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Epistemic Logics with Structured Knowledge

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Abstract

Multi-agent Dynamic Epistemic Logic, as a suitable modal logic to reason about knowledge evolving systems, has been emerged in a number of contexts and scenarios. The agents knowledge in this logic is simply characterised by valuations of propositions. This paper discusses the adoption of other richer structures to make these representations, as graphs, algebras or even epistemic models. This method of building epistemic logics over richer structures is called "Epistemisation". On this view a parametric method to build such Epistemic Logics with Public Announcements is introduced. Moreover, a parametric notion of bisimulation is presented, and the modal invariance of the proposed logics, with respect to this relation, are proved. Some interesting application horizons opened with this construction are stated.

Keywords: Dynamic Epistemic Logic; Structured States; Parametric construction of Logics

1 Introduction

Multi-agent epistemic logic has been investigated in Computer Science [17,10,28] to represent and reason about agents, or groups of agents, knowledge and beliefs. Models for these logics are Epistemic Kripke Structures - multi-modal Kripke structures whose states formalize informations and, an equivalence relation of edges for each agent, relating indistinguishable states from their perception. Typically, the states information is characterised

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by a set of propositions. This paper introduces a method to build dynamic epistemic logics to deal with scenarios where the representation of knowledge needs richer presentations than a simply set of propositions. The method to derive these logics (called 'epistemisations'), in a demand driven fashion, is parametric to the (logic) formalism that better fits the nature of the knowledge involved. We illustrate the approach representing knowledge by means of three core structures of computer science: propositions, graphs and abstract data types. This is figured out in three instances of the method, namely the 'epistemisations' of propositional logic (that captures Dynamic Epistemic Logic (DEL)), of Equational Logic (as the classic formalism for abstract data types specification) and of Hybrid Logic with Binders (as the standard logic for graphs representation).

There are works that use epistemic models in which states have a structure [1,16]. In [1] it is introduced a multi-agent epistemic logic in which states are positions in $\mathbb{R}^n$ and the accessibility relations represent the possible states (positions) compatible with the current one. Also in [16,26,27], (dynamic) epistemic logics based on the notion of visibility or observability of propositional variables are presented, i.e, some propositional variable are observable and others are not.

Other works deal with values in an epistemic setting [29,30,31]. Here, states are equipped with register that can store values ([29]) or with constants that can have their values updated ([30,31]).

In the case of the epistemisation of equational logic it reminds first order modal logic [14], where epistemic states are relational structures.

The combination of logics is an active research topic in modern logic. In addition of the well known symmetric combination of logics with methods as fibring, product and fusion of logics [7], combinations where the particular features of a logic are combined 'on top' of another one have been considered in several contexts. The Epistemisation process proposed in this paper can be regarded as a one of these combinations. As other examples of this kind of combinations we would like to refer the works in 'modalisation' of logics, which endows systematically logics with Kripke semantics (w.r.t. the standard $\Box$ and $\Diamond$ modalities) presented in [11,9]. These works were extended by the authors with the 'hybridisation' construction [8], by enriching these logics with the hybrid logic machinery. The 'temporalisation' of logics [13] and the 'probabilisation' of logics [2] are other remarkable examples on this kind of combinations. Other proposals in the literature abstract the combination pattern by considering the 'top logic' itself arbitrary. Such is the case of what is called *parametrisation of logics* in [6]. In brief, a logic is parametrized by another one if an atomic part of the first is replaced by the second. Therefore, the method distinguishes a parameter to fill (the atomic part), a parametrised logic (the 'top' logic) and a parameter logic (the logic inserted within). The method of *importing logics* [25] aims at formalising this kind of asymmetric combinations resorting to a graph-theoretic approach. A more general account of this combinations can be found in [21].

The remaining of this paper is organised as follows: we start by introducing the notion of *knowledge representation framework*, i.e. a generic notion of logic that will be used to specify/support structured states information. Based on this parameter, we introduces the building *'epistemisation' method*, with three paradigmatic illustrations. These logics are then enriched with *public announcements*. Finally, we trace a research agenda to follow up from this work, discussing a set of potential applications and research lines.

2 Representation of Structured Knowledge

In order to represent knowledge of structured states a generic notion of logic is used. This formalism will be the parameter for the construction introduced in the next section, and is defined as follows:

Definition 2.1 A *knowledge representation framework* consists of a tuple $\mathcal{L} = (\mathrm{Fm}_{\mathcal{L}}, \mathrm{Mod}_{\mathcal{L}}, \models_{\mathcal{L}})$, where

- $\mathrm{Fm}_{\mathcal{L}}$ is a countable set of formulas,
- $\mathrm{Mod}_{\mathcal{L}}$ is the set of models for $\mathcal{L}$ and
- relation $\models_{\mathcal{L}} \subseteq \mathrm{Mod}_{\mathcal{L}} \times \mathrm{Fm}_{\mathcal{L}}$ is a relation called *satisfaction relation*.

The usual notion of elementary equivalence is also used in this work. Let us recall it:

Definition 2.2 The *elementary equivalence in* $\mathcal{L}$ is the relation $\equiv_{\mathcal{L}} \subseteq \mathrm{Mod}_{\mathcal{L}} \times \mathrm{Mod}_{\mathcal{L}}$ defined by

$$\equiv_{\mathcal{L}} = \{(M, M') \mid \text{ for any } \varphi_0 \in \mathrm{Fm}_{\mathcal{L}},\, M \models_{\mathcal{L}} \varphi_0 \text{ iff } M' \models_{\mathcal{L}} \varphi_0\}$$

As examples of Knowledge representation frameworks we can enumerate all the logics with a complete calculus. Relevant for the present work we have the standard Classic Propositional and Equational Logic, and Hybrid Logic with Binders [4].

3 Parametric construction of Epistemic Logics with structured States

Let us fix a knowledge representation framework $\mathcal{L}$. We introduce in the following a generic construction to building the logic $\mathcal{E}(\mathcal{L})$, the *epistemisation of* $\mathcal{L}$.

Definition 3.1 The formulas for the $\mathcal{L}$-epistemic logic for a finite set of agents $\mathcal{A}$, in symbols $\mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$, is defined by the following grammar:

$$\varphi ::= \varphi_0 \mid \mathbf{tt} \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid K_a\varphi \mid C_G\varphi$$

where $\varphi_0 \in \mathrm{Fm}_{\mathcal{L}}$, $a \in \mathcal{A}$ and $G \subseteq \mathcal{A}$.

The standard connectives can be presented as abbreviations, namely $\mathbf{ff} \equiv \neg\mathbf{tt}$, $\varphi \vee \phi \equiv \neg(\neg\varphi \wedge \neg\phi)$, $\varphi \to \phi \equiv \neg(\varphi \wedge \neg\phi)$, $B_a\varphi \equiv \neg K_a\neg\varphi$ and $E_G\varphi \equiv \bigwedge_{a\in G} K_a\varphi$.

The intuitive meaning of the modal formulas are:

- φ_0 - is an assertion about the (structured) epistemic states, expressed in the knowledge representation framework $\mathcal{L}$;
- $K_a\varphi$ - agent a knows φ;
- $E_G\varphi$ - every agent $a \in G$ knows φ;
- $C_G\varphi$ - it is common knowledge among all members of group G that it is the case that φ.

Definition 3.2 An $\mathcal{L}$-*epistemic model* for a finite set of agents $\mathcal{A}$, $\mathcal{A}$-model for short, is a tuple $\mathcal{M} = (W, \sim, M)$ where

- W is a non-empty set of states;
- $\sim$ is an $\mathcal{A}$-family of equivalence relations $(\sim_a \subseteq W \times W)_{a\in\mathcal{A}}$; and
- $M : W \to \mathrm{Mod}_{\mathcal{L}}$ is a function, that assigns the knowledge structure of each state.

We also consider the relations $\sim_G = \bigcup_{a\in G} \sim_a$ and $\sim_G^* = (\sim_G)^*$, where $(\sim_G)^*$ is the reflexive, transitive closure of $\sim_G$. The set of $\mathcal{L}$-epistemic models for a set of agents $\mathcal{A}$ is denoted by $\mathrm{Mod}_{\mathcal{E}(\mathcal{L})}$.

Definition 3.3 For any $\mathcal{A}$-model $\mathcal{M} = (W, \sim, M)$, for any $w \in W$, and $\varphi \in \mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$, the satisfaction relation

$$\models \subseteq \mathrm{Mod}_{\mathcal{E}(\mathcal{L})} \times \mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$$

is recursively defined as follows:

- $\mathcal{M}, w \models \varphi_0$ iff $M(w) \models_{\mathcal{L}} \varphi_0$
- $\mathcal{M}, w \models \neg\phi$ iff $\mathcal{M}, w \not\models \phi$
- $\mathcal{M}, w \models \phi \wedge \psi$ iff $\mathcal{M}, w \models \phi$ and $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models K_a\phi$ iff for all $w' \in W : w \sim_a w' \Rightarrow \mathcal{M}, w' \models \phi$
- $\mathcal{M}, w \models C_G\phi$ iff for all $w' \in W : w \sim_G^* w' \Rightarrow \mathcal{M}, w' \models \phi$

We write $\mathcal{M} \models \varphi$ whenever, for any $w \in W$, $\mathcal{M}, w \models \varphi$.

It is easy to see that

- $\mathcal{M}, w \models E_G\phi$ iff for all $w' \in W$ we have $w \sim_G w' \Rightarrow \mathcal{M}, w' \models \phi$, and
- $\mathcal{M}, w \models B_a\phi$ iff there is a $w' \in W$ such that $w \sim_a w'$ and $\mathcal{M}, w' \models \phi$.

Now we are in condition to introduce the first illustration of the paper that suggests that the standard propositional epistemic logic can be captured with our epistemisation method:

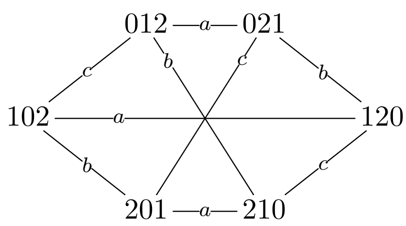
Example 3.4 This example is adapted from [28]. Suppose that a father has three envelopes, each containing: **0**, **1** and **2** dollars inside respectively. The father has three children: **a**nna, **b**ob and **c**lara. Each child receives one envelope and do not know the content of the envelopes of the other children.

In order to represent this scenario, we adopt $\mathcal{E}(\mathcal{PL})$, for $\mathcal{PL}$ the classic propositional logic. Note that this logic corresponds exactly to the standard *Multi-Agent Epistemic Logic* (e.g. [28]).

Hence, for the construction of the base formulas (in $\mathrm{Fm}_{\mathcal{PL}}$) we use propositional sentences over the set of variables $\mathrm{Var} = \big\{0_n, 1_n, 2_n \mid n \in \{a, b, c\}\big\}$ meaning "child n has envelope **0**, **1** or **2**. Moreover, the knowledge in each epistemic state will be simply structured as valuations $\mathbf{2}^{\mathrm{Var}}$. We represent each state by the envelope that each child has in that state, for instance 012 is the state where child **a** has **0**, child **b** has **1** and child **c** has **2**; hence $W = \{012, 021, 102, 120, 201, 210\}$. This notation also supports the information of the local models Knowledge representation. For instance, the knowledge structure in state 012 is a function $M(012)$ defined, for each $C_n \in \mathrm{Var}$, as:

$$M(012)(C_n) = \begin{cases} \mathtt{tt} & \text{if } C = 0 \text{ and } n = a \\ \mathtt{tt} & \text{if } C = 1 \text{ and } n = b \\ \mathtt{tt} & \text{if } C = 2 \text{ and } n = c \\ \mathtt{ff} & \text{otherwise} \end{cases}$$

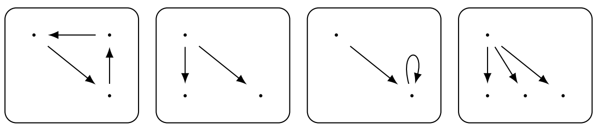
Finally, we take the $\{a, b, c\}$-epistemic relation $\sim$ as expected. For instance, we have, for Ana, the relation $\sim_a = \{(012, 012), (012, 021), (021, 021), \dots\}$. The graphical representation of $\mathcal{M} = (W, \sim, M)$ is given as follows:



Observe that $\mathcal{M}, 012 \models B_b 0_a$ since, $012 \sim_b 012$ and $\mathcal{M}, 012 \models 0_a$ (because $M(012)(0_a) = \mathtt{tt}$, i.e. $M(012) \models_{\mathcal{PL}} 0_a$). Moreover, it is not difficult to see that $021 \models E_{ac} 2_b$ does not hold.

The next two examples introduce two new logics on means of two instantiations of our epistemisation method:

Example 3.5 Let us now consider the following game: from the universe of structures

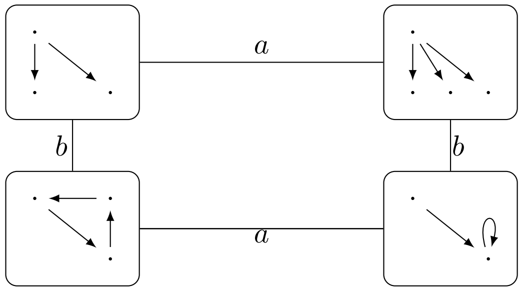


one graph is chosen. Anna and Bob will play with this hidden graph with the following additional information:

- Anna knows that the graph is deterministic
- Bob knows that the graph has exactly 3 nodes

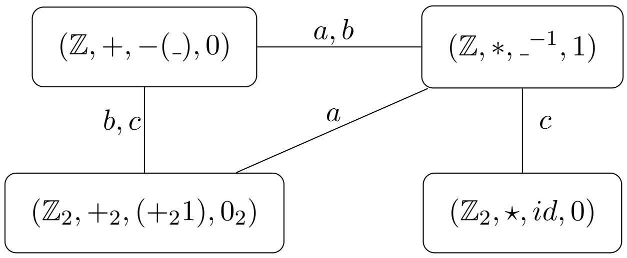
This knowledge perception can be represented directly by means of an epistemic structured model, whose epistemic states consist precisely in these four possible graphs. As it is well known, *hybrid logic with binders* is a very good candidate to express (local) properties about these structures (e.g. [12]). Very succinctly, the set of hybrid formulas over a signature $(Prop, Nom)$, $Prop$ the set of propositions and Nom the set of nominals, enriched the standard modal language over $Prop$ with the formulas i, which only hold at the state named by the nominal i, $@_i \rho$, which asserts that formula ρ holds in the state named by the nominal i and the formulas $\downarrow x.\rho$ that are true in a given state w, if ρ is true in w whenever all the occurrences of x in φ refers to w.

Hence, let us build the logic $\mathcal{E}(\mathcal{H}(\downarrow, @))$ in order to deal with this scenario. Hence, the intended $H(\downarrow, @)$-epistemic structure $\mathcal{M}$ is represented as follows:



Using the features of hybrid logic, we are able to express the players information with the sentences $K_a\big(\downarrow x.(\diamond \mathtt{tt} \longrightarrow \diamond \downarrow y.@_x\Box y)\big)$ and $K_b\big((i\vee j\vee k)\wedge(\neg@_j k\wedge\neg@_i k\wedge\neg@_i j)\big)$ Other formulas valid in this model are *Anna knows, that Bob does not knows, that the graph is deterministic* expressed in the sentence $K_a\big(\neg K_b(\downarrow x.(\diamond \mathtt{tt} \longrightarrow \diamond \downarrow y.@_x\Box y)\big)$.

Example 3.6 In order to illustrate the 'epistemisation' of equational logic, the logic $\mathcal{E}(\mathrm{EQ})$, let us consider a similar game of the one used in the previous example. From the four algebras (in a signature with a binary symbol $\odot$, an unary operation symbol inv and a constant symbol e) depicted in the model $\mathcal{N}$ represented bellow, one algebra is chosen.



The operations of the algebras of the top line are the usual integers sum and product and the respective inverses. The operations of the bottom left algebra are the standard sum and inverse on the cyclic field $\mathbb{Z}_2$. The binary operation $\star$ of the bottom right algebra is defined by

$$\star(x, y) = \begin{cases} 1 & \text{if } x = 0, y = 1 \\ 0 & \text{otherwise} \end{cases}.$$

The following properties are valid in this model: *Anna knows that $\odot$ is associative*, expressed by the sentence $K_a\big((x \odot y) \odot z = x \odot (y \odot z)\big)$; *Bob knows that e is a neutral element* expressed by $K_b\big(x \odot e = x \wedge e \odot x = x\big)$; and finally, *Clara knows that every element has an inverse*, expressed by the sentence $K_c\big(x \odot x^{-1} = e \wedge x^{-1} \odot x = e\big)$.

4 Public Announcements

The present section enrich 'epistemisations' with the public announcements. These are announcements that are made publicly to all agents and their content is common knowledge among all agents.

Definition 4.1 The formulas for the *$\mathcal{L}$-epistemic logic with public announcement* (for a set of agents $\mathcal{A}$) , in symbols $\mathrm{Fm}^{\mathrm{pub}}_{\mathcal{E}(\mathcal{L})}$, is defined by the following grammar:

$$\varphi ::= \varphi_0 \mid \top \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi \mid C_G\varphi \mid [\varphi]\varphi$$

where $\varphi_0 \in \mathrm{Fm}_{\mathcal{L}}$, $a \in \mathcal{A}$ and $G \subseteq \mathcal{A}$.

This grammar extends the on presented in Definition 3.1 with the formulas $[\varphi]\varphi'$ expressing the situation 'after the public announcement of φ, formula φ' is true'.

Definition 4.2 Given $\mathcal{L}$-epistemic *model* for a set of agents $\mathcal{A}$, $w \in W$, and $\varphi \in \mathrm{Fm}^{\mathrm{pub}}_{\mathcal{E}(\mathcal{L})}$, the satisfaction relation

$$\models^{\mathrm{pub}} \subseteq \mathrm{Mod}_{\mathcal{E}(\mathcal{L})} \times \mathrm{Fm}^{\mathrm{pub}}_{\mathcal{E}(\mathcal{L})}$$

extends the definition of the relation $\models$ with:

- $\mathcal{M}, w \models^{\mathrm{pub}} [\chi]\varphi$ iff $\mathcal{M}, w \models^{\mathrm{pub}} \chi$ implies $\mathcal{M}|_{\chi}, w \models^{\mathrm{pub}} \varphi$

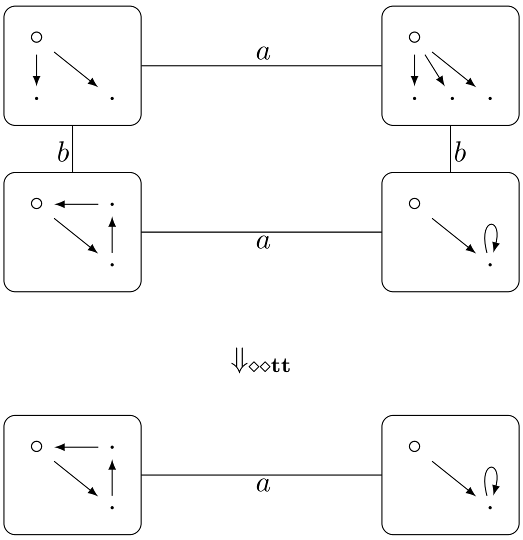
where $\mathcal{M}|_{\chi} = (W|_{\chi}, \sim|_{\chi}, M|_{\chi})$ is the the $\mathcal{L}$-epistemic *model* for a set of agents $\mathcal{A}$ where:

- $W_{\chi} = \{w \mid \mathcal{M}, w \models^{\mathrm{pub}} \chi\}$;
- for any $a \in \mathcal{A}$, $(\sim_a)|_{\chi} = \sim_a \cap (W_{\chi} \times W_{\chi})$;
- $M|_{\chi}$ is the restriction of M to W_{χ}, i.e. the function defined for any $w \in W_{\chi}$ by $M|_{\chi}(w) = M(w)$.

As usual we use $\langle\chi\rangle\varphi$ to denote its dual $\neg[\chi]\neg\varphi$. Semantically, we have

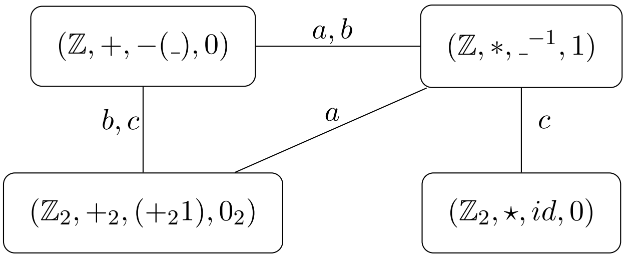
$$\mathcal{M}, w \models^{\mathrm{pub}} \langle\chi\rangle\varphi \text{ iff } \mathcal{M}, w \models^{\mathrm{pub}} \chi \text{ and } \mathcal{M}|_{\chi}, w \models^{\mathrm{pub}} \varphi$$

Example 4.3 Let us suppose, in the context of Example 3.5, that it is announced that 'We can execute two consecutive transitions' $\diamond\diamond\mathbf{tt}$. This entails that the updated model $\mathcal{M}|_{\diamond\diamond\mathbf{tt}}$ depicted as follows:

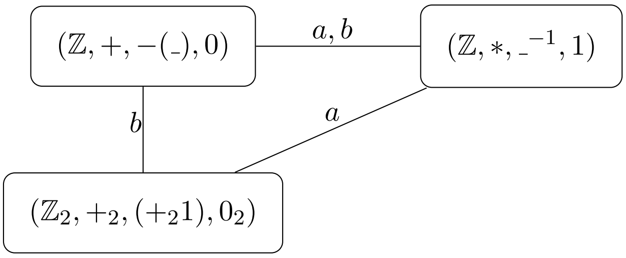


Now, we have, for instance that $\mathcal{M} \models \neg K_b(\Diamond(\downarrow x.\Diamond\Diamond\Diamond x))$ but $\mathcal{M} \models [\diamond\diamond\mathbf{tt}]K_b(\Diamond(\downarrow x.\Diamond\Diamond\Diamond x))$.

Example 4.4 In the situation of Example 3.6, let us suppose that it is announced that the algebra chosen is a group (let us assume Φ as the conjunction of the equational axiomatization of the (variety) of Groups e.g.[5]). The model $\mathcal{N}|_{\Phi}$ is represented in the following diagram:



$$\Downarrow_\Phi$$



Now, we have for instance that $\mathcal{N} \models [\Phi]K_c(a \odot (b \odot c) = (a \odot b) \odot c$ but $\mathcal{N} \not\models K_c(a \odot (b \odot c) = (a \odot b) \odot c$.

5 Bisimulation Invariance

In this section we introduce a parametric notion of bisimulation that preserves properties in 'epistemisations'.

Definition 5.1 [Bisimulation and bisimilarity] Let $\mathcal{L}$ be a Knowledge representation framework and $\mathcal{M} = (W, \sim, M)$ and $\mathcal{M}' = (W', \sim', M')$ two $\mathcal{L}$-epistemic models for the set of agents $\mathcal{A}$. A *bisimulation* between $\mathcal{M}$ and $\mathcal{M}'$ is a relation $R \subseteq W \times W'$ such that, for each $a \in \mathcal{A}$, and for any $w, v \in W$ with $(w, w') \in R$:

(atom) $M(v) \equiv_{\mathcal{L}} M(v')$;

(zig) for any $v \in W$, if $w \sim_a v$, then there is a $v' \in W'$ such that $w' \sim'_a v'$ and $(v, v') \in R$;

(zag) for any $v' \in W'$, if $w' \sim'_a v'$, then there is a $v \in W$ such that $w \sim_a v$ and $(v, v') \in R$.

Whenever a bisimulation between $w \in W$ and $w' \in W'$ exists, we say that w and w' are bisimilar and we write $(M, w) \leftrightarrows (M', w')$.

Note that, as it is well known, the *bisimilarity* relation $\leftrightarrows$ is itself a bisimulation. The modal invariance result for the Epistemic Logic is extended for the generic $\mathcal{L}$-Epistemic logics:

Theorem 5.2 (Invariance) *Let $\mathcal{M} = (W, \sim, M)$ and $\mathcal{M}' = (W', \sim', M')$ two $\mathcal{L}$-epistemic models for the set of agents $\mathcal{A}$ and $w \in W$ and $w' \in W'$. Then, if $(\mathcal{M}, w) \leftrightarrows (\mathcal{M}', w')$, we have that, for any formula $\varphi \in \mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$,*

$$\mathcal{M}, w \models \varphi \textit{ iff } \mathcal{M}', w' \models \varphi$$

Proof. We prove this result by induction over the structure of $\mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$. For the atomic sentences $\varphi_0 \in \mathrm{Fm}_{\mathcal{L}}$, we just have to observe that

$$
\begin{aligned}
& \mathcal{M}, w \models \varphi_0 \\
\Leftrightarrow \quad & \{ \models \text{defn} \} \\
& M(w) \models_{\mathcal{L}} \varphi_0 \\
\Leftrightarrow \quad & \{ \text{By Defn 5.1 } M(w) \equiv_{\mathcal{L}} M'(w') \} \\
& M'(w') \models_{\mathcal{L}} \varphi_0 \\
\Leftrightarrow \quad & \{ \models \text{defn} \} \\
& \mathcal{M}', w' \models \varphi_0
\end{aligned}
$$

The proof for other cases is done as for the Epistemic Logic (eg. [28]). $\square$

Let us consider the following auxiliary result:

Lemma 5.3 *Let $\mathcal{M} = (W, \sim, M)$ and $\mathcal{M}' = (W', \sim', M')$ two $\mathcal{L}$-epistemic models for the set of agents $\mathcal{A}$, $w \in W, w' \in W'$ and $\varphi \in \mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$. Then, if $(\mathcal{M}, w) \leftrightarroweq (\mathcal{M}', w')$ and $\mathcal{M}, w \models \varphi$, we have $(\mathcal{M}|_{\varphi}, w) \leftrightarroweq (\mathcal{M}'|_{\varphi}, w')$.*

Proof. Let us assume a bisimulation $B \subseteq W \times W'$ with $(w, w') \in B$. Let us suppose that $\mathcal{M}, w \models \varphi$. Hence, in order to prove $(\mathcal{M}|_{\varphi}, w) \leftrightarroweq (\mathcal{M}'|_{\varphi}, w')$, it is enough to show that $B|_{\varphi} = B \cap (W|_{\varphi} \times W'|_{\varphi})$ is a bisimulation. Let us start to see $(\mathbf{zig})$. For any $a \in \mathcal{A}$ and $v \in W|_{\varphi}$,

$$
\begin{aligned}
& w\, (\sim_a)|_{\varphi} v \\
\Rightarrow \quad & \{ |_{\varphi} \text{ defn} \} \\
& w \ \sim_a \ v \\
\Rightarrow \quad & \{ \text{Since } B \text{ is bisimulation} \} \\
& \exists v' \in W' \text{ such that } w' \sim'_a v' \text{ and } (v, v') \in B \\
\Rightarrow \quad & \{ \text{By Thm 5.2, } w' \in W'|_{\varphi} + |_{\varphi} \text{ dfn.} \} \\
& w'\, (\sim'_a)|_{\varphi} v' \text{ and } (v, v') \in B
\end{aligned}
$$

The proof for $(\mathbf{zag})$ is analogous and for $(\mathbf{atom})$ is trivial. $\square$

Theorem 5.4 *Let $\mathcal{M} = (W, \sim, M)$ and $\mathcal{M}' = (W', \sim', M')$ two $\mathcal{L}$-epistemic models for the set of agents $\mathcal{A}$ and $w \in W$ and $w' \in W'$. Then, if $(\mathcal{M}, w) \leftrightarroweq (\mathcal{M}', w')$, we have that, for any formula $\varphi \in \mathrm{Fm}^{\mathrm{pub}}_{\mathcal{E}(\mathcal{L})}$,*

$$\mathcal{M}, w \models^{\mathrm{pub}} \varphi \textit{ iff } \mathcal{M}', w' \models^{\mathrm{pub}} \varphi$$

Proof.

This proof extends the one of Theorem 5.2 with the proof of invariance of the sentences $[\varphi]\rho$. On this view, we just have to observe that

$$
\begin{aligned}
& \mathcal{M}, w \models^{\text{pub}} [\varphi]\rho \\
\Leftrightarrow \quad & \{ \models^{\text{pub}} \text{defn} \} \\
& \mathcal{M}, w \models^{\text{pub}} \varphi \Rightarrow \mathcal{M}|_{\varphi}, w \models^{\text{pub}} \rho \\
\Leftrightarrow \quad & \{ \text{Lemma 5.3 + H.I.} \} \\
& \mathcal{M}', w' \models^{\text{pub}} \varphi \Rightarrow \mathcal{M}'|_{\varphi}, w' \models^{\text{pub}} \rho \\
\Leftrightarrow \quad & \{ \models^{\text{pub}} \text{defn} \} \\
& \mathcal{M}', w' \models^{\text{pub}} [\varphi]\rho
\end{aligned}
$$

□

6 Research agenda

This paper introduced a method to build suitable epistemic logics to deal with scenarios where the representation of knowledge needs richer presentations than sets of propositions. The illustrations provided shown the genericity of the method by presenting epistemic logics which states are represented with three paradigmatic structures of computer science - sets of propositions, graphs and algebras. Moreover, a parametric notion of bisimulations, invariant to the 'epistemisations' was also introduced. As happens in other modal logics, this is a very important result, not only to identify equivalent epistemic models, but also to base the development of algorithms for the minimisation of models (a crucial aspect to scale up this approach to the analysis of real practical scenarios).

Next step would be to provide a method to axiomatise our "epistemizations" and to establish (under some conditions) their completeness, soundness, finite model property, decidability and complexity. We also would like to extend our method with the common knowledge operator.

It is important to notice that this approach can be potentially useful in a wide range of application domains. For instance, in order to modelling a given autonomous hybrid system, we can understand sensors as agents that partially know the state of the system (e.g. some of them known the vertical acceleration, some other the current position etc.). The 'epistemisation' of differential dynamic logic [24], is a good candidate to perform this analysis. By specifying, as common knowledge, the classic Newtonian laws of mechanics (on means of ordinary differential equations), we are able to reason about the whole system behaviour, to formalise cooperative strategies, etc.

Another natural application domain for this research would be the analysis of cryptographic protocols, in particular asymmetrical cryptography protocols, by assuming public keys as common knowledge and private keys as specific agent knowledge. The structured representation of epistemic states, for instance as algebras, endows the method with expressibility to analyse more complex cryptographic schemes dealing with partial shared information.

Other interesting, less conventional, logics can emerge for this method. For instance, it would be interesting to derive the 'epistemisation' of the probabilistic [23] or of the fuzzy logic (e.g. [15]) and analyse their relation with the probabilistic and multi-valued epistemic logics in the literature (e.g. [18,3]).

Finally, we note that the parametric nature of the method can also pave the way for the development of their own computational supporting tools. Our previous experience in demand driven generation of specification logics, parametric to the specificities of some classes of complex systems (e.g. [19,20,3]) will certainly hint this research. For instance, in the line of what we have done on the parametric construction of hybrid logics [19], we expect to introduce a method to derive logic calculus for 'epistemisation', parametric to the calculus of the base logic (cf. [22]). This would provide the proof support necessary to apply this research to the analysis of real case scenarios.

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Questions of the Reviewers

We reply to the comments of the 3 referees pointing out how they have been taken into account. All this issues are commented in detail below. Our answers to the reviewers remarks are in blue.

7 Reviewer 1

7.1 Evaluation

The paper presents a framework for the epistemisation of logical languages. It shows its applicability on three case studies. The work further enrich the language with public announcement operators and prove some results about bisimulation invariance under their new constructs.

The paper is generally well written and the technical results are sound. The presented work is interesting, but I think it lacks of motivation. I do miss in the paper further discussion on what the authors achieve with their approach that other combination methods cannot achieve. It seems to me that fusions of the intended logics with DEL would produce exactly the same logics obtained by the epistemisation construction given in the paper, as it is clear to me that such construction does not yield to any interaction between the logics. I suggest to extended the discussion about related works and, in particular, enlighten on the differences between the construction given here and some more traditional approaches.

We have added a paragraph in tne Section 1 Introduction, paragraph 5, explaining better the motivations, some related work and how our approach differ from other methods of combination of logics

7.2 Minor comments

$\surd$ All the minors have been fixed as requested by the referee.

8 Reviewer 2

8.1 Evaluation

Authors enrich systems of Epistemic Logics by providing structure to the representation of agents knowledge. They present a parametric method to build Epistemic Logics with Public Announcements with structured knowledge together with a parametric notion of bisimulation to these systems. They prove the bisimulation invariance result for the new class of logics. The parametric presentation of the new class of logical systems will make it easier to obtain the meta-properties results needed to the applications.

The text needs revision. I pointed out only some of the typos and mistakes I have identified.

8.2 Typos

$\surd$ All the typos have been fixed as requested by the referee.

9 Reviewer 3

9.1 Evaluation

This paper investigates the use of richer structures to define dynamic epistemic logics. In particular, instead of considering set of valid propositions in a given state/world, the authors search for a formalism that better fits the nature of the knowledge involved. Instances of the framework include graphs and abstract data types.

First, a general knowledge representation framework L is proposed, which is not more than a set of formulas and their models/semantics. Then, an L-epistemic logic uses such an L to build modal formulas dealing with knowledge of agents (the usual K and C modalities for knowledge and common knowledge respectively). (Atomic) Propositions in this logic are L formulas. The semantics of the logic considers an equivalence relation on words and a function that maps words into models of L. Later, L-epistemic logics are enriched with a public announcement operator. Finally, the authors propose a notion of bisimilarity that it is not far from what it is expected. Here, the L-models must be the same (for states in the graph) and then, bisimilar models must make true the same formulas.

All in all I think the idea is interesting: introducing more structure on words induces a more suitable frameworks to specify and verify systems. The question that remains is whether the resulting system can be expressed as the usual set of propositions in words: this looks clear for example 3.4 and 3.6. However, the use of hybrid logic with binders in Example 3.5 makes things more interesting.

We have added a paragraph in tne Section 1 Introduction, paragraph 5, explaining better the motivations, some related work and how our approach differ from other methods of combination of logics

9.2 Typos and Minors

$\surd$ All the typos and minors have been fixed as requested by the referee.