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Economic Model Predictive Control and Process Equipment: Control-Induced Thermal Stress in a Pipe

Helen Durand

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***Abstract*— Recent work on economic model predictive control (EMPC) has indicated that some processes may be operated in a more economically-optimal fashion under a time-varying operating policy than under a steady-state operating policy. However, a concern for time-varying operation is how such a change in operating policy might impact the equipment within which the processes being controlled are carried out. While under steady-state operation, the operating conditions to which equipment would regularly be exposed can be estimated, this would be more difficult to assess thoroughly *a priori* under time-varying operation. It could be explored whether the EMPC could be made aware of any impacts the control actions that it chooses might have on equipment, and then to seek to impose constraints on these impacts. This would require explicit consideration of equipment design, material properties/behavior, and material loading at the EMPC design stage. This work provides an initial exploration of this topic by seeking to extract principles related to the integration of equipment material fidelity considerations and EMPC through an example accounting for a simple preliminary case of thermal stresses in a pipe at equilibrium conditions.**

I. INTRODUCTION

EMPC [1], [2], [3] is an optimization-based control design that selects control actions for a process that are economically-optimal, subject to constraints, with respect to a profit metric over a prediction horizon. In many works on EMPC, this profit metric is based on factors such as production rate or the cost of using an input. However, there is a potential that the use of EMPC could increase capital costs, just as time-varying operation for power plants has been expected to lead to increased capital and maintenance costs [4]. An important question with regard to EMPC, therefore, is whether, when the potential for increased capital and maintenance costs are accounted for in the analysis of the total profit, EMPC remains economically attractive. Integration of statistics, optimization, and/or optimization-based control [5] with equipment concerns or maintenance scheduling has been addressed [8], [9], [10] (e.g., for actuators [6] and transformers [7]).

To account for equipment degradation caused by EMPC, one could attempt to model both the process and material/equipment behavior [11] to allow the material degradation to be predicted by the EMPC as a function of the operating conditions it sets up. The goal would then be to develop appropriate constraints for the EMPC that force it to compute control actions which would prevent material failure but are economically-optimal with respect to traditional profit metrics based on instantaneous operating costs/profit subject to such a constraint. A first step in achieving this goal is to clarify new considerations that may arise when seeking to create explicit materials-based constraints for EMPC. This work provides preliminary results in this direction by examining a simplified example regarding equilibrium thermal stresses in a pipe due to changes in the temperature of the flow through the pipe resulting from control of an upstream process by EMPC.

II. PRELIMINARIES

A. Notation

The Euclidean norm of a vector is denoted by $|\cdot|$. A class $\mathcal{K}$ function $\alpha : [0, a) \to [0, \infty)$ is strictly increasing with $\alpha(0) = 0$. The transpose of a vector x is denoted by x^T. Set subtraction is signified by " / " such that $x \in A/B := \{x \in R^n : x \in A, x \notin B\}$. A level set of a positive definite function V is denoted by $\Omega_\rho := \{x \in R^n : V(x) \leq \rho\}$.

B. Class of Systems

We consider systems of the following form:

$$\dot{x}(t) = f(x(t), u(t), w(t)) \tag{1}$$

where f is a locally Lipschitz nonlinear vector function, $x \in R^n$ is the state vector, $u \in U \subset R^m$ is the vector of manipulated inputs, and $w \in W \subset R^l$ is the vector of bounded disturbances ($W := \{w \in R^l : |w| \leq \theta\}$). We consider that $f(0, 0, 0) = 0$ and that the system is stabilizable in the sense that there exists a sufficiently smooth positive definite Lyapunov function $V : R^n \to R_+$, functions $\alpha_j(\cdot)$, $j = 1, \ldots, 4$, of class $\mathcal{K}$, and a controller $h_1(x)$ that can asymptotically stabilize the origin of the closed-loop system of Eq. 1 in the absence of disturbances such that:

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \tag{2}$$

$$\frac{\partial V(x)}{\partial x} f(x, h_1(x), 0) \leq -\alpha_3(|x|) \tag{3}$$

$$\left|\frac{\partial V(x)}{\partial x}\right| \leq \alpha_4(|x|) \tag{4}$$

$$h_1(x) \in U \tag{5}$$

$\forall\ x \in D \subset R^n$ (D is an open neighborhood of the origin). We call a level set $\Omega_\rho \subset D \cap X$ of V the stability region.

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C. Economic Model Predictive Control

EMPC is described by the optimization problem:

$$\min_{u(t)\in S(\Delta)} \int_{t_k}^{t_{k+N}} L_e(\tilde{x}(\tau),u(\tau))\,d\tau \tag{6a}$$

$$\text{s.t.}\quad \dot{\tilde{x}}(t)=f(\tilde{x}(t),u(t),0) \tag{6b}$$

$$\tilde{x}(t_k)=x(t_k) \tag{6c}$$

$$\tilde{x}(t)\in X,\ \forall\, t\in[t_k,t_{k+N}) \tag{6d}$$

$$u(t)\in U,\ \forall\, t\in[t_k,t_{k+N}) \tag{6e}$$

where $u(t)$ is a piecewise-constant input vector trajectory with N pieces (N is the prediction horizon), where each piece is held for a period Δ (i.e., $u(t) \in S(\Delta)$). The economics-based stage cost L_e of Eq. 6 is evaluated throughout the prediction horizon using predictions $\tilde{x}$ of the process state from the model of Eq. 6b (i.e., the model of Eq. 1 without disturbances) initialized from the state measurement at t_k (i.e., Eq. 6c). The constraints of Eqs. 6d-6e are state and input constraints, respectively. The first of the N pieces of the input vector trajectory that is the optimal solution to the optimization problem is applied to the process. The optimal solution at t_k is denoted by $u^*(t_i|t_k)$, where $i=k,\dots,k+N$ (reflecting sample-and-hold input implementation).

In this work, we will also add two Lyapunov-based stability constraints to Eq. 6 as follows [12]:

$$V(\tilde{x}(t))\le\rho_e,\ \ \forall\, t\in[t_k,t_{k+N}),\quad \text{if } x(t_k)\ \in\ \Omega_{\rho_e} \tag{7a}$$

$$\frac{\partial V(x(t_k))}{\partial x}f(x(t_k),u(t_k),0) \le \frac{\partial V(x(t_k))}{\partial x}\,f(x(t_k),h_1(x(t_k)),0)\quad \text{if }\ x(t_k)\in\Omega_\rho/\Omega_{\rho_e} \tag{7b}$$

where $\Omega_{\rho_e}\subset\Omega_\rho$ is a subset of the stability region that makes Ω_ρ forward invariant under the controller of Eqs. 6-7.

III. Accounting for EMPC Impacts on Equipment

Traditional formulations of EMPC predict only the future states of the process. The impacts of the process on the materials, however (for example, impacts of changes in pressure or temperature in the fluid flow on stresses or strains in the equipment), are not typically taken into account when examining the use of EMPC for a chemical process, though these impacts also can be modeled dynamically (e.g., stresses and strains can vary in both time and space in a material). Following recent work on including valve dynamics in EMPC [13], it would be desirable to consider how accounting for the behavior of materials in response to the operating policies set up by an EMPC might affect the solutions for the EMPC. In the following sections, we utilize a simple pipe flow example to explore some of the differences between traditional EMPC design thinking and EMPC accounting explicitly for material behavior.

A. Illustrative Example: Accounting for Equipment Behavior in EMPC

In this section, we describe the illustrative example that will be utilized in subsequent sections to clarify some of the new considerations that must be addressed by an EMPC that explicitly accounts for equipment behavior. Specifically, we consider a continuous stirred tank reactor (CSTR) which has been examined in a number of works (e.g., [12], [14]) but for which we explicitly consider a piping element which follows closely after the reactor such that we consider that the fluid temperature is the same at the entrance to this pipe as it was in the outflow of the CSTR. This pipe is considered to be rigidly fixed at one end with a bellows joint on the other and is insulated. The pipe is assumed to have negligible impact on the mixing in the tank, and is taken to have an inner radius of 0.05115 m, an outer radius of 0.05715 m, and a length L of 2.54 m. Furthermore, it is considered to be made of an alloy with an ultimate strength of 400 MPa, a yield strength of 270 MPa, a Young's Modulus of 200 GPa, and a thermal expansion coefficient of 12.5×10^{-6} K^{-1} [15]. Within the CSTR, the exothermic second-order reaction $A \to B$ occurs. The manipulated inputs for the CSTR are the concentration C_{A0} of the reactant in the feed and the heat rate Q which can be added or removed by a heating/cooling jacket. The dynamics of the process are as follows, with process parameters from [14]:

$$\dot{C}_A=\frac{F}{V}(C_{A0}-C_A)-k_0e^{-\frac{E}{R_gT}}C_A^2 \tag{8}$$

$$\dot{T}=\frac{F}{V}(T_0-T)-\frac{\Delta Hk_0}{\rho_LC_p}e^{-\frac{E}{R_gT}}C_A^2+\frac{Q}{\rho_LC_pV} \tag{9}$$

where C_A and T represent the reactant concentration and temperature in the reactor, R_g is the ideal gas constant, E is the reaction activation energy, ΔH is the enthalpy of reaction, and k_0 is the pre-exponential constant. The inlet/outlet volumetric flow rate F is considered fixed, as are the liquid density ρ_L, heat capacity C_p, and liquid volume V. Vectors of deviation variables for the states C_A and T and inputs C_{A0} and Q from their steady-state values $C_{As}=1.22$ kmol/m^3, $T_s=438.2$ K, $C_{A0s}=4$ kmol/m^3, and $Q_s=0$ kJ/h, respectively, are $x=[x_1\ x_2]^T=[C_A-C_{As}\ T-T_s]^T$ and $u=[u_1\ u_2]^T=[C_{A0}-C_{A0s}\ Q-Q_s]^T$. This steady-state was not selected based on economic optimality.

The temperature of the fluid through the outlet pipe changes due to changes in the inputs C_{A0} and Q over time as computed by the EMPC, which adjusts C_{A0} and Q in a manner that seeks to optimize the production rate of B as follows:

$$L_e=-k_0e^{-\frac{E}{R_gT(\tau)}}C_A(\tau)^2 \tag{10}$$

Because the volumetric flow rate F is constant, optimizing the time integral of the production rate (in kmol/m^3 h) of the desired product is related to optimizing the mol or mass of product produced per unit time. Input constraints are also enforced, requiring that $0.5\le C_{A0}\le 7.5$ kmol/m^3 and $-5\times10^5\le Q\le 5\times10^5$ kJ/h.

Furthermore, Lyapunov-based stability constraints of the form in Eq. 7 are imposed (the constraint of Eq. 7a was enforced at the end of each sampling period when $x(t_k) \in \Omega_{\rho_e}$ and also at the end of every sampling period after the first when $x(t_k) \in \Omega_\rho/\Omega_{\rho_e}$ to constrain the state after the first sampling period). These Lyapunov-based stability constraints are developed using $V = x^T P x$, where $P = [1200\ 5; 5\ 0.1]$. The Lyapunov-based control law $h_1(x)$ was developed such that its first component $h_{1,1}(x)$ was fixed at 0 kmol/m^3 for simplicity, whereas its second component $h_{1,2}(x)$ was computed via Sontag's control law [16] as follows:

$$h_{1,2}(x) = \begin{cases} -\frac{L_{\tilde{f}}V + \sqrt{L_{\tilde{f}}V^2 + L_{\tilde{g}_2}V^4}}{L_{\tilde{g}_2}V}, & \text{if } L_{\tilde{g}_2}V \neq 0 \\ 0, & \text{if } L_{\tilde{g}_2}V = 0 \end{cases} \tag{11}$$

but with the value of $h_{1,2}(x)$ from Eq. 11 saturated at its bounds if these bounds were reached. $\tilde{f}$ in Eq. 11 represents the vector-valued function that is not related to the inputs in the deviation variable form of Eqs. 8-9, and $\tilde{g}$ represents the matrix-valued function that multiplies the input vector in the deviation variable form of the process model equations ($\tilde{g}_2$ represents its second column). $L_{\tilde{f}}V$ and $L_{\tilde{g}_2}V$ represent the Lie derivatives of V with respect to $\tilde{f}$ and $\tilde{g}_2$. Using a discretization of the state-space between $C_A = 0$ kmol/m^3 and 4 kmol/m^3 and between $T = 340$ K and 560 K, $\rho = 300$ was selected so that the value of T was able to become significantly larger than the steady-state value within the allowable operating region Ω_ρ (for the purpose of the thermal strain analyses to be presented below). ρ_e was arbitrarily set to 75% of ρ. The process state was initialized from $x_{init} = [-0.4$ kmol/m^3 $\ 8$ K$]^T$, N was set to 10, and Δ was set to 0.01 h. An integration step of 10^{-4} h was utilized to numerically integrate Eqs. 8-9. The simulations were performed for one hour of operation using MATLAB (MathWorks®) and fmincon. In the optimization problem, the value of u_2 was scaled down by 10^5 to account for the magnitude of this term, and the initial guess for the decision variables was the steady-state values of the inputs at each t_k.

The trajectories of the states throughout the one hour of operation are depicted in Fig. 1, where the temperature of the stream leaving the CSTR increases to approximately 490.2 K and remains there thereafter. This indicates that the temperature of the insulated pipe downstream of the CSTR should eventually reach 490.2 K as well after sufficient time passes, if the EMPC was to continue to operate the process at 490.2 K. We will explore the thermal stresses in the pipe when it reaches this new temperature and compare them with the thermal stresses for steady-state operation (i.e., $T = 438.2$ K).

We consider that the stresses in the pipe are only in the axial direction and that they result from the constraints on the ends of the pipe. Furthermore, we assume that we will operate the process in a manner where the stress remains less than the elastic limit. The strain ϵ (i.e., material deformation, or the fractional change in length of a material compared to its original length) is typically considered to be induced by

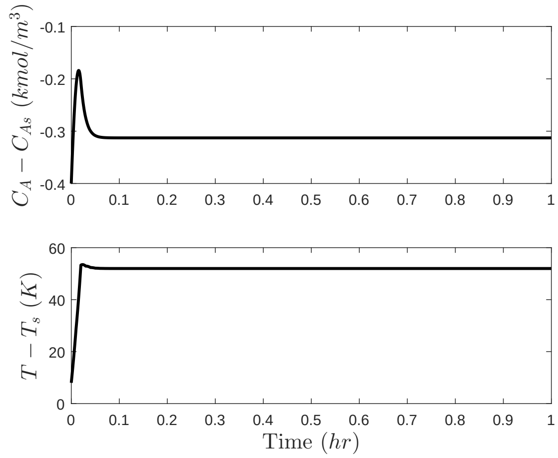

Fig. 1. States over one hour of operation for the process of Eqs. 8-9 under the EMPC that does not account for limitations on u_1.

both temperature T (i.e., thermal expansion) and stress σ (i.e., the force on a solid per unit area). Assuming that the material has a proportional relationship between differential changes in stress and strain below the elastic limit equal to the Young's modulus E (i.e., $E = \left(\frac{\partial \sigma}{\partial \epsilon}\right)_T$), that E and the thermal expansion coefficient for linear thermal expansion in the axial direction α can be approximated as constant in the range of operating conditions considered, and that the zero-strain condition is at $T = 293.15$ K, the following equation relates stress and strain in the pipe after the temperature of the pipe has reached T_f uniformly throughout [15]:

$$\sigma = E\epsilon - \alpha E(T_f - 293.15) \tag{12}$$

where all terms are in SI units (e.g., T_f is in K). This is an equilibrium relationship and therefore this does not capture the time-dependence of stress/strain as the pipe is heated; however, by examining the equilibrium situation as a first case, we can gain a number of insights for how EMPC for a process should be considered in light of equipment material limitations.

We first explore how the EMPC's control action decisions, when implemented without the EMPC being aware of the equipment material limitations, might impact the stress/strain in the pipe. To analyze this, we utilize the techniques for computing equilibrium thermal compressive stress in a pipe for various pipe equipment constructions from [15]. When the pipe is at 293.15 K (i.e., it has not experienced thermal strain), the force from the stress on the bar is equal to the force on the pipe from the bellows joint (modeled like a spring force with spring constant k_s), with the result that the stress in the bar is given by [15]:

$$\sigma = \frac{-\alpha E(T_f - 293.15)}{1 + \frac{AE}{k_s L}} \tag{13}$$

where A is the area of the pipe in contact with the wall/bellows joint ($A = 0.002041$ m^2) and a negative stress indicates a compressive stress. We now explore the implications of the EMPC's behavior given different designs for the bellows joint (i.e., different values of k_s). We first will consider an extreme case in which we see what would happen if the value of k_s was set assuming a steady-state operating policy where it was considered that the maximum temperature that might be reached in the pipe is 10 K above the steady-state value (i.e., 448.2 K). In that case, the maximum value of k_s that would be needed to ensure that the stress was no greater than a design value of 10^8 Pa would be 5.58×10^7 N/m (from Eq. 13). Therefore, if $k_s = 5.50 \times 10^7$ N/m, then even when the temperature in the pipe reaches 448.2 K, the stress magnitude is still less than 10^8 Pa. However, if this bellows joint is used and the steady-state-enforcing controller is replaced with an EMPC that operates the process as shown in Fig. 1, the stress in the pipe when the temperature reaches 490.2 K becomes 1.26×10^8 Pa (above the desired threshold). Alternatively, if the spring constant for the bellows is much lower (e.g., $k_s = 4.4 \times 10^5$ N/m, which [15] cites as a more typical spring constant for a bellows), then even the higher temperatures reached under operation with the EMPC are unlikely to cause the stress to reach a high level (in this example, the equilibrium stress magnitude when $k_s = 4.4 \times 10^5$ N/m and $T_f = 490.2$ K would be 1.34×10^6 Pa, which is significantly lower than the 10^8 Pa threshold).

Returning to the extreme case with $k_s = 5.50 \times 10^7$ N/m for the sake of illustration, we can consider several alternatives for preventing the thermal stress from exceeding the design value at any condition where the temperature remains constant for an extended period of time. One method would be to add a constraint on the equilibrium stress to the EMPC. Because the stress is given by Eq. 13, where all terms on the right-hand side are fixed except the value of T_f, a constraint on the stress (e.g., $\sigma > -10^8$ Pa, $\forall t \in [t_k, t_{k+N})$) could be expressed as a constraint on the temperature at an equilibrium condition (i.e., $T < 450$ K). The results with a constraint requiring $T < 450$ K are shown in Fig. 2 (this constraint is enforced numerically at every integration step, and the constraint requiring $V(\tilde{x}) \leq \rho_e$ is also enforced at the end of every integration step both when Eq. 7a is used and when Eq. 7b is used). The oscillatory behavior in T toward the beginning of the time of operation is selected by the EMPC because it generates a higher profit than would be obtained in that time period by, for example, values of C_{A0} and Q fixed at the values which they take for the majority of the time of operation in Fig. 3.

Another means for seeking to keep the stress magnitude sufficiently low with the stiff bellows joint would be to find a stability region in which the value of T does not exceed 450 K and to then modify the EMPC to include Lyapunov-based stability constraints that are compatible with such an approach. The time-averaged value of the stage cost of Eq. 10 over the hour of operation is 13.88 for operation at the steady-state, 32.85 for the EMPC in Fig. 1, and 29.40 for the EMPC in Fig. 2. It is noted that the solutions obtained are only guaranteed to be local minima; no attempt was made to determine the global optimum solution. The specific trajectories obtained in each case are thus reflective of the most optimal solutions that fmincon found numerically for the given optimization problems.

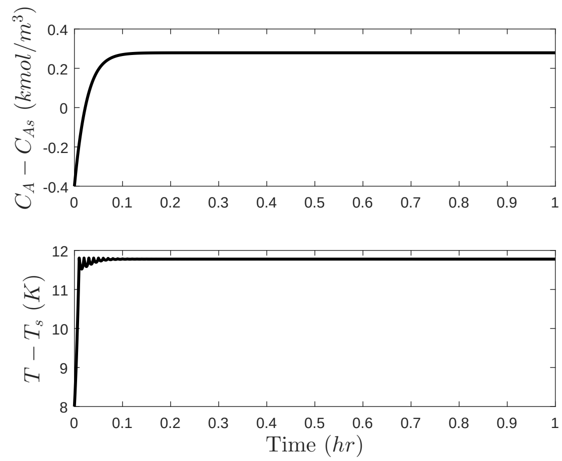

Fig. 2. States over one hour of operation for the process of Eqs. 8-9 under the EMPC with a hard constraint on T.

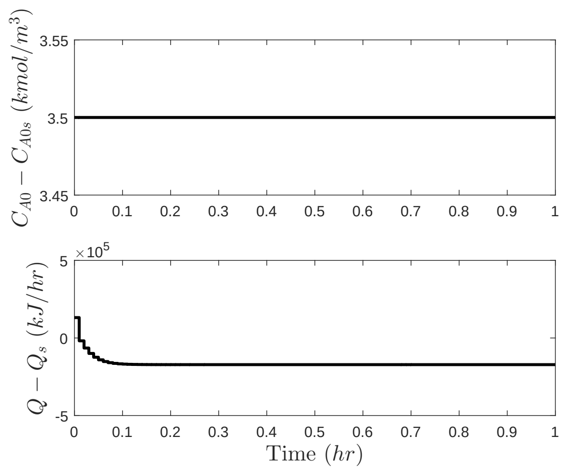

Fig. 3. Inputs over one hour of operation for the process of Eqs. 8-9 under the EMPC with a hard constraint on T.

This analysis, though for a heavily simplified situation and with a spring constant that is likely much stiffer than those which would typically be installed, nevertheless indicates a number of significant conclusions about potential practical implications of the use of EMPC for a chemical process: 1) The details of equipment design (in this example, factors such as the pipe length, whether there is a bellows joint and what its spring constant is, and whether the pipe is rigidly fixed on any side) for units in a process under EMPC may play a significant role in the analysis of whether the use of EMPC for such a process in place of a controller that enforces steady-state operation may be considered without some level of control/equipment co-design, such as incorporating the equipment limitations in the EMPC by modifying the constraints. The example indicates that though typical values of k_s for a bellows joint would cause the piping to have no issues with the added thermal stress due to the EMPC-induced temperature change in the pipe compared to the expected thermal stress at steady-state conditions, a specialty equipment design could be negatively impacted by the added stress. 2) Equipment selection and design may need to be performed in light of control. For example, consider the results above, where when the temperature is restricted to prevent thermal stress, the profit is less (at least for the local minimum found by the optimizer) than it would be if the closed-loop state was free to take any value within the stability region Ω_ρ when $\rho = 300$ (i.e., where in this Ω_ρ, the upper limit on the temperature is higher than 450 K). To be able to utilize a larger Ω_ρ without allowing the stress magnitude to exceed the design stress, a less stiff bellows joint could be utilized as discussed above. This work therefore indicates that a future direction in the EMPC literature may need to be not only how to account for the impacts of the operating conditions of an EMPC on equipment life, but also how to co-design controllers and equipment so that an optimal mechanical design, control design, and materials selection is performed, recognizing that there may be greater profits which can be made long-term for certain processes by designing equipment to require less stringent constraints in the controller. 3) The decisions made by an EMPC upstream of a process will impact downstream processes, and this may result in constraints in EMPC intended to account for how the upstream process impacts the downstream process, including units such as pipes which might otherwise not be considered in detail. 4) The need to consider equipment conditions in EMPC requires that modifications to equipment during routine work such as maintenance may require updates to the control design.

In the EMPC design for Fig. 1, the EMPC essentially drove the closed-loop state to a more profitable steady-state that was not the operating steady-state. We can also explore a case where the connection to the steady-state with $u_1 = 0$ kmol/m^3 is more explicit. In this case, we consider the same process as above except that we consider that we would like the time-averaged amount of reactant fed to the reactor in every hour of operation to be as close as possible to that which would be fed at steady-state, meaning that we would like the following equation to be satisfied as closely as possible:

$$\frac{1}{1 \text{ h}} \int_{t=0 \text{ h}}^{t=1 \text{ h}} u_1(\tau) d\tau = 0 \text{ kmol/m}^3 \tag{14}$$

This constraint prevents the EMPC from driving the closed-loop state to a new operating condition and thereafter maintaining it at such a condition. For this simulation, because the constraint of Eq. 14 is not guaranteed to be satisfied, it was implemented with slack variables to ensure feasibility of the optimization problem at every sampling time, which resulted in the time-averaged value of u_1 being equal to 0.4007 kmol/m^3 (this is much closer to the desired value of 0 kmol/m^3 than what would be achieved in the case in Fig. 1 without the constraint on the input, where the time-averaged value of u_1 reaches 3.5 kmol/m^3). Specifically, the constraint was implemented in the manner described in [1]

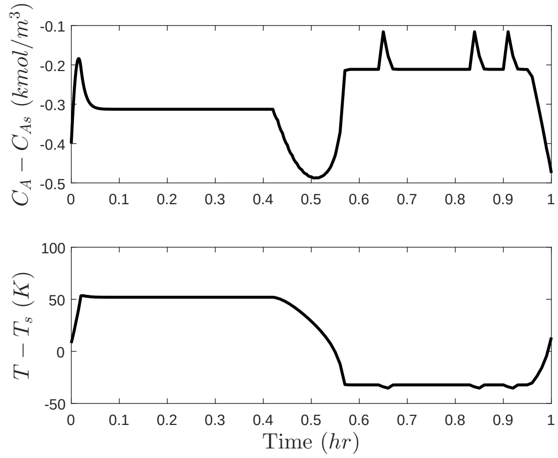

Fig. 4. States over one hour of operation for the process of Eqs. 8-9 under EMPC with the slack variables in the constraints of Eqs. 15-16.

but with slack variables s_1 and s_2 as follows:

$$\begin{aligned} s_1 \geq & \sum_{i=0}^{k-1} (u_1^*(t_i|t_i)) + \sum_{i=k}^{k+N_k} (u_1(t_i|t_k)) \\ & - 3.5\delta(100 - \frac{t_k}{\Delta} - N) \end{aligned} \tag{15}$$

$$\begin{aligned} s_2 \geq & - \sum_{i=0}^{k-1} (u_1^*(t_i|t_i)) - \sum_{i=k}^{k+N_k} (u_1(t_i|t_k)) \\ & - 3.5\delta(100 - \frac{t_k}{\Delta} - N) \end{aligned} \tag{16}$$

where $N_k = N$ and $\delta = 1$ for $t_k < 0.9$ h, and $\delta = 0$ and N_k is set to the number of sampling periods remaining in the operating period of 1 h when $t_k \geq 0.9$ h. Fig. 4 shows the states in this case. The guess of the slack variables provided to the optimization solver was 0 at each sampling time, and they were effectively unbounded in the optimization problem (the upper and lower bounds of s_1 and s_2 were 2×10^{19} and -2 $\times$ 10^{19}, respectively). The objective function minimized in this case with the slack variables was as follows:

$$\int_{t_k}^{t_{k+N}} \left[-k_0 e^{-\frac{E}{R_g T(\tau)}} C_A(\tau)^2 \right] d\tau + 100(s_1^2 + s_2^2) \tag{17}$$

Figs. 4-5 show the state and input trajectories under the EMPC accounting for the added constraints in Eqs. 15-16 and the modified objective function of Eq. 17. In Fig. 4, the value of T increases again to around 490 K and remains there for some time. This indicates that there may be EMPC cases where, depending on the heat transfer coefficient and thermal conductivity, short-term equilibrium conditions might be estimated for some processes under EMPC where considerations like equilibrium thermal stress could be relevant, even if the process state is not fixed at that condition permanently. Fig. 4 also indicates, however, that modeling and failure scenarios which account for how time-varying behavior impacts materials will also be important.

Remark 1: Accounting for materials in control will require adequate modeling of the impacts of operating policies

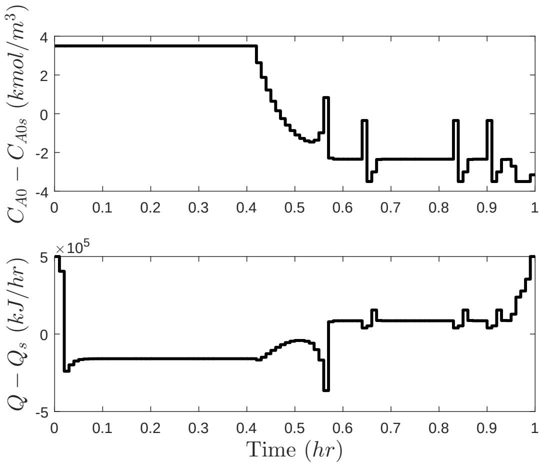

Fig. 5. Inputs over one hour of operation for the process of Eqs. 8-9 under EMPC with the slack variables in the constraints of Eqs. 15-16.

determined by controllers on equipment, which will require more work to be performed in the direction of EMPC for systems described by partial differential equations. An important approximation of the results in the prior section was that an equilibrium thermal stress analysis could be utilized to predict the thermal stress in the pipe. However, when the temperature changes in the fluid, there is some time period before the temperature in the pipe reaches this new temperature uniformly (i.e., until thermal equilibrium is reached) that depends on the heat transfer coefficient between the flow and the pipe and also on the thermal conductivity of the pipe material. The transient between when the temperature of the flow changes and the temperature of the pipe reaches the temperature of the flow would need to be modeled with partial differential equations even in the simple case of radial variation uniformly, despite that the chemical process (i.e., the model of Eqs. 8-9) is approximated by ordinary differential equations. Furthermore, as there is some lag time due to the heat transfer resistance at the fluid-solid interface and the time that it takes for conduction to occur, frequent changes in the flow temperature would require the conduction equations to be utilized in determining the impact of the flow temperature changes on the temperature of the solid pipe wall.

Remark 2: It is not required that Lyapunov-based stability constraints be utilized in an EMPC [2]. A benefit of the use of such constraints when examining equipment considerations, however, is that they provide an explicitly characterizable (*a priori*) region in state-space within which the process state will remain under sufficient conditions. When the constraints related to materials can be expressed as process state constraints as in this example, it is beneficial to be able to ensure, *a priori*, that under the sufficient conditions, the closed-loop state of the process will never reach values that could compromise the equipment (at least for the failure mechanisms being considered; there are many which are not considered in this work), as that would pose a safety hazard that must be avoided under EMPC.

IV. CONCLUSIONS

This work developed a preliminary analysis of potential implications of EMPC for process equipment material fidelity by exploring how the changes that an EMPC controlling the production rate of a product in a reactor may make in the temperature of the outflow of that reactor compared to the temperature that would be expected under steady-state operation might impact thermal stress in downstream equipment.

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