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Authenticity in mathematics lessons from a computational linguistics perspective

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1. Introduction

Authenticity has become a key concept in contemporary mathematics education research. Scholars in this field have paid attention to the authenticity of mathematical tasks posed in actual classrooms (e.g., English, 2010; Sethole, 2005; Uegatani & Koyama, 2016). It is impossible to realize perfect authenticity in classrooms (Palm, 2008); it is also questionable whether situations that are too authentic are educationally effective (Vos, 2018). Keeping in mind that Weiss, Herbst, and Chen (2009) reveal that the term authenticity has at least four meanings in the mathematics education field, we need to consider which authenticity we want to focus on and to what extent we intend to realize it in a balanced manner in our classrooms (Hattori & Uegatani, 2019; Uegatani, 2017). There are, however, only abstract and qualitative models for capturing authenticity. In order to continue constructive discussions, we need a quantitative model for capturing the degree of authenticity.

This paper aims to propose a computational linguistics perspective-based quantitative model for rectifying this issue. A computational linguistics approach is a kind of statistical approach, which analyzes natural language in a computational manner. Recently, there has been an increase in the number of studies using a computational linguistics approach in mathematics education research (Uegatani, 2018; Uegatani & Otani, in press, 2019). In this paper, we will review the existing literature in order to obtain the key to quantifying authenticity in mathematics lessons and propose an idea for connecting authenticity research with a computational linguistic approach.

2. Literature review

2.1. Types of authenticity

Weiss, Herbst, and Chen (2009), who investigated mathematics teachers' views regarding authentic mathematics, argued that there are at least four types of authenticity in mathematics education: authentic mathematics related to the real *W*orld (AM_W), to the *D*iscipline of mathematics (AM_D), to experts' *P*ractices (AM_P), and to *S*tudents' points of view (AM_S). The differences between them stem depending on what the given mathematics is similar to (Uegatani, 2017). AM_W can be assessed by examining its similarity to the mathematics found in the real world. AM_D can be assessed by examining its similarity to the mathematics found in the academic discipline of mathematics. An activity that involves performing AM_P can be assessed by examining the similarity to experts' (typically mathematicians') way of doing mathematics. However, the last type differs slightly from the others. AM_S can be assessed based on the extent to which the students construct their knowledge by themselves with the teacher's support.

In other words, we can say that AM_S is created on the basis of the students' experiences, though an opportunity for its creation is provided by the teacher or the textbook. This type of authentic mathematics is considered students' mathematics from a constructivist perspective (Weiss et al., 2009). AM_S is considered to be especially important and thus prioritized compared to the other three types of authentic mathematics because of this asymmetry (Uegatani, 2017). The other types of authenticity can be assessed primarily based on the observer's perspective, while AM_S can be assessed from a learner's perspective. Authentic mathematics establishes its true value only when at least one from among AM_W, AM_D, and AM_P is completed with AM_S together in classrooms.

2.2. Research on humor

In order to consider a practical way of realizing authentic mathematics in classrooms, some insightful research on humor in mathematics education may be useful. The existing literature points out that humor in classrooms influences students' understanding of mathematical contents (Shmakov & Hannula, 2010). For example, van Dooren, Lem, De Wortelaer, and Verschaffel (2019) have shown that a humorous illustrative interpretation of a mathematical problem-solving process's results prompts students to interpret it in more real world contexts. For example, Hattori and Uegatani (in press) illustrate that posing a mathematical problem with a vague scope of consideration in a humorous way encourages the students to expand their scope of consideration by themselves. A teacher's humorous presentation can signal to students that humorous behavior is acceptable in the classroom. A behavior that is both humorous and mathematical in their classrooms is always extra for the students. Since their teacher does not explicitly require them to behave in such a way, it is they themselves who often want to behave in such a way in many cases. Therefore, a behavior that is humorous as well as mathematical is also always intentional. Since the teacher never formally provides instructions regarding how to behave humorously, when the students want to behave humorously, they must always produce their humorous behaviors based on their own experiences. Therefore, if their behaviors are not only humorous but also mathematical, then they can be necessarily assessed as AM_S.

2.3. Research on complaint

In this paper, we introduced a recent pioneering work in moral and art education in Japan. Michita and Sakai (2019) argue that complaining provides an opportunity for critical thinking. They propose this idea based on their actual lessons regarding morality and art. Although they only point out one potential effect of complaints and provide limited empirical evidence, their idea is highly compatible with the abovementioned implications drawn from research on humor. When the students want to complain, they must always produce their complaints based on their own experiences. Therefore, this idea is suggestive for us; that is, if their complaints are related to mathematics, then they can be necessarily assessed as AM_S.

3. Connecting with a Computational linguistics approach

A computational linguistics approach is a type of text mining approach. Softwares using this approach enable us to automatically calculate the statistics of a given textual data in a natural language. For example, what terms appear in a given data, how many times each term appears in the data, and to what degree those terms occur together in the data. KH Coder is a software for conducting computational linguistics analysis in multiple languages including English and Japanese: (http://khcoder.net/en/, see also Higuchi, 2014). Co-occurrence networks analysis, which indicates to what degree and what terms tend to occur in a given text data, is especially powerful. We can identify the mutual relationships between the concepts used—the relationships that are apt to be overlooked only from a qualitative perspective. Thus, aided by computation through software, we can also deal with large-scale text data. A computational linguistics approach offers one more advantage: the analysis of text data in a reproducible manner. The approach strongly functions, if it is difficult for us to deal with a given research topic in a reproducible way when using different approaches. In this section, we consider authentic mathematics using a computational linguistics perspective.

Especially with regard to AM_S, we have an interest in examining the extent to which the students construct their knowledge by themselves. Since knowledge construction unavoidably requires some inference-making, we need to consider the extent to which the students' inferences were based on their own experiences. If they only follow a given normative inference rule, it can be concluded that their inferences are based not on their own experiences but on the basis of the given rule. On the other hand, let us suppose that they are partially following a given inference rule. In such a case, we can say that their inferences are based on their own experiences *to some degree*. In other words, if a consequence of a student's inference is not explainable by using only mathematically or grammatically normative rules, it can be concluded that her inference depends on her experience to some degree. As the literature review provided in the previous section suggests, we focus on extra words created by students. Therefore, if we can quantify a degree of unexplainability with regard to such normative rules, then the degree can be considered as an indicator of authenticity from the student's point of view.

Let us explain this idea using an illustrative example. The following vignette is from the introduction of a seventh-grade lesson[1), 2)]. The topic of the lesson is movement in plane geometry: translation, rotation, and reflection.

1. T: Today, let's enjoy the topic of movement. Can you move?
2. S1: Yes. On foot.
3. T: Oh, aha. You can move on foot. Yes. Well, I will change my question slightly. Can you move something?
4. S2: Because we have our hands, we can!
5. S3: Move what?
6. T: Move what…. Well, a triangle.
7. S2: We have one chance!
8. T: One chance!
9. Ss: (laugh)[3)]
10. S3: It depends on the weight of the object.
11. S1: Not a triangle in everyday life but a triangle in geometry?[4)]
12. T: Ah, not a triangle in everyday life but a triangle in geometry.
13. S1: Aha.
14. S2: One chance. One chance….
15. S4: If in space geometry…?
16. S5: On paper?
17. T: On paper?
18. S1: We can behave as if we have moved the object (on paper).
19. T: As if we have moved it? So, can't we really move it?

20. S2: No, it is impossible!

21. S1: It is….

22. S2: But, but, but… we can, if we draw it!

This vignette illustrates authenticity for the students. For example, in line 4, S2 uses the word "hands." This word choice reflects his interpretation of the teacher's question. Because the teacher's question in line 3 is a yes/no question, it is grammatically sufficient for S2 to answer with a "yes" or a "no." However, he adds the extra word "hands." This addition is impossible without his experience of moving something. Thus, we can judge that he makes his interpretation, at least partially, on the basis of his own experience. The same holds in the case of S3's utterance in line 10, in the case of S4's utterance in line 15, or in the case of S5's utterance in line 16.

Simply speaking, if a word is used by a student for the first time in the vignette, then we can consider the word to be authentic. Of course, this is a sufficient condition for establishing authenticity. For example, let us suppose that a student answers only with a "yes" when the teacher asks her if she can move something. Then, her answer of "yes" may stem from her own experience. However, if she has a piece of grammatical knowledge on how to answer a yes/no question, it is also possible that she just answers "yes" without any serious thought. For this reason, it is difficult to assess the authenticity of a given response without knowing which word has been used for the first time . In this way, authenticity can be quantified. We can determine a student's degree of authenticity by answering the question regarding how many new words she produces in the vignette. However, note that this is just a quantification of authenticity and not that of authentic mathematics. A student's authentic behavior is not always mathematical. In order to assess how mathematical such a behavior can be, let us consider mathematics as a social construction. If a student's authentic behavior is mathematical, it must be socially acknowledged. From this point of view, we can determine a student's degree of authentic mathematics by answering the question regarding how many already established mathematical terms her own authentic word is used with. Co-occurrence networks analysis based on a computational linguistics approach will enable us to investigate the student's degree of authentic mathematics.

4. Conclusion

For the purpose of using a computational linguistics perspective to propose a quantitative model, we reviewed the existing literature and discussed the connection between authenticity and the computational aspects of students' utterances recorded during mathematics lessons. As a result, the existing literature suggests that one clue regarding the quantification of authenticity can be found in the extra words produced by students. Based on this finding, we proposed that a word appearing for the first time in the vignette could be considered as an indicator of authenticity. Counting such words created by a student and investigating how many already established mathematical terms are used along with such words indicates the degree of authentic mathematics used by the student. However, this paper's proposal remains just a potential idea. We need to provide sample analysis with regard to authentic mathematics.

Notes

1) This lesson was conducted at a junior high school attached to a national university in Japan in 2019. The teacher is the first author of this paper. He usually teaches mathematics to the students in the class throughout the year.
2) All utterances were spoken originally in Japanese. This paper's authors have translated them into English with

the help of an English editing advisor.

3) “One chance” is a translation of a teen slang term in Japanese. The students laugh because S2 uses such a slang word, albeit in a mathematics lesson.

4) In Japanese, “Sankaku”(a triangle in everyday life) and “Sankakkei”(a triangle in geometry) have slightly different connotations. The difference is implicit for many Japanese people, and Japanese mathematics textbooks use the two terms in a manner that is conscious of this difference.

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Summary in Japanese

計量言語学の視座から見た数学の授業の真正性

数学の授業で真正な数学を実現するにあたって，真正な数学の質的なモデルが提案されている．しかしながら，量的なモデルを提案した研究は管見の限り見当たらない．本稿では，真正な数学の 1 種である，生徒から見た数学の真正性に関する量的なモデルを構築するためのアイディアを，計量言語学の視座から提案する．本稿では，先行するユーモアや愚痴に関する研究から得られる示唆を手がかりとして，生徒達にとっての真正性が，その文脈において初めて使用される単語を計量することによって量化される可能性を示した．本稿では，具体的な授業中の発言に基づきながらこのアイディアの可能性を示したが，その一方で，実際に量的モデルを構築するには至っていない．この点について，量的モデルを定式化し，実際の授業分析に応用する作業は，今後の課題である．