Desenvolvimento Curricular e Didática

Using mathematical magic to improve student's motivation in mathematics classes

O recurso à magia matemática para aumentar a motivação dos estudantes nas aulas de Matemática

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Abstract:

One of the problems identified in the failure of mathematics teaching is the lack of motivation that students feel towards the discipline. Considering that it is imperative to make the teaching and learning processses of mathematics more stimulating, taking into account modern society and student's interests, the authors have developed several strategies to increase the interest and improve the success of students in mathematics. Thus, they decided to use mathematical magic tricks in their classes to raise the motivation of the students, captivating them and stimulating their interest in mathematics. In this work, a case study to evaluate how mathematical magic can contribute to increase students' motivation for learning mathematics is presented. In order to develop this experience, the techniques of inquiry, direct observation and analysis of documents were applied and the following instruments were used: questionnaires and respective analysis grids; production of a battery of tasks of a diversified nature and field notes. The analysis of the collected data allows us to conclude that the use of mathematical magic tricks in the classroom, with the purpose of motivating the students to learn mathematics, revealed to be effective.

Keywords: mathematics; mathematical education; recreational mathematics; mathematical magic; higher education.

Resumo:

Um dos problemas identifcados no fracasso do processo de aprendizagem da Matemática é a falta de motivação que os estudantes sentem em relação à disciplina. Sendo evidente que é imperativo que o processo de ensino da Matemática se torne mais estimulante, tendo em conta a sociedade moderna e os interesses dos estudantes, as autoras desenvolveram diversas estratégias para aumentar o interesse e promover o sucesso dos estudantes relativamente à Matemática. Assim, decidiram, nas suas aulas, recorrer a truques de magia matemática

para aumentar a motivação dos estudantes, cativando-os e estimulando o seu interesse pela Matemática. Neste artigo, é apresentado um estudo de caso para avaliar como a magia matemática pode contribuir para aumentar a motivação dos estudantes para aprenderem Matemática. Para desenvolver a experiência referida, privilegiaram-se as técnicas de inquirição, observação direta e análise de documentos e os seguintes instrumentos: questionários e respetivas grelhas de análise; produções de uma bateria de tarefas de natureza diversificada e notas de campo. A análise dos dados recolhidos permite concluir que o recurso a truques de magia matemática em contexto de sala de aula, com o intuito de motivar os alunos para a aprendizagem da Matemática, se revelou eficaz.

Palavras-chave: matemática; educação matemática; matemática recreativa; magia matemática; ensino superior.

Resumen:

Uno de los problemas identificados en el fracaso del proceso de enseñanza-aprendizaje de las matemáticas es la falta de motivación que los estudiantes sienten en relación a la disciplina. Siendo evidente que es imperativo que el proceso de enseñanza y aprendizaje de las matemáticas se vuelva más estimulante, teniendo en cuenta la sociedad moderna y los intereses de los estudiantes, las autoras desarrollaron diversas estrategias para aumentar el interés y promover el éxito de los estudiantes con respecto a las matemáticas. Así, decidieron, en sus clases, recurrir a trucos de magia matemática para aumentar la motivación de los estudiantes, cautivándolos y estimulando su interés por las matemáticas. En este artículo, se presenta un estudio de caso para evaluar cómo la magia matemática puede contribuir a aumentar la motivación de los estudiantes para aprender matemáticas. Para desarrollar esta experiencia, favoreció a las técnicas de investigación, la observación directa y el análisis de los documentos y las siguientes herramientas: cuestionarios y las respectivas rejillas de análisis; producciones de una batería de tareas diversificadas y notas de campo. El análisis de los datos recogidos muestran que el uso de trucos de magia matemática en el contexto del aula, con el fin de motivar a los estudiantes para el aprendizaje de las matemáticas, ha demostrado ser eficaz.

Palabras clave: matemáticas; educación matemática; matemáticas recreativas; magia matematica; enseñanza superior.

Introduction

Nowadays, we live in an ever-changing society. The educational context is no exception, and requires a renewal of paradigms (Morin, 2002). Often, teachers are faced with discouraged students: students with glassy eyes, heads down on desks, students who don´t remember content taught to them the day before! Mathematics classes are no exception. Quite the opposite!

Mathematics, considered one of the basic areas of various education settings, has been the subject of concern for many authors and researchers due to its enormous academic and

educational failure (Araújo & Cabrita, 2012; Pais, Cabrita & Anjo, 2011). Its importance in day-to-day life and the education of individuals is irrefutable (Earls & Holbrook, 2007). One of the problems identified in the failure of mathematics teaching is the lack of motivation that students feel towards the subject. According to Matos (2011) and Hall & Pais (2018) motivation is an essential factor for learning since the quality of learning is not only related to the capacity to learn, but also to the level of motivation that we have to carry out this same learning.

Profound changes to the role and function of both teachers and students are particularly vital. Active, cooperative and participative methodologies of learning that allow a break from magisterial education and the mere transmission of knowledge must be privileged. The greatest challenge that teachers face today is to engage students, motivate them to learn and take an active role in their learning process (Felder & Brent, 2005). If students care about mathematics they are far more likely to remember what they learn. If students develop motivation to learn a subject, they are much more likely to develop long-term understanding (Lester, 2013).

The Joint Mathematical Council of the United Kingdom (2011) states that it is of the utmost importance that the teaching and learning process of mathematics become more stimulating, taking into account modern society and the students' interests.

The present world is constantly changing. New challenges emerge every day. Students should be prepared to face the uncertainty and constant change of the future. However, the density and rigidity of most math curricula point towards the opposite direction of what is needed. Nassim Taleb (2014) points out the importance of the unexpected, in today's world. The author argues that human beings tend to focus on what they already know, preferring the particular over the general. As in many other contexts, school curricula reflect our tendency to control and predict. Elliot Eisner (2002) argues that, although understandable at a practical level, this tendency must be inverted:

> *"Opening oneself to the uncertain is not a pervasive quality of our current educational environment. I believe it needs to be among the values we cherish. ... How can the pursuit of surprise be promoted in a classroom?" (p.11).*

Being a teacher in the current context poses challenges that go beyond a solid knowledge of the subjects to be taught and the didactic issues associated with them. Being a teacher also means being creative in finding appropriate answers for today's students, who, due to being exposed to an immense diversity of sources of information and entertainment means, impose an increased demand on the educational actors, making the teaching and learning process more demanding than ever before.

Considering that it is imperative to make the teaching and learning process of mathematics more stimulating (JMC, 2011) and taking into account modern society and student's interests, the authors have developed several strategies to increase the interest and improve the success of students in mathematics. Thus, and because they believe that exploring the relationship between magic and mathematics has an enormous potential in developing activities in the classroom to promote the interest in mathematics, they decided to use mathematical magic tricks in their classes to raise the motivation of the students, captivating them and stimulating their interest

in mathematics. In this paper, the authors present a case study to evaluate how mathematical magic can contribute to increase the students' motivation regarding mathematics learning.

Mathematical Magic and Mathematics Education

The issue of academic failure in mathematics is at the order of the day. Knowing its causes and finding ways to combat it is a priority at all levels of education. Year after year, teachers are faced with discouragement, detachment and consequent student failure. It is imperative to change the educational model (Alarcão, 2002; D'Hainaut, 1980; Freire, 1983; Morin, 2002; Roldão, 1998, 2000, 2001; among others); It is necessary to have another look at the way in which one learns and, consequently, how one must teach (Alarcão, 2002; Sá-Chaves, 2003; Roldão, 1998, 2000, 2001). Researchers, such as Nóvoa (2009), argue that the educational process should be focused on the student's learning. The student is challenged to abandon his role as a passive recipient of information and to assume a central role in the teaching and learning process, adopting a more critical, active, regulatory and dynamic attitude in the personal construction of his own knowledge. Reversing the negative general feeling towards mathematics is essential. It is crucial to motivate, arouse interest, thus promoting students' success in mathematics. One way to do this is through recreational mathematics, which David Singmaster (1992) defines as math that is fun, popular, and of pedagogical value. There are several areas of mathematics in which recreation and seriousness are easily interlinked as for instance through games involving probability, graph theory, number theory, topology, geometry, among others.

Recreational mathematics is quite often linked to game theory. Mathematical magic is another of its important facets. At first glance, mathematics and magic do not seem to have much in common, but, looking more closely, we realize that this relationship has been around for a long time with valuable benefits for both parties. Five hundred years ago, Luca Pacioli described the first math-related card trick known in his manuscript De Viribus Quantitatis (Blasco, 2011). Since then, much work has been developed applying mathematical ideas in creating different types of magic tricks, including numerical tricks, card tricks, and visual effects based on geometric relationships or topological features. Among the authors of recreational mathematics, Martin Gardner (1914-2010) deserves special mention for his important role in building bridges between the world of magic and mathematics. Martin Gardner published close to 200 books, including the book Math, Magic and Mystery (Gardner, 1956), which has influenced so much magicians and mathematicians ever since. Colm Mulcahy is one of the mathematicians inspired by Martin Gardner. Mulcahy created a bimonthly column, Card Colm, on the American Mathematical Association's website, from 2004 to 2014, exclusively dedicated to mathematical card tricks (http://cardcolm.org/CardColms.html). More recently, he published a book where he gathered many of the effects described in his column (Mulcahy, 2013).

Several other mathematicians have dedicated themselves to the dissemination of mathematical magic and its promotion both to its students and to society in general. Diaconis and Graham (2011) in Canada have published a wonderful book that stands out for the mathematical depth of their magic tricks. Fernando Blasco of the Polytechnic University of Madrid holds numerous sessions of mathematical magic. Blasco has a website

and a blog to share his ideas (http://www.fblasco.com/index.html) and published a book on mathematical magic (Blasco, 2007). On his website, Blasco says:

> *"I am fully convinced that if everyone knew more details about what mathematics involves, they would become interested in this subject. ... The world of mathematics offers as much seriousness and rigor as illusion and entertainment. Many are convinced that mathematics is something austere and boring, but in spite of these strong convictions, things are not always what they seem. "Magic is just that: to make things look like what they are not, or to make them look differently from what they are, producing constant surprise, wonder and fun. Ultimately, the astonishment is that all of this can be done using mathematics."*

Karakasheva (2017) describes a learning environment based on mathematical magic trics and on the use of sophisms in order to trigger positive emotions in the process of learning mathematics, thus stimulating students to achieve better results.

Exploring the relationship between magic and mathematics has an enormous potential in developing activities in the classroom to promote the interest in mathematics. Performing a magic trick and exploration its mathematical content can be a source of reasoning and mathematical learning which can be empowered by the magic setup.

Method

Methodological options

The research question underlying this study is how mathematical magic can contribute to increase students' motivation regarding mathematics learning. Particularly, we want to analyse the influence of the use of mathematical magic tricks in math classes, on a regular basis, in:

- promoting students' motivation to learn mathematics;

- the development of teaching methodologies that can elicit more positive attitudes towards mathematics in students.

In this perspective, a mixed case study was developed (quantitative and qualitative, based on a logic of complementarity), grounded on a pragmatic paradigm and case study design (Carmo & Ferreira, 1998; Ponte, 2006; Yin, 1994). The study was undertaken in a higher education institution of central Portugal involving undergraduate students. The teacher responsible for the course in question is simultaneously one of the researchers of this study.

In order to develop this experience, the techniques of inquiry, direct observation and analysis of documents were applied and the following instruments were used: field notes, students' portfolio and final questionnaire. The students' portfolio consisted on a production of a battery of tasks of a diversified nature. The main objective of the questionnaire was to collect the students' opinion on the use of mathematical magic in the classroom.

Statistical treatment of the data, specifically descriptive statistics (frequency distributions, mean, median, standard deviation and crosstabulation), was performed using the Statistical Package for the Social Sciences (SPSS), version 25.0.

Case study description

This case study took place during the classes of the course entitled *Conceitos de Matemática I* (CMI - Mathematical Concepts I) in the first semester of the academic year 2018-19. This course is part of the 1st year syllabus of the undergraduate programme "Licenciatura em Educação Básica" (LEB) which is a three year higher education 1st cycle programme for teacher training, form kindergarten to grade 6 (pre-school, 1st and 2nd Cycles of Basic Education in Portugal). This course has five hours contact every week, divided into two classes of 2 and 3 hours, during a whole semester (15 weeks).

The topics of this course are numbers and operations. In the first part of the course different numeral systems are addressed and the students learn how to represent (and convert) numbers in different numeral systems and bases. The four basic operations are explored (addition, subtraction, multiplication and division) and their algorithms are detailed not only in the decimal system but also in other bases. The second part of the course is devoted to elementary number theory. Multiples/divisors, divisibility rules, prime number decomposition and applied problems are addressed. The third and last part of the course is related to decimal numbers, exploring the four operations both with fractions and/or decimal representations. Problem solving is also addressed using different approaches with a strong emphasis in the use of diagrams.

In the beginning of every class some minutes (about 5 to 10) were dedicated to the performance of a mathematical magic trick which, whenever possible, was related to the topic of the class. The explanation of the mathematics underlying the trick was usually presented later during the class or in the next class. In the last case, students were challenged to think about the trick at home. Most of the times, they couldn't find out the explanation by themselves. However, sometimes they brought ideas that were good clues to the full explanation.

In order to illustrate the kind of tricks performed in the classes we describe a selection of four tricks directly related to the topics involved in the classes. The first trick – Guessing a birthday – is related to the representation of numbers in the binary system. The second trick – Guessing a word – is related to the standard addition and subtraction algorithms and constitutes a good example of algebraic reasoning. The third trick – Crystal ball – is related to the concept of multiples and is a good example of a simple algebraic proof. The fourth trick – Guessing a number – is related to the divisibility rule for nine.

Guessing a birthday – A volunteer is shown a set of four lists of months and another set of 5 lists of numbers shown in Figure 1. One by one, he is asked if his month (and later day) of birth is, or is not, in the given lists. Based on the answers the magician/teacher "guesses" his date of birth. Without realizing it, the answers given by the volunteer (yes/no) constitute the digits of the number of the month and the day of his birthday in binary code (yes – 1; no – 0).

Figure 1. Tables for guessing a birthday.

Guessing a word – The volunteer is asked to guess a word, previously hidden in the room, from a thick book (we used one of the Harry Potter books). In order to guess the word he is asked to find a code. For that purpose he is asked to choose (at random) a three digit number of his choice (digits must be different). He is then asked to reverse the digit order and subtract the smallest number from the largest. He is then asked to reverse the order of the digits of the difference and sum this number to the difference. What he doesn't know is that the final sum is always 1089, irrespectively of the starting number. This number is presented as the code to the word in the book – the participant is asked to look up at page 10, line 8 and word 9.

When explaining the mathematics of this trick the challenge is to prove that starting with any 3 digit number $\overline{xyz}$ (the first digit must be different from the last one: $x \neq z$), reversing its order, subtracting the smallest from the largest ($x > z$), reversing the order of the sum digits and adding the sum with its reversed version always gives 1089 as the result, irrespectively of the initial choice. This algebraic proof can be surprisingly simplified using the usual subtraction and addition algorithms (in Portugal subtraction is usually performed using the equal additions algorithm as below). This trick explanation is a good example to work algebra in middle school classes, providing students with a simple proof and showing them that variables do not appear only in equations and can be used in other contexts.

We start with the subtraction: since $x > z$ the units digit of the minuend has to be increased ($z + 10$) and the tens digit of the subtrahend is increased accordingly ($y + 1$) so that the difference is kept unchanged. Now the tens digit of the minuend (y) is again smaller than that of the subtrahend ($y + 1$) and the same procedure is applied to the tens and hundreds (underlined grey text).

Hundreds	Tens	Units
x	y _+10_	$z + 10$
$-\quad z\underline{+1}$	$y + 1$	x
$x - z - 1$	9	$z + 10 - x$

Next we use the addition algorithm after reversing the order of the digits of the difference. For the units we have $(z + 10 - x) + (x - z - 1) = 9$, for the tens we have $9+9 = 18$ leaving the 8 in the tens and passing the unit to the hundreds. For the hundreds we have $(x - z - 1) + (z + 10 - x) + 1 = 10$. The final result is always 1089.

Hundreds	Tens	Units
$x - z - 1$	9	$z + 10 - x$
$z + 10 - x$	9	$x - z - 1$
$+$		
10	$\,+\,8$	9

Crystal Ball – Everybody in the classroom is asked to think of a two digit number, sum its digits and subtract the sum from the initial number. Then, by looking up the result in a projected table of numbers and symbols (see Figure 2), the magician/teacher is able to guess which symbol is attached to everybody's results. This trick is based on the flash applet "The Flash Mind Reader" [9] and is based on the divisibility by nine. The proof of the result is a simple example of algebraic reasoning: starting with a two digit number, $\overline{xy} = 10x + y$, and subtracting the sum of the digits always gives a multiple of nine: $(10x + y) - (x + y) = 10x + y - x - y = 9x$.

Figure 2. Example of a table for Crystal ball.

Guessing a number – The volunteer is presented with the following table of numbers:

$$\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}$$

He is asked to choose any line or column or diagonal and make a three digit number with the three numbers in the line/column/diagonal chosen. Next he is asked to repeat the procedure obtaining another three digit number. He is asked to multiply both numbers and for each digit in the product he must pick a card from a standard deck of cards with the same number. Zero digits should be discharged since there are no zero cards. With the set of cards in his hand the volunteer is asked to hide one of the cards and show the remaining ones in any desired order to the magician/teacher, who immediately reveals the value of the hidden card.

This trick is based on the divisibility rule for nine, since, whatever choice is made, the three digit numbers obtained are always multiples of three and therefore the product is a multiple of nine. To show that every choice leads to a multiple of three, several properties can be explored. For instance, choosing a line means choosing three consecutive natural numbers. The sum of three consecutive natural numbers is always a multiple of three:

$x + (x + 1) + (x + 2) = 3x + 3 = 3(x + 1).$

Results

In this section we report a preliminary analysis of the questionnaire delivered towards the end of the term. The questionnaire was filled in by forty two students (37 female and 5 male), corresponding to the number of students that attended the course regularly.

Students were asked (on a five point Likert scale: 1- not at all, 5- very much) if they enjoyed the performance of mathematical magic trick in the classes. The vast majority of the students gave a positive reply: 42.9% (17 out of 39) replied 4 - "I liked" and 52.4% (19 out of 39) replied 5 - "very much". Only 4.7% (3 out of 39) students replied 3 - "neutral", and none gave a negative answer (however, three students didn't answer this question).

Students were also asked to rate on a five point Likert scale their agreement regarding several statements (full results can be seen in Table 1).

One of the statements was "The use of mathematical magic in the classroom contributes to a more positive attitude towards mathematics". Once more, the vast majority of the students gave a positive reply: 85.7% (36 out of 42) of the students gave a positive answer.

Another of these statements was "The use of mathematical magic tricks in math classes makes mathematics more boring and demotivating". None of the students agreed with the statement.

Concerning the statement "The use of mathematical magic tricks in math classes makes the learning process more challenging, interesting and stimulating", 80.0% (32 out of 40)of the students gave a positive answer, 20% (8 out of 40) replied 3 – neutral, and none gave a negative answer.

Regarding the statement "The use of mathematical magic tricks in math classes contributes to better understand the importance of mathematics", 62.5% (25 out of 40) agreed with it, 35.0% (14 out of 40) were neutral and only one (2.5%) respondent partially disagreed with it (no one was in total disagreement).

With regard to the statement "The use of mathematical magic tricks in math classes provides a less rigid method of learning, making it more interactive and interesting", a large majority of students gave a positive reply (78.9% agreed with it – 30 out of 38), 18.4% (7 out of 38) were neutral and only one student (2.6%) partially disagreed.

Another statement was "The use of mathematical magic tricks in math classes serves only to entertain for a while". Almost every one (88% - 37 out of 42)) disagreed with it and only one student (2.4%) claimed to partially agree with the statement.

Overall, students gave a "positive" (expected) agreement to all statements: those stated in a positive form have a high mean answer, between 3.8 and 4.3 (greater that 2.5), and those stated in a negative form have a small mean answer, between 1.1 and 1.5 (smaller than 2.5). Standard deviations are relatively small, corresponding to a small dispersion of answers and thus reflecting the fact that students revealed the same type of agreement (either positive or negative) in all statements (see Table 1 for detailed values).

Table 1. Distribution (absolute frequency; mean and St. Dev.) of the answers on the agreement with some given statements.
(1 – total disagreement; 2- partial disagreement; 3 – neutral; 4 – partial agreement; 5 – total agreement; NOp. – No answer)

Statement: The use of mathematical magic tricks in math classes…	1	2	3	4	5	NOp	mean	st.dev.
contributes to a more positive attitude towards mathematics.	0	0	6	17	19	0	4.3	0.7
makes mathematics more boring and demotivating.	38	2	2	0	0	0	1.1	0.5
makes the learning process more challenging, interesting and stimulating	0	0	8	17	15	2	4.2	1.1
contributes to better understand the importance of mathematics	0	1	14	16	9	2	3.8	1.1
provides a less rigid method of learning, making it more interactive and interesting	0	1	7	16	14	4	4.1	1.4
serves only to entertain for a while	25	12	4	1	0	0	1.5	0.8

When questioned if "the use of mathematical tricks in the classes is relevant?", 98% (almost all of the respondents – 41 out of 42 students) said "Yes" and only one student said "No". When asked to specify, amongst a given list, the reasons for this relevance, the most selected ones were:

- The classes are more interesting – 70.7% of the respondents selected this option.

- It stimulates the like for mathematics – 58.5% of the respondents selected this option.

As for "other reasons" specified by the respondents (open answers) we outline one which stated "It provides pre-service teachers with other teaching methodologies".

Students were asked to further specify the importance of using mathematical magic tricks in the classes, by rating it on a 5 point Lickert scale (1 – not important; 5 – very important): 23.8% (10 out of 42) of the students replied 5; 45.2% (19 out of 42) rated it as 4; 31.0% (13 out of 42) as 3; none replied 1 or 2. Although the mode is 4 and not 5, all students used positive rate, 3 to 5. The overall average rate is 3.9 which is clearly positive, though not very close to the maximum. The use of mathematical magic tricks in the classroom is a complementary tool and not the primary methodology for teaching mathematics. It is therefore expected that students don't rate it as a very important resource.

When asked if the use of magic tricks in math classes helped them to enjoy mathematics more, the great majority of the students – 66.7% (28 out of 42)- answered "Yes" (33.3% (14 out of 42) answered "No").

The authors compared the degree of enjoyment of the performance of mathematical magic tricks in the classes between male and female students. Through graphical gender analysis (Graph 1) it can be seen that the differences between male students and female students are not relevant. However, whereas for the male gender, no student answered "neutral", with regard to the female gender, 5.4% of the students chose this option. In contrast, the percentage of female students who chose the option "Very Much" is higher than the percentage of male students who chose the same option. Regarding the "Liked" option, the percentage of female students choosing this option is lower (by about 20%) than the percentage of students of the male gender.
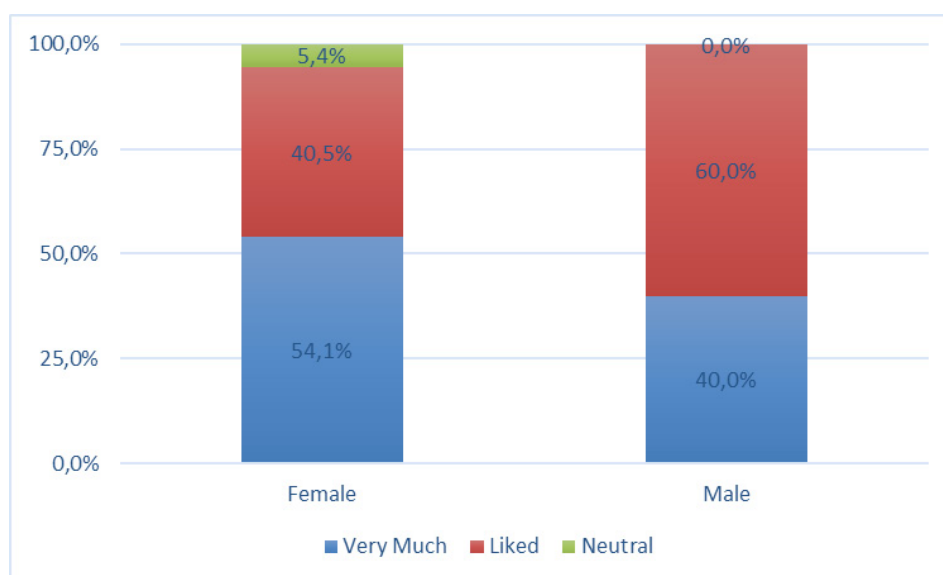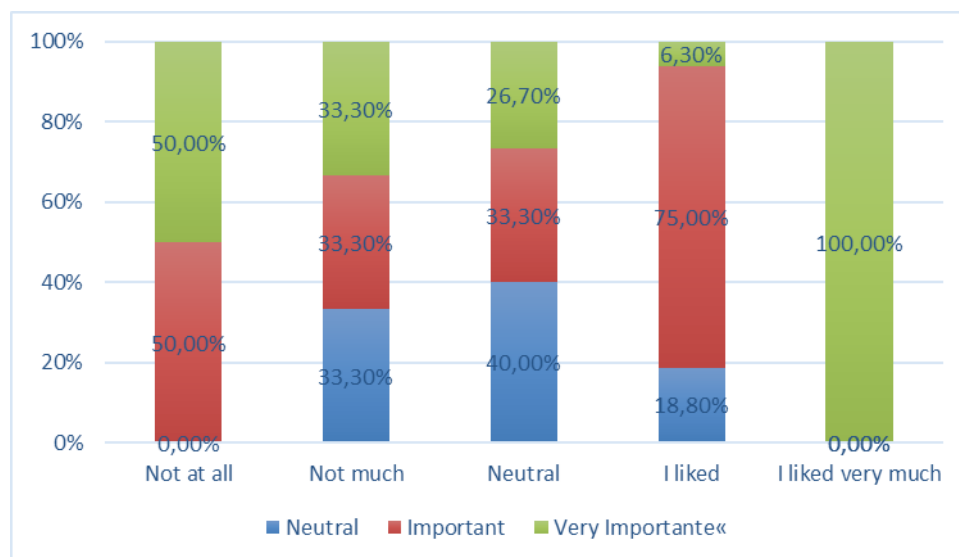

Graph 1. Crosstabulation between the variables "enjoy mathematical magic tricks in classes" and "Gender".

In order to confirm the idea that there are no significant differences between the two genders, a chi-square test of independence was applied and the null hypothesis was formulated - There is no relation between the variable "Gender" and the variable "Do you enjoy the performance of mathematical magic trick in the classes ". From the results obtained ($X^2=0.826$, p = 0.662) and considering the significance level to be $a= 0.05$, we conclude that there is no reason to reject the null hypothesis, that is, there are no statistically significant differences between the two genders regarding the answers given to the question "Do you enjoy the performance of mathematical magic trick in the classes".

The relation between the degree of importance given to the use of mathematical magic tricks in the classes and the students' taste for mathematics was also analysed. Through a graphical analysis (Graph 2), it was observed that although the great majority of the students found it

important to use magic tricks in classes, the students who like math more also find it more important to use the tricks. In order to confirm that idea, a chi-square test of independence was also applied and the null hypothesis was formulated - There is no relation between the variable "Taste for mathematics" and the variable "Importance of using mathematical magic tricks in classes".

Graph 2. Crosstabulation between the variables "Taste for mathematics" and "Importance of using mathematical magic tricks in classes".

From the results obtained (X^2=14.255, p = 0.075) and considering the significance level to be a = 0.05, we conclude that there is no reason to reject the null hypothesis, that is, there are no statistically significant differences between the two variables.

Conclusions

As Hall and Pais (2018) say, finding ways to engage students in the classroom and motivate them for learning mathematics is a challenge that most mathematics teachers face today. All contributions towards this goal are welcome and should be disseminated across these professionals. There is no single solution to this challenge and as in many other complex problems, several solutions must be combined and adapted to each context.

In this paper the authors propose one particular way to contribute to this goal: using mathematical magic tricks in the classroom. The case study here described, continues the preliminary work described in Hall and Pais (2018). As in Hall and Pais (2018), the analysis of the field notes and the students' portfolios allows us to conclude that:

- In all classes there were always several students very interested in volunteering for the performance of the mathematical magic tricks, which always involved volunteers. This reveals that the students liked the activity and liked to participate in it.

- Quite often, several students were very surprised and intrigued with the magic trick and tried to figure out what was behind it. Sometimes they explicitly asked the teacher not to reveal the secret immediately so they could think on it. At other times the curiosity was so strong that they asked the teacher to explain it immediately. Either way the students were curious and were developing their mathematical reasoning and engaging with mathematics.

- The topics taught which were subject to mathematical magic gained more meaning after performing the tricks because they had concrete examples of their application, with an extremely attractive purpose.

- Whenever the teacher forgot or intentionally began the lesson with other things than the magic trick, some students inquired if the teacher had forgotten the trick for this class.

The analysis of the collected information, through the questionnaire applied in the end of the term, the field notes and the students' portfolios, allows us to conclude that the use of mathematical magic tricks in the classroom, with the aim of engaging students and motivating them to learn mathematics, turned out to be effective in math courses for pre-service teachers, namely CM I within the undergraduate degree LEB in a Portuguese university. Based on the opinion of the students involved in the study, we can conclude that, the use of mathematical magic in classes made the learning of mathematics more interesting, stimulating and challenging. It further contributed to a more positive view of mathematics, to better understand the importance of mathematics, and to like mathematics more.

These finding are in line with those of other researchers such as Karakasheva (2017), who proposed several tasks based on mathematical magic in the classroom, in order to trigger positive emotions in students. The author concludes that "the smart and balanced use of such groups of tasks contributes to attracting and retaining student's attention, but also encourages learner activation and, as a result, contributes to the continuous learning of new mathematical knowledge" (p. 22-23).

Nevertheless, the use of mathematical magic, in spite of being useful and welcome by the students participating in the present study, was not considered as an extremely important methodology (average rate 3.9 out of 5) corroborating the idea that no single methodology provides a full answer to the teaching challenges and that the use of mathematical magic tricks is mostly a complementary tool with high potential to increase the students' motivation towards mathematics.

The statistical tests applied allow us to conclude that there were no statistically significant diferences:

- between the variable "Gender" and the variable " Do you enjoy the performance of mathematical magic trick in the classes". Nevertheless, the most students enjoyed using mathematical magic tricks in classes;

- between the variable "Taste for mathematics" and the variable "Importance of using mathematical magic tricks in classes". Although the observed differences were not significant, most students found it important to use mathematical magic tricks in classes.

As a final remark and reinforcing the preliminary conclusions obtained by Hall and Pais (2018), we believe that the use of mathematical magic tricks on a regular basis throughout math courses is an added value in the process of learning with positive effects on both the motivation for learning and the learning itself.

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