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The formulation of the foot-ground contact model influences the computation time of biomechanical optimal control problems

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Introduction

Musculoskeletal simulations can be useful to have a better understanding of the forces involved in human movements and their interactions with the environment. Recent advancements in the integration of algorithmic differentiation tools in powerful dynamic engines allow the simulations, based on the resolution of an optimal control problem, to be much faster than using traditional methods to compute derivatives, such as finite differences.
The formulation of the optimal control problem could also have an impact on the convergence of the optimization (regarding number of iterations and computational time), especially when some variables are very sensitive to small changes of design variables. In this study we present how the use of algorithmic differentiation [1] can help to analyse the influence of slightly different formulations on the convergence of an optimization framework [2] to calibrate contact models (foot-ground and subject exoskeleton) and then predict movement with their forces.

Material and methods

The problem is based on two phases. The details of the problem and the formulation have been described in [2]. In Phase A, we calibrate the foot-ground and subject-exoskeleton contact models so that they track experimental data (kinematics, ground reaction forces – GRF –, joint torques and subject-exoskeleton contact forces) using three trials at the same time. In Phase B, we estimate the kinematics as well as the subject-exoskeleton contact forces, while tracking experimental GRF and joint torques. In each phase, an optimal control problem is solved using a direct-collocation method in order to find the optimal states and controls, as well as the contact model parameters in Phase A.
In Phase A, we optimize the stiffness and damping parameters of the foot-ground contact model, the position of the spheres with respect to the foot, and the stiffness parameters of the spring-damper systems between subject and exoskeleton (*p*). In both phases, the state vector (*x*) contains the coordinates, velocities and accelerations of a multibody planar model consisting of the human and the exoskeleton. Human and exoskeleton kinematic chains are connected at the foot. The model has nine degrees of freedom (DoF): two translations and one rotation between the foot and the ground, and one DoF at the ankle, knee and hip joints of the human and exoskeleton. The subject-exoskeleton contact is modeled with three spring-damper systems at the pelvis, thigh and shank segments, and the foot-ground contact between two spheres and the ground plane with a smooth Hunt-Crossley contact model (Figure 1).
An implicit dynamic formulation is used, therefore, the control vector (*u*) contains joint torques (u_T) and jerks (u_J). Subject-exoskeleton contact forces are also added as controls (u_{SEF}). GRF are very sensitive to the state values, for this reason these variables could be treated as controls (u_{GRF}), as in [2,3]. In order to test the influence of the different treatment of GRF, two formulations, with (Form 1) and without GRF (Form 2) as controls were tested.

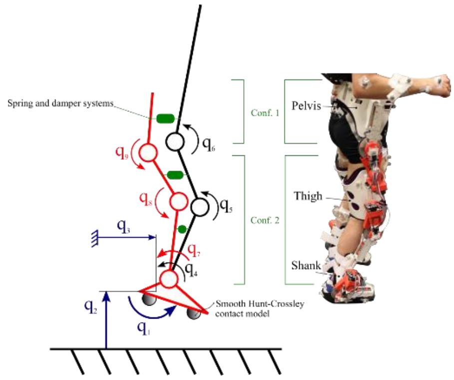

Figure 1. Left: Subject-exoskeleton model. q_1, q_2 and q_3: DoFs of the foot with respect to ground; q_4, q_5 and q_6: relative DoFs of the human; q_7, q_8 and q_9: relative DoFs of the exoskeleton. Conf. 1 and 2 indicate the locations where we had experimental contact forces.

The objective function of the optimization has terms to track experimental data: GRF, joint torques, and, only in Phase A, kinematics and subject-exoskeleton contact forces. It also has terms to minimize variables such as jerk and contact energy.
Due to the implicit dynamic formulation, equations of motion calculated using OpenSim [2,4] are set as path constraints (algebraic constraints):

$$f\left(x,u,p\right)=0 \qquad (1)$$

and dynamic constraints are set to impose continuous derivatives of the states.
In Form 1 GRF are considered control variables $\left(GRF_{u_{GRF}}\right)$ in both the objective function (GRF tracking terms) and in the constraint vector (path constraints, Eq (1)); whereas in Form 2, GRF are calculated using the contact model (as a function of states and contact parameters). In addition, path constraints are added in

Form 1 so that the model GRF (GRF_{model}) matches the GRF calculated using controls ($GRF_{u_{GRF}}$):

$$GRF_{model}(q, \dot{q}, p) - GRF_{u_{GRF}} = 0 \quad (2)$$

Doing so, the objective function is less sensitive to small changes of state values in Form 1, since GRFs are not dependent on states directly.

Experimental data used in Phase A to calibrate the contact models consist of motion capture, GRF and subject-exoskeleton contact pressures in three sit-to-stand trials with the exoskeleton in passive mode. Pelvis contact forces are measured in one of these trials and, thigh and shank contact forces in two trials. In addition, six sit-to-stand trials with the exoskeleton in assistive mode are used in Phase B to predict the kinematics and interaction forces of the collaborative subject-exoskeleton movement (three trials with contact forces at the pelvis and three at the thigh and shank).

The optimal control problem is solved using 200 mesh intervals with four collocation points per interval.

Results

Both formulations (Form 1 and 2) led to almost the same local optima. For example, root mean square differences (RMSD) between formulations in phase A were 0.43 ± 0.26 degrees, 1.73 ± 1.00 degrees/s for angular coordinates and velocities, 6.5 ± 8.8 N for GRF, and 3.6 ± 3.9 N for subject-exoskeleton contact forces.

In Phase B, both formulations also led to almost the same solutions (RMSD coordinates = 0.10 ± 0.12 degrees, RMSD angular velocities = 0.30 ± 0.35 degrees/s, RMSD contact forces = 0.01 ± 0.01 N).

Form 1 led to a non-linear programming (NLP) problem bigger than Form 2. For example, in Phase A, the optimal control problem had 78972 variables and 73443 constraints with Form 1, and 76560 variables and 71031 constraints with Form 2.

However, the number of iterations and, hence, the computational time, were lower in Form 1 (Figure 2).

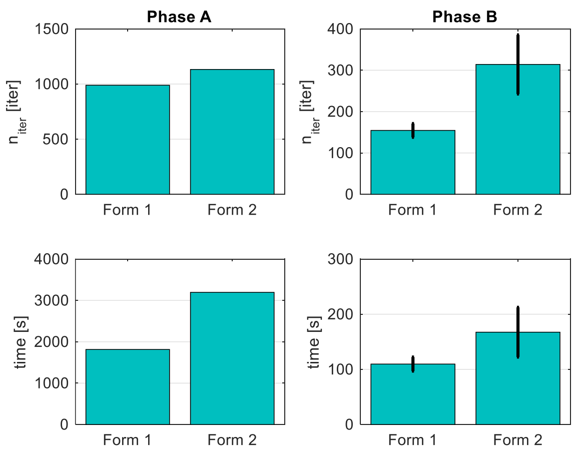

Figure 2. Number of iterations (upper row) and computational time (lower row) spent to solve the optimization in Phases A (left) and B (right). Note that the vertical scales in the two phases are not the same.

When using GRF as controls (Form 1), the optimization took 989 iterations to solve Phase A, versus 1132 without using GRF as controls (Form 2), which corresponds to 1814 s and 3195 s using Form 1 and 2, respectively. In Phase B, the number of iterations spent using Form 2 was also higher in all six trials (314 ± 71 iterations versus 155 ± 17 iterations), which corresponds to a computational time of 167.4 ± 45.1 s using Form 2 and 109.6 ± 12.8 s using Form 1. The results were obtained with a regular laptop Intel Core i7-6700HQ CPU @2.6 GHz.

Discussion

In this study we used an algorithmic differentiation [1] framework recently developed to solve biomechanical optimal control problems, and we did an analysis of the impact of formulating GRF as controls, since these forces are known to be highly sensitive to state values. Although the introduction of GRF as controls led to a bigger non-linear problem, the number of iterations as well as the computational time were smaller than when these control variables were not used. This was due to the fact that the introduction of these controls prevent the gradient of the objective function from being dependent on the model GRF, which could reach huge values just by slightly changing state values. Since the search direction of the optimization algorithm is gradient-based, this fact could explain why Form 1 needs less iterations than Form 2 to find an optimal solution.

The algorithmic differentiation framework allowed us to obtain the results with less than half an hour even when calibrating both models with three sit-to-stand trials at the same time. Otherwise, using finite differences, every iteration of the optimization would take much more time. For example, one iteration takes around 5 minutes only for Phase B (without the contact model calibration and simulating only one trial) with finite differences, versus around 1 second with algorithmic differentiation. Future work will be focused on testing different formulations of contact models to decrease the computational time of the resolution of the optimization even more.

Conclusion

The take-home message of this study is two-fold. On the one hand, the used algorithmic differentiation framework allows to analyse different formulations of a large NLP with a relatively short computation time. On the other hand, it is shown that the use of GRF as controls leads to both a lower number of iterations and computational time.

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