Scale-invariant alternatives to general relativity. III. The inflation-dark energy connection

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We discuss the cosmological phenomenology of biscalar-tensor models displaying a maximally symmetric Einstein-frame kinetic sector and constructed on the basis of scale symmetry and volume-preserving diffeomorphisms. These theories contain a single dimensionful parameter Λ_0—associated with the invariance under the aforementioned restricted coordinate transformations—and a massless dilaton field. At large field values these scenarios lead to inflation with no generation of isocurvature perturbations. The corresponding predictions depend only on two dimensionless parameters, which characterize the curvature of the field manifold and the leading-order behavior of the inflationary potential. For $\Lambda_0 = 0$ the scale symmetry is unbroken and the dilaton admits only derivative couplings to matter, evading all fifth-force constraints. For $\Lambda_0 \neq 0$ the field acquires a runaway potential that can support a dark-energy-dominated era at late times. We confront a minimalistic realization of this appealing framework with observations using a Markov chain Monte Carlo approach, with likelihoods from present baryon acoustic oscillation, type Ia supernova, and cosmic microwave background data. A Bayesian model comparison indicates a preference for the considered model over ΛCDM, under certain assumptions for the priors. The impact of possible consistency relations among the early and late Universe dynamics that can appear within this setting is discussed with the use of correlation matrices. The results indicate that a precise determination of the inflationary observables and the dark energy equation of state could significantly constrain the model parameters.

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I. INTRODUCTION

We have entered an era of precision cosmology. Cosmological parameters are measured with unprecedented accuracy [1] and, in addition to electromagnetic probes, gravitational-wave observations [2–5] tightly constrain a plethora of modified gravity scenarios [6–10].

In spite of the undeniable success of modern cosmology, the origin of the present accelerated expansion of the Universe remains unknown. The next decade of observations—with an upcoming generation of galaxy redshift surveys such as Euclid [11,12] or LSST [13]—will be of crucial importance for determining whether this phase arises due to an inert cosmological constant or rather a dynamical dark energy (DE) component. The combination of these surveys with stage-4 cosmic microwave background (CMB) observations [14] will also pin down the inflationary parameters, setting the stage for more fundamental questions on the relation between the early and late Universe. Indeed, although inflation and dark energy are usually treated as two independent epochs, they might be closely related as happens, for instance, in quintessential inflationary models [15–21] or in certain theories invariant under dilatations [22–24]. A potential confirmation of this appealing hypothesis might completely change our understanding of modern cosmology.

In the last few years there has been renewed interest in the implications of scale and conformal symmetries, and many of their aspects—both formal and phenomenological—have been thoroughly investigated [19,22–72]. In this paper, we focus on the cosmological consequences of a general class of biscalar-tensor models first introduced in Ref. [33] which are invariant under volume-preserving

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diffeomorphisms[1] and display spontaneous breaking of scale symmetry. We restrict ourselves to theories that contain at most two derivatives of the fields, such that the particle spectrum comprises only healthy degrees of freedom (d.o.f.). In addition, we are interested only in models whose Einstein-frame target manifold is maximally symmetric during inflation and, more precisely, globally hyperbolic [50,73].[2] Moreover, we require the equations of motion governing the dynamics of the theories under consideration to admit Minkowski, de Sitter, and anti–de Sitter vacuum solutions, since these might be essential for the eventual quantization of the theory. Finally, when needed, we assume the existence of a hierarchy between the inflationary and particle-physics scales, similar to that between the Planck and electroweak scales.

Even though we will go into further details in what follows, let us spell out some of the most intriguing features of these specific models. On general grounds, these theories contain a single dimensionful parameter Λ_0 associated with the invariance of the action under volume-preserving coordinate transformations. For $\Lambda_0 = 0$, one of the scalar fields—which we will call the *dilaton*—becomes the Goldstone boson of the spontaneously broken scale symmetry. The combination of gravity and dilatation invariance forces this field to have only derivative couplings to matter. Consequently, the fifth-force effects associated with the dilaton are highly suppressed in this particular context [22,31,53,67]. Additionally, scale invariance forbids the generation of isocurvature perturbations during the inflationary stage due to the presence of a conserved (scale) current that effectively reduces the biscalar theory to a single-field scenario. Interestingly, these theories also admit an "α-attractor" solution [74–76] for the spectral tilt and tensor-to-scalar ratio [50]. For $\Lambda_0 \neq 0$, the dilatation symmetry is explicitly broken. The combination of this specific symmetry-breaking term with the omnipresent nonminimal coupling to gravity of scalar-tensor theories leads to a *unique* quintessential potential for the dilaton field [31]. For sufficiently small values of Λ_0, all the inflationary properties mentioned previously are approximately realized and the dilaton remains an almost massless d.o.f. potentially responsible for the current accelerated expansion of the Universe. This, in turn, can lead to a set of nontrivial consistency conditions between the inflationary observables and the dark energy equation-of-state parameter, which could be tested with future cosmological observations.

The paper is organized as follows. In Sec. II, we introduce the notion of transverse diffeomorphisms and, closely following Ref. [33], we construct the most general class of scale-invariant biscalar models invariant under this type of transformations. In Sec. III, we recast the obtained set of models in the Einstein frame, where the gravitational part of the action takes the usual Einstein-Hilbert form. After discussing the general features above, we focus on models involving a maximally symmetric field manifold in the Einstein frame. The cosmological consequences of this broad class of theories are considered in Sec. IV, while in Sec. V we make use of a Markov chain Monte Carlo approach to confront a particular realization of our scenario with present data sets and discuss the chances of differentiating it from other cosmological scenarios such as ΛCDM. Finally, our conclusions are presented in Sec. VI.

II. SCALE-INVARIANT BISCALAR MODELS

Our current understanding of the gravitational interaction is based on a massless spin-2 field: the graviton. In general relativity, this d.o.f. is associated with general coordinate transformations or diffeomorphisms (Diffs). At the infinitesimal level, these transformations take the form

$$x^\mu \to x^\mu(x) + \delta x^\mu(x), \tag{1}$$

where δx^μ is arbitrary. In spite of this "traditional" association, the minimal group leading to graviton excitations *is not* the group of general coordinate transformations, but rather the subgroup spanned by the transverse vectors

$$\delta x^\mu = \xi^\mu, \quad \text{with} \quad \partial_\mu \xi^\mu = 0. \tag{2}$$

In what follows we will refer to these transformations as volume-preserving, restricted, or transverse diffeomorphisms (TDiff) interchangeably. It should be clearly stated that, in general, theories invariant under Eq. (2) propagate an extra scalar d.o.f. related to the metric determinant on top of the two graviton polarizations.

Contrary to what happens in diffeomorphism-invariant theories, the requirement of invariance under TDiffs (2) does not completely determine the form of the action. In particular, it is always possible to include arbitrary functions of the metric determinant $g \equiv -\det(g_{\mu\nu})$ in the Lagrangian density, since this quantity transforms as a scalar under volume-preserving diffeomorphisms. As shown in Ref. [33], the most general TDiff action that is also invariant under the scale transformations [$g_{\mu\nu}(x)$ is the metric and $\phi(x)$ is a scalar field with scaling dimension one]

$$g_{\mu\nu}(x) \mapsto g_{\mu\nu}(\lambda x), \quad \phi(x) \mapsto \lambda\phi(\lambda x), \quad \lambda = \text{const}, \tag{3}$$

takes the form

[1] The precise definition of these transformations can be found in the next section.

[2] The reasons for this choice will become apparent later, but the essence is that under this assumption the arbitrariness in the construction of the corresponding action is greatly reduced.

$$\mathcal{S}=\int d^4x\sqrt{g}\left\{\frac{\phi^2 f(g)}{2}R-\frac{\phi^2}{2}[G_1(g)(\partial g)^2 -2G_2(g)(\partial g)(\phi^{-1}\partial\phi)+G_3(g)(\phi^{-1}\partial\phi)^2]-\phi^4 v(g)\right\}, \tag{4}$$

where f, G_1, G_2, G_3, and v are arbitrary functions of the metric determinant. For general choices of these *theory-defining functions*, the action (4) contains three propagating d.o.f. on top of the scalar field ϕ: the two graviton polarizations and a new scalar associated with the metric determinant.[3] The existence of this additional d.o.f. can be made explicit by rewriting the above action in a Diff-invariant form. To this end, we first transform Eq. (4) to an arbitrary coordinate frame [i.e., we perform a *general* coordinate transformation with Jacobian $J(x)\neq 1$] to obtain [33]

$$\mathcal{S}=\int d^4x\sqrt{g}\left\{\frac{\phi^2 f(\frac{g}{a})}{2}R-\frac{\phi^2}{2}\left[G_1\left(\frac{g}{a}\right)(\partial g/a)^2 - 2G_2\left(\frac{g}{a}\right)(\partial g/a)(\phi^{-1}\partial\phi)+G_3\left(\frac{g}{a}\right)(\phi^{-1}\partial\phi)^2\right] - \phi^4 v\left(\frac{g}{a}\right)-\frac{\Lambda_0}{\sqrt{g/a}}\right\}, \tag{5}$$

where $a(x)\equiv J(x)^{-2}$ and Λ_0 is a unique scale symmetry-breaking term that arises as an integration constant in the original TDiff formulation.[4] Promoting $a(x)$ to a (dynamical) compensator field transforming under Diffs as

$$\delta_\xi a=\xi^\mu\partial_\mu a+2a\partial_\mu\xi^\mu, \tag{6}$$

the Lagrangian density in Eq. (5) can equivalently be written as

$$\frac{\mathcal{L}}{\sqrt{g}}=\frac{\phi^2 f(\tilde\theta)}{2}R-\frac{\phi^2}{2}[G_1(\tilde\theta)(\partial\tilde\theta)^2+2G_2(\tilde\theta)(\partial\tilde\theta)(\phi^{-1}\partial\phi) + G_3(\tilde\theta)(\phi^{-1}\partial\phi)^2]-\phi^4 v(\tilde\theta)-\frac{\Lambda_0}{\sqrt{\tilde\theta}}, \tag{7}$$

with $\tilde\theta\equiv g/a>0$ [33,47]. This expression is, by construction, invariant under general coordinate transformations and reduces to the TDiff form (4) in the $a=1$ gauge. Given the (classical) equivalence of the TDiff- and Diff-invariant formulations [31,33], we will work in what follows with the more familiar diffeomorphism-invariant form. Note also that small choices of Λ_0 are technically natural [86] (see also Refs. [84,85]). Indeed, for $\Lambda_0=0$, the action associated with Eq. (7) is invariant under scale transformations, which are now internal. This means that the coordinates are kept fixed, while the various fields change as[5]

$$g_{\mu\nu}(x)\mapsto\lambda^2 g_{\mu\nu}(x),\quad \phi(x)\mapsto\lambda\phi(x),\quad \tilde\theta(x)\mapsto\tilde\theta(x). \tag{8}$$

III. EINSTEIN-FRAME FORMULATION

The phenomenological consequences of the theories under consideration are most easily studied in the *Einstein frame*, in which the gravitational part of the action takes a "canonical form." Requiring the existence of a well-defined graviton propagator at all field values, i.e., $\phi^2 f(\tilde\theta)>0$, we can perform the Weyl rescaling $g_{\mu\nu}\to M_P^2/(\phi^2 f(\tilde\theta))g_{\mu\nu}$, to rewrite Eq. (7) as

$$\frac{\mathcal{L}}{\sqrt{g}}=\frac{M_P^2}{2}R-\frac{1}{2}[M_P^2K_1(\tilde\theta)(\partial\tilde\theta)^2+2M_PK_2(\tilde\theta)(\partial\tilde\theta)(\partial\tilde\Phi) + K_3(\tilde\theta)(\partial\tilde\Phi)^2]-U(\tilde\theta)-\frac{\Lambda_0}{f^2(\tilde\theta)\sqrt{\tilde\theta}}e^{-4\tilde\Phi/M_P}, \tag{9}$$

where $M_P=2.48\times10^{18}$ GeV is the reduced Planck mass. In the above expression, we introduced the following $\tilde\theta$-dependent functions:

$$K_1(\tilde\theta)\equiv\frac{G_1(\tilde\theta)}{f(\tilde\theta)}+\frac{3}{2}\left(\frac{f'(\tilde\theta)}{f(\tilde\theta)}\right)^2,\qquad K_2(\tilde\theta)\equiv\frac{G_2(\tilde\theta)}{f(\tilde\theta)}+3\frac{f'(\tilde\theta)}{f(\tilde\theta)}, \tag{10}$$

$$K_3(\tilde\theta)\equiv 6+\frac{G_3(\tilde\theta)}{f(\tilde\theta)},\qquad U(\tilde\theta)=\frac{M_P^4 v(\tilde\theta)}{f^2(\tilde\theta)}, \tag{11}$$

and used primes to denote derivatives with respect to $\tilde\theta$. Note that the rescaled field

$$\tilde\Phi\equiv M_P\ln\left(\frac{\phi}{M_P}\right) \tag{12}$$

is defined in such a way that the scale transformations (8) act on it as a shift.

The nondiagonal kinetic terms in Eq. (9) can be diagonalized by considering an additional field redefinition [33,50],

$$\tilde\Phi\to\Phi=\tilde\Phi-M_P\int^{\tilde\theta}d\tilde\theta'\frac{K_2(\tilde\theta')}{K_3(\tilde\theta')}. \tag{13}$$

[3]The additional d.o.f. is only absent for very particular choices of the theory-defining functions, leading either to general relativity or to unimodular gravity [77–83]. Interestingly, these two limiting cases are completely equivalent at the classical level but not necessarily when quantum corrections are taken into account [84,85].

[4]For more details on this point, the interested reader is referred to Ref. [33].

[5]This should be compared with the transformation (3).

Once this is performed, we obtain the Lagrangian density

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2}R - \frac{1}{2}[M_P^2 K(\tilde{\theta})(\partial\tilde{\theta})^2 + K_3(\tilde{\theta})(\partial\Phi)^2] - U(\tilde{\theta}) - U_{\Lambda_0}(\tilde{\theta},\Phi), \tag{14}$$

with

$$U_{\Lambda_0}(\tilde{\theta},\Phi) = \Lambda_0 K_\Lambda(\tilde{\theta})e^{-4\Phi/M_P}, \tag{15}$$

and

$$K(\tilde{\theta}) = \frac{K_1(\tilde{\theta})K_3(\tilde{\theta}) - K_2^2(\tilde{\theta})}{K_3(\tilde{\theta})},$$
$$K_\Lambda = \frac{1}{f^2(\tilde{\theta})\sqrt{\tilde{\theta}}}\exp\left(4\int\frac{K_2(\tilde{\theta}')}{K_3(\tilde{\theta}')}d\tilde{\theta}'\right). \tag{16}$$

In order to ensure the absence of ghost-like excitations and to prevent the potential appearance of anti–de Sitter regimes, we will require the $\tilde{\theta}$-dependent functions in these expressions to be positive at all field values, namely,[6]

$$K(\tilde{\theta})>0, \quad K_3(\tilde{\theta})>0, \quad U(\tilde{\theta})\geq 0, \quad \Lambda_0 K_\Lambda(\tilde{\theta})\geq 0. \tag{17}$$

Note that these conditions do not restrict the derivatives of the corresponding functions, which could be negative for particular field ranges, allowing for instance for limited tachyonic instabilities.

For $\Lambda_0 = 0$, the Lagrangian density (14) acquires an emergent shift symmetry $\Phi \to \Phi + M_P C$, where C is a constant. This symmetry is nothing else than a manifestation of the nonlinear realization of the original scale symmetry (3) [or equivalently Eq. (8)] that the theory exhibits in the scaling frame (7). The field Φ is therefore identified as the Goldstone boson or *dilaton* associated with the spontaneous breaking of scale invariance. For $\Lambda_0 K_\Lambda(\tilde{\theta}) > 0$, the symmetry is explicitly broken and the dilaton acquires the runaway potential (15).

Given the Lagrangian density in the form (14), it is still possible to perform additional field redefinitions to modify the precise structure of the theory-defining functions $K(\tilde{\theta})$, $K_3(\tilde{\theta})$, etc. For instance, if $K(\tilde{\theta}) \neq 0$, we can introduce a variable[7]

$$\theta = \int_{\tilde{\theta}_0}^{\tilde{\theta}} d\tilde{\theta}'\sqrt{\left|\frac{K_1(\tilde{\theta}')K_3(\tilde{\theta}') - K_2^2(\tilde{\theta}')}{K_3(\tilde{\theta}')}\right|}, \tag{18}$$

in terms of which the kinetic term of $\tilde{\theta}$ becomes canonical.[8] In this case, we get the following Lagrangian density:

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2}R - \frac{1}{2}[M_P^2(\partial\theta)^2 + K_3(\theta)(\partial\Phi)^2] - U(\theta) - U_{\Lambda_0}(\theta,\Phi). \tag{19}$$

This freedom to perform field redefinitions can be trivially understood once the scalars are viewed as the coordinates of the two-dimensional field manifold. In fact, this interpretation allows to rewrite the Einstein-frame Lagrangian in the explicitly covariant form

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2}R - \frac{1}{2}\gamma_{ab}g^{\mu\nu}\partial_\mu\varphi^a\partial_\nu\varphi^b - V(\varphi). \tag{20}$$

Here, the latin indices $a, b, \ldots = 1, 2$ denote the two real scalars present in the model, γ_{ab} is the metric in this field space, and

$$V(\varphi) = U(\varphi) + U_{\Lambda_0}(\varphi). \tag{21}$$

The variation of the action associated with the Lagrangian density (20) leads to the Einstein and Klein-Gordon equations, respectively,

$$M_P^2 G_{\mu\nu} = -\gamma_{ab}\left(\partial_\mu\varphi^a\partial_\nu\varphi^b - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\varphi^a\partial_\sigma\varphi^b\right) + Vg_{\mu\nu}, \tag{22}$$

$$\Box\varphi^c + g^{\mu\nu}\mathcal{G}^c_{ab}\partial_\mu\varphi^a\partial_\nu\varphi^b = \gamma^{cd}V_{,d}, \tag{23}$$

where $G_{\mu\nu}$ is the Einstein tensor computed from the Einstein-frame *spacetime metric* $g_{\mu\nu}$ and $\mathcal{G}^c_{ab}$ is the (symmetric) affine connection computed from the Einstein-frame *field-space metric* γ_{ab}, i.e.,[9]

$$\mathcal{G}^c_{ab} = \frac{1}{2}\gamma^{cd}(\gamma_{da,b} + \gamma_{db,a} - \gamma_{ab,d}). \tag{24}$$

A. Scale current and single-field dynamics

In the absence of the dimensionful parameter Λ_0, the scale invariance of the theories under consideration leads to the existence of a (covariantly) conserved current, which can be obtained from Noether's theorem. In the Einstein frame, it reads

$$J^\mu = -\gamma_{ab}\partial^\mu\varphi^a\Delta\varphi^b, \tag{25}$$

[6]The shift (13) excludes the $K_3(\tilde{\theta}) = 0$ case. The choice $K(\tilde{\theta}) = 0$ is also excluded, since in such a case the target manifold would be one dimensional. This means that one of the two propagating d.o.f. becomes nondynamical.

[7]Here, $\tilde{\theta}_0$ is an arbitrary integration constant ensuring that $\theta(\tilde{\theta}_0) = 0$.

[8]In general, the kinetic sector can always be diagonalized. However, the kinetic mixing among the fields *cannot* be removed, unless the target manifold is flat.

[9]As customary, commas denotes partial derivatives.

with $\Delta\varphi^a$ denoting the infinitesimal action of dilatations on the fields. Note that both the explicit form of $\Delta\varphi^a$ and the current depend on the variables under consideration. For instance, for the variables in Eq. (9) we have $\Delta\varphi^a \equiv (\Delta\tilde{\theta}, \Delta\tilde{\Phi}) = (0, M_P)$, and

$$J^\mu = -M_P(K_3(\tilde{\theta})\partial^\mu\tilde{\Phi} + M_P K_2(\tilde{\theta})\partial^\mu\tilde{\theta}). \tag{26}$$

For the ones in Eq. (14), we see that the infinitesimal transformation corresponds to $\Delta\varphi^a = (\Delta\theta, \Delta\Phi) = (0, M_P)$, and the current is given by

$$J^\mu = -M_P K_3(\theta)\partial^\mu\Phi. \tag{27}$$

From either Eq. (26) or Eq. (27), we find that the (covariant) divergence of the scale current takes the form

$$\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}J^\mu) = 4U_{\Lambda_0}, \tag{28}$$

clearly showing that the above indeed vanishes only for $\Lambda_0 = 0$. For homogeneous fields in the cosmologically relevant Friedmann-Lemaître-Robertson-Walker background, this equation takes the very simple form

$$\frac{1}{a^3}\frac{d}{dt}(a^3\gamma_{ab}\dot{\varphi}^a\Delta\varphi^b) = 4U_{\Lambda_0}, \tag{29}$$

where $a = a(t)$ is the scale factor and the dots stand for derivatives with respect to the coordinate time t. For small Λ_0 (and/or sufficiently large dilaton expectation values), the contribution of the symmetry breaking term on the right-hand side of this equation can be safely neglected. In this limit, the quantity $a^3\gamma_{ab}\dot{\varphi}^a\Delta\varphi^b$ becomes approximately conserved, such that $\gamma_{ab}\dot{\varphi}^a\Delta\varphi^b$ approaches zero as the Universe expands. For the particular set of variables in Eq. (14), this statement takes the intuitive form

$$\frac{d\Phi}{dN} \propto \frac{1}{HK_3(\theta)}e^{-3N}, \tag{30}$$

where H is the Hubble parameter and N is the number of e-folds. An immediate consequence of this equation is that $d\Phi/dN = 0$ is actually an attractor solution, leading to an effective constraint in the $\{h, \chi\}$ plane [22]. The existence of this attractor is of course a physical statement independent of the frame in which the scale current is computed. In particular, one could perform the same computation in the scaling frame (7). In this case, it is simpler to obtain the precise expression for the current from Noether's theorem,

$$J^\mu = \frac{\delta\mathcal{L}}{\delta(\partial_\mu g_{\nu\lambda})}\Delta g_{\nu\lambda} + \frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)}\Delta\phi^i. \tag{31}$$

Taking into account the infinitesimal form of Eq. (8), namely, $\Delta g_{\mu\nu} = -2g_{\mu\nu}$ and $\Delta\phi = \phi$, we get

$$J^\mu = -\frac{1}{2}[(G_3(\tilde{\theta}) + 6f(\tilde{\theta}))\partial^\mu\phi^2 + 2\phi^2(G_2(\tilde{\theta}) + 3f'(\tilde{\theta}))\partial^\mu\tilde{\theta}]. \tag{32}$$

This expression is nothing else than the conformally transformed version of the Einstein-frame current (26), as can be easily verified by taking into account the Weyl rescaling of the metric together with Eqs. (10)–(13).

IV. INFLATION AND DARK ENERGY IN A SINGLE SHOT

The kinetic sector of Eq. (14) constitutes a nonlinear sigma model. The associated (Gauss) curvature of the Einstein-frame target manifold in Planck units is given by

$$\kappa(\tilde{\theta}) = \frac{K_3'(\tilde{\theta})F'(\tilde{\theta}) - 2F(\tilde{\theta})K_3''(\tilde{\theta})}{4F^2(\tilde{\theta})}, \tag{33}$$

with $F(\tilde{\theta}) \equiv K(\tilde{\theta})K_3(\tilde{\theta})$. It should be obvious at this point that without specifying the various theory-defining functions, it is not possible to extract any detailed information about the dynamics of the theory. However, for inflationary models in which $\kappa(\tilde{\theta})$ is constant—corresponding to a maximally symmetric target manifold—the situation simplifies considerably. The reason is that in that case the above equation can be straightforwardly integrated to obtain [50]

$$K(\tilde{\theta}) = -\frac{K_3'^2(\tilde{\theta})}{4K_3(\tilde{\theta})(\kappa K_3(\tilde{\theta}) + c)}, \tag{34}$$

where c is an arbitrary constant. Assuming that both U and K_Λ are analytic functions of $\tilde{\theta}$ (such that they can be expressed in term of K_3), we can rewrite Eq. (14) as

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2}R - \frac{1}{2}\left[-\frac{M_P^2(\partial\Theta)^2}{4\Theta(\kappa\Theta + c)} + \Theta(\partial\Phi)^2\right] - U(\Theta) - \Lambda_0 K_\Lambda(\Theta)e^{-4\Phi/M_P}, \tag{35}$$

where we have defined the variable $\Theta \equiv K_3(\theta)$ to stress the fact that the function K_3 *itself* plays the role of a *dynamical d.o.f.*. The requirement that both fields have healthy kinetic terms imposes the restrictions

$$\kappa\Theta + c < 0, \qquad \Theta > 0. \tag{36}$$

Maximally symmetric scale-invariant models can naturally support inflation, while providing a unique dark-energy-dominated era. To understand this, let us focus on the pole structure of Eq. (35). The kinetic term for the Θ field in this expression contains two poles, located at $\Theta = 0$ and $\Theta = -c/\kappa$, respectively. The presence of these poles

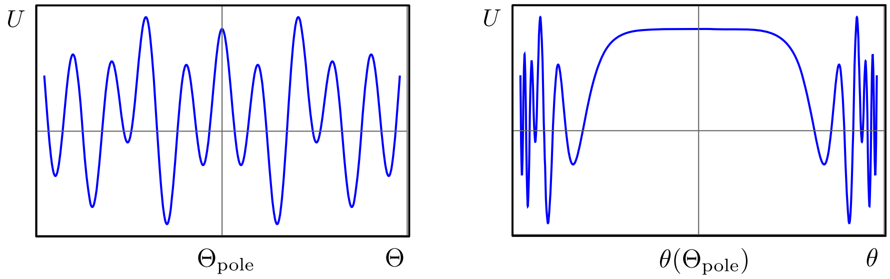

FIG. 1. The effect of the Einstein-frame kinetic pole structure in Eq. (35) for a generic potential $U(\Theta)$. The presence of a pole at the value $\Theta_{\rm pole}$ translates into an effective stretching of the canonically normalized field θ in Eq. (37) and the associated flattening of the potential around $\theta(\Theta_{\rm pole})$. This allows for inflation with the usual slow-roll conditions even if the original potential was not sufficiently flat.

translates into an effective stretching of the canonically normalized field θ, namely,[10]

$$\theta = \int^{\theta} \frac{d\Theta}{\sqrt{-4\Theta(\kappa\Theta + c)}}$$
$$\to \Theta = \begin{cases} \exp\left(-2\sqrt{-\kappa}\theta\right), & \text{for } c = 0, \\ \frac{c}{-\kappa}\cosh(\sqrt{-\kappa}\theta), & \text{for } c \neq 0. \end{cases} \tag{37}$$

For $c = 0$, the two poles coincide and the stretching in θ is exponential, with $\Theta = 0$ corresponding to $\theta = \infty$. For $c \neq 0$, the stretching of θ is restricted to a compact field range around $\theta = 0$.

This flattening of the potential for the canonically normalized field θ allows for inflation with the usual slow-roll conditions [50,76,87], cf. Fig. 1. For sufficiently small values of Λ_0 (and/or sufficiently large values of the dilaton field Φ), the contribution of the U_{Λ_0} term in Eq. (35) is subdominant and can be safely neglected. In the absence of this symmetry-breaking term, the conservation of the dilatation current (25) leads to the attractor behavior (30) and forces the dilaton to freeze at a given value, say Φ_0, during the whole inflationary evolution. As first proved in Ref. [22], this reduces the number of dynamical variables by one and avoids the generation of dangerous isocurvature fluctuations (see also Ref. [63]).

For potentials allowing a graceful inflationary exit, the inflaton field Θ will undergo damped oscillations after the end of inflation and will eventually relax to the ground state of $U(\Theta)$ via particle production. Although the shape of the potential in this transition phase is *a priori* arbitrary, its precise low-energy form can be restricted on phenomenological grounds. To see this, let us neglect for the time being the term proportional to Λ_0 in Eq. (35) and consider the existence of stable solutions involving constant field values $\Phi = \Phi_0$ and $\Theta = \Theta_0$. Demanding $U'(\Theta_0) = 0$, we obtain [cf. Eq. (11)]

$$f(\Theta_0)v'(\Theta_0) - 2f'(\Theta_0)v(\Theta_0) = 0. \tag{38}$$

The Ricci scalar associated with this field configuration can be easily determined by tracing the Einstein equation (22) over spacetime indices. Taking into account that the contributions from the field derivatives in this expression vanish for constant field values together with the second relation in Eq. (11), we obtain

$$R = -4M_P^2 \frac{v(\Theta_0)}{f^2(\Theta_0)}. \tag{39}$$

This expression allows us to distinguish three cases depending on the value of $v(\Theta_0)$. For $v(\Theta_0) = 0$, the background is obviously Minkowski, while for $v(\Theta_0) < 0$ or $v(\Theta_0) > 0$, it becomes de Sitter (dS) or anti–de Sitter (AdS), respectively. While an AdS scenario can be excluded on purely phenomenological grounds, the dS case could potentially lead to a late-time acceleration of the Universe in agreement with the observations. Note, however, that a scale-invariant theory with spontaneous symmetry breaking always contains a massless Goldstone mode, which is known to generate instabilities as far as dS is concerned [89,90]; see also Refs. [91–101]. We are therefore left with a unique scenario that might be phenomenologically viable, namely, the one in which the *induced cosmological constant* following from the potential $U(\Theta_0)$ is appropriately *fine-tuned* to be zero by requiring

$$v(\Theta_0) = v'(\Theta_0) = 0. \tag{40}$$

Note that, although we set $\Lambda_0 = 0$ in the above derivation for the sake of simplicity, this is not a necessary condition. Indeed, even if $\Lambda_0 \neq 0$, the late-time evolution of the Universe will be eventually dominated by a constant component if the condition (40) is not satisfied, giving rise to an eternal de Sitter expansion and to the resurgence of instabilities.

Reinserting the Λ_0 contribution, the Lagrangian (35) at the minimum (40) boils down to

[10]For a detailed discussion on the connection between this approach and the existence of stationary points along the inflationary trajectory, see Refs. [87,88].

$$\frac{\mathcal{L}}{\sqrt{g}} \simeq \frac{M_P^2}{2} R - \frac{1}{2}\Theta_0(\partial\Phi)^2 - \Lambda_0 K_\Lambda(\Theta_0) e^{-4\Phi/M_P}, \quad (41)$$

which must be supplemented with that of the particles produced during the heating stage. If $\Lambda_0 K_\Lambda(\Theta_0) > 0$, the potential term in this expression is of a runaway type. In order to not overclose the Universe, the energy density in the dilaton field should be rather small, namely,

$$\frac{1}{2}\Theta_0(\partial_0\Phi)^2 + \Lambda_0 K_\Lambda(\Theta_0) e^{-4\Phi/M_P} \lesssim 10^{-120} M_P^4, \quad (42)$$

with the right-hand side of the above inequality standing for the present critical energy density. Given this restriction, the expansion rate of the Universe will be initially dominated by the radiation and matter components generated during the heating stage. The field Φ behaves essentially as a thawing quintessence field [26,102–104]. In particular, it stays frozen at the value Φ_0 inherited from inflation until the moment in which the decreasing energy density of the heating products becomes comparable to its approximately constant energy density. When that happens, the dilaton starts rolling towards $\Phi \to \infty$, while driving the present-day accelerated expansion.

A. A worked-out example

To illustrate the cosmological consequences of the general Lagrangian density (35), we will restrict ourselves to a simple scenario involving a maximally symmetric *hyperbolic* field manifold ($\kappa < 0$) and the following set of potentials:

$$U(\Theta) = U_0\left(1 - \frac{\Theta}{\Theta_0}\right)^2, \qquad K_\Lambda(\Theta) = \Theta^2. \quad (43)$$

This choice is done for illustrative purposes only. Indeed, as should be clear from Fig. 1, the field stretching in the vicinity of the kinetic poles makes the observables almost insensitive to the details of the potentials as long as inflation is concerned [50]. Interestingly, the constant Θ_0 denoting the position of the Θ minimum in this example can be reabsorbed into the definition of the dilaton Φ. Indeed, by performing the transformations

$$\Phi \to \gamma\Phi, \qquad \Theta \to \gamma^{-2}\Theta, \qquad c \to \gamma^{-2} c, \quad (44)$$

with $\gamma \equiv \Theta_0^{-1/2}$, we obtain

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{1}{2}\left[-\frac{M_P^2(\partial\Theta)^2}{4\Theta(\kappa\Theta + c)} + \Theta(\partial\Phi)^2\right] - U(\Theta) - U_{\Lambda_0}(\Theta, \Phi), \quad (45)$$

with

$$U(\Theta) = U_0(1-\Theta)^2, \quad U_{\Lambda_0}(\Theta, \Phi) \equiv \frac{\Lambda_0}{\gamma^4}\Theta^2 e^{-4\gamma\Phi/M_P}. \quad (46)$$

Written in this form, the dilaton field Φ becomes canonically normalized at late times (i.e., when $\Theta \to 1$).

1. Inflation

As argued in the previous section, for a phenomenologically viable choice of Λ_0, both the symmetry-breaking term U_{Λ_0} and the dilaton field Φ can be safely neglected during the inflationary stage. Therefore, we are left with a single Θ component, whose scalar and tensor perturbations can be computed using the standard techniques. To this end, we parametrize the spectra of these fluctuations in the almost scale-invariant form [105]

$$P_s = A_s\left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln\left(\frac{k}{k_*}\right)}, \qquad P_t = A_t\left(\frac{k}{k_*}\right)^{n_t}, \quad (47)$$

and compute the inflationary observables

$$A_s = \frac{1}{24\pi^2 M_P^4}\frac{U}{\epsilon}, \qquad n_s = 1 + 2\eta - 6\epsilon, \quad (48)$$

$$\alpha_s = 8\epsilon(2\eta - 3\epsilon) - 2\delta^2, \qquad r \equiv \frac{A_t}{A_s} = -8n_t = 16\epsilon. \quad (49)$$

In the above we have introduced the standard slow-roll parameters, but appropriately adapted to the noncanonical scalar field Θ,

$$\epsilon \equiv \frac{M_P^2}{2K}\left(\frac{U_{,\Theta}}{U}\right)^2, \qquad \eta \equiv \frac{M_P^2}{\sqrt{K}U}\left(\frac{U_{,\Theta}}{\sqrt{K}}\right)_{,\Theta},$$
$$\delta^2 \equiv \frac{M_P^4 U_{,\Theta}}{K U^2}\left[\frac{1}{\sqrt{K}}\left(\frac{U_{,\Theta}}{\sqrt{K}}\right)_{,\Theta}\right]_{,\Theta}, \quad (50)$$

where $K \equiv K(\Theta)$ can be read from Eq. (34) or equivalently from the Lagrangian densities (35) and (45). The quantities A_s, n_s, α_s, and r in Eqs. (48) and (49) should be understood as evaluated on the field value $\Theta_* \equiv \Theta(N_*)$, at which the reference pivot scale k_* in Eq. (47) exits the horizon, or in other words at $k_* = a_* H_*$. Here,

$$N_* = \frac{1}{M_P}\int_{\Theta_{\rm E}}^{\Theta_*} \frac{\sqrt{K} d\Theta}{\sqrt{2\epsilon}} = \frac{1}{8c}\ln\left[\frac{\Theta_*}{\Theta_{\rm E}}\left(\frac{\kappa\Theta_{\rm E} + c}{\kappa\Theta_* + c}\right)^{1+\frac{c}{\kappa}}\right] \quad (51)$$

stands for the corresponding number of inflationary e-folds, and

$$\Theta_{\rm E} = \frac{1 - 4c - 2\sqrt{4c^2 - 2c - 2\kappa}}{1 + 8\kappa} \quad (52)$$

denotes the value of the Θ field at the end of inflation. As usual, this is defined by the condition $\epsilon(\Theta_{\rm E}) \equiv 1$. By inverting Eq. (51), we can express the inflationary

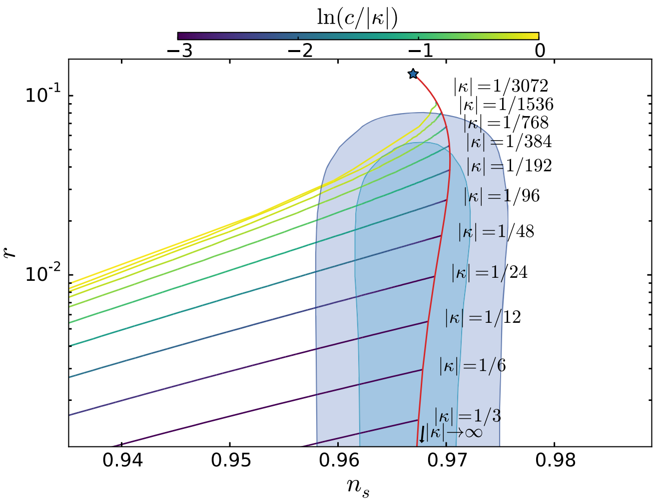

FIG. 2. The (n_s, r) plane for a biscalar model with a maximally symmetric hyperbolic kinetic sector and the theory-defining functions in Eq. (43). The red line follows from the exact analytic expressions in Eqs. (57) and (58), corresponding to the $c = 0$ case. This line interpolates between the chaotic $m^2\phi^2$ inflationary predictions (65) at small $|\kappa|$, and the Higgs/Starobinsky inflation predictions (63) at large $|\kappa|$. The shaded regions mark the Planck 2018 constraints at 68% and 95% C.L. obtained for a ΛCDM model [106]. As evident from the plot, the bounds on the tensor-to-scalar ratio constrain the field-space curvature κ.

observables as functions of the model parameters and N_*. For general values of c and κ, this inversion cannot be analytically performed and one must rely on numerical methods. The values of the spectral tilt n_s and the tensor-to-scalar ratio r, following from a numerical treatment of the potential (37), are presented in Fig. 2.

The qualitative behavior of these observables can be understood by considering two limiting cases in parameter space.

(1) *Quadratic pole limit*: For $c = 0$, the kinetic pole in Eq. (45) becomes quadratic. In this limit, Eq. (51) yields

$$N_* = \mathcal{N}_* - \frac{1}{8|\kappa|}\left(\frac{1}{\Theta_\mathrm{E}} + \ln\Theta_\mathrm{E}\right), \qquad (53)$$

with

$$\mathcal{N}_* = \frac{1}{8|\kappa|}\left(\frac{1}{\Theta_*} + \ln\Theta_*\right), \quad \text{and} \quad \Theta_E = \frac{1}{1+\sqrt{8|\kappa|}}. \qquad (54)$$

Using the above, it is straightforward to see that [87]

$$\Theta_* = -\frac{1}{\mathcal{W}_{-1}[-e^{-8|\kappa|\mathcal{N}_*}]}, \qquad (55)$$

where $\mathcal{W}_{-1}$ is the lower branch of the Lambert $\mathcal{W}$ function. Inserting Eq. (55) into Eq. (50) and taking into account Eqs. (48) and (49), we obtain analytic expressions for the amplitude of the primordial spectrum of scalar perturbations,

$$A_s = \frac{U_0}{192|\kappa|\pi^2 M_P^4}\frac{(1+\mathcal{W}_{-1})^4}{\mathcal{W}_{-1}^2}, \qquad (56)$$

for the spectral tilt and its running,

$$n_s = 1 - 16|\kappa|\frac{1-\mathcal{W}_{-1}}{(1+\mathcal{W}_{-1})^2},$$
$$\alpha_s = -128|\kappa|^2\frac{\mathcal{W}_{-1}^2 - 3\mathcal{W}_{-1}}{(1+\mathcal{W}_{-1})^4}, \qquad (57)$$

and, finally, for the tensor-to-scalar ratio

$$r = \frac{128|\kappa|}{(1+\mathcal{W}_{-1})^2}. \qquad (58)$$

Note that the above quantities are nontrivially related as

$$r = \frac{(1-n_s)^2}{2|\kappa|Y_1^2}, \qquad \alpha_s = -\frac{1}{2}(1-n_s)^2 Y_2, \qquad (59)$$

with Y_1 and Y_2 given by

$$Y_1 \equiv \frac{1}{2}(1+\sqrt{1+y}), \qquad Y_2 \equiv \frac{(y+2Y_1)(y+8Y_1)}{(2Y_1)^4}, \qquad (60)$$

and

$$y \equiv \frac{1-n_s}{2|\kappa|}. \qquad (61)$$

The inflationary observables (57) and (58) display an interesting attractor behavior at large $|\kappa|N_*$, very similar to that appearing in the α-attractor scenarios [74–76]. Indeed, by taking into account the Lambert function bound [107],

$$\mathcal{W}_{-1}[-e^{-8|\kappa|\mathcal{N}_*}] > -8|\kappa|\mathcal{N}_* - \sqrt{2(8|\kappa|\mathcal{N}_* - 1)}, \qquad (62)$$

we obtain

$$n_s \simeq 1 - \frac{2}{\mathcal{N}_*}, \quad r \simeq \frac{2}{|\kappa|\mathcal{N}_*^2}, \quad \alpha_s = -|\kappa| r, \qquad (63)$$

at $8|\kappa|N_* \gg 1$. In this limit—namely, for $1-n_s \ll 2|\kappa|$, or equivalently $y \ll 1$—the functions Y_1

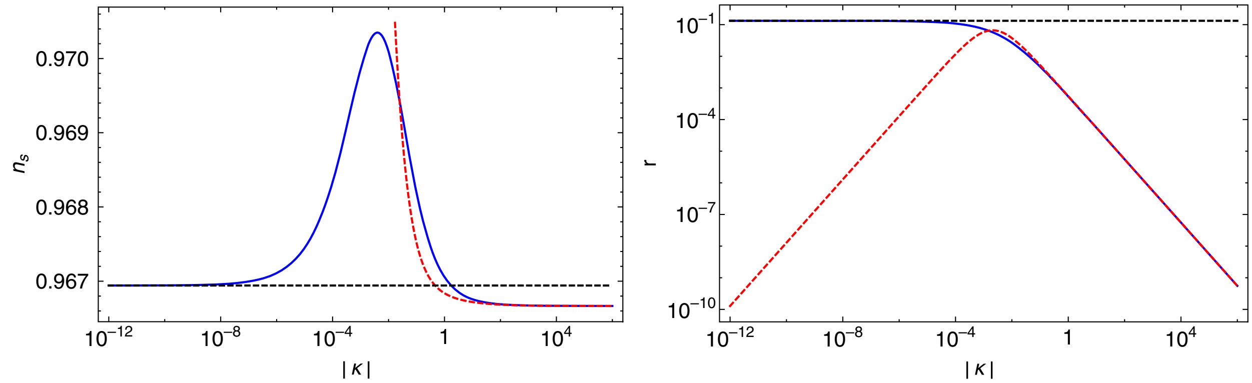

FIG. 3. Comparison between the approximate expressions (63) (red dashed line) and (65) (black dashed line) for the spectral tilt n_s and tensor-to-scalar ratio r and the most general expressions in Eqs. (57) and (58) (blue solid line).

and Y_2 approach their minimal value $Y_1 = Y_2 = 1$, as can be immediately verified from Eq. (60). Consequently, we have

$$r = \frac{(1-n_s)^2}{2|\kappa|}, \qquad \alpha_s = -\frac{1}{2}(1-n_s)^2. \tag{64}$$

Interestingly, the tensor-to-scalar ratio approaches zero at $|\kappa| \to \infty$.

In the opposite limit, i.e., for $|\kappa| \to 0$ (which should of course be taken with care when $c \to 0$), the predictions coincide with those of the $m^2\phi^2$ chaotic inflationary scenario,[11]

$$n_s \simeq 1 - \frac{4}{1+2N_*} \simeq 1 - \frac{2}{N_*},$$
$$r \simeq \frac{16}{1+2N_*} \simeq \frac{8}{N_*}, \qquad \alpha_s = -|\kappa| r, \tag{65}$$

and the relations in Eq. (59) become

$$r = 4(1-n_s), \qquad \alpha_s = -(1-n_s)^2. \tag{66}$$

The comparison of the approximate expressions (63) and (65) for the spectral tilt and tensor-to-scalar ratio with the most general ones given in Eqs. (57) and (58) is shown in Fig. 3.

(2) *The quadratic-to-linear pole transition:* For $c \neq 0$, the inflationary pole at $\Theta = 0$ is no longer reachable and we are left with a linear pole at $\Theta = c/|\kappa|$. To understand the consequences of this pole, let us consider the inversion of Eq. (51) in the limit $c/|\kappa| \ll 1$ and $4|\kappa| N_* \gg 1$. We obtain

$$\mathcal{N}_* \simeq \frac{1}{8c} \ln \frac{\Theta_*}{\Theta_* - c/|\kappa|} \to \Theta_*(\mathcal{N}_*) \simeq \frac{c}{|\kappa|} \frac{e^{8c\mathcal{N}_*}}{e^{8c\mathcal{N}_*} - 1}, \tag{67}$$

with

$$\mathcal{N}_* \equiv N_* - \frac{1}{8c} \ln\left(1 - \frac{c}{|\kappa| \Theta_E}\right). \tag{68}$$

To the lowest order in $c/|\kappa|$, the inflationary observables become

$$A_s = \frac{\lambda \sinh^2(4c\mathcal{N}_*)}{1152\pi^2 \xi_{\rm eff}^2 c^2}, \quad n_s = 1 - 8c \coth(4c\mathcal{N}_*), \tag{69}$$

$$\alpha_s = -32c^2 \sinh^{-2}(4c\mathcal{N}_*), \quad r = \frac{32c^2}{|\kappa|} \sinh^{-2}(4c\mathcal{N}_*), \tag{70}$$

where we introduced the effective coupling

$$\xi_{\rm eff} \equiv \frac{1}{\sqrt{6a^2|\kappa|}} \tag{71}$$

and defined

$$a = -\frac{1-6|\kappa|}{|\kappa|}. \tag{72}$$

For $4c\mathcal{N}_* > 1$, the spectral tilt decays linearly and the tensor-to-scalar ratio approaches zero asymptotically, i.e.,

$$n_s \simeq 1 - 8c, \qquad r \simeq 0. \tag{73}$$

[11]Note that now the expressions contain N_* rather than $\mathcal{N}_*$.

2. Dark energy

After the end of inflation, the field Θ will perform oscillations around the minimum of its effective potential, heating the Universe and eventually relaxing to $\Theta = 1$. When that happens, the Lagrangian boils down to

$$\frac{\mathcal{L}}{\sqrt{g}} \simeq \frac{M_P^2}{2} R - \frac{1}{2}(\partial\Phi)^2 - \frac{\Lambda_0}{\gamma^4} e^{-4\gamma\Phi/M_P}. \tag{74}$$

It is therefore clear that the dilaton can drive an accelerated expansion of the Universe for suitable values of Λ_0 and γ. At early times, the potential of Φ is small compared to the Hubble rate. This prevents the motion of the field and forces it to stay frozen at the value that it inherited from inflation. At late times, the dilaton starts evolving and the system approaches an effective equation-of-state parameter [26,108,109]

$$1 + w = \frac{16\gamma^2}{3} F(\Omega_{\rm DE}), \quad \text{with}$$
$$F(\Omega_{\rm DE}) = \left[\frac{1}{\sqrt{\Omega_{\rm DE}}} - \Delta \tanh^{-1}\sqrt{\Omega_{\rm DE}}\right]^2, \tag{75}$$

which leads to acceleration ($w < -1/3$) if $\gamma < 1/(2\sqrt{2})$. Here,

$$\Omega_{\rm DE} \equiv \frac{1}{1 + \Delta_0 a^{-3}} \tag{76}$$

stands for the dark energy abundance associated with the dilaton component and

$$\Delta \equiv \frac{1 - \Omega_{\rm DE}}{\Omega_{\rm DE}}, \qquad \Delta_0 \equiv \frac{1 - \Omega_{\rm DE,0}}{\Omega_{\rm DE,0}}, \tag{77}$$

where the subscript 0 refers to quantities evaluated today.

3. Connecting inflation and dark energy

Until this point, we have assumed that the parameters κ, c, and γ in our example are independent. If these quantities were related, the set of scale-invariant maximally symmetric TDiff theories will also lead to nontrivial connections between the inflationary and DE observables. This is what happens, for instance, in the simplest scale-invariant model that can be constructed out of two scalar fields ϕ_1 and ϕ_2, namely [31],

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{\xi_1\phi_1^2 + \xi_2\phi_2^2}{2} R - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2$$
$$- \frac{\lambda}{4}(\phi_1^2 - \alpha\phi_2^2)^2 + \Lambda_0, \tag{78}$$

where ξ_1, ξ_2, λ, and α are positive dimensionless couplings and Λ_0 is constant. This Diff-equivalent Lagrangian density follows from the TDiff-invariant one in Eq. (4) after restoring the full symmetry with the Stückelberg trick of Sec. II and with the following choice of theory-defining functions (see also Ref. [33] for more examples):

$$G_1(g) = \beta^2 g^{2(\beta-1)}, \quad G_2(g) = \beta g^{2\beta-1}, \quad G_3(g) = 1 + g^{2\beta},$$
$$f(g) = \xi_1 + \xi_2 g^\beta, \quad v(g) = \frac{\lambda}{4}(1 - \alpha g^{2\beta})^2. \tag{79}$$

Here, β is an arbitrary constant, and to obtain Eq. (78) we have identified $\phi = \phi_1$ and introduced $\phi_2 = \phi_1 g^\beta$. When transformed to the Einstein frame and rewritten in terms of variables

$$\gamma^{-2}\Theta \equiv \frac{(1+6\xi_1)\phi_1^2 + (1+6\xi_2)\phi_2^2}{\xi_1\phi_1^2 + \xi_2\phi_2^2},$$
$$\exp\left[\frac{2\gamma\Phi}{M_P}\right] \equiv \frac{\kappa_c}{\kappa} \frac{(1+6\xi_1)\phi_1^2 + (1+6\xi_2)\phi_2^2}{M_P^2}, \tag{80}$$

with

$$\gamma \equiv \sqrt{\frac{\xi_2}{1+6\xi_2}}, \tag{81}$$

the Lagrangian density (7) approximately[12] reduces to the form (45) [24], with $U(\Theta)$ and $K_\Lambda(\Theta)$ given by Eq. (43) and

$$U_0 \equiv \frac{\lambda M_P^4}{4}\left(\frac{1+6\kappa}{\kappa}\right)^2, \qquad \kappa \equiv \kappa_c\left(1 - \frac{\xi_2}{\xi_1}\right),$$
$$\kappa_c \equiv -\frac{\xi_1}{1+6\xi_1}, \qquad c \equiv \frac{\kappa}{\kappa_c}\gamma^2. \tag{82}$$

A simple inspection of these expressions reveals that the parameters κ, c, and γ in this particular scenario are not independent. This allows us to obtain a set of consistency relations among the inflationary and dark energy observables. An analytic form for these consistency relations can be obtained in the limit $|\kappa| \approx |\kappa_c|$, corresponding to an inflationary dynamics essentially dominated by the ϕ_1 component, i.e., with $\xi_1 \gg \xi_2$. Indeed, combining the expression (75) with those for the spectral tilt, its running, and the tensor-to-scalar ratio in Eqs. (69) and (70), we obtain [24]

$$n_s = 1 - \frac{2}{\mathcal{N}_*} X \coth X, \qquad r = \frac{2}{|\kappa_c|\mathcal{N}_*^2} X^2 \sinh^{-2} X,$$
$$\alpha_s = -|\kappa_c| r, \tag{83}$$

with

[12]The main difference is associated to an additional pole $\Theta = 1$ in the Einstein-frame kinetic sector of Eq. (78). This "Minkowski" pole is, however, irrelevant for the cosmological phenomenology discussed in this paper; for details, cf. Refs. [24,50,87].

$$X \equiv 4c\mathcal{N}_* = \frac{3\mathcal{N}_*(1+w)}{4F(\Omega_{\rm DE})}. \tag{84}$$

Given the value of λ at the inflationary scale, the constant $|\kappa|$ can be determined from the amplitude of the primordial power spectrum (69). For not too small values of λ, the effective coupling is typically rather large, $\xi_{\rm eff} \simeq \xi_1 \gg 1$, leading to values of $|\kappa|$ very close to $1/6$. In this limit, the expressions in Eq. (83) reduce to those first found in the context of the *Higgs-dilaton model* [22].

V. COMPARISON WITH PRESENT DATA SETS

To interpret the existing data in light of scale-invariant models, we perform a Markov chain Monte Carlo (MCMC) analysis similar to those in Refs. [23,24]. In particular, we sample the posterior probability distribution $P = p(\theta|x, M)$ of cosmological parameters θ given the data x and a model M by means of Bayes' theorem

$$P = \frac{p(\theta|M)}{E}\mathcal{L}, \tag{85}$$

where $p(\theta|M)$ is the prior distribution of parameters given the model, and $\mathcal{L} = p(x|\theta, M)$ is the likelihood. The evidence $E = p(x|M) = \int d\theta p(x|\theta, M)p(\theta|M)$ follows as a normalization factor. Once the likelihood and the priors are given, the MCMC algorithm constructs a chain of points whose density is proportional to the posterior probability distribution $p(\theta|x, M)$. For the likelihood, we include the following observational data sets:[13]

(i) The 2015 Planck high-multipole TT likelihood [110].[14]
(ii) The 2015 Planck low-multipole polarization and temperature likelihoods [110].
(iii) The 2015 Keck/Bicep2 likelihood data release [111].
(iv) The Joint Lightcurve Analysis data [112].
(v) The baryon acoustic oscillation data from 6dF, BOSS, LOWZ, BOSS CMASS, and SDSS [113–115].

For the maximally symmetric scale-invariant models under consideration we vary c and $-\ln(|\kappa|)$ in the ranges [0, 1] and $[-\ln(1/6), 8]$, with the logarithmic parametrization chosen only for numerical convenience and the intervals motivated by the particular example in Sec. IV A 3. For each pair of values, we numerically solve the inflationary trajectory and compute the spectral tilt n_s and the tensor-to-scalar ratio r.

While the details of the heating stage after inflation remain to be specified, here we adopt a conventional estimate that turns out to be reasonable in many heating scenarios [116–118]. In particular, we restrict the number of inflationary e-folds to a Gaussian distribution with mean 60 and standard deviation 2.5. Additionally, we vary the customary cosmological parameters using flat and unrestrictive priors. The prior ranges can be found in Table I of Ref. [24].

A. Maximally symmetric model without consistency conditions

To discuss how the model parameters can be constrained by *pure inflationary physics* we first study a particular realization of Eq. (45) with c and κ completely unrelated to γ^2. In other words, we assume that the inflationary and dark-energy-dominated eras are completely independent. The results of the MCMC analysis for this particular scenario are presented in Fig. 4, both in terms of the parameters c and κ and in terms of the observable quantities n_s and r. As is evident from this figure, the allowed values for the spectral tilt and the tensor-to-scalar ratio mostly correspond to a restricted version of ΛCDM, with the curvature of the Einstein-frame kinetic sector closely related to r and the parameter c constrained by the spectral tilt n_s for fixed κ. The mean values of these parameters are

$$n_s = 0.9686^{+0.0026}_{-0.0015}, \qquad r = 0.040^{+0.016}_{-0.025}, \\ c = 0.23^{+0.06}_{-0.23}, \qquad -\ln(|\kappa|) = 5.5^{+1.4}_{-1.1}, \tag{86}$$

with the errors denoting the 68% C.L. We emphasize that these constraints should be taken with a grain of salt for two reasons. On the one hand, our parametrization in terms of c and $-\ln(|\kappa|)$ is not suitable for large $|\kappa|$ values. On the other hand, given the present data sets, it turns out to be quite challenging to numerically explore the $|\kappa| \to 0$ limit at relatively large c values, or, correspondingly, the region of large tensor-to-scalar ratios and small spectral tilts. As it can be seen from the contours in Fig. 4, the viability of relatively large tensor-to-scalar ratios still prevents us from identifying the full 95% confidence regions for c and κ. However, this is expected to improve significantly with the eventual release of the Planck 2018 likelihood.

Both the Planck 2018 likelihood and other future CMB experiments are expected to set tight bounds on the Einstein-frame kinetic curvature. In particular, a decreasing limit on the tensor-to-scalar ratio would directly translate into an increasing lower bound on $|\kappa|$. This becomes apparent when one considers, for instance, the latest bound on r, namely, $r < 0.064$ [106].[15] This value translates into a restriction $-\ln(|\kappa|) < 6.5$, therefore excluding a large part of the Planck 2015 (c, κ) parameter space. Note, however, that no upper bound on $|\kappa|$ follows from present data sets. Indeed, only an eventual detection of primordial gravitational waves could provide an upper limit on it.

[13]We assume that these data sets are independent and do not model cross correlations among them.

[14]The 2018 Planck likelihood is not yet publicly available.

[15]Note that this bound was derived in a ΛCDM cosmology and it should be reevaluated for scale-invariant models after the eventual release of the Planck 2018 likelihood.

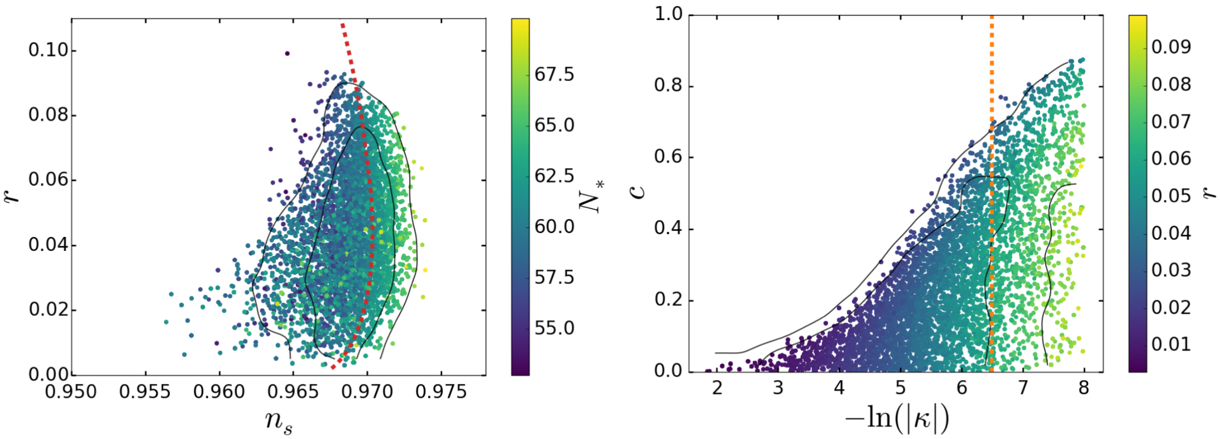

FIG. 4. Left: MCMC samples for the spectral tilt n_s and tensor-to-scalar ratio r in a scale-invariant model *without consistency relations*. The color coding indicates the number of e-folds N_* to the end of inflation. The red dashed line corresponds to the limit $c \to 0$ for $N_* = 60$. Right: MCMC samples for the model parameters $-\ln(|\kappa|)$ and c in the same scenario. The color coding now indicates the tensor-to-scalar ratio r. For a fixed value of κ, the parameter c is tightly constrained by the spectral tilt. Note that small values of $-\ln(|\kappa|)$ are permitted only for tiny values of c. This tail corresponds to the bottom-left corner in the $r - n_s$ plot, which is not properly explored in our parametrization. Black lines mark the 68% and 95% C.L. regions. The orange dashed line corresponds to the expected 95% bound following from the Planck/BICEP 2018 data release.

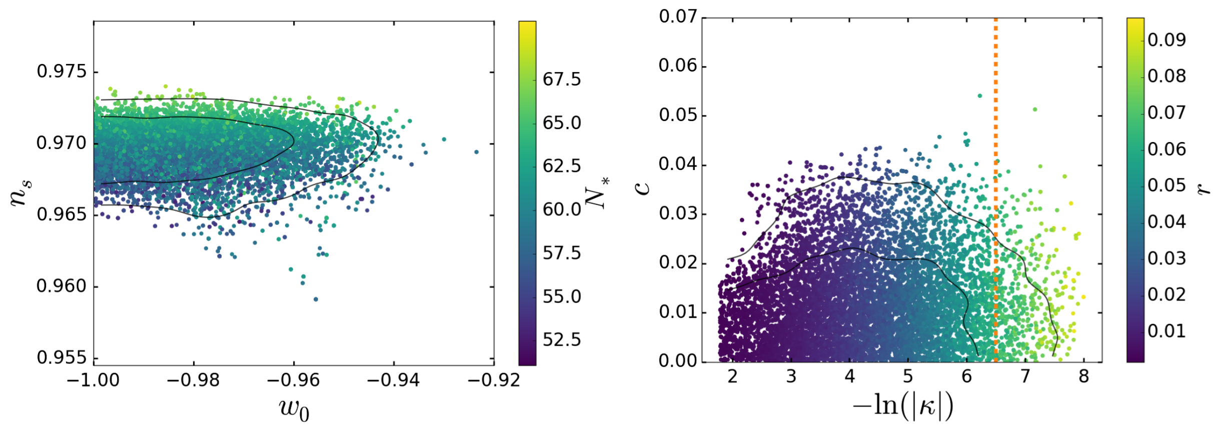

FIG. 5. Left: MCMC samples for the spectral tilt n_s and the equation-of-state parameter w_0 in a scale-invariant model *with the consistency relation* $\gamma^2(c) = c$. The color coding indicates the number of e-folds N_* to the end of inflation. Measurements of the equation-of-state parameter strongly constrain the spectral tilt. Right: MCMC samples for the model parameters $-\ln(|\kappa|)$ and c in the same scenario. The color coding now indicates the tensor-to-scalar ratio r, while the orange dashed line marks the expected 95% C.L bound following from the Planck/BICEP 2018 data release. Note that if the early and late Universe observables are related by a consistency relation, the constraints on c are much tighter than in the absence of it (note the different scale for c in Fig. 4).

B. Maximally symmetric model with consistency conditions

To illustrate the impact of a potential connection between the early and late Universe we now consider a realization of Eq. (45) involving the consistency relation $\gamma^2(c) = c$. This choice is motivated by the simple biscalar scenario presented in Sec. IVA 3 and should be understood as just a particular example of the different consistency relations that could appear in this type of models. As shown in Fig. 5, the existing constraints on the present equation-of-state parameter effectively constrain the spectral tilt and significantly reduce the 68% C.L. ranges of c and κ,[16]

$$n_s = 0.9695^{+0.0019}_{-0.0013}, \qquad r = 0.026^{+0.007}_{-0.024},$$
$$c = 0.013^{+0.003}_{-0.013}, \qquad -\ln(|\kappa|) = 4.28^{+1.27}_{-1.56}. \qquad (87)$$

[16] The cut on the left-hand side of the (c, κ) plot is due to our prior restriction $|\kappa| < 1/6$.

TABLE I. The maximum likelihood estimate of the logarithm of the Bayes factor $\ln B(M)$ with respect to a baseline ΛCDM model. Although the comparison remains inconclusive, the scale-invariant model without consistency relations appears to be slightly disfavored with respect to ΛCDM. On the contrary, a scale-invariant model with the consistency condition $\gamma^2(c) = c$ is preferred over the concordance model.

	ΛCDM	Without consistency rel.	With consistency rel.
No. of parameters	8	10	9
$\ln B$	0	−1.73	2.44

This parameter-space reduction is expected to become stronger in the near future. On the one hand, galaxy redshift surveys such as Euclid or LSST will provide percent-level measurements of the dark-energy equation-of-state parameter. On the other hand, stage-4 CMB observers such as LiteBIRD will determine the value of the tensor-to-scalar ratio with an unprecedented 10^{-3}–10^{-4} accuracy.

C. Bayesian evidence and correlation matrices

To quantify how the scale-invariant models above compare to ΛCDM we calculate the Bayes factor, defined as the evidence ratio for a model M and a ΛCDM scenario given the data x, namely,[17]

$$B(M) = \frac{p(M|x)}{p(M_{\Lambda\mathrm{CDM}}|x)}. \tag{88}$$

We compute this quantity from the obtained MCMC chains using the method proposed in Refs. [119,120] and interpret the result according to the Kass and Raftery scale [121], where a value $|\Delta \ln B| > 3$ is understood as a strong statistical preference. As shown in Table I, the scale-invariant model without consistency relations appears to be slightly disfavored with respect to ΛCDM. On the contrary, the scale-invariant model with consistency relations seems to be preferred over the concordance model. Although these quantitative results should not be taken at face value,[18] they illustrate two important points. First, the strong preference for a scale-invariant model with consistency relations over the one without them stresses the importance of these conditions when dealing with existing and future data sets. Second, the *positive evidence* [121] for the model with consistency relations over ΛCDM indicates that scale-invariant scenarios can be on equal footing with—and in some cases superior to—the concordance model.[19]

The impact of the consistency relations is also reflected in the correlation among different cosmological parameters. This interesting feature is shown in Fig. 6, where we display the MCMC covariance matrices obtained from current data sets, converted into correlation matrices. The left and right panels correspond to a model without and with consistency relations, respectively. In these plots we have defined $\hat{\kappa} \equiv -\ln(|\kappa|)$ for visualization purposes.

Without consistency relations, there exists a positive correlation among c and $\hat{\kappa}$, which matches very well with the behavior displayed in Fig. 4 where, for a constant value of the tensor-to-scalar ratio, an increase in c corresponds to an increase in $\hat{\kappa}$. In this case, the equation-of-state parameter w_0 is an independent parameter, which—leaving aside the fact that σ_8 is a derived quantity depending on all parameters affecting the growth of structures—is only anticorrelated with the reduced Hubble rate h due to the expansion of the Universe.

When including the consistency relations, w_0 is no longer an independent parameter, but rather a derived one, totally correlated with c. This means that c has now taken the role of a dark-energy equation-of-state parameter and, consequently, is now anticorrelated with the reduced Hubble rate h. Additionally, we observe strong positive correlations between $\hat{\kappa}$ and the standard inflationary parameters n_s and r. Moreover, c and $\hat{\kappa}$ are basically uncorrelated and independent of each other. Both of these features are reflected in the right panel of Fig. 5, the former by noting that for a constant value of c the tensor-to-scalar ratio increases for increasing $\hat{\kappa}$ and the latter by the observation that the $\hat{\kappa} - c$ contours are almost circular.

The above findings are summarized in the bottom panel of Fig. 6, where we display the difference between the absolute values of the correlation coefficients in the model with and without consistency relations. Red (blue) elements correspond to parameters that are more (less) correlated with (without) consistency relations. In the presence of consistency relations, we see three main features: (i) c and $\hat{\kappa}$ become independent of each other, (ii) c takes the role of w_0, and (iii) the spectral index n_s is more correlated with the number of e-folds N_*, the curvature $\hat{\kappa}$, and the tensor-to-scalar ratio r. This leads to the conclusion that future CMB and galaxy redshift surveys measuring the parameters n_s, r, and w_0 with precision should be able to test a scale-invariant model with consistency conditions. The inflationary observables alone would then fix the values of the model parameters c and κ, while the measurement of w_0 would provide an independent test of the consistency relation.

[17]This implicitly assumes that all models are equally probable *a priori*.

[18]In particular, the parameter basis is varied when comparing the three models. Additionally, the prior on $-\ln(|\kappa|)$ restricts the available parameter volume. This renders the value of the Bayes factor prior dependent [119,120]. A change of the prior volume by a factor λ would induce a change $\ln \lambda$ in the Bayes ratio.

[19]Note that a Bayes factor $\ln B = 2.44$ corresponds to a relative probability of approximately 11∶1 for the scale-invariant model over ΛCDM.

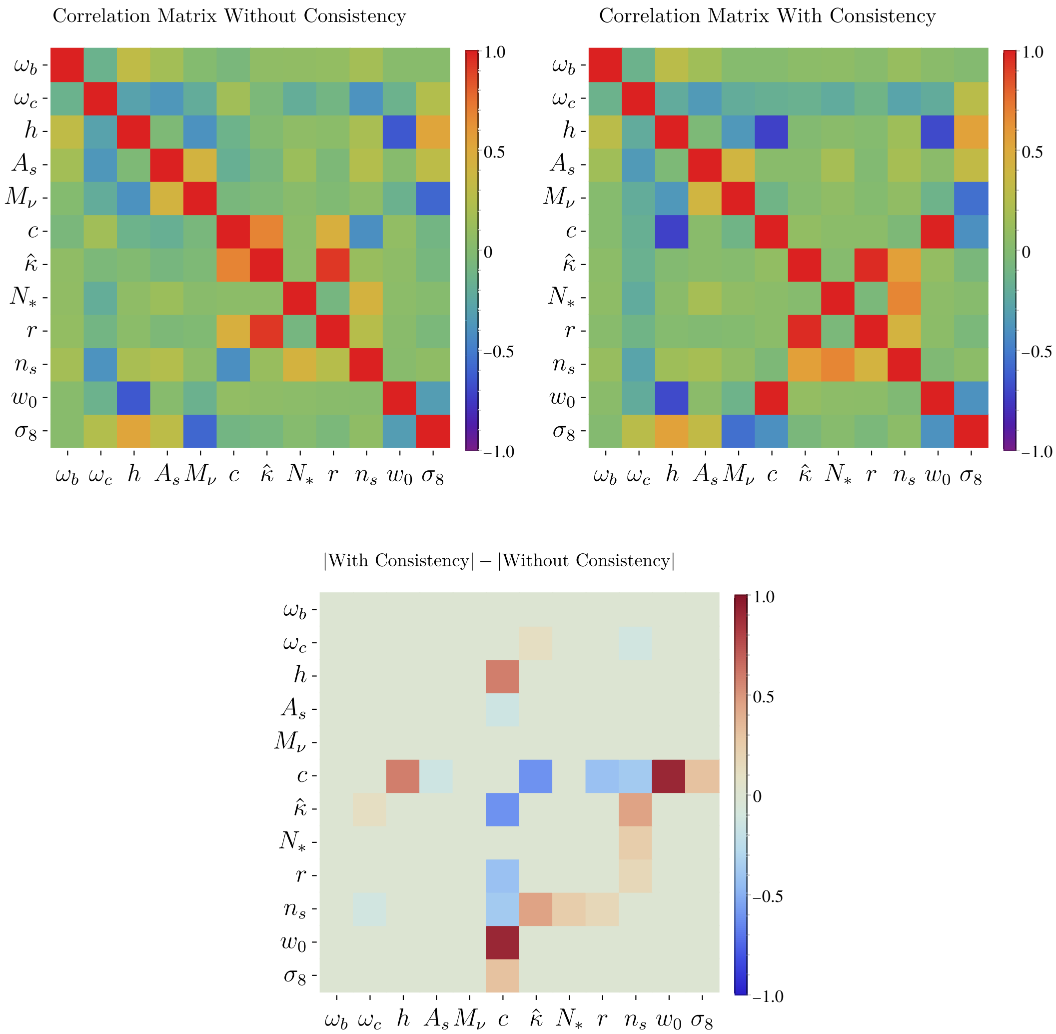

FIG. 6. (Top) Correlation matrices obtained from the covariance matrices of the MCMC runs for a model without (left) and with (right) consistency relations. The $+1$ and -1 limits stand for totally correlated and totally anticorrelated, respectively. (Bottom) Half difference of the absolute values of the correlation coefficients with the $+1$ and -1 limits now indicating whether the "correlation strength" has increased or decreased, respectively. In this figure we define $\hat{\kappa} \equiv -\ln(|\kappa|)$ for visualization purposes.

VI. CONCLUSIONS

Biscalar theories invariant under scale transformations and volume-preserving diffeomorphisms can accommodate an inflationary expansion of the Universe followed by a standard hot big bang evolution and a dark-energy-dominated era.

The scalar character of the metric determinant under volume-preserving diffeomorphisms together with the requirement of classical scale invariance leads to a very specific particle spectrum containing two graviton polarizations and two scalar d.o.f. on top of the standard matter content. A Lagrangian constructed within this framework contains in general arbitrary functions of the ratio of these two scalar fields.

In spite of their apparent arbitrariness, the resulting theories turn out to be predictive. On the one hand, the existence of an effectively conserved current related to dilatations makes these models essentially indistinguishable from single-field inflationary scenarios, from which they "inherit" all their virtues. On the other hand, the symmetries of the Einstein-frame kinetic sector significantly restrict the inflationary observables. More specifically, if this target space is maximally symmetric, the arbitrary functions in the Lagrangian become related in a rather nontrivial way. As a result, the dynamics is governed by the pole structure of the Einstein-frame kinetic sector, making the inflationary predictions universal and almost insensitive to the details of the potential.

At low energies, the invariance under volume-preserving diffeomorphisms gives rise to a *unique* runaway potential for the dilaton, which can play the role of dynamical dark energy. Interestingly, the early and late Universe dynamics may become intertwined in some particular scenarios, leading to nontrivial consistency relations among the inflationary and dark-energy observables. The comparison of particular realizations of our paradigm with present data reveals a strong preference for maximally symmetric models with consistency relations over those without them. Surprisingly, there is also *positive evidence* for the former class of models over the concordance ΛCDM model given the present data sets.

The results of this paper illustrate the strong impact that our assumptions concerning the early and late Universe dynamics could have on the interpretation of cosmological data sets. This poses an interesting question for future CMB observations and galaxy redshift surveys: are inflation and dark energy independent processes in the expansion history of the Universe, or rather two sides of a single underlying principle?

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