated by other methods, such as the Minimaks method). A periodic function of this type is selected for a model again:

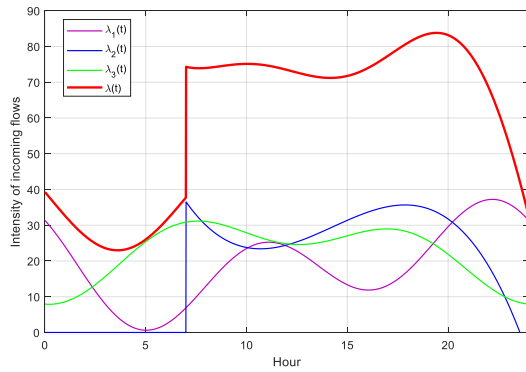


Fig. 5. The densities of the three types of flows  $\lambda_1(t), \lambda_2(t), \lambda_3(t)$ , as well as the cumulative flow  $\lambda(t) = \lambda_1(t) + \lambda_2(t) + \lambda_3(t)$

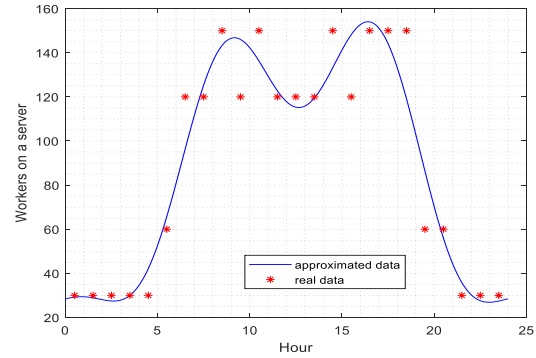


Fig. 6. Density of passenger service from buses passing through the bus stop

Table 4

Number of vehicles and maximum capacity for a given interval of the day

Interval of daytime $[t_i, t_{i+1}]$ with beginning and end $t_i^{-th}$ and $t_{i+1}^{-th}$ hour	Part of $p_i$ of inbound flow for the time interval of $[t_i, t_{i+1}]$ , proportional to arriving trains	Average number of requests $p_i A_1$ for the time interval of $[t_i, t_{i+1}]$
[0,1]	1	30
[1,2]	1	30
[2,3]	1	30
[3,4]	1	30
[4,5]	1	30
[5,6]	2	60
[6,7]	4	120
[7,8]	4	120
[8,9]	5	150
[9,10]	4	120
[10,11]	5	150
[11,12]	4	120
[12,13]	4	120
[13,14]	4	120
[14,15]	5	150
[15,16]	4	120
[16,17]	5	150
[17,18]	5	150
[18,19]	5	150
[19,20]	2	60
[20,21]	2	60
[21,22]	1	30
[22,23]	1	30
[23,24]	1	30

$$\begin{aligned} \tilde{\mu}(t) = & a_0 + a_1 \cos\left(\frac{2\pi t}{24}\right) + b_1 \sin\left(\frac{2\pi t}{24}\right) + \\ & + a_2 \cos\left(\frac{4\pi t}{24}\right) + b_2 \sin\left(\frac{4\pi t}{24}\right) + a_3 \cos\left(\frac{6\pi t}{24}\right) + b_3 \sin\left(\frac{6\pi t}{24}\right). \end{aligned} \quad (15)$$

Because it is approximated, just as in the flow  $\lambda_2(t)$ , a normative condition for the whole volume is imposed:

$$\tilde{C} \int_{t_1}^{t_5} \tilde{\mu}(t) dt = M. \quad (16)$$

For  $\mu(t)$ , we have:

$$\begin{aligned} \mu(t) = & 1.3611(90 - 60.4475 \cos\left(\frac{2\pi t}{24}\right) - 15.0990 \sin\left(\frac{2\pi t}{24}\right) - 16.9037 \cos\left(\frac{4\pi t}{24}\right) + \\ & - 5.4767 \sin\left(\frac{4\pi t}{24}\right) + 15.7716 \cos\left(\frac{6\pi t}{24}\right) + 11.1522 \sin\left(\frac{6\pi t}{24}\right). \end{aligned} \quad (17)$$

In the following figures (Figs. 7 - 9), the results of the system solution (2) are reflected in the waiting queue time  $T_w = \frac{1}{3}h = 20$  min, i.e. the reciprocal wait time  $\nu = \frac{1}{T_w} = 3$ . From Fig. 7, it follows that the probability of no traveller at the stop is greater for the whole day, with the probability of about 7 hours being over 70%.

The probability of a passenger number between 4 and 7 in the observed stop is shown in Fig. 8. Most likely, over 6% is observed around 7 or 8 p.m.

The probability of having exactly 10, 20, 30 or 40 passengers is the greatest, over 3%, after 8 p.m. and is presented in Fig. 9. This is due to the sharp reduction in the number of vehicles serving public transport. When analysing the waiting time of the passengers at the stop during the 24-hour day, Fig. 10, the results show that between 5 a.m. and 7 p.m., the waiting time of the passengers is within the permissible waiting period. Outside this interval, if passengers wish to be serviced, they have to wait a lot longer (from 9 p.m. to 4 a.m.) and they will actually choose an alternative.

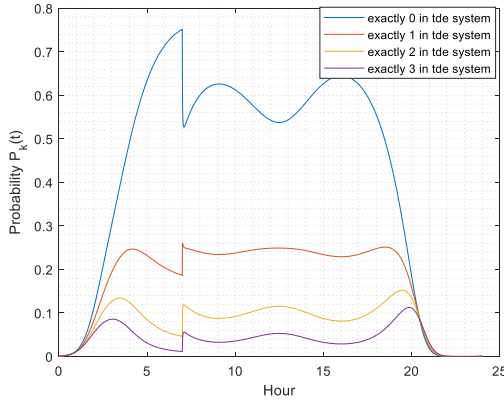


Fig. 7. The probability of having exactly  $k \in [0, 3]$  seated in the system

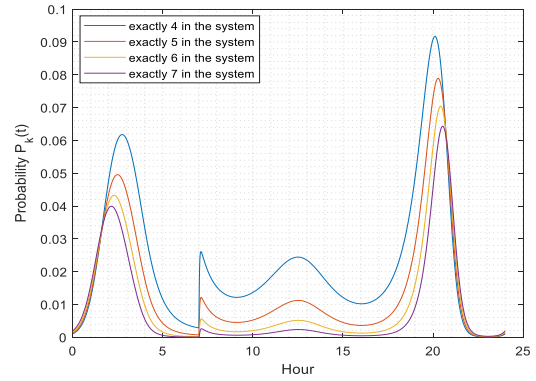


Fig. 8. The probability of having exactly  $k \in [4, 7]$  seated in the system

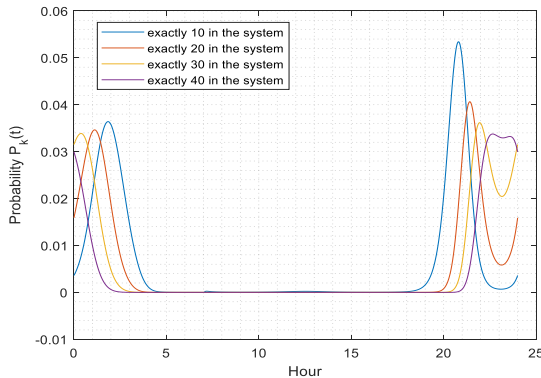


Fig. 9. The probability of having exactly  $k \in \{10, 20, 30, 40\}$  seated in the system

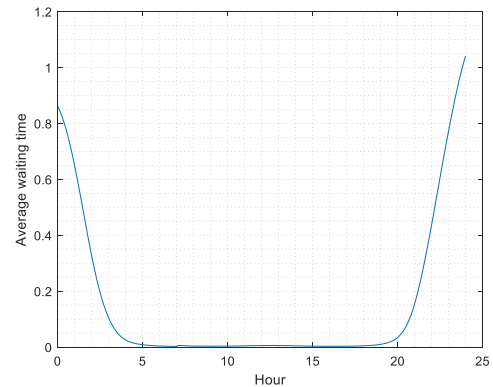


Fig. 10. Average waiting time for one passenger at the stop every day

The average number of passengers who wish to find an alternative way is presented in Table 5. For the specified intervals between: 0:00 and 7:00 a.m., the passengers who have given up waiting are 31,43%; from 7:00 a.m. to 7 p.m. they are 1.64%; and from 7 p.m. to 12:00 a.m. they are

66.91%. To accommodate passengers who have not been serviced, it is necessary to provide public transport vehicles with a timetable between 7 p.m. and 7 a.m. This good practice has been organized in Sofia and since May 2019 in Ruse. Under the project "CIVITAS ECCENTRIC – Innovative solutions for sustainable mobility of people in the suburban city districts and emission free freight logistics in urban centers", in the city of Ruse a trolley bus line has been established that runs at an interval of one hour at night to accommodate customers [22].

Table 5

Average number of non-served queries for a given time interval in the day

Interval of daytime $[t_i, t_{i+1}]$ with beginning and end $t_i^{-th}$ and $t_{i+1}^{-th}$ hour	Average number of undelivered queries
[0,7]	49,233
[7,10]	0,817
[10,16]	1,773
[16,19]	1,017
[19,24]	104,804

## CONCLUSION

The service of passengers arriving at one stop of rail and bus terminals and passengers living near the area of the passenger stop of the public transport for one route has been analysed. Passenger incoming flows are separated individually and ultimately merged into a single incoming flow.

When analysing the incoming flow of passengers at a given stop from mass urban transit, it can be assumed that the flow is ordinarily uninterrupted (Poisson flow) and, in general, is non-stationary due to uneven arrivals of passengers during the day.

At the speed of service of the passengers on the route considered from the stop, the most characteristic features are the range of traffic of the vehicles serving the route, as well as their capacity. Therefore, mathematical models with twenty-four hour periodicity have been proposed that describe the total incoming flow of passengers and the service speed accordingly. As a result of the specifics of waiting time for passengers, the system is considered to be a mass service system with errors at a time that is described by a system of differential–algebraic equations. To solve these equations, a program was created in Matlab software.

After solving the model system differential–algebraic equations, all probabilities have been obtained that indicate the probability  $P_k(t)$  of having exactly  $k$  number of passengers in the system at time  $t$  during the day. These probabilities have determined the average waiting time of one passenger per day and the average number of passengers who have given up waiting, specified five subintervals. The results show that between 5 a.m. and 7 p.m., the waiting time for passengers is within the permissible range. Outside of this interval, if passengers wish to be serviced, they have to wait a lot longer (after 9 p.m. and until 4 a.m.) and they would actually give up waiting for the service offered by mass urban passenger transport by choosing an alternative. For the specified intervals between 12 a.m. and 7 a.m., the passengers who have given up waiting are 31,43%; from 7 a.m. to 7 p.m. for the three intervals are the total of 1.64%; and from 7 p.m. to 12 a.m., those who have given up are 66.91%. For the satisfaction of those passengers, it is necessary to provide additional vehicles for mass urban passenger transport between 7 p.m. and 7 a.m.

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