Horizontal mergers on platform markets: cost savings v. cross-group network effects?

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Horizontal mergers on platform markets: cost savings v. cross-group network effects?*

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Abstract

We study the impact of cost savings on the outcome of horizontal mergers between two-sided platforms. We consider four symmetrically differentiated platforms located equidistantly on the unit circle and competing in membership fees. Users on both sides single-home, and we allow for both positive and negative cross-group externalities. We find that the impact of merger cost savings on prices is generally not monotonic, and that synergies are necessary for horizontal platform mergers to be Pareto-improving. Furthermore, the merger may benefit users on one side while harming users on the opposite side, which raises some interesting questions for the enforcement of merger control on two-sided markets.

Keywords: horizontal merger, two-sided markets, cost savings, indirect network effects, merger control

JEL codes: L41, D43, K21

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1 Introduction

This is a theoretical paper discussing the consequences of horizontal mergers on two-sided markets. We conduct an equilibrium analysis in a four-platform framework allowing for horizontal differentiation: we determine post-merger outcomes (prices, profits and welfare levels) taking into account the cost savings from the merger as well as the pricing behavior of outsider platforms. We find that the impact of merger cost savings on prices is generally not monotonic, and that it takes high enough cost savings for horizontal platform mergers to be Pareto-improving. In addition, for a given level of synergies, the merger may benefit users on one side while harming users on the opposite side. This raises interesting questions for competition authorities when assessing mergers between rival platforms on two-sided markets.

Starting with Rochet and Tirole (2002, 2003), two-sided markets have been the object of increasing research focus. These industries often exhibit business strategies and outcomes that would be sub-optimal on traditional/one-sided markets. For instance, setting prices below cost on one side can be profit-maximizing thanks to the resulting increase in demand on the opposite side through the indirect (or cross-group) network effects. Furthermore, from a normative point of view, this outcome should not be subject to antitrust vetoing, as predatory pricing would be. And while much has been accomplished regarding unilateral pricing strategies on two-sided markets[1], much is left to be done for the study of coordinated behavior, in particular pricing following horizontal mergers.

Mergers between rivals typically give rise to enhanced market power and higher prices, thus harming customers. But on two-sided markets another effect may reverse this outcome: the cross-group externalities granting users increased utility from having access to a greater pool of business partners on the other side of the platform may neutralize, and even outweigh the utility loss due to a price increase. As a result, the merger could actually be welfare enhancing rather than welfare detrimental (Evans, 2003). Moreover, it is even possible for post-merger prices to

[1] See OCDE 2018, Part IV, for a recent review.

be lower than before merger, due to the fact that the merging platforms internalize the effect of a price increase on the partner platform. In other words, the same indirect externality may reverse the typical post-merger incentive to increase prices to exploit market power (Chandra and Collard-Wexler, 2009, and Leonello, 2010). It is now quite generally acknowledged that insights from traditional merger analysis do not directly apply to platform markets (see e.g. Wright, 2004, Evans and Noel, 2008, Evans and Schmalensee, 2013 or Affeldt et al., 2013).

Understanding and correctly predicting the outcome of mergers on two-sided markets is increasingly relevant from the public policy viewpoint (Economides 2008, 2010). Interestingly enough, various agencies have addressed this challenge in different ways, and the two-sided nature of the market does not systematically play a role in the decision.[2] For the 2004 UK merger between two weekly local newspapers, Archant and Independent News and Media[3], the Competition Commission looked into the impact on advertisers but ignored that on readers. Doing the same, i.e., focusing on the advertising side of the market, allowed the DoJ and the FCC to clear in 2008 the merger between the only two US satellite digital radio services, which would have been considered a 2-to-1 merger, unless the relevant market had been widened to include, on the advertising side, other kinds of broadcast.[4] In Norway, the media merger between Edda Media and A-Pressen[5], the second and the third largest media houses in the country, was cleared in 2012 conditional on structural remedies/asset divestitures of local newspapers on two overlapping local/geographic markets, after the two sides of the market, readers and advertisers, had been considered independently, but with a discussion of the network externality from readers to advertisers. The European Commission cleared in 2007 the merger between Travelport and Worldspan thanks to the lack of negative effect on travel agents because the platforms were arguably likely to build a large enough network to attract agents on the other side (travel service

[2] Incidentally, although a new edition of the US Merger Guidelines was released in 2010, replacing the previous Guidelines from 1992, there was still no mention of platform markets.

[3] UK Competition Commission, "A report on the acquisition by Archant Limited of the London Newspapers of Independent News and Media Limited,"2004.

[4] Belleflamme and Peitz, 2015.

[5] Konkurranse Tilsynet Case 2011/0925 MAB BMBE.

providers).[6] Later on, the European Commission prohibited the 2012 stock exchanges merger between Deutsche Börse and NYSE Euronext due to insufficient cost savings to compensate for the supposedly likely post-merger price increase.[7]

Despite the surge in platform merger cases for competition authorities, the latter still lack clear guidance on how to assess them, because the literature on this topic is still relatively scarce.[8] Chandra and Collard-Wexler (2009) study post-merger pricing in a modified Hotelling duopoly of newspaper platforms with single-homing readers and multi-homing advertisers. The key finding is that a monopolist platform may not necessarily lead to higher prices on either side as long as readers are heterogeneous with respect to the value they bring to advertisers, and the less-valuable readers are also those who are more price-sensitive. This result is however conditioned on pricing below marginal cost on the reader side: if newspapers sell their content at a price below marginal cost, then additional readers are only valuable to the extent that the revenues which could be made by selling their attention to advertisers are greater than the subsidy.[9] In a model of differentiated products *à la* Salop (1979) on both sides of the market, where the side consuming content does not pay, Malam (2011) finds that a merger to monopoly between ad-sponsored platforms lacks the incentive to increase prices on the advertiser side. Leonello (2010) considers the alternative setting of two newspapers located at the extreme points of a Hotelling line. In a merger-to-monopoly scenario, the monopolist will offer advertisers in one

[6] Case No. M.4523.

[7] Case No. M.6166.

[8] The empirical evidence available focuses on media markets, and provides somewhat mixed results. Chandra and Collard-Wexler (2009) assess mergers in the Canadian newspaper market to conclude that greater concentration did not lead to higher prices for either readers or advertisers. Filistrucchi et al. (2012) test the unilateral effect of a hypothetical merger in the Dutch newspaper industry to find only a small effect on subscription prices and readers' welfare, but a substantial increase in the price paid per reader by advertisers. Jeziorski (2014) uses data from the 1996-2006 US radio industry merger wave to estimates a structural supply-and-demand model and finds on a a 11% drop in advertisement volumes and a 6% increase in per viwer ad prices. Fan (2013) uses a structural model of the US daily newspaper market to simulates a merger in the Minneapolis market and concludes that newspaper prices increase and circulation decreases, thus leading to welfare losses for both readers and advertisers.

[9] The fact that the data on the Canadian newspaper industry corroborates the absence of price increase post-merger does not however rule out alternative explanations some mergers may have been driven by motives other than increased market power, such as for instance empire-building or political motives (Anderson and McLaren, 2012).

newspaper the opportunity to advertise also in the other one: for a single price, the advertiser can now reach twice as many consumers as before, which is referred to as "interoperability". Leonello (2010) shows that the introduction of such advertising bundling by the monopolist increases the incentives to keep prices low on at least one side of the market, because the interoperability increases the margin which the newspaper can charge on advertising, and it thereby becomes profitable to reduce prices on the consumer side in order to stimulate demand. Overall, welfare could increase following a merger, and this result is obtained absent efficiency gains.[10] Unlike these contributions which suggest some policy implications based on merger-to-monopoly analyses, Tan and Zhou (2019) examines the impact of platform competition on equilibrium prices and customer participation rates in a general model of price competition among n multi-sided platforms providing differentiated services to single-homing users. Their model incorporates possibly non-linear externalities, as well as random utility maximization by customers on each side of the market, which makes all platforms equidistant to each other. Tan and Zhou (2019) concludes that a horizontal merger always has incentives to raise prices in the absence of cost-related efficiency gains. Finally, and in stark contrast to all these contributions, Correia-da-Silva et al. (2018) devise a model of Cournot competition among two-sided platforms offering a homogenous service, in order to examine the effects of mergers which do not affect the average marginal costs of the industry. The paper identifies sufficient conditions for such mergers to benefit users on either side of the market, namely the fact that the externality-adjusted pre-merger price be below the average marginal cost on the corresponding side. Moreover, when the externality-adjusted prices on both sides exceed the average marginal cost, the market power effect of mergers is shown to dominate the potential efficiency gains stemming from a larger participation on each platform.

We, in contrast, focus on the role played by the merger cost savings for the post-merger pricing strategy of all platforms on the market, both insiders and outsiders, and thereby ultimately

[10] See also Weyl (2010) for the possibility for horizontal mergers to increase market power and thereby lower participation and welfare on both sides in Armstrong's (2006) framework. His model considers, again, a duopoly setting, i.e. merger to monopoly, and without any cost savings.

for the merger's impact on users, profits and welfare. This is arguably relevant for competition authorities that need to explicitly balance the efficiency gains from a merger against its anti-competitive effect in order to decide whether to ban or to clear it[11]. Our model displays four symmetrically differentiated platforms located equidistantly on the unit circle market and competing in membership fees. Users on both sides single-home, and we allow for both positive and negative cross-group externalities. The merger involves the complete and costless interoperability between the merging platforms, leading to the situation where "buying" from one merged platform grants access to users of the merging partner. We compute and compare optimal post-merger prices set by both insider and outsider platforms in two different cases: bilateral merger between adjacent, closely substitutable platforms, or between distant, less substitutable ones. We also assess the mergers' impact on all platforms' profits, as well as on users' consumer surplus and total welfare.

Starting with the post-merger pricing, we find that the adjacent merger will only lead to lower prices iff it generates enough cost savings. This holds for both sides of the market, and does not hinge on the sign of the cross-group externality. In contrast, cost savings may not be necessary for the distant merger to achieve this outcome: post-merger prices always drop on the side that values relatively less the opposite side, i.e., regardless of the merger synergies, but this does not hold on the other side. There, insider or outsider platforms may only drop the price post-merger if the merger generates enough cost savings, depending on the valuation this side gets from the first one (positive or negative). Note thus that as far as the pricing behavior is concerned, our results confirm the intuition that when merging platforms become "interoperable", then each of the merging platforms will have incentives to lower prices to benefit from the increase in demand on the other platform. But the market power effect of the adjacent merger goes against this effect, so it takes enough synergies to achieve a price drop. The result we obtain for the distant merger highlights in turn the substitutability between cost savings

[11] See for instance the 2010 US Horizontal Merger Guidelines and the European Commission Merger Regulation 139/2004.

and indirect network effects: the former are not necessary for prices to drop on one side of the market, the one that is valued relatively more.[12] On the other side, it all depends on the sign of the indirect externality, because the latter is crucial for the opportunity cost that platforms face when contemplating a drop in their price to serve additional demand. A further result on the optimal post-merger pricing holds for both types of merger: on the side that values less the presence of the other side, all prices drop for lower levels of merger synergies than on the side exhibiting a higher relative valuation for the opposite side. It is quite straightforward that platforms have incentives to lower their prices "first", i.e., for a lower amount of cost savings, on the side that will give them the highest return in terms of additional cross-sides demand.

Our analysis also provides some insights for the merger profitability: we actually extend in this respect the results obtained in the standard, one-sided framework, i.e., the fact that merging is always profitable for neighboring firms/platforms, but for distant firms/platforms only if there are enough cost savings.[13] The intuition relies again on the presence (or lack) of a market power effect, which is the crucial difference between the two types of merger considered.

Regarding the welfare impact of the mergers, we find that the distant merger always improves consumers' surplus, i.e., on both sides and regardless of the amount of merger cost savings. On the contrary, the adjacent merger only benefits users on either side for high enough efficiency gains. The additional, and even more important conclusion, is that for a given amount of cost savings from the merger between neighboring platforms, users' surplus on one side increases, while that of users on the opposite side decreases.[14] This result is driven by the different relative valuation between sides, and is obtained with the underlying assumption of positive margins on

[12]Recall that in contrast, on the circular one-sided market a merger between distant firms does not affect prices if post-merger costs stay the same (Levy and Reitzes, 1992, Brito, 2003, 2005).

[13]Correia-da-Silva et al. (2018) provides, to our knowledge, the only other profitability analysis for horizontal mergers between platforms: in their Cournot setting with linear demand, platform mergers are profitable, in the absence of cost savings, as long as there are no more than four platforms, marginal costs are not too high, and the network effects are large enough.

[14]This is a confirmation, by means of an equilibrium analysis, i.e. taking into account the optimal reply of outsider platforms, of a similar conclusion but in terms of mere incentives to raise price post-merger, obtained by Cosnita-Langlais and al. (2018). They show that an upward pricing pressure on one side of the market may result in downward price pressure on the other side.

both sides of the market.[15] Moreover, it is relevant for the practice of competition agencies enforcing merger control, that may, as a consequence, need to balance one side's gain against the other side's loss. This is far from being the current practice for competition authorities, but the issue is all the more worth addressing given recent court decisions[16] questioning whether effects on one or both sides of the market should be relevant for the final decision. Finally, we show that both types of platform merger, adjacent and distant, are Pareto-improving only for high enough cost savings.

Before going on to our model, let us note that we perform a significantly more general analysis than what has been previously done in the literature. To start with, we allow for outsiders and no longer consider merger to monopoly. We thus provide an equilibrium analysis, since generally speaking, a merger's final impact on prices, profits and welfare hinges on the outsiders' conduct. Thirdly, we consider different types of merger: between neighbor, or, on the contrary, distant platforms. Finally, despite the limitations imposed by some specific assumptions such as localized competition among platforms and two-sided single-homing, the scope of our results is wider than it may seem at first, because we investigate a somewhat "worst case scenario" for users/consumers. On the one hand, the localized competition involves captive users for adjacent platforms, thus enabling clear and strong incentives for a price increase after merger. And on the other hand, assuming single-homing on both sides guarantees stronger price competition among platforms than with multi-homing users, and hence also the largest reduction in competition and strongest increase in market power through the merger.[17]

The rest of the paper is organized as follows: first we introduce the framework, then present the pre-merger equilibrium. We go on to determine optimal post-merger prices, profits and welfare, and compare them with the pre-merger levels. We conclude by discussing the policy

[15] Note that this is actually different from the seemingly similar result obtained by Correia-da-Silva et al. (2018), since their merger preserves the average marginal cost in the industry, and it takes an externality-adjusted pre-merger price below the marginal cost for prices to fall after the merger.

[16] See in particular Ohio v. American Express Co., 138 S. Ct. 2274 (2018) and the Mastercard decision by the European Court of Justice, C-382/12 P (previously Case T-111/08 EC).

[17] Recall that since multi-homing consumers choose how many platforms to join, the value of a platform for a multi-homing users is reduced to its incremental value, so a direct price comparison may not be relevant for her.

relevance of our results and listing the robustness checks that could be performed. The proofs for our results, obtained with Maple, are provided in a separate Technical Appendix available upon request.

2 Framework

We study bilateral mergers between platforms where customers on both sides can join at most one platform (i.e., two-sided single-homing)[18]. We consider a four-platform market and adapt the Hotelling-based framework developed by Armstrong (2006) to have the platforms compete on the Salop (1979) unit circle. Each platform faces two groups of customers, located on the opposite sides of the platform, enabling them to interact. The platforms, denoted $k \in \{A, B, C, D\}$, compete in access or membership fees[19]: they simultaneously set an access price for each group, and upon observing these prices, agents choose which unique platform to visit. We assume equidistant exogenous locations: A is located in $0(1)$, B in $\frac{1}{4}$, C in $\frac{1}{2}$ and D in $\frac{3}{4}$. The four platforms compete to attract buyers on both sides bearing the same constant unit production cost c.

On each side $i \in \{1, 2\}$ there is a unit mass of customers uniformly distributed. Denote v the reservation price, the same for all buyers. They all have inelastic demand, buying only one unit of the good/service if the total price is lower than the reservation price. Each customer will choose the platform offering the lowest total price, equal to the sum of the transportation cost and the membership fee. We assume linear and symmetric transportation costs on each side: we normalize to 1 the constant unit transportation cost, and denote d the distance from a platform to the customer's location. Thus, the net utility for a customer on side i joining platform k is

[18] See Belleflamme and Peitz (2015) for a reminder that single-homing environments are representative of many real-life situations.

[19] We focus on these so as to capture the competition for users/customers rather than the impact on the volume of transactions of the platforms' pricing decisions. Note that membership fees are the natural assumption to make when platforms cannot monitor transactions, so implicitly we focus on such platforms for our analysis. As an alternative, platforms may charge transaction fees (see Rochet and Tirole, 2003). For an insightful discussion of the use of different price instruments, see Rochet and Tirole (2006).

equal to:

$$U_i^k = v + a_i x_j^k - p_i^k - d_{i,k} \tag{1}$$

where p_i^k is the price set by platform k on side i, a_i the indirect network externality (a customer patronizing platform k on side j induces consumer utility equal to a_i on side i), x_j^k the total demand reached by platform k on side j, and $d_{i,k}$ the distance between the customer and the platform k. Let $a_i \geq 0$, and assume the intrinsic utility v to be high enough to always guarantee full market coverage.

We allow for a single source of exogenous asymmetry between the two sides: let $a_1 = a$ but $a_2 = \beta \times a$, where $\beta \in [-1, 1]$. Parameter β measures the asymmetry between sides in term of valuation of the other side's presence. For instance, with $\beta = 0$, side 2 does not value the presence of side 1, although the latter does value the presence of side 2. On the contrary, for $\beta = 1$, the cross-group externalities are identical for the two sides of the market, meaning that users on each side of the market value the other side's participation with the same positive intensity. Finally, for $\beta = -1$, users on side 2 dislike the presence of users on side 1 as intensely as users on side 1 like side 2.

Before presenting the pre-merger equilibrium, let us note that several features of our framework fit well the case of newspapers as two-sided markets. Horizontal differentiation is common among newspapers, and single-homing appears to be a reasonable assumption.[20] Furthermore, there is no agreement in the literature on the tastes of readers regarding advertising[21], and we do account for this by allowing for both positive and negative cross-group externalities.

[20] Argentesi and Filistrucchi (2007) find that "On each day of the week, more than 84% of advertisers put an advertising message in only one of the four newspapers, and only 10% in two of them." Kaiser and Wright (2006) also found that in the German magazine market, from 1972 to 2003, only 8% of the readers and 17% of the advertisers multihome.

[21] Argentesi and Filistrucchi (2007), van Cayselee and Vanormelingen (2018) and Fan (2013) find no effect of advertising on the sales of daily newspapers in Italy, Belgium and the US respectively. Kaiser and Song (2009) report that readers of magazines do not dislike advertising, and may even like it depending on the type of magazine. It seems that ads are mainly disliked when they cannot be avoided, as it is typically the case for TV and radio - see for instance Wilbur (2008).

3 Pre-merger analysis

Each platform k will maximize the total profit it makes on both sides of the market by optimally setting prices p_i^k. In order to establish the pre-merger price equilibrium, the demand system must be determined first, and for this it is necessary to consider the demand served by platform k on each side. We proceed as follows.

Since platforms are located on a circle, each platform k has two neighbors: k_l, on its left, and k_r, on its right. Then, the marginal customer x_i^{k,k_l} between platform k and its neighbor k_l on side i is defined by

$$v + a_i x_j^k - p_i^k - x_i^{k,k_l} = v + a_i x_j^{k_l} - p_i^{k_l} - \left(\frac{1}{4} - x_i^{k,k_l}\right), \tag{2}$$

while the marginal customer between k and k_r on the same side of the market is defined by:

$$v + a_i x_j^k - p_i^k - x_i^{k,k_r} = v + a_i x_j^{k_r} - p_i^{k_r} - \left(\frac{1}{4} - x_i^{k,k_r}\right). \tag{3}$$

The total demand for platform k on side i is therefore given by $x_i^{k,k_l} + x_i^{k,k_r}$, where

$$x_i^{k,k_{l,r}} = \frac{4a_i x_j^k - 4p_i^k - 4a_i x_j^{k_{l,r}} + 4p_i^{k_{l,r}} + 1}{16}, \text{ with } k_{l,r} = k_l, k_r. \tag{4}$$

Solving the system of total demands yields the individual demand x_i^k for each platform k and for each side i of the market as functions of all prices (see the Technical Appendix for the detailed derivation).

To ensure that the demand system yields a unique interior solution, some restrictions on our parameters are required[22]. More precisely, it is necessary to make sure that participation on each side is a decreasing function of the access fee on the same side.[23] We henceforth assume

[22] See Armstrong (2006, eq.8) for instance. See Filistrucchi and Klein (2013) for a more general discussion of such conditions in the case of an oligopoly platform market.

[23] Given the demand system displayed in the Technical Appendix, and since $\beta \in [-1, 1]$, this comes down to

that the following condition holds.

Assumption: let $a < 0.5$. This is the standard assumption stating that, given our normalization of the transport cost parameter, the intensity of the cross-group network effects is lower than the strength of the horizontal differentiation between platforms.

Given the individual demands for each platform, we can now write down the total profit (from both sides) made by platform k:

$$\Pi^k = (p_i^k - c)x_i^k + (p_j^k - c)x_j^k. \tag{5}$$

Each platform k maximizes its individual profit by simultaneously setting its prices p_i^k and p_j^k. Hence, the vector of equilibrium prices $\left(\left(p_i^k\right)^*, (p_j^k)^*\right)_{k=A,B,C,D}$ solves the system of the FOCs, i.e.,

$$\left(p_i^k\right)^* = \arg\max_{p_i^k} \left(x_i^k(p_i^k - c) + x_j^k(p_j^k - c)\right), i,j = 1,2, k = A,B,C,D. \tag{6}$$

In the Technical Appendix we derive the SOCs[24], which are all satisfied for $a < 0.5$.

Due to symmetry we obtain that before merger the four platforms set the same price on each side:

$$p_1^A = p_1^B = p_1^C = p_1^D = c + \frac{2 - 3a\beta - 5a^2\beta + 6a^3\beta^2}{2\left(4 - 9a^2\beta\right)}, \tag{7}$$

$$p_2^A = p_2^B = p_2^C = p_2^D = c + \frac{2 - 3a - 5a^2\beta + 6a^3\beta}{2\left(4 - 9a^2\beta\right)}, \tag{8}$$

whereas all individual (consolidated across sides) pre-merger profits equal

$$\Pi = \frac{1}{8}\frac{6a^3\beta^2 + 6a^3\beta - 10a^2\beta - 3a\beta - 3a + 4}{4 - 9a^2\beta}. \tag{9}$$

Lemma 1 *Pre-merger price on side 2 is decreasing in the intensity of the cross-group external-*

$\frac{1}{2}\frac{5\beta a^2 - 2}{1 - 5\beta a^2 + 4\beta^2 a^4} < 0$, or $a^2\beta < 0.25$.

[24] See Appendix A1 at the end of the paper.

ity. Pre-merger price on side 1 is decreasing in the intensity of the cross-group externality if both sides value the presence of the other side, but increasing with it if one side dislikes the other. Stand-alone pre-merger profit decreases with the level of cross-group externality unless one side dislikes the other very much.

Proof. It is straightforward to check that $\frac{\partial p_1^i}{\partial a} < (>)\ 0$ for $\beta > (<)\ 0$, that $\frac{\partial p_2^i}{\partial a} < 0\ \forall \beta$, and that $\frac{\partial \Pi}{\partial a} < 0$ for $\beta > 0$, but $\frac{\partial \Pi}{\partial a} \lesseqgtr 0$ for $\beta < 0$, in particular $\frac{\partial \Pi}{\partial a} > 0$ only when $\beta \to -1$ and $a \to 0.5$. ■

Lemma 1 states first that prices on both sides are decreasing in the level of cross-group externality when both sides value the presence of the other side. The intuition is of course that a stronger cross-group externality provides incentives to lower the price and attract more customers on both sides of the market: dropping the price on one side increases demand on the other side, which provides incentives to lower the price on this same side to further increase demand. But this gives further incentives to drop the price on the first side through the cross-side effect. Note also that even when side 2 dislikes side 1, the price on side 2 is still decreasing in the level of the cross-group externality, simply because dropping the price on side 2 still increases demand on side 1. However, the price on side 1 is now increasing with the externality: the negative valuation that users on side 2 have for side 1 writes off the opportunity cost of increasing the price on side 1 in order to capture more of the side 1 users' utility.

Lemma 1 also makes clear the impact of the cross-group network effect on pre-merger profits: the standalone profit decreases with the cross-group externality for positive or weakly negative valuation between sides (which is perfectly intuitive given the overall drop in prices), but increases with it otherwise. More precisely, individual profit only increases with the level of indirect externality when side 2 strongly dislikes side 1 and the cross-group network effect itself is very strong. In such a case, the corresponding increase in the price on side 1 more than compensates the drop in price on side 2 in terms of resulting profit.

The question we tackle next is the impact of a horizontal merger. Market concentration typically yields market power for merging firms, which seize the opportunity to increase profits by extracting higher prices from their combined residual captive demand. Outsider firms benefit from a price-increasing merger, since they also increase their prices, leaving customers clearly worse off. However, mergers may also generate cost savings, which may mitigate all this. Merger cost savings increase the insiders' efficiency relative to their rivals, enabling the insiders to lower their prices, attract more customers, and thereby increase their profits. The net price effect for the insider firms depends on the amount of cost savings, and the outsiders' own prices will take into account the insiders' pricing strategy.

Below we characterize the post-merger market equilibrium and compare it to the pre-merger situation, so as to study the role played by the cross-group externality for this market-power v. cost savings trade-off.

4 Merger analysis

4.1 Preliminary remarks

Before going on to solve the post-merger equilibrium, we first provide a brief overview of the method of resolution. As compared with the pre-merger situation, the post-merger setting exhibits a third parameter that impacts prices, profits and welfare: the (exogenous) amount of cost savings generated by the merger. In order to compare the post- and pre-merger situations, we compute the threshold values of merger cost savings that keep prices, profits and welfare (both consumers' and total) constant. This way we are able to state how much efficiency gains a given merger type (between neighbor or distant platforms) needs to generate in order to be profitable, to also benefit consumers, and to improve welfare. Since these threshold values of cost savings are likely to depend on the cross-group externality and the relative valuation between sides, we also pinpoint the impact of these parameters (a and β) for the comparison between the post- and pre-merger situations.

For tractability reasons, we ignore the merger's possible impact on locations, i.e., we rule out post-merger product repositioning, as well as the possibility for the merger to trigger changes in the number of active outlets or products.[25] Also, we assume that although the merger does give access to more users/business partners on the other side of the market, this will not change their willingness to pay or intrinsic utility from joining a platform[26]. In other words, we assume that the merger only changes the ownership pattern in the industry, thus leading to a joint pricing decision for the merging platforms, but it will not alter the type of equilibrium that the platforms play, nor the users' preferences.

We obtain throughout closed-form post-merger solutions for the eight equilibrium prices[27], four equilibrium profits, and also consumers' and total welfare expressions. However, they are far too much space-consuming to be displayed in the text, so we relegate all this to the Technical Appendix. The latter also contains the detailed proofs for all results in the rest of the paper. In the main text we only provide the sketches for the proofs of all our results. Finally, for illustrative purposes, the Appendix A2 at the end of the paper provides the detailed resolution in a particular case, that of perfect symmetry between the two market sides ($\beta = 1$).

4.2 Joint profit maximization

As before mentioned, our four-platform framework makes possible the analysis of two types of (exogenous) mergers: either between adjacent/neighbor or between distant platforms. Below we characterize the post-merger market equilibrium in terms of prices, profits, and resulting (consumers' and total) welfare in both cases, while recalling first that they are not outcome-equivalent in the "standard"/one-sided market analysis.

[25] When firms sell differentiated products, maintaining all outlets after the merger is less of an issue than if they sell homogenous goods. This actually points at the question of interoperability between the merged platforms when they are kept separate after the merger - recall for instance that after the acquisition of Waze, Google allowed direct navigation between Google Maps and Waze.

[26] We do not consider an exogenous increase in the intrinsic customer valuation after the merger, but focus instead on the increases in the net utility from joining a platform that comes from the fact that the change of ownership through merger provides access to a greater pool of business partners on the other side.

[27] Since we assume that the merger does not close down one of the participating platforms, post-merger there are still eight equilibrium prices to be determined.

In particular, a merger without cost savings between neighbor firms competing in prices on the circular market leads to price increases for all firms on the market, but the farther away the outsiders, the weaker their price increase (Levy and Reitzes, 1992). In turn, the merger between distant firms does not affect prices, nor profits, in the absence of cost savings, simply because for the market-power price-increasing effect to arise, the two merger firms need some captive demand. This effect is missing when they are not neighbors. Thus, the spatial literature on horizontal mergers between firms competing in prices on the circular market (Levy and Reitzes, 1992, Brito, 2003, 2005) concludes that a bilateral merger between neighbors, which necessarily involves a unilateral market power effect, is always profitable, whereas a merger between non adjacent firms can only be motivated either by cost savings or possibly to better sustain collusion.

We, in contrast, allow for merger cost savings: the merged platform will operate with a lower constant unit cost, the same across sides[28], as compared with the remaining outsider platforms. As a result, the merger turns each side of the circular market into a cost-asymmetric triopoly. Nonetheless, for both types of mergers (between adjacent or distant platforms), the resulting post-merger triopoly preserves the symmetry between the two insiders and the two outsiders respectively.

Let us first consider the case of the merger between adjacent platforms, say A and B. Denote $c-\delta$ the unit cost for the group $A+B$, with $\delta \geq 0$. Then the merged entity solves

$$\max_{p_1^A,p_2^A,p_1^B,p_2^B} \left(\Pi^A+\Pi^B\right) = \left(\begin{array}{c} (p_1^A-c+\delta)\left(x_1^{A,D}+x_1^{A,B}\right)+(p_2^A-c+\delta)\left(x_2^{A,D}+x_2^{A,B}\right) \\ +(p_1^B-c+\delta)\left(x_1^{B,C}+x_1^{B,A}\right)+(p_2^B-c+\delta)\left(x_2^{B,C}+x_2^{B,A}\right) \end{array} \right), \tag{10}$$

[28] We assume the same cost reduction through merger on both sides because we only consider merger synergies, i.e. the type of cost savings that competition agencies recognize as capable of being transferred, through lower prices, to consumers/users. One essential feature of marginal cost reductions to be recognized as merger synergies is their specificity: they should not be available other than through the submitted merger (see the EU Merger Regulation 139/2004 for instance). In our context, assuming the same cost savings on both sides satisfies this constraint, since our assumption clearly makes these efficiencies merger-, and not side-, specific.

whereas the platforms C and D keep on maximizing their stand-alone profits:

$$\max_{p_1^C, p_2^C} \Pi^C = (p_1^C - c)\left(x_1^{C,D} + x_1^{C,B}\right) + (p_2^C - c)\left(x_2^{C,D} + x_2^{C,B}\right), \tag{11}$$

$$\max_{p_1^D, p_2^D} \Pi^D = (p_1^D - c)\left(x_1^{D,A} + x_1^{D,C}\right) + (p_2^D - c)\left(x_2^{D,A} + x_2^{D,C}\right). \tag{12}$$

All SOCs are satisfied for $a < 0.5$ (see Appendix A1 at the end of the paper).

Let us now consider the alternative type of merger, i.e., between distant platforms, say A and C. Again, assuming equal cost savings between sides, the merged platform will operate with a lower constant unit cost as compared with the remaining outsider platforms. Denote now $c - \Delta$ the unit cost for $A + C$, with $\Delta \geq 0$. Then the merged entity will solve

$$\max_{p_1^A, p_2^A, p_1^C, p_2^C} \left(\Pi^A + \Pi^C\right) = \left(\begin{array}{c} (p_1^A - c + \Delta)\left(x_1^{A,D} + x_1^{A,B}\right) + (p_2^A - c + \Delta)\left(x_2^{A,D} + x_2^{A,B}\right) \\ +(p_1^C - c + \Delta)\left(x_1^{C,D} + x_1^{C,B}\right) + (p_2^C - c + \Delta)\left(x_2^{C,D} + x_2^{C,B}\right) \end{array} \right), \tag{13}$$

whereas the platforms B and D go on maximizing their stand-alone profits:

$$\max_{p_1^B, p_2^B} \Pi^B = (p_1^B - c)\left(x_1^{B,C} + x_1^{B,A}\right) + (p_2^B - c)\left(x_2^{B,C} + x_2^{B,A}\right), \tag{14}$$

$$\max_{p_1^D, p_2^D} \Pi^D = (p_1^D - c)\left(x_1^{D,A} + x_1^{D,C}\right) + (p_2^D - c)\left(x_2^{D,A} + x_2^{D,C}\right). \tag{15}$$

Again, all SOCs are satisfied for $a < 0.5$ (see Appendix A1 at the end of the paper).

We further check that the post-merger equilibrium is actually a triopoly and not a monopoly, i.e., that the merger cost savings do not force the outsider platforms out of the market. Since the outsiders' market share on each side of the market is linear and decreasing in the efficiency gains parameter whatever the merger type (see the Technical Appendix), the following holds:.

Remark *For each type of merger and for each side of the market there exists a strictly* *positive level of merger efficiency gains such that the outsider platforms hold each (through symmetry) a zero market share.*

In the Technical Appendix we check that all relevant thresholds that we discuss in the remaining of the paper, i.e., for merger profitability as well as price- and welfare-effects, are always below this upper bound of merger cost savings.

5 Results

We begin the analysis of the merger's outcome, and first discuss the impact on prices, before moving on to profits and finally welfare.

5.1 Price effect of mergers on two-sided markets

Generally speaking, the post-merger prices of insider platforms will result from a trade-off between two opposing effects. On the one hand, the indirect externality gives rise to a demand increase, which provides incentives to increase price. But on the other hand, giving access to users or consumers to both outlets encourages the latter to drop the price (each outlet internalizing the new users that it serves). Outsider platforms will take into account the insiders' incentives to modify prices when determining their own. We establish below to what extent the factoring in of merger cost savings impacts this trade-off:

Lemma 2 *Whatever the merger type (between adjacent or distant platforms), there is a unique threshold of merger cost savings on each side of the market such that the insiders' (outsiders') price is the same as before merger. This threshold is generally non-monotonic in the level of cross-group externality and the relative valuation between sides.*

Sketch of proof. All post-/pre-merger price differentials (i.e., on each side of the market, and for both insider and outsider platforms) are linear in the efficiency gains parameter.

■

In the Technical Appendix we compute the best reply functions for both insider and outsider platforms on each side and for both types of merger. They are always linear in the efficiency gains parameter, which explains the uniqueness result provided by Lemma 2. Furthermore, the unique threshold keeping the insiders'/outsiders' prices constant on each side of the market turns out to be not monotonic w.r.t. either a or β. This is due to the fact that the insiders'/outsiders' best reply functions on either side are decreasing in the insiders'/outsiders' own price on the opposite side, but increasing with the rivals'(outsiders'/insiders') prices on both sides.

The resulting impact of the merger on equilibrium prices is provided below by Propositions 1 and 2 (for the adjacent and distant mergers respectively):

Proposition 1 *The adjacent merger only leads to lower prices, for both insider and outsider platforms, and on each side of the market, iff it generates enough cost savings.*

Sketch of proof. The synergy thresholds keeping prices constant (for both insider and outsider platforms, and for each side of the market) are strictly positive.

■

Proposition 1 states the optimal post-merger price behavior in case of adjacent merger. It basically extends the standard trade-off between cost savings and market power for post-merger pricing behavior to the two-sided framework. To better grasp this result, recall that the cross-group externality favors customers to the extent that it provides incentives for firms to lower their prices so as to better benefit from the ensuing increase in demand. But in the case of an adjacent merger, the merger also involves a market-power effect, pushing the price upwards for the insider platforms, which fully internalize the fact that they enjoy captive demand on both sides from users located between them. The market-power effect of the merger dominates the incentives to lower the price provided by the cross-group externality, and it takes high enough cost savings for the insider platforms to eventually lower their price. Note that the outsider platforms also lower their prices: they do not benefit from cost savings, so the price drop is the only strategy enabling them to minimize demand loss due to the insiders' aggressive pricing.

Turning now to the merger between distant platforms, we find that the following holds:

Proposition 2 *The distant merger generally leads to lower prices, absent any efficiency gains:*

(i) On the side that values relatively less the presence of the other side, both the insider and the outsider platforms always lower their prices (i.e., regardless of the amount of merger synergies);

(ii) Optimal post-merger pricing on the side that values relatively more the presence of the other side depends on the valuation it gets from the other side: when side 1 is valued by side 2 ($\beta \geq 0$), the insider platforms may need enough cost savings to drop their price after merger, but not the outsider platforms (they always lower their price); instead, when side 1 is disliked by side 2 ($\beta < 0$), the insiders always need cost savings to drop their prices post-merger, but the outsider platforms may not.

Sketch of proof. (i) the thresholds of merger synergies keeping prices constant (both for the insider and the outsider platforms) on side 2 are strictly negative;

(ii) result obtained for side 1 by signing the threshold levels of merger synergy keeping prices constant for both merging and non-merging platforms when $\beta \geq 0$ and $\beta < 0$ respectively.

■

Proposition 2 provides the equilibrium price behavior after a distant merger, depending not only on the amount of merger cost savings, but also on the market side and the relative valuation between sides. More precisely, it is necessary to distinguish between the side that is more valued and the side that values relatively more in order to understand the rationale behind the optimal price-setting behavior after the distant merger.

Part (i) of Proposition 2 deals with the optimal post-merger pricing on the side that is relatively more valued. Recall that distant firms share no captive demand between them, so on traditional, one-sided markets, such a merger would not affect prices absent cost savings. This is not the case here: on side 2, the insider platforms do not need any merger synergies to drop their price, and this is due to the cross-group externality: given that side 1 values side 2, a price drop on side 2 will trigger increased demand on (and higher revenue from) side 1. The optimal reply from the outsider platforms will be similar: not only do they lack cost savings to cope with the insiders' demand increase, but they are stuck between the insiders as well, meaning they lose demand both to the left and to the right on side 1. Note that part (i) of Proposition 2 equally highlights the extent to which cost savings and indirect network effects are substitutable for the merging platforms: the former are not necessary for prices to drop on one side of the market, the one that is valued relatively more.

Part (ii) of Proposition 2 deals with optimal post-merger pricing behavior on side 1, the one which exhibits the higher valuation for the opposite side. The mechanism explaining it is actually more complex, because it hinges on whether side 2 users value or not the presence of side 1 users.

Whenever side 2 does value the presence of side 1 ($\beta \geq 0$), insider platforms may need high enough cost savings to drop their price on side 1. More precisely, this happens when the relative valuation between sides is quite low ($\beta \to 0$): in fact, the lower β, the more cost savings are necessary for the insider platforms to lower their price post-merger. In contrast, outsider platforms always drop their price on side 1 when $\beta \geq 0$. The intuition is the following: there is now an opportunity cost for the insiders to lower their price on side 1, because the induced demand gain on side 2 is very weak (β is very low). This trade-off is only solved through the presence of enough cost savings making it optimal to lower the price. In turn, it is always optimal for the outsider platforms to respond by dropping their price on side 1, precisely because they lack cost savings, so this is the only way to cope with the insiders' aggressive pricing on side 2.

In the opposite case, where side 2 dislikes side 1 ($\beta < 0$), the insider platforms always drop their price on side 1 for high enough cost savings, while the outsiders do so only if the relative valuation between sides, β, is negative enough. To see this, recall the opportunity cost that the merging platforms face when they contemplate lowering their price on side 1: there is now no additional demand from side 2, because side 2 users do not value side 1 users. So basically insiders face now a one-sided-market situation: any price drop on side 1 will only bring about a relatively small gain in demand, i.e., on side 1 only, so they need high enough cost savings to compensate for the negative price effect and make this price drop become optimal. The outsiders' response however is now more complex. For a weak dislike of side 2 for side 1 (β only weakly negative), the outsider platforms drop their price on side 1 regardless of the merger's efficiency gains: this enables them to amplify the effect of their own price drop on side 2, i.e., a demand increase on side 1. In other words, the weak dislike of side 2 for side 1 gives rise to a situation where the outsiders face a high opportunity cost if they do not price aggressively on side 1. In turn, with very strong dislike of side 2 for side 1 (β strongly negative, close to -1), this opportunity cost for outsiders is much lower: as a result, they will now mimic the insiders' price behavior, i.e., drop their price only for high enough cost savings. Note that for the outsider platforms this basically comes down to facing a one-sided-market situation as well, due to the strong asymmetry in terms of valuation between sides: they now focus on side 1 exclusively and given the less aggressive pricing by the insider platforms (only drop the price for high enough cost savings), they can afford to do the same.

Finally, note that for both the adjacent and distant merger, the following holds:

Corollary 1 *Regardless of the intensity of the cross-group externality and relative valuation between sides, both the insiders and the outsiders drop their price "first" on side 2 (i.e., for a lower amount of cost savings generated by the merger) and only "afterwards" on side 1 (i.e., for a higher amount of merger synergies).*

Sketch of proof. Result obtained by signing the difference between the respective synergy thresholds.

■

Corollary 1 further clarifies the post-merger optimal pricing behavior: on the side that values less the presence of the other side, all prices drop for lower levels of merger synergies than on the side exhibiting a higher valuation for the opposite side. The intuition is quite straightforward actually, and runs as follows. The insiders start lowering their price "first", meaning for a lower amount of cost savings, on the side that will give them the highest return in terms of additional cross-sides demand, here side 2. The strategic reason is that the latter is more valued by side 1, so dropping the price "first" on side 2 gives them a higher gain on side 1 than they would obtain on side 2 if instead they dropped the price "first" on side 1. The outsiders will do the same through strategic complementarity.

Before going on to discuss the resulting impact of these pricing strategies on profits, let us summarize the takeaways so far:

(i) for the merger between adjacent platforms, the analysis of post-merger pricing behavior yields quite similar results to the case of one-sided markets, to the extent that cost savings are always necessary for prices to drop; this is not generally the case for the merger between distant platforms (Propositions 1 and 2);

(ii) for both types of merger, a price drop on one side of the market from all firms is compatible, for a given amount of merger cost savings, with a simultaneous price increase (for all platforms) on the opposite side (Corollary 1).[29]

5.2 Merger profitability on two-sided markets

It is important to look into the outcome of pricing decisions for the profits of the merged platforms for at least one obvious reason: consumers would not be affected whatsoever by the merger if the latter is not submitted in the first place. The following proposition provides the results we obtain for the (internal) profitability of mergers on two-sided markets:

Proposition 3 *(i) Merging is always profitable for neighboring platforms, regardless of the amount of cost savings;*

[29] The only exception occurs for $\beta = 1$, i.e. identical positive valuation between sides, in which case the thresholds for price changes are the same for the insider and outsider platforms, whatever the side considered. See Appendix A2 at the end of the paper.

(ii) Distant platforms find it profitable to merge only for enough cost savings.

Sketch of proof. Whatever the merger type, the insiders' joint profit is convex in the efficiency gains parameter, with two real roots, one of each is always strictly negative. The other root is therefore the relevant threshold for the profitability analysis: it represents the minimum amount of cost savings that guarantee merger profitability for the insider platforms. For the adjacent merger, the second root is also negative, hence the merger increases the insiders' profit for any level of positive merger synergies. For the distant merger, the second root is positive, meaning that distant platforms need enough cost savings for their merger to be profitable.

■

Note that Proposition 3 provides a direct extension of profitability results obtained in the one-sided case (Levy & Reitzes 1992, Brito 2005). The intuition is straightforward: adjacent firms benefit from captive demand located between them and will find it profitable to merge thanks to this market power effect, even without cost savings. In turn, distant insiders have no captive demand between them, therefore their merger cannot be profitable through the exploitation of a market power effect. Then the only way left for this merger to be profitable is through merger cost savings. Recall that following the distant merger the insider platforms lower their prices (on both sides), which triggers additional demand for them. Hence it takes high enough cost savings to make the distant merger profitable (by guaranteeing a high enough increase in demand to compensate for the lower margin).

5.3 Welfare impact of mergers on two-sided markets

In what follows we examine the impact of mergers on welfare: both consumers' and total. Although most competition agencies enforce a consumer surplus standard for merger assessment, some allow for the total welfare standard to play a role in their decision. We find that:

Proposition 4 *(i) The adjacent merger only increases consumers' surplus on either side for high enough cost savings;*

(ii) the distant merger always increases consumers' surplus on both sides.

Sketch of proof. Whatever the merger type, the consumers' surplus functions (one for each side of the market, and consolidated across sides) are convex in the efficiency gains parameter, with two real roots, one being always strictly negative. For the adjacent merger, the second root is always strictly positive, whatever the side considered, whereas for the distant merger it is always strictly negative, regardless of the side considered.

■

Proposition 4 summarizes results for the impact of each type of merger on consumers' surplus: analyzing side by side, the distant merger always improves consumers' welfare, whereas the adjacent merger only does so for high enough cost savings. This is actually quite intuitive, since consumers' surplus is decreasing in the price paid by users.

Let us start with the distant merger: in this case, all post-merger prices drop most of the time. They actually always drop on the side which values less the presence of the other side (side 2), i.e., regardless of the level of cost savings, hence post-merger consumers' surplus necessarily always increases on this side. However, on the side that values more the presence of the other side (side 1), post-merger prices may go down for both the insiders and the outsiders only for high enough cost savings, in particular when $\beta < 0$. But then it is worth recalling that users' utility is positively affected by the cross-group externality, which is positive from side 1 to side 2. In other words, Proposition 4 establishes that on side 1, the additional utility derived from interacting with more users on the opposite side more than compensates for the non-decrease in price whenever the cost savings are not high enough to trigger a price drop, thus yielding a consumers' surplus increase as well.

Note that the very same situation is potentially present for the adjacent merger on either side: a consumers' surplus trade-off between the additional utility derived from interacting with more users on the opposite side on the one hand, and the non-decrease in price whenever the cost savings are not high enough to trigger a price drop on the other hand. However, and in contrast to the distant merger case, consumers' surplus does not increase after the adjacent merger unless the merger synergies are high enough, despite the positive utility effect of the cross-group externality. This is due to the fact that the amount of cost savings needed to makes prices drop post-merger is higher for the adjacent merger than for the distant merger[30], and this is to be put down to the presence of a market-power effect for the adjacent merger.

For the latter, an additional result holds:

Corollary 2 *For the adjacent merger, a given amount of cost savings can lead to a consumer surplus increase on one side but a consumer surplus decrease on the other.*

Sketch of proof. Result obtained by signing the difference between the synergy levels keeping constant the consumers' surplus for each side. ■

Corollary 2 states that the thresholds of merger synergies keeping consumers' surplus constant after the adjacent merger may differ across sides. In other words, for the same amount of cost savings, users on one side may benefit from the merger while those on the opposite side may be hurt by it. This is due to the fact that the constant-price thresholds of cost savings are different across sides for both insider and outsider platforms.[31] As a result, for the purposes of the final decision to be made by the competition authority, the merger's net effect on customers would likely involve balancing one side's gain against the other side's loss.

Turning now to the total welfare outcome of the two types of mergers, we first note the following:

Lemma 3 *(i) The outsiders to the adjacent merger incur a profit loss as soon as the merger cost savings are high enough.*

(ii) The outsiders to the distant merger always incur a profit loss, i.e., whatever the level of merger cost savings.

[30] See the Technical Appendix.

[31] Unless $\beta = 1$ of course, in which case these thresholds are identical on each side - see the explicit example in the Appendix A2 at the end of the paper.

Sketch of proof. (i) The profit differential for the outsider platforms to the adjacent merger is convex in the efficiency gains parameter. Both roots are positive, but one is above the threshold of efficiency gains for which demand served by the outsider platforms becomes zero. Hence, post-merger the outsiders only make a higher profit as long as the merger's synergy level is lower than the first root of their profit differential; for higher values they make a lower profit than before merger;

(ii) The profit differential for the outsider platforms to the distant merger is convex in the efficiency gains parameter. One root is always positive and higher than the threshold of efficiency gains for which demand served by the outsider platforms becomes zero. The other root is always lower than the minimum threshold of efficiency gains for which the merger is internally profitable. Thus the outsider platforms always make a lower profit than before merger.

■

Both parts of Lemma 3 are quite intuitive, and stem from the post-merger equilibrium price behavior. To see this, recall that the only way for the outsiders to benefit from a merger, given their lack of cost savings, is through the merger's strategic, market-power effect, which enables them to raise prices without losing demand. But when the outsider platforms are forced to lower their prices, in response to the aggressive pricing by the more cost-efficient insiders, they end up making a lower margin and also serving a lower demand too. Hence the outsiders cannot but make a lower profit than before merger. The post-merger price drop occurs if the merger cost savings are high enough in the case of adjacent insiders, and always (i.e., regardless of the amount of merger efficiencies) in the case of distant insiders.

Based on Propositions 3 and 4 as well as on Lemma 3, Proposition 5 below concludes on the total welfare impact of the two merger types:

Proposition 5 *Both types of merger increase total welfare for high enough cost savings.*

Sketch of proof. Total welfare functions (one for each type of merger) are convex in the efficiency gains parameter, with two real roots, one being always strictly negative. The second root is always strictly positive in the case of the adjacent merger. For the distant merger, the second root becomes negative only for high enough values of a and β.

■

Proposition 5 states that both types of merger, and perhaps a little unexpectedly not only the adjacent one, will be Pareto-improving only for high enough synergies. Recall that after the adjacent merger enough cost savings are required to ensure that consumers' surplus increases. However, insiders' profits always increase afterwards, whereas the outsiders incur a profit loss if the cost savings are high enough. Thus, for high enough cost savings, the adjacent merger's positive impact on consumers and insiders more than compensates the outsiders' profit loss, and hence total welfare increases. A similar trade-off plays out in the case of the distant merger: the insiders' profit only increases for high enough efficiency gains, but the outsiders' profit always drops, while consumers' surplus always increases. Again, there is a trade-off, and for high enough cost savings, the distant merger's positive impact on consumers and insiders more than compensates the outsiders' profit loss, allowing for total welfare to increase. Note that this trade-off completely vanishes for a high enough positive reciprocal externality between sides (a and β high enough): this is actually quite intuitive, since it is for such parameter values that the parties that benefit from the interoperability between the merging platforms, that is the users and the merging firms, actually do so to the fullest.

6 Discussion and concluding remarks

This paper focuses on the role played by the merger cost savings for the outcome of horizontal mergers between two-sided platforms. Our model considers four symmetrically differentiated platforms located equidistantly on the unit circle market and competing in membership fees. Users on both sides single-home, and we allow for both positive and negative cross-group externalities, as well as for bilateral mergers between either adjacent, closely substitutable platforms, or between distant, less substitutable ones.

We find that the adjacent platform merger will only lead to lower prices on either side iff it generates enough cost savings, regardless of the relative valuation sides exhibit for one another.[32] In contrast, cost savings may not be necessary to achieve this outcome for the distant merger: post-merger prices always drop on the side that values relatively less the opposite side. Moreover, for both types of merger, all platforms lower their prices "first", i.e., for a lower amount of merger's cost savings, on the side that is relatively more valued, because it will give them the highest return in terms of additional cross-sides demand. Our analysis also provides some insights for the merger profitability: we actually extend in this respect the results obtained in the standard, one-sided framework, in particular the fact that merging is always profitable for neighboring firms/platforms, but for distant firms/platforms only if there are enough cost savings. Regarding the welfare impact of the mergers, we find that the distant merger always improves consumers' surplus, i.e., on both sides and regardless of the amount of merger cost savings. On the contrary, the adjacent merger only benefits users on either side for high enough efficiency gains. In terms of aggregate welfare effect, both types of platform mergers are found to be Pareto-improving only for high enough cost savings. Finally, we show in the case of merger between adjacent platforms that the two sides may face post-merger opposite price and consumers' surplus effects, for a given amount of cost savings from the merger, with one side benefitting from the merger while the other one being harmed by it. This raises some interesting questions for the practice of competition agencies when enforcing merger control on two-sided markets.

[32] To a certain extent, this result provides a formal background for the EC's prohibition decision in the Deutsche Börse - NYSE Euronext merger case (EC Case No. M.6166). The EC considered that the efficiency gains argued by the merging parties would be insufficient to compensate for the likely anticompetitive concern raised by the expected increase in concentration (up to 90%) on the European market for exchange-traded derivatives (for an evaluation of horizontal merger efficiencies on multi-sided markets, in particular for stock exchange mergers such as Deutsche Börse - NYSE Euronext, see OECD 2018, Part V, Ch. 7). In our framework, this would mirror the case of closely-substitutble platforms with positive reciprocal valuation between sides, for which cost savings are necessary in order for prices (on either side) to not increase.

To start with, both in the US and in the EU, competition law prohibits mergers that raise anti-competitive concerns in the relevant market, even when these concerns may be outweighed by efficiencies in another market.[33] Accordingly, even if the benefit for consumers outside the market defined as relevant is higher than the welfare loss of consumers in that relevant market, the merger would not be cleared even though it may be welfare-increasing. This general principle on how to take into account merger efficiencies, in particular the out-of-market ones, turns out to be crucial in a two-sided context, where two or more set of users are necessarily and above all closely interconnected.[34] Moreover, it draws attention to the proper definition of the "relevant market" in case of platform mergers, given that recent front-page judicial decisions recognized that both sides of platform markets must be considered when assessing market power and establishing the existence of adverse effects on competition.[35] Furthermore, if a platform merger were to be only conditionally cleared due to opposite consumer effects on the two sides, the question of appropriate, side-wise, remedies would need to be addressed as well.

Arguably, several robustness checks may be contemplated in relation with the formal setting we used for our analysis. It would be relevant to consider alternative assumptions, such as multi-homing customers[36] for instance, or non-localized competition among platforms, which could also be non symmetrically differentiated, and so on. By the same token, usage fees or

[33] The EU merger regulation completely disregards out-of-market efficiencies, since it establishes that efficiencies should, in principle, benefit consumers in those relevant markets where it is likely that competition harm would occur (Commission Notice, Guidelines On The Assessment Of Horizontal Mergers Under The Council Regulation On The Control Of Concentrations Between Undertakings, OJ 2004 C31/03, at para 79). In the US, the latest version of US Horizontal Merger Guidelines, from 2010, allows to some extent to take into account cross-market efficiencies, since it explicitly states that the analysis should focus on whether "the merger is likely to benefit customers overall" (see US Department of Justice and Federal Trade Commission, Horizontal Merger Guidelines (2010), at 30, note 14). However, the evaluation of out-of-market efficiencies is still conditioned on the "prosecutorial discretion" of the antitrust agencies. In stark contrast, the Canadian Merger Enforcement Guidelinesclearly state (section 12.23) that "In addition to direct effects in the relevant market, the Bureau also considers price and non-price effects in interrelated markets."

[34] For a discussion of out-of-market effciencies and competition analysis of two-sided platform markets, see Ducci (2016).

[35] See United States v. Am. Express Co., 838 F.3d 179, 197-98, 202-05 (2d Cir. 2016), and Case C-67/13 P, Groupement des Cartes Bancaires (CB) v. European Commission, 2014 E.C.R.2204.

[36] The nature of competition among platforms would be substantially changed in this case, and the literature on media markets indicates that the merger effects are different from when marginal consumers single-home (Anderson et al., 2018).

two-part tariffs would be interesting to assume instead of our linear access fees.[37] The industrial organization literature on two-sided platforms has yet to produce such a more general but still tractable framework for the analysis of horizontal mergers among two-sided platforms, and we leave these robustness checks for later research.

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7 Appendix

7.1 Appendix A1 - The SOCs

Before merger, the SOCs require that $\frac{5\beta a^2-2}{1-5\beta a^2+4\beta^2 a^4}<0$ and $\left(-\frac{1}{4}\right)\frac{6\beta^2 a^3-10\beta a^2-3\beta a+4+6\beta a^3-3a}{(4\beta a^2-1)^2}\frac{6\beta^2 a^3-4-3\beta a-3a+10\beta a^2+6\beta a^3}{(\beta a^2-1)^2}>0$ respectively.

After the adjacent merger, they require that:

for the merged platforms, A **and** B:

$\frac{5\beta a^2-2}{1-5\beta a^2+4\beta^2 a^4} < 0$, $\left(-\frac{1}{4}\right)\frac{(6\beta^2 a^3-10\beta a^2-3\beta a+4+6\beta a^3-3a)\left(6\beta^2 a^3-4-3\beta a-3a+10\beta a^2+6\beta a^3\right)}{(4\beta a^2-1)^2(\beta a^2-1)^2} > 0$,
$\left(-\frac{1}{4}\right)\frac{(44a^6\beta^4+88a^6\beta^3-49\beta^3 a^4+44a^6\beta^2-218\beta^2 a^2+14\beta^2 a^2-49a^4\beta+136\beta a^2+14a^2-24)}{(4\beta a^2-1)^2(\beta a^2-1)^3} < 0$ and
$\frac{1}{16}\frac{(\beta a+2+a)(\beta a-2+a)\left(8\beta^2 a^3-6-5\beta a-5a+12\beta a^2+8a^3\beta\right)\left(8\beta^2 a^3+6-5\beta a-5a-12\beta a^2+8a^3\beta\right)}{(4\beta a^2-1)^2(\beta a^2-1)^4} > 0.$

for the outsider platforms, either C or D:

$\frac{5\beta a^2-2}{1-5\beta a^2+4\beta^2 a^4} < 0$ and $\left(-\frac{1}{4}\right)\frac{(6\beta^2 a^3-10\beta a^2-3\beta a+4+6\beta a^3-3a)\left(6\beta^2 a^3-4-3\beta a-3a+10\beta a^2+6\beta a^3\right)}{(4\beta a^2-1)^2(\beta a^2-1)^2} > 0.$

After the distant merger, the SOC require that:

for the merged platforms, A and C:

$\frac{5\beta a^2-2}{1-5\beta a^2+4\beta^2 a^4} < 0$, $\left(-\frac{1}{4}\right)\frac{(6\beta^2 a^3-10\beta a^2-3\beta a+4+6\beta a^3-3a)\left(6\beta^2 a^3-4-3\beta a-3a+10\beta a^2+6\beta a^3\right)}{(4\beta a^2-1)^2(\beta a^2-1)^2} > 0$,
$-\frac{\left(8\beta^3 a^4+16\beta^2 a^4-5\beta^2 a^2+8a^4\beta-30\beta a^2+8-5a^2\right)}{(4\beta a^2-1)^2(\beta a^2-1)^2} < 0$, and $\frac{4(\beta a+2+a)(\beta a-2+a)(\beta a-1+a)(\beta a+1+a)}{(4\beta a^2-1)^2(\beta a^2-1)^2} > 0.$

for the outsider platforms, either B or D:

$\frac{5\beta a^2-2}{1-5\beta a^2+4\beta^2 a^4} < 0$ and $\left(-\frac{1}{4}\right)\frac{(6\beta^2 a^3-10\beta a^2-3\beta a+4+6\beta a^3-3a)\left(6\beta^2 a^3-4-3\beta a-3a+10\beta a^2+6\beta a^3\right)}{(4\beta a^2-1)^2(\beta a^2-1)^2} > 0.$

7.2 Appendix A2 - The perfectly symmetric case

We consider here the case where $\beta = 1$ and solve analytically the post-merger equilibrium. Below we provide the detailed computations for equilibrium profits and prices, while distinguishing the two types of merger we consider, between adjacent and distant platforms.

- **Merger between adjacent platforms (A+B merger)**

With $\beta = 1$, post-merger profits for the insiders and outsiders write respectively as follows: $\widetilde{\Pi}^{A+B} = \frac{1}{2}\frac{\left(4\delta-11a-6a\delta+4+7a^2\right)^2}{(1-a)(5-8a)^2}$ and $\widetilde{\Pi}^{C} = \widetilde{\Pi}^{D} = \frac{1}{4}\frac{(2a-1)}{1-a}\frac{(3a-2)(3a-3+2\delta)^2}{(5-8a)^2} \equiv \widetilde{\Pi}^{O}$.

SOCs

Whenever the SOCs are satisfied the Hessian matrices for all profit functions are negative definite (ND). We define below the Hessian matrix H^{A+B} for the merged firm and $H^{C,D}$ for the outsider profits:

$$H^{A+B} = \frac{N}{1-4a^2}\begin{bmatrix} \frac{1}{-a^2+1}\left(5a^2-4\right) & 3\frac{a}{-a^2+1}\left(2a^2-1\right) & 1 & 2a \\ 3\frac{a}{-a^2+1}\left(2a^2-1\right) & \frac{1}{-a^2+1}\left(5a^2-2\right) & 2a & 1 \\ 1 & 2a & \frac{1}{-a^2+1}\left(5a^2-2\right) & 3\frac{a}{-a^2+1}\left(2a^2-1\right) \\ 2a & 1 & 3\frac{a}{-a^2+1}\left(2a^2-1\right) & \frac{1}{-a^2+1}\left(5a^2-2\right) \end{bmatrix}$$

and $H^{C,D} = \frac{1}{(1-a^2)(1-4a^2)}\begin{bmatrix} 5a^2-2 & 3a\left(2a^2-1\right) \\ 3a\left(2a^2-1\right) & 5a^2-2 \end{bmatrix}$.

In order for H^{A+B} and $H^{C,D}$ to be ND the following principal minor conditions need to be fulfilled:

- for the merged firm: $\left|H_1^{A+B}\right| = \frac{5a^2-2}{4a^4-5a^2+1} < 0$, $\left|H_2^{A+B}\right| = -\frac{9a^2-4}{4a^4-5a^2+1} > 0$, $\left|H_3^{A+B}\right| = -\frac{11a^2-6}{4a^4-5a^2+1} < 0$, $\left|H_4^{A+B}\right| = \frac{16a^2-9}{4a^4-5a^2+1} > 0$;

- for each outsider: $\left|H_1^{C,D}\right| = \frac{5a^2-2}{4a^4-5a^2+1} < 0$, $\left|H_2^{C,D}\right| = -\frac{9a^2-4}{4a^4-5a^2+1} > 0$.

The SOCs are satisfied for $a < 0.5$.

Equilibrium prices

Due to the symmetry between the insider platforms, as well as between the outsiders, the following optimal post-merger prices obtain for the insider and outsider platforms respectively: $\widetilde{p}^{A+B} = c + \frac{10a\delta-6\delta-11a+7a^2+4}{10-16a}$ and $\widetilde{p}^{C} = \widetilde{p}^{D} = c + \frac{4a\delta-2\delta-9a+6a^2+3}{10-16a}$. Note that $\widetilde{p}^{A+B} - c + \delta \geq 0$, $\forall\delta \in [0, c]$, but $\widetilde{p}^{C} - c \geq 0$ iff $\delta \leq \frac{3}{2}(1-a) \equiv \overline{\delta}$.

Let $f(\delta, a)$ stand for the post-/pre-merger price difference for the insider platforms on either side of the market. Then $f(\delta, a) = \widetilde{p}^{A+B} - p^* = \frac{(3-5a)\left(6a\delta-4\delta-2a+1+a^2\right)}{(4-6a)(5-8a)}$. By the same token, the outsider-platform price difference between merger and no-merger on either side of the market writes $h(\delta, a) = \widetilde{p}^{C} - p^* = \frac{1}{2}\frac{(1-2a)\left(6a\delta-4\delta-2a+1+a^2\right)}{(2-3a)(5-8a)}$. It is straightforward to check that $\frac{\partial}{\partial\delta}f(\delta, a) = \frac{3-5a}{8a-5} < 0$ and $\frac{\partial}{\partial\delta}h(\delta, a) = \frac{1-2a}{8a-5} < 0$. Note also that $f(\widetilde{\delta}, a) = h(\widetilde{\delta}, a) = 0$ for $\widetilde{\delta} = \frac{(a-1)^2}{2(2-3a)} > 0$, where $\widetilde{\delta} < \overline{\delta}$ for any $a < 0.5$. Thus $f(0, a) \leq 0$ and $h(0, a) \leq 0$ for $\delta \in \left[\widetilde{\delta}, \overline{\delta}\right]$.

Equilibrium profits

The profit differentials (post/pre-merger) for insiders and outsiders are given by

$\widetilde{\Pi}^{A+B} - 2\Pi^* = \frac{4(2-3a)^3\delta^2+4(4-7a)(1-a)(2-3a)^2\delta+\left(19a^2-23a+7\right)(1-a)^3}{2(1-a)(8a-5)^2(2-3a)}$ and

$\widetilde{\Pi}^O - \Pi^* = (1-2a)\frac{(4-6a)^2\delta^2 - 12(1-a)(3a-2)^2\delta + (1-a)^3(11-17a)}{4(1-a)(2-3a)(8a-5)^2}$ respectively.

We have that $\widetilde{\Pi}^{A+B} - 2\Pi^* = 0$ if $\delta = \widetilde{\delta}_1$ or $\delta = \widetilde{\delta}_2$, where both roots are negative for $a < 0.5$, i.e. $\widetilde{\Pi}^{A+B} - 2\Pi^* > 0$.

Similarly, $\widetilde{\Pi}^O - \Pi^* = 0$ for $\widetilde{\delta}_1^O = \frac{(1-a)^2}{2(2-3a)} = \widetilde{\delta}$ and $\widetilde{\delta}_2^O = \frac{(11-17a)(1-a)}{2(2-3a)}$.

We have that $0 < \widetilde{\delta}_1^O < \overline{\delta} < \widetilde{\delta}_2^O$ for any $a < 0.5$, and thus $\widetilde{\Pi}^O - \Pi^* \leq 0$ for $\delta \in \left[\widetilde{\delta}, \overline{\delta}\right]$.

Consumers' surplus

We can at first compare the difference of consumers' surplus between post and pre-merger on each side (note that since $\beta = 1$, the difference is the same for side 1and side 2)

$g(\delta, a) = CS_i^{A+B} + 2CS_i^o - CS$ with $i = 1, 2$, where CS_i^{A+B},CS_i^o and CS respectively the consumers' surplus from the merger $A+B$, the consumers' surplus when users are connected to the outsiders, and the consumers' surplus in the pre-merger case. Note that the overall difference in consumers' surplus is just $2\left(CS_i^{A+B} + 2CS_i^o - CS\right)$. We can prove that $g(0, a) < 0$.

Basically, for $g(\delta, a) = 0$ there exist two roots $\delta_1^{CSAB} < 0$ and $\delta_2^{CSAB} > 0$, where $\delta_2^{CSAB} = -\frac{1}{4}\frac{\left(112\,a^3 - 240\,a^2 + 164\,a - 36 + \sqrt{(196\,a^4 - 616\,a^3 + 717\,a^2 - 364\,a + 68)(-5+8\,a)^2}\right)(a-1)}{(3\,a-2)^2}$.

And we can conclude that the adjacent merger improve the consumers' surplus on each side ($\beta = 1$) for high enough cost savings, i.e. for $\delta \geq \delta_2$. Moreover, the overall difference in consumers' surplus is just $2\left(CS_i^{A+B} + 2CS_i^o - CS\right)$, so we reach the same conclusion.

Welfare comparison

The difference between the post and the pre-merger welfare provides a function $W^{A+B}(\delta, a)$, for which there exists two roots $\delta_1^{WAB} < 0$ and $\delta_2^{WAB} > 0$, with $W^{A+B}(\delta, a) = 0$, and $W^{A+B}(0, a) < 0$, where $\delta_2^{WAB} = \frac{1}{4}\frac{\left(124\,a^3 - 272\,a^2 + 192\,a - 44 + \sqrt{(244\,a^4 - 798\,a^3 + 961\,a^2 - 502\,a + 96)(-5+8\,a)^2}\right)(a-1)}{(3\,a-2)(10\,a^2 - 19\,a + 8)}$.

We can conclude that the adjacent merger is welfare improving for high enough cost savings.

- **Merger between distant platforms (A+C merger)**

For $\beta = 1$ post-merger equilibrium profits write as follows:

$\widehat{\Pi}^{A+C}$ =76a$^5 - 1008a^4\delta - 196a^4 + 2712a^3\delta + 187a^3 + 432\delta^2 a^3 - 864a^2\delta^2 - 78a^2 - 2704\delta a^2 + 1184a\delta + 12a + 576a\delta^2 - 192\delta - 128\delta^2 \frac{}{16(3a-2)(1-2a)(4\,a-3)^2}$

$\widehat{\Pi}^B = \widehat{\Pi}^D = (3a-2)(a-1)(6a+4\delta-3)^2 \frac{}{16(1-2a)(4a-3)^2} \equiv \widehat{\Pi}^O.$

SOCs

Whenever the SOCs are satisfied the Hessian matrices for all profit functions are negative definite (ND). We define below the Hessian matrices for the merged entity and for the outsiders:

- for the merged entity $(A+C)$:

$$H^{A+C} = \frac{1}{(1-a^2)(1-4a^2)} \begin{bmatrix} 5a^2-2 & 3a\left(2a^2-1\right) & -3a^2 & -a\left(2a^2+1\right) \\ 3a\left(2a^2-1\right) & 5a^2-2 & -a\left(2a^2+1\right) & -3a^2 \\ -3a^2 & -a\left(2a^2+1\right) & 5a^2-2 & 3a\left(2a^2-1\right) \\ -a\left(2a^2+1\right) & -3a^2 & 3a\left(2a^2-1\right) & 5a^2-2 \end{bmatrix}$$

- and for the outsiders (B or D):

$$H^O = \frac{1}{(1-a^2)(1-4a^2)} \begin{bmatrix} 5a^2-2 & 3a\left(2a^2-1\right) \\ 3a\left(2a^2-1\right) & 5a^2-2 \end{bmatrix}.$$

This leads the following SOCs conditions:

-for the insider platforms: $\left|H_1^{A+C}\right| = -\frac{2-5a^2}{4a^4-5a^2+1} < 0$, $\left|H_2^{A+C}\right| = \frac{4-9a^2}{4a^4-5a^2+1} > 0$, $\left|H_3^{A+C}\right| = -\frac{8}{4a^4-5a^2+1} < 0$, $\left|H_4^{A+C}\right| = \frac{16}{4a^4-5a^2+1} > 0$;

- for the outsiders: $\left|H_1^{B,D}\right| = -\frac{2-5a^2}{4a^4-5a^2+1} < 0$, $\left|H_2^{B,D}\right| = \frac{4-9a^2}{4a^4-5a^2+1} > 0$.

Thus the SOCs are satisfied for $a < 0.5$.

Equilibrium prices

As before, due to the symmetry between the insider platforms, as well as between the outsiders, we obtain the following optimal post-merger prices: $\widehat{p}^{A+C} = c + \frac{20a\Delta-16\Delta-19a+14a^2+6}{8(3-4a)}$,and $\widehat{p}^B = \widehat{p}^D = c + \frac{(a-1)}{4}\frac{(6a+4\Delta-3)}{3-4a}$. Note that: $\widehat{p}^{A+C} - c + \Delta \geq 0, \forall\Delta \in [0,c]$, but $\widehat{p}^B - c \geq 0$ iff $\Delta \leq \frac{3}{2}\left(\frac{1}{2}-a\right) \equiv \overline{\Delta}$.

Price differences (post/pre-merger) for the insiders (A and C) and the outsiders (B and D) respectively are now equal to $g(\Delta,a) = \frac{(4-5a)\left(12a\Delta-8\Delta-a+2a^2\right)}{8(3a-2)(4a-3)}$ and $l(\Delta,a) = \frac{(1-a)\left(12a\Delta-8\Delta-a+2a^2\right)}{4(3a-2)(4a-3)}$. Then $\widehat{\Delta} = \frac{1}{4}\frac{(2a-1)a}{2-3a}$ solves for $g(\Delta,a) = l(\Delta,a) = 0$, where $\widehat{\Delta} \leq 0$ for any $a < 0.5$. Moreover, $\frac{\partial}{\partial\Delta}g(\Delta,a) = \frac{4-5a}{8a-6}$ and $\frac{\partial}{\partial\Delta}l(\Delta,a) = \frac{1-a}{4a-3}$, therefore both price differences are decreasing in Δ. Thus $g < 0$ and $l < 0$ for any $a < 0.5$.

Equilibrium profits

The profit differential for the merging and non-merging platforms equals respectively:

$\widehat{\Pi}^{A+C} - 2\Pi^* = \left(-\frac{1}{16}\right) \frac{16\Delta^2(3a-2)^3 + a(2a-1)^2\left(19a^2-30a+12\right) - 8\Delta(2a-1)(3a-2)^2(7a-6)}{(3a-2)(2a-1)(4a-3)^2}$

and $\widehat{\Pi}^{O} - \Pi^* = \left(-\frac{1}{16}\right) \frac{(a-1)\left(\Delta^2 16(3a-2)^2 + \Delta 24(2a-1)(3a-2)^2 + a(17a-12)(2a-1)^2\right)}{(3a-2)(2a-1)(4a-3)^2}$.

We have that $\widehat{\Pi}^{A+C} - 2\Pi^* = 0$ if $\Delta = \Delta_1$ or $\Delta = \Delta_2$, where

$\Delta_1 = \frac{(2a-1)}{2(3a-2)^2}\left(\frac{1}{2}(7a-6)(3a-2) - \sqrt{2}(4a-3)\sqrt{(a-1)(3a-2)}\right)$

and $\Delta_2 = \frac{(2a-1)}{2(3a-2)^2}\left(\frac{1}{2}(7a-6)(3a-2) + \sqrt{2}(4a-3)\sqrt{(a-1)(3a-2)}\right)$.

It can be easily checked that $\Delta_1 < 0 < \Delta_2 < \overline{\Delta}$ for $a < 0.5$, leading to $\widehat{\Pi}^{A+C} - 2\Pi^* \geq 0$ whenever $\Delta \in \left[\Delta_2, \overline{\Delta}\right]$.

Similarly, it is straightforward to check that $\widehat{\Pi}^{O} - \Pi^* = 0$ for $\widehat{\Delta}_1 = -\frac{1}{4}a\frac{2a-1}{3a-2} < 0$ and for $\widehat{\Delta}_2 = -\frac{1}{4}\frac{2a-1}{3a-2}(17a-12) > 0$. Since $\widehat{\Delta}_1 < \overline{\Delta} < \widehat{\Delta}_2$, we have that $\widehat{\Pi}^{O} - \Pi^* \leq 0$ for $\Delta \left[0, \overline{\Delta}\right]$.

Consumers' surplus

We can at first compare the difference of consumer surplus between post and pre merger on each side (note that since $\beta = 1$, the difference is the same for side 1 and side 2)

$z(\Delta, a) = CS_i^{A+C} + 2CS_i^o - CS$ with $i = 1, 2$.

$z(\Delta, a) = \frac{1}{64} \frac{\left(448a^4 - 1162a^3 + 36a^2\Delta + 1113a^2 - 466a - 48a\Delta + 16\Delta + 72\right)\left(2a^2 + 12a\Delta - a - 8\Delta\right)}{(2a-1)^2(4a-3)^2(3a-2)}$

We can note that with $z(0, a) > 0$. There exist two roots for $z(\Delta, a) = 0$, and both are negative. As the difference for the overall consumer's surplus is just given by $2z(\Delta, a)$, we have the same roots. That means the merger between distant platforms always increases the consumers' surplus.

Welfare comparison

The difference between welfare in post and pre-merger cases is given by $W^{A+C}(\Delta, a) =$

$-\frac{1}{32} \frac{-16a^5 - 1984a^4\Delta + 20a^4 + 960a^3\Delta^2 + 4944a^3\Delta - 8a^3 - 4600a^2\Delta - 2032a^2\Delta^2 + a^2 + 1888a\Delta + 1408a\Delta^2 - 288\Delta - 320\Delta^2}{(4a-3)^2(2a-1)^2}$,

where

$W^{A+C}(0, a) > 0$ if the cross-group externality is high enough, i.e. for $a > \frac{1}{4}$. Again, there exist two roots for $W^{A+C}(\Delta, a) = 0$, Δ_1^{WAC} and Δ_2^{WAC}, where $\Delta_2^{WAC} < 0$ and

$\Delta_1^{WAC} = \frac{1}{4}\frac{\left(124a^3-247a^2+164a-36+2\sqrt{(2a-1)(122a^3-239a^2+160a-36)(4a-3)^2}\right)(2a-1)}{(3a-2)(20a^2-29a+10)}$, with $\Delta_1^{WAC} \lesseqgtr 0$ if $a \gtreqless \frac{1}{4}$. So the merger between distant platforms is always welfare improving if $a > \frac{1}{4}$. When $a < \frac{1}{4}$ the merger is welfare improving if there exist high enough cost savings, $\Delta > \Delta_1^{WAC}$.