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Editorial

Entropic Aspects of Nonlinear Partial Differential Equations: Classical and Quantum Mechanical Perspectives

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There has been increasing research activity in recent years concerning the properties and the applications of nonlinear partial differential equations that are closely related to nonstandard entropic functionals, such as the Tsallis and Renyi entropies. It is well known that some fundamental partial differential equations of applied mathematics and of mathematical physics—such as the linear diffusion equation—are closely linked to the standard, logarithmic Boltzmann–Gibbs–Shannon–Jaynes entropic measure.

This link can be extended to the realm of nonlinear partial differential equations via the aforementioned nonstandard (or generalized) entropic functionals. In particular, nonlinear diffusion and Fokker–Planck equations endowed with power-law nonlinearities in the diffusion term (Laplacian term) admit exact time-dependent solutions exhibiting the form of Tsallis maximum entropy distributions, where the Tsallis measure is optimized under a small number of appropriate constraints. The connection between nonlinear Fokker–Planck equations and nonstandard entropies proved to be a remarkably fertile field of research, leading to a set of ideas and techniques that has been successfully applied to the analysis of a variegated family of systems or processes in physics, biology, and other areas. As examples we can mention applications to the behavior of granular media and to the study of the motility of biological microorganisms. Most of the above developments concern purely classical (as opposed to quantum mechanical) concepts.

However, new results arising in recent years have made it clear that the connection between nonstandard entropies and nonlinear partial differential equations can also be extended to new nonlinear wave equations inspired on quantum mechanics. Nonlinear versions of the celebrated Klein–Gordon and Dirac equations have been discovered that admit exact time-dependent soliton-like solutions having the forms of the so-called q-plane waves. These q-plane waves are power-law generalizations of the complex exponential plane wave solutions of the linear Klein–Gordon and Dirac equations. The q-plane waves are complex-valued counterparts of the real-valued functions arising from the generalized Tsallis or Renyi maximum entropy principles. The aforementioned nonlinear extensions of the relativistic wave equations exhibit interesting physico-mathematical properties that have been explored in the recent research literature. Non-relativistic complex wave equations have also been incorporated to these research efforts, in terms of a parameterized family of power-law nonlinear Schrödinger equations.

The field of research discussed above is currently experiencing rapid development and is giving rise to new lines of enquiry, in many cases of a multi-disciplinary character. It thus seems necessary to devote a special issue of the Journal *Entropy* to these subjects. It would constitute a very useful, timely, and unique contribution to the current scientific literature. It should stimulate further research in the field, and we can expect it to be highly cited. A brief outline of the present ten contributions follows, and makes for exciting reading.

1. Tehseen Abbas, Muhammad Ayub, Muhammad Mubashir Bhatt, et al. examine entropy generation on viscous nanofluid through a horizontal Riga plate [1].
2. Yun Zhao and Fengqun Zhao focus on obtaining analytical solutions for d-dimensional, parabolic Volterra integro-differential equations with different types of frictional memory kernel [2].
3. Bo Liang, Xiting Peng, and Chengyuan Qu study the existence and uniqueness of solutions for an initial boundary problem of a nonlinear fourth-order parabolic equation with variable exponent [3].
4. T.D. Frank proposes a thermostatistic framework for active Nambu systems and uses the so-called free energy Fokker–Planck equation approach to describe stochastic aspects of these Nambu systems [4].
5. Hiroaki Yoshida considers the diffusion flows of probability measures associated with the Fokker–Planck partial differential equation [5].
6. Javier Zamora, Mario C. Rocca, Angelo Plastino, et al. discuss non-linear generalization of both Schrödinger's and Klein–Gordon's equations via a perturbative approach [6].
7. Fernando D. Nobre, Marco Aurélio Rego-Monteiro, and Constantino Tsallis review recent developments on the generalizations of two fundamental wave equations, namely the Schrödinger and Klein–Gordon equations. These generalizations present nonlinear terms, characterized by exponents depending on an index q, in such a way that the standard linear equations are recovered in the limit $q = 1$ [7].
8. Ervin K. Lenzi, Luciano R. da Silva, Marcelo K. Lenzi, et al. investigate an intermittent process obtained from the combination of a nonlinear diffusion equation and pauses. They consider the porous media equation with reaction terms related to the rate of switching the particles from the diffusive mode to the resting mode or switching them from the resting to the movement [8].
9. Angel R. Plastino and Roseli S. Wedemann advance two nonlinear wave equations related to the nonextensive thermostatistical formalism based upon the power-law nonadditive S_q entropies. The wave equations that they analyze in this work illustrate new possible dynamical scenarios leading to time-dependent q-Gaussians [9].
10. Renio dos Santos Mendes, Ervin Kaminski Lenzi, Luis Carlos Malacarne, et al. investigate a nonlinear random walk related to the porous medium equation (a special case of the nonlinear Fokker–Planck equation). This random walk is such that when the number of steps is sufficiently large, the probabilities of finding the walker in different positions approximate a q-Gaussian distribution [10].

Summing up, this Special Issue is devoted to the intense activity currently going on concerning the properties and the applications of nonlinear partial differential equations that are closely related to nonstandard entropic functionals, such as the Tsallis and Renyi entropies. This research extends and generalizes some fundamental aspects of important partial differential equations of applied mathematics and of mathematical physics—such as the linear diffusion and Fokker–Planck equations—that are closely linked to the standard logarithmic Boltzmann–Gibbs–Shannon–Jaynes entropic measure. These kinds of connections can be extended to the realm of nonlinear partial differential equations via the aforementioned generalized entropic functionals. This Special Issue looks for new developments and/or new results regarding the connection between nonstandard entropies and nonlinear partial differential equations.

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