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Complex Valued Graphs for Soft Computing

Florentin Smarandache
University of New Mexico, smarand@unm.edu

W.B. Vasantha Kandasamy

Ilanthenral K

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K. ILANTHENRAL
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PREFACE

In this book authors for the first time introduce in a systematic way the notion of complex valued graphs, strong complex valued graphs and complex neutrosophic valued graphs. Several interesting properties are defined, described and developed. Most of the conjectures which are open in case of usual graphs continue to be open problems in case of both complex valued graphs and strong complex valued graphs.

We also give some applications of them in soft computing and social networks. At this juncture it is pertinent to keep on record that Dr. Tohru Nitta was the pioneer to use complex valued graphs in neural networks in particular and soft computing in general. However in this book authors define and develop mathematical models using the complex valued directed graphs akin to Fuzzy Cognitive Maps model and Fuzzy Relational Maps models.

Further it is pertinent to record in real world problems the values or impulse or synaptic relations can be imaginary or indeterminate or both. These situations can be studied using complex valued graphs or strong complex valued graphs or neutrosophic complex valued graphs.

Thus using complex valued graphs one can construct the Fuzzy Complex Cognitive Maps model and using complex valued bigraphs we can construct Fuzzy Complex Relational Maps model analogous to FCMs model and FRMs model respectively. Likewise for other complex neutrosophic valued graphs. Authors feel this book will be a boon to researchers in computer science and social sciences.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

W.B.VASANTHA KANDASAMY
ILANTHENRAL K
FLORENTIN SMARANDACHE

Chapter One

INTRODUCTION

In this book authors for the first time introduce the notion of complex valued graphs. When we say complex valued graphs we simply mean that at least one of the edge weights must be a complex value. Study in this direction is innovative and interesting. Several properties of complex valued graphs are derived and developed.

Further if G is a complex valued graph and if some of its vertices are also complex then we define the graph to be strong complex valued graph.

The following facts are very important.

In the first place we know the only closed algebraic field is the field of complex numbers C. So even if our equations are to solved we need basically $C = \{a + bi / a, b \in R, i^2 = -1\}$.

It is pertinent to record that researchers have presented a novel approach for the simultaneous modeling and forecasting of wind where by the wind field is considered as a vector of its speed and direction components in the field of complex numbers. They have recently introduced framework of augmented complex statistics. Augmented Complex Least Mean Squares (ACLMS) algorithm is introduced and its usefulness in wind forecasting is analyzed in 2017 [6].

Further augmented CRTRL for complex-valued recurrent neural networks was introduced in 2007 [5]. However to ones surprise we are not in a position to find a systematic development of complex valued graphs.

Still it is interesting to note the notion of complex valued neural networks which is defined as an extension of real valued neural networks have already been introduced in [2]. They have developed in [2] the singularity and its effect on learning dynamics in the complex valued neural networks. They have obtained simulation results on learning dynamics of the three layered real valued and complex valued neural networks in the neighbourhood of singularities which supports the analytical results.

A modified error back propagation algorithm for complex valued neural networks introduced in [10] is one of the early papers published work on this topic.

They have listed the inherent properties of complex valued neural networks.

1. Ability to learn 2-dimensional affine transformations.

The complex-valued neural network can transform geometric figures, for instance rotation, similarity transformation and parallel displacement of straight lines, circles etc.

Examples of the applications of the ability to learn 2-dimensional affine transformations

a) Application to the estimation of optical flows in the computer vision. Please refer [4].
b) Application of the generation of fractal images. [4]

2. Orthogonality of decision boundaries. A decision boundaries of a complex valued neural network basically consists of two hyper surfaces that intersect orthogonally and divides a decision region into four equal sections. Several problems that cannot be solved with a single real-valued neuron can be solved with a single complex-valued neuron using the orthogonal property.

3. Structure of critical points. The critical points (satisfying a certain condition) of the complex - valued neural network with one output neuron caused by the hierarchical structure are all saddle points not local minima, unlike the real valued case where a critical point is a point at which the derivative of the loss function equal to zero. [7]

Though there is some sort of soft computing done with complex valued neural networks still one is surprised to see any form of systematic development about complex valued graphs is absent in literature.

Here we define and develop complex valued graphs in a very systematic way.

Further we realize in the working of the human brain in the neuron anatomy and comparing with the Artificial Neural Network here the term synapse is only a signal it can be at the inactive state, active state where some real signal passes and action takes place or it can be an imaginary signal where some imaginary information is processed. It is unfortunate that till date such imaginary signal and its processing is not described in artificial neural networks. However with the exception of Dr. Tohru Nitta [9-11] who has analyzed such models though he has not represented them explicitly.

End of the day when we say the synapse is imaginary it does not mean the signal between two neurons is imaginary it only means the information passed from two or more neurons are imaginary; we do not say the connecting synapse to be imaginary.

Since to some extent there are use of complex values in ANN like Back propagation and so on we do not intended to give applications of complex valued graphs to soft computing in that direction. However as no development of complex valued graphs are mentioned in the systematic way in this book complex valued directed graphs and complex valued bigraphs are defined in Chapter II and they are used in the construction of new Fuzzy Imaginary Cognitive Maps Models and Fuzzy Imaginary Relational Maps Models. This is explained in Chapter III of this book.

Thus the notion of complex valued graphs can play a vital role in soft computing of the unsupervised data provided the problem under investigation involves an amount of imaginary concepts or relations we can use these complex valued graphs and obtain these models.

So these complex valued graphs can play a vital role in soft computing under the condition the problem involves some imaginary concepts and relations like the medical diagnostics of a hypochondria patient or the finding symptoms and decease model of a hypochondria patient.

Finally in the last chapter of this book we have introduced the notion of complex neutrosophic valued directed graph and complex neutrosophic valued bigraphs. It is further proved if any problem in hand has some indeterminate as well as complex values (that is imaginary) then we can using these graphs analogously define some new models.

Thus we have defined Fuzzy Complex Neutrosophic Cognitive Maps (FCNCMs) model or Fuzzy Imaginary Neutrosophic Cognitive Maps (FINCMs) model so we can replace imaginary by complex and vice versa.

Similarly we have defined using the fuzzy complex neutrosophic valued bigraph the new model viz. Fuzzy Complex Neutrosophic Relational Maps model (FCNRMs model). This model will be appropriate when the nodes / concepts associated with the problem can be separated into two distinct classes and the concepts and relations involve both imaginary and indeterminate notions.

Study in this direction is carried out in the last chapter. Finally for these new models we have defined the notion of combined disjoint Fuzzy Complex (Imaginary) Cognitive maps model and Combined Disjoint Fuzzy Complex Neutrosophic Cognitive Maps model.

Further Combined Overlap Fuzzy ICMs model and Combined Overlap Fuzzy Complex Neutrosophic Relational Maps model are defined and described.

Thus for the notions of Fuzzy Cognitive Maps, Fuzzy Relational Maps and their neutrosophic analogue please refer [15]. For neutrosophic logic refer [12, 13]. For combined disjoint FCMs and combined overlap FCMs refer [18].

Chapter Two

COMPLEX VALUED GRAPHS AND THEIR PROPERTIES

In this chapter for the first time authors introduce the new notion of complex valued graphs in a systematic way and give their applications. Throughout this chapter C denotes the complex numbers, that is

$$C = \{a + bi \,/\, a, b \in Z \text{ or } Q \text{ or } R \text{ with } i^2 = -1\}.$$

It is important to note only the edge weights of these graphs are complex values.

Further these structures will find applications in ANN, Fuzzy Cognitive Maps models and Fuzzy Relational Maps models.

First we give examples of them.

Example 2.1. Let G = {V, E} be the graph with edge weights from C given by the following figure;

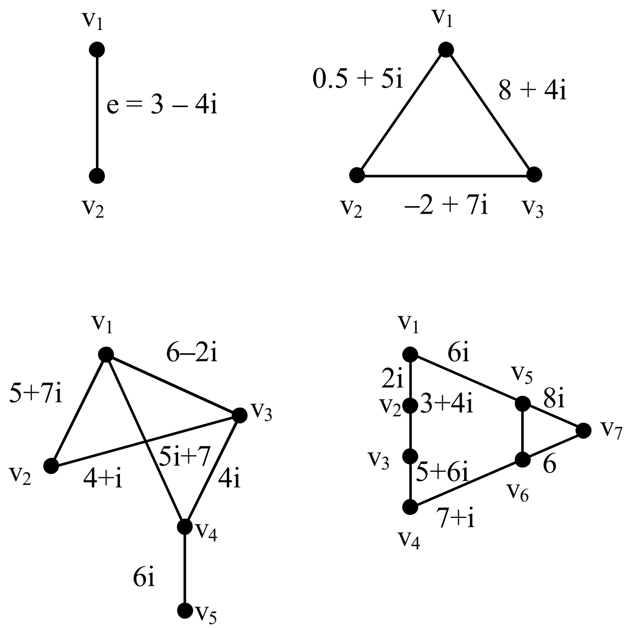


Figure 2.1

In view of this we make the following definition of a complex valued graph.

Definition 2.1. *Let $G = \{V, E\}$ be a graph whose edge values E are from $C = \{a + bi / a, b \in R$ or Q or Z with $i^2 = -1\}$ be defined as the complex valued graph provided there is atleast one edge which is a complex number.*

The following facts are important. In reality certain edges in neural network or be a relational map or any cognitive maps we can have the existence of some edges to be imaginary. This can be represented by complex values. So at the first step we are very much justified in giving edge weights to be complex numbers in graphs. We have already provided some examples, now we will develop this new and innovative notion in a systematic way.

First of all the graphs given in Figure 2.1 are mixed imaginary for the edge weights are $a + bi \in C$ or real ($a \neq 0$ and $b = 0$).

Now we can have graphs which have edge weights to be purely imaginary and some real.

We obtain first results about some complex valued graphs.

Example 2.2. Let G = (V, E) be a graph with two vertices and one edge given by the following figure;

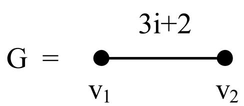


Figure 2.2

Clearly G is a imaginary valued graph.

Example 2.3. Let K = (V, E) be a graph with three vertices and three edges given by the following figure;

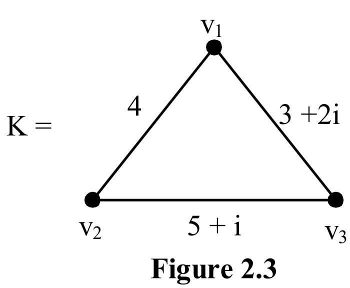


Figure 2.3

We now see $K - v_1$ gives the graph.

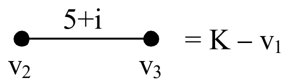


Figure 2.4

So the complex valued graph K becomes a purely imaginary graph by the removal of one vertex v_1.

We now find $K - v_3$; $K - v_3$ is given by the figure 2.5.

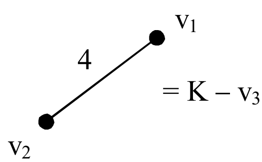


Figure 2.5

Clearly the removal of the vertex v_3 makes the complex valued graph to be real graph.

Now we find the graph obtained by removal of the vertex v_2.

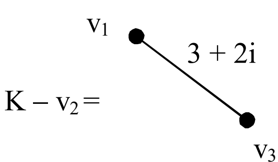


Figure 2.6

Clearly the removal of the vertex v_2 from the complex valued graph K makes $K - v_2$ a purely imaginary graph.

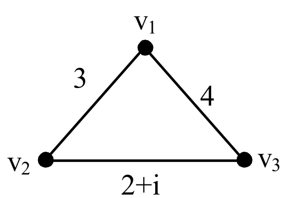
So it is pertinent to record the following:

i) In general a complex valued graph by a removal of one or two vertices may become a real graph.

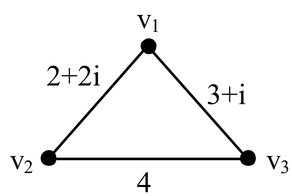
ii) In general we cannot say two complex valued graphs with same number of edges and same number of vertices to be isomorphic.

The following example proves these facts.

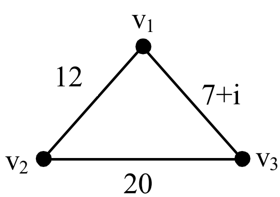
Example 2.4. Let us consider all complex valued graphs with 3 edges and 3 vertices given by the following figures.



(a)



(b)



(c)

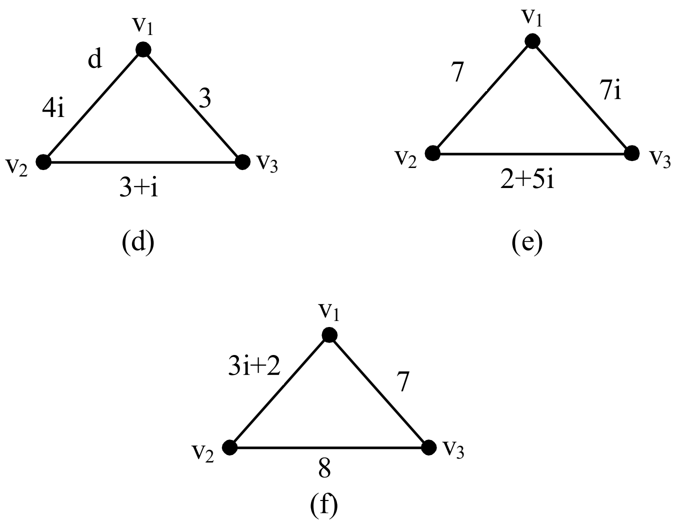


Figure 2.7

We see none of these complex, valued graphs given in Figure 2.7 are isomorphic.

For if v_1 is removed in a, b, c, d, f and e;

we see in case of a, d, and e the removed of vertex v_1 results in an pure imaginary graph whereas in case of f, b and c the graphs after removal of vertex v_1 is real.

Now for the complex graphs a, b, c, d and c remove vertex v_2. The complex valued graphs f, a and d are real and the complex valued graphs b, c and e are pure complex graphs.

Now the removal of the vertex v_3 from the complex valued graphs a, b, c, d, e and f we see a, c and e are real and the complex valued graphs f, b and d are pure complex.

We first tabulate the results for comparison.

Graphs	Removal of vertex v_1	Removal of vertex v_2	Removal of vertex v_3
a	Pure imaginary	Real	Real
b	Real	Pure imaginary	Pure imaginary
c	Real	Pure imaginary	Real
d	Pure imaginary	Real	Pure imaginary
e	Pure imaginary	Pure imaginary	Real
f	Real	Real	Pure imaginary

We observe each of the rows.

All the six rows are distinct which clearly proves all the six graphs are distinct.

Further we see a and b behave in the opposite way so we define a to be the quasi dual of b and vice versa. c and d behave in the opposite way so c is the quasi dual of d and d is a quasi dual of c.

e and f behave as a quasi dual pair.

Thus we cannot say the complex valued graphs with same number of edges and vertices are isomorphic.

Example 2.5. Let G_1, G_2, G_3, G_4 and G_5 be 5 complex valued graphs having same number of vertices and same number of edges which is given in the following figure 2.8.

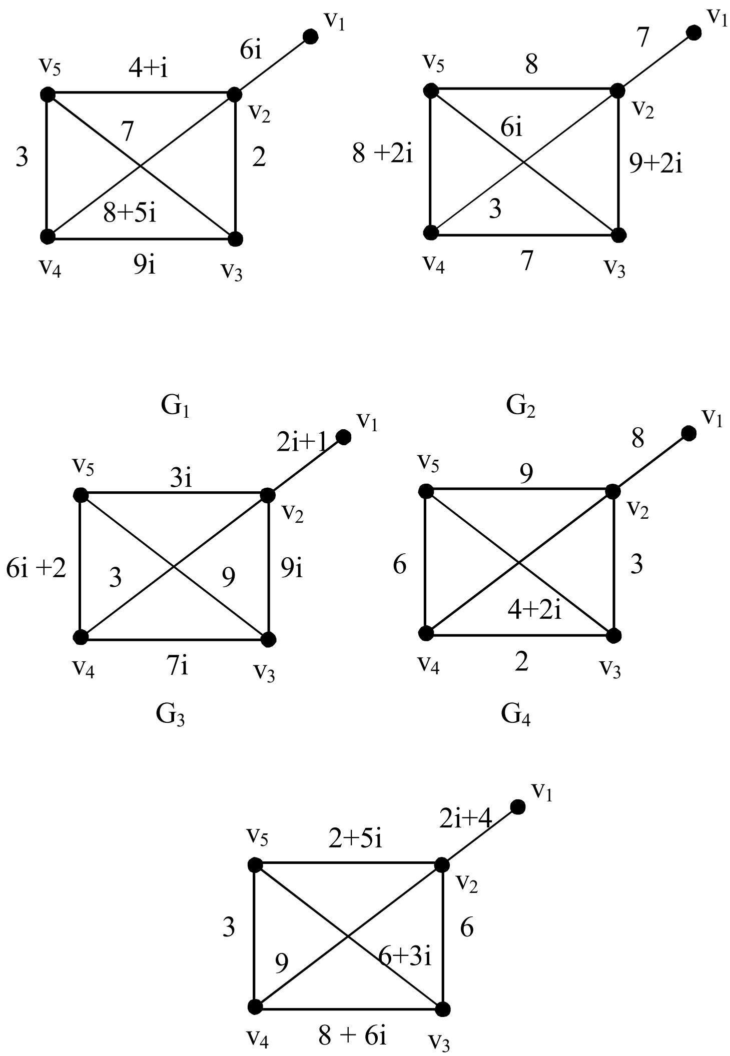


Figure 2.8

We see how many subgraphs of G_1 are real and how many are pure imaginary how many are complex valued subgraphs.

Consider the following subgraphs of G_1

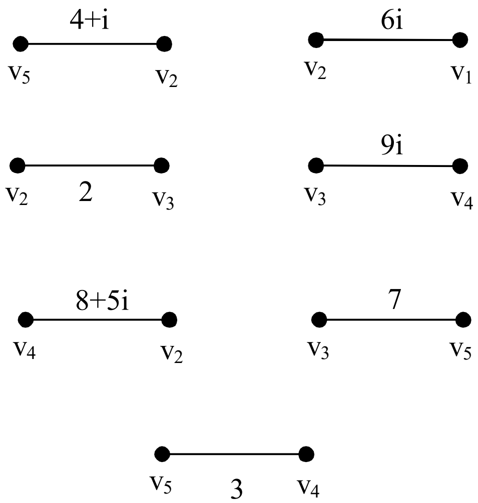


Figure 2.9

We give these seven subgraphs in figure 2.9 with two vertices and one edge (it pertinent to mention here that we don't accept vertex set without edges for all vertex sets will trivially fall under the real graphs).

We see there are 4 subgraphs which are pure imaginary and only three subgraphs which are real.

No questions of complex valued graphs arises in this case as there is only one edge connecting the two vertices which can either be real or complex. Hence the claim.

The subgraphs with three vertices and two edges or three edges of G_1 is as follows.

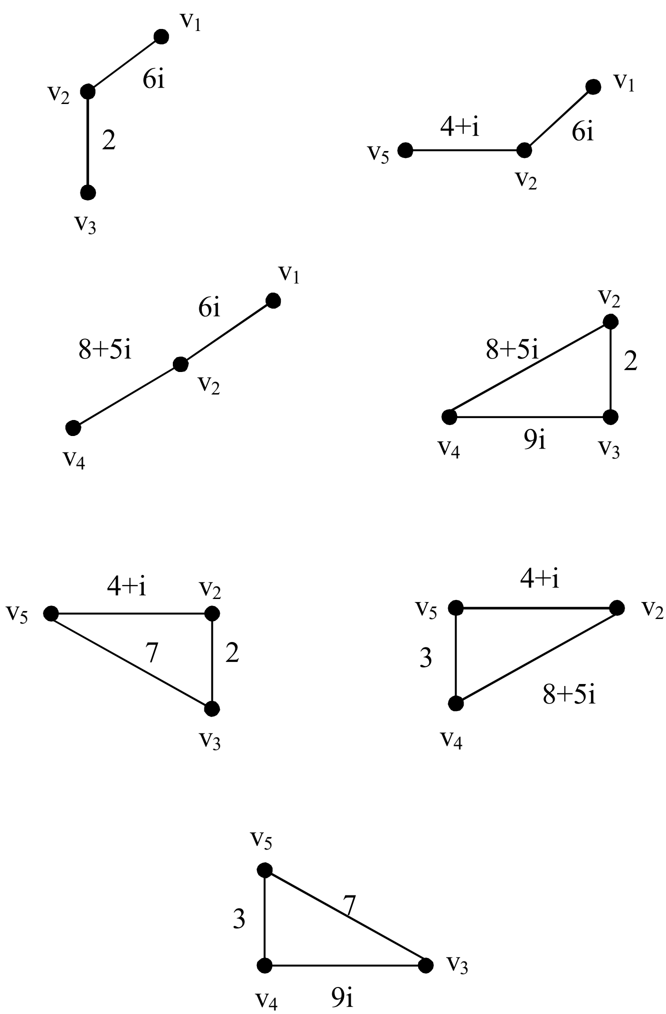


Figure 2.10

There are 7 subgraphs of G, with three vertices with 2 edges or 3 edges.

Only out of these 7 subgraphs two are pure imaginary and 5 others are complex valued subgraphs there is no subgraph with real edges so there is no subgraph with 3 or two edges and 3 vertices which is a real graph.

Subgraphs with four vertices and six edges or four edges or five edges are given below.

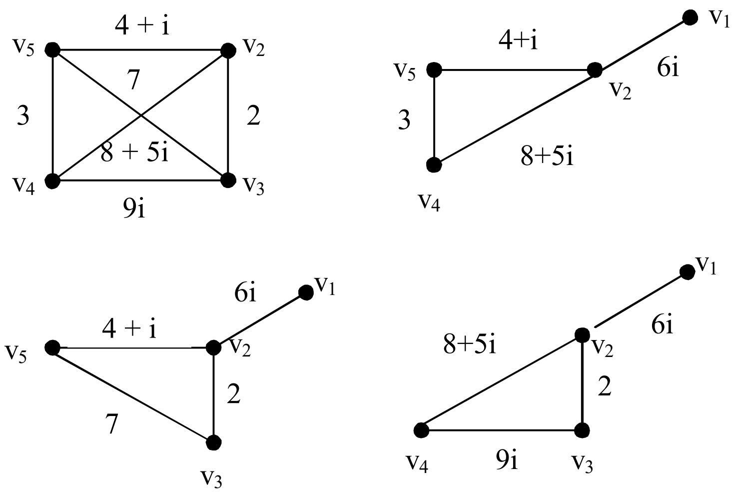


Figure 2.11

We see all the four subgraphs of G_1 are complex valued subgraphs so it is pertinent to put forth a simple problem for the reader, will all four vertices subgraphs of G_1 be only a complex valued subgraph?

Interested reader can work with the complex valued graphs G_2, G_3 and G_4 and find their respective subgraphs.

Further can there be quasi dual subgraphs of G_1, G_2, G_3 etc? If so find them and compare them.

Now for the graph G_1 we remove one of the vertices and see how they behave. If we remove v_1 from G_1 we see $G_1 \setminus v_1$ is as follows;

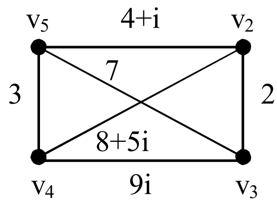


Figure 2.12

Clearly $G_1 \setminus v_1$ is again a complex valued subgraph of G_1.

Next we remove the vertex v_2 from G_1. We find $G_1 \setminus v_2$ and the related figure which is given in the following;

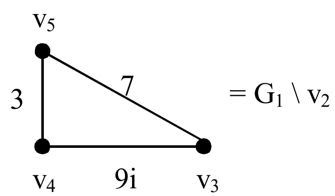


Figure 2.13

This subgraph is also a complex valued subgraph of G_1. Further this also removes vertex v_1 from G_1.

We now remove the vertex v_3 from G_1, $G_1 \setminus v_3$ is as follows;

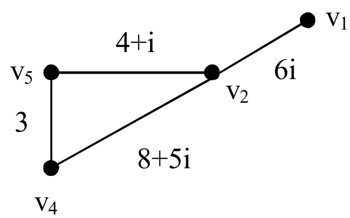


Figure 2.14

This is also a complex valued subgraph of G_1.

We now find $G_1 \setminus v_4$ which is as follows.

$G_1 \setminus v_4 =$

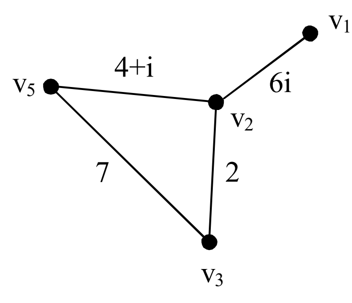


Figure 2.15

We see $G_1 \setminus v_4$ is again a complex valued subgraph of G_1.

Next we find the subgraph of G_1 by removing the vertex v_5.

$G_1 \setminus v_5 =$

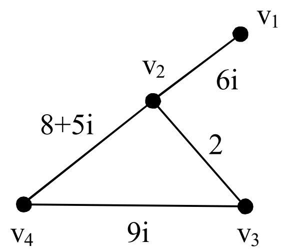


Figure 2.16

The related subgaph of $G_1 \setminus v_5$ is again a complex valued subgraph of G_1.

We now remove two of the vertices of G_1 and study the resulting structure $G_1 \setminus \{v_1, v_2\}$ is given by the following figure.

$G_1 \setminus \{v_1, v_2\} =$

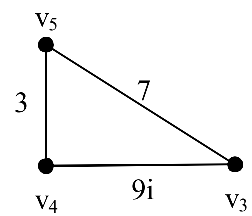


Figure 2.17

Clearly $G_1 \setminus \{v_1, v_2\}$ is again a complex valued subgraph of G_1.

We now find $G_1 \setminus \{v_1, v_3\}$ the subgraph is given in figure 2.18.

$G_1 \setminus \{v_1, v_3\} =$

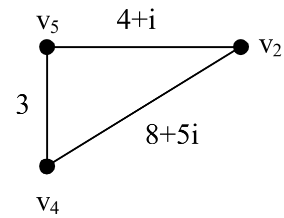


Figure 2.18

Thus $G_1 \setminus \{v_1, v_3\}$ is again a complex valued subgraph of G_1.

We now find the graph $G_1 \setminus \{v_1, v_4\}$ which is as follows.

$G_1 \setminus \{v_1, v_4\} =$

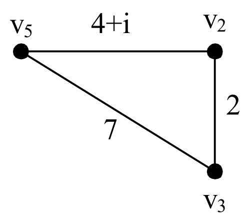


Figure 2.19

We now find $G_1 \setminus \{v_1, v_5\}$. The related graph is given in Figure 2.20.

$G_1 \setminus \{v_1, v_5\} =$

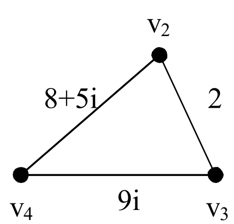


Figure 2.20

This is also a complex valued subgraph G_1.

Next we find the subgraph $G_1 \setminus \{v_2, v_3\}$

$G_1 \setminus \{v_2, v_3\}$ =

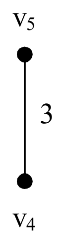


Figure 2.21

Clearly removal of two vertices dismantles v_1 from G_1 and however the resulting subgraph of G_1 is a real subgraph.

We find $G_1 \setminus \{v_2, v_4\}$ in the following Figure 2.22.

$G_1 \setminus \{v_2, v_4\}$ =

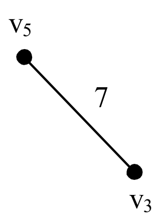


Figure 2.22

This also removes the vertex v_1 and the resultant subgraph is a real subgraph of G_1.

In the following figure we give the subgraph given by $G_1 \setminus \{v_3, v_4\}$.

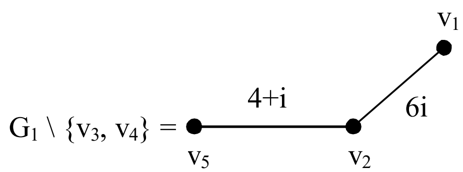


Figure 2.23

Clearly $G_1 \setminus \{v_3, v_4\}$ is a pure imaginary subgraph of G_1.

Now we find $G_1 \setminus \{v_3, v_5\}$ and describe it in the following Figure 2.24.

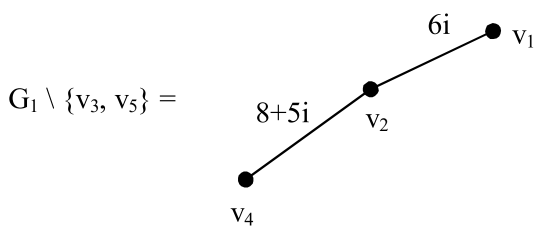


Figure 2.24

Clearly this subgraph is pure imaginary.

Now we find $G_1 \setminus \{v_4, v_5\}$;

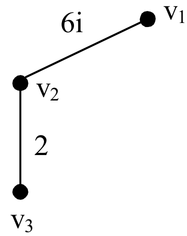
$G_1 \setminus \{v_4, v_5\} =$ 

Figure 2.25

Clearly the subgraph $G_1 \setminus \{v_4, v_5\}$ is a complex valued subgraph of G_1.

Now we find the subgraphs obtained by $G_1 \setminus \{v_1, v_2, v_3\}$ which is given by the following Figure 2.26.

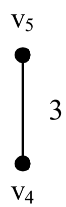
$G_1 \setminus \{v_2, v_1, v_3\} =$ 

Figure 2.26

We see $G_1 \setminus \{v_1, v_2, v_3\}$ is a real subgraph which is the same as $G_1 \setminus \{v_2, v_3\}$ given in Figure 2.21.

Now we find $G_1 \setminus \{v_1, v_3, v_4\}$ and that is described by the following Figure 2.27.

$G_1 \setminus \{v_1, v_3, v_4\} =$

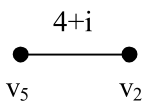


Figure 2.27

Clearly $G_1 \setminus \{v_1, v_3, v_4\}$ is a pure imaginary subgraph of G_1.

Next we find $G_1 \setminus \{v_1, v_2, v_4\}$ which is given by the following Figure 2.28.

$G_1 \setminus \{v_2, v_2, v_4\} =$

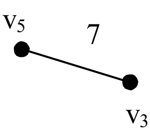


Figure 2.28

Clearly $G_1 \setminus \{v_1, v_2, v_4\}$ is a real subgraph of G_1.

Consider $G_1 \setminus \{v_1, v_2, v_5\}$, which is given by the following graph.

$G_1 \setminus \{v_1, v_2, v_5\} =$

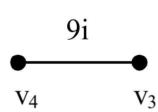


Figure 2.29

Clearly $G_1 \setminus \{v_1, v_2, v_5\}$ is a pure imaginary subgraph of G_1.

Consider $G_1 \setminus \{v_1, v_3, v_5\}$

$G_1 \setminus \{v_1, v_3, v_5\}$ =

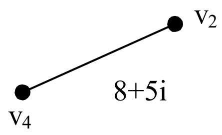


Figure 2.30

We see $G_1 \setminus \{v_1, v_3, v_5\}$ is a pure imaginary subgraph of G_1.

Now we give the figure of the subgraph $G_1 \setminus \{v_1, v_4, v_5\}$.

$G_1 \setminus \{v_1, v_4, v_5\}$ =

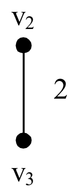


Figure 2.31

We see $G_1 \setminus \{v_1, v_4, v_5\}$ is a real subgraph of G_1.

We find the subgraph $G_1 \setminus \{v_2, v_3, v_4\}$ which is given by the following figure 2.32.

$G_1 \setminus \{v_2, v_5, v_4\}$ =

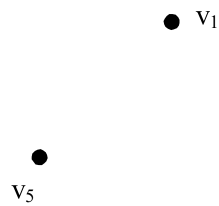


Figure 2.32

We see the subgraph is just vertex subgraph v_1,v_5.

Now we find out $G_1 \setminus \{v_2, v_3, v_5\}$ which is given by the following Figure 2.33.

$G_1 \setminus \{v_2, v_3, v_5\}$ =

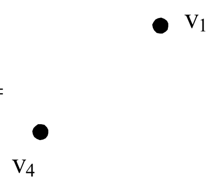


Figure 2.33

This subgraph is also just the vertex v_1, v_4 subgraph of G.

Finally we find out $G_1 \setminus \{v_2, v_4, v_5\}$ which is given by the following Figure 2.34.

$G_1 \setminus \{v_2, v_4, v_5\}$ =

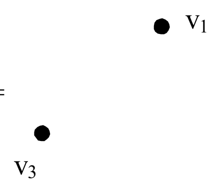
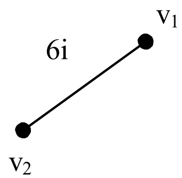


Figure 2.34

This subgraph is also a double vertex.

Usually we ignore the single vertices presence throughout our discussion we do not consider one vertex alone as subgraph.

Now $G_1 \setminus \{v_3, v_4, v_5\}$ is given by the following Figure 2.35.



$= G_1 \setminus \{v_3, v_4, v_5\}$

Figure 2.35

Finally $G_1 \setminus \{v_3, v_4, v_5\}$ is a pure imaginary complex subgraph of G_1.

Now we wish to state if from G_1 a set of four vertices is removed the resultant here is just a single point subgraph of G_1.

Having seen subgraphs got by removal of a vertex we now study yet other properties of these complex valued graphs.

One of the natural and interesting question is how many complex valued graphs with four vertices and six edges with no loops can be obtained is analysed in the following Figure 2.35 before we proceed to describe them we will just describe the notations $\bar{a}$ will denote a complex number of the form x + iy where $y \neq 0$; a, b c or d etc.; denotes a real number.

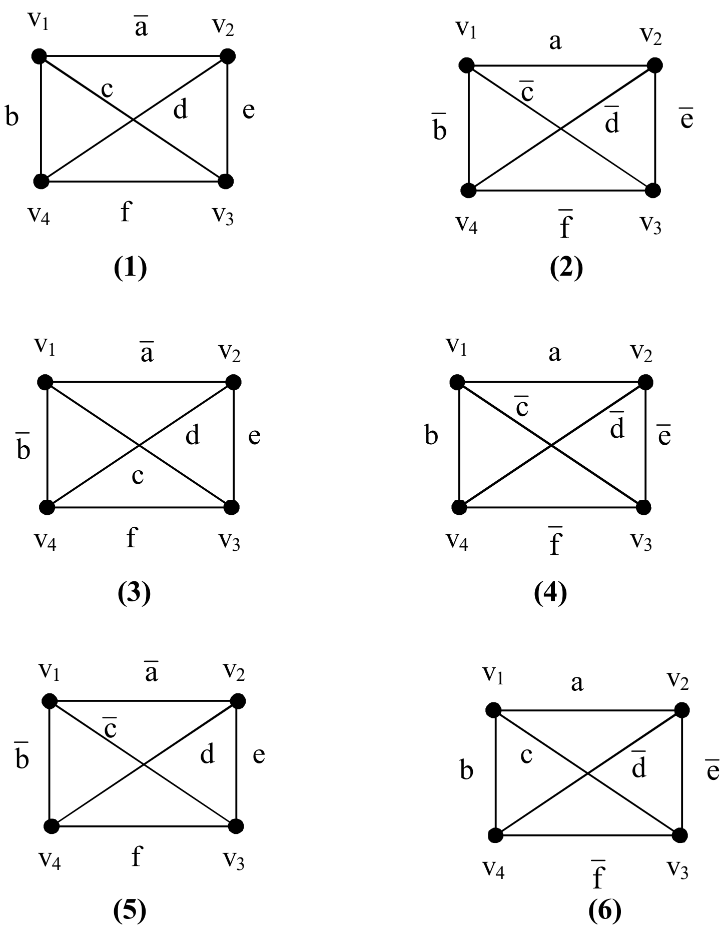


Figure 2.36

We see (1) and (2) the complex valued graphs which are complex complements of each other.

Likewise (3) and (4) complex valued graphs which are complex complements of each other.

(5) and (6) are complex valued graphs that are complements of each other.

We can have this concept in the case of complex valued graphs only.

Next we give first some examples of conjugates of complex valued graphs before we proceed to define them in a systematic way.

Example 2.6. Let G be the complex valued graph given by the following Figure 2.37.

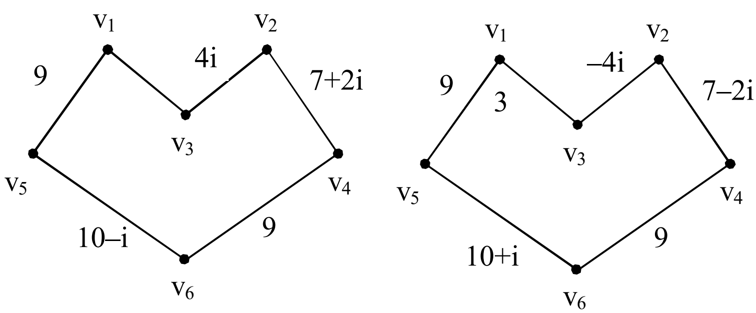


Figure 2.37

We define the complex valued graph B to be the conjugate of the complex valued graph A and vice versa.

We further wish to record that we call a usual weighted graph to be a complex valued graph if atleast one of its edge weights is a complex number from $C = \{a + bi / a, b \in R, i^2 = -1\}$.

Throughout this book we study only complex valued graphs which are finite. That is those graphs which has atleast one of the edge weights to be a complex number that is they are complex valued graphs.

For in many problems we give weights for instance ANN, FCMs, FRMs and so on. Here it is pertinent to record that the two nodes which are connected by given weight in reality may be a imaginary connection.

This sort of occurrence are most common in medical diagnostics, personality evaluation and in study of social problems or social networking.

In these places complex valued graphs will play vital role. Secondly in case of unsupervised data one expert who assigns a value for edge weight between nodes may give a imaginary value for the other expert may feel no such connection to be possible. So in our opinion these graphs will be a boon to a researcher.

However the authors felt when they wanted to study ANN in which they encountered with possible imaginary relation between networks they could not proceed as the concept of complex valued graphs or graphs with complex edges weights, was not in practice and did not exist so authors felt it mandatory, first to develop such graphs and then go for the study.

Further it is pertinent to record at this conjucture that in some cases one may have the very nodes (some of them) to be also imaginary. So in all these cases the complex valued graphs will serve the better purpose.

We now define systematically a complex valued subgraph.

Definition 2.2. *Let G be a complex valued graph. A subgraph H of G is defined as a complex valued subgraph of G if H itself is a complex valued graph.*

It is interesting note the following.

In general all subgraphs of a complex valued graph G need not in general be a complex valued graph it can be pure imaginary subgraph or a real subgraph.

However if G is a pure imaginary graph then every subgraph of G is also a pure imaginary subgraph.

We will prove an example where a complex valued graph G has subgraphs which are real subgraphs which are pure imaginary subgraphs that are complex valued.

Example 2.7. Let G be the complex valued graph given by the following figure;

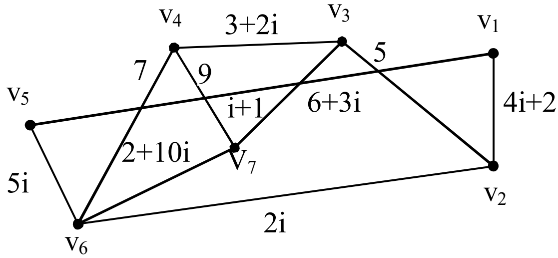


Figure 2.38

Consider the subgraph H_1, given by the following Figure.

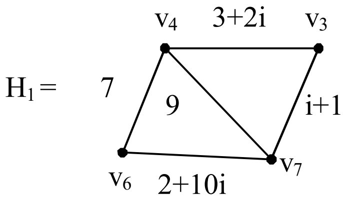


Figure 2.39

Clearly H_1 is a complex valued subgraph of G.

Let H_2 be the subgraph of G given by the following Figure;

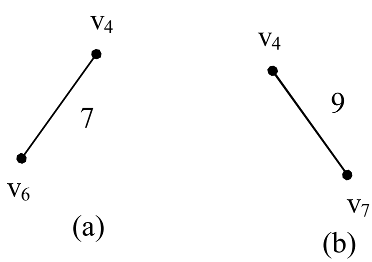


Figure 2.40

Both (a) and (b) are subgraphs of G which are real subgraphs. Consider H_3 the subgraph of G given by the following Figure 2.41.

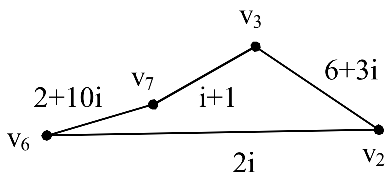


Figure 2.41

Clearly H_3 is a pure imaginary subgraph of G.

Hence the claim.

A walk of a complex valued graph G is an alternating sequence of points and lines or complex valued lines v_0, v_1, $v_2, \ldots, v_{n-1}$, x_n, v_n, beginning and ending with points in which each line is incident with the two points immediately preceding and following it.

So in case of complex valued graphs the walk is imaginary only in parts it will be real.

Here when we define degree of a vertex or point v_1 of a complex valued graph G we call it instead as real degree of v_i of G and complex number degree of v_i of G as the number of real lines (edges) incident with it as real degree of v_i and complex value degree of v_i of G is the number of complex valued edges or lines incident with it.

We will first illustrate this situation by an example or two.

Example 2.8. Let G be the complex valued graph given by the following figure;

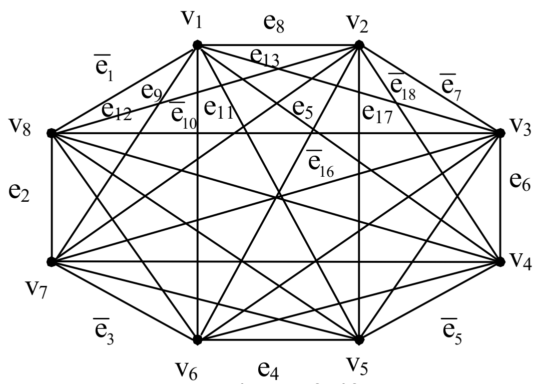


Figure 2.42

Here $\overline{e}_1$ denotes the edge is a complex valued one where as e_j denotes the edge is a real valued one.

Now the real degree of v_1 is 5 the complex valued degree of v_1 is 2.

The real degree of v_2 is 4.

The complex degree of v_2 is 3.

The real degree of v_3 is 5.

The complex degree of v_3 is 2

The real degree of v_4 is 4.

The complex degree of v_4 is 3.

The real degree of v_5 is 5

The complex degree of v_5 is 2.

The real degree of v_6 is 3.

The complex degree of v_6 is 4.

The real degree of v_7 is 5.

The complex degree of v_7 is 2.

The real degree of v_8 is 5.

The complex degree of v_8 is 5.

We see on the whole the complex valued graph G has 17 real edges and 11 imaginary edges.

Example 2.9.

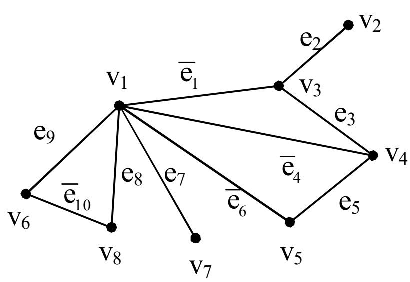


Figure 2.43

The real degree of v_1 is 3 and the complex degree of v_1 is 3.

The real degree of v_2 is 1 and the complex degree of v_2 is 0.

The real degree of v_3 is 2.

The complex degree of v_3 is 1.

The real degree of v_4 is 2.

The complex degree of v_4 is 1.

The real degree of v_5 is 1.

The complex degree of v_5 is 1.

The real degree of v_6 is 1.

The complex degree of v_6 is 1.

The real degree of v_7 is 1.

The complex degree of v_7 is 0.

The real degree of v_8 is 1.

The complex degree of v_8 is 1.

Thus we see in case of graph given in example 2.8 some of the degrees of real and complex of each vertex v_i is 7.

However in the example 2.9 given in figure 2.43 we see the sum of the degrees of each vertex v_i is different.

For instance sum of the degrees of vertex v_1 is 6.

The sum of the degrees of vertex v_2 is 1.

The sum of the degrees of vertex v_3 is 3.

The sum of the degrees of vertex v_4 is 3.

The sum of the degrees of vertex v_5 is 2.

The sum of degrees of vertex v_7 is 1.

The sum of the degrees of vertex v_6 is 2.

The sum of the degrees of vertex v_8 is two.

Thus we see in case of K_n complex value of graphs the sum of the real degree and complex degree adds upto $n - 1$.

In case of the other graph all the vertices may not have the same degree for every vertex.

However in case of complex valued graphs, the Ulms conjecture is stated as, if G is a complex dual graph with some r vertices and H is another complex valued graph with r vertices with $r \geq 3$.

If for each vertex v_i the subgraphs of $G_i = G - v_i$ and $H_i = H - v_i$ are isomorphic then the complex valued graphs G and H are isomorphic remain open for complex valued graphs also.

All theorems in general are true in case of complex value graphs regarding degrees of the vertices of a graph.

However path and walk will have imaginary lines.

We claim if G is a complex valued graph and H is a real graph then the product of two graphs G and H.

$G \times H$ is only a complex valued graph with H as a real subgraph.

We will first illustrate this situation by some examples.

Example 2.10. Let

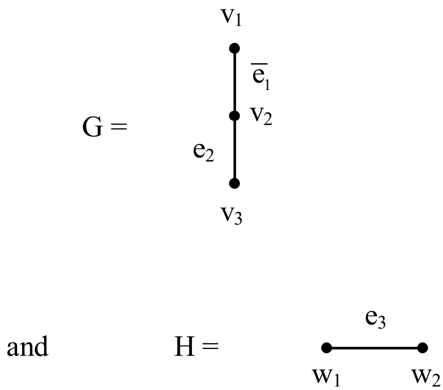


Figure 2.44

Be the complex valued graph and real graph respectively.

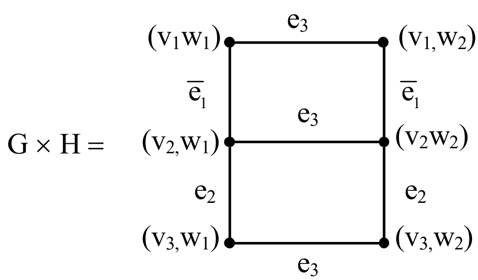


Figure 2.45

Clearly G × H is a complex valued graph.

We find composition of graphs G[H] and H[G] in the following.

A natural question is will G[H] be the same as H[G].

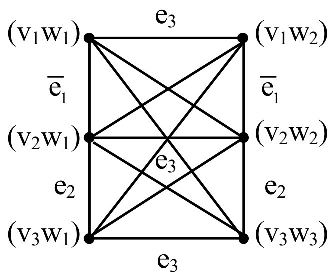
G[H] = 

Figure 2.46

H[G] is given by the following

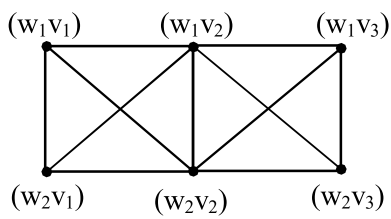


Figure 2.47

Clearly in general G(H) ≠ H(G) evident from the figures 2.46 and 2.47.

It is difficult to derive the notion of regular graphs in case of complex valued graphs as it is very different to define degree of a vertex = sum of (complex degree + real degree) so only in few cases regularity will be got and further most results dependent on regularity are not true in case of complex valued graphs.

Hence we make two types of definitions.

We say a complex valued graph G is complex regular if

i) every vertex has same degree.

ii) The complex degree of each vertex are equal.

iii) The real degree of each vertex are equal.

We will first illustrate this situation by an example.

In case of point graph that is no lines we do not have the notion of complex valued graphs so we do not have the definition of 0-regular graphs.

In case of 1-regular complex valued graphs we see they are only pure imaginary graphs and so on.

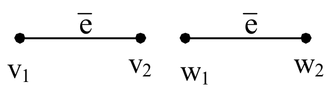


Figure 2.48

Once again we recall if we put $\bar{e}$ for the edge it means $\bar{e}$ = a + bi where b ≠ 0 a can be zero or non zero.

Theorem 2.1. *If G is a one regular complex valued graph then G is a pure imaginary graph.*

Proof. The one regular complex valued graphs are of the form

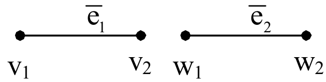


Figure 2.49

so it can only be a pure imaginary graph.

Now we first provide an example or two of regular complex valued graphs.

***Example 2.11*.** Consider the following complex valued graphs.

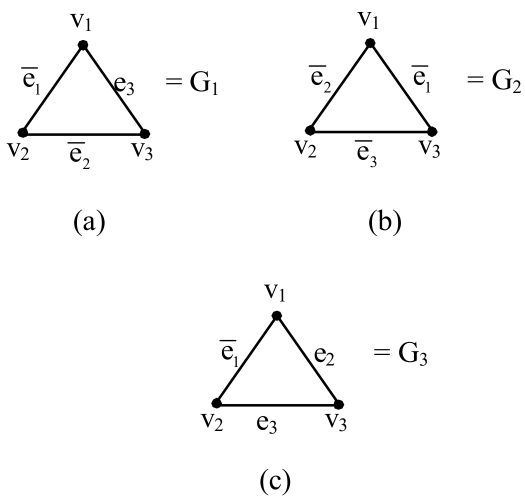


Figure 2.50

We see we can have only three complex valued graphs with three vertices which has same degree at each of vertices.

We see from the three graphs only one of them is regular namely (b), but (b) is a pure imaginary graph.

We see in case of (a) a deg (v_1) = one complex degree + one real degree

deg (v_2) = two complex degree

deg (v_3) = one complex degree + one real degree.

So the complex valued graph (a) is not regular according to the definition given.

Consider the complex valued graph (b). Clearly degree of each vertex is equal to two in which complex degree is two and real degree is zero.

In case of the complex valued graph (c) the task of proving G_3 is not regular is given to the reader.

Consider the following graph K given below.

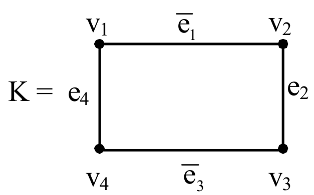


Figure 2.51

Clearly it is verified K is complex 2-regular and 2-regular. Consider J given by the following figure;

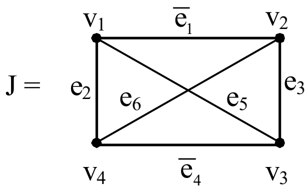


Figure 2.52

It is easily verified J is also complex 3-regular.

Let W be the complex valued graph given by the following figure;

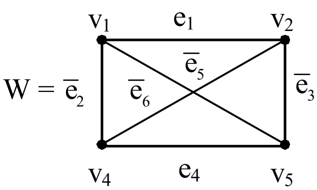


Figure 2.53

It is easily verified W is also 3 complex regular.

However it is clear through both W and J are 3 complex regular yet they are not the same or isomorphic we call these two graphs as dual regular graphs or to be more specific J is the dual regular graph of W and vice versa.

Consider the complex valued graph H given by the following figure.

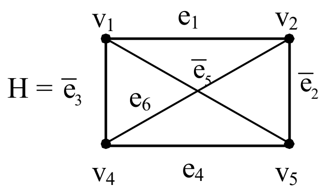


Figure 2.54

We see the complex valued graph H is not complex regular it is only regular in a different way.

We call such complex valued graphs as quasi 3-regular graphs. So (a) and (c) in figure 2.50 are defined as quasi 2-regular graphs.

Theorem 2.2. *Let G and G_1 be two complex valued graphs with same number of vertices and same number of edges, which are complex n-regular. They are in general not isomorphic.*

Proof. The proof is only by contradiction. Consider the complex 3-regular graphs J and W given in figures 2.52 and 2.53 respectively. They have all conditions of the theorem to be true yet they are not isomorphic.

We now give examples of quasi complex (4, 1)-regular graphs. Consider the complex valued graph B given by the following figure;

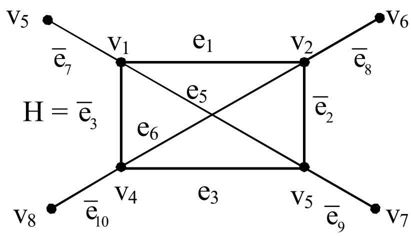


Figure 2.55

We see B is a quasi complex (4, 1) regular graph. The complex degree of each of the vertices is four with degree of real edges = 2 and degree of complex edges also is 2.

We observe the following two facts.

i) Degree (complex) of each vertex is even.

ii) The complex degree of v_i = the real degree of v_i = 2, i = 1, 2, 3, 4.

iii) Four other vertices have degree 1.

We observe this situation can occur only in case of even complex degree graph. However it is pertinent to keep on record that in general for all appropriate complex even degree graph such facts may be satisfied.

We call such complex regular graphs as quasi complex equally regular graphs.

We will show we may have complex quasi (4, 1) regular graphs which may not be quasi complex (4, 1) equally regular graphs by the following example. Consider the complex valued graph D given by the following figure.

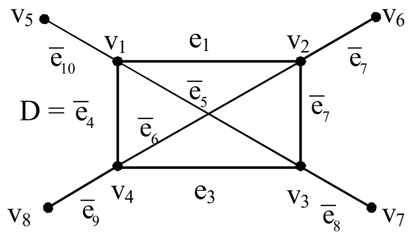


Figure 2.56

Consider the complex valued graph given by the following figure.

Clearly D is a quasi complex (4,1) regular graph.

Degree of each v_i = 4 = sum of (real degree + complex degree), $1 \leq i \leq 4$.

Thus D is not a quasi complex (4, 1) equally regular graph as real degree of vertex $v_i \neq$ complex degree of vertex v_i, $1 \leq i \leq 4$.

Consider M the complex value graph given by the following figure;

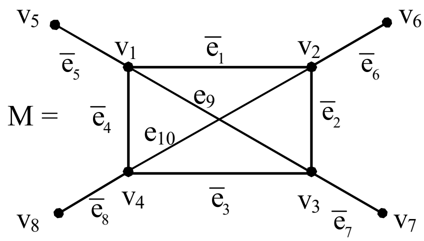


Figure 2.57

We see clearly M is a quasi complex (4, 1) regular graph degree of each v_i = sum of (real degree 1 + complex degree 3) for i = 1, 2, 3, 4.

However still this M is not isomorphic with D given in figure 2.56.

Consider the complex valued graph N given by the following figure;

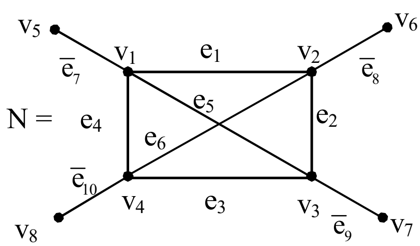


Figure 2.58

Clearly N is also a quasi complex (4, 1) regular graph for four of the vertices are complex (4, 1) regular and other four vertices 1 - regular.

In view of this we make the following definition.

Definition 2.3. *Let V be a any complex valued graph with even number of vertices say 2 n vertices. We say V is a quasi complex (s, 1) regular graph if n vertices are complex s-regular and remaining n-vertices 1 regular.*

We have given examples of quasi (4, 1) regular complex graphs with 8 vertices.

We see

$\times =$

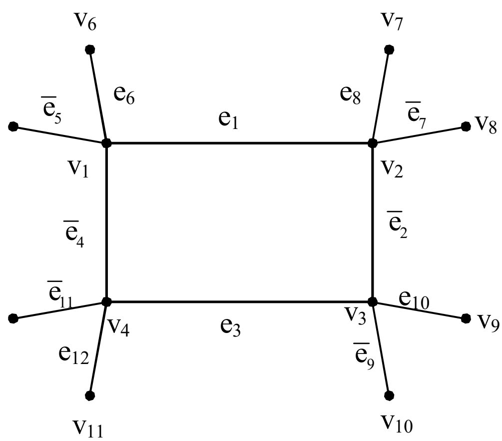


Figure 2.59

a complex valued graph. This is not complex regular for the number of degrees at each vertices are not the same. Further these cannot be classified under quasi (s, 1) regular complex graphs though we have even number of vertices.

For in the first place 8 vertices and of degree 1 and four vertices are degree four.

So we cannot say all complex valued graphs with even 2n number of vertices with one set with equal degrees and another set with equal degrees but the cardinality of the sets different from n are quasi (s, 1) complex regular.

Next we provide some examples of complex 4-regular graphs.

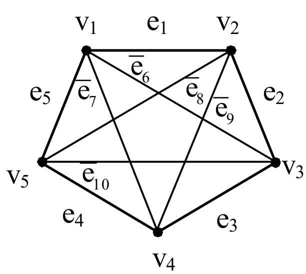


Figure 2.60

We see P is a complex 4 regular graph. Infact complex 4-equally regular graph.

We see degree v_i = sum of (real degree of v_i + complex degree of v_i) = 2 + 2 = 4. For i = 1, 2, 3, 4, 5.

We see real degree v_i = complex degree v_i = 2, $1 \leq i \leq 5$.

Consider the complex valued graph Q given by the following Figure;

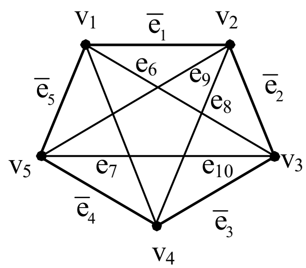


Figure 2.61

Clearly Q is a complex valued graph with five vertices and the complex degree of v_i is 2 = real degree of v_i which is 2, for i = 1, 2, 3, 4, 5.

We see however Q and P are not isomorphic, but both of them are complex 4 equally regular graphs.

Next to we give yet another example.

R =

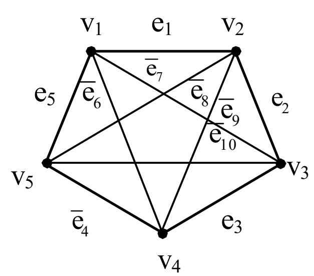


Figure 2.62

We see R is again a complex valued graph with 5 vertices however there are only 5 imaginary edges and five real edges but R is not complex regular graph.

For we see degree of vertex v_1 = sum of (real degree 3 + imaginary degree 1) = 4.

degree of vertex v_2 = sum of (real degree 2 + imaginary degree 2) = 4.

degree of vertex v_3 = sum of (real degree 2 + imaginary degree 2) = 4.

degree of vertex v_4 = sum of (real degree 2 + complex degree 2) = 4

degree of vertex v_5 = sum of (real degree 1 + sum of complex degree 3) = 4.

We see vertex v_1 and vertex v_5 behave exactly in an opposite way for real degree of v_1 is three and that of complex degree of v_1 is three whereas real degree of v_5 is one and that of the complex degree is 3.

Further for the vertices v_2, v_3 and v_4 we see that their real degree is 2 and that of the complex degree is 2.

Hence R is not a complex 4-regular graph though the degree of each vertex is of degree 4.

Now we give yet another example of a complex valued graph with five vertices and the degree of each vertices being only 4 given in the following.

Let T be a complex valued graph given in the figure 2.63.

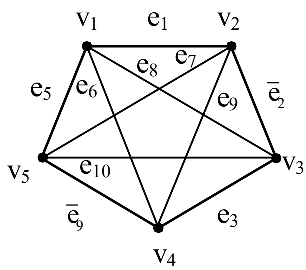


Figure 2.63

We see though the degree of each vertex is four the vertex v_1 has all real edges and none of the edges is imaginary.

The remaining four vertices are such that three edges are real and only one edge is imaginary so T is not a complex valued 4-regular graph.

We now give an example of complex 5-regular graphs. Let M be the complex valued graph.

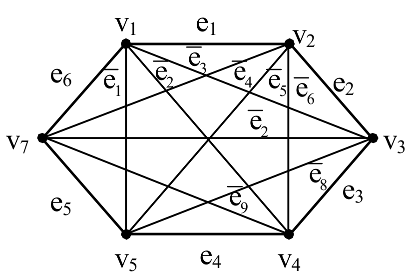


Figure 2.64

Clearly M is a complex 5-valued regular graph as every vertex v_i is such that complex degree v_i = sum of (real degree v_i = 2 + complex degree v_1 is 3) = 5, for i = 1, 2, 3, 4, 5, 6. Hence the claim.

Now consider the complex valued graph A given by the following figure.

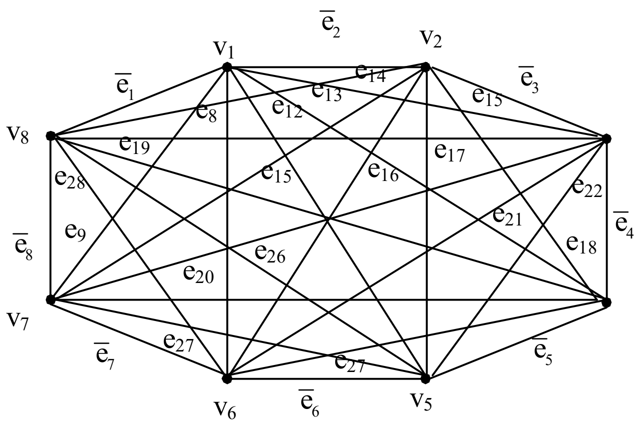


Figure 2.65

Complex degree v_i = sum of (real degree v_i which is 5 + complex degree 2) = 7. This is true for i = 1, 2, 3, …, 8.

Thus A is a complex degree 7-regular graph.

Consider the complex valued graph B given by the following figure;

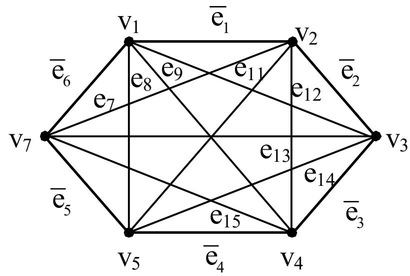


Figure 2.66

Consider the complex valued graph M of figure 2.64 and compare it with the complex valued graph B of figure 2.65.

We see the outer edges of M are all real and the inner edges are all complex.

On the contrary in the complex valued graph given in Figure 2.66. We see all the outer edges of B are complex valued whereas all the edges inside the graph are real.

However both M and B are complex 5-regular with general degree at each vertex being 5.

Further the complex valued graphs M and B are not isomorophic but they are quasi dual complex valued graphs pair. That is the complex valued graph B is the quasi dual of the complex valued graph M and vice versa.

Thus we see in case of complex valued graphs K_8 there are atleast two complex (n – 1) regular graphs which are dual of each other.

Now consider the complex valued graph E given by the following figure;

E=

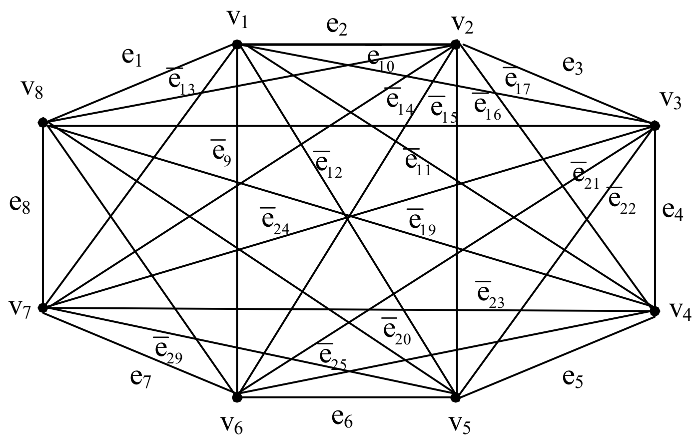


Figure 2.67

We see E has 8 vertices the number edges incident at each vertex v_i is seven for i =1, 2, …, 8.

Further complex degree of v_i = sum of (real degree v_i + complex degree of v_i) = sum of (2 + 5) = 7 for i = 1, 2, 3, …, 8.

Thus E is a complex 7-regular graph.

In view of all these examples we have the following theorem.

Theorem 2.3. *Let $\{K_n\}$ be any complex valued graphs ($n \geq 4$). There are atleast two complex valued graphs P and Q which are complex (n – 1) regular graphs such that P is the quasi dual of Q and vice versa.*

Proof. Given P and Q are two K_n complex valued graphs. P is a complex valued graph such that each of the outer edge is complex valued and all the inner valued edges are real.

That is P is described by the following figure;

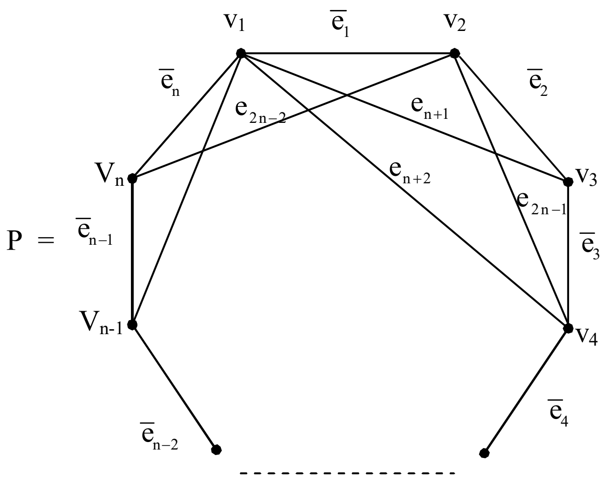


Figure 2.68

Clearly the number of edges adjacent to each vertex v_i is n – 1 of which two are imaginary and n – 3 are real. This is true for i = 1, 2, …, n.

Thus P is a complex (n – 1) regular graph. Consider Q a K_n complex valued graph from the $\{K_n\}$.

Consider the outer edges to be real and all the inner edges to be complex valued given by the following figure;

Q =

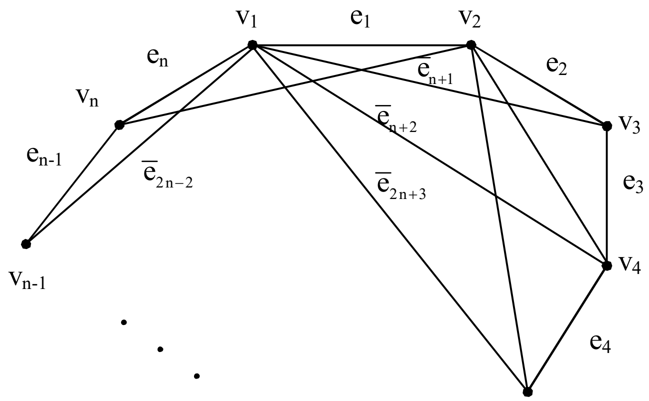


Figure 2.69

Clearly all the outer edges the complex valued graph Q are real whereas all the inner edges of Q are imaginary.

The complex degree of each vertex v_i = sum of (real degree of $v_i = 2$ and imaginary degree of $v_i = n - 3$) $= n - 1$ true for all $i = 1, 2, \ldots, n$.

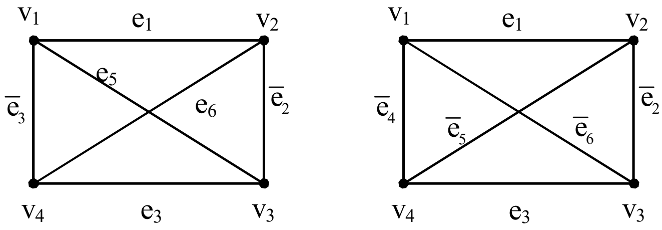
Thus Q is a complex $n - 1$ regular graph.

Infact Q is the quasi dual of P and vice versa. Hence the claim.

Now a natural question would be can we have other quasi dual pairs of K_n.

We will excavate this by some examples.

Consider the following two complex valued graph given by the Figure 2.70.

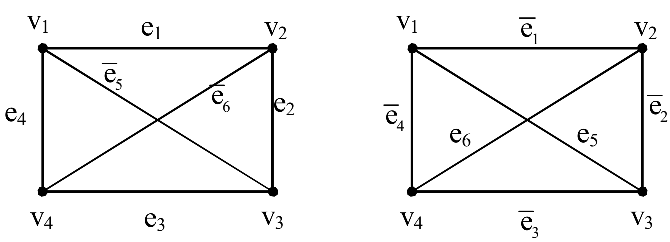


(a) (b)

Figure 2.70

We see both the complex valued graph given by the figures 2.70 (a) and 2.70 (b) are both complex 3-regular graphs which are quasi dual of each other. They are different from the complex valued graphs described in the theorem.

The analogous of the complex valued 3-regular graphs given in that theorem is described in the following figure.

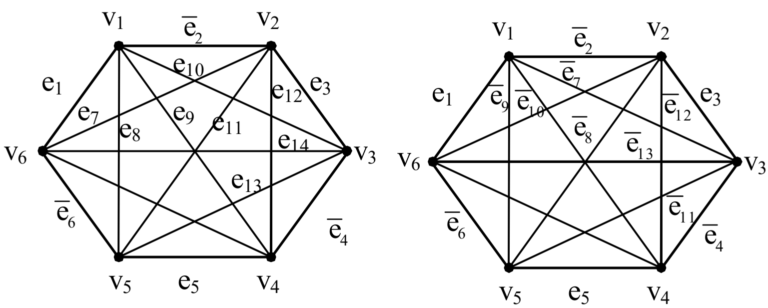


(a) (b)

Figure 2.71

Clearly complex valued graphs (a) and (b) given in Figure 2.69 are dual of each other described in theorem 2.

Consider the following figures which represents the complex valued graphs.

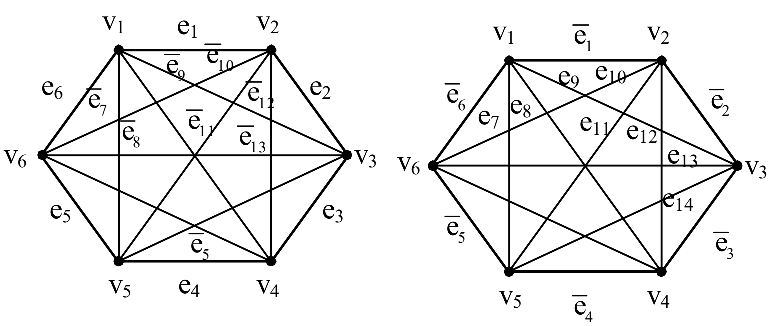


(a) (b)

Figure 2.72

We see the complex valued graphs (a) and (b) are complex 5 regular but they are quasi dual of each other and they are not the one's mentioned in the theorem.

Consider



(a) (b)

Figure 2.73

Clearly the complex valued graphs (a) and (b) given in figure 2.73 are dual of each other described in theorem.

Now we just give the complex valued graphs.

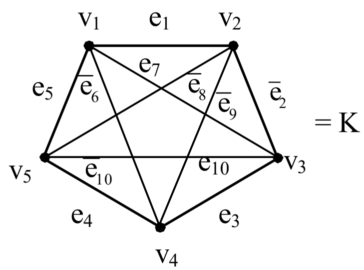


Figure 2.74

We see K is not a complex valued 4 regular graph.

We make the following observation if the complex valued graph K_n in which n is odd then we find it difficult to get quasi dual graphs.

Only when n is even we have quasi dual graphs other than the ones described in the theorem.

Now we give examples of complex valued 4 regular graphs other than the K_5 complex valued graphs.

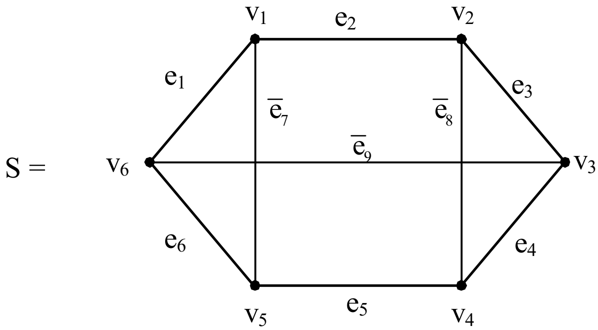


Figure 2.75

S is a complex valued 3-regular graph.

We see each vertex has three edges incident to it of which two of them are real and one is imaginary.

Thus S is a complex valued 3-regular graph.

Consider the complex valued graph F given by the following figure.

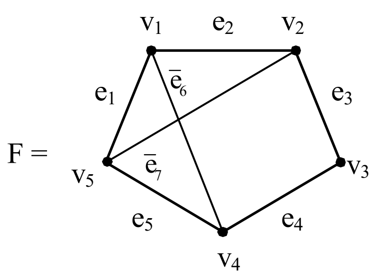


Figure 2.76

We see the complex valued graph in the figure is complex 3-regular for four of the vertices and one vertex does not satisfy it. We define such complex valued graphs as complex nearly 3-regular graph if degree of (n – 1) vertices are the same and only one vertex is different.

We describe the relevant definition in the following.

Definition 2.4. *G be a any complex valued graph with n vertices if the complex degree + real degree of all (n – 1) vertices are equal and the sum is r and only one of the vertex has different degree then we define G to be a complex valued nearly r-regular complex valued graph.*

The complex-valued graph F given in figure 2.76 is a nearly complex valued n-regular graph here r = 3.

We will provide more examples of the situation.

Let G be the complex valued graph given by the following figure;

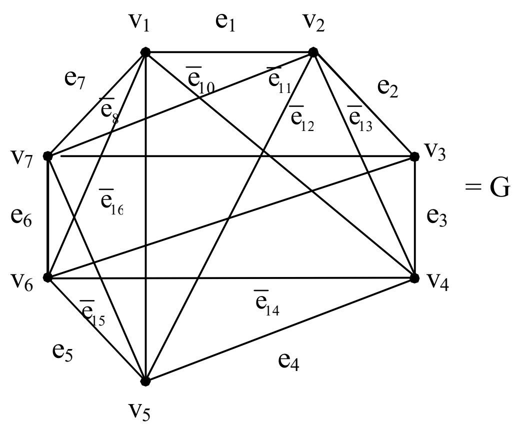


Figure 2.77

We see G is a complex valued graph with seven vertices. Except vertex v_3 all the other vertices have the same number of complex edges and same number of real edges adjacent to it.

We will now give the exact number of the complex degree vertices.

The real edges incident to each of v_1, v_2, v_4, v_5, v_6 and v_7 are two.

The complex edges incident to each of v_1, v_2, v_4, v_5, v_6 and v_7 are three.

Thus the complex degree of each of the vertices = sum of (real degree + complex degree = 2 + 3 = 5.

Thus G is a near complex 5-regular graph.

Consider the following complex valued graph K given by the following figure;

K =

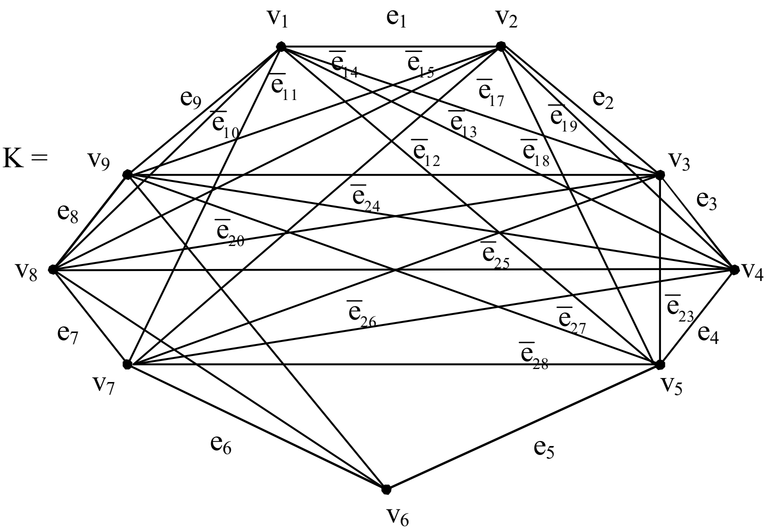


Figure 2.78

We see these are vertices for this complex valued graph K.

Further the complex edges incident to each of the vertices v_1, v_2, v_3, v_4, v_5, v_7, v_8 and v_9 are 5.

The real edges incident to each of the vertices v_1, v_2, v_3, v_4, v_5, v_7, v_8, and v_9 are 2.

Hence the complex degree of each of the vertices v_1, v_2, v_3, v_4, v_5, v_7, v_8 and v_9 are 7.

Thus K is a complex nearly 7-regular graph.

In view of all these we can have the following theorem.

Theorem 2.4. *Let H be complex valued graph with (2n + 1) vertices forming (2n + 1) polygon or (2n+1)-gon.*

There is a complex valued nearly 2n – 1 regular graph which can be built using the (2n +1) - gon in a very special way.

Proof. Let H complex valued graph whose vertices form a (2n +1) - gon. Make the outer edges of H to take real values.

Make 2n – 3 complex edges incident to 2n of the vertices so that only one vertex has 2n – 4 complex edges. Let the vertex which as only 2n – 4 complex edges be labeled as v_i.

Then the resultant graph has for each of its vertices v_i (i ≠ t) two real edges incident to it and 2n – 3 complex edges incident with it there by the 2n vertices of the complex valued graph has 2n – 1 edges adjacent to it (two real edges + 2n – 3 complex edges) only vertex v_t does not enjoy this property.

Thus H is a complex 2n – 1 nearly regular graph.

Hence the claim.

We have illustrated this situation with five vertices complex valued graph which forms a 5-polygon, 7-polygon and 9 polygon.

Thus the class of complex nearly regular graphs is non empty.

Next we provide some more examples of this situation.

Consider a complex valued graph S given by the following figure;

S =

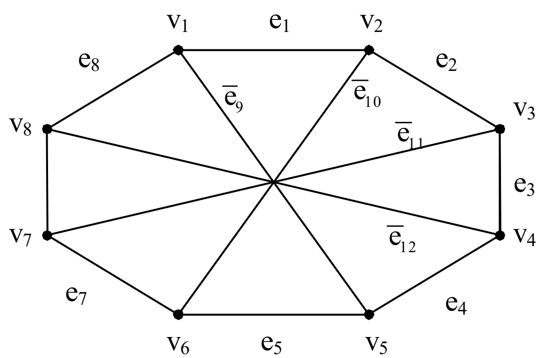


Figure 2.79

Clearly S is a complex 3-regular graph where real degree is 2 and complex degree is one.

Consider the complex valued graph R given by the following figure;

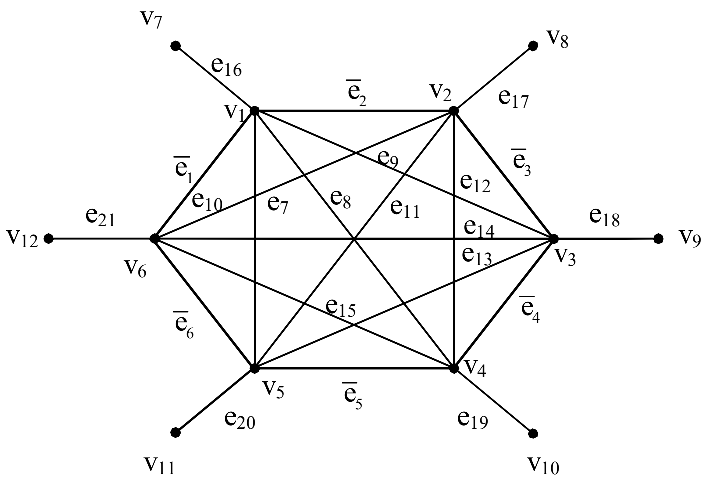


Figure 2.80

Clearly the complex valued graph is a quasi complex (5, 1) equally regular graph.

Now we proceed onto describe we define a complex valued wheel of either K_1 or C_{n-1} is a pure imaginary graph 'or' is used in the mutually exclusive sense.

We will give examples of complex wheels.

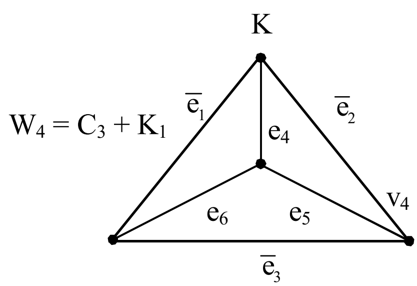


Figure 2.81

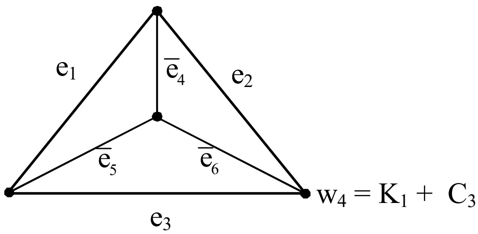
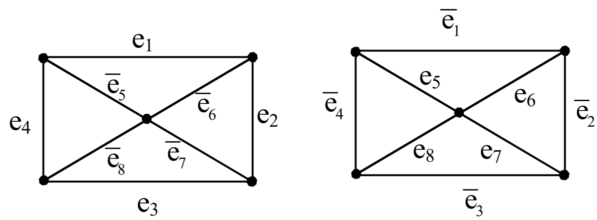


Figure 2.82

Both complex graphs given in Figures 2.81 and 2.82 are complex valued wheels they are dual of each other.



(a) **(b)**

Figure 2.83

$W_5 = K_1 + C_4$ given in figure 2.83 are complex valued wheels

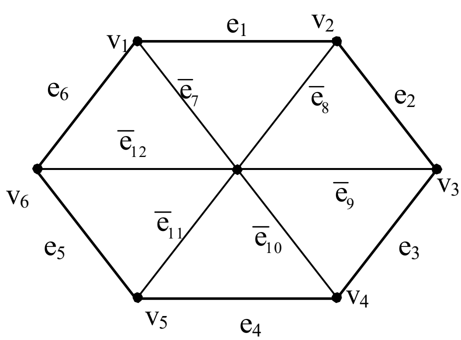


Figure 2.84

$C_6 + K_1 = W_7$ and

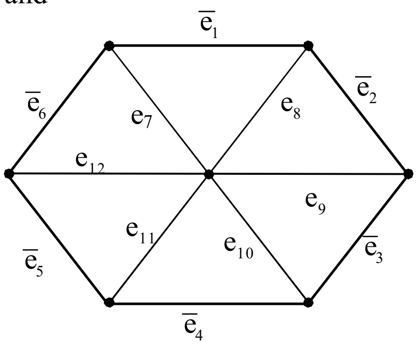


Figure 2.85

There are two distinct complex wheels $C_{n-1} + K_1 = W_n$ where one is the quasi dual of the other.

Consider.

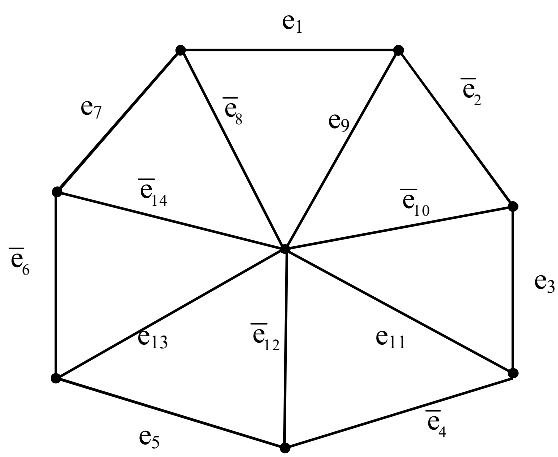


Figure 2.86

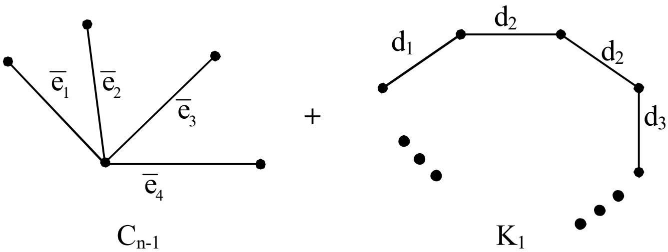
Clearly the complex valued graph G is not a complex valued wheel it is just a complex valued graph.

However we see a complex valued wheel occur as quasi dual pairs.

The following result is very interesting and is different from the classical property of wheel $W_n = K_1 + C_{n-1}$; $n \geq 4$.

Theorem 2.5. *Let $W_n = K_1 + C_{n-1}$ be a complex valued wheel then there is for every W_n a quasi dual wheel which is also a complex valued wheel.*

Proof. We know a complex valued wheel $W_n = C_{n-1} + K_1$ given by



or

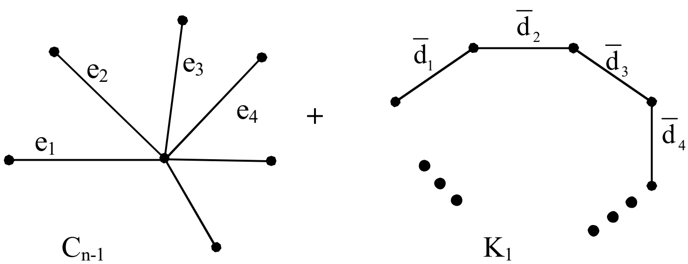
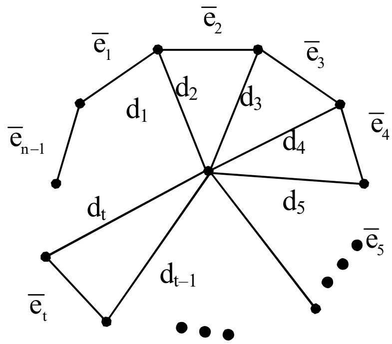


Figure 2.87

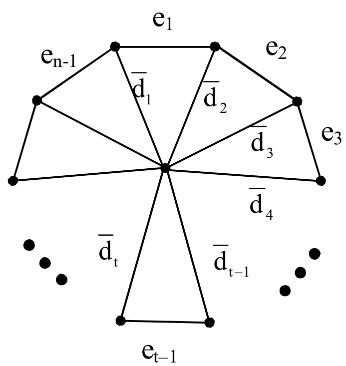
$W_n = C_{n-1} + K_1$



W_n

(a)

and $C_{n-1} + K_1$



W_n

(b)

Figure 2.88

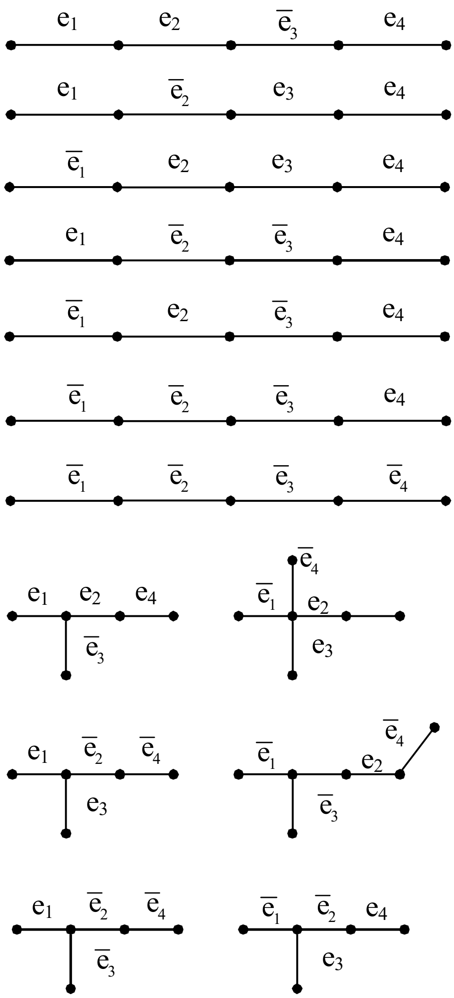
So we have two complex valued wheels by the very definition of complex valued wheels.

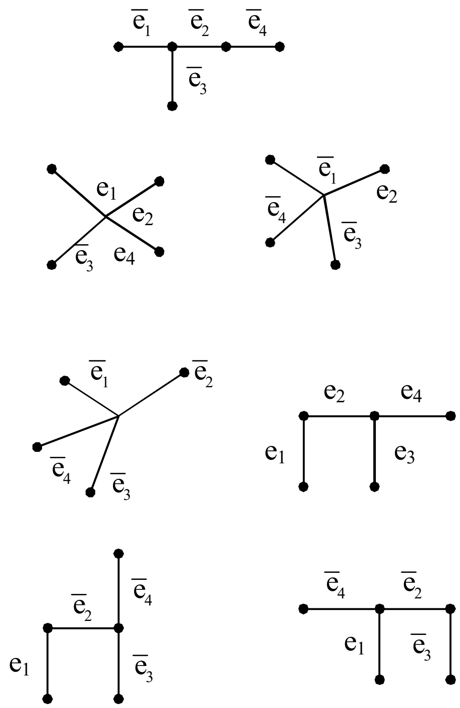
We see they are quasi dual pairs. Hence the claim.

This is the major distinction between real wheels and complex wheels.

Now we proceed onto give examples of complex valued trees.

Example 2.12. Here we give all the trees which are complex valued trees with five points by the following figure.





and so on.

Figure 2.89

Infact it is left as an exercise for the reader to determine all complex valued trees with five vertices.

Further we see if positioning of the vertices are also taken into account for a give set of vertices we certainly have more number of complex valued trees than the general or classical trees for the same number of points.

However the definition of a tree is the same with a simple modification that in case of complex valued trees we must have atleast one path to be imaginary.

While discussing about complex valued graphs in general or trees in particular it is defined as either connected imaginarily or connected in the real sense.

It is important to record at this juncture that the path remaining imaginary is not artificial for the graphs find their applications in artificial neural networks and these networks behave more like neurons of the brain.

Certainly neurons in the brain have several synapse which imagine things it is not only the synapse has imaginary functioning also the neurons induce such imaginary functioning.

For the neural network of the brains of a child or that of a poet, a writer, a painter have lots of imagined concepts these imagined ones if is to depicted in reality common man may think it as madness but of course in general the imaginary world functions in several cases more perfectly than the real world.

For before a scientist put forth a postulates he mimics or imagines take the simple case of human trying to fly.

The imagined wings of large size etc. so when we put a imaginary edge it mainly depicts at that particular time the path connecting those two nodes are imaginary after a time period it may become real or it may continue to remain imaginary.

We have functions in the complex plane. The related graphs of these imaginary functions are described in the complex plane.

So it has become mandatory to define and develop the concept of complex valued graphs in a systematic way for they will play a vital role in almost all mathematical models.

Secondly it is pertinent to a record at this juncture when we find roots of the equations we get the value of roots to be imaginary and it is proved algebraically the complex roots occur in conjugate pair.

Further it is basics that the complex field is algebraically closed. That is all roots of all equations find their values in the field of complex numbers when this is the case with algebra and analysis where they have imaginary values why cannot we have graphs with imaginary paths. Such study can find place in ANN and medical diagonistics.

No one ever said the graphs are real so it is a paradigm of shift which necessities researchers to use complex valued graphs to cater to more sensitive and real or true situation of the problem in hand.

Further calculus has been developed in case of complex functions. We have derivation and integration of all complex valued functions. We study occurrences, of events in the complex plane assume the solutions for them. More so we have partial differentials also associated with complex functions.

Hence the study, to develop and describe these complex valued graphs has become mandatory. Once we have such systematic development of such graphs we will be in a position to build fuzzy complex models and there will be a paradigm of shift in the very field of soft computing.

Be it fuzzy cognitive models or fuzzy relational maps model or artificial neural networks or perceptron or auto associative memories any other soft computing basically exploits the very notion of directed graphs or graphs only real weights are given so far no trial with imaginary weights are ever made so if we need to accept imaginary weights we ought to have imaginary value for edges which means our graphs basically must be a complex valued graphs.

However we have not made the vertices imaginary but, that sort of study will be carried out in the later part of this chapter.

Here we only concentrate on the edge weights to be imaginary and we develop the related structures in asystematic way for this type of study is new and to the best of our knowledge is not present in the literature.

So now we proceed onto describe the weighted matrices of these complex valued graphs before we proceed to describe or develop the notion of bipartite graphs or bigraphs.

Example 2.13. Let G be the complex valued graph given by the following figure;

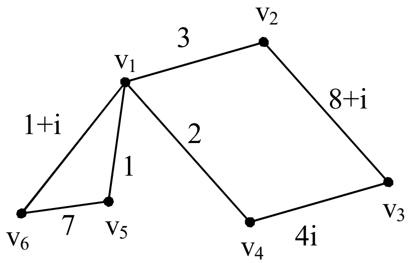
G = 

Figure 2.90

The weight matrix M associated with this graph G is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 2 & 1 & 1+i \\ 3 & 0 & 8+i & 0 & 0 & 0 \\ 0 & 8+i & 0 & 4i & 0 & 0 \\ 2 & 0 & 4i & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 7 \\ 1+i & 0 & 0 & 0 & 7 & 0 \end{bmatrix} \end{array}.$$

Clearly the following observations are important.

i) The complex valued graph G given by figure 2.90 is not a directed one.

ii) Since G has 6 vertices we see the weight matrix or the weight complex matrix M associated with G is 6 × 6 matrix. Clearly M has both real and complex entries.

iii) Since the complex valued graph has no loops we see the diagonal entries of the 6 × 6 square matrix is zero.

iv) Further as the complex valued graph is not a directed graph the related weight matrix M is symmetric about the main diagonal.

We can perform operations on M by multiplying M with a 1×6 row vector and so on.

Such sort of operations will be described in the next chapter of this book.

Now in the following example we describe a complex valued directed graph.

Example 2.14. Let H be the directed complex valued graph given by the following figure;

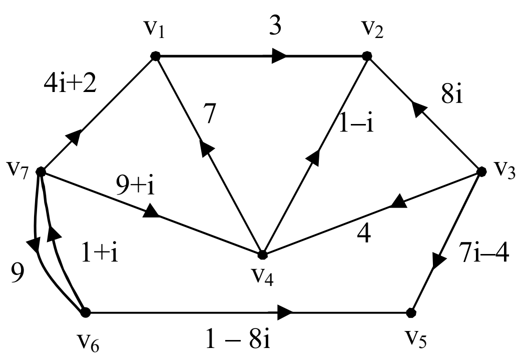


Figure 2.91

The weight matrix N associated with the complex valued graph H is as follows.

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8i & 0 & 4 & 7i-4 & 0 & 0 \\ 7 & 1-i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-8i & 0 & 1+i \\ 4i+2 & 0 & 0 & 9+i & 0 & 9 & 0 \end{bmatrix} \end{array}.$$

Clearly the following observations are important

i) N is not a symmetric complex matrix.

ii) The diagonal entries are zero as the complex valued graphs has no loops.

iii) However the main diagonal entries are zero and N is a 7 × 7 square complex matrix.

Operations performed on the weight matrix and the purpose of defining them will be discussed in the last chapter of this book.

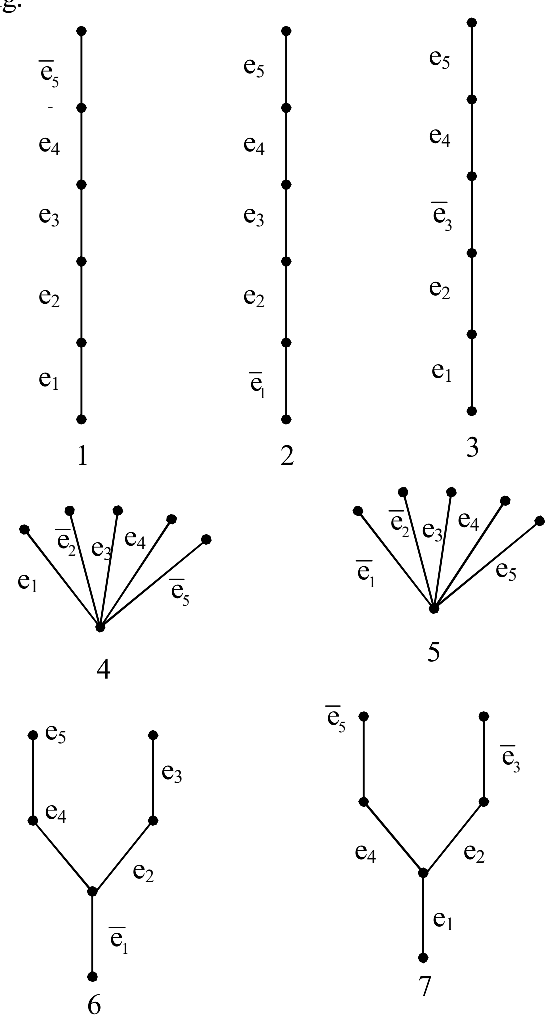
Now we have seen the weight matrix of a complex valued graph which is directed as well not directed has been described.

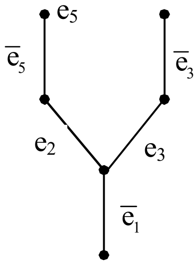
It is important to mention at this juncture that the adjacency matrix will be the same for this complex valued matrix.

This is considered as a matrix of routine and left as an exercise to the reader.

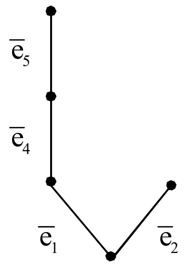
Next we proceed onto briefly describe complex valued roots trees.

We first describe routed trees with six vertices in the following.

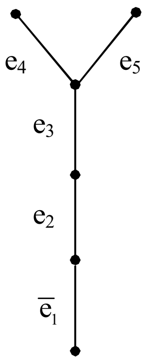



e_5
$\bar{e}_5$
$\bar{e}_3$
e_2
e_3
$\bar{e}_1$

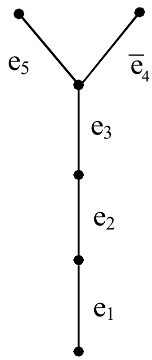
8


$\bar{e}_5$
$\bar{e}_4$
$\bar{e}_1$
$\bar{e}_2$

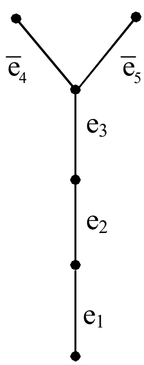
9


e_4
e_5
e_3
e_2
$\bar{e}_1$

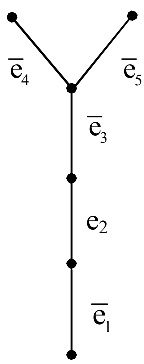
10


e_5
$\bar{e}_4$
e_3
e_2
e_1

11


$\bar{e}_4$
$\bar{e}_5$
e_3
e_2
e_1

12


$\bar{e}_4$
$\bar{e}_5$
$\bar{e}_3$
e_2
$\bar{e}_1$

13

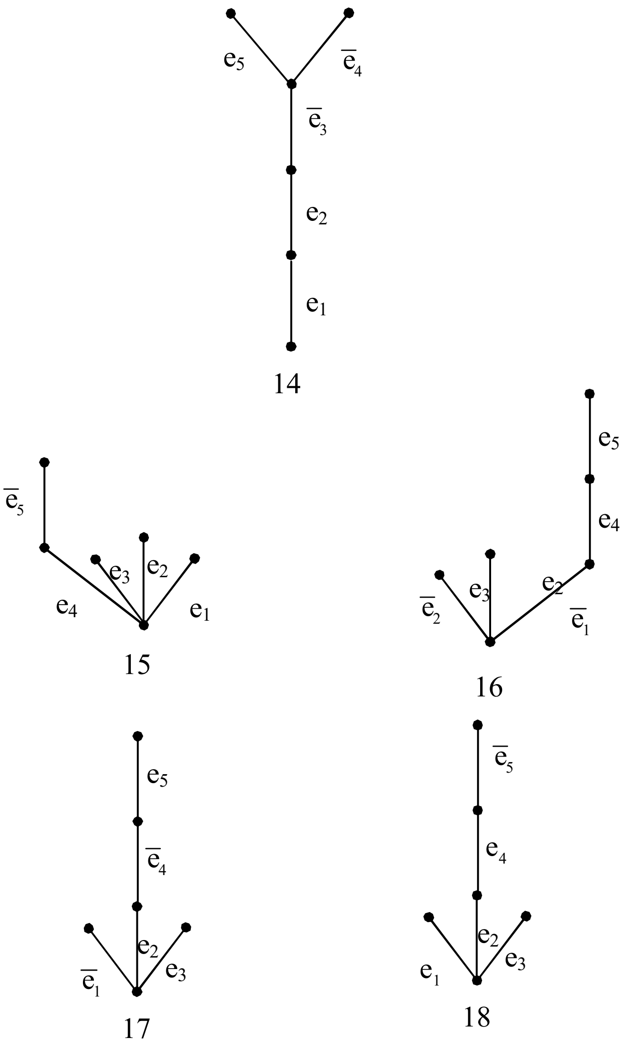


Figure 2.92

We can have many more such complex valued rooted trees some of whose graphs are described.

Consider the complex valued graphs (complex valued rooted trees given in Figure 2.92, 2, graphs6, 8, 9, 10 and 13. We see the roots are imaginary. We call such complex valued rooted trees as complex valued imaginary rooted trees.

In case of Figure 2.92 of (9) we see both the edges connecting the roots are imaginary which is different from others.

Further we see Figure 2.92 are not complex valued imaginary rooted trees for some edges are real and some are imaginary which connects the root.

We call such complex valued rooted trees as complex valued partially imaginary rooted trees.

Figures 4, 5 and 17 of 2.92 are examples of complex valued partially rooted trees.

Next we speak about a leaf (or buds) or children of the complex valued rooted trees.

We in this book call them only as leaf nodes if the edge connecting the parent node to the leaf node is imaginary we call those trees as complex valued trees with imaginary leaf nodes.

Figures 1, 7, 8, 12, 13, 15 and 18 of 2.90 are complex valued rooted trees with imaginary leaf nodes.

We can also using the conventions of computer scientists give the complex valued trees in the following way.

Rooted complex valued tree

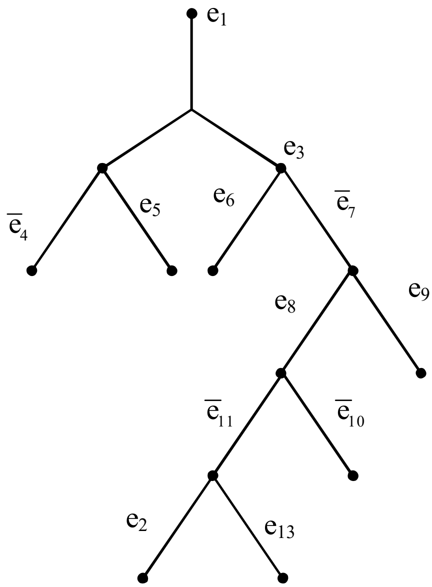


Figure 2.93

Clearly this complex rooted tree has six layers, first two layers are real.

Third layer is partially real and partially complex. Fourth layer is real. Fifth layer is complex and six layer is real.

So we can analyse each of the layers.

If the last layer is fully complex then we call the rooted complex value tree to be a tree with imaginary leaves.

If the first layer is complex we call the complex valued tree with imaginary root.

We can also study all types of rooted complex valued trees.

We will not describe the complex valued rooted trees and its constituent complex valued rooted trees.

It is interesting to record here that for a given constituent rooted trees of a complex valued rooted tree need not in be general complex valued.

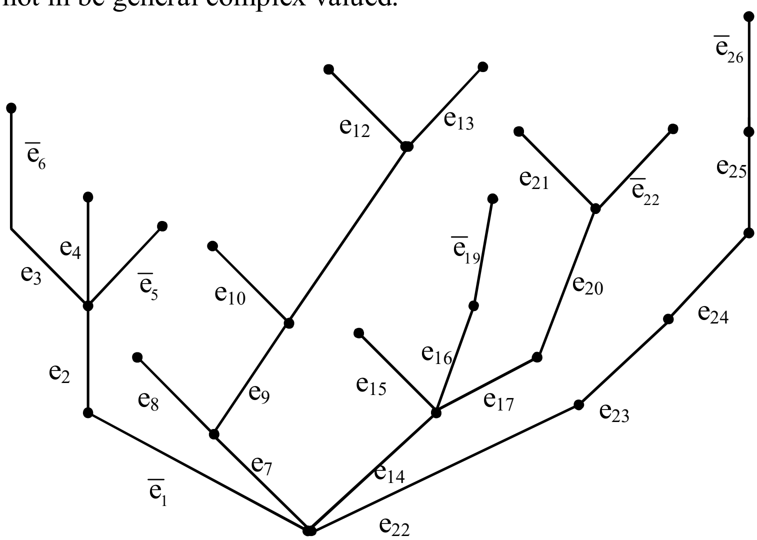


Figure 2.94

The complex valued rooted tree.

We give below the constituent rooted trees.

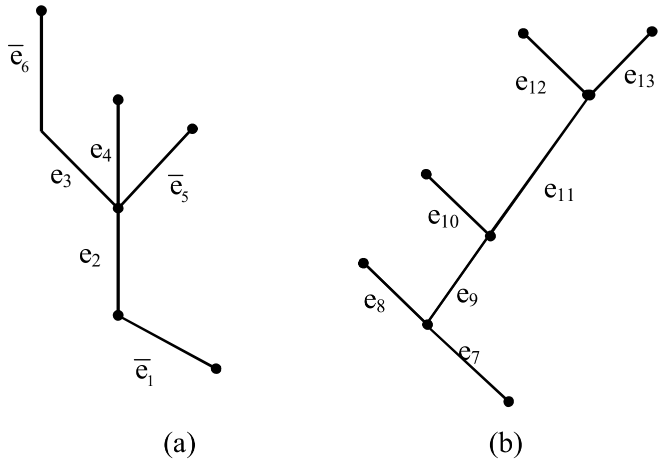


Figure 2.95

Clearly (a) and (b) are two of the four constituent rooted trees of T.

However (a) is a complex valued rooted tree whereas (b) is only a constituent rooted tree not a complex valued one.

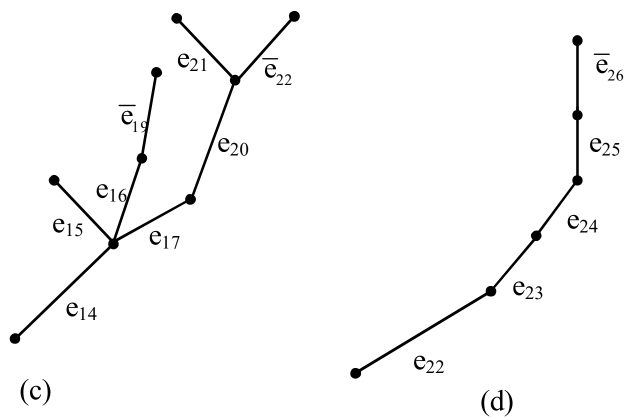


Figure 2.96

Clearly the constituent rooted trees (c) and (d) of T are complex valued rooted tree.

A given complex valued rooted tree T may not be imaginary rooted tree but however some of the constituent rooted trees of T can be a complex valued rooted tree which is imaginary rooted. For a constituent rooted tree to be a imaginary rooted tree it is mandatory the constituent rooted tree must be a complex valued constituent rooted tree. Only the complex valued constituent rooted tree (a) alone is a imaginary rooted tree of the complex valued rooted tree given in Figure 2.94.

We can also for a given tree find the complex valued rooted tree and its quasi dual complex valued rooted tree. It is pertinent to record that for a given complex valued rooted tree we can always find the quasi dual complex valued rooted tree.

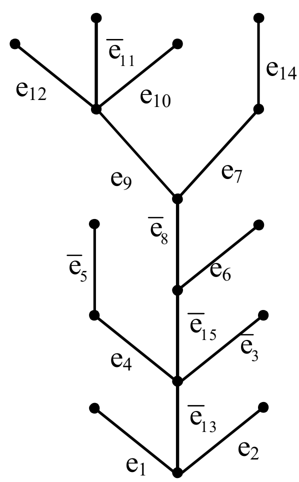


Figure 2.97

Let T be the complex valued rooted tree given in Figure 2.97.

We will now give the quasi dual S of T given in the Figure 2.97.

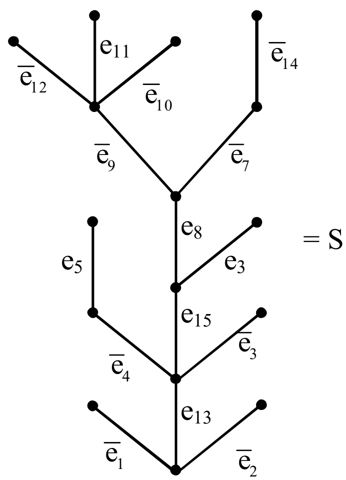


Figure 2.98

Clearly S is the dual of the complex valued rooted tree T and vice versa.

Interested reader can find the constituted complex valued trees of S and T.

Now we illustrate by examples strong complex valued rooted trees and vertex complex valued rooted trees.

Example 2.15. Let M be a complex valued rooted free.

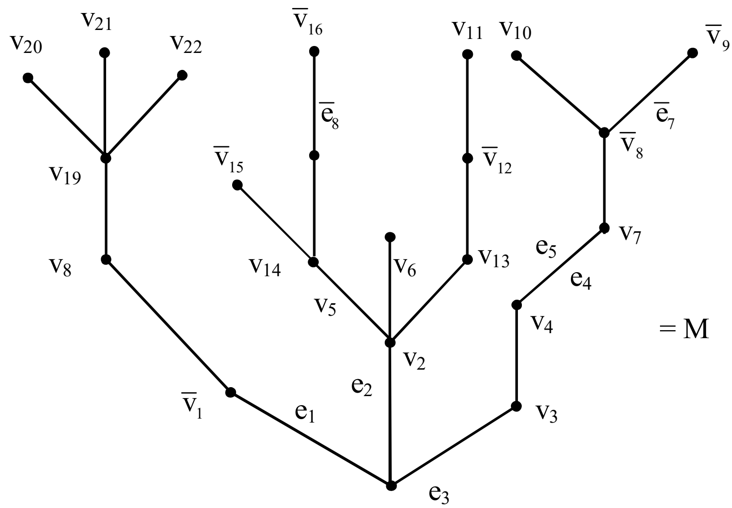


Figure 2.99

Clearly M is a strong complex valued rooted tree given in Figure 2.99.

We now give the vertex complex valued tree. Here a caution is to be made if v_i is connected to v_j then only one of v_i or v_j can be complex vertex both cannot be a complex vertex. Unless this is followed the resultant rooted tree cannot be a vertex complex valued rooted tree.

We will illustrate this situation by an example.

Example 2.16. Let W be a vertex complex valued rooted tree given by the following figure 2.100.

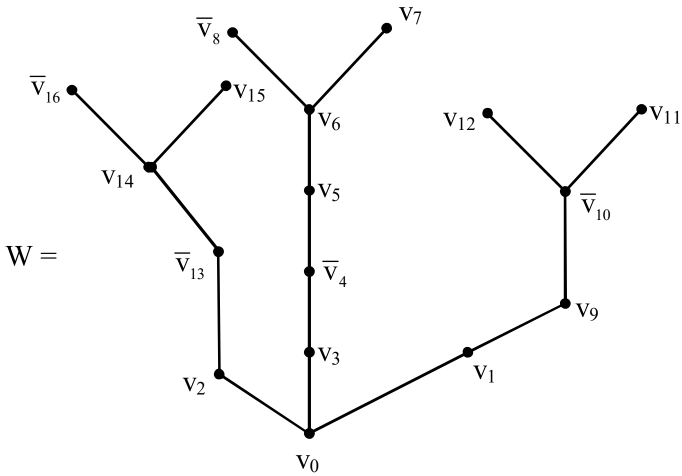


Figure 2.100

Clearly W is only a vertex complex valued rooted tree as all the edge values in this case are given to be only real values.

We cannot define in general quasi dual of a vertex complex valued rooted tree graph for in many cases it may cease to be a vertex complex valued rooted tree graph. So they cannot be called a tree or say complex valued graph itself as the edges connecting two of the imaginary vertices may be real.

So we do not give or define the concept quasi dual of vertex complex rooted trees.

With these limitations we next proceed to define and develop the new notion of complex valued bipartite or bigraph and the related weighted matrix of them.

We also show in the last chapter of this book the probable applications of these concepts.

Next we proceed on the describe bigraph or bipartite graph. We in practical situations may not use a complete bigraph for in models that is not always mandatory.

First we provide some examples of them and then make the routine definition.

Example 2.17. Let H be the complex valued graph given by the following figure. This h as two sets of vertices which form a disjoint classes

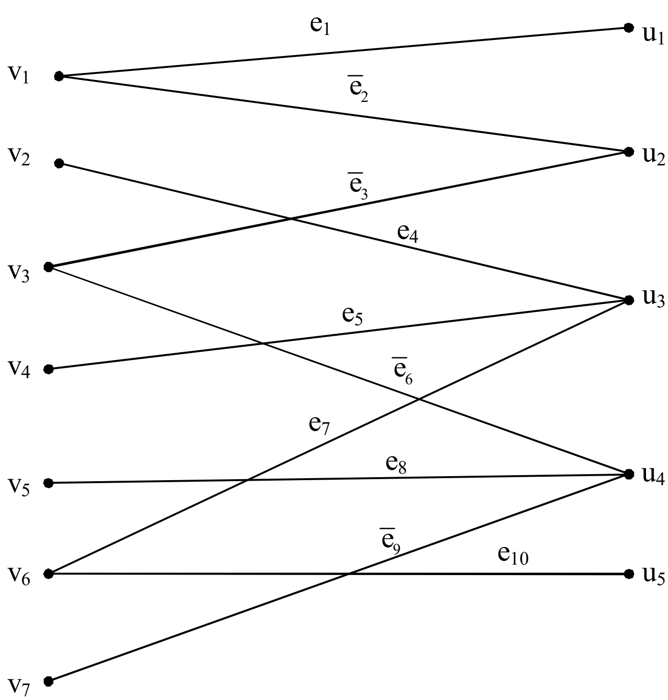


Figure 2.101

The bigraph given in Figure 2.101 is defined as the complex valued bigraph or bipartite graph.

This is not directed we are these types of graphs in fuzzy relation maps, fuzzy relational equations and artificial neural networks.

If we have only feed forward neural networks then we will use the directed graphs only in one direction if they are both ways then there is no meaning in calling them as directed graph.

However it is pertinent to keep on record that the complex valued bipartite graph or bigraph have not been described or defined in literature.

The main purpose of describing such complex valued bigraphs are for we have given enough representation to the concept of indeterminancy but however we have totally failed to study the concept imaginary in real valued problems.

For take the simple case of personality text and the related results.

The person who is taking the personality test may aspire to live in a utopian set of personality but in reality he may be a lazy, poorly performed and not with good degrees or anything as a positive credit.

But he may be a criminal or culprit so in order to get the job becomes a imaginary character of personification and answers the test as an ideal person.

So each and every question he has answered is an imaginary one here we cannot call it as an indeterminate one as he has lived or mocked a perfect personality to get the job and has answered the personality test.

So here the concept of imaginary or complex valued graph will play a role.

For the first question they will put forth is whether the answers imaginary or real.

They have to text him differently to map his real personality from the imaginary personality.

In another place where the complex valued graphs can play a vital role in in relationships.

A couple may have imaginary grievances or in a love relationship they may have imaginary feelings which may not have been at any time construed in that way.

In such study also complex valued graphs may play a vital role.

It is important to note that while studying fuzzy model a study of indeterminate was included and studied but however in no place the imaginary concept was discussed or used in these models.

Someone may like to study a literary work using mathematical models. Then the notion of imaginary concept becomes mandatory.

Now we give the complex valued directed bigraphs by some examples.

Example 2.17. Let B denote the complex valued directed graph given by the following figure;

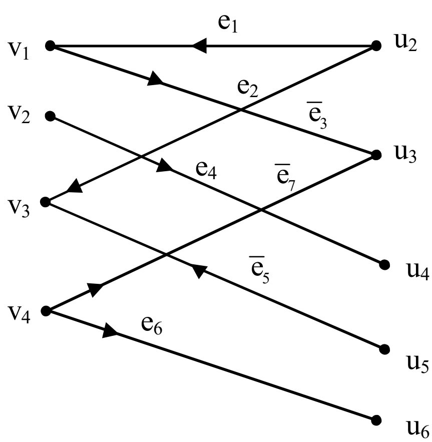


Figure 2.102

Now we give the method of finding the weight matrix of the complex valued bigraph by an example.

Example 2.18. Let B the complex valued bigraph given by the following Figure.

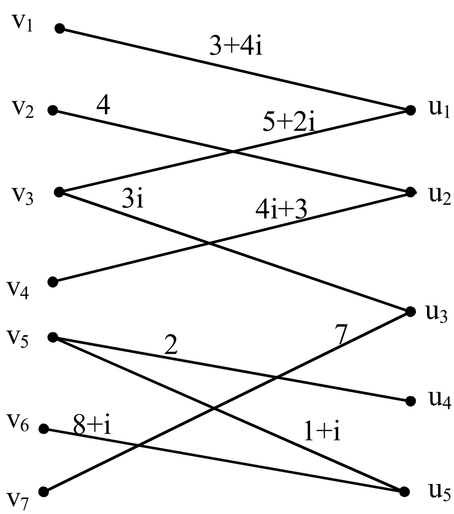


Figure 2.103

Let M be the 7 × 5 matrix associated with the complex valued graph.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} 3+4i & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 5+2i & 0 & 3i & 0 & 0 \\ 0 & 4i+3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1+i \\ 0 & 0 & 0 & 0 & 8+i \\ 0 & 0 & 7 & 0 & 0 \end{bmatrix} \end{array}$$

Thus the related matrix M of a complex valued bigraph is a matrix with complex entries.

Appropriate operations can be done in these connected or weight matrices which will be discussed and described in the last chapter of this book.

Now we study by an example how a directed bigraph is constructed and different type of matrix is needed and if both are the same.

This is just to show the real situation of directed bigraphs has no meaning for we only have to describe them. This will have meaning when both input nodes (vertices) and output vertices are the same.

Now having seen complex valued graphs, complex valued trees and complex valued bigraphs we proceed onto study a situation where both vertices and edges can be imaginary.

It is mandatory that if the vertices are imaginary then we cannot have the edges to be real they will also be imaginary. First we provide an example or two of this situation before we proceed onto describe them abstractly.

It is important to note the following notation.

We have so far used $\overline{e}_i$ to denote the edge $\overline{e}_i$ is imaginary. So always v_i denotes the real vertex then $\overline{v}_j$ will denote the imaginary vertex.

Example 2.19. Let G be a complex valued graph which has some of its vertices to be imaginary also.

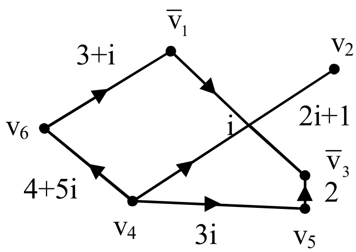


Figure 2.104

The following observations are mandatory $\overline{v}_i$ to another imaginary vertex $\overline{v}_j$ is always imaginary.

However we can have real edges from a imaginary vertex to a real vertex and vice versa. This is the simple condition which we have to follow.

Example 2.20. Let G be the complex valued graph given by the following Figure.

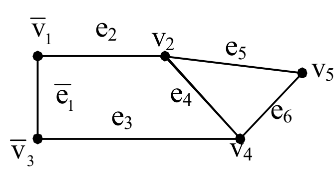


Figure 2.105

Now we make the following definition.

Definition 2.5. *Let G be a graph which has some of vertices $\overline{v}_i$ to be imaginary and v_j to be some real vertices. Some edges are imaginary and some of them real.*

Then we define G to be a strong complex valued graph. We also call it asvertex complex graph.

We call it vertex complex graph if only some vertices are imaginary and all the edges are real.

We will first illustrate this situation by some examples.

Example 2.21. G be the graph given by the following figure.

G =

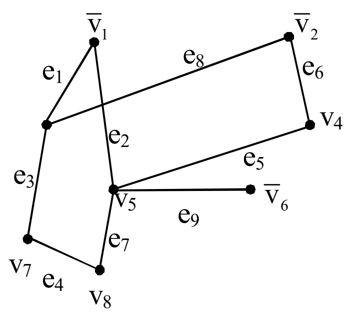


Figure 2.106

We see the imaginary vertices in this graph are $\overline{v}_1$, $\overline{v}_2$ and $\overline{v}_6$ and they are not connected with each other so we see all the remaining edges are only given to be real.

In this situation we define G to be a vertex complex graph. Clearly G is not a complex valued graph.

In view of all these we have the following results.

Theorem 2.6. *Let G be a strong complex valued graph then G is a complex valued graph.*

Proof is direct and hence left as an exercise to the reader.

However if G is a graph with only complex vertices and none of the edges are complex valued then we define G to be just a complex vertex graph and it is not a strong complex valued graph.

Converse of theorem is not true for a complex valued graph h as no complex vertices.

Now we can also have the notion of strong complex valued bigraphs which we will illustrate by some examples.

Further we have complex vertex bigraphs also which is neither complex valued nor a strong complex valued graph.

Example 2.22. Let G be a strong complex valued graph given by the following figure;

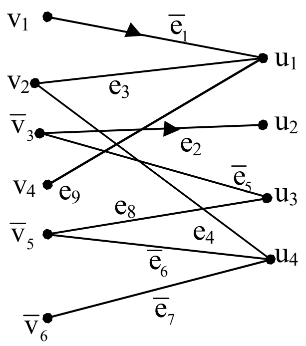


Figure 2.107

Clearly G is a strong complex valued bigraph as G has both complex vertices as well as complex valued edges.

Example 2.23. Consider the following example given in Figure 2.108. A graph H which is a bigraph.

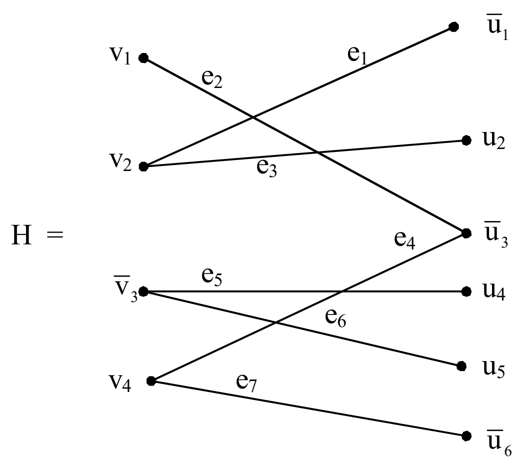


Figure 2.108

Clearly H is not a complex valued bigraph as none of the edges is imaginary. So H is not a strong complex valued bigraph. Further some of the vertices are imaginary so H is a complex vertex bigraph. Hence the claim.

Now we can in case of strong complex valued bigraphs give the adjacency matrix associated with it.

Further the weight matrix in case of them will also be provided. We give examples of weight matrix first in case of a strong complex valued graph and strong complex valued directed graph in the following.

***Example 2*.24** Let G be a strong complex valued directed graph given by the following figure 2.109;

G =

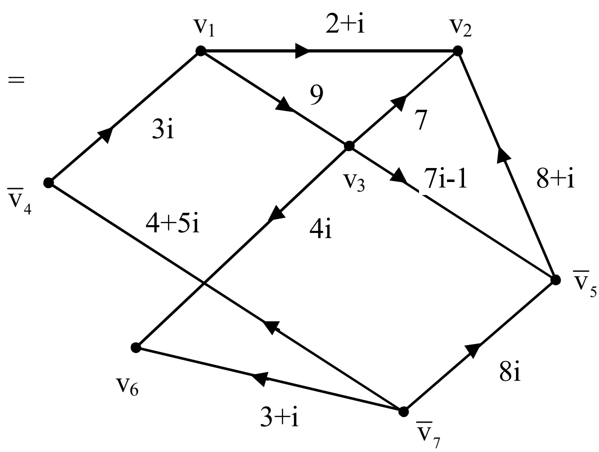


Figure 2.109

Let M be the weight matrix associated with the strong complex valued directed graph G which is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \bar{v}_4 \\ \bar{v}_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 2+i & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 & 7i-1 & 4\bar{i} & 0 \\ 3i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8+i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4+5i & 8i & 3+i & 0 \end{bmatrix} \end{array}.$$

The following observations are mandatory.

i) As there are no loops we see the main diagonal entries of M are zero. Further M is a square matrix.

ii) The matrix M is not symmetric about the main diagonal as the strong complex valued graph G is only a directed graph.

Next we proceed onto give example of strong complex valued graph H which is not a directed graph.

Example 2.25. Let H be the strong complex valued graph given by the following figure 2.111.

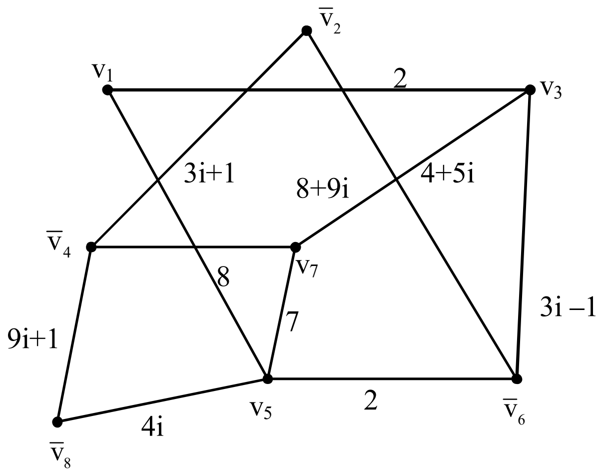


Figure 2.110

Clearly H has 8 vertices of which four are complex and the rest are real. The weight matrix P associated with the strong complex valued graph H is as follows.

$$P = \begin{array}{c} \\ v_1 \\ \bar{v}_2 \\ v_3 \\ \bar{v}_4 \\ v_5 \\ \bar{v}_6 \\ v_7 \\ \bar{v}_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & \bar{v}_2 & v_3 & \bar{v}_4 & v_5 & \bar{v}_6 & v_7 & \bar{v}_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3i+1 & 0 & 4+5i & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 3i-1 & 8+9i & 0 \\ 0 & 3i+1 & 0 & 0 & 0 & 0 & 8 & 9i+1 \\ 8 & 0 & 0 & 0 & 0 & 2 & 7 & 4i \\ 0 & 4+5i & 3i-1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 8+9i & 8 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9i+1 & 4i & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We make the following observations.

i) P is a square matrix

ii) As there are no loops P has its diagonal entries to be zero.

iii) P is symmetric about the main diagonal

We can easily observe the difference between strong complex directed graph and the strong complex valued graph.

Now we proceed onto describe complex vertex valued graph and the weighted matrix associated with them.

Example 2.26. Let K be the complex vertex valued graph given by the following Figure 2.110.

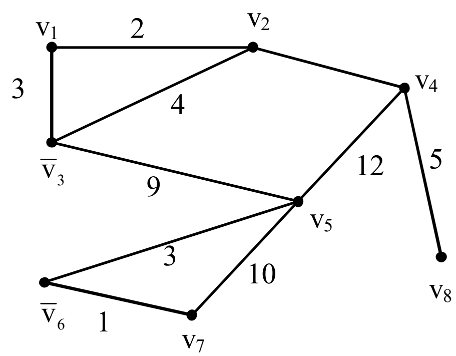


Figure 2.111

Let B be the weight matrix associated with the complex vertex valued graph K.

$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ \bar{v}_3 \\ v_4 \\ v_5 \\ \bar{v}_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & \bar{v}_3 & v_4 & v_5 & \bar{v}_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 4 & 7 & 0 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 9 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 9 & 12 & 0 & 3 & 10 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 10 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

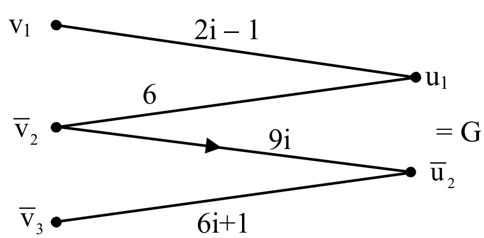
i) Clearly B is a square real matrix with diagonal entries to be zero as these are no loops in the graph K.

ii) B is symmetric about the origin as K is not a complex vertex valued directed graph.

iii) Clearly B is only a real matrix as K is not a strong complex valued graph.

Next we proceed onto describe the weight matrix of a strong complex valued bigraph and complex vertex valued bigraph in the following.

Let G be the strong complex valued bigraph given bythe following Figure 2.112.



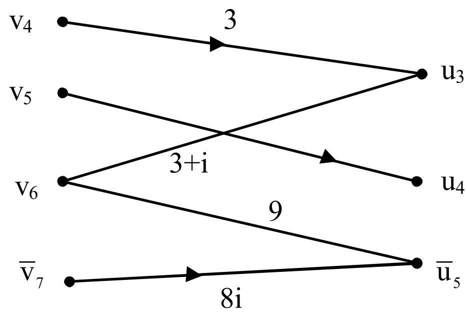


Figure 2.112

Let W be the weight matrix associated with the strong complex valued bigraph.

$$W = \begin{array}{c} \\ v_1 \\ \bar{v}_2 \\ \bar{v}_3 \\ v_4 \\ v_5 \\ v_6 \\ \bar{v}_7 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & \bar{u}_2 & u_3 & u_4 & \bar{u}_5 \end{array} \\ \begin{bmatrix} 2i-1 & 0 & 0 & 0 & 0 \\ 6 & 9i & 0 & 0 & 0 \\ 0 & 6i+1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2+i & 0 \\ 0 & 0 & 3+i & 0 & 9 \\ 0 & 0 & 0 & 0 & 8i \end{bmatrix} \end{array}.$$

Clearly W is a complex valued matrix which is a 7 × 5 matrix.

Example 2.27. Let H be a complex vertex value at bigraph given by the following Figure 2.113.

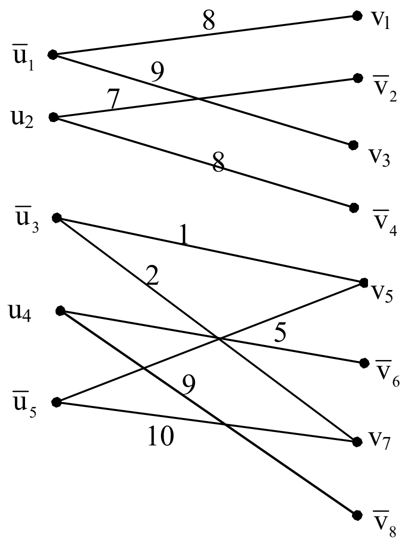


Figure 2.113

Let R be the weight matrix associated with the complex vertex valued bigraph H.

$$R = \begin{array}{c} \\ \bar{u}_1 \\ u_2 \\ \bar{u}_3 \\ u_4 \\ \bar{u}_5 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & \bar{v}_2 & v_3 & \bar{v}_4 & v_5 & \bar{v}_6 & v_7 & \bar{v}_8 \end{array} \\ \begin{bmatrix} 8 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 9 \\ 0 & 0 & 0 & 0 & 0 & 5 & 10 & 0 \end{bmatrix} \end{array}.$$

Clearly the weight matrix is a 5×8 matrix with only real entries so all the edge weights are real.

Let W be a complex valued graph and V be the quasi dual of W given by the following Figures.

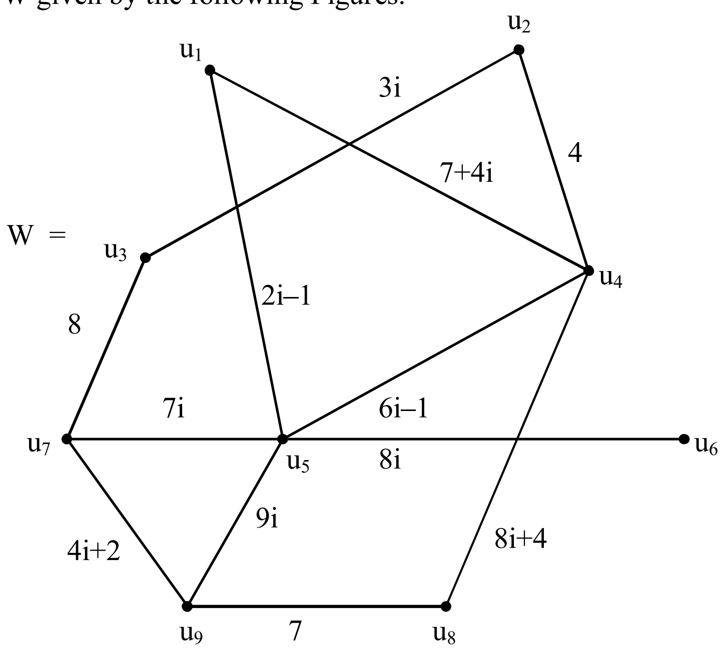


Figure 2.114

Let V be the quasi dual of the complex valued graph W given by the following figure 2.115.

V =

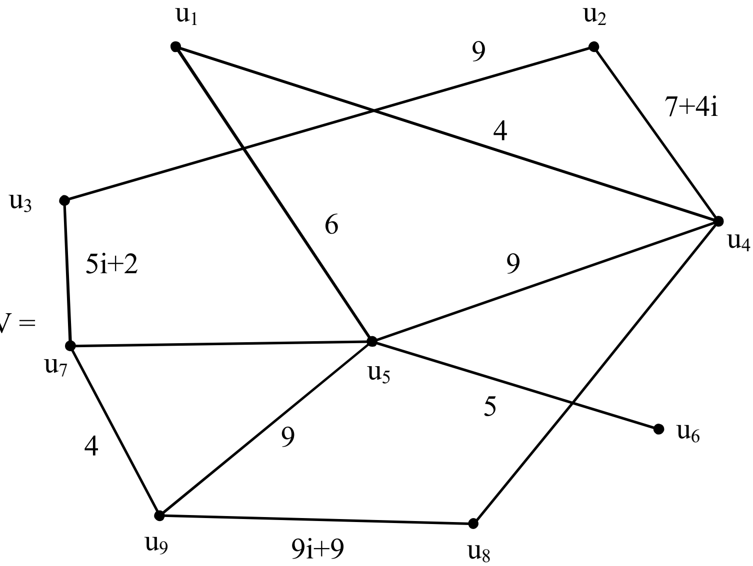


Figure 2.115

We now give the matrices P and Q of W and V respectively in the following

$$P = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \left[\begin{array}{ccccc} 0 & 0 & 0 & 7+4i & 2i-1 \\ 0 & 0 & 3i & 4 & 0 \\ 0 & 3i & 0 & 0 & 0 \\ 7+4i & 0 & 0 & 0 & 7i-1 \\ 2i-1 & 0 & 0 & 6i-1 & 0 \\ 0 & 0 & 0 & 0 & 6i-1 \\ 0 & 0 & 0 & 0 & 7i \\ 0 & 0 & 8 & 8i+4 & 0 \\ 0 & 0 & 0 & 0 & 9i \end{array} \right. \end{array}$$

$$\begin{array}{cccc} u_6 & u_7 & u_8 & u_9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 8i+4 & 0 \\ 6i-1 & 7i & 0 & 9i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4i+2 \\ 0 & 0 & 0 & 7 \\ 0 & 4i+2 & 7 & 0 \end{array}\Bigg] .$$

Clearly P is a symmetric matrix with diagonal entries zero.

Now we give the matrix Q associated with V.

Q =

$$\begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{array}
\begin{array}{c} \begin{array}{ccccccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 \end{array} \\
\begin{bmatrix} 0 & 0 & 0 & 4 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 7+4i & 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 & 5i+2 & 0 & 0 \\ 4 & 7+4i & 0 & 0 & 9 & 0 & 0 & 2 & 0 \\ 6 & 0 & 0 & 9 & 0 & 5 & 8 & 0 & 9 \\ 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5i+2 & 0 & 8 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 9i+9 \\ 0 & 0 & 0 & 0 & 9 & 0 & 4 & 9i+9 & 0 \end{bmatrix} \end{array}$$

Q is also a symmetrix complex valued matrix with diagonal entries zero.

It is pertinent to make the following observations.

i) The matrices P and Q are both 9×9 square matrices and symmetric about the origin.
ii) Let $P = (a_{ij})$ and $Q = (b_{ij})$ we see if a_{ij} is complex then b_{ij} is real as P and Q are quasi dual complex valued matrices $i \neq j$, $1 \leq i, j \leq 9$.

Next we define conjugate of complex valued graphs and bigraphs.

Before we proceed onto define the above said concepts we wish to bring out the following facts.

Suppose A is any symmetric $n \times n$ matrix with diagonal entries as zero and entries are either zero or complex or real numbers then we can associate a complex valued graph with no loops.

We will just illustrate this situation by a line or two.

Example 2.28. Let A be a 4×4 matrix given in the following.

$$A = \begin{bmatrix} 0 & 3+i & 2 & 0 \\ 3+i & 0 & 0 & 1+4i \\ 2 & 0 & 0 & 12 \\ 0 & 1+4i & 12 & 0 \end{bmatrix}$$

We see A is a complex valued matrix. Now corresponding to this matrix A we have the following complex valued graph G described by the following figure 2.115.

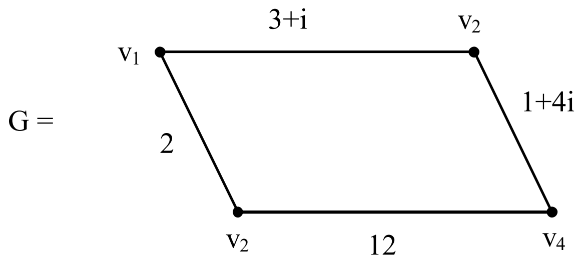


Figure 2.116

Now we try to get the weighted matrix M associated this complex valued graph G in thefollowing;

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 3+i & 2 & 0 \\ 3+i & 0 & 0 & 1+4i \\ 2 & 0 & 0 & 12 \\ 0 & 1+4i & 12 & 0 \end{bmatrix} \end{array}.$$

It is clearly seen the matrix A and M are identical. Next we give yet another example.

$$\text{Let } B = \begin{bmatrix} 0 & 3 & 1+i & 0 & 2i & 1 \\ 3 & 0 & 2 & 1+i & 0 & 5i \\ 1+i & 2 & 0 & 0 & 4 & 1+2i \\ 0 & 1+i & 0 & 0 & 1+i & 0 \\ 2i & 0 & 4 & 0 & 0 & 7 \\ 1 & 5i & 1+2i & 1+i & 7 & 0 \end{bmatrix}$$

We the 6 × 6 complex valued matrix. We get the corresponding complex valued graph which is as follows.

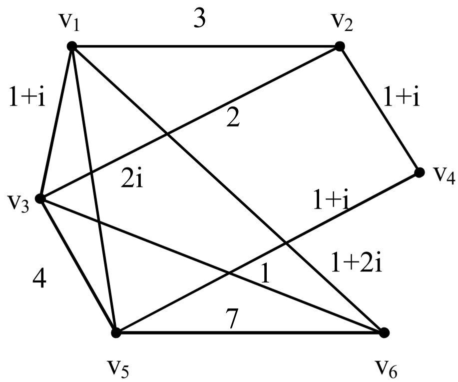


Figure 2.117

Clearly if we obtain the weighted matrix associated with the complex valued graph K then we see the weighted matrix and the complex valued matrix B are identical. In view of all these we give the following theorem.

Theorem 2.7. *Let M be a n × n symmetric complex valued matrix with diagonal entries as zero. Then there is a unique complex valued graph with n vertices whose weighted matrix is identical with M and vice versa.*

Proof is direct and hence left as an exercise to the reader.

Now let us consider a 7 × 7 complex valued matrix D whose diagonal terms are zero where

$$D = \begin{bmatrix} 0 & 2 & 2+i & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 5i & 0 & 3+7i & 0 \\ 0 & 2i & 0 & 3i & 1 & 0 & 5+2i \\ 4+i & 0 & 0 & 0 & 0 & 4i & 2 \\ 1 & 4i+2 & 1+i & 3i & 0 & 1+i & 0 \\ 3i & 0 & 2i+5 & 0 & 0 & 0 & 7 \\ 4 & 7i+1 & 0 & 5i & 2+i & 2i+1 & 0 \end{bmatrix}$$

Now using this D as the weighted matrix we can obtain a complex valued directed graph H with seven vertices given by the following figure.

H =

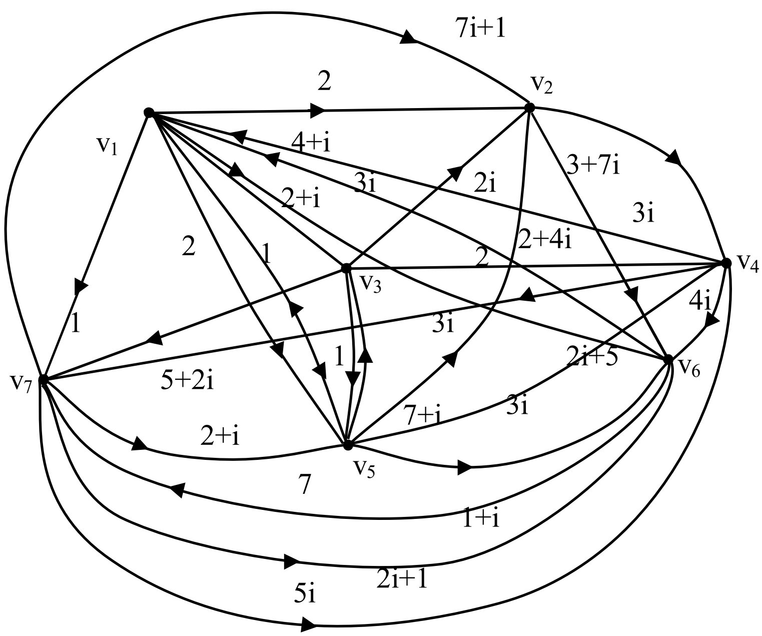


Figure 2.118

It is easily verified that we see the weighted matrix M of the complex valued graph H given in figure is such that M = D.

Hence the claim clearly H is a directed graph so only the matrix D is not symmetric about the main diagonal. However as all the main diagonal entries are zero we see the complex valued directed graph got using D has no loops.

Before we make relevant conclusions give yet another simple example.

Let E be a 5 × 5 complex valued matrix with all the main diagonal entries to be zero given in the following.

$$E = \begin{bmatrix} 0 & 2+i & 0 & 0 & 7 \\ 0 & 0 & 7+i & 2 & 0 \\ 3 & 0 & 0 & 0 & 5+i \\ 6+i & 0 & 1+i & 0 & 0 \\ 0 & 4 & 0 & 5i+2 & 0 \end{bmatrix}.$$

Now the complex valued graph S associated with the complex valued matrix E as the weighted matrix is as follows.

S =

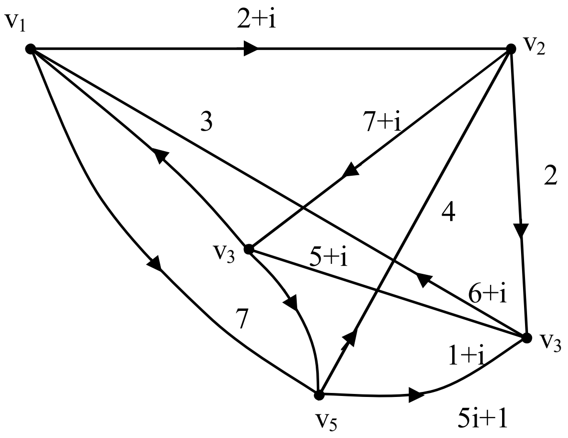


Figure 2.119

Clearly the weighted matrix of the complex valued directed graph S will have the matrix identical with the complex valued matrix E.

In view of all these we have the following theorem.

Theorem 2.8. *Let B a n × n complex valued matrix all of whose diagonal entries are zero and the matrix B is not symmetric about the diagonal. To each such B there is a complex value directed graph H and vice versa.*

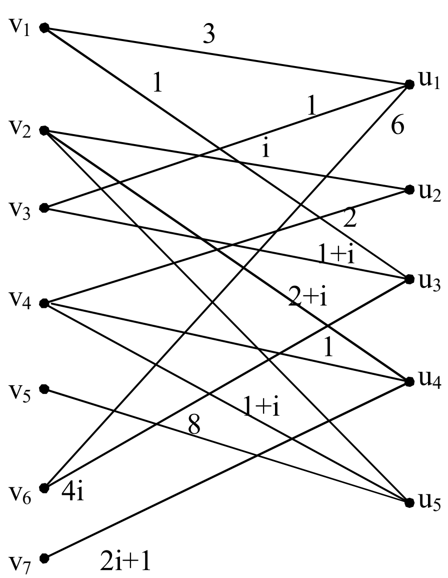
Proof is direct and hence left as an exercise to the reader.

Now we this concept proceed to define and develop similar concept for complex valued bigraph in the following.

Suppose we have a 7 × 5 complex valued matrix P given in the following.

$$P = \begin{bmatrix} 3 & 0 & 1 & 0 & 0 \\ 0 & i & 0 & 2+i & 1 \\ 1 & 0 & 1+i & 0 & 0 \\ 0 & 2 & 0 & 1 & 1+i \\ 0 & 0 & 0 & 0 & 8 \\ 6 & 0 & 4i & 0 & 0 \\ 0 & 0 & 0 & 2i+1 & 0 \end{bmatrix}$$

we now show we can have a unique complex valued bigraph associated with it.



= K

Figure 2.120

It is easily seen that the weight matrix of the K is identical with the matrix P.

We now consider the complex valued matrix Q is as follows.

$$Q = \begin{bmatrix} 2+i & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 8+i \\ 0 & 0 & 1+i & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 7i & 0 & 0 & 0 & 2 & 0 \\ 0 & 5 & 0 & 0 & 8+i & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1+i & 0 & 1 & 6 \\ 0 & 1+2i & 0 & 9 & 1+3i & 0 & 7 & 0 & 0 \\ 9i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 1+i & 0 & 0 & 0 & 1+i & 0 \end{bmatrix}$$

Clearly Q is a complex valued matrix.

Now we obtain the complex valued bigraph H associated with the complex valued matrix Q.

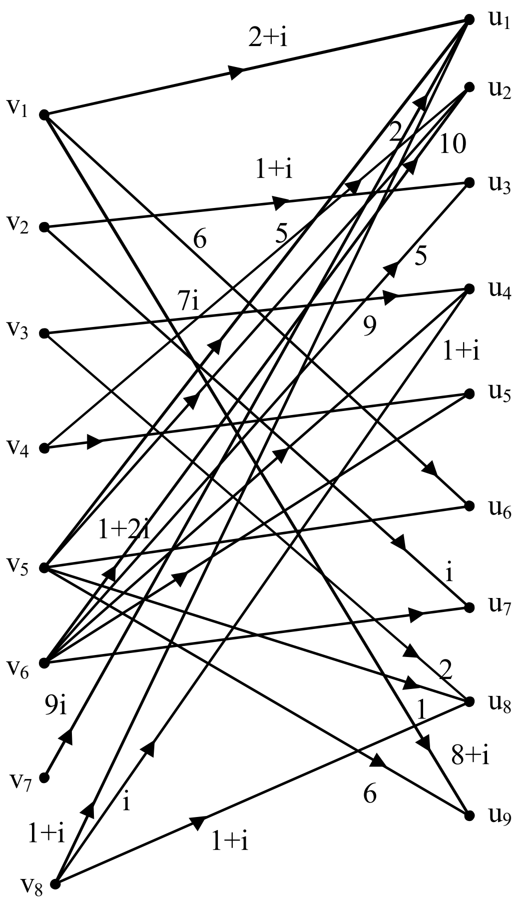


Figure 2.121

It is easily verified that the weight matrix associated with the complex valued bigraph H is a complex valued matrix identical with Q.

In view of all these we give the following result.

Theorem 2.9. *Let M be a m × n matrix (m ≠ n) which is complex valued. Related with M is a complex valued bigraph G for which M serves as the weight matrix of G and vice versa.*

The reader is left with the task of proving the result.

It is interesting note that only in case of complex valued graphs with weights we have a provision to define the conjugate of the complex valued graph.

We will first illustrate this situation by some examples.

Example 2.29. Let G be the complex valued graph with weights be given by the following figure. We first find the weight matrix M which is complex valued.

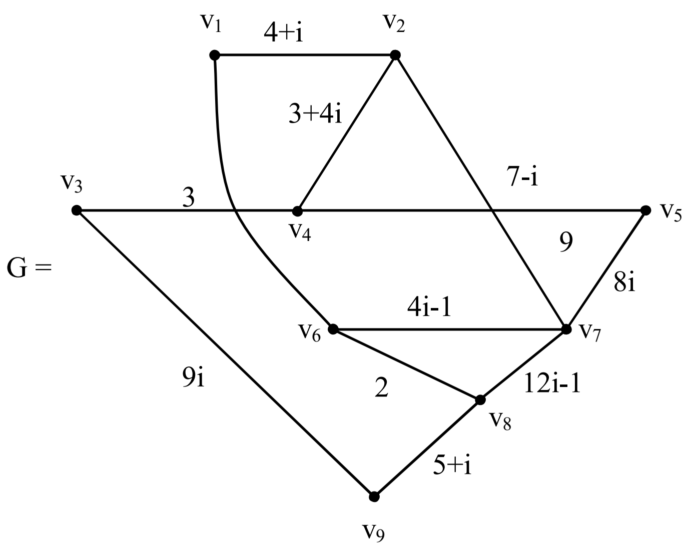


Figure 2.122

The complex valued weight matrix M associated with the complex valued graph G is as follows.

$$
M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 \end{array} \\
\begin{bmatrix}
0 & 4+i & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
4+i & 0 & 0 & 3+4i & 0 & 0 & 9i & 0 & 0 \\
0 & 0 & 0 & 5+2i & 0 & 0 & 0 & 0 & 9i \\
0 & 3+4i & 5+2i & 0 & 9 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 9 & 0 & 0 & 8i & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 9 & 0 & 0 & 8i & 0 & 0 & 12i-1 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 12i-1 & 0 & 5+i \\
0 & 0 & 9i & 0 & 0 & 0 & 0 & 5+i & 0
\end{bmatrix}
\end{array}
$$

is the complex valued weight matrix of the complex valued graph G.

We define complex conjugate of the complex valued graph G denoted by $\overline{G}$ is got by finding the complex conjugate of the complex valued matrix M, since M is symmetric about the main diagonal are need not get the transpose of it just the complex conjugate will do. Thus in case of a undirected complex valued graph G we see its complex conjugate is just the complex valued graph got using the conjugate matrix of M.

Now we find the complex conjugate of the matrix M say $\bar{M}$.

$$\bar{M} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 \end{array} \\ \left[\begin{array}{ccccccccc} 0 & 4-i & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 4-i & 0 & 0 & 3-4i & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 5-2i & 0 & 0 & 0 & 0 & -9i \\ 0 & 3-4i & 5-2i & 0 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & -8i & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 9 & 0 & 0 & -8i & 0 & 0 & -12i-1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -12i-1 & 0 & 5-i \\ 0 & 0 & -9i & 0 & 0 & 0 & 0 & 5-i & 0 \end{array}\right] \end{array}$$

We see $\overline{M}$ is just the complex conjugate of M.

Now we proceed onto give the complex valued graph H = ($\overline{G}$) associated with $\overline{M}$ the matrix in the following.

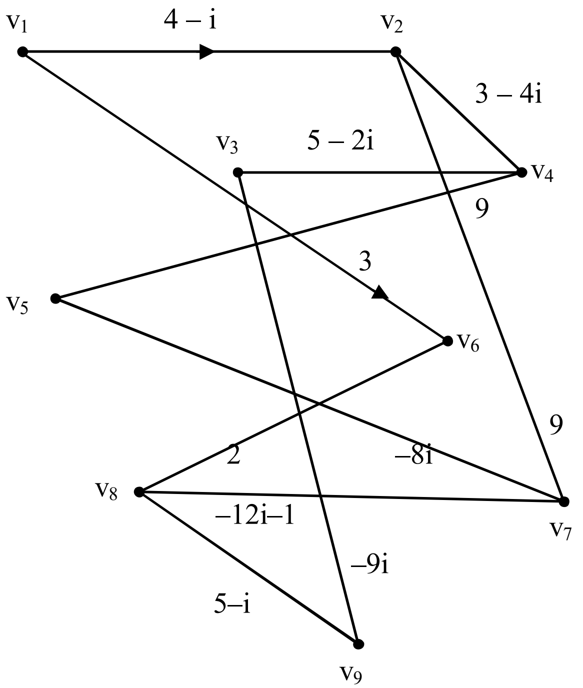


Figure 2.123

We see both the graphs look alike only there are changes in the edge weights.

Let H be the complex valued graph K given by the following Figure.

K =

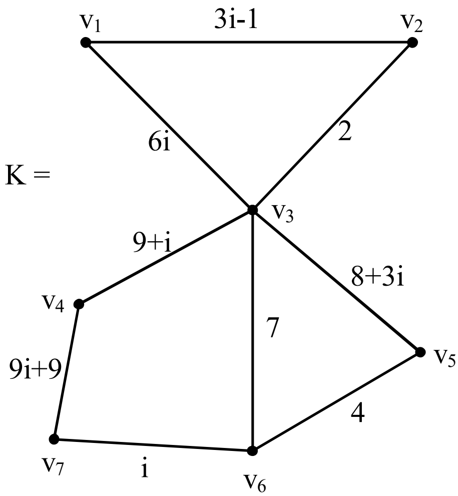


Figure 2.124

The weight matrix M associated with the complex valued graph K is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 3i-1 & 6i & 0 & 0 & 0 & 0 \\ 3i-1 & 0 & 2 & 0 & 0 & 0 & 0 \\ 6i & 2 & 0 & 9+i & 8+3i & 7 & 0 \\ 0 & 0 & 9+i & 0 & 0 & 0 & 9+9i \\ 0 & 0 & 8+3i & 0 & 0 & 4 & 0 \\ 0 & 0 & 7 & 0 & 4 & 0 & i \\ 0 & 0 & 0 & 9+9i & 0 & i & 0 \end{bmatrix} \end{array}$$

Now we find the complex conjugate of M. Let $\overline{M}$ be the complex conjugate of M.

$$\bar{M} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & -1-3i & -6i & 0 & 0 & 0 & 0 \\ -1-3i & 0 & 2 & 0 & 0 & 0 & 0 \\ -6i & 2 & 0 & 9-i & 8-3i & 7 & 0 \\ 0 & 0 & 9i & 0 & 0 & 0 & 9-9i \\ 0 & 0 & 8-3i & 0 & 0 & 4 & 0 \\ 0 & 0 & 7 & 0 & 4 & 0 & -i \\ 0 & 0 & 0 & 9-9i & 0 & -i & 0 \end{bmatrix} \end{array}.$$

Now we give the complex valued graph associated with $\bar{M}$ which will be known as the complex conjugate graph $\bar{K}$ of the graph K which is the complex conjugate of the complex valued graph K.

$\bar{K}$ =

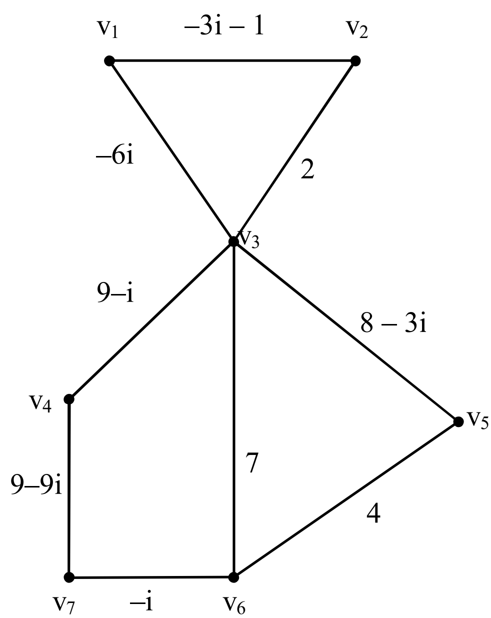


Figure 2.125

Which is the complex conjugate of the complex valued graph K.

In view of all this we have the following result.

Theorem 2.10. *Let K be any complex valued graph. If M is the complex valued matrix associated with K and* $\overline{M}$ *the complex conjugate of M then* $\overline{K}$ *is the corresponding conjugate complex valued graph associated with* $\overline{M}$ *.*

So to every complex valued graph G we have the conjugate complex valued graph and $\overline{G}$ vice versa.

Now we proceed onto find the complex valued directed graph.

Example 2.30. Let G be a complex valued directed graph given by the following Figure.

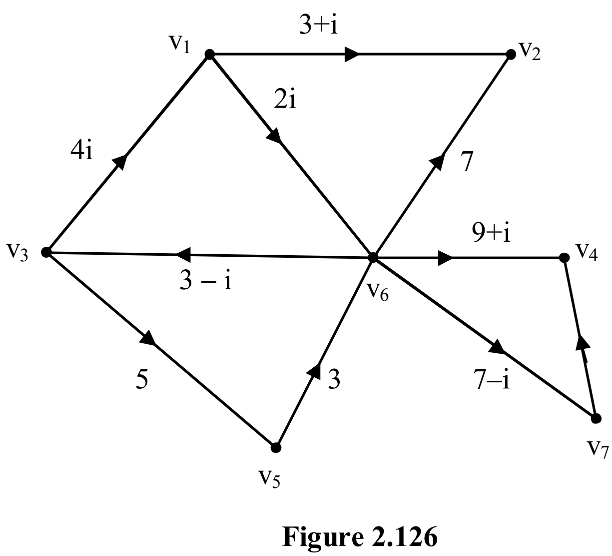


Figure 2.126

Let M be the complex valued weighted matrix associated with G given in the following

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 3+i & 0 & 0 & 0 & 2i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 3 & 0 \\ 0 & 7 & 3-i & 9+i & 0 & 0 & 7-i \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The complex conjugate matrix $\overline{M}$ of M is as follows.

$$\overline{M} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & -4i & 0 & 0 & 0 & 0 \\ 3-i & 0 & 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 5 & 3+i & 0 \\ 0 & 0 & 0 & 0 & 0 & 9-i & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2i & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7+i & 0 \end{bmatrix} \end{array}$$

Now we find the complex valued directed graph H associated with $\overline{M}$.

H =

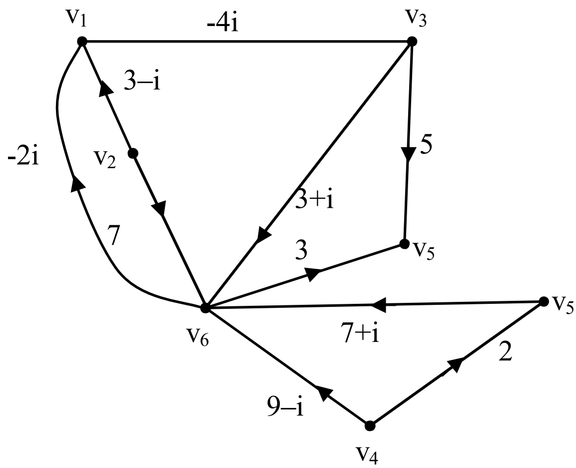


Figure 2.127

H is define as the complex conjugate directed graph of the complex valued directed graph G and usually it is denoted by $\bar{G}$

We just give one more example of this situation.

Example 2.31. Let K be the complex valued directed graph given by the following Figure.

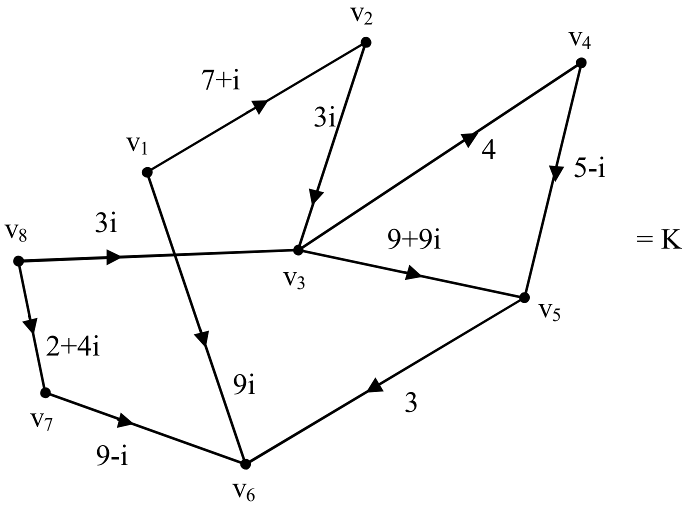


Figure 2.128

Let S be the weighted complex valued matrix associated with the complex valued directed graph K.

$$S = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 7+i & 0 & 0 & 0 & 9i & 0 & 0 \\ 0 & 0 & 3i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 9+9i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5-i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9-i & 0 & 0 \\ 0 & 0 & 3i & 0 & 0 & 0 & 2+4i & 0 \end{bmatrix} \end{array}$$

It is clear S is only a complex valued matrix with diagonal entries zero. However S is not a symmetric matrix. Now we find the conjugate of the complex valued matrix S.

Further as the diagonal entries are zero both K and the complex conjugate directed graph of K will have no loops.

Further $\overline{S} = (\overline{S}^t)$.

We now find the transpose conjugate of S and denote it by S-.

$$\overline{S} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7-i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3i & 0 & 0 & 0 & 0 & 0 & -3i \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9-9i & 5\_i & 0 & 0 & 0 & 0 \\ -9i & 0 & 0 & 0 & 3 & 0 & 9+i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2-4i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

We now describe the complex conjugate directed graph $\overline{K}$ associated with $\overline{S}$ by the following Figure.

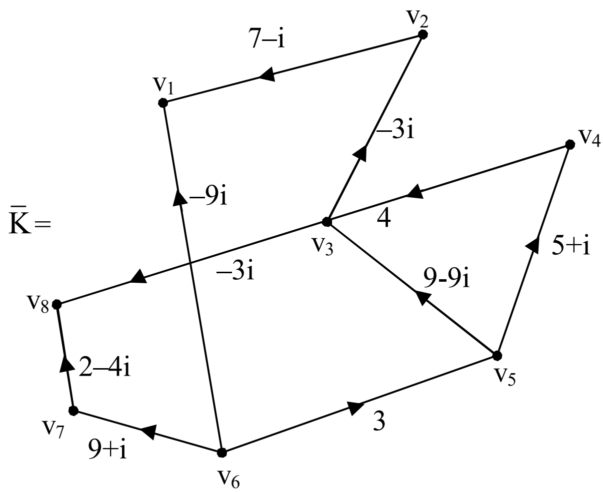


Figure 2.129

We make the following observation when we compare K and $\bar{K}$ we see the direction are oriented in the opposite way from vertex to vertex. For instance in K we have from vertex v_1 to v_2 with edge weight 7 + i where as in $\bar{K}$ we see the direction is from vertex v_2 to v_1 with edge weight 7 - i.

Similar change in orientation of each and every node is observed. Also the complex values in $\bar{K}$ are the complex conjugate of the values in K.

Thus we see given any complex valued directed graph K we can always get a conjugate complex directed graph $\bar{K}$ of K.

Next we study the complex conjugate of a complex valued bigraph G.

Here also we wish to keep on record given any s × t complex matrix we can always find the related complex valued bigraph and vice versa.

Let P be the complex valued weighted bigraph given by the following Figure.

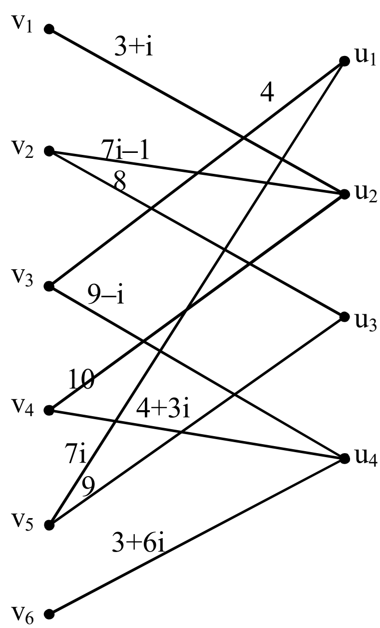


Figure 2.130

The related complex valued weighted matrix M associated with the complex valued bigraph P is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\ \begin{bmatrix} 0 & 3+i & 0 & 0 \\ 0 & 7i-1 & 8 & 0 \\ 4 & 0 & 0 & 9-i \\ 0 & 10 & 0 & 4+3i \\ 7i & 0 & 9 & 0 \\ 0 & 0 & 0 & 3+6i \end{bmatrix} \end{array}$$

Now M is the weight complex valued matrix of the complex valued weighted bigraph P.

We find $\overline{M}$ the complex conjugate of M.

$$\overline{M} = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 4 & 0 & -7i & 0 \\ 3-i & -1+7i & 0 & 10 & 0 & 0 \\ 0 & 8 & 0 & 0 & 9 & 0 \\ 0 & 0 & 9+i & 4-3i & 0 & 3-6i \end{bmatrix} \end{array}$$

Now $\overline{M}$ is a complex valued matrix the complex valued weighted bigraph $\overline{P}$ associated with $\overline{M}$ is as follows.

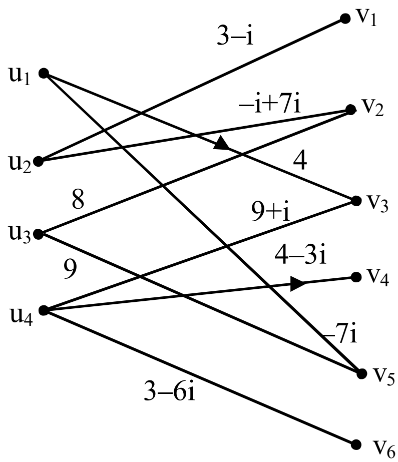


Figure 2.131

We see in case of complex valued bigraph P the complex conjugate of P yield a 4×6 complex valued weighted matrix.

Study in this direction is interesting and this work is left as an exercise to the reader.

Now one is interested in the following if G is a complex valued weighted graph with associated complex valued matrix M and $\bar{G}$ the complex conjugate graph of G and $\bar{M}$ the conjugate complex valued weight matrix of $\bar{G}$. We want to study the product matrix G $\bar{G}$ and $\bar{G}$G and the corresponding graphs.

This we illustrate by an example.

Example 2.31. Let G be a complex valued weighted graph given by the following figure.

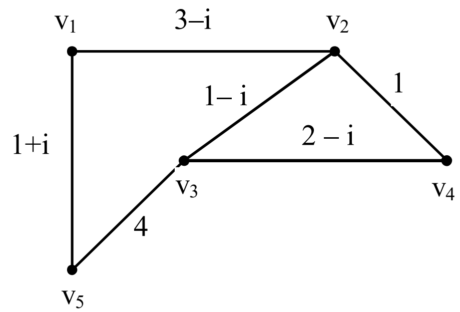


Figure 2.132

Let M be the complex valued weighted matrix associated with G.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 3-i & 0 & 0 & 1+i \\ 3-i & 0 & 1-i & 1 & 0 \\ 0 & 1-i & 0 & 2-i & 4 \\ 0 & 1 & 2-i & 0 & 0 \\ 1+i & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The complex conjugate of M denoted by $\bar{M}$;

$$\bar{M} = \begin{bmatrix} 0 & 3+i & 0 & 0 & 1-i \\ 3+i & 0 & 1+i & 1 & 0 \\ 0 & 1+i & 0 & 2+i & 4 \\ 0 & 1 & 2+i & 0 & 0 \\ 1-i & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We now find $M \times \bar{M}$ and $\bar{M} \times M$

$$M \times \bar{M} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 12 & 0 & 4+2i & 3-i & 0 \\ 0 & 1-3 & 2+i & 3-i & 6-8i \\ 4 & 2-i & 7 & 1-i & 0 \\ 3+i & 3+i & 1+i & 6 & 8-4i \\ 0 & 2+2i & 0 & 0 & 2 \end{bmatrix} \end{array}$$

We see $M \times \bar{M}$ results in a complex matrix whose complex value graph will lead to loops. So at this juncture we say this process of finding conjugate and producting may not in general end finitely.

Thus we have only limitations to find them.

Now having worked with complex valued graphs its conjugate we now proceed onto develop or just make a mention of finite complex number graphs in a systematic way and justify how at times finite complex number graphs may be better than the usual complex numbers.

Throughout this chapter $C(Z_n) = \{a + bi_F / a, b \in Z_n\}$;$C(Z_n) = \{a + bi_F / i_F^2 = 1$ $a, b \in \{0, 1\}\} = \{0, 1, i_F, 1 + i_F\}$, $C(Z_3) = \{a + bi_F / i_F^2 = 2, a, b \in Z_3\}$ and so on.

We see if v_1 and v_2 are two vertices then

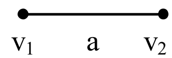


Figure 2.133

where $a \in C(Z_5)$, a can be complex or real only condition being that $i_F^2 = 4$, $a = x$, and $b = yi_F \in C(Z_5)$ $x, y \in Z_5$.

So we will as in case of usual complex valued graphs denote the complex edge by $\overline{e_i}$, $\overline{e_i} \in C(Z_n)$ and $e_i \in Z_n$ if e_i has no line on it.

We see

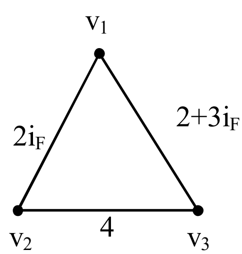


Figure 2.134

where $2i_F$, 4, $2 + 3i_F \in C(Z_6) = \{a + bi_F / a, b \in Z_6,\ i_F^2 = 5\}$.

Similarly,

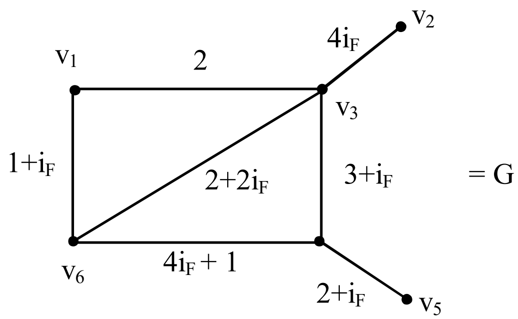


Figure 2.135

The edge values are from $C(Z_7) = \{a + bi_F / a, b \in Z_7, i_F^2 = 6\}$.

Clearly G is a complex valued weighted graph which is not directed.

We give the weight matrix M which is complex valued associated with the graph G.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 1+i_F \\ 0 & 0 & 4i_F & 0 & 0 & 0 \\ 2 & 4i_F & 0 & 3+i_F & 0 & 2+2i_F \\ 0 & 0 & 3+i_F & 0 & 2+i_F & 0 \\ 0 & 0 & 0 & 2+i_F & 0 & 0 \\ 1+i_F & 0 & 2+2i_F & 0 & 0 & 0 \end{bmatrix} \end{array}$$

It is clear the weight complex matrix is a symmetric matrix with diagonal entries to be zero. M is a symmetric complex valued matrix.

New consider the directed weighted finite complex number graph H with edge weights from $C(Z_9) = \{a + bi_F / a, b \in Z_9, i_F^2 = 8\}$ given by the following figure;

H =

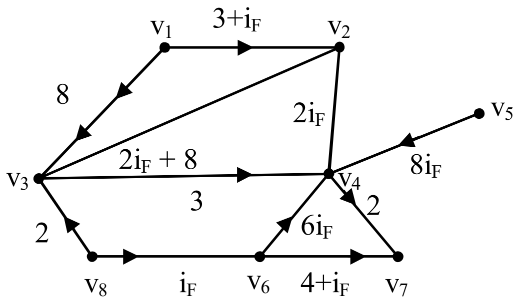


Figure 2.136

The complex valued weighted matrix N associated with the complex valued graph H is as follows.

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 3+i_F & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2i_F & 0 & 0 & 0 & 0 \\ 0 & 2i_F+8 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 8i_F & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6i_F & 0 & 0 & 4+i_F & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & i_F & 0 & 0 \end{bmatrix} \end{array}$$

Clearly N is a complex valued matrix with diagonal entries to be zero, however this matrix is not symmetric along the main diagonal.

We next proceed onto give some illustrations of complex valued weighted bigraph using weights from $C(Z_{10})$ by the following Figure.

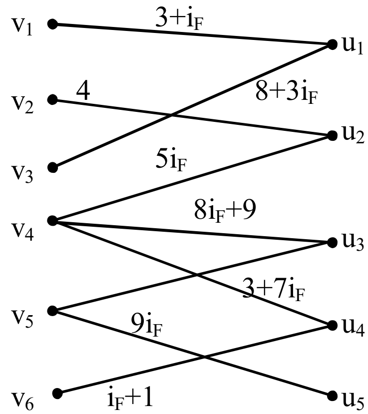


Figure 2.137

The corresponding complex valued weight matrix B associated with the Figure.

$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \left[\begin{array}{ccccc} 3+i_F & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 8+3i_F & 0 & 0 & 0 & 0 \\ 0 & 5i_F & 8i_F+9 & 3+7i_F & 0 \\ 0 & 0 & 2i_F & 0 & 9i_F \\ 0 & 0 & 0 & 1+i_F & 0 \end{array}\right] \end{array}$$

Now all properties associated with the complex valued weighted bigraphs can be obtained for finite complex valued weighted bigraphs.

We now find the complex conjugate of the weight matrix associated with the finite complex valued weighted graph.

Let S be the finite complex valued weighted graph G given by the Figure. Weighted are from $C(Z_{10})$

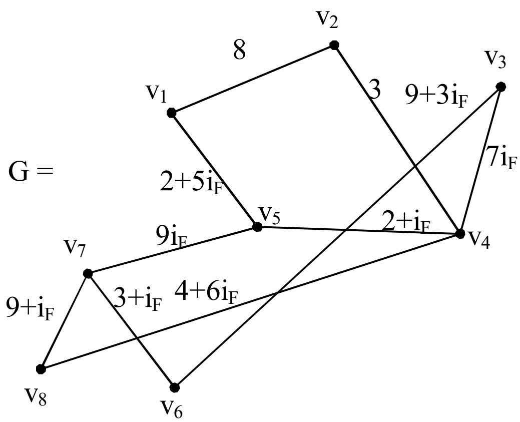


Figure 2.138

The corresponding complex valued weight matrix M associated with the Figure.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \left[\begin{array}{ccccc} 0 & 8 & 0 & 0 & 2+5i_F \\ 8 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 7i_F & 0 \\ 0 & 3 & 7i_F & 0 & 0 \\ 2+5i_F & 0 & 0 & 0 & 0 \\ 0 & 0 & 9+3i_F & 0 & 0 \\ 0 & 0 & 0 & 0 & 9i_F \\ 0 & 0 & 0 & 4+6i_F & 0 \end{array}\right. \end{array}$$

$$
\begin{array}{c}
\begin{array}{ccc} v_6 & v_7 & v_8 \end{array} \\
\left.\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
9+3i_F & 0 & 0 \\
0 & 0 & 4+6i_F \\
0 & 9i_F & 0 \\
0 & 3+i_F & 0 \\
3+i_F & 0 & 9+i_F \\
0 & 9+i_F & 0
\end{array}\right]
\end{array}
$$

Clearly M is a symmetric complex valued matrix with diagonal elements zero.

Let $\overline{M}$ be the complex conjugate of the complex valued matrix.

$$
\overline{M} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array}
\begin{array}{c}
\begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\
\left[\begin{array}{ccccc}
0 & 8 & 0 & 0 & 2+5i_F \\
8 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 3i_F & 0 \\
0 & 3 & 3i_F & 0 & 0 \\
2+5i_F & 0 & 0 & 0 & 0 \\
0 & 0 & 9+7i_F & 0 & 0 \\
0 & 0 & 0 & 0 & i_F \\
0 & 0 & 0 & 4+4i_F & 0
\end{array}\right.
\end{array}
$$

$$\begin{array}{ccc} v_6 & v_7 & v_8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9+7i_F & 0 & 0 \\ 0 & 0 & 4+4i_F \\ 0 & i_F & 0 \\ 0 & 3+9i_F & 0 \\ 3+9i_F & 0 & 9+9i_F \\ 0 & 9+9i_F & 0 \end{array}\Bigg]$$

Now we give the weighted complex valued graph $\bar{G}$ associated the complex conjugate matrix $\bar{M}$ is as follows.

$\bar{G}$ =

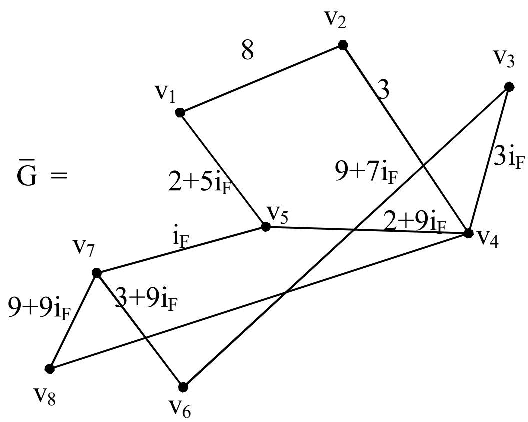


Figure 2.139

We see all the complex valued weights get the conjugate value except the weight $2 + 5i_F$ as $\overline{2+5i_F} = 2 + 5i_F$ since $\overline{5i_F} = 5i_F$ in $C(Z_{10})$. Thus we can as in case of usual complex valued graphs get the conjugate graphs get the conjugate in case of finite complex valued graph also.

Next we give the example of finite complex valued weighted directed graphs in the following.

Let G be the finite complex valued weighted directed graph with weights from $C(Z_9) = \{a + bi_F / a, b \in Z_9; i_F^2 = 8\}$ given by the following figure.

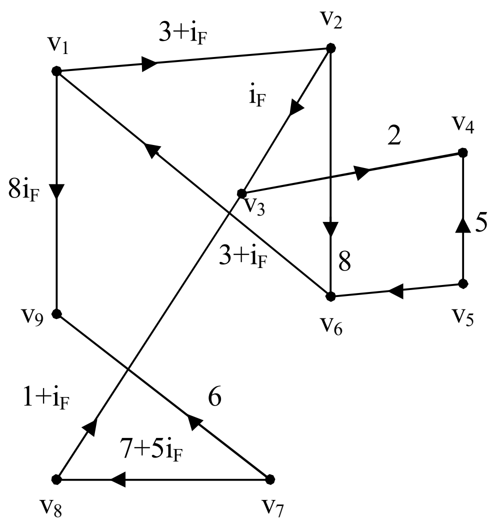
H = 

Figure 2.140

Let M be the weighted complex valued matrix associated with H.

$$
M = \begin{array}{c c}
 & \begin{array}{cccccccccc} v_1 & v_2 & v_3 & v_4 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{array} &
\left[\begin{array}{cccccccccc}
0 & 3+i_F & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8i_F \\
0 & 0 & i_F & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3i_F+1 & 0 & 0 & 0 \\
0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\
3i_F+1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7+5i_F & 6 \\
0 & 0 & 1+i_F & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
$$

Clearly main diagonal elements of M are zero.

However the weighted complex valued matrix M of H is not symmetric about the main diagonal.

Now we find the finite complex conjugate of M and denote it by $\overline{M}$.

We also determine the finite complex conjugate directed weighted graph associated with $\overline{M}$ and it is nothing but the finite complex valued graph $\overline{H}$ which is the conjugate of H.

H =

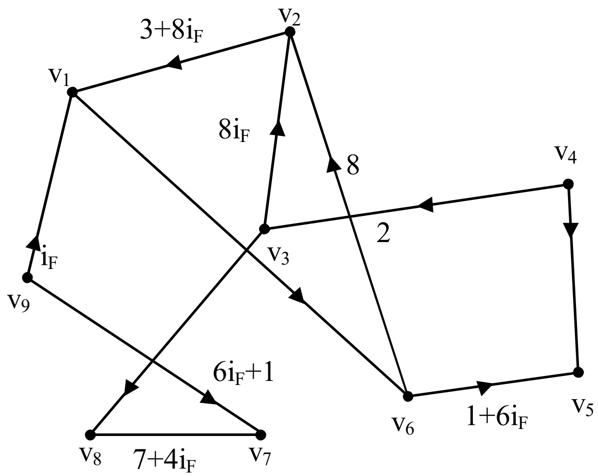


Figure 2.141

Now $\overline{M}$ the complex valued weighted matrix of $\overline{H}$ is the same as the finite complex conjugate of the matrix M.

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 \end{array} \\ \left[\begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 0 & 6i_F+1 & 0 & 0 & 0 \\ 3+8i_F & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8i_F & 0 & 0 & 0 & 0 & 0 & 1+8i_F & 0 \\ 0 & 0 & 2 & 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 1+6i_F & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7+4i_F & 0 & 0 \\ i_F & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \end{array}\right] \end{array}$$

We see M has also the main diagonal entries to zero.

Further $\overline{M}$ is not symmetric as the complex valued graph is a weighted directed one.

Next we give one example of a finite complex valued weighted bigraph K with entries from $C(Z_{12})$ and describe it by the following figure.

We also find the finite complex valued weighted matrix associated with K.

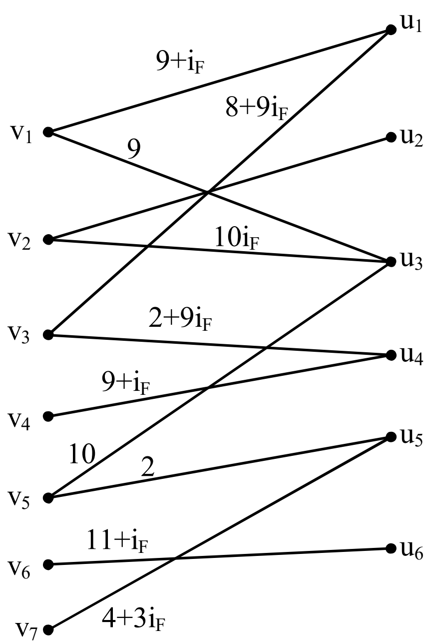


Figure 2.142

Now we give the finite complex valued weighted matrix N of the finite valued weighted bigraph K.

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 9+i_F & 0 & 9 & 0 & 0 & 0 \\ 0 & 2i_F & 10i_F & 0 & 0 & 0 \\ 8+9i_F & 0 & 0 & 2+9i_F & 0 & 0 \\ 0 & 0 & 0 & 9+i_F & 0 & 0 \\ 0 & 0 & 10 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11+i_F \\ 0 & 0 & 0 & 0 & 4+3i_F & 0 \end{bmatrix} \end{array}.$$

Clearly N is a complex valued weighted matrix of the complex valued weighted bigraph. Now we find the complex conjugate of N.

$$\bar{N} =$$

$$\begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 9+11i_F & 0 & 8+3i_F & 0 & 0 & 0 & 0 \\ 0 & 10i_F & 0 & 0 & 0 & 0 & 0 \\ 9 & 2i_F & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 2+3i_F & 9+11i_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 4+9i_F \\ 0 & 0 & 0 & 0 & 0 & 11+11i_F & 0 \end{bmatrix} \end{array}$$

We now find the complex valued weighted bigraph associated with $\bar{N}$. Let $\bar{K}$ be that bigraph

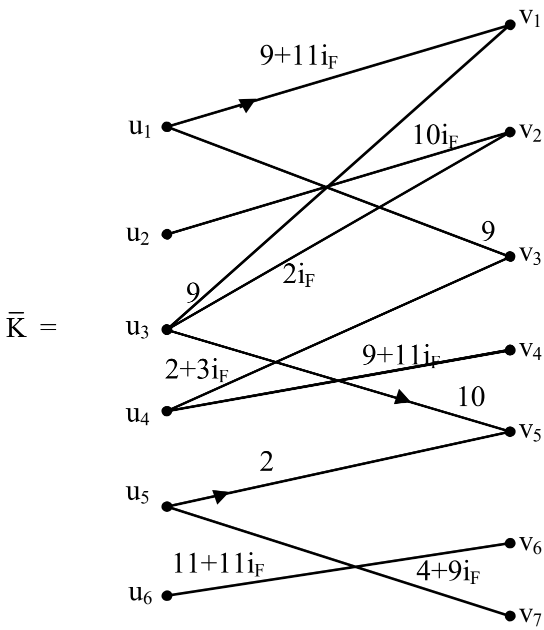


Figure 2.143

Thus for any finite valued weighted bigraph H we can always find a conjugate finite valued weighted bigraph $\overline{H}$ and the correspond complex conjugate matrix.

Now having see examples of finite complex valued weighted bigraphs we now proceed onto describe the concept of finite complex valued wheel by some examples.

Let W_5 =

$\overline{K}$

and

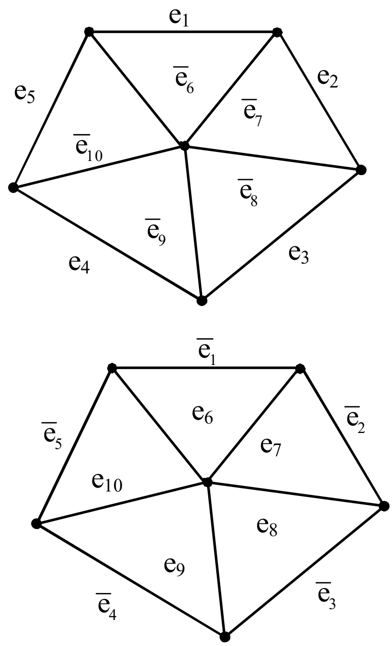


Figure 2.144

Where $e_i \in Z_n$ and $\overline{e}_i \in C(Z_n) \setminus Z_n$.

We know Figure 2.141 (a) is the quasi dual of (b) and vice versa. Here also as in case of complex number in case of finite complex numbers also they occur in pairs viz the wheel and quasi dual component of it.

The major difference between the complex valued matrices and the finite complex valued matrices is these matrices after producing with each other become a fixed point or a limit cycle however this property is not guaranteed in case of complex valued graphs.

We can akin to complex valued rooted trees can build also finite complex valued rooted trees.

We will illustrate this situation by an example or so example.

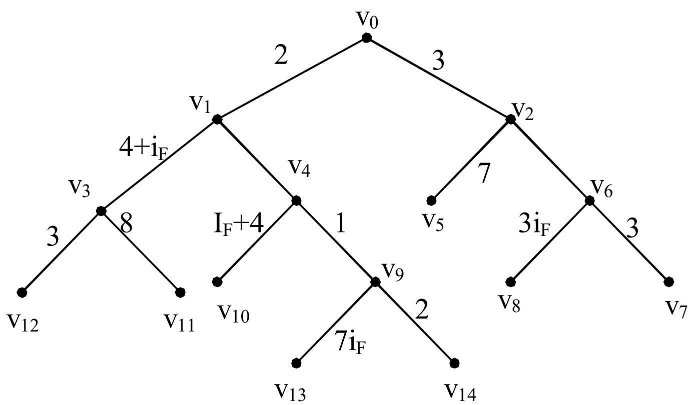


Figure 2.145

We see the complex valued rooted tree has real roots and T is not an imaginary rooted tree.

Clearly this finite complex valued rooted tree is not a complex or imaginary rooted binary tree.

The constituent part of these finite complex valued rooted trees are

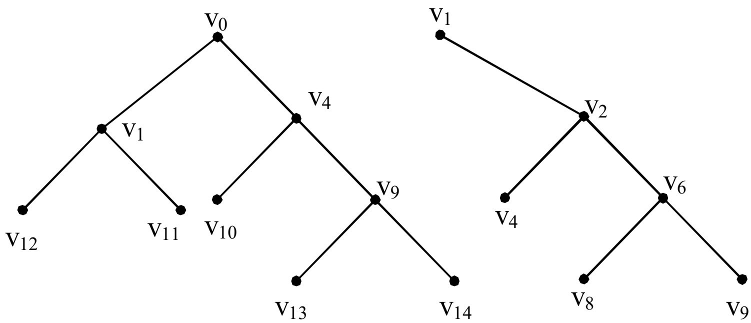


Figure 2.146

If the finite complex valued rooted tree is a binary tree then certainly it will have only two constitutional parts.

So it goes without saying if T is a finite complex valued rooted trinary tree it will have three constitutional trees associated with it hence for any finite complex valued rooted trinary tree will have n constitutional trees associated with it.

Consider the finite complex value trinary tree.

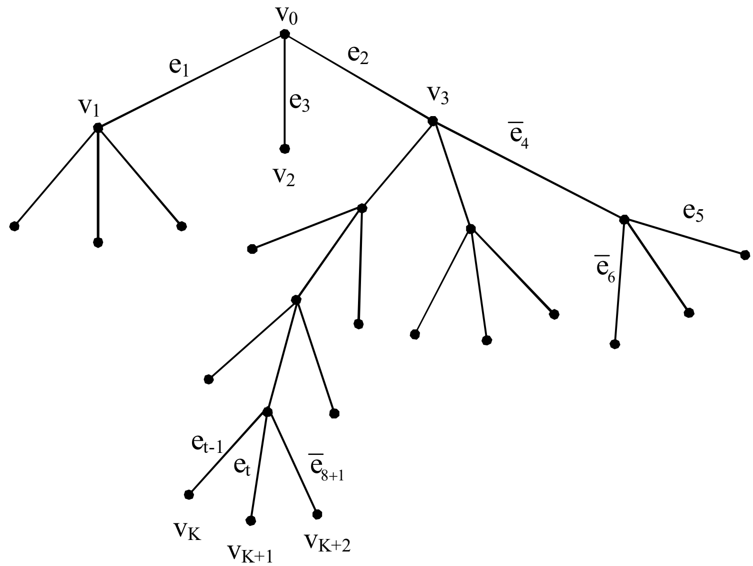


Figure 2.147

Interested reader can study finite complex valued graphs in an analogous way without any difficulty.

Further we can also develop all other properties.

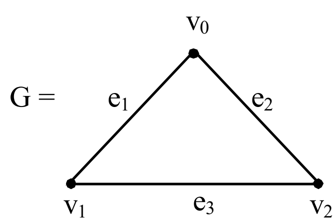


Figure 2.148

The only major difference between the complex valued graphs and finite complex valued graphs is that in the case of complex valued graphs we can have infinite number of them with

$e_1, e_2, e_3 \in C = \{a + bi / a, b \in R, i^2 = -1\}$.

However if the some G is taken with the edge values e_1, e_2, e_3 $\in C(Z_{10})$ then we can have finite number of them. This is the marked difference between the finite complex valued weighted graphs and the usual complex valued graphs.

The above results also hold good in case of complex valued bigraphs suppose we have finite complex valued weighted graph we have a finite complex valued matrix M associated with it.

Conversely given a finite complex valued weighted graph we will have the weighted matrix associated with it.

The above result is also true in case of finite complex valued bigraphs.

In the following we propose some problems to the reader.

Problems

1. Find all non isomorphic complex valued graphs with same number of edges and same numbers vertices (Assume six vertices and K_6 complex graph)

2. Show for any complex valued graph can in general have subgraphs which are real provided the complex valued graph is not a pure complex valued graph.

3. Give examples of complex valued graph with four vertices and all possible types of edges.

   a) How many such complex valued graphs exists?

   b) Show these complex valued subgraphs can be real.

   c) Show these complex valued subgraphs can also be pure imaginary.

4. Let G = {V, E} be the complex valued graph.

G =

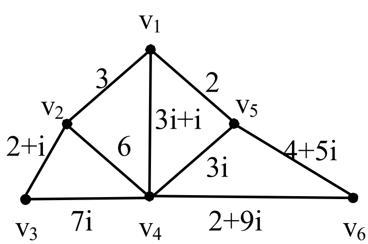


Figure 2.149

   i) Find all subgraphs of G which are real.

ii) Find all subgraphs of G which are pure imaginary.
iii) Find all subgraphs of G which are complex valued.

5. Let G be the complex valued weighted graph given by the following figure;

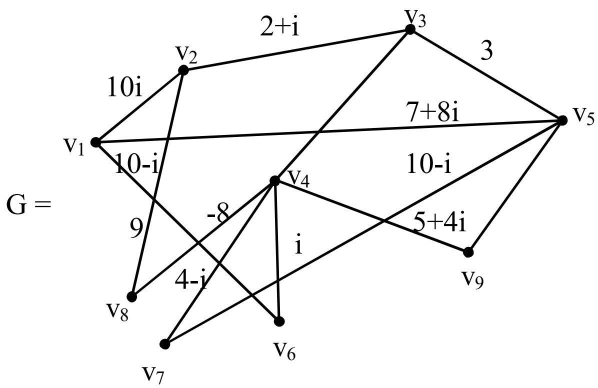


Figure 2.150

i) Find all real subgraphs of G.
ii) Find all complex valued subgraphs of G.
iii) Find the complex valued weighted matrix M associate with G.
iv) Find the complex conjugate matrix $\overline{M}$ of the complex valued matrix M.
v) Find the complex conjugate graph of G and prove $\overline{M}$ is the weighted complex matrix associated with it.

6. Let K be the complex valued weighted graph given by the following figure;

K

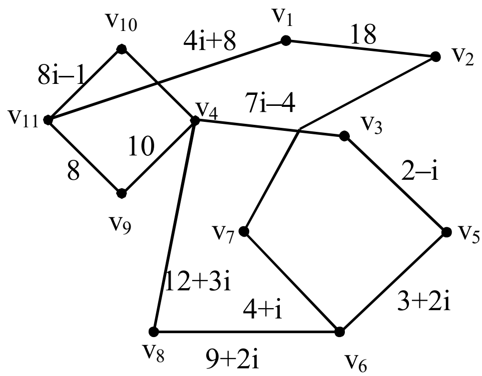


Figure 2.150

i) Find the quqsi dual complex valued graph of K.

ii) How many quasi dual complex valued graphs are associated with the complex valued graph K?

iii) Study questions (i) to (v) of problem (5) for this K.

7. Given any complex valued symmetric matrix M with zero diagonal entries prove we have a unique complex valued weighted graph associated with it.

8. Let P be the complex valued matrix (symmetric with diagonal entries zero).

i) Find the complex valued weighted graph G associated with P.

$$P = \begin{bmatrix} 0 & 3+i & 7i & 0 & 9i+1 & 2 \\ 3+i & 0 & 0 & 10i-1 & 0 & 8i+4 \\ 7i & 0 & 0 & 0 & 7i & 8 \\ 0 & 10i-1 & 0 & 0 & 2i-1 & 4 \\ 9i+1 & 0 & 7i & 2i-1 & 0 & -3i \\ 2 & 8i+4 & 8 & 4 & -3i & 0 \end{bmatrix}.$$

ii) Find the complex conjugate of P and the associated complex valued graph K.

iii) Is $K = \overline{G}$? Justify !

9. Prove if G is a complex valued directed graph then the complex valued matrix associated is not a symmetric one.

10. Let G be a complex valued weighted directed graph given by the following figure?

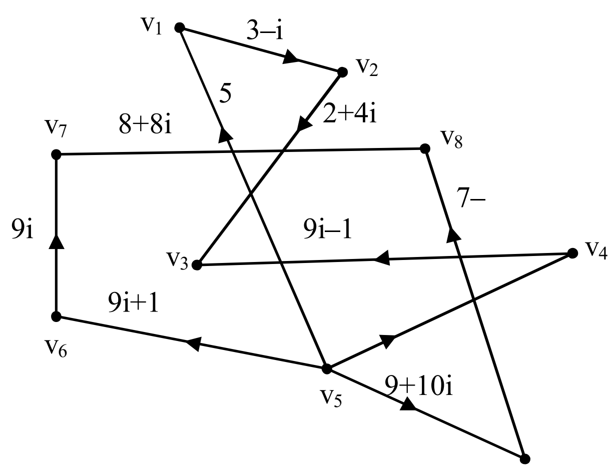


Figure 2.151

i) Findthe complex valued weighted matrix M associated with G.

ii) Prove M is not a complex valued symmetric matrix.

iii) Find the complex conjugate matrix $\overline{M}$ of the matrix M.

iv) Find the complex valued directed graph K associated with the matrix $\overline{M}$.

v) Prove K = $\overline{G}$.

11. Let G be the complex valued weighted bigraph given by the following Figure 2.152.

a) Find the complex valued matrix P associated with the bigraph with G.

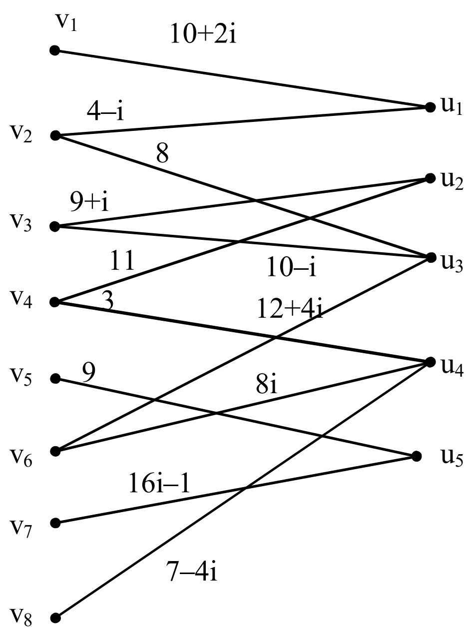


Figure 2.152

i) Find the complex valued weighted matrix M associated with G.

ii) Find the complex conjugate matrix $\overline{M}$ of the matrix M.

iii) Is $M \times \overline{M}$ defined?

iv) Find the complex conjugate bigraph $\overline{G}$ associated with the complex conjugate matrix $\overline{M}$.

v) Obtain any other special feature associated with the complex valued bigraph.

12. Let H be the complex valued matrix given in the following.

$$H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 3+4i & 0 & 2 & 0 & 0 & 7i-1 \\ 0 & 4+i & 0 & 8i+1 & 7 & 0 \\ 0 & 12 & 9+i & 0 & 8-i & 9 \\ 10 & 0 & 0 & 3 & 0 & 0 \\ 7+8i & 0 & 2-i & 2 & 3-i & 5i \\ 0 & 6i & 7 & 8i & -1 & 4 \\ 7i & 0 & 0 & 0 & 0 & 8i-1 \\ 0 & 5i-1 & 2+i & 0 & 3+i & 0 \\ 1+i & 0 & 0 & 1-i & 0 & 4i \\ 0 & 2 & 3 & 4i & 1 & 3 \\ 1+6i & 0 & 8i+1 & 0 & 7+i & 0 \end{bmatrix} \end{array}.$$

a) Find the complex valued weighted bigraph associated with the complex valued matrix H.

b) Find $\overline{H}$ the complex conjugate of the complex matrix H.

c) Find the complex valued bigraph P associated with the complex valued matrix $\overline{H}$.

d) Is P the complex conjugate bigraph of the bigraph K?

e) Obtain any other special feature associated with P, K and H.

13. Prove wheels of the form $W_n = K_1 + C_{n-1}$ occur in pairs in case of complex valued edges.

14. Find the two wheels. $W_{20} = K_1 + C_{19}$.

15. What are the special and interesting features one can associate with complex valued graphs?

16. Obtain some special applications of these complex valued graphs.

17. If some of the vertices of a complex valued graph is also complex what are the distinct features that can be associated with it in comparison with usual complex valued graphs whose vertices are real.

18. Can we have a directed complex valued bigraph?

19. What are the special and distinct features associated with complex valued rooted tree?

20. Can complex valued rooted trees be used in data mining?

21. What are the special applications one can associate with complex valued rooted trees?

22. Give an example of a complex valued rooted binary tree with imaginary root.

23. Give an example of a complex valued rooted tree which has all its leaves to be imaginary.

24. Let T be a complex valued rooted tree given by the following figure.

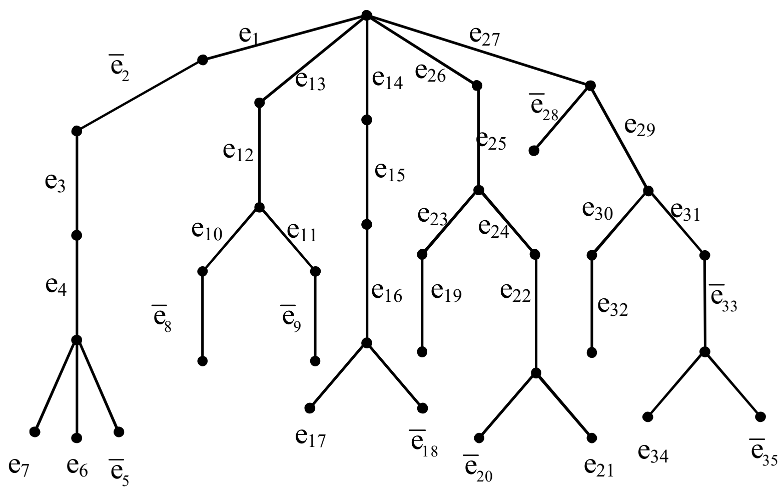


Figure 2.153

a) Find all the constituted trees associated with T.
b) How many leaves are imaginary.

c) How many of the constitutional trees have imaginary roots?

25. Find some innovative applications of complex valued graphs.

26. Is every edge we use in our graphs practical models a real edge?

27. Why neutrosophic edge that is edges which cannot be defined exists, in graphs how can one define graphs with edge values to be complex valued?

28. Can we say in social networking complex valued weighted edge has a major role to play? Justify your answer!

29. Prove complex valued graphs can find applications in the medical diagonistic problems.

30. Let G be the finite complex valued graph given by the following figure with edge weights from $C(Z_{15})$.

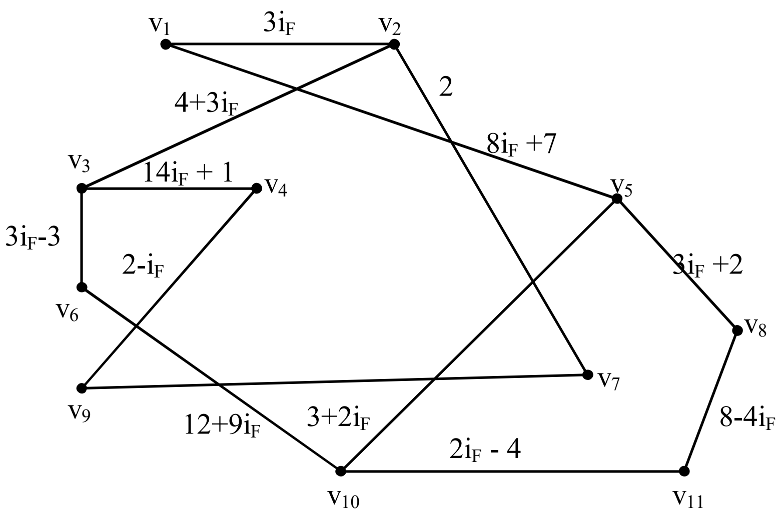


Figure 2.154

i) Find the complex valued weighted matrix M associated with G.

ii) Find the complex conjugate matrix $\overline{M}$ of M.

iii) Find the complex valued graph associated with $\overline{M}$.

iv) Find $M \times \overline{M}$ and $\overline{M} \times M$.

v) Is $M \times \overline{M} = \overline{M} \times M$?

vi) Find are subgraph of G.

vii) How many subgraphs of G are real?

viii) How many subgraphs of G are complex?

31. Let G be the finite complex valued graph given by the following figure with entries from $C(Z_5) = \{a + bi_F / a, b \in Z_5, i_F^2 = 4\}$

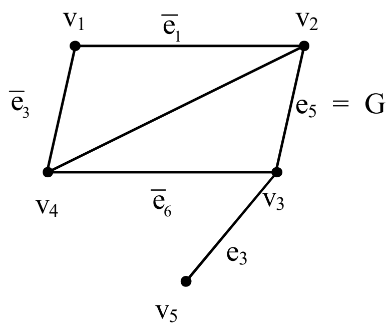


Figure 2.155

where $\bar{e}_1$, e_2, $\bar{e}_3$, e_5, $\bar{e}_6$ and $e_4 \in C(Z_5)$.

i) Keeping the complex edge as a complex edge and real edge as a real edge how many graphs can be obtained similar to G.

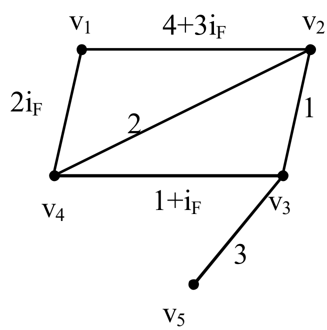


Figure 2.156

(Hint we say G is similar to K if that is edge weights which are complex continue to be complex and those are real continue to be real.

The number of edges and vertices are maintained to be same only values of the edges vary)?

ii) Prove the collection in (i) is only finite.

32. Let G be a complex valued weighted directed graph given by the following figure with edge weights from $C(Z_6) = \{a + bi_F / a, b \in Z_6;\ i_F^2 = 5\}$

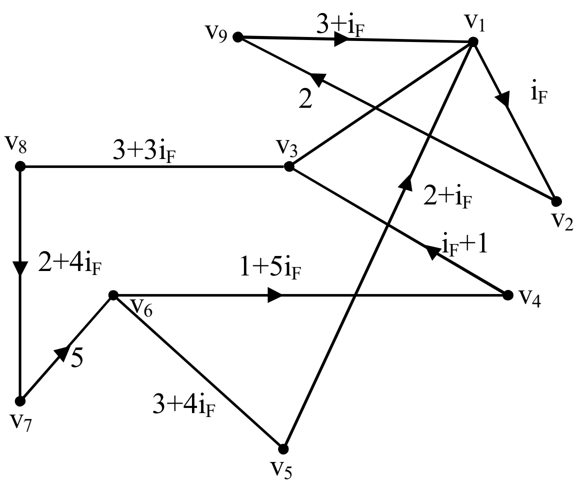


Figure 2.157

i) Find the weighted finite complex valued matrix M of G.

ii) Find all real valued subgraphs of G.

iii) Find all complex valued subgraphs of G.

iv) Find $\overline{M}$ the finite complex conjugate matrix of the finite complex matrix M.

v) Find the complex valued graph H associated with the finite complex valued matrix $\overline{M}$.

vi) Is H the complex conjugate graph of G?

vii) Find all real valued subgraphs of H.

viii) Is the number of real valued subgraphs of G and H the same?

ix) Will the number of complex valued subgraphs of both G and H be the same?

x) Will the subgraphs of G be the same as the subgraphs of H? Justify !

xi) Obtain any other special feature associated with the complex valued graphs G and H.

xii) Will all the subgraphs of G (and H) be again a directed subgraphs? Substantiate your claim.

33. Let K be a finite complex valued graph with edge weights from $C(Z_8) = \{a + bi_F / a, b \in Z_8, i_F^2 = 7\}$ given by the Figure 2.153.

K =

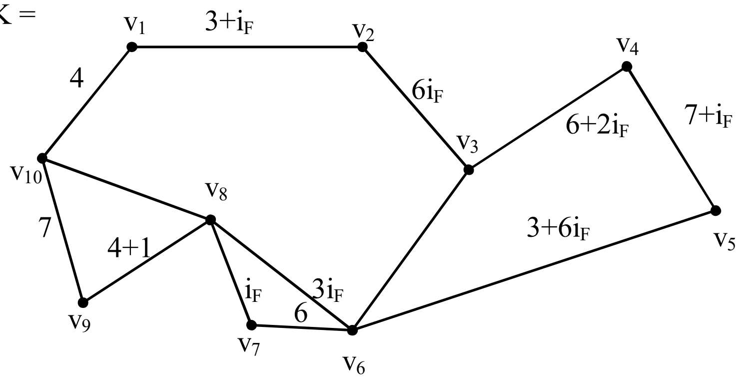


Figure 2.158

i) Find the weighted matrix M associated with K.

ii) Find all subgraphs of K which has real edges.

iii) Find all subgraphs of K which has complex edge weights.

iv) Find $\overline{M}$ the complex conjugate matrix of M.

v) Find the complex valued graph associated with M.

vi) Find $\underbrace{M \times M \times \cdots \times M}_{10 \text{ times}}$.

vii) Find $\underbrace{\overline{M} \times \overline{M} \text{ K} \cdots \times \overline{M}}_{10 \text{ times}}$.

34. Let S be the finite complex valued directed weighted graph given by the following example.

S =

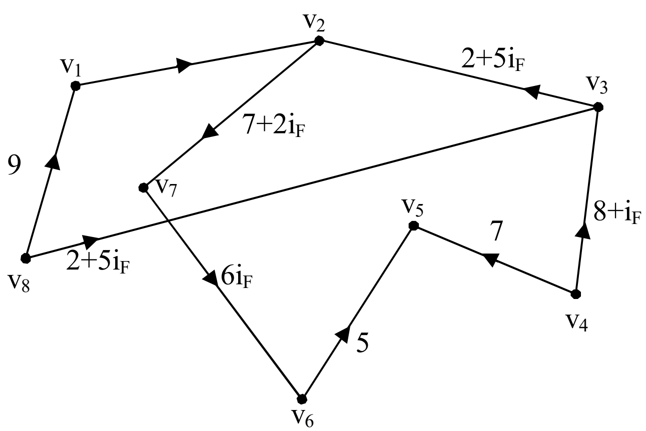


Figure 2.159

The finite complex valued directed weighted graph takes the weights form $C(Z_{10}) = \{a + bi_F / a, b \in Z_{10}; i_F^2 = 9\}$

i) Find the finite complex valued weighted matrix M associated with the finite complex valued directed weighted graph S.

ii) Find the finite complex conjugate $\overline{M}$ of the matrix M.

iii) Determine the finite complex valued graph K associated with $\overline{M}$.

iv) Is $K = \overline{S}$?

v) Is K a directed finite complex valued graph?

35. Let M be a finite complex valued matrix given in the following with entries from $C(Z_{18})$.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 3+2i_F & 0 & 0 & 14i_F & 2+10i_F & 9 \\ 0 & 9+9i_F & 3+i_F & 0 & 0 & 0 \\ 2+17i_F & 0 & 0 & 8 & 0 & 1+i_F \\ 9 & 2i_F & 8 & 1+3i_F & 9i_F & 0 \\ 10i_F & 0 & 4i_F & 0 & 7 & 2i_F \\ 0 & 1+i_F & 0 & 1 & 0 & 3 \\ 2 & 0 & 2+i_F & 0 & 1+i_F & i_F \\ 1+i_F & 1+9i_F & 9i_F & 2+i_F & 0 & 1+9i_F \end{bmatrix} \end{array}$$

i) Find the finite complex valued weighted bigraph G associated with M.

ii) Find the finite complex conjugate $\overline{M}$ of M.

iii) Find the finite complex valued weighted bigraph K associated with $\overline{M}$.

iv) Is K the complex conjugate bigraph of G and vice versa?

v) Obtain any other special feature associated with M, $\overline{M}$, K and G.

36. Let P be a non symmetric finite complex number matrix i given in the following.

$$P = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 3+i_F & 0 & 9+2i_F & 0 & 8i_F & 0 \\ 2i_F & 0 & 8+i_F & 0 & 7 & 0 & 1+i_F \\ 0 & 1+i_F & 1 & 0 & 1+3i_F & 2 & 4 \\ 1+5i_F & 0 & 2i_F & 7i_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 2i_F & 1+i_F & 9i_F & 0 \\ 3+8i_F & 4+i_F & 0 & 0 & 0 & 0 & 7+i_F \\ 2+i_F & 0 & 3+i_F & 1+5i_F & 2 & 4 & 0 \end{bmatrix} \end{array}$$

where the entries are from $C(Z_{10}) = \{a + bi_F / a, b \in Z_{10}, i_F^2 = 9\}$

i) Find the complex valued graph G associated with the matrix P.

ii) Does the graph G contain loops?

iii) Is the graph G a directed one?

iv) Find $\overline{P}$ the complex conjugate of P.

v) Find the complex valued graph H associated with $\overline{P}$.

vi) Is H the conjugate of G and vice versa?

37. Let M be the finite complex valued graph with entries from $C(Z_9) = \{a + bi_F / a, b \in Z_9, i_F^2 = 8\}$ given in the following.

M =

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array}
\begin{array}{c}
\begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\
\begin{bmatrix}
0 & 2+i_F & 0 & 8i_F & 0 & 7 & 0 \\
1+i_F & 0 & 2i_F & 0 & 7+8i_F & 0 & 4 \\
0 & 8 & 0 & 1+i_F & 0 & i_F & 0 \\
0 & 0 & 2+5i_F & 0 & 4i_F & 0 & 1+i_F \\
8i_F+1 & 0 & 0 & 3+i_F & 0 & 1+4i_F & 0 \\
0 & 3+i_F & 2 & 0 & 4 & 0 & 5i_F \\
0 & 0 & 0 & 2i_F+1 & 0 & 1+4i_F & 0
\end{bmatrix}
\end{array}$$

i) Find the complex valued graph G associated with the complex valued matrix M.

ii) Is G a directed complex valued graph?

iii) Find $\overline{M}$ the complex conjugate of M.

iv) Find the directed complex valued graph H associated with $\overline{M}$.

v) Is H the complex conjugate graph of the graph H?

38. Define and describe the complex valued wheels.

39. Prove there are two such complex wheels W_n for any given positive n.

40. Show in case of real wheel W_n it is easily one.

41. Show in case of finite complex valued wheel also we have two W_n's for a given n.

42. Are these complex valued wheels quasi dual complex valued ones? Justify.

43. Give an example of a complex valued wheel W_{11}.

44. Give an example of a finite complex valued wheel W_{18} entries from $C(Z_n) = \{a + bi_F / a, b \in Z_n, i_F^2 = (n - 1)\}$.

45. Give an example of a disconnected complex valued finite graph.

46. Prove for any given complex valued symmetric matrix M with diagonal zero we have a weighted complex valued graph G which is not directed and vice versa.

47. Prove if M in problem 46 has diagonal entries then the graph G has loops.

48. Prove the complex matrix M in problem (46) has a unique complex conjugate $\overline{M}$.

49. Prove $\overline{M}$ the complex conjugate of M has a unique complex valued graph associated with it.

50. Let S be a finite complex value matrix given in the following with entries from $C(Z_{12}) = \{a + bi_F / a, b \in Z_{12}, i_F^2 = 11\}$;

$$S = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} 10+i_F & 0 & 3i_F & 0 & 6 \\ 0 & 3i_F+4 & 0 & 4i_F & 0 \\ 2 & 6 & 2+i_F & 0 & 1+i_F \\ 0 & 0 & 0 & 1+8i_F & 0 \\ 7i_F & 1+9i_F & 4 & 0 & 3i_F \\ 0 & 0 & 0 & 8i_F+7 & 0 \\ 4+3i_F & 4i_F & 2+i_F & 0 & 7+7i_F \end{bmatrix} \end{array}$$

i) Find $\overline{S}$ the complex conjugate of S.

ii) Find the complex valued bigraph H associated with S.

iii) Find the complex valued bigraph K associated with $\overline{S}$.

iv) Prove if S is the complex conjugate of $\overline{S}$ then the complex valued bigraph H is the complex conjugate of the bigraph K and vice versa.

51. Give examples of perfect complex valued graphs.

52. Give an example of a complex imaginary rooted tree.

53. Give an example of a complex tree which has no imaginary leaves.

54. Give an example of a comlex valued tree which has all of its leaves to be imaginary.

55. Give an example of a complex valued trinary tree with 6 layers.

56. Give an example of a complex valued n-ary tree T (n = 7) with 3 layes.

    i) How many nodes are in T?

    ii) How many of them are leaf nodes?

    iii) Enumerate any of the special features associated with T.

57. Give an example of a K_8 with complex values as edge weights.

58. How many K_4 exists if the edge weights are from $C(Z_5) = \{a + bi_F / a, b \in Z_5, i_F^2 = 4\}$?

59. How many of the complex valued graphs K_4 given in problem 58 are quasi dual of each other?

60. Enumerate all special and interesting features associated with complex valued graphs K_n ($n \geq 3$).

61. Enumerate all special features associated with the complex valued graph $K_{3,3}$.

62. How many such K_5's and their dual can be made if the edge weights are from $C(Z_8)$?

63. Define complex valued outer maximal planar graphs and illustrate them by examples.

64. How many complex valued trees exists with six vertices?

65. How many trees with six vertices exist?

66. Compare the problems (64) and (65) for the trees.

67. How are these complex valued rooted trees different from the usual trees which are not rooted?

68. Define the notion of complex semi irreducible and a complex irreducible graph.

69. Can we give edge weights for graphs in question (68)?

70. Give a real world example were the edge weights can be complex.

71. Let G be a complex valued graph given by the following Figure.

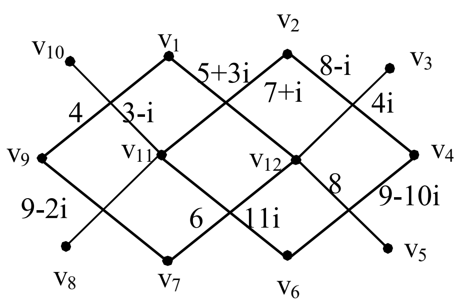


Figure 2.160

i) Find the complex conjugate of the graph G.

ii) Find the weighted matrix M of the complex valued graph G and its complex conjugate $\overline{M}$ of M.

iii) Find the complex valued graph associated with $\overline{M}$.

iv) Prove in general $M \times \overline{M} \neq \overline{M} \times M$ and the complex valued graphs associated with $\overline{M} \times M$ (or $M \times \overline{M}$) do not inherit all properties of the graph G.

81. Let $K_{5,5}$ be the complex valued weighted graph.

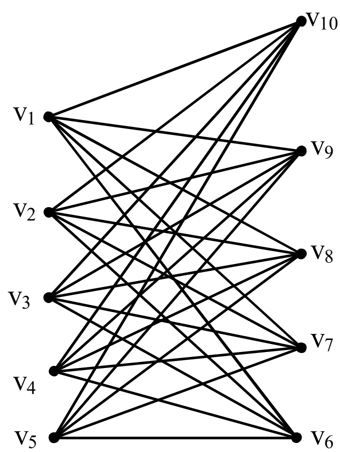


Figure 2.161

i) Give edge weights as elements from C and find the complex conjugate of $K_{5,5}$.

ii) For a given complex weight graph find the quasi dual.

iii) Give weights for the edge such that the subgraphs $K_{4,4}$ and $K_{3,3}$ are pure complex that is purely imaginary.

82. Obtain any other special feature associated with finite complex valued weighted graphs and find all possible application.

83. Construct strong complex valued graphs which has both the vertices and edges to be complex.

84. Can complex weighted rooted tree be used in image processing?

85. Give an example of complex weighted regular bigraph G.

86. Is such G's 1-factorable? Justify your glaim.

87. Define and describe a complete n-partite complex weighted (edge) graphs. Is it possible they these graphs satisfy the classical properties?

88. Define union of any two distinct imaginary walks joining two vertices a cycle contains.

89. Describe and develop the notion of a complex valued graph and its dique is also complex valued.

90. Describe by an example a complex valued out tree and the converse in tree.

91. If G is the complex valued out tree will the in tree associated with G be real?

92. Give an example of a complex valued graph with 5 components.

93. Give an example of a connected complex valued graphs.

94. When a walk is imaginary between some two vertices. What are the possibilities it can be used in the applications?

95. Prove when graph theory is used in sociology complex valued graphs are more appropriate than real valued graphs.

96. Can complex valued graphs (that is edge weights complex) be used in biology?

97. Can complex valued graphs be used in problems in social media?

98. Can complex valued graphs be used in linguistics?

99. Can complex vertex graphs be used in artificial neural networks?s

100. Prove the cognitive maps can also have complex vertices and complex edges!.

Chapter Three

APPLICATIONS OF COMPLEX VALUED GRAPHS

In this chapter we authors give the possible and probable applications of complex valued graphs and strong complex valued graphs.

A systematic study of complex valued graphs was carried out in the earlier chapter. Our concentrations in this study is mainly focused on how this new structure can be applied to mathematical complex models.

So in the first place we will see how the cognitive structure can have complex nodes and complex edges.

Infact authors wish to state that when one can indeterminate edges why not imaginary edges. Imagined feelings or imagined feelings of suffering from diseases, imagined grievances against people or against relations etc.; exists in human actions as well as in the feelings. So pain can be imaginary, hatered can be imaginary, sometimes the very thought processing at a particular period and for a particular situation may be imaginary.

So the concept of imaginary node (vertex of a graph) and imaginary edge weight (in the graph) that is connecting two nodes is also possible.

So such situations arise in the case of social networks analysis, soft computing, medical diagnostics and personality assessments.

Another place where the imaginary nodes function in the human are redefined as follows. When we say the neuron is imaginary it does not mean the existence of the neuron is imaginary. It is only imagined what that node has the capacity to imagine so many things which may not even exist only in imagination it is said to be imaginary.

For instance this is the apt situation in case of an artist or poet or a writer for that matter. So in his brain there will be neurons which perform imaginary activities and the neurons are also defined by us to be connected imaginarily and produce imaginary impulses.

So with this explanation we instead of fuzzy cognitive maps have also fuzzy imaginary cognitive maps where the directed graphs associated with this new structure will utilize only strong complex valued weighted directed graphs.

This will be first mathematical model which we are developing using the strong complex valued weighted graphs.

We define a graph to be a strong complex valued weighted directed graph if in the usual directed graph some of the vertices are imaginary and some of the edge weights are imaginary.

Example 3.1. The strong complex valued graph is given by the following figure;

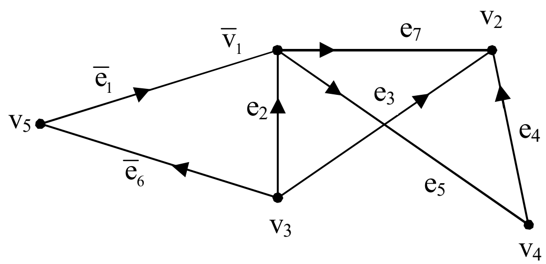


Figure 3.1

The vertex $\bar{v}_1$ is complex and the rest of the vertices are real. Only the edges $\bar{e}_1$ and $\bar{e}_6$ are complex and the rest of the edge weights are real.

Now we will be using this sort of strong directed complex valued graphs to describe, define and develop the notion of fuzzy imaginary cognitive maps.

In the first place we want to once again recall and reregister that fuzzy imaginary cognitive maps functions akin to fuzzy cognitive maps with a difference the state of a node can be imaginary i or 1 + i and the edge weights can also be i or 1 + i. To this end we make the following definition.

Throughout our discussions of Fuzzy Imaginary Cognitive Maps (FICMs) model we assume the state can be ‘0’ off; 1, imaginary 1 + i and purely imaginary i.

We define Fuzzy Imaginary Cognitive Maps (FICMs) model as fuzzy imaginary signed directed graphs with feed back.

The directed edge e_{ij} from causal concept C_i to concept C_j measures how much C_i causes C_j, it may be pure imaginary i or imaginary 1 + i or real 1 or no relation 0. The time - varying concept C_i (t) can measure the non negative occurrence of some fuzzy event perhaps the medical investigation of a hypochondria (patient) or a criminal investigation or a caste based education system. FICMs model the world as a collection of classes and causal relations between classes.

For more about FCMs and the development of FCMs please refer [15]. However it is pertinent to keep on record that the notion of Neutrosophic Cognitive Maps was developed in [15] and has been used in several real world problems [15].

Simple FCMs have edge values {-1, 0, 1} and Fuzzy Imaginary Cognitive Maps take edge weights from {0, i, 1, 1 + i} or S = {0, -i, -1, i, 1, 1 – i, -1 – i, 1 + I, -1 + i}.

This study is interesting for certainly there are certain concepts in which C_i causes C_j in an imaginary way. Here also feedback precludes the graph search technique used in artificial intelligence expert systems and causal imaginary trees.

FICMs feedback also allows causal adaptation laws.

$$\dot{m}_{ij} = -m_{ij} + S_i S_j + \dot{S}_i \dot{S}_j \quad (1)$$

$$\dot{m}_{ij} = -m_{ij} + \dot{S}_i \dot{S}_j \quad (2)$$

where S_j and S_j are binary signal functions, m_{ij} are synaptic coefficient FICMs take the edge values from S.

In case of FICMs also the causal feedback loops abound in thick tangles. Feed back makes possible the graph search techniques used in artificial - intelligence expert systems and causal trees.

Akin to FCMs, FICMs feedback allows experts to freely draw causal pictures of their problem which allows causal adaptation laws mentioned in (1) and (2) with appropriate modifications to infer causal links from sample data. FICMs feedback also forces us to discard graph search forward and specially backward chaining. Instead one can view FICMs also as a dynamical system and its equilibrium behavior as a forward evolved inference. As in case of FCMs one can also add two or more FICMs to produce a new FICMs as law of large numbers ensures the reliability of information with the experts sample size.

As in case of FCMs in FICMs also we pass on a vector forming a result after each pass.

The result (resultant) vector is thresholded and updated at each pass which settles down as a fixed point or a limit cycle or hidden pattern.

The limit cycle inference as in case of FCMs in case of FICMs also summarizes the joint effects of all the interacting fuzzy imaginary knowledge.

Suppose we have 6×6 causal connection matrix M that represents the FICM in the following figure;

G =

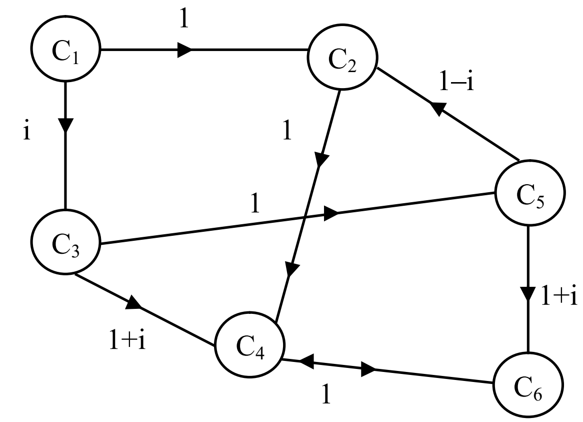


Figure 3.2

where C_1, C_2, C_3, C_4, C_5 and C_6 are nodes associated with an hypochondria patient who complains of the following symptoms who had been admitted in the hospital for fracture near the wrist.

In the following we briefly describe each of the nodes.

C_1 - pain in the hand

C_2 - Pain in shoulder + neck

C_3 - Feels giddy

C_4 - Complains of stomach upset

C_5 - Complains of problems related to heart

C_6 - Feels tired or depressed.

Clearly we see some of the nodes or concepts associated with this problem is imaginary or purely imaginary.

Secondly when the nodes / concepts itself happens to be imaginary or purely imaginary the causal relation can also be imaginary or purely imaginary.

Now we find the effect of the on state of some state vector from.

$X = \{(a_1, .., a_6) / a_i \in \{0, 1, i, 1 + i\}; 1 \leq i \leq 6\}$

Let $x = (1, 0, 0, 0, i, 0) \in X$.

To find the effect of x on the dynamical system M got using the complex valued directed graph G.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 1 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1+i & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1-i & 0 & 0 & 0 & 1+i \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

We find the effect of x on M.

$xM = (0, 2 + i, i, 0, 0, i - 1) \rightarrow (1, 1, i, 0, i, i) = x_1$ (say)

($\rightarrow$ denotes the resultant vector has been updated that is the vectors which were initially in on state is kept in the onstate be it a imaginary state or real state).

Now we threshold as if a + ib = $\begin{cases} = & 1 & \text{if } a > b \text{ and } a > 0 \\ = & i & \text{if } a < b \text{ and } b > 0 \\ = & 0 & \text{if } a < 0 \text{ and } b < 0 \\ = & 1+i & \text{if } a = b, \text{ and } a > 0 \end{cases}$

Now we find the effect of the resultant vector x_1 on M

$x_1M = (0, 2 + i, i, 2i, i, i - 1) \rightarrow (1, 1, i, i, i, i) = x_2$ (say).

Now we find the effect of x_2 on M

$x_2M = (0, 2 + i, i, 3i - 1, i\ 3i - 1) \rightarrow (1, 1, i, i, i, i, i) = x_3$ (say).

Clearly as $x_2 = x_3$ the hidden pattern of the state vector (1, 0, 0, 0, i, 0) is a fixed point given by (1, 1, i, i, i, i).

So only the state the patient may have pain in the shoulder and neck may be true and all other symptoms experienced by the hypochondria patient is imaginary as seen from the hidden pattern.

Interested reader can work with other nodes. Thus the FICMs model can be used in medical diagnostics when some imaginary symptoms are suffered by the patient.

Such study may also be extended in patients who are mentally ill in case of paranoia patients and so on where they suffer from imaginary persecution.

For we can more say the symptoms are imaginary for they believe they are so.

So in medical diagnostics the FICMs can play a vital role when an element of imaginary feelings or sufferings are associated with the patient.

Thus we have described the new fuzzy mathematical model which can also measure the imaginary trait in it. For in reality the functioning of the brain that the synaptic connections from one neuron to another can also be imaginary.

Till date the FCMs or TAMs or FAMs or BAMs models can only measure the real trait of the problem However the indeterminancy concept when involved have been well studied using NCMs [15] etc. For more about the above mentioned concept please refer [15].

Now we work on the FICMs model where we analyse some C_1, C_2, C_3, C_4, C_5 and C_6 concepts or nodes which can at any arbitrary time take the value 0 or 1 or i state only, that is zero state or off state on state, 1 and pure imaginary state i and the imaginary state $1 + i$.

Now using these six concepts / nodes C_1, C_2, C_3, C_4, C_5 and C_6 we give the corresponding complex valued the directed graph G associated with the dynamical system.

Once again we state that the edge weights can be from the set $\{-1, 1, -i, i, 0, 1 + i, -1 + i, 1 - i, -1 - i\}$

We see the state vectors are from the space $\times = \{(a_1, a_2, a_3, a_4, a_5, a_6) / a_i \in \{0, 1, i, 1 + i\}, i = 1, \ldots, 6\}$.

The elements in X will be called as the state vectors of the dynamical system.

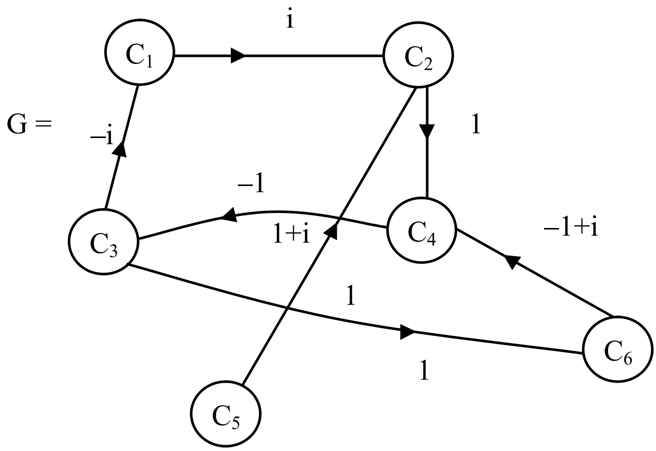


Figure 3.3

The connection matrix of this fuzzy imaginary dynamical system is as follows.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1+i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+i & 0 & 0 \end{bmatrix} \end{array}.$$

Consider an initial state vector x = (1, 0, 0, 0, i, 0) $\in$ X. To find the effect of x on the dynamical system M.

$$xM = (0, 2i - 1,0, 0, 0, 0) \to (1, i, 0, 0, i, 0) = y_1$$

($\to$ denotes the state vectors has been updated and thresholded as follows (if in a + bi a $\geq$ 2 put 1 and if b $\geq$ 2 put 1 if a is –ve put 0 and if b is negative put 0)).

Now we find the effect of y_1 on the dynamical system M.

$y_1M = (0, 2i – 1, 0, i, 0, 0) \to (1, i, 0, i, i, 0) = y_2$ (say)

$y_2M = (0, 2i – 1, - i, i, 0, 0) \to (1, i, 0, i, i, 0) = y_3$ (= y_2 say)

We see the resultant is a fixed point given by (1, i, 0, i, i, 0).

So the on state of C_1 and C_5 state is imaginary we only get the C_4 state on which is pure imaginary.

Consider a initial state vector x = (0 1 0 0 1 0), to find the effect of x on M is as follows.

$xM = (0, 1 + i, 0, 1, 0, 0) \to (0\ 1 + i, 0, 1, 1, 0) = y_1$ (say)

$y_1M = (0, 1 + i, –1, 1 + i, 0, 0) \to (0, 1 + i, 0, 1 + i, 1, 0)$

$= y_2$ (say)

$y_2M = (0, 1 + i, –1 – i, 1 + i, 0, 0) \to (0, 1 + i, 0, 1 + i, 1, 0)$

$= y_3$ (say)

But $y_3 = y_1$ so the resultant vector is a fixed point given by (0, 1 + i, 0, 1 + i, 1, 0).

We see the on real state C_2 takes the value 1 + i, C_4 comes to on state and takes the value 1 + i.

Interested reader can work with other initial state vectors and find the resultant to be either a fixed point or a limit cycle.

Thus we call the resultant fixed point or the limit cycle to be the hidden pattern of the complex dynamical system.

We are sure after a finite number of iterations we will to arrive at a fixed point or a limit cycle.

We will give yet another example before we proceed onto describe the complex - neutrosophic cognitive maps model.

Example 3.2. Let G be a directed complex valued graphs with 7-concepts / nodes given by C_1, C_2, C_3, …, C_7 with edge weights from the set $\{-1, 1, -i, i, 0, 1 + i, -1 + i, -1 - i, 1 - i\}$ given by the following figure (it is pertinent to record at this juncture that all these examples do not cater to any real valued problems they are only examples).

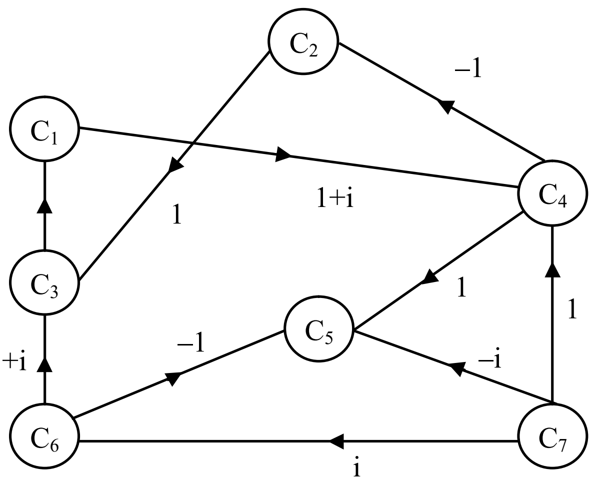


Figure 3.4

Now we find the complex valued connection matrix M which serves as the dynamical system of G.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1+i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+i & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -i & i & 0 \end{bmatrix} \end{array}.$$

Now we use X = $\{(a_1, a_2, a_3, a_4, a_5, a_6, a_7) / a_i \in \{0, 1, i, 1 + i\}, 1 \leq j \leq 7\}$ to be the collection of all initial state vectors.

Let x = (1, 0, 0, 0, 0, 0, 0) $\in$ X. To find the effect of x on the dynamical system M.

xM = (0, 0, 0, 1 + i, 0, 0, 0) $\to$ (1, 0, 0, 1 + i, 0, 0, 0) = x_1 (ay)

x_1M = (0, –1, –1, 1 + i, 1 + i, 0, 0) $\to$ (1, 0, 0, 1 + i, 1 + i, 0, 0)

= x_2 (say)

x_2M = (0, -1 – i, 0, 1 + i, 1 + i, 0, 0) $\to$

(1, 0, 0, 1 + i, 1 + i, 0, 0) = x_3 (say).

Clearly the hidden pattern of the initial state vector is a fixed point given by (1, 0, 0, 1 + i, 1 + i, 0,0).

Let x = (i, 0, 0, 0, 0, 0, 0) to find the effect of x on the dynamical system M.

xM = (0, 0, 0, i – 1, 0, 0, 0) $\to$ (i, 0, 0, i, 0, 0, 0) = y_1

y_1M = (0, –i, 0, i – 1, i, 0, 0) $\to$ (i, 0, 0, i, i, 0, 0) = y_2 (say);

$y_2M = (0, -i, 0, 1 - 1, i, 0, 0) \to (i, 0, 0, i, i, 0, 0) = y_3$ (say).

Clearly $y_2 = y_3$ hence the hidden pattern is a fixed point given by (i, 0, 0, i, i, 0, 0). When we compare if $C_1 = 1$ then the nodes C_4 and C_5 become imaginary takes values 1 + i and 1 + i only.

If $C_1 = i$ then the nodes C_4 and C_5 comes to on state which is purely imaginary taking values i and i respectively.

Let us consider the initial state vector $x = (0, 0, 1, 0, 0, 1, 0) \in X$.

To find the effect of x on M.

$xM = (i, 0, 1 + i, 0, -1, 0, 0) \to (i, 0, 1 + i, 0, 0, 1, 0) = y_2$.

We now find the effect of y_2 on M.

$y_2M = (i - 1, -i, 1 + i, i - 1, i - 1, 0, 0) \to (i, 0, 1 + i, i, i, 1, 0)$

$= y_3$.

$y_3M = (i - 1, -i, 1 + i, i - 1, i - 1, 0, 0) \to (i, 0, 0, 1 + i, i, 1, 0)$

$= y_4$ (say).

Clearly $y_4 = y_3$ hence the hidden pattern is a fixed point given by (i, 0, 1 + i, i, i, 1, 0).

This is the way we can obtain the resultant vectors for a given initial state vector.

Let us consider $x = (0, 0, 0, 1 + i, 0, 0, 0) \in X$; to find the effect of x on M.

$xM = (0, -1-i, 0, 0, 1+i, 0, 0)$

$y_1 \to (0, 0, 0, 1+i, 1+i, 0, 0) = y_1$

$y_1M = (0, -1-i, 0, 0, 1+i, 0, 0) \to (0, 0, 0, 1+i, 1+i, 0, 0)$

$= y_2$ (say);

Clearly $y_2 = y_1$ so the hidden pattern is a fixed point given by $(0, 0, 0, 1+i, 1+i, 0, 0)$.

Now we find the hidden pattern of the initial state vector

$x = (0, 0, 0, 0, 0, 0, 1) \in X$.

To find the effect of x on the dynamical system M

$xM = (0, 0, 0, 1, -I, i, 0) \to (0, 0, 0, 1, -I, I, 0) \to (0, 0, 0, 1, 0, I, 1) = y_1$

$y_1M = (0, -1, i-1, 1, 1-i-i, i, 0) \to (0, 0, i, 1, 1, i, 1)$

$= y_2$ (say)

$y_2M = (-1, -1, i-1, 1, 1, i, 0) \to (0, 0, i, 1, 1, i, 1)$

$= y_3$ (say)

Thus the hidden pattern is a fixed point given by

$(0, 0, i, 1, 1, i, 1)$.

We can in case of FICMs also define both the notion of combined disjoint FICM, and combined overlap FICMs models.

Suppose we have some 8 attributes say $C_1, C_2, C_3, C_4, C_5, C_6$ C_7 and C_8 and two experts work on the problem.

First expert works with the attributes C_1 C_4 C_5 C_7 and C_8 and the second expert works with C_2, C_3 and C_6 attributes then we can get the combined disjoint FICMs as $\{C_1, C_4, C_5, C_7, C_8\} \cap \{C_2, C_3, C_6\} = \phi$ in the following way.

Let M, be the matrix of the complex valued graph G, given by

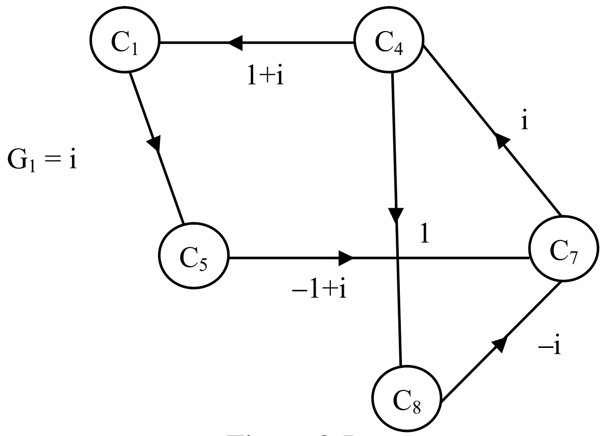


Figure 3.5

Let M_1 the connection matrix associated with G_1 be as follows.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_4 \\ C_5 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_4 & C_5 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 0 & 0 & i & 0 & 0 \\ 1+i & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1+i & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 \end{bmatrix} \end{array}$$

Let G_2 be the complex valued directed graph given by the second expert which is as follows.

G_2 =

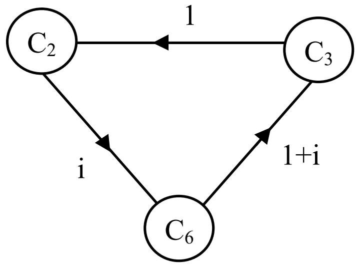


Figure 3.6

Let M_2 be the connection matrix associated with G_2

$$M_2 = \begin{array}{c} \\ C_2 \\ C_3 \\ C_6 \end{array} \begin{array}{c} \begin{array}{ccc} C_2 & C_3 & C_6 \end{array} \\ \begin{bmatrix} 0 & 0 & i \\ 1 & 0 & 0 \\ 0 & 1+i & 0 \end{bmatrix} \end{array}$$

Using the connection matrices M_1 and M_2 we combine them in a special way and obtain M which serves as the dynamical system of the combined disjoint FICMs.

Fuzzy (complex) imaginary cognitive maps model. For the construction and more on the structure of them and the working of it please refer [Ele. Socio].

Now we give M in the following $M = M_1 \oplus M_2$

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1+i & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1+i & 0 \\ 0 & 0 & 1+i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \end{bmatrix} \end{array}.$$

Now interested reader can find the hidden pattern for any initial state vector using the disjoint combined FICMs.

Here we briefly describe the overlap combined FICMs by an example.

Suppose there are some six attributes C_1, C_2, C_3, C_4, C_5, C_6 associated with the problem P. Let two experts work on the problem. The first expert uses the nodes $S_1 = \{C_1, C_3, C_4$ and $C_6\}$ and the second expert uses the nodes $S_2 = \{C_2, C_5, C_3$ and $C_4\}$. Clearly $S_1 \cap S_2 = \{C_3, C_4\}$ so the nodes attributes of the two experts overlap. Now we give the directed complex valued graph G_1 given by the first expert.

$G_1 =$

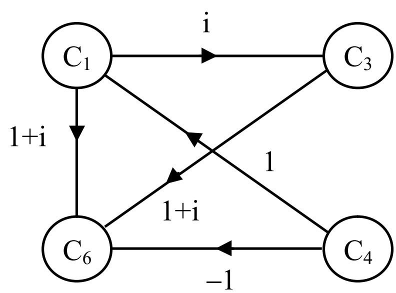


Figure 3.7

Let G_2 be the weighted directed complex valued graph given by the second expert which is as follows.

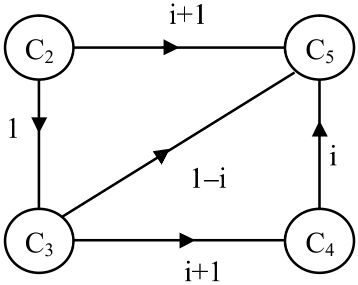
$G_2=$ 

Figure 3.8

Let N_1 be the connection matrix associated with the graph G_1

$$N_1 = \begin{array}{c} \\ C_1 \\ C_3 \\ C_4 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_3 & C_4 & C_6 \end{array} \\ \begin{bmatrix} 0 & i & 0 & 1+i \\ 0 & 0 & 0 & 1+i \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

It N_2 be the connection matrix associated with the graph G_2

$$N_2 = \begin{array}{c} \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{cccc} C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1+i \\ 0 & 0 & 1+i & 1-i \\ 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now the overlap matrix N_1 and N_2 is given by N. The obtaining of N using N_1 and N_2 is elaborately described in [18].

$$N = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 0 & i & 0 & 0 & 1+i \\ 0 & 0 & 1 & 0 & 1+i & 0 \\ 0 & 0 & 0 & 1+i & 1-i & 1+i \\ 1 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \end{array}.$$

N will serve as the overlap combined FICMs dynamical system. Interested reader can work the resultant using initial state vectors from $Y = \{(x_1, \ldots, x_6) / x_j \in \{0, 1, i, 1 + i\}; 1 \leq j \leq 6\}$.

Thus at this juncture the authors want to make clear that something is imaginary is very different from something is indeterminate so such type of fuzzy models have become mandatory.

Next we proceed onto describe the Fuzzy Imaginary Relational Maps model (FIRMs model). This model is akin to Fuzzy Relational Maps model and Neutrosophic Relational Maps model. Here some of the concepts may be imaginary, purely imaginary apart from some of them being real.

We first describe them. Already in the earlier chapter we have defined complex valued bipartite graph.

Now we construct / describe methodically the Fuzzy Imaginary Relational Maps (FIRMs) model.

For more about FRMs and NRMs model refer [15].

Fuzzy Imaginary Relational Maps (FIRMs) are built as in Fuzzy Imaginary Cognitive Maps (FICMs) model just described in this chapter. In FICMs the causal associations between concepts nodes imaginary or otherwise is among concurrently active units is analysed.

Only as in case of FRMs and FIRMs we demand that the very causal associations to be divided into two disjoint units. For instance the relation between a mental patient and a psychiatrist or the relation between a doctor and a hypochondria patient. Novelist and the imaginary characters, children and their imaginary world and so on.

All novels are not real several times the role is played by the imaginary characters. Thus the need for Fuzzy Imaginary Relational Maps (FIRMs) model is mandatory.

Thus we can abstractly define a Fuzzy Imaginary Relational Maps (FIRMs) model to be a complex valued graph or a map from D to R where $D = \{D_1, \ldots, D_n\}$ denotes the domain space of nodes and $R = \{R_1, R_2, \ldots, R_m\}$ denotes the range space of nodes. The node of $D = \{(x_1, \ldots, x_n)\} \mid x_j = 0$ or i or 1 or $1 + i$ for $j = 1, 2, \ldots, n\}$ and $Y = \{(y_1, \ldots, y_m) \mid y_k = 0$ or 1 or i or $1 + i$, $k = 1, 2, \ldots, m\}$ denotes the on state 1 or off state 0 or imaginary state $1 + i$ or the purely imaginary state i.

Let D_i and R_j denote the two nodes of a FIRM. The directed edge from D_i to R_j denotes the causality of D_i on R_j called relations. Every edge of an FIRMs model is weighted with a value in the set $\{0, i, 1, 1 + i\}$. Let e_{ij} denote the edge weight, if $e_{ij} = 0$ then there is no effect of D_i on R_j, if $e_{ij} = 1$ the effect of D_i on R_j is such that increase or decrease in D_i causes increase or decrease in R_j respectively of $e_{ij} = i$ then the causality of D_i on R_j is purely imaginary. If $e_{ij} = 1 + i$ then the causality of D_i on R_j is imaginary. We do not go for values like -1, i, $-1 + i$, $-1 - i$, and $1 - i$. However it is also possible to define relations or causalities in that form also.

The nodes of a FIRM are called as fuzzy imaginary nodes with edge weights from the set $\{0, 1, i, 1 + i\}$ will be defined as simple FIRMs.

Let $D_1, \ldots, D_n$ be the nodes of the domain space D and $R_1, R_2, R_3, \ldots, R_m$ be the nodes of the range space R of an FIRMs model.

Let the complex valued matrix E be defined as $E = (e_{ij})$ where e_{ij} is the weight of the directed edge $D_i R_j$ (or $R_j D_i$) taking entries from the set $\{0, 1, i, 1 + i\}$. E is defined as the complex valued relational matrix of the FIRMs model.

Also E can be often termed as the dynamical system associated with the FIRMs.

As in case of FRMs model and FIRMs model also for the given nodes $D_1, D_2, \ldots, D_n$ and $R_1, \ldots, R_m$; $A = \{(a_1, \ldots, a_n) / a_j \in \{0, i, 1, 1 + i\}; 1 \leq j \leq n\}$ is defined as the instantaneous state vector which shows the on or off or imaginary or purely imaginary position of the nodes at any instant.

Similarly $B = \{(b_1, \ldots, b_m) / b_j \in \{0, 1, i, 1 + i\}; 1 \leq j \leq m\}$ is defined as the instantaneous state vector which shows the on or off or imaginary or purely imaginary state of the nodes at that instant.

Let $D_1, \ldots, D_n$ and $R_1, R_2, \ldots, R_m$ be the nodes of an FIRMs model. Let D_jR_j (or R_jD_i) be the edges of an FIRMs model, j = 1, 2, …, m and i = 1, 2, …, n. Let the edges form a directed cycle.

An FIRM is said to be a cycle if it possesses a directed cycle. An FIRM is said to be a cyclic if it does not possess any directed cycle.

An FIRM as in case of a FRM is said to be FIRM with cycles is defined to be a FIRM with feedback and when there is a feedback in the FIRM that is when the causal relations flow through a cycle in a revolutionary manner the FIRM is called a complex dynamical system.

Let D_i R_j (or R_j D_i), ($1 \leq i \leq n$ and $1 \leq j \leq m$) when D_i or R_j is switched on and if causality flows through edges of the cycle and if it again causes D_i (or R_j) we say the complex dynamical system goes round and round. This is true for any node R_j (or D_i); $1 \leq i \leq n$ and $1 \leq j \leq m$. The equilibrium state of this complex dynamical system is defined as the hidden pattern.

The equilibrium state of the complex dynamical system is a unique pair of state vector then it is a fixed point pair. If the FIRM settles down with on state vector or respectively in form then this equilibrium is a limit cycle pair.

We will describe this by an example.

Example 3.3. Let $\{D_1, D_2, D_3, D_4, D_5, D_6\}$ = D denotes the six symptoms suffered by an hypochondria patient.

R = {R_1, R_2, R_3, R_4, R_5} are the four possible medication the doctor wishes to prescribe.

The description of nodes D_1, …, D_6 is described.

D_1 - The patient has fever.

D_2 - The patient suffers from chronic cold and cough.

D_3 - The patient says he feels giddy but has not fainted even once in the past medical history.

D_4 - He says he has heart problems but his ECG and BP are very normal.

D_5 - The patient complains of urinary infections but the tests showed negative.

D_6 - The patient complains of disturbed sleep.

The descriptions of the range nodes R_1, R_2, …, R_5 are as follows.

R_1 - Medication to stop fever.

R_2 - Medication for old and cough.

R_3 - Medical to help over come heart problems.

R_4 - Medication for urinary infection.

R_5 - Medication for calming down the patient.

It is important to keep on record doctor cannot fully rely on the patient feelings as h e is an hypochondria. Secondly the doctor cannot say to him his symptoms are imaginary.

At the same times should treat him with least damage to health as well with some care to his real symptoms. So the symptoms D_3, D_4, D_5 and D_6 can be purely imaginary or imaginary. D_3 can be a real symptoms as he suffers symptoms D_1 and D_2.

The expert doctor follows the following simple directed complex valued bigraph G relating to the nodes D_1, …, D_6 and $R_1, R_2, \ldots, R_5$.

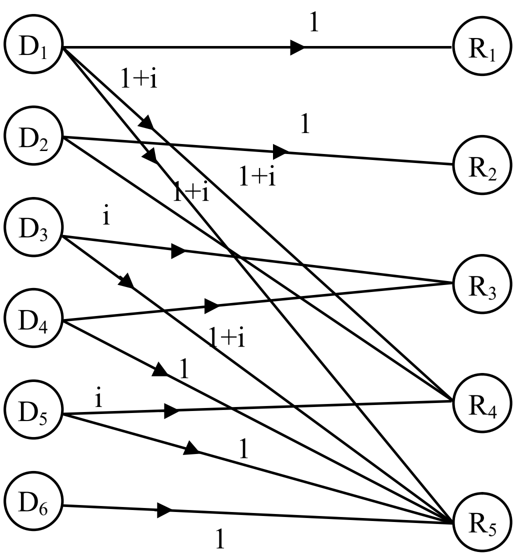


Figure 3.9

This expert who is a doctors feels so and supplies the complex valued bigraph G.

Let S denote the related complex valued weighted matrix of the bigraph G which is as follows.

$$S = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1+i & 1+i \\ 0 & 1 & 0 & 1+i & 0 \\ 0 & 0 & i & 0 & 1+i \\ 0 & 0 & 1+i & 0 & 1 \\ 0 & 0 & 0 & i & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}.$$

This expert feels might be the patient cough (or throat problem) with fever may be related to urinary problems so connects D_1 with R_4 as well as with R_5 to put him to rest so that temperature may come down. However the doctor feels that it may be 50% real and 50% imaginary in both cases.

Likewise we can interpret the experts view on the problem. However in mind the doctor may feel the nodes D_3, D_4, D_5 and D_6 must be purely imaginary or imaginary that is values i and 1 + i respectively.

Now we try to work with the on state on the node

$x = (1, 0, 0, 0, 0, 0) \in A$ that the node D_1 is on with real value 1 and rest of the nodes are in the off state.

Consider the effect of x on S;

$xS = \quad (1, 0, 0, 1 + i, 1 + i) = y_1$ say

$y_1S^t = \quad (1 + 4i, 2i, 2i, 1 + i, 2i, 1 + i) \to$

$$(i, i, i, 1 + i, i, 1 + i) = x_1 \text{ (say)}$$

$$x_1S = (i, i, 4i, -3 + 2i, 5i) \to (i, i, i, i, i) = y_2 \text{ (say)}$$

$$y_2S^t = (3i - 2, 2i - 1, i - 2, 2i - 1, -1 + i, i) \to$$

$$(i, i, i, i, i, i) = x_2 \text{ (say)}$$

$$x_2S = (i, i, -2 + i, -3 + 2i, 5i - 2) \to (i, i, i, i, i) = y_3 \text{ (say)}$$

Clearly $y_3 = y_2$.

Now we see the on state of the node the hypochondria patient suffers from fever results in imaginary state of each and every node.

At this juncture we wish to express the very idea that the hypochondria patient is suffering from fever itself is imaginary consequently the all other symptoms given by the patient are only imaginary when discussed with some experts about this issue they said one can suffer mild temperature if they feel and think all the time they suffer from the fever symptom.

As mind rules the body in general this is always possible. Further the doctor has to be very careful in prescribing medication. He can choose to provide some vitamins and avoid giving heavy dosage of medication for fever. It can be a low dosage. Only this can be a solution to treat this patient.

Only the hidden pattern pair when the only state fever was on resulted in the imaginary pair $\{(i, i, i, i, i, i), (i, i, i, i, i)\}$.

So all the symptoms are imaginary so the doctor has to deal this situation in a very very careful way.

For when the fever is caused by feeling any sort of high medication to lower the fever may lower the normal temperature which may result in the risk of patients' health.

Under these condition the doctor should judiciously use his expertise and be careful in treating such patients. Further the resultant is not even a limit cycle pair only a fixed point pair.

This FIRMs maps can play a vital role in medical diagnosis as the data when applied in the FIRMs model are handy and the latter model is used when the concepts under discussion can be divided into two disjoint classes.

Further the FIRMs model can be realized as the generalization of FICMs model and the graphs associated with FICMs will be complex valued weighted directed graphs whereas in case of FIRMs the graphs are complex valued weighted bigraphs. In all cases the study is innovative and important as there are always imaginary concepts in the cognitive structure of the human mind as well as the connection or the edge weights can also be imaginary. Infact the use of strong complex valued directed graphs (or bigraphs) has become mandatory.

Thus these two models FICMs and FIRMs can be realized as the generalization of FCMs and FRMs respectively where the concepts / nodes can be imaginary as well as the edge weights can be imaginary.

Next we proceed onto define for the first time the notion of complex neutrosophic valued graphs whose edge weights are from the set $\{ai + bI + ci + diI / a, b, c, d \in R, i^2 = -1, I^2 = I, (iI)^2 = -I\} = S = \langle R \cup i \cup I \rangle$.

Before the systematic definition is provided we give the some examples of this situation.

Example 3.4. Let G be a graph given by the following figure which has $v_1, v_2, \ldots, v_9$ to be the vertices and edge weights are from S.

G =

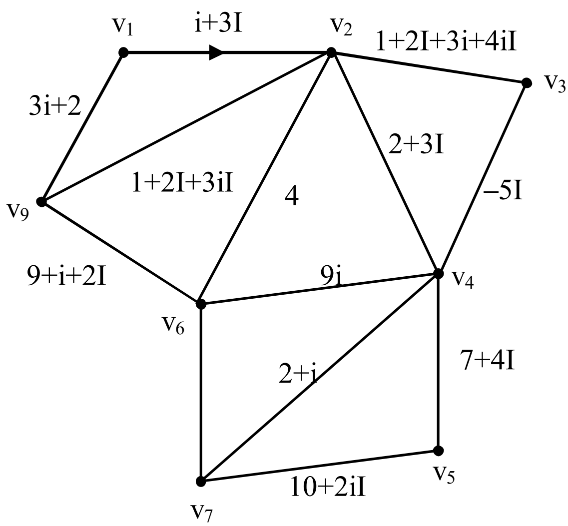


Figure 3.10

Clearly G has no loops further G is not a directed graph. The edge weights are from S so it can be real, complex indeterminate or complex indeterminate.

For neutrosophic graphs in general please refer [21].

Clearly G is not a directed graph it has no loops.

We prove one example of a complex neutrosophic (neutrosophic complex) valued directed graph which will have the edge weights from

S = {⟨R ∪ I ∪ I⟩} = {a + bi + cI + diI / d, a, b, c ∈ R, $i^2 = -1$ and $I^2 = I$}. G will have no loops.

Example 3.5. Let G be a directed graph with vertices v_1, v_2, v_3, v_4, v_5 and v_6 and edge weights from S given by the following figure;

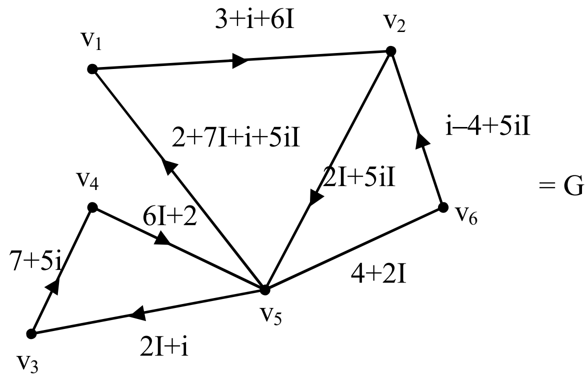


Figure 3.11

Clearly G is a complex - neutrosophic directed graph with edge weights from S.

In view of all these we now present the definition of the neutrosophic-complex valued graph systematically.

Definition 3.1. *Let G be a graph with n vertices v_1, v_2, ...,v_n and some m edges with edge weights from {⟨R ∪ i ∪ I⟩ = {a + bi + cI + diI / a, b, c, d ∈ R, $i^2 = -1$, $I = I^2$ and $(iI)^2 = -I$}.*

We call G to be a complex - neutrosophic (neutrosophic complex) edge weights graphs if atleast one edge takes weights a + bi + cI + diI where not all of b c or d zero atleast any two from the set {b, c, d} are non zero.

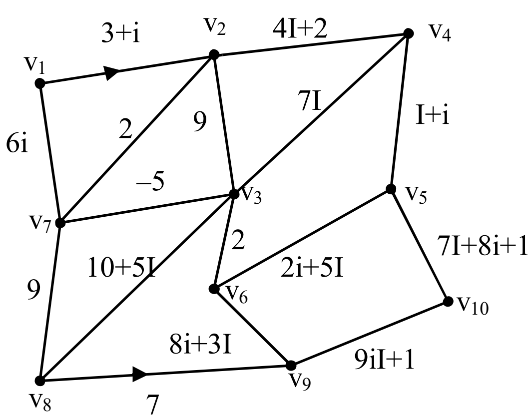
We do not demand this for all edges if atleast one edge takes weight as a + bi + cI + diI (a, b, c, d $\in$ R \ {0}) or a + bi + cI + diI (b, c, d $\in$ R \ {0} or c, d $\in$ R \ {0} or d, b $\in$ R \{0}).

The edges are directed and weighted we call the graph to be a neutrosophic-complex valued directed graph.

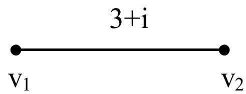
We can for any complex-neutrosophic valued graph find subgraphs. It is pertinent to record that all subraphs need not be complex-neutrosophic valued graph they can be real edged graphs or complex edged graph or neutrosophic edged graph or complex - neutrosophic edged graphs.

Example 3.6. Let G be the complex-neutrosophic valued graph given by the following Figure. G has 10 distinct vertices and the edge weights are from S = {$\langle R \cup I \cup I \rangle$ = a + bi + cI + diI where a, b, c, d $\in$ R, $i^2 = -1$, $I^2 = I$ and $(iI)^2 = -I$}.

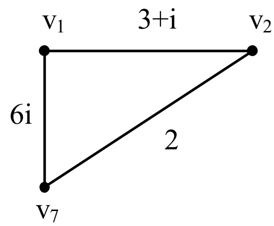
G =



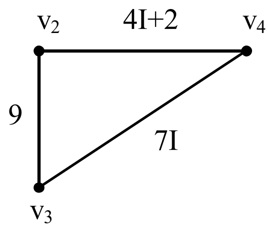
We now enumerate some of the graphs of G in the following;



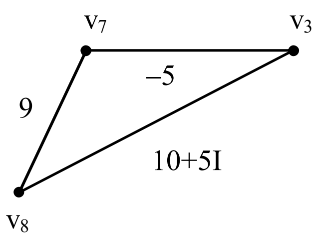
$= H_1$



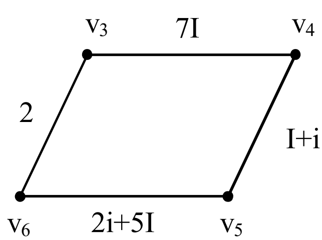
$= H_2$



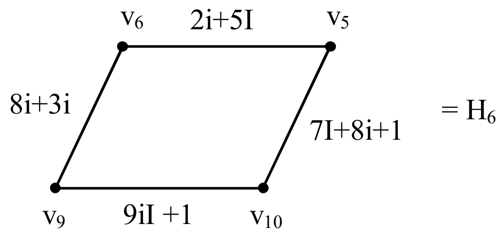
$= H_3$

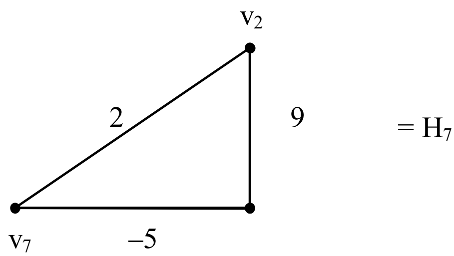


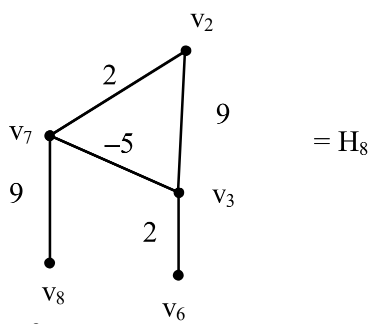
$= H_4$



$= H_5$


v6
2i+5I
v5
8i+3i
7I+8i+1
= H6
v9
9iI +1
v10


v2
2
9
= H7
v7
–5


v2
2
9
v7
–5
= H8
9
v3
2
v8
v6

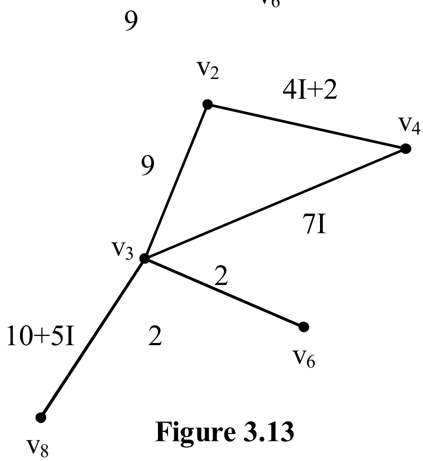

9
v2
4I+2
v4
9
7I
v3
2
10+5I
2
v6
v8

Figure 3.13

and so on.

We make the following observations.

H_1 is a pure complex valued subgraph.

H_2 is a complex valued subgraph.

H_3 is a neutrosophic valued subgraph of G.

H_4 is again a neutrosophic valued subgraph of G.

H_5 is a complex-neutrosphic valued subgraph of G.

H_6 is also a complex neutrosophic valued subgraph of G.

H_7 is a real valued subgraph.

H_8 is again a real valued subgraph of G.

H_9 is a neutrosophic valued subgraph of G.

From this the following observations are made;

i) If G is a complex-neutrosophic graphs all subgraphs need not be complex-neutrosophic subgraphs evident from some of H_8, H_9, H_7, H_1, H_2, H_3 and H_4 are not complex - neutrosophic subgraph.

ii) G has real valued subgraphs evident from subgraphs H_7 and H_8.

iii) G has pure complex valued subgraphs. H_1 is one such subgraph.

iv) G has neutrosophic valued subgraphs evident from subgraphs given in the figures H_3 and H_4.

v) We observe G has all types of subgraphs.

Hence we have the following theorem.

Theorem 3.1. *Let G be any complex-neutrosophic valued graph. All subgraphs of G in general need not be a complex neutrosophic valued subgraphs.*

The reader can give proof by constructing examples.

We have already given one example to this effect.

Now we give the related weighted matrix of neutrosophic-complex valued graphs.

It is pertinent to recall that a matrix is said to be a complex neutrosophic valued matrix M if at least one of its entries is of the form a + bi + cI + dIi with {b and c} or {c and d} or {b and d} or {b and c} $\in R \setminus \{0\}$. Otherwise M is not a complex neutrosophic valued graph.

Now we will give the association of a weight complex neutrosophic matrix with a complex-neutrosophic valued graph G.

Example 3.7. Let G be a complex neutrosophic valued graph given by the following figure.

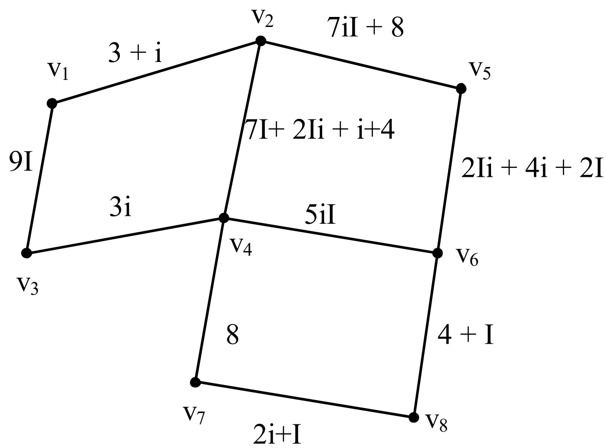


Figure 3.14

The weighed matrix M associated with the complex neutrosophic valued graph G is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{llll} v_1 & v_2 & v_3 & v_4 \end{array} \\ \left[\begin{array}{llll} 0 & 3+i & 9I & 0 \\ 3+i & 0 & 0 & 7I+2Ii+i+4 \\ 9I & 0 & 0 & 3i \\ 0 & 7I+2Ii+i+4 & 3i & 0 \\ 0 & 8+7iI & 0 & 0 \\ 0 & 0 & 0 & 5iI \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right. \end{array}$$

$$\begin{array}{cccc} v_5 & v_6 & v_7 & v_8 \end{array}$$

$$\left.\begin{array}{llll}
0 & 0 & 0 & 0 \\
8+7iI & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 5iI & 8 & 0 \\
0 & 4i+2I+2Ii & 0 & 0 \\
4I+2I+2Ii & 0 & 0 & 4+I \\
0 & 0 & 0 & 2i+I \\
0 & 4+I & 2i+I & 0
\end{array}\right] .$$

We see M is a 8×8 complex-neutrosophic valued weighted matrix associated with M.

Clearly M is symmetric about the main diagonal and the diagonal entries are zero. Thus we can record without any hesitation that if G is just a neutrosophic complex valued graph which is not directed the associated weighted matrix M of G is a symmetric matrix and the main diagonal entries are zero.

Now we proceed to describe the complex neutrosophic valued weighted matrix of a complex neutrosophic valued directed graph K.

Example 3.8. Let K be a complex neutrosophic valued weighted directed graph given by the following figure;

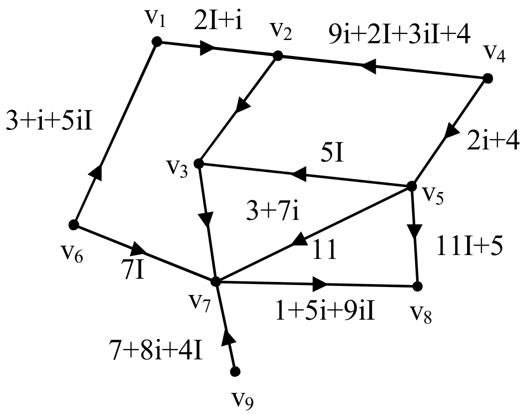


Figure 3.15

Let N be the associated weight matrix of the complex valued neutrosophic graph K.

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \left[\begin{array}{lllll} 0 & 2I+i & 0 & 0 & 0 \\ 0 & 0 & 3i+8 & 9i+2I+3iI+4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2i+4 \\ 0 & 0 & 5I & 0 & 0 \\ 3+i+5I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right. \end{array}$$

$$\begin{array}{cccc} v_6 & v_7 & v_8 & v_9 \\ \end{array}$$

$$\left.\begin{array}{llll}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 3+7i & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 11 & 11I+5 & 0 \\
0 & 7I & 0 & 0 \\
0 & 0 & 1+5i+9iI & 0 \\
0 & 0 & 0 & 0 \\
0 & 7+8i+4I & 0 & 0
\end{array}\right]$$

Clearly N is a neutrosophic complex valued matrix which is not symmetric and the main diagonal entries of N are zero.

Now we proceed onto describe the conjugate of a complex-neutrosophic numbers in $S = \{\langle R \cup i \cup I \rangle\} = \{a + bi + cI + dIi \mid a, b, c, d \in R, i^2 = -1, I^2 = I, (iI)^2 = -I\}$.

Let $x = a + bi + cI + diI \in S$ to find the conjugate of x. We denote the conjugate by $\overline{x}$ and $\overline{x} = a - bi + cI - diI$. Clearly if $x = 4 + 8I$ then $\overline{x} = 4 + 8I$ if $x = 9$ then $\overline{x} = 9$.

If $x = I - 3i + 9$ then the conjugate of x is given by

$$\overline{x} = I + 3i + 9.$$

Thus given any neutrosophic complex valued graph we can find the conjugate graph.

To this end we have to find the weight matrix of the graph and find. The conjugate of that complex – neutrosophic valued matrix then obtain the corresponding graph of that matrix else if one is well versed in finding the conjugate of the graph it can be done directly also.

For instance if G is the complex valued graph.

The conjugate graph of G is as follows.

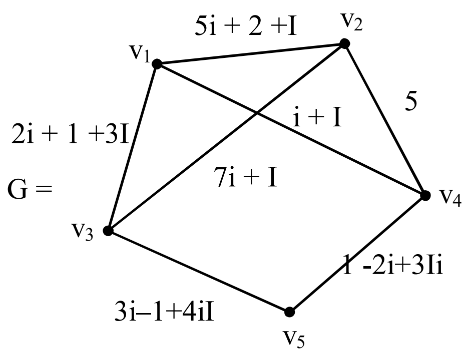


Figure 3.16

The conjugate graph of G is as follows.

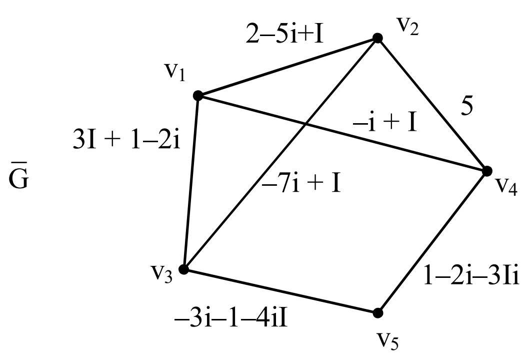


Figure 3.17

Clearly $\bar{G}$ is the conjugate of G.

Now we proceed onto give the neutrosophic - complex valued weighted matrix M associated with G. Then we will find $\bar{M}$ the conjugate matrix of M and also the matrix N of $\bar{G}$.

M =

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \left[\begin{array}{lllll} 0 & 5i+2+I & 2i+1+3I & i+I & 0 \\ 5i+2+I & 0 & 7i+I & 5 & 0 \\ 2i+1+3I & 7i+I & 0 & 0 & 3i-1+4iI \\ i+I & 5 & 0 & 0 & 2i+3iI \\ 0 & 0 & 3i-1+4iI & 2i+3iI & 0 \end{array}\right] \end{array}$$

Now we find $\bar{M}$ the conjugate of M.

$\bar{M}$ =

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \left[\begin{array}{lllll} 0 & 2-5i+I & 1-2i+3I & -i+I & 0 \\ 2-5i+I & 0 & -7i+I & 5 & 0 \\ 1-2i+3I & -7i+I & 0 & 0 & -3i-1-4iI \\ -i+I & 5 & 0 & 0 & -2i-3Ii \\ 0 & 0 & -1-3i-4iI & -2i-3iI & 0 \end{array}\right] \end{array}$$

Next we find the weighted neutrosophic - complex valued matrix N associated with $\bar{G}$.

N =

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 2-5i+I & 3I+1-2i & -i+I & 0 \\ 2-5i+I & 0 & -7i+I & 5 & 0 \\ 3I+1-2i & -7i+I & 0 & 0 & -3i-1-4iI \\ -i+I & 5 & 0 & 0 & -2i-3Ii \\ 0 & 0 & -3i-1-4iI & -2i-3iI & 0 \end{bmatrix} \end{array}$$

It is easily verified that N = $\bar{M}$. Thus we see either we can generate the conjugate complex - neutrosophic matrix using M and get the corresponding conjugate neutrosophic complex graph or get the conjugate neutrosophic - complex graph and then obtain the corresponding conjugate neutrosophic - complex valued matrix.

However it is pertinent to keep on record that the conjugate neutrosophic-complex graph is easily got if it is not directed however one has to be careful if the complex-neutrosophic graph is directed.

This is illustrated by the following example.

Example 3.9. Let G be a complex-neutrosophic directed graph H given by the following figure;

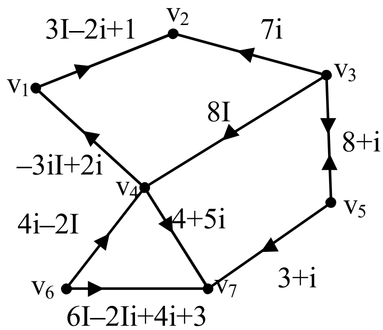


Figure 3.18

Now we proceed onto obtain the corresponding complex neutrosophic weighted matrix M associated with the graph H.

K =

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & 3I-2i+I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7i & 0 & 8I & 8+i & 0 & 0 \\ -3iI+2i & 0 & 0 & 0 & 0 & 0 & 4+5i \\ 0 & 0 & 8+i & 0 & 0 & 0 & 3+i \\ 0 & 0 & 0 & 4i-2I & 0 & 0 & \begin{array}{l} 6I-2Ii \\ +4i+3 \end{array} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

We now find the related conjugate complex -neutrosophic matrix $\overline{K}$ of K in the following.

$$\overline{K} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 3iI-2i & 0 & 0 & 0 \\ 3I+2i+1 & 0 & -7i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8-i & 0 & 0 \\ 0 & 0 & 8I & 0 & 0 & -4i-2I & 0 \\ 0 & 0 & 8-i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4-5i & 3-i & \begin{matrix} 6I+2Ii \\ +3-4i \end{matrix} & 0 \end{bmatrix} \end{array}.$$

Now we find the corresponding complex-neutrosophic conjugate graph $\overline{H}$ associated with matrix $\overline{K}$.

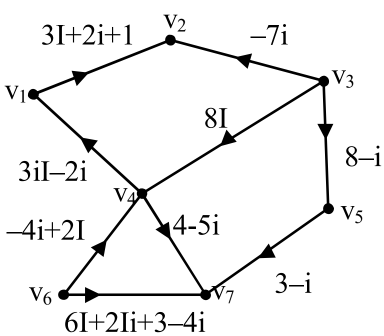


Figure 3.19

We see the orientation of $\overline{H}$ is reversed in direction in comparison with H.

However the directed edges remain the same the direction is changed and weights also are changed they happen to be the complex conjugate of the weights given in H.

Thus we can see the conjugate of complex-neutrosophic graph which are directed behave differently from the other graphs.

Now having seen examples of complex-neutrosophic graphs both directed and otherwise we will proceed onto give their applications in appropriate models.

Example 3.10. Let us consider the problem in which the synaptic relation connecting any of the concepts can be real or imaginary or neutrosophic. Suppose there are some 8 concepts say C_1, C_2, C_3, C_4, …,C_8. The directed neutrosophic complex weighted graph G with C_1, C_2, …,C_8 as nodes are given by the following figure;

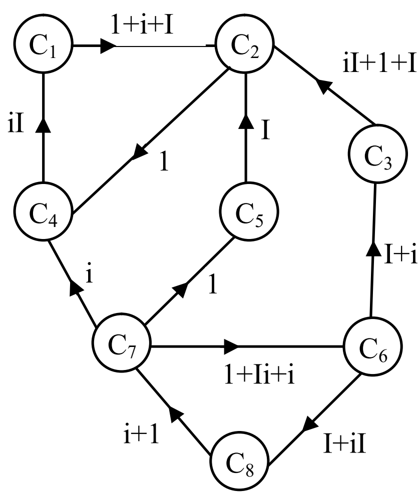


Figure 3.20

Let M denote the complex neutrosophic valued (weighted) connection matrix M associated with the complex-neutrosophic valued directed weighted graph G.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 0 & 1+i+I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1+I+iI & 0 & 0 & 0 & 0 & 0 & 0 \\ iI & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i+I & 0 & 0 & 0 & 0 & I+iI \\ 0 & 0 & 0 & i & 1 & 1+i+Ii & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1+i & 0 \end{bmatrix} \end{array}$$

Let X = {(a_1, a_2, a_3, …, a_8) / $a_i \in$ {0, 1, I, 1 + I, i, i + 1, iI, 1+iI, 1 + I + I, 1 + I + iI, 1 + I + iI, I + I + iI, 1 + I + i + iI I + I, I + iI, I + iI}; $1 \leq i \leq 8$} be the collection of all complex - neutrosophic valued state vectors} associated with the complex-neutrosophic valued model M.

Let x = (1, I, 0, 0, 0, 0, 0, iI) be the initial state vector from the collection of complex neutrosophic valued state vectors X.

To find the effect of x on M.

xM = (0, 1 + i + I, 0, I, 0, 0, iI – I, 0) $\to$

(1, 1 + i + I, 0, I, 0, 0, iI, iI) = x_1 (say)

"$\to$" denotes the complex-neutrosophic valued state vector has been updated.

We now consider

$x_1M = (iI, 1 + i + I, 0, 1 + i, iI, iI – 1 – I, iI – I\ \ 0) \rightarrow$
$(iI, 1 + i + I, 0, 1 + I, iI, iI, iI, iI) = x_2$ (say)

Now we find the effect of the complex - neutrosophic valued vector x_2 on the dynamical system M.

$x_2M = (iI – I, iI – I + iI, iI, iI – I, 1 + i + I – I, iI, iI – I – I, iI – I, iI – I) \rightarrow (iI, iI, iI, 1 + i, iI, iI, iI, iI) = x_3$ (say).

We find the effect of x_3 on M and so on.

We follow this procedure until we arrive at a fixed point or a limit cycle.

This depending on the hidden pattern we interpret the complex - neutrosophic valued resultant vector.

Since the elements in the set x is finite we are sure to arrive at the hidden pattern after a finite number of iterations.

Thus interested reader can find the hidden pattern for different sets of complex neutrosophic valued state vectors.

Let $x = (0, 1, 0, 0, 0, 0, 0, 0) \in X$, to find the effect of x on M.

$xM = (0, 0, 0, 1, 0, 0, 0, 0) \rightarrow (0, 1, 0, 1\ 0, 0, 0, 0) = x_1$ (say)

$x_1M = (iI, 0, 0, 1, 0\ 0\ 0\ 0) \rightarrow (iI, 1, 0, 1, 0, 0, 0, 0) = x_2$ (say)

$x_2M = (iI, iI – I + iI, 0, 1, 0, 0, 0, 0) \rightarrow (iI, iI, 0, 1, 0, 0, 0, 0) =$ x_3 (say)

$x_3M = (iI, iI - I + iI, 0, iI, 0, 0, 0, 0) \to (iI, iI, 0, iI, 0, 0, 0, 0)$

$x_4M = (-I, iI - I + iI, 0, iI, 0, 0, 0, 0) \to (0, iI, 0, iI, 0, 0, 0, 0)$
$= x_5$ (say)

$x_5M = (-I, 0, 0, iI, 0, 0, 0, 0) \to (0, 1, 0, iI, 0, 0, 0, 0) = x_6$ (say)

$x_6M = (-I, 0, 0, 1, 0, 0, 0, 0) \to (0, 1, 0, 1, 0, 0, 0, 0) = x_7$ (say).

We see $x_7 = x_1$ so the hidden pattern in this case is a limit cycle given by

$$x_1 \to x_2 \to x_3 \to x_4 \to x_5 \to x_6 \to x_1 \to \ldots$$

Thus the hidden pattern is a real state vector from x.

So on state of x_2 makes only x_4 to on state however no state takes up the neutrosophic value or a complex value or a complex neutrosophic value.

We call this type of model as fuzzy complex - neutrosophic cognitive maps model shortly as FCNCMs model.

We have seen the hidden pattern of the FCNCMs model can be real, complex, neutrosophic or complex neutrosophic depending on the initial state vector.

It is given as a simple problem for the reader to prove if $x \in X$ is a real state vector will the resultant vector be also real?

Let us now consider the initial state vector which is purely imaginary.

$x = (0, i, 0, 0, 0, 0, 0, 0) \in X.$

The effect of x on M is given by

$xM = (0, 0, 0, i, 0, 0, 0, 0) \to (0, i, 0, i, 0, 0, 0, 0) = y_1$ (say)

$y_1M = (-I, 0, 0, I, 0, 0, 0, 0) \to (0, i, 0, i, 0, 0, 0, 0) = y_2$ (say)

Clearly $y_1 = y_2$ so the hidden pattern is a fixed point given by (0, i, 0, i, 0, 0, 0, 0) which is all pure imaginary.

Now consider the initial state vector

$x = (0, I, 0\ 0\ 0\ 0\ 0\ 0) \in X$ which is pure neutrosophic

$xM = (0, 0, 0\ I, 0, 0, 0, 0) \to (0, I, 0, I, 0, 0, 0, 0) = y_1$ (say)

$y_1M = (iI, 0, 0, I, 0, 0, 0, 0) \to (iI, I, 0, I, 0, 0, 0, 0) = y_2$ (say)

$y_2M \quad = (iI, iI - I + iI, 0, I, 0, 0, 0, 0) \to (iI, iI, 0, I, 0, 0, 0, 0)$

$= y_3$ (say)

$y_3M \quad = (I, iI - I + iI, 0, iI, 0, 0, 0, 0) \to (iI, iI, 0, iI, 0, 0, 0, 0)$

$= y_4$ (say).

Clearly $y_3 = y_4$ hence the hidden pattern is a fixed point given by

y_3 = (iI, iI, 0, iI, 0, 0, 0, 0) which is pure complex neutrosophic however we started the initial state vector (0, I, 0, 0, 0, 0, 0, 0) which is only pure neutrosophic.

Next we proceed onto work with the initial state vector which is pure neutrosophic complex, given by x = (0, iI, 0, 0, 0, 0, 0, 0) $\in$ X.

We find the hidden pattern related with this x.

$xM = (0, 0, 0, iI, 0, 0, 0, 0) \to (0, iI, 0, iI, 0, 0, 0, 0) = y_1$ (say).

$y_1M = (–I, 0, 0, iI, 0, 0, 0, 0) \to (0, iI, 0, iI, 0, 0, 0, 0) = y_2$ (say).

Clearly $y_1 = y_2$ so the hidden pattern of the state vector x is a fixed point given by (0, iI, 0, iI, 0, 0, 0, 0).

Consider the resultant it is also pure complex neutrosophic.

Next we consider

$x = (0, 1 + I, 0, 0, 0, 0, 0, 0) \in X$ is the initial state vector which is neutrosophic.

The effect of x on M is given by $xM = (0, 0, 0, 1 + I, 0, 0, 0, 0) \to (0, 1 + I, 0, 1 + I, 0, 0, 0, 0) = y_1$ (say).

The effect of y_1 on M is $y_1M = (iI + iI, 0, 0, 1 + I, 0, 0, 0, 0) \to (iI, 1 + I, 0, 1 + I, 0, 0, 0, 0) = y_2$ (say).

$y_2M = (iI + iI, iI – I + iI, 0, 1 + I, 0, 0, 0, 0) \to (iI, iI, 0, 1 + I, 0, 0, 0, 0) = y_3$ (say).

$y_3M = (iI + iI, iI – I + iI, 0, iI, 0, 0, 0, 0) \to (iI, iI, 0, iI, 0, 0, 0, 0) = y_4$ (say)

$y_4M = (-I, iI – I + iI, 0, iI, 0, 0, 0, 0) \to (0, iI, 0, iI, 0, 0, 0, 0)$

$= y_5$ (say).

$y_5M = (-I, 0, 0, iI, 0, 0, 0, 0) \to (0, 1 + I, 0, iI, 0, 0, 0, 0)$

$= y_6$ (say)

$y_6M \quad = (\text{-}I, 0, 0, 1 + I, 0, 0, 0, 0) \to (0, 1 + I, 0, 1 + I, 0, 0, 0, 0)$

$= y_7$ (say)

Clearly $y_1 = y_7$ thus the hidden pattern of the given initial state vector is a limit cycle given by (0, 1 + I, 0, 1 + I, 0, 0, 0, 0).

Thus $y_1 \to y_2 \to y_3 \to y_4 \to y_5 \to y_6 \to y_7 (= y_1) \to$

Let x = (0, I + 1, 0, 0, 0, 0, 0, 0) be the initial state vector given by the imaginary on state of C_2 and all other nodes are zero.

Let us find the hidden pattern of x using on the dynamical system M.

$xM \quad = (0, 0, 0, 1 + i, 0, 0, 0, 0) \to (0, 1 + i, 0, 1 + i, 0, 0, 0, 0)$

$= y_1$ (say)

$y_1M = \quad (iI – I, 0, 0, 1 + i, 0, 0, 0, 0)$

$\to (iI, 1 + I, 0, 1 + i, 0, 0, 0, 0) = y_2$ (say).

$y_2M = (iI – I, iI – I + iI, 0, 1 + i, 0, 0, 0, 0)$

$\to (iI, iI, 0, 1 + I, 0, 0, 0, 0) = y_3$ (say).

$y_3M = (iI – I, iI – I + iI, 0, iI, 0, 0, 0, 0) \to (iI, iI, 0, iI, 0, 0, 0, 0)$

$= y_4$ (say).

$y_4M = (–I, iI – I + iI, 0, iI, 0, 0, 0, 0) \to (0, iI, 0, iI, 0, 0, 0, 0)$

$= y_5$ (say).

$y_5M = (–I, 0, 0, iI, 0, 0, 0, 0) \to (0, iI, 0, iI, 0, 0, 0, 0) = y_6$ (say).

Clearly $y_6 = y_5$ hence the hidden pattern is a fixed point which is purely complex neutrosophic different from the on state of the initial state vector.

Let x = (0, i + I, 0, 0, 0, 0, 0, 0) be the initial state vector in X.

To find the effect of x on M

$xM = (0, 0, 0, i + I, 0, 0, 0,0) \to (0, i + I, 0, i + I, 0, 0, 0, 0)$

$= y_1$ (say).

$y_1M = (-I + iI, 0, 0, i + I, 0, 0, 0, 0) \to (iI, I + I, 0, i + I, 0, 0, 0, 0) = y_2$ (say).

$y_2M = (iI – iI, iI – I + iI, 0, i + I, 0, 0, 0,0) \to (iI, i + I, 0, i + I, 0, 0, 0, 0) = y_3.$

Clearly $y_2 = y_3$ thus the hidden pattern of the state vector x is a fixed point given by (iI, i + I, 0, i + I, 0, 0, 0, 0).

Let x = (0, 1 + i + I, 0, 0, 0, 0, 0, 0) $\in$ X, be the initial state vector given. To find the effect of x on M

$xM = (0, 0, 0, 1 + i + I, 0, 0, 0, 0) \to (0, 1 + i + I, 0, 1 + i + I, 0, 0, 0, 0) = y_1$ (say).

$y_1M = (iI – I + iI, 0, 0, 1 + i + I, 0, 0, 0, 0) \to (iI, 1 + i + I, 0, 1 + i + I, 0, 0, 0, 0) = y_2$ (say).

$y_2M = (iI – I + iI, iI – I + iI, 0, 1 + i + I, 0, 0, 0, 0) \to (iI, iI, 0, 1 + i + I, 0, 0, 0, 0) = y_3$ (say).

y_3M = (iI – I + iI, iI – I + iI, 0, iI, 0, 0, 0, 0) → (iI, iI, 0, iI, 0, 0, 0, 0) = y_4 (say).

y_4M = (–I, iI – I +iI, 0, iI, 0, 0, 0, 0) → (0, iI, 0, iI, 0, 0, 0, 0)

= y_5 (say).

y_5M = (–I, iI – I + iI, 0, iI, 0, 0, 0, 0) → (0, iI, 0, iI, 0, 0, 0, 0)

= y_6 (say).

Clearly y_6 = y_5 thus the hidden pattern is a fixed point given by (0, iI, 0, iI, 0, 0, 0, 0).

Now we find the hidden pattern of the initial state vector.

x = (0, 1 + i + I + iI, 0, 0, 0, 0, 0, 0) ∈ X.

xM = (0, 0, 0, 1 + i + I + iI, 0, 0, 0, 0) → (0, 1 + i + I + iI, 0, 1 + i + I + iI, 0, 0, 0, 0) = y_1 (say).

y_1M =(iI – I + iI – I, 0, 0, 1 + i + I + iI, 0, 0, 0, 0) → (iI, 1 + I + I + iI, 0, 1 + i + I + iI, 0, 0, 0, 0) = y_2 (say).

y_2M = (iI + iI – I, 0, 0, 1 + iI + i + I, 0, 0, 0, 0) → (iI, 1 + i + I + iI, 0, 1 + I + I + iI, 0, 0, 0, 0) = y_3 (say).

Clearly since y_3 = y_2 the hidden pattern is a fixed point given by (iI, i + 1 + I + iI, 0, 1 + i + I + iI, 0, 0, 0, 0).

Consider x = (0, 1 + I + I, 0, 0, 0, 0, 0, 0) ∈ X; to find the effect of x on M.

xM = (0, 0, 0, 1 + I + i, 0, 0, 0, 0) → (0, 1 + i + I, 0, 1 + I + I, 0, 0, 0, 0) = y_1 (say)

$y_1M = (iI - I + iI, 0, 0, 1 + i + I, 0, 0, 0, 0) \to (iI, 1 + i + I, 0, 1 + i + I, 0, 0, 0,) = y_2$ (say)

$y_2M = (iI + iI - I, Ii - I + Ii, 0, 1 + i + I, 0, 0, 0, 0) \to (iI, iI, 0, 1 + i, + I, 0, 0, 0, 0) = y_3$ (say)

$y_3M = (iI - I + iI, iI - I + iI, 0, iI, 0, 0, 0, 0) \to (iI, iI, 0, iI, 0, 0, 0, 0) = y_4$ (say).

$y_4M = (-I, iI - I + iI, 0, iI, 0, 0, 0, 0) \to (0, iI, 0, iI, 0, 0, 0, 0)$

$= y_5$ (say).

$y_5M = (-I, iI - I + iI, 0, iI, 0, 0, 0, 0) \to (0, iI, 0, iI, 0, 0, 0, 0)$

$= y_6$ (say).

Clearly $y_6 = y_5$ so the hidden pattern is a fixed point given by (0, iI, 0, iI, 0, 0, 0, 0).

It is interesting and important to make the following observations.

i) In case of FCMs we can for a on state of a node get only on state of the other nodes and the node which is in the on state initially.

ii) In the case of FINCMs we see the on state of a node can after a series of iterations arrive at a complex state 1 + i or pure imaginary state i, neutrosophic state I + 1 or pure neutrosophic state I or neutrosophic imaginary state 1 + i + I + iI or 1 + i +I or pure neutrosophic imaginary state i + I, iI or i + I + iI.

Thus the node which was on as pure imaginary i may at the point of limit cycle or a fixed point result in any one of the terms from the set

P = {1, i, 1 + i, I, 1 + I, i + I, 1 + I + i, 1 + i + iI, iI, 1 + iI + I, i + i + iI, 1 + i + iI + I, I + i, i + iI, I + iI}.

Thus we see the advantage of using FCNCMs model at appropriate places.

The above example has clearly shown how the hidden patterns vary in the set P for any one of the elements of P as the state of the node.

Now at this juncture we wish to mention about the concept of Combined Fuzzy Complex Indeterminate Cognitive Maps model analogous to the Combined Fuzzy Cognitive Maps model.

This is the case when two or more experts work on the same problem with the same number of concepts or nodes. More about such study please refer [15].

Next we proceed onto study the notion of complex neutrosophic valued weighted bigraphs. Consider the following definition.

Let G be a weighted bigraph. If the edge weights are from the set S ={a + bi +cI + diI / a, b, c, d $\in$ R; $i^2 = -1$, $I^2 = I$ and $(iI)^2 = -I$} then we define G to be a complex-neutrosophic valued weighted bigraph.

We will provide some examples of them.

Example 3.11. Let K be a complex neutrosophic valued weighted bigraph with edge weights from S = {a + bi + cI + diI / a, b, c, d $\in$ R; i^2 = -1, I^2 = I and $(iI)^2$ = -I} given in the following.

K =

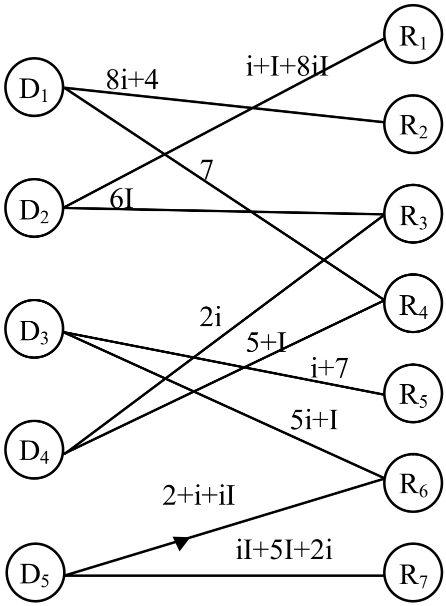


Figure 3.21

The connection matrix M associated with the bigraph K is as follows

M =

$$\begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{array}\begin{array}{c} \begin{array}{ccccccc} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 \end{array} \\ \begin{bmatrix} 0 & 8i+4 & 0 & 7 & 0 & 0 & 0 \\ i+I+8iI & 0 & 6I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i+7 & 5i+I & 0 \\ 0 & 0 & 2i & 5+I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+i+iI & iI+5I+2i \end{bmatrix} \end{array}.$$

We can for this matrix M find the conjugate which is denoted by $\overline{M}$ is as follows.

$$\overline{M} = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \end{array} \begin{array}{c} \begin{array}{ccccc} D_1 & D_2 & D_3 & D_4 & D_5 \end{array} \\ \begin{bmatrix} 0 & I - i - 8iI & 0 & 0 & 0 \\ -8i + 4 & 0 & 0 & 0 & 0 \\ 0 & 6I & 0 & -2i & 0 \\ 7 & 0 & 0 & 5 + I & 0 \\ 0 & 0 & 7 - i & 0 & 0 \\ 0 & 0 & I - 5i & 0 & 2 - i - iI \\ 0 & 0 & 0 & 0 & -iI + 5I - 2i \end{bmatrix} \end{array}$$

Now we find the bigraph $\overline{K}$ associated with the complex valued weighted (connection) matrix $\overline{M}$.

$\overline{K} =$

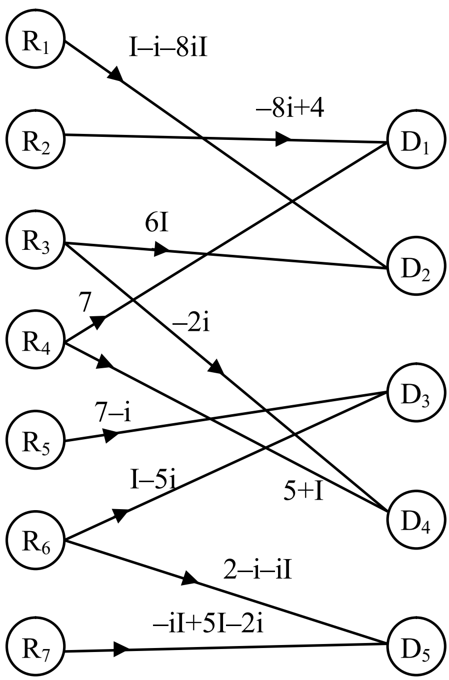


Figure 3.22

We also in the models which we are going to construct use the notion of transpose of a connection matrix M of the bigraph K.

Now let M^t be the transpose of the matrix M.

$$M^t = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \end{array} \begin{array}{c} \begin{array}{ccccc} D_1 & D_2 & D_3 & D_4 & D_5 \end{array} \\ \begin{bmatrix} 0 & i+I+8iI & 0 & 0 & 0 \\ 8i+4 & 0 & 0 & 0 & 0 \\ 0 & 6I & 0 & 2i & 0 \\ 7 & 0 & 0 & 5+I & 0 \\ 0 & 0 & i+7 & 0 & 0 \\ 0 & 0 & 5i+I & 0 & 2+i+iI \\ 0 & 0 & 0 & 0 & iI+5I+2i \end{bmatrix} \end{array}.$$

Clearly M^t is not the same as $\overline{M}$ the conjugate of M.

Now based on all these we proceed onto define the notion of Fuzzy Complex Neutrosophic Relational Maps (FCNRMs) model.

Definition 3.2. *Fuzzy Complex Neutrosophic Relational Maps (FCNRMs) models are constructed analogous to Fuzzy Complex Neutrosophic Cognitive Maps (FCNCMs) models described and discussed earlier in this chapter.*

In FCNCMs we promote correlations between causal associations among concurrently active units. But in FCNRMs we divided just like FRMs the very causal associations into two disjoint units for example the relation between a teacher and a student or relation between an employee or employer or a relation between a doctor and a patient.

Here when we use FCNRMs model the causal relation between a drug addict and a counselor or a doctor and a mental patient and so on.

Such study is very innovative and new for we have introduced Neutrosophic Relations Maps (NRMs) model, Neutrosophic Complex Relational Maps model and so o n.

Here we introduce Fuzzy Complex Neutrosophic Relational Maps model (FCNRMs model).

A FCNRM is a directed bigraph or a map from D to R where D is the domain space of nodes; $D = \{(x_1, x_2, \ldots, x_n) / x_j \in \{0, 1, i, I, 1 + i, 1 + I, iI, 1 + iI, i + I, i + iI, I + iI, 1 + i + I, 1 + i + iI, 1 + I + iI, 1 + I + i + iI, I + i + iI\}, 1 \leq j \leq n\}$ and $R = \{(y_1, y_2, \ldots, y_m) / y_i \in \{0, 1, I, I, iI, 1 + I, 1 + I, 1 + iI, I + I, I + iI, I + iI, 1 + i + I, 1 + i + iI, 1 + I + iI, i + I + iI, 1 + i + I + iI\}; 1 \leq i \leq m\}$. This represents causal relation between D and R.

Let D_i and R_j denote that two nodes of a FCNRM. The directed edges from D_i to R_j denotes the causality of D_i on R_j called relations. Every edge e_{ij} in the FCNRM is weighted with a number in the set $\{0, 1, i, I, iI, 1 + i, 1 + I, 1 + iI, i + I, i + iI, I + Ii, 1 + I + i, 1 + I + iI, 1 + i + iI, i + I + iI, 1 + I + iI + i\} = P$.

Let e_{ij} be the weight of the edge D_iR_j, $e_{ij} \in P$.

The weight of the edge D_iR_j is determined as that of FRM or NRM or FCRM.

Let $D_1, \ldots, D_n$ be the nodes / concepts of the domain space D of an FCNRM and $R_1, R_2, \ldots, R_m$ be the nodes of the range space R of an FCNRM.

Let the complex neutrosophic valued matrix with $E = (e_{ij})$ where e_{ij} is the weight of the edge D_iR_j (or $R_j\ D_i$) E is defined as the relational complex neutrosophic matrix of the FCNRM.

Like FCRMs all operations can be performed on $E = (e_{ij})$ using $A = \{(a_1, \ldots, a_n) / a_i \in \{0, 1, i, I, iI, 1 + I, 1 + i, 1 + iI, i + I, i + iI, I + iI, 1 + I + i, 1 + I + iI, 1 + iI + I, i + I + iI, 1 + i + I + iI\}$ $1 \leq i \leq n$ the instantaneous state complex-neutrosophic vectors of the domain space which, denotes the on, off, imaginary neutrosophic, pure imaginary, pure neutrosophic, imaginary - neutrosophic, pure imaginary neutrosophic position of the nodes.

Likewise $B = \{(b_1, b_2, \ldots, b_m) / b_i \in \{0, 1, i, I, 1 + i, 1 + I, iI, 1 + iI, i + I, i + iI, I + iI, 1 + i + I, 1 + i + iI, 1 + I + iI, I + i + iI, 1 + i + I + iI\}, 1 \leq i \leq m\}$ are instantaneous state vector of the range space and it denotes to on-off, imaginary, neutrosophic, etc. The functioning of FCNRMs are akin to FRMs, NRMs and so on.

Similarly we see FCNRMs is said to be cyclic if it possess is directed cycle. An FCNRMs is said to be acyclic if it does not possess any directed cycle.

An FCNRMs with cycles is said to be a FCNRMs with feedback when there is a feedback in the FCNRMs when the causal relations flow through a cycle in a revolutionary manner the FCNRM is called a dynamical system.

The equilibrium state of the dynamical system is a unique state vector, then it is called a fixed point. Consider an FCNRM with $R_1, R_2, \ldots, R_m$ and $D_1, D_2, \ldots, D_n$ as nodes. For example let us start the dynamical system by switching on R_1 (or D_1).

Let us assume the FCNRM settles down with R_1 and R_m (or D_1 and D_n) on; that is the state vector remains as $(1 + i + I, 0, 0, 0, I + iI + 1)$ in R (or $(1 + i + iI, 0, 0, \ldots, 0, iI + 1 + i)$ in D). This state vector is called the fixed point.

Similarly limit cycle of a FCNRM is defined akin to FRMs.

Further the method of determining the hidden pattern of FCNRMs is similar to that of FRMs.

Another important factor which we wish to define in case of FCNRMs is the combined FCNRMs. Like finite number of FRMs can be combined to get a joint effect of all the FRMs we can combine a finite number of FCNRMs to get a joint effect of all FCNRMs.

Let $E_1, E_2, \ldots, E_s$ be fuzzy complex or imaginary neutrosophic relational matrices of s FCNRMs (FINRMs) with nodes $R_1, R_2, \ldots, R_m$ and $D_1, D_2, \ldots, D_n$ then the combined FINRM (RCNRM) is represented by the fuzzy imaginary neutrosophic relational matrices $E = E_1 + E_2 + \ldots + E_s$.

Thus as in case of combined FRMs we can also work with combined FINRMs (or FCNRMs) in the same with appropriate simple modification.

Mostly this model will find more applications in the medical diagnostics. This will be explained by the following model.

Example 3.12. Let us consider the problem of counseling a drug addict we give D_1, D_2, D_5 to be attributes symptoms associated with the drug addict and R_1, R_2, R_3, R_4 and R_5 are the remedies supplied by the psychiatrist.

We will describe each of the nodes / concepts by a line or two.

D_1	Dazzled state	All the time in the drowsy state
D_2	Suffers from illusion imaginary suffering	Attempted to murder his girl friend when questioned said he was attacked by wolves - lives on illusions
D_3	Never does any work	All the line under the influence of drugs so fails to do any routines even basic hygiene brushing teeth etc.
D_4	Does not eat well	As liver and vital organs are damaged does not eat regularly craves only for drugs.
D_5	Mood Swing	Mood swing is so high never polite with family or elders of the family easily takes every thing as offence steals money and valuables from home and friends.
D_6	Cause why he leads this way of life	This remains an indeterminate for parents and close friends who are not drug addicts are not in a position to unravel his mind. Some think it may be some imaginary grievance.

Now we briefly describe the attributes related to the treatment.

R_1	Isolation	The patient must be isolated from the bad company and bad habits. So isolation becomes mandatory
R_2	Recovery	This is not easily predictable only an indeterminate
R_3	Real problems	The real problems faced by the patient may be imaginary or and n indeterminate one.
R_4	Councilling etc or treatment	Can be executed only if the patient is in a partial state of good mind. If absolutely in a different state nothing can be done.
R_5	Family support	Unless family support is obtained it is not possible for any form of treatment a friend or family member must be with the drug addict throughout the treatment as a moral support.

Now we just describe the model in the form of a complex neutrosophic weighted bigraph.

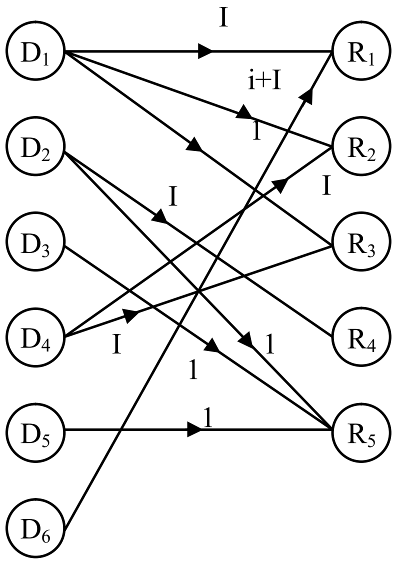


Figure 3.23

Taking this experts which is a complex neutrosophic valued bigraph now we obtain the connection matrix M which serves as the dynamical system of the problem.

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{bmatrix} I & i+I & 1 & 0 & 0 \\ 0 & 0 & 0 & I & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & I & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Consider the initial state vectors X and Y of the dynamical system.

X = {(a_1, a_2, a_3, a_4,a_5, a_6) / a_i ∈ {0, 1, i, 1 + i, I, iI, 1 + I, 1+ iI, i + I, i + iI, I + iI, 1 + I + I, 1 + I + iI, 1 + i + iI, i + I + iI, 1 + I + i + I + iI}, $i^2 = -1$, I2 = I, $(iI)^2 = -I$, 1 ≤ i ≤ 6} and Y = {(b_1, b_2, b_3, b_4, b_5) / b_j ∈ {0, 1, i, 1 + i, iI, I, 1 + I, 1 + iI, i + I, i + Ii, I + iI, 1 + I + i, 1 + I + iI, 1 + iI, i, i + I + iI, 1 + I + i + iI}, 1 ≤ j ≤ 5}.

Let x = (1, 0, 0, 0, 0, 0) ∈ X; to find the effect of x on the dynamical system M

$xM = (I\ i + I, 1, 0, 0) = y_1$

$y_1M^t = (2I + 2iI, 0, 0, iI + 2I, 0, I) \to (I + iI, 0, 0, I, 0, I) = x_1$

$x_1M = (2I + iI, 2Ii + I, 2I + iI, 0, 0) \to (I, Ii, I, 0, 0) = y_2$ (say)

$y_2M^t = (Ii + I, 0, 0, I + Ii, 0, I) \to (Ii + I, 0, 0, I + Ii, 0, I)$

$= x_2$ (say)

$x_2M = (2I + Ii, 3Ii + I, 2I + 2Ii, 0, 0) \to (I, Ii\ I + Ii, 0, 0) = y_3$ say

$y_3M^t = (2Ii + I, 0, 0, 2Ii + I, 0, I) \to (Ii, 0, 0, Ii, 0, I) = x_3$ (say)

$x_3M = (Ii + I, -I + Ii + Ii, 2I, 0, I) \to (I + Ii, Ii, Ii, 0, I) = y_4$ (say)

$y_4M^t = (I + Ii - I + Ii + Ii, I, I, 2Ii, I\ I + Ii) \to (Ii, I, I, Ii, I, I + Ii)$ $= x_4$ (say)

Interested reader is left with the task of finding the hidden pattern which may be fixed point pair or a limit cycle pair.

Infact if we have several such FINRMs (FCNRMs) with same number of domain nodes and same number of range nodes then as in case of FRMs we can find the combined FCNRMs (or FINRMs) and work with the combined FCNRMs to obtain the hidden pattern pairs.

Other types of combined FICMs and FINCMs are defined in the following.

As in case of FCMs we in case of FICMs define a special type of combined FICMs which we will describe. We can also obtain a special type of combined FICMs also analogous to FCMs or NCMs.

Let us consider some n attributes $C_1, C_2, \ldots, C_n$ where C_j can take the states 0 or 1 or i or 1 + i; $j \in \{0, 1, 2, \ldots, n\}$.

Suppose on the problem some take s_1 attributes, some other take s_2 attributes and the s_r^{th} expert takes s_r number of attributes $3 \leq s_1, s_2, \ldots, s_r \leq n$ then we obtained the special type of combined FICMs in the following way; which is first describe by the following example.

Let us consider some 16 attributes / nodes associated with the problem. Let the nodes be denoted by $C_1, C_2, \ldots, C_{12}$ where C_j's can take values 0 or i or 1 or 1 + i only; $1 \leq j \leq 12$.

Suppose there are 3 experts who wish to work on the problem and all of them choose to work with the FICMs only but select only a fixed number of attributes from the set of 12 attributes.

Now we apply the combined disjoint block FICMs model with the only difference FICMs replaced by the FICMs.

The first expert works with the nodes $\{C_1, C_3, C_8, C_{10}\}$, the second expert has choosen the nodes $\{C_4, C_7, C_{11}, C_{12}\}$ to work with the problem.

Finally the 3rd expert has choosen to work with $\{C_2, C_5, C_6, C_9\}$ as the nodes.

Now we give the complex - neutrosophic directed graphs associated with each of the 3 experts in the following

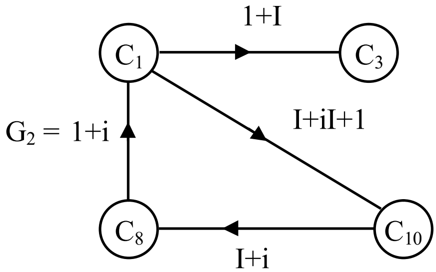


Figure 3.24

G_1 is the complex-neutrosophic valued directed graph given by the first expert.

G_2 is the complex - neutrosophic valued directed graph given by the second expert in the following.

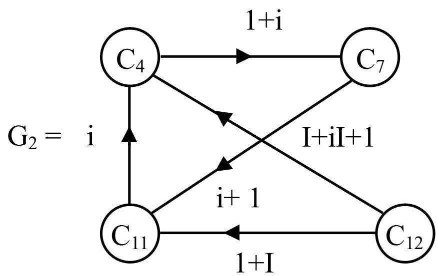


Figure 3.25

Let G_3 be the complex valued directed bigraph given in the following using the nodes $\{C_2, C_5, C_6, C_9\}$;

$G_3 =$

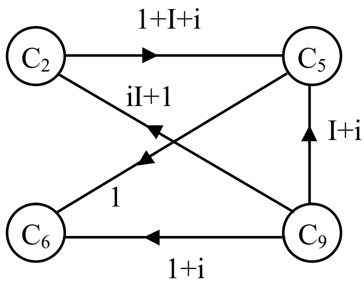


Figure 3.26

The complex - neutrosophic valued matrices associated with the graph G_1, G_2 and G_3 are supplied below.

Let M_1 be the complex valued matrix associated with the graph G_1.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_3 \\ C_8 \\ C_{10} \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_3 & C_8 & C_{10} \end{array} \\ \begin{bmatrix} 0 & 1+I & 0 & 1+I+iI \\ 0 & 0 & 0 & 0 \\ 1+i & 0 & 0 & 0 \\ 0 & 0 & I+i & 0 \end{bmatrix} \end{array}$$

Let M_2 be the connection complex neutrosophic valued matrix associated with the complex neutrosophic directed weighted graph G_2

$$M_2 = \begin{array}{c} \\ C_4 \\ C_7 \\ C_{11} \\ C_{12} \end{array} \begin{array}{c} \begin{array}{cccc} C_4 & C_7 & C_{11} & C_{12} \end{array} \\ \begin{bmatrix} 0 & 1+i & 0 & 0 \\ 0 & 0 & i+1 & 0 \\ i & 0 & 0 & 1+I \\ 1+I+iI & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let M_3 be the complex-neutrosophic valued connection matrix associated with the complex-neutrosophic weighted value graph G_3.

$$M_3 = \begin{array}{c} \\ C_2 \\ C_5 \\ C_6 \\ C_9 \end{array} \begin{array}{c} \begin{array}{cccc} C_2 & C_5 & C_6 & C_9 \end{array} \\ \begin{bmatrix} 0 & 1+I+i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1+iI & i+I & 1+i & 0 \end{bmatrix} \end{array}.$$

We will use the complex neutrosophic valued dynamical systems and find the effect of some initial state vectors.

Let $X_1 = \{(a_1, a_2, a_3, a_4) / a_j \in \{1 + i, i, I, iI, 0, 1 + I, i + I, I + iI, 1 + iI, I + iI, 1 + i + I, 1 + i + iI, 1 + I + iI, i + I + iI, 1 + i + I + iI\}, 1 \leq j \leq 4\} = C_1 \times C_3 \times C_8 \times C_{10}$

Let $X_2 = \{(a_1, a_2, a_3, a_4) / a_j \in \{0, 1, iI, I, I, 1 + I, 1 + I, 1 + iI, I + I, I + iI, i + Ii, 1 + I + I, 1 + I + iI, 1 + iI + I, I + iI + i, 1 + I + iI + i\}\ 1 \leq j \leq 4\} = C_4 \times C_7 \times C_{11} \times C_{12}$

Let $X_3 = \{(a_1, a_2, a_3, a_4) / a_j \in \{0, 1, I, i, iI, 1 + I, 1 + i, iI + 1, i + I, i + iI, iI + I, 1 + i + I, 1 + i + iI, I + i + iI, 1 + I + iI, 1 + i + I + iI\};1 \leq j \leq 4\} = C_2 \times C_5 \times C_6 \times C_9$.

Now just for the sake of illustration we find the effect of $x \in X_1$ on the dynamical system M_1.

Let $x = (i, 0, 0, 0) \in X$ to find the effect of x on M_1

$xM_1 = (0, i + I, 0, i – 1 – I) \to (i, i + I, 0, i) = y_1$ (say)

$y_1M_1 = (0, i + iI + 2I, Ii – 1, i – 1 – I) \to (i, I, Ii, i) = y_2$ (say)

$y_2M_1 = (Ii – I, i + iI, –1 + iI, i – 1 – I) \to (Ii, i + Ii, Ii, i)$ \

$= y_3$ (say)

$y_3M_1 = (Ii – I, Ii + Ii, -1 + iI, Ii – I – I)\to (Ii, Ii, iI, Ii) = y_4$ (say)

$y_4M = (iI – I, Ii + Ii, iI – I, Ii – I – I) \to (iI, Ii, Ii, iI) = y_5$ (say)

Since $y_4 = y_5$ we see the hidden pattern is a fixed point given by

(iI, Ii, Ii, Ii) I

For the node C_1 in the pure imaginary state i.

Next let us work with the on state of the node C_7 in the state of initial vectors X_2 using the dynamical system M_2.

Given $a = (0, 1, 0, 0) \in X_2$ that C_7 state is on with 1. To find the effect of a on the dynamical system M_2.

$aM_2 = (0, 0, 1 + I, 0) \to (0, 1, 1 + i, 0) = a_1$

$a_1M_2 = (i – 1, 0\ 1 + I\ 1 + iI + I + i) \to (i, 1, 1 + I, 1 + iI + I + i)$

$= a_2$ (say).

The effect of a_2 on the dynamical system M_2 is given by

$a_2M_2 = (2i + 5iI – I, i – 1, i + 1, 1 + I + i + iI) \to (iI, I, i + 1,$

$1 + i + I + iI) = a_3$ (say)

The effect of a_3 on the dynamical system M_2 is given by

$a_3M_2 = (i – 1 + 1 + i + I + iI + I + iI + iI + iI + iI – I + iI – I,$

$iI – I, i – 1, 1 + I + i + iI) \to (iI, iI, i, 1 + I + i + iI) = a_4$ (say)

The resultant of a_4 on M_2, $a_4M_2 = (-1 + 1 + I + i + iI + I + I + I + Ii + Ii + iI + iI – I – I, iI – I, I – 1, i + iI) \to (iI, iI, i, i + iI) = a_5$ (say).

The effect of a_5 on M_2 is $a_5M_2 = (–1 + i + I – I + iI + iI – I,$

$iI – I, –1 + i, i + iI) \to (iI, iI\ i, i + iI) = a_6$ (say)

Clearly $a_6 = a_5$ so the hidden pattern is a fixed point given by

$(iI, iI, i, i + iI)$ – II

when C_7 was on with 1.

Next we find the effect of the on state of C_9 in the indeterminate state I on M_3.

Let $b = (0, 0, 0, 0, I) \in X_3$, to find the effect of b on the dynamical system M_3

$bM_3 = (I + iI, iI + I, I + Ii, 0) \to (I + iI, I + iI, I + iI, I) = b_1$ (say).

We find the effect of b_1 on the dynamical system M_3

b_1M_3 = (I +iI, I + I + iI + iI + iI – I + iI + I + iI + I, I + iI + iI – I, 0) $\rightarrow$ (I + iI iI, iI, I) = b_2 (say).

The effect of b_2 on M_3 is given by

b_2M_3 = (I + iI, –1 + iI + iI + iI + iI + I, iI – I 0) $\rightarrow$ (I + iI, iI, iI, I) = b_3 (say)

Since $b_3 = b_2$ clearly the hidden pattern is given by a fixed point

(I + iI, iI, iI, I) III

When the node C_9 is in the indeterminate state I.

Now we find the combined block fuzzy imaginary - neutrosophic cognitive maps model connection matrix M using the matrices M_1, M_2 and M_3.

$$
\text{So } M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \end{array}
\begin{array}{l}
\begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\
\left[\begin{array}{cccccc}
0 & 0 & 1+I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1+i+I & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1+i & 0 & 0 & 0 & 0 & 0 \\
0 & 1+iI & 0 & 0 & i+I & 1+i \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & i & 0 & 0 \\
0 & 0 & 0 & 1+I+iI & 0 & 0
\end{array}\right.
\end{array}
$$

$$
\begin{array}{l}
\begin{array}{cccccc} C_7 & C_8 & C_9 & C_{10} & C_{11} & C_{12} \end{array} \\
\left.\begin{array}{cccccc}
0 & 0 & 0 & 1+i+iI & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1+i & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1+i & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & I+i & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1+I \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right].
\end{array}
$$

Now we call M the disjoint block dynamical system of the disjoint block combined FINCMs model.

Now the initial state vectors associated with M are given by

$X = \{(a_1, a_2, \ldots, a_{12}) / a_j \in \{0, 1, I, iI, 1 + I, 1 + iI, 1 + I, I + I, I + iI, i + iI, 1 + I + i, 1 + I + iI, 1 + i + iI, i + I + iI, 1 + i + I + iI\}; 1 \leq j \leq 12\}$.

Now we find the effect of $s = (i, 0, 0, 0, 0, 0, 1, 0, I, 0, 0, 0) \in X$.

The effect of s on the dynamical system M is given by

$sM = (0, I + iI, i + iI, 0, I + iI, I + iI, 0, 0, 0, i - 1 - I, 1 + i, 0) \to (i, I + iI, i + iI, 0, I + iI, I + iI, 1, 0, I, i, 1 + I, 0) = s_1$ say

$s_1M = (0, I + iI, i + iI, i - 1, I + iI + I + iI - I + iI + I + iI + iI + I, I + iI, 0, iI - 1, 0, i - 1 - I, 1 + i, 1 + i + I + iI) \to (i, I + iI, I + iI, i, iI, I + iI, 1, iI, I, i, 1 + i, 1 + i + I + iI) = s_2$ (say).

Now we find the effect of s_2 on M.

$s_2M = (iI - I, I + iI, i + iI, i - 1 + 1 + I + I + iI + I + iI + I + iI + iI - I + iI - I, 5iI + 3I, I + iI, i - 1, iI - 1, 0, I - 1 - I, 1 + I, 1 + I + I + iI) \to (iI, I + iI, i + iI, iI, iI, I + iI, i, iI, I, i, i, 1 + i + I + iI) = s_3$ (say).

We now find s_3M;

$s_3M = (iI - I, I + iI, iI + i + iI + iI, i - 1 + 1 + i + I + iI + I + iI + I + iI + iI - I + iI - I, I + iI + I + iI - I + iI + I + iI + I + iI, I + Ii, iI - I, iI - 1, 0, iI - 1 - I, i - 1, i + iI \to (iI, I + iI, iI, iI, I + Ii, iI, iI, I, iI, i, I + i + 0) = s_4$ (say).

One can calculate the effect of s_4 on M and so on.

Interested reader can compare the resultant or hidden pattern of s on the dynamical system M and compare it with equations I, II and III given earlier.

Let us consider t = (0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0) $\in$ X.

To find the effect of t on the dynamical system M

tM = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 + I, 0) $\to$ (0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1 + I, 0) = t_1 (say).

We find the resultant of t_1 on M.

t_1M = (0, 0, 0, i – 1, 0, 0, 0, 0, 0, 0, 1 + I, 1 + i + I + iI) $\to$ (0, 0, 0, i, 0, 0, 1, 0, 0, 0, 1 + i, 1 + i + I + iI) = t_2 (say).

The effect of t_2 on M;

t_2M = (0, 0, 0, i – 1 + 1 + i + I + iI + I + iI + I + iI + iI – I + iI – I, 0, 0, 1 – i, 0, 0, 0, 1 + i, 1 + I + i + iI) $\to$ (0, 0, 0, iI, 0, 0, 1, 0, 0, 0, 1 + I, 1 + I + i + iI) = t_3 (say).

t_3M = (0, 0, 0, i – 1 + 1 + I + I + iI + I + I + iI + iI + iI + iI – I – I, 0, 0, iI – I, 0, 0, 0, 1 + I, 1 + I + iI + i) $\to$ (0, 0, 0, iI, 0, 0, iI, 0, 0, 0, 1 + I, 1 + I + iI + i) = t_4 (say).

We now determine t_4M = (0, 0, 0, i – 1 + 1 + i + I + iI + I + iI + I + iI + iI + iI – I – I, 0, 0, iI – I, 0, 0, 0, iI – I, 1 + I + i + iI) $\to$ (0, 0, 0, iI, 0, 0, iI, 0, 0, 0, iI, 1 + i + I + iI) = t_5 (say)

One can find the effect of t_5 on M and find the hidden pattern to be a fixed point.

Next we will just describe the overlap combined FINCMs briefly.

Let $C_1, C_2, \ldots, C_n$ be n attributes associated with the problem. Suppose 'r' experts work with the problem using some subset Si of attributes from the set $\{C_1, C_2, C_3, \ldots, C_n\}$ such that $S_i \cap S_j \neq \phi$ for all i, j $\in$ {1, 2, …, r} where $S_1, S_2, \ldots, S_r$ are the subsets associated with the 'r' experts using FINCM's model. As in case of combined overlap FCMs we will in the case of the combined overlap FINCMs model also obtain the resultant. FCMs model takes the state vectors values from {0, 1} where as in case of FINCMs model the entries are from{0, 1, i, I, iI, 1 + i, 1 + I, 1 + iI, i + I, i + Ii I + Ii, 1 + I + I, 1 + i + Ii, Ii + I + 1, 1 + I + I + Ii, I + I + Ii}.

With this appropriate difference we can build this new model and find the hidden pattern.

This task is left as an exercise to the reader.

On similar lines we can define combined disjoint block FINRMs also.

However disjoint combined FRMs and combined disjoint NRMs models have been defined and developed in [15].

It is further contemplated that we can adopt these fuzzy imaginary valued graphs in the Artificial Neural Networks (ANN), Back Propagation Network (BPN), perceptron etc.; with appropriate modifications.

As weights are from $C = \{a + bi / a, b \in R, i^2 = -1\}$ the complex field we see the sighmoidal functions be it binary or bipolar with complex values can be adopted without any difficulty.

The main difference in this case being that the connection matrix or the weight matrix being complex valued.

Here it is pertinent to keep on record that if we use complex - neutrosophic valued graphs then we cannot in general use them in building the some these soft models which exploit the sigmoidal functions as derivative of the indeterminate I cannot be defined. However these can be used when the model is discrete one. This is the only limitations at this juncture.

These types of models in case of both supervised and unsupervised data are elaborately dealt in the forthcoming book.

In this book we have applied the imaginary valued directed graphs and bigraphs to soft models like FICMs and FIRMs.

However it is pertinent to record at this juncture we can use the complex neutrosophic valued directed graphs as well as bigraphs in the construction of soft models like FINCMs and FINRMs where mainly unsupervised data is involved.

Such models will be a boon in the study or analysis of medical diagnostics or evaluation of personality tests and so on.

In this chapter appropriate examples are used to illustrate this situation. However we wish to keep on record that these examples are not related with any real world data we have given them only for illustrations.

Further it is a challenge for the researchers to find more applications of these new models.

Certainly these models will be very well suited in the social network problems and community net work problems, where the analysis as well as the unsupervised data is full of complex (imaginary notions) and indeterminate concepts / relations. So only in this books such types of graphs are constructed mainly for this purpose.

Here we suggest a set of problems for the reader which will enhance the understanding of this work in a better way.

Problems

1. Give an example of a complex valued directed graph G with 5 vertices and 8 edges.

   i) How many such graphs can be obtained?

   ii) Find the weighted complex matrix M of G.

   iii) Find $\overline{M}$ the conjugate of M.

   iv) Show $\overline{G}$ the directed complex valued graph associated with $\overline{M}$ is the conjugate of the graph G.

   v) Obtain any other special feature associated with G and M.

2. Let G be the directed complex valued graph associated with a FICMs model given in the following.

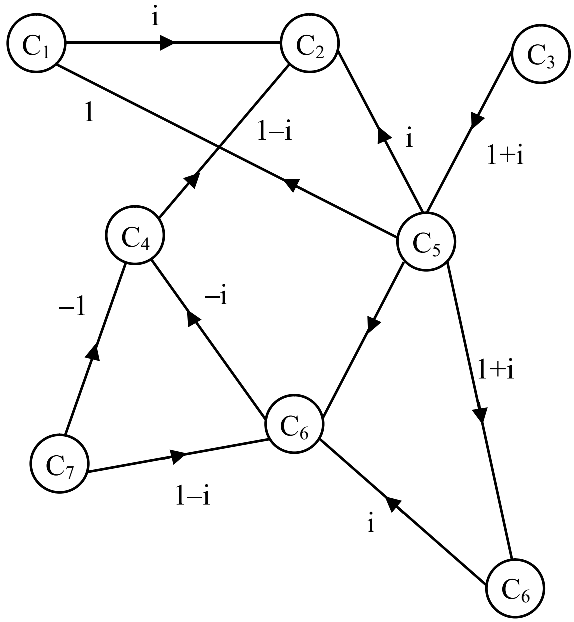


Figure 3.27

i) Find the complex dynamical system M, that is the connection complex valued matrix associated with G.

ii) If x = (1, 0, I, 0, 0, 0, 0,1) ∈ {(a_1, a_2, …, a_8) where a_i ∈ {0, 1, I, 1 + i}, 1 ≤ i ≤ 8} = X find the effect of x on M.

a) Is the resultant or hidden pattern a fixed point or a limit cycle?

iii) Find the hidden pattern of a = (0, 0, 1, i, i, 0, 0, 0) ∈ X using M.

3. Use the FICM's model in a practical problem and derive the importance of the same.

4. What are the special features associated with the FICMs model?

5. Prove in certain medical diagnostics models FICMs plays a better role than FCMs and NCMs.

6. Give an illustrative real world model in which FICMs play a better role than FCMs and NCMs.

7. Give an example of a problem in which only FCMs model alone can be used and FICMs model has no role to play.

8. Give an example of a real world problem in which only NCMs model alone can be used and FICMs model has no role to play.

9. Can we say FICMs model can play a role in getting the mental map of a drug addict?

10. What are the main problems encountered in solving or using FICMs model?

11. Describe a Fuzzy Imaginative (Imaginary or complex) Relational Maps (FIRMs) model.

12. Compare a FIRMs model with FRMs model.

13. Give a real world problem in which FRMs model cannot be replaced by FIRMs model.

14. Give a real world problem in which FIRMs model alone is mandatory.

15. Give a real world problem in which only NRM model is appropriate. That is FRM and FIRMs cannot be used.

16. Determine all the draw backs as well as merits of the FIRMs model.

17. Suppose 3 experts work on a problem using FICMs model with same number of concepts with associated complex valued matrices M_1, M_2, M_3.

    Prove $M = M_1 + M_2 + M_3$ can be defined as the Combined FICMs (CFICMs) model analogous to CFCMs model.

18. Apply the CFICMs model in a real world problem using atleast more than six experts.

19. Define and develop the notion of Combined Fuzzy Imaginary Relational Maps (CFIRMs) model.

20. Illustrate CFIRMs model using atleast some five experts.

21. What are the advantages of using CFIRMs?

22. Prove if there are say some n domain attributes and m range attributes and if r experts work using the FIRMs and s of the experts work using FRMs then also we can combine them and get the CFIRMs model related with the r + s experts.

23. Illustrate this situation in problem 22 by some examples from the real world problem.

24. Let G be a complex - neutrosophic weighted directed graph given by the following figure.

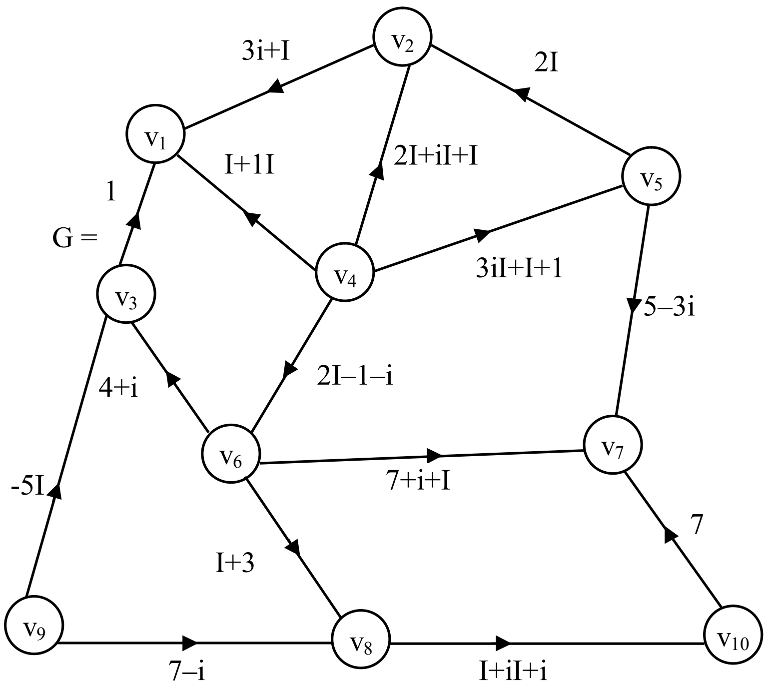


Figure 3.28

i) Find all subgraphs of G.

ii) Which of the subgraphs of G complex-neutrosophic valued?

iii) Find all subgraphs which are real.

iv) Find all neutrosophic valued subgraphs.

v) Find all complex valued subgraphs.

vi) Find the weighted matrix M associated with G.

vii) Find the conjugate of $\overline{M}$ of M.

viii) Find K the complex neutrosophic valued graph associated with $\overline{M}$

ix) Find $\bar{G}$ of G. Prove $\bar{G}$ = K.

x) Obtain any other special feature associated with G.

xi) Find M × $\bar{M}$ and $\bar{M}$ × M.

xii) Is M $\bar{M}$ = $\bar{M}$ × M?

xiii) What is the nature of the graphs associated with $\bar{M}$ ×M?

25. Let S be the complex - neutrosophic value directed weighted graph given by the following figure.

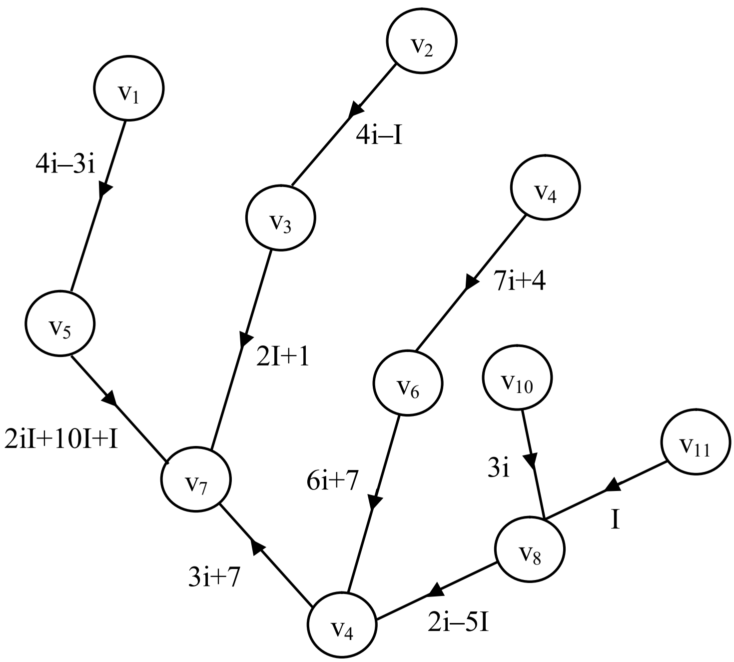


Figure 3.29

Study questions (i) to (xiii) of problem (25) for this graph S.

26. Let R be the complex neutrosophic weighted bigraph given by the following figure;

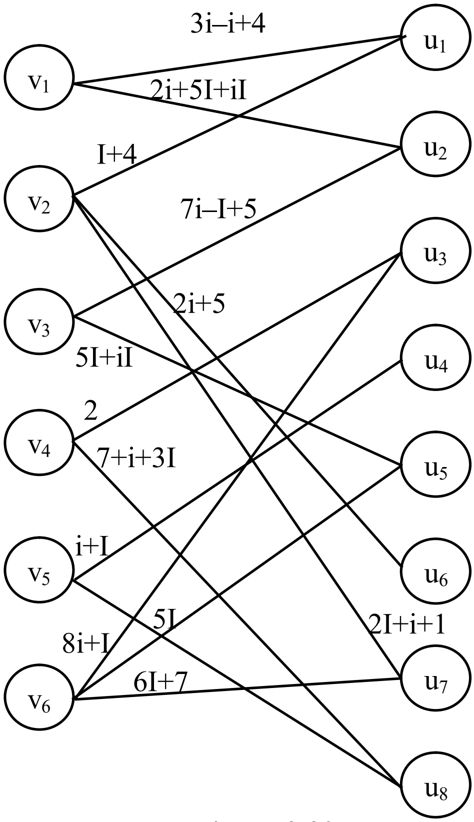


Figure 3.30

i) Study questions (i) to (xiii) of problem (25) for this R.

ii) Prove every subgraph of R is also a bigraph.

iii) Associate with R a Fuzzy Imaginary Neutrosophic Relational Maps connection matrix M.

iv) Give a set of domain space and range space of state vectors find the resultant.

v) Find M^t. Is $M^t = \bar{M}$? Justify

27. Find a real world problem where FIRMs models are more suited than the FRMs models.

28. Give an illustration in the real world problem where FINRMs models are more suited than the FIRMs model and NRMs models.

29. Show in many medical diagnostic problems FIRMs and FINRMs are more suited by constructing a real world problem.

30. Give an example of a real world problem in which only FCMs models are well suited than FICMs or FINCMs or NCMs model.

31. Give an example of a real world problem in which only NCMs models are best suited than FNICMs and FICMs models.

32. Enumerate the distinct and special features associated with FICMs and FINCMs models.

33. Find the main difference between the FICMs and FINCMs models.

34. Give an example of a real world problem where FIRMs model is very apt than other models.

35. When complex numbers play a vital role in all problems how can one justify the absence of complex valued graphs?

36. When we use any graph structure in mathematical model do we have the graph in a concrete way?

37. Give a real world problem in which FINRMs model is more apt from the FRMs and NRMs models.

38. Let H be the complex neutrosophic valued weighted graph given by the following figure.

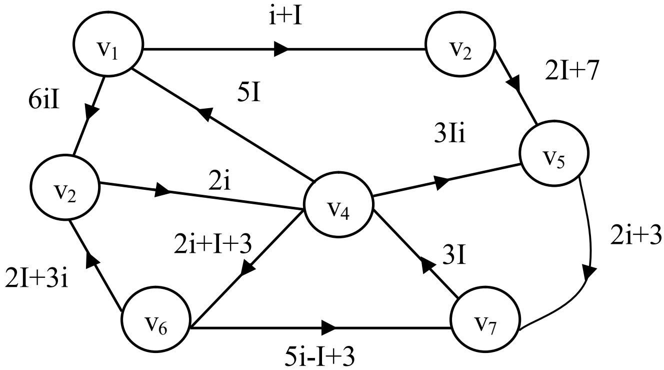


Figure 3.31

i) Find all subgraphs of H.

ii) Prove H cannot have subgraphs with real weights.

iii) Find the weight matrix or connection matrix M of the complex neutrosophic weighted directed graph H.

iv) Find the conjugate $\overline{M}$ of M.

v) Find the complex neutrosophic weighted graph conjugate to the graph H.

39. Define the notion of complex neutrosophic valued wheel, W_n

40. Compare complex valued wheel W_n with the complex neutrosophic valued wheel W_n.

41. Can we say complex neutrosophic weighted wheels W_n also occurs in pairs for a fixed n?

42. Give an example of a complete complex neutrosophic valued graph.

43. Derive all special features associated with complex neutrosophic weighted complete graph and compare them with usual read valued graph.

44. Let S = $\{C_1, C_2, \ldots, C_{20}\}$ be two attributes. Suppose 5 experts work using subsets of S adopt FICMs model such that the subsets are disjoint?

    i) Obtain the combined disjoint FICMs model using S.

    ii) How many different such combined disjoint FICMs model can be obtained?

    iii) Compare the resultant of each experts separately with the combined resultant using the disjoint combined FICMs model.

45. Derive all special features associated with combined disjoint FICMs.

46. Describe the important features associated with the combined overlap FICMs model.

47. Let S = $\{C_1, C_2, \ldots, C_{18}\}$ be the set of attributes whose state can be 0 or 1 or I or 1 + i.

    Let $S_1 = \{C_1\ C_5\ C_{10}\ C_{18}\}$, $S_2 = \{C_2, C_7, C_{15}, C_9, C_{16}, C_{17}\}$,

    $S_3 = \{C_1\ C_3, C_6, C_7, C_5, C_8, C_{11}\}$and

    $S_4 = \{C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{10}\}$ be the attribute the four experts work using FICMs.

    i) Find the combined overlap FICMs model and find the resultant of
        i) x = (1, 0, i, 0, …, 0) and
        ii) y = (0, 0, 0, …, 1 + i, 0, 1).

48. Apply in a real world problem the notion of disjoint combined FINCMs model.

49. Discuss the merits and demerits of disjoint combined FINCMs in comparison with combined FINCMs and overlap combined FINCMs.

50. Does there exists real world problems for which overlap combined FINCMs are more suited than disjoint combined FINCMs and combined FINCMs model ?s

51. Obtain all special and distinct features enjoyed by overlap combined FINCMs.

52. Show if there are 5 different experts working on a problem using FINCMs such that the collection forms a disjoint combined FINCMs prove if N_1, N_2, N_3, N_4 and N_5 are 5 distinct dynamical systems associated with the FINCMs and if N is the disjoint combined FINCMs dynamical system and if x_i is an initial state vector with C_i node in the on state.

    Will the effect of C_i node on a N_i give the same set of on state vector as that N? Justify.

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ABOUT THE AUTHORS

Dr.W.B.Vasantha Kandasamy is a Professor in the School of Computer Science and Engg., VIT University, India. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 694 research papers. She has guided over 100 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 123rd book.

On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

She can be contacted at vasanthakandasamy@gmail.com
Web Site: http://www.vasantha.in

Dr. K. Ilanthenral is Assistant Professor in the School of Computer Science and Engg, VIT University, India. She can be contacted at ilanthenral@gmail.com

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Dr. Florentin Smarandache is a Professor of Mathematics at the University of New Mexico in USA. He published over 75 books and 200 articles and notes in mathematics, physics, philosophy, psychology, rebus, literature. In mathematics his research is in number theory, non-Euclidean geometry, synthetic geometry, algebraic structures, statistics, neutrosophic logic and set (generalizations of fuzzy logic and set respectively), neutrosophic probability (generalization of classical and imprecise probability). Also, small contributions to nuclear and particle physics, information fusion, neutrosophy (a generalization of dialectics), law of sensations and stimuli, etc. He got the 2010 Telesio-Galilei Academy of Science Gold Medal, Adjunct Professor (equivalent to Doctor Honoris Causa) of Beijing Jiaotong University in 2011, and 2011 Romanian Academy Award for Technical Science (the highest in the country). Dr. W. B. Vasantha Kandasamy and Dr. Florentin Smarandache got the 2012 New Mexico-Arizona and 2011 New Mexico Book Award for Algebraic Structures. He can be contacted at smarand@unm.edu

In this book authors define, describe and develop the notion of complex valued graphs, complex neutrosophic valued graphs and mod complex valued graphs in a systematic way. However complex valued neural networks have been analysed and studied as early as 2003. This book gives several applications of them in medical diagnostics, soft computing and so on. This will be of interest to researchers in these areas.

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