Business Model Innovation, Social Interactions, and Behavioral Decision Making

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First of all, I would like to thank my advisors Dr. Guangwen Kong and Dr. Saif Benjaafar. It has been a great honor working with them. This thesis would not have been possible without their motorship and guidance. Dr. Guangwen Kong's continuous support kept me going throughout this long journey. Her courage for research and positive attitude made my time as a Ph.D. student enjoyable and my research more productive. A significant portion of my academic growth should be attributed to Dr. Saif Benjaafar's tireless guidance and feedback on my works. His experience was a treasure that I was fortunate to have access to.

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Dedication

I dedicate this thesis to

Narges who introduced me to true happiness,

Ali & Khadijeh who helped me to be a better person,

Hans who taught me invaluable lessons,

and to all who inspired me to learn.

Abstract

This thesis consists of three parts. All chapters are centered around the behavioral decision making and business model innovation. In the first essay, we study a supply chain with a supplier selling products to a retailer who is boundedly rational. Under this setting, we study the impacts of the retailer's bounded rationality on the supplier's choice of contract and the supply chain efficiency. We develop a behavioral model that incorporates the human retailers bounded rationality in a supply chain setting. We then conduct a series of laboratory experiments to test whether the model's predictions are still salient even when the supplier is not necessarily rational.

In contrast to a supply chain with a fully rational retailer, where a wholesale price contract usually cannot perform better than more complicated nonlinear contracts, we find that when the retailer is boundedly rational a wholesale price contract can dominate commonly used nonlinear contracts such as buy-back and revenue sharing contracts. We characterize the conditions under which a wholesale price contract outperforms more sophisticated non-linear price contracts for the supplier. In both theoretical model and the experiments, we find that a wholesale price contract is more likely to be implemented by the supplier when the supply chain profit margin is low, the retailer is less rational, the demand variance is high, and the retailer's reservation value is high. The results can explain the prevalence of wholesale price contract in business practice when the rationality of retailers cannot always be guaranteed. We also find that the retailer's bounded rationality plays a more important role in determining supply profit than the supplier's bounded rationality.

In the second essay, we consider a setting which involves a service provider who sells access to a service or a product to a unit mass of heterogenous consumers. Such s business model is gaining popularity in recent years. With this growth comes opportunities for peer-to-peer trading marketplaces to emerge. However, there is a debate on whether or not peer-to-peer trading of excess capacity is beneficial to service providers and consumers. The second essay in this thesis aims to shed light on this debate and identifies conditions under which the existence of such marketplaces can be a win-win situation for all parties. We develop a game-theoretic model in which consumers participate in a simultaneous coordination game. Consumers are strategic and take into account the opportunity of purchasing or selling extra capacity on the trading market. Our model captures the heterogeneity of consumers' demand and the service provider's ability to modify service plans in view of this trading among consumers. We compare equilibrium outcomes with and without trading and show that outcomes with regard to service provider profit, consumer surplus, and social welfare are crucially dependent on service cost and trading price. A service provider would benefit from trading as long as the trading price is not too low (a low trading price encourages more consumers to opt for the low plan) and the service cost is not too high (a high service cost makes increased consumption due to trading too costly). A trading price that is too low can decrease consumer surplus and social welfare. Hence, a social planner would be interested in inducing a moderate or high trading price. In settings where the service provider can modify prices, consumers are no longer guaranteed to benefit from trading. In this case, trading can hurt consumers if the trading price is either sufficiently high (resulting in consumers paying a higher price for the higher plan) or sufficiently low (resulting in less consumption because more consumers opt for the low plan). Our results provide guidance to service providers, consumers, and policy makers as to when peer-to-peer trading may or may not be beneficial. The results highlight the important interplay between trading price and cost of service in determining various outcomes. For policy-makers, the results can be useful in pointing out when such trading improves outcomes

for consumers or social welfare and to potential policy levers that could be deployed to affect outcomes.

Finally, in the last essay, we study the interaction between information asymmetry and the reciprocity in a financial crowdfunding setting. Most of the crowdfunding platforms encourage entrepreneurs to tap into their social network and bring investors from their social networks to their crowdfunding campaigns. This is done with the intention of creating the early momentum which appears to be the key to running a crowdfunding campaign. However, the incentives and information of those investors who are attracted to crowdfunding campaign from the entrepreneur's social network could be different from other investors who do not have a social tie with the entrepreneur. On the other hand, the regular investors do not have a social tie with the entrepreneur and their sole investment motivation is financial. In the last essay of this thesis, we develop a signaling game to better understand the interaction between the reciprocity and the information flow in a financial crowdfunding setting. Our main result indicates that the reciprocity may create a situation in which the informed investor (those from the entrepreneur's social network) cannot signal their type via distorting her investment.

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Chapter 1

Is Simplicity the Ultimate Sophistication?

1.1 Introduction

The emerging literature in behavioral and experimental economics has acknowledged that people have limited cognitive abilities in optimizing their performances (Simon, 1955). In particular, human decision makers may deviate from the the optimal decision and exhibit errors in making decisions (Su, 2008; Wu and Chen, 2014). In behavioral operations management, numerous experiments have been conducted to verify the finding that a human newsvendor facing uncertain demand may not be able to place the optimal order quantity as predicted by the normative theory. The non-optimizing behavior in this chapter is referred to as *boundedly rational*.

The literature on supply chain contracts has suggested adopting more complicated non-linear contracts, instead of simple wholesale price contracts, to align the incentives of a rational retailer with that of the supply chain. Two most common contracts of this kind are buy-back and revenue sharing contracts. Despite of the praise of those complicated forms of contracts in theory, simple wholesale price contracts are still commonplace in many businesses. Our main goal is to address such a puzzle by incorporating a retailer's boundedly rational behavior in the supply chain contracting game. In particular, we adapt a behavioral newsvendor order quantity model from the behavioral operations literature in a standard two echelon supply chain involving one supplier and one retailer, and examine if this would make an impact on a supplier's choice of contract.

In the case that the retailer is fully rational, the retailer tends to order less than the supply chain optimal level of inventory under wholesale price contracts. However, the supply chain optimal order quantity can be achieved by nonlinear contracts such as buy-back and revenue sharing contracts. Take buy-back contract as an example, there exist pairs of buy back contracts (w, b) under which the supply chain is coordinated. In addition, arbitrary supply chain profit allocation between the retailer and the supplier can be achieved by changing the buy-back contract parameters. The supplier's profit increases with the buy-back price b. Given that the retailer has certain bargaining power in a supply chain (e.g., the retailer has a reservation value or outsider option), by choosing the buy-back contract at maximum buy-back price b, the manufacture will extract all the surplus above reservation value from the supply chain. The same results apply to revenue sharing contract or other non-linear contracts that coordinates the supply chain.

However, the results may not continue to hold when the retailer is boundedly rational. Specifically, when the retailer is boundedly rational, there is a systematic deviation from the optimal order quantity and the order quantity place by the retailer may not be the best response to a given contract. Our work addresses a few questions in supply chain contracting with boundedly rational retailers. How does a supplier's choice of contract change when dealing with a boundedly rational newsvendor? Are nonlinear contracts such as buy-back or revenue sharing contracts still preferred over wholesale price contracts when the retailer is boundedly rational? How is the supplier's choice of contract affected by factors such as profit margin, demand variance, the retailer's bounded rationality and reservation value? We find that, when facing a boundedly rational retailer, the supplier may not necessarily gain more profit under a nonlinear contract, such as buy-back contract or revenue sharing contract, than under a wholesale price contract. This is because, in contrast to the case of fully rational retailer, a supplier may suffer more loss from the retailer's biases under nonlinear contracts where the supplier's profit is affected by the retailer's ex-post performance. Hence, the supplier's profit may not always increase with the buy-back price or revenue sharing rate. As a result, there exists a wide parameter region in which a wholesale price contract outperforms nonlinear contracts.

To further support our behavioral models prediction and show that the wholesale price contract is indeed preferable for human suppliers, we conduct a series of controlled laboratory experiments in which human subjects play the role of suppliers. We develop hypotheses to test whether insights derived from our behavioral model sustain in a controlled laboratory environment. Our experimental results show that changes in human suppliers' contractual preferences are consistent with what our behavioral model predicts. That is, human suppliers are more likely to choose a wholesale price contract when the retailer's rationality is low, the profit margin is low, and demand variation is high.

The rest of this chapter is organized as follows: Section 1.2 is devoted to the relevant literature; Section 1.3 presents our behavioral supply chain model with a boundedly rational retailer; Section 1.4 characterizes the conditions under which a wholesale price contract performs better than buy-back contracts and investigates how the supplier's preference to a wholesale price contract changes under different situations; Section 1.5

presents laboratory experiments that provide empirical support to our theoretical findings. Section 1.6 revisits some of our assumptions and checks the robustness of our results. Section 1.7 discusses implications and concludes this chapter.

1.2 Literature Review

There are two main areas of research related to our study: (1) supply chain contracting (2) behavioral studies of newsvendor models, and (3) supply chain contracting. A classic result in the supply chain contracting literature is that a wholesale price contract cannot coordinate a supply chain involving different decision makers that aim to maximize their own profits due to *double marginalization*. However, Pasternack (1985) shows that a buyback contract can coordinate a supply chain and allows for an arbitrary profit division between a retailer and a supplier. Cachon and Lariviere (2005) propose a revenue sharing contract and show that it can be equivalent to a buyback contract in terms of allocating profits between a retailer and a supplier. Beside buyback and revenue-sharing contracts, there are also other contracts aiming to resolve the misalignment of incentives between supply chain partners when facing stochastic demand. Examples are penalty (Lariviere, 1999) and sales-rebate (Taylor, 2002) that achieve supply chain coordination. A comprehensive review of different coordination mechanisms for a supply chain with a newsvendor retailer can be found in Lariviere (1999), Tsay et al. (1999), Kaya and Özer (2011), and Cachon (2003). There are other studies that extend the classic coordination problem by considering alternative assumptions and richer problem setting such as information sharing and feedback (Ha et al., 2011), competition (Tsay and Agrawal, 2000; Chen et al., 2008), leakage of information (Kong et al., 2013), and fairness (Cui et al., 2007).

A common feature of the aforementioned literature is that retailers are fully rational in that they are able to make optimal decision to maximize their expected profits. However, the emerging literature in the behavioral operations management suggests that this assumption does not necessarily hold. Evidences from laboratory experiments have indicated that human subjects may exhibit different behavioral biases and their decisions may deviate from the expected profit-maximizing order quantity(Sterman, 1989; Diehl and Sterman, 1995; Croson and Donohue, 2002; Bendoly et al., 2006). Carlson and O'Keefe (1969) report that subjects made "*almost every kind of mistake*" in a newsvendor experiment with human subjects. Schweitzer and Cachon (2000) observe that a human newsvendor's order quantities show a significant *pull-to-center* effect. That is, a human newsvendor places an order quantity in between the theoretical prediction and the mean demand, which results in an under-order of high-margin products and an over-order of low-margin products. The robustness of this observation is repeatedly confirmed by other studies, including Bostian et al. (2008), Kremer et al. (2010), Ho et al. (2010), and Bolton and Katok (2008), among others.

While the above literature mostly focuses on investigating the behavioral bias itself, in recent years, there is an emerging literature continues to explore the optimal design of contracts incorporating human biases. Loch and Wu (2008) examine the effect of social preferences such as relationship and status seeking under wholesale price contracts chosen by wholesalers. Özer et al. (2011) explore trust and trustworthiness among supply chain partners and show that the demand information can be effectively revealed under wholesale price contracts, contradicting to the cheap talk theory that babbling equilibrium always occurs. Ho et al. (2014) consider wholesale price contracts offered to sequential retailers with fairness concern. Most of the aforementioned papers focus on newsvendors' behavior under wholesale price contracts. Recently, a few papers start looking into subjects' behavioral biases under more complicated forms of contracts. Becker-Peth et al. (2013) find that subjects place significantly different order quantities under contracts with different parameters with the same critical ratio. They propose a behavioral model which takes predicable irrationalities such as anchoring, loss aversion, and mental accounting into account in the design of buyback contract and show that their model can fit the experimental data accurately. Li et al. (2016) study the effect of a newsvendor's overconfidence in a competitive setting. They show that when profit margin is high, overconfidence bias can lead to first-best outcome.

There are only a few papers that consider how a supplier's choice of contract is affected when a supply chain partner exhibits bounded rationality. Zhang et al. (2015) examine the contract preferences of a loss-averse supplier and show that, from the point of view of a loss-averse supplier, buyback and revenue-sharing contracts are not always equivalent. Their theoretical results as well as experimental studies confirm that a loss-averse supplier prefers buyback contracts in a low critical ratio setting and revenue-sharing contracts in a high critical ratio setting. Although they examine the supplier's contract preferences, the retailer in their setting is rational and orders according to the normative theory. In contrast to their setting where a retailer is fully rational, we focus on the impact of a retailer's bounded rationality on a supplier's choice of contract.

A few papers examine the impact of a retailer's behavioral bias on a supplier's contracting decision. (Kalkanci et al. (2011), Kalkancı et al. (2014), and Katok and Wu (2009)). They find that the performance gap between the wholesale price contract and more complex contracts is not as large as it is predicted by the normative theory. In contrast, our work points out that a simple wholesale price contract can perform *better* than the more complicated nonlinear contracts (e.g. buyback and revenue sharing) believed to coordinate supply chain due to retailers' bounded rationality.

1.3 Model Description: Notations and Setup

We consider a dyadic supply chain with a rational supplier (he) and a boundedly rational retailer (she) who sells a product at market price p. The downstream demand of the product denoted by $\mathbf{D}$, is random and is uniformly distributed according to $\mathbf{D} \sim \mathcal{U}(\mu(1-\Delta), \mu(1+\Delta))$, where μ is the mean of the demand and Δ is the scale parameter that captures the variations of the demand.

The supplier offers a take-it-or-leave-it contract to the retailer and, before the demand is realized, the retailer places an order with the supplier. Production takes place at the supplier at per unit cost c and the salvage value is normalized to zero. The retailer observes the realized demand at the end of the period. In the following, we first present the retailer's behavioral ordering model in Section 1.3.1 and then introduce the class of nonlinear contracts considered in Section 1.3.2.

1.3.1 The Behavioral Order Quantity from a Boundedly Rational Retailer

In a classic newsvendor model with a fully rational retailer, the retailer always places an optimal order quantity q^* that maximizes her expected profit. The behavioral model departs from standard newsvendor model in that the retailer is not always able to choose the optimal order quantity. The deviation from the optimal newsvendor order quantities observed in laboratory experiments is called as "pull-to-center" effect. Researchers have suggested three different behavioral ordering models in the literature: demand chasing (Schweitzer and Cachon, 2000; Bostian et al., 2008), mean-anchoring (Schweitzer and Cachon, 2000; Bostian et al., 2008), and Logit choice (Su, 2008), which will be incorporated in this chapter using a generic newsvendor behavioral ordering model. The behavioral ordering model consists of the optimal order quantity and a behavioral bias term. That is, under a given contract (w, u, v), the order quantity placed by a

Table 1.1: Newsvendor Behavioral Order Quantity Heuristics

Ordering Heuristic	$Q = \lambda\, q^* + (1-\lambda)\, \tilde{q}$	generic model in §1.3.1
Mean-anchoring	$\tilde{q} = \mu$	see §1.6.1
Logit model	$\tilde{q} \sim \text{Truncated}\mathcal{N}(\mu_{\mathcal{N}}, \sigma^2_{\mathcal{N}})$	see §1.6.1
Demand chasing	$\tilde{q} \sim \mathcal{U}\big(\mu(1-\Delta),\, \mu(1+\Delta)\big)$	base model in §1.4

boundedly rational retailer is given by

$$Q(w,u,v) = \lambda\, q^*(w,u,v) + (1-\lambda)\, \tilde{q}, \tag{1.1}$$

where $q^*(w,u,v)$ is the optimal order quantity that satisfies $\mathbb{P}(q^*(w,u,v) \geq \mathbf{D}) = \frac{c_u^r}{c_u^r + c_o^r} = \frac{p-w+u}{p-v+u}$. The term $\tilde{q}$ captures the deviation from the optimal order quantity and λ is the retailer's *rationality coefficient*, with $\lambda = 1$ representing a fully rational retailer. As shown in Table 1.1, different behavioral models studied in the literature could be considered as special cases of the behavioral order quantity model in (1.1). For example, if $\tilde{q}$ is equal to the mean (or median) demand, then Q captures the mean-anchoring decision heuristic; if $\tilde{q}$ follows a truncated Normal distribution with proper mean and standard deviation, then Q expresses the behavioral order quantity induced by the Logit choice model introduced in Su (2008) (see Lemma 3 in the Appendix A for the formal statement and proof); and finally if $\tilde{q}$ follows the same distribution as demands, then Q represents the demand chasing as it is seen from the supplier's point of view.

In the base model, we consider the demand chasing heuristic. That is, the retailer's behavioral orders is a weighted average of the optimal order quantity q^* and a random error $\tilde{q}$, where $\tilde{q}$ has a distribution that is identical to the demand's distribution, i.e., $\tilde{q} \sim \mathcal{U}(\mu(1-\Delta),\, \mu(1+\Delta))$. Therefore, this captures demand chasing as seen by the supplier. Later, in Section 1.6.1, we will show that our results also hold for the other two behavioral ordering models as well.

For notation simplicity, let $\xi = 2(\frac{p-w+u}{p-v+u}) - 1 = \frac{p-2w+u+v}{p-v+u}$ and notice that the non-negative (negative) values of ξ corresponds to high (low) margin scenarios. Under a uniformly distributed demand, the optimal order quantity is $q^* = \mu(1+\Delta\xi)$ and the behavioral order quantity observed by the supplier is

$$Q(w,u,v) = \lambda\mu\big(1+\Delta\,\xi\big) + (1-\lambda)\tilde{q}, \tag{1.2}$$

with $\mathbb{E}[Q] = \mu(1+\lambda\,\Delta\,\xi)$. When $\lambda = 1$, the behavioral order quantity Q is reduced to the classic newsvendor order quantity q^* from a rational retailer, and when $\lambda = 0$, Q is the order quantity from an irrational retailer whose decision is random and does not respond to the contract parameter (w,u,v). When $0 < \lambda < 1$, the retailer is boundedly rational and the order quantity has a random term $\tilde{q}$. Hence, when deciding on the contract to offer to the retailer, the supplier faces two different sources of randomness: demand uncertainty and a noisy order quantity by the boundedly rational retailer.

Based on the behavioral order quantity model, we first derive the expected leftover $\mathbb{E}[Q-\mathbf{D}]^+$ and the service level $\mathbb{P}(Q \geq \mathbf{D})$ as follows (see Lemma 5 in the Appendix A).

$$\begin{aligned} \mathbb{E}[Q-\mathbf{D}]^+ &= \frac{\mu\Delta}{12}\left(3+6\lambda\xi+\left(1+\lambda\left(-2+\lambda+3\lambda\xi^2\right)\right)\right) && (1.3)\\ \mathbb{P}(Q \geq \mathbf{D}) &= \frac{1+\lambda\xi}{2} && (1.4) \end{aligned}$$

Recall that $\xi = 2(\frac{p-w+u}{p-v+u}) - 1 \geq 0$ if and only if the critical ratio $\frac{p-w+u}{p-v+u} \geq \frac{1}{2}$. When the retailer is boundedly rational (i.e. $\lambda < 1$) and $\frac{p-w+u}{p-v+u} \geq 0.5$, the retailer's probability of overstocking increases with the retailer's rationality coefficient λ. In contrast, if $\frac{p-w+u}{p-v+u} < 0.5$, the retailer's service level decreases with the retailer's bounded rationality. In other words, a boundedly rational retailer is more likely to place an order larger (smaller) than the optimal order quantity q^* when the profit margin is high (low). These results are consistent with the "pull-to-center effect" observed in the literature.

1.3.2 Supply Chain Contract and Profit Functions

We introduce a general framework that covers a wide range of supply chain contracts that are shown to effectively reduce the double marginalization in the supply chain literature. Without providing the specific contract terms, we first represent the supply chain profit, the retailer's profit, and the supplier's profit with underage and overage costs. Later in this section we introduce a general contract framework which incorporate a wide range of supply chain contracts as its special case.

Denote the retailer's behavioral order quantity by Q and the random demand by $\mathbf{D}$. The supply chain profit can be written as

$$\Pi_c = c_u^c\, \mathbb{E}\big[Q\big] - (c_u^c + c_o^c)\, \mathbb{E}\big[Q - \mathbf{D}\big]^+,$$

where $c_u^c = p - c$ and $c_o^c = c$ are supply chain's underage and overage costs, respectively. In a similar way, the retailer's profit (Π_r) and the supplier's profit(Π_s) can be represented as

$$\Pi_r = c_u^r\, \mathbb{E}\big[Q\big] - (c_u^r + c_o^r)\, \mathbb{E}\big[Q - \mathbf{D}\big]^+, \quad \text{and} \tag{1.5}$$

$$\Pi_s = c_u^s\, \mathbb{E}\big[Q\big] - (c_u^s + c_o^s)\, \mathbb{E}\big[Q - \mathbf{D}\big]^+. \tag{1.6}$$

where $c_u^r(c_u^s)$ and $c_o^r(c_o^s)$ are the retailer's (supplier's) underage and overage costs, respectively. Since $\Pi_c = \Pi_r + \Pi_s$ always holds, we have $c_u^s + c_u^r = c_u^c(= p - c)$ and $c_o^s + c_o^r = c_o^c(= c)$. Many supply chain contracts can be phrased using the above framework (see Table 1.2). For example, under a wholesale price contact the retailer's underage cost is $c_u^r = p - w$ and we have $c_u^r + c_o^r = p$. Furthermore, the manufacture's underage cost is $c_u^s = w - c$ and $c_u^s + c_o^s = 0$.

In order to reduce double marginalization, a nonlinear contract aims to increase the retailer's order quantity by (1) increasing the retailer's underage cost and/or (2) decreasing her overage cost. Using this framework, we can categorize supply chain contracts into four mutually exclusive categories which cover a wide range of supply chain contracts (see also Chen and Özer (2017)).

1. *Downside-protection.* A contract in this category reduces the retailer's overage cost compared to that of the wholesale price contract while the underage cost stays unchanged. In our notations, under a downside-protection contract we have $c_u^r = p - w$ and $c_o^r \leq w$. We define $c_o^r = w - v$ with $v \geq 0$.

2. *Upside-protection.* A contract in this category increases the retailer's underage cost compared to that of the wholesale price contract while the overage cost stays unchanged. In our notations, under an upside-protection contract we have $c_u^r \geq p - w$ and $c_o^r = w$. We define $c_u^r = p - w + u$ with $u \geq 0$.

3. *Two-sided protection.* A contract in this category simultaneously reduces the retailer's overage cost and increases the retailer's underage cost compared to those of the wholesale price contract. In our notations, under a downside-protection contract we have $c_u^r \geq p - w$ and $c_o^r \leq w$ where we define $c_u^r = p - w + u$ and $c_o^r = w - v$.

4. *No-protection.* A contract in this category does not change the retailer's overage and underage costs comparing to those of the wholesale price contract, i.e. $c_u^r = p - w$ and $c_o^r = w$. The contracts in this category, such as two-part tariff and wholesale price, are usually used to share channel profit between the retailer and the supplier without changing the inventory risk from one party to another.

Let $c_u^r = p - w + u$ and $c_o^r = w - v$ and observe that retailer's critical ratio is $\frac{c_u^r}{c_u^r + c_o^r} = \frac{p-w+u}{p-v+u}$. Here, w is the wholesale price, u captures the changes in the retailer's underage cost (comparing to its base value $p-w$), v indicates the changes in the retailer's overage cost (comparing to its base value w), and the sum of overage and underage costs decreases by $v-u$ from its base value p in the wholesale price contract. We now introduce our general nonlinear contract (w, u, v). This contract increases (decreases) retailer's underage(overage) cost from its base value by $u(v)$. According to (1.5) and (1.6) for a contract (w, u, v), the retailer's and the supplier's profit functions can be written as

$$\Pi_r(w, u, v) \quad = \quad (p - w + u)\,\mathbb{E}\big[Q\big] - (p - v + u)\,\mathbb{E}\big[Q - \mathbf{D}\big]^+, \qquad (1.7)$$

and

$$\Pi_s(w, u, v) \quad = \quad (w - c - u)\,\mathbb{E}\big[Q\big] - (v - u)\,\mathbb{E}\big[Q - \mathbf{D}\big]^+, \qquad (1.8)$$

respectively, where Q denotes the retailer's order quantity under the contract (w, u, v) which is specified in Section 1.3.1. The equation (1.7) suggests that the supplier's profit does not depend on the ex-post demand if and only if $u = v$. A contact is a wholesale price contract if and only if $u = v$ holds[1]. On the other hand, manufacturer's profit under a nonlinear contract depends on ex-post realization of demand implying $u \neq v$. As we have mentioned, the contract (w, u, v) includes most of well-known supply chain contracts as its special cases. Table 1.2 lists common supply chain contracts and shows how they can be represented as special cases of a general contract (w, u, v). For example, $(w, u, v) = (w, 0, 0)$ represents a wholesale price contract with wholesale price w, a buyback contract with a buyback price b can be written as $(w, u, v) = (w, 0, b)$,

[1]A wholesale price contract is defined as $u = v = 0$. However, for any $u = v > 0$, the contract (w, u, v) can be transformed into an equivalent wholesale price contract $(w - u, 0, 0)$. See Lemma 6 in the Appendix A for the formal proof.

Table 1.2: Special Cases of Contract (w, u, v)

Contracts		u	v	c_u^r	c_o^r	Critical Ratio
Supply chain		–	–	$p-c$	c	$\frac{p-c}{p}$
Wholesale	w	0	0	$p-w$	w	$\frac{p-w}{p}$
Buyback	(w,b)	0	b	$p-w$	$w-b$	$\frac{p-w}{p-b}$
Revenue sharing	(w,f)	0	$(1-f)p$	$fp-w$	w	$\frac{fp-w}{fp}$
Rebate	(w,r)	r	0	$p-w+r$	w	$\frac{p-w+r}{p+r}$
Penalty	(w,z)	z	0	$p-w+z$	w	$\frac{p-w+z}{p+z}$

and a revenue sharing contract with a sharing rate f can be written as $(w, u, v) = (w, 0, (1-f)p)$.[2]

We denote the set of all feasible contracts that the supplier may offer to the retailer by

$$\mathcal{F} = \{(w, u, v) \, : \, 0 \leq w \leq p, \;\; -(p-w) \leq u \leq w, \;\; -(p-w) \leq v \leq w\}.$$

The condition is derived by assuming that the value of the wholesale price w, the retailer's overage cost c_o^r, and the retailer's underage cost c_u^r all need to be non-negative and less than p. In addition, the critical ratio under a contract (w, u, v) is between its minimum value induced by the wholesale price contract, i.e. $\frac{p-w}{p}$, and the maximum value by the integrated supply chain, i.e. $\frac{p-c}{p}$.

We study the supply chain problem from the supplier's point of view subject to the constraint that the retailer needs to earn at least her reservation utility α.[3] Hence, the

[2] We adopt the same way to define u and v as in Chen and Özer (2017) by comparing the overage and underage cost of revenue sharing with the wholesale price contract $(w + (1-f)p)$.

[3] Even though the retailer has limited ability in choosing the optimal decision, the supplier would offer the retailer a contract that gives her at least the reservation utility (in expectation) to keep her in business. This is because the retailer can evaluate the expected outcome based on her long-run interaction with the supplier. Our model can be generalized to the case in which the participation constraint is replaced with $\mathbb{P}(\Pi_r \geq \alpha) \geq \beta > 0$, where β represents the retailer's capability in making participation decisions.

supplier's problem can be expressed as

$$\max_{w,u,v} \left\{ \Pi_s(w,u,v) \quad \text{s.t.} \quad \Pi_r(w,u,v) \geq \alpha, \;\; (w,u,v) \in \mathcal{F} \right\}. \qquad (\mathcal{P}_s)$$

We also formulate the contract design problem from the supply chain's point of view and study the problem

$$\max_{w,u,v} \left\{ \Pi_c(w,u,v) \quad \text{s.t.} \quad \Pi_r(w,u,v) \geq \alpha, \;\; (w,u,v) \in \mathcal{F} \right\} \qquad (\mathcal{P}_c)$$

as a benchmark.

1.4 Model Analysis and Results

In this section, we consider the supplier's choice of contracts among all nonlinear contracts in the form of (w,u,v). In Section 1.4.1 we derive the optimal value of w for a given (u,v). In Section 1.4.2, we endogenize the supplier's decision of u and v and show that among all possible nonlinear contracts (w,u,v), a contract with no protection, such as a simple wholesale price contract, can perform better than a contract with $u, v \neq 0$.

1.4.1 Optimal Wholesale Price

We first investigate the supplier's optimal choice of w for given u and v by solving

$$\Phi_s(u,v) = \max_w \left\{ \Pi_s(w,u,v) \quad \text{s.t.} \quad \Pi_r(w,u,v) \geq \alpha, \;\; (w,u,v) \in \mathcal{F} \right\}. \qquad (\mathcal{P}_{sw})$$

Similar to the analysis of the fully rational case [4], we can show that the supplier fully extracts the retailer' surplus up to her reservation value. That is, the constraint $\Pi_r(w,u,v) \geq \alpha$ in $\mathcal{P}_{sw}$ is binding.

[4] See the proof of Lemma 2, part (iii) in Appendix A.

Lemma 1 *The optimal solution of* $\mathcal{P}_{sw}$ *is the unique* $w_\alpha(u,v)$ *which solves the equation* $\Pi_r(w_\alpha(u,v),u,v)=\alpha$*. In addition,* $w_\alpha(u,v)$ *increases in* $u-v$*.*

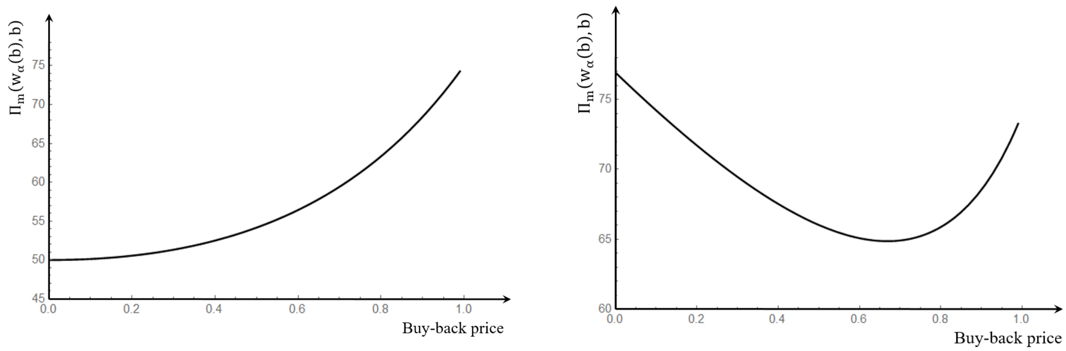
Lemma 1 implies that the solution to $\mathcal{P}_{sw}$ is unique and the retailer earns, in expectation, her reservation value. This simplifies the problem $\mathcal{P}_{sw}$ and enables us to characterize the optimal contract in §1.4.2. As the following lemma indicates, the above result also enables us to examine the impact of retailer's bounded rationality on her service level.

Lemma 2 *The retailer's service level* $\mathbb{P}(Q \geq \mathbf{D}) = \frac{1+\xi}{2}$ *under the optimal wholesale price* $w_\alpha(u,v)$ *does not change with* $u-v$ *when* $\lambda = 1$*, and is strictly increasing in* $u-v$ *when* $0 \leq \lambda < 1$*.*

Intuitively, as long as the supplier leaves the same profit α to the fully rational retailer, he has to induce the same critical ratio by providing the contract $(w_\alpha(u,v),\ u, v)$ so that the retailer places the same order quantity. However, for a boundedly rational retailer, $w_\alpha(u,v)$ increases with $u-v$ at a lower rate because the retailer is less responsive to the contract term. As a result, the retailer's service level increases in $u-v$.

1.4.2 Optimal Protections

We now endogenize the supplier's decision on (u,v) and solve the optimization problem $\mathcal{P}_s$. When a retailer is rational, a contract (w,u,v) with non-zero u or v allows the risk sharing between the supplier and the retailer. The goal in this section is to show that the optimal choice for a supplier contracting with a boundedly rational retailer can be in the form of no-protection contract, i.e. $u=v=0$. As a no protection contract with $u=v=0$ induces the same overage and underage costs as the wholesale price contract, the outcome of any such a contract is identical to the wholesale price contract.



(a) Fully rational retailer ($\lambda = 1$) (b) Boundedly rational retailer ($\lambda = 0.65$)

Figure 1.1: Examples of Fully Rational and Boundedly Rational Cost Function $\Pi_s(w_\alpha(b), b) = \Pi_s(w, b, 0)$ with Parameters of $p = 1$, $c = 0.5$, $\mu = 200$, $\Delta = 0.5$ for a Buyback Contract.

When the retailer is fully rational, the supplier's profit function $\Pi_s(w_\alpha(u, v), u, v)$ is convex and strictly increasing in $v - u$. However, when the retailer is boundedly rational, although $\Pi_s(w_\alpha(u, v), u, v)$ is still convex, it is not always increasing anymore. Figure 1.1b illustrates this point using an example of buyback contract. Recall that the contract $(w, u, v) = (w, 0, b)$ represents the buyback contract with wholesale price w and buyback price b. As in Figure 1.1b, the supplier's profit $\Pi_s(w_\alpha(0, b), 0, b)$ first decreases as b $(=v - u)$ increases and then increases. This subtle difference between the rational retailer and the boundedly rational retailer gives rise to our main result. Next, we characterizes the conditional under which a wholesale price contract is preferable to other non-linear contracts.

Since the optimal solution to the problem $\mathcal{P}_{sw}$ is $w_\alpha(u, v)$ and we have $\Pi_c(w_\alpha(u, v), u, v) = \Pi_s(w_\alpha(u, v), u, v) + \alpha$, solving $\mathcal{P}_s$ is equivalent to solving

$$\max_{u,v}\{\Pi_c(w, u, v) \text{ s.t. } w = w_\alpha(u, v), \ (w, u, v) \in \mathcal{F}\}. \tag{1.9}$$

As shown in Lemma 7 in the Appendix A, the supply chain profit in (1.9) depends on contract parameters only through the intermediate variable ξ, i.e. we can write the supply chain's profit as

$$\Pi_c(\xi) = \mu\,(p - c(1+\Delta\lambda\xi)) - \frac{\mu\,p}{12}\Big(\Delta\big(\lambda(\lambda + 3\,\xi(\lambda\xi - 2) - 2) + 4\big)\Big). \tag{1.10}$$

Since ξ is a well-defined function with maximum value 1 and minimum value $\underline{\xi} = 1 - 2\frac{w_\alpha(0,0)}{p}$, we can write the problem (1.9) as $\max_\xi\{\Pi_c\big(\xi\big)$ s.t. $\underline{\xi} \le \xi \le 1\}$. Because $\Pi_c(\xi)$ is concave, the solution of this problem is either the interior solution $\frac{1}{\lambda}\left(\frac{p-2c}{p}\right)$ solved by the first order condition or one of the boundary points. The following proposition shows that when the condition $\frac{1}{\lambda}\left(\frac{p-2c}{p}\right) < 1 - 2\frac{w_\alpha(0,0)}{p}$ holds the optimal choice of contract by the supplier, among *all* feasible contracts (w, u, v), will be a wholesale contract.

Proposition 1 *If the condition*

$$0 \le w_\alpha(0,0) \le \frac{1}{2}\left(p - \frac{1}{\lambda}\,(p - 2c)\right) \tag{1.11}$$

holds, the wholesale contract $(w^*, u^*, v^*) = (w_\alpha(0,0), 0, 0)$ *is optimal among all feasible nonlinear contracts defined in* $\mathcal{F}$*.*

This proposition is in sharp contrast with the classical results in the literature. For example, Pasternack (1985) shows that, under a buyback contract (w, b), the supplier's profit is always increasing in buyback price b. This implies that, when dealing with a fully rational retailer, the supplier always prefers a buyback contract with $b > 0$ (i.e., the downside-protection with $u = 0$ and $v = b > 0$) to a wholesale price contract.

Note that the condition (1.11) holds only when the retailer is boundedly rational. When the retailer is fully rational, i.e. when $\lambda = 1$, the condition (1.11) can never be satisfied because $\frac{1}{\lambda}\left(\frac{p-2c}{p}\right) > \underline{\xi} = \frac{p-2\,w_\alpha(0,0)}{p}$ cannot hold as long as $w_\alpha(0,0) \ge c$. Hence, Proposition 1 confirms the classic result that nonlinear contracts, such as buyback, revenue sharing, and rebate, dominate wholesale price contracts when the retailer is fully rational. Intuitively, when the retailer is rational, a nonlinear contract serves as a mechanism to motivate the retailer to place a larger order quantity. Hence, a nonlinear contract can always implement the supply chain's optimal order quantity by having the supplier sharing the risk with the retailer. However, when the retailer is boundedly rational, her order can be already larger than the supply chain optimal order quantity due to the errors that the retailer may make under a wholesale price contract. Adopting a nonlinear contract in such cases may harm the supplier's profit, especially when the profit margin is low. Taking the buyback contract as an example, consider an extreme case in which the retailer is fully irrational, i.e. $\lambda = 0$. That is, the retailer's order quantity is a random number that is independent of the supplier's contract. In such a case, the best strategy for a supplier is to set the highest possible wholesale price while setting the buyback price to be zero. The next two corollaries present sufficient and necessary conditions for the optimality of a wholesale price contract based on the intuition discussed above.

Corollary 1 (Necessary Condition) *The wholesale price contract $w^* = w_\alpha(0, 0)$ is optimal among all feasible nonlinear contracts only if $p - 2c \leq 0$.*

Corollary 1 states that the superiority of a wholesale price contract happens only when the supplier produces low margin profit products. The result is consistent with real-world observation that nonlinear contracts are adopted mainly in high margin industries, such as revenue sharing contracts in DVD rental (Cachon and Lariviere, 2005), rebate contracts in high tech companies such as Microsoft, Hewlett Packard, Lotus (Kaya and Özer, 2011) and pharmaceutical companies (Dunlop et al., 2018).On the other hand, wholesale price contracts are more often observed in low margin industries such as grocery stores. The sufficient condition in the next corollary indicates that if the profit margin is low and the retailer is sufficiently irrational, a wholesale price contract is optimal for any values of α and Δ.

Corollary 2 (Sufficient Condition) *The wholesale price contract* $w^* = w_\alpha(0,0)$ *is always optimal among all feasible nonlinear contracts if* $\frac{2c}{p} > 1 + \lambda$.

Notice that the condition $\frac{2c}{p} > 1 + \lambda$ in Corollary 2 holds only when the condition $p - 2c \leq 0$ in Corollary 1 holds as well.

1.4.3 Comparative Statics

In this section, we examine how the supplier's choice of contracts is affected when the parameters in the model change using buyback contracts as an example.

Proposition 2 *For given parameters* p, c, μ, Δ, *and* α, *there exists* $0 \leq \bar{\lambda} \leq 1$ *such that a wholesale price contract is optimal among all nonlinear contracts if and only if* $\lambda \leq \bar{\lambda}$. *In addition, the threshold* $\bar{\lambda}$ *is nondecreasing in* c.

Figure 1.2 shows the supplier's optimal choice of contract for pairs (λ, c) where $p = 12$, $\mu = 200$, $\Delta = 0.5$, and $\alpha = 20$. As the rationality coefficient λ decreases, the supplier chooses a wholesale price contract for a wider range of unit production cost c. For example, given $c = 9$, the supplier prefers a buyback (nonlinear) contract for $\lambda = 0.75$, whereas the supplier's preference shifts to a wholesale price contract if the rationality coefficient decreases to $\lambda = 0.45$ or below. When λ is relatively low, the retailer is not quite responsive to a nonlinear contract and the supplier prefers a wholesale price contract because he otherwise has to share a high risk with the retailer whose order is very random under a nonlinear contract. When λ is relatively high, the supplier always prefers a nonlinear contract because it is effective in aligning the retailer's incentive to that of supply chain

Proposition 3 *For given parameters p, λ, μ, Δ, and α, there exists $0 \leq \bar{c} < p$ such that a wholesale price contract is optimal if and only if $c \geq \bar{c}$. In addition, the threshold $\bar{c}$ is decreasing in λ.*

As shown in Figure 1.2, given a rationality coefficient $\lambda = 0.6$, a buyback (nonlinear) contract is preferred if $c = 7$, whereas the preference is shifted to a wholesale price contract when c increases to $c = 10$. As we have stated in Corollary 1, a wholesale price contract is preferred only in the low margin cases, i.e., $\frac{c}{p} \geq \frac{1}{2}$. If $\frac{c}{p}$ is more than $\frac{1}{2}$ (i.e., the area below the dashed line in Figure 1.2), a buyback contract with a positive buyback price is preferred for all feasible values of λ, μ, Δ, and α. Proposition 4 shows how the demand variation or the accuracy of demand information can affect the supplier's contracting preferences.

Proposition 4 *For given parameters p, λ, c, μ, and α, there exists $0 \leq \bar{\Delta} \leq 1$ such that a wholesale contract is optimal if and only if $\Delta \geq \bar{\Delta}$. In addition, the threshold $\bar{\Delta}$ is increasing in λ.*

As shown in Figure 1.3a, the accuracy of the demand information matters when λ is medium. As Δ decreases, the accuracy of the demand information improves. Figure 1.3a indicates that the supplier is more likely to adopt a wholesale price contract when the demand is less accurate (i.e., Δ is large), but is less likely to do so when the demand information is more precise (i.e. Δ is small).

Proposition 5 *For given parameters p, λ, c, μ, and Δ, there exists $\bar{\alpha} > 0$ such that a wholesale contract is optimal if and only if $\alpha \geq \bar{\alpha}$. In addition, the threshold $\bar{\alpha}$ is increasing in λ.*

Proposition 5 and Figure 1.3b show that the retailer's reservation value (or outside option) α can affect the supplier's choice of contract. When α increases, the supplier

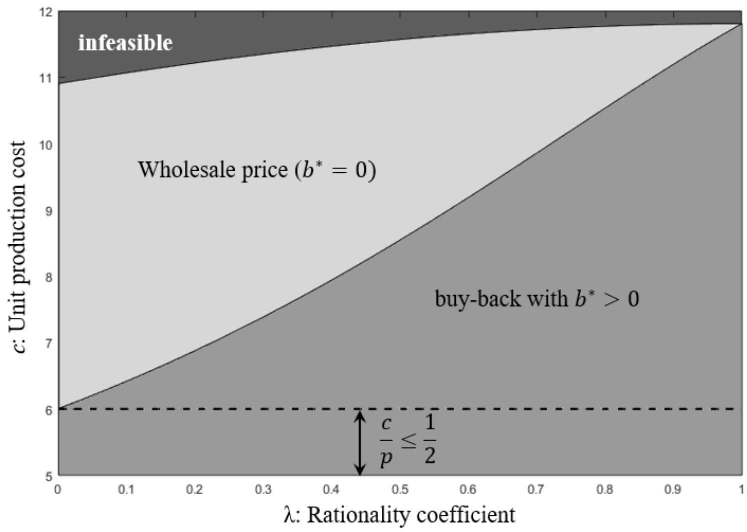
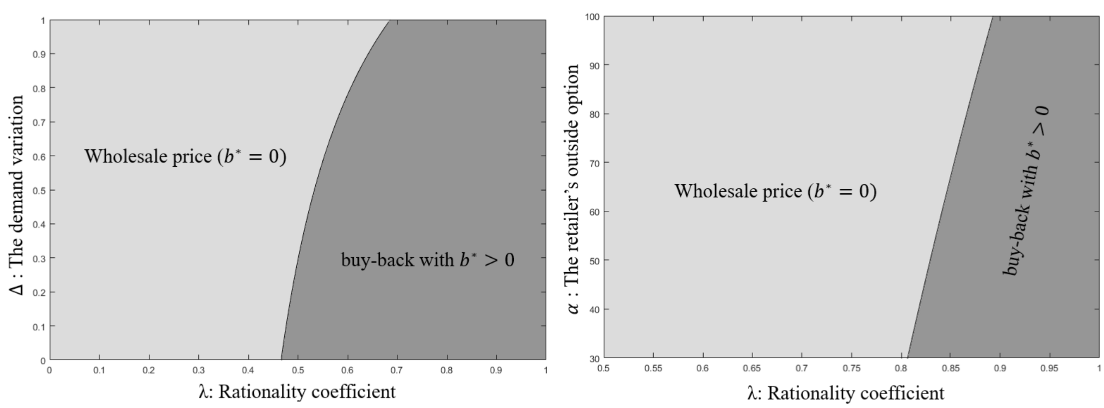


Figure 1.2: Effects of Changes in λ and c on the Supplier's Choice of Contract



(a) $c = 8.75$ and $\alpha = 20$ (b) $\Delta = 0.5$ and $c = 10.5$

Figure 1.3: Effects of Changes in Δ and α on the Supplier's Choice of Contract ($p = 12$ and $\mu = 200$)

favors the wholesale price contract more. This is because the supplier has to lower the wholesale price in order to leave more reservation value to the retailer which makes nonlinear contracts less likely to be profitable.

1.5 Laboratory Experiments

To examine whether human decision makers exhibit a similar contractual preferences as those predicted by the behavioral model, we conduct a series of laboratory expe riments. As so, we have human subjects play the role of a supplier who needs to offer a contract to a boundedly rational retailer. The purpose of the experiments is not to tease out a human supplier's behavioral biases. Instead, we use experiments as a robustness check and to validate the results obtained in the behavioral model. That is, a suppliers may prefer a wholesale price contract when facing a boundedly rational retailer despite of other potential biases that we haven't considered in the model.

1.5.1 Hypotheses

We first study how changes in parameters affect subjects' decisions. Our behavioral model suggests that wholesale price contracts become more favorable when retailer's rationality decreases (Proposition 2), production cost increases (Proposition 3), and variance of demand increases (Proposition 4). These results lead to Hypotheses 1.

Hypothesis 1 *When the retailer's rationality decreases, production cost increases, or demand variation increases, human suppliers are more likely to choose a wholesale price contract.*

Our behavioral model predicts that, when the retailer is boundedly rational, the supplier may gain higher profits under a wholesale contract than under any other non-linear contract (a contract with non-zero u and v). Although human suppliers' choices

Table 1.3: Summary of Treatments

Treatments	$\lambda\times$ **100%**	**c**	$\boldsymbol{\Delta}$
1. BASE : Baseline	75%	9	0.5
2. LOWR : Low Rationality	55%	9	0.5
3. HPCO : High Production Cost	75%	10.5	0.5
4. HDVAR : High Demand Variation	75%	9	0.8

may not completely follow our behavioral model's prediction, we expect that majority of subjects choose a wholesale price contract when the condition in Proposition 1 is satisfied. We examine this in Hypotheses 2 which serves both as a robustness check of our behavioral model and a sanity check of our experiments.

Hypothesis 2 *Where the wholesale price contract is predicted to be optimal by the behavioral model, a larger portion of human suppliers choose a wholesale price contract.*

1.5.2 Design and Implementation

Our experimental design consists of 4 treatments. Treatments differ in the values that the bounded rationality coefficient λ, the production cost c, and the demand variation Δ take. In particular, we consider $\lambda \in \{55\%\,, 75\%\}$, $c \in \{9,\,10.5\}$, and $\Delta \in \{0.5\,, 0.8\}$, $\mu = 200$. In our first treatment (*baseline* or BASE), we keep the values of c and Δ the same as in BASE treatment, but decrease λ to $= 0.55$. Similarly, to form the third treatment (*high production cost* or HPCO), we increase c to 10.5. Finally, in the fourth treatment (*high demand variation* or HDVAR), we increased Δ from 0.5 to 0.8 which corresponds to the demand distribution $D \sim \mathcal{U}(40, 360)$. Table 1.3 summarizes our experimental design and four treatments.

Table 1.4: The Menu of Contracts and Their Payoffs

	Contract		Expected Payoff				Rational q^*	
	w	b	BASE	LOWR	HPCO	HDVAR	others	HDVAR
A	10.75	0	246	35	80	173	121	65
B	11	7.75	249	11	15	143	147	110
C	11.25	0	302	**101**	**156**	**199**	127	75
D	11.5	8.25	**316**	98	104	188	113	51

Human subjects took the role of suppliers who would offer a contract to a computerized boundedly rational retailer. Subjects were informed that the retailer was computerized and placed an order quantity according to Equation (1.1). we provided subjects with the rational order quantity so they do not need to calculate q^*(see Figure B1 in the Appendix B)[5]. Table 1.4 shows the list of contracts we used in our experiments. We used the buyback contract as the nonlinear contract. In addition, to have a fair comparison, the number of buyback contracts and wholesale price contracts were equal and the list of contracts is kept the same for all treatments. The list of contracts was constructed such that it includes a contract close to the optimal contract in each treatment with numbers rounded to the closest quarter. While the contract $(w, b) = (11.5, 8.25)$ results in the highest payoff in the BASE treatment, the contract $(w, b) = (11.25, 0)$ is the most profitable choice in other three treatments.

We used between-subjects design, i.e., each subject was assigned to only one treatment, and all experimental sessions followed the same procedure. In each round, subjects chose a contract to offer to the retailer before the retailer's order quantity and the random demand were realized. Then, subjects observed the retailer's order quantity, the realized demand, and their own payoffs before moving to the next decision round.

[5]This is to reduce the cognitive load of their task and to avoid any possible calculation errors. This ensures us that our results are not driven by limited conative ability of subjects to predict q^*. We conduct experiments to examine if the results would be affected if the supplier is not provided with the rational optimal order quantity In Section 1.5.4

Experiments were programmed and conducted with Z-tree (Fischbacher, 2007). Each subject first played 3 practice rounds, in which their performance did not affect their monetary payments, followed by 30 rounds of the same game in which they accumulated credits toward their final monetary payment. Subjects were paid $4 show-up fee plus a performance-based payment. The sessions took 70 to 90 minutes and payments ranged from $15 to $27. We conducted the experiments at a university in North America where a total of 63 students participated in our study across four treatments. Participants were given written instructions and a quiz to ensure that they understood the instructions (see Appendix C).

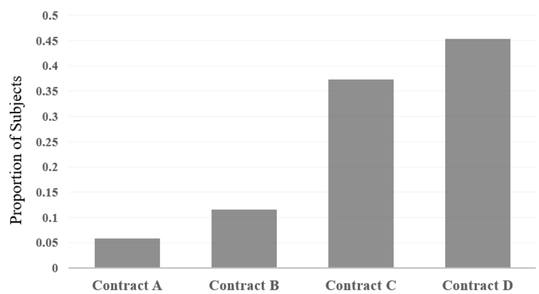
1.5.3 Results

Before providing statistical analysis, we present a summary of the experimental results. Figure 1.4 illustrates the percentage of subjects choosing each contract. In almost all treatments, the suppliers' contract preferences follow the same order as the expected payoffs (see Table 1.4)[6]. On average, 45% of subjects have chosen the optimal buyback contract $(11.5, 8, 25)$ in BASE treatment, whereas roughly half of the subjects (51%, 48%, and 44%, respectively, in LOWR, HPCO, HDVAR treatments) have chosen the optimal wholesale price contract $(11.25, 0)$ in the other three treatments. This indicates that the overall trend is consistent with the behavioral model's prediction.

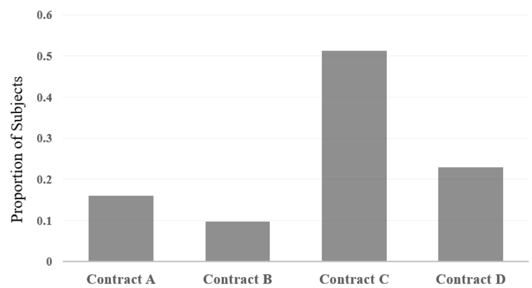
We use *multinomial logistic regression* to compare the outcomes between treatments[7]. Let π_i^t represent the probability that a subject offers contract $i \in \{A, B, C, D\}$ in treatment $t \in \{2(\text{LOWR}),\ 3(\text{HPCO}), 4(\text{HDVAR})\}$. Set the contract D as the base

[6]The only exception is HPCO treatment in which more subjects have chosen contract B than contract A although the expected payoff of contract A is higher. Even though this order is not consistent with the order of the expected payoffs, these two contracts (A and B) together contributes to only 12% of all choices and the effect is not significant in our analysis.

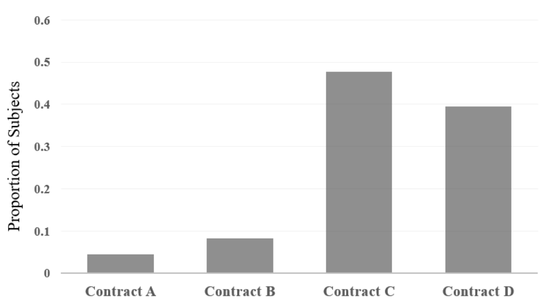
[7]Here we only presents the results for contracts C and D. In the Appendix B, we follow the same analysis but aggregate contracts with the same type (wholesale price vs. buyback). We can get the similar results that wholesale price contracts are more likely to be offered in LOWR treatment, HPCO and HDVAR compared to BASE treatment.



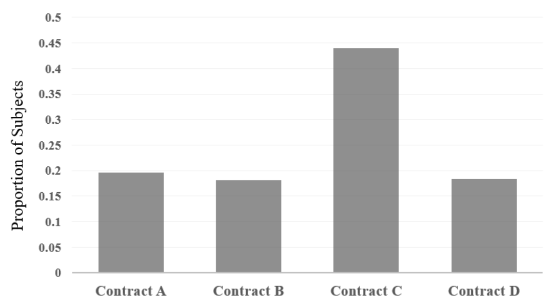
(a) BASE



(b) LOWR



(c) HPCO



(d) HDVAR

Figure 1.4: Summary of Experimental Results for Each Treatment

category and define

$$\mathrm{I}_k^t = \begin{cases} 1 & \text{if } k = t, \\ 0 & \text{if } k \neq t, \end{cases}$$

as the dummy variables for treatment t with $k = 2, 3, 4$. The multinomial logistic regression model can then be formulated as[8]:

$$\log\left(\frac{\pi_j^t}{\pi_D^t}\right) = \alpha_j^t + \sum_{k=2}^{4} \beta_j^t \, \mathrm{I}_k^t, \qquad j \in \{A, B, C\}. \tag{1.12}$$

We conduct three comparisons: (1)LOWR vs. BASE, (2)HPCO vs. BASE, and (3)HDVAR vs. BASE. The coefficient β_j^t represents the treatment effect between offering contract j to contract D in treatment t compared to the BASE treatment. Table 1.5 shows the estimation results of $\log\left(\frac{\pi_C^t}{\pi_D^t}\right)$ (for estimation results of $\log(\pi_A^t/\pi_D^t)$ and $\log(\pi_B^t/\pi_D^t)$ see Appendix B). We reject the null hypothesis that there is no treatment effect in the likelihood of offering contract C versus D, i.e. $H_0 : \beta_C^t = 0$, with *p-value*< 0.05. According to Table 1.5, the probability of offering the contract C in LOWR treatment is $exp(1.021) = 2.775$ times larger than that in BASE treatment. In other words, when the retailer's rationality decreases from 0.75 to 0.55, keeping everything else unchanged, we expect that a supplier is 2.775 times more likely to offer the contract C. In the HPCO treatment and HDVAR treatment, we observe the same trend, i.e., the supplier is 1.466 more likely to offer a wholesale price contract C when the production cost c is increased from 9 to to 10.5, or 2.912 more likely to offer a wholesale price contract C when the demand variation is increases from 0.5 to 0.8. These results provide strong supports for Hypothesis 1. Also, while all factors including rationality coefficient, demand variation, and production cost matter in determining the possibility of offering wholesale

[8]We use the aggregate data from all periods. In Appendix B, we present the model with an additional independent variable of time period and show that the results do not change if we account for the time period effect This also confirms that there is no learning effect.

Table 1.5: Estimation Results for Multinomial Logistic Regression
$^{***}p-value < 0.001$; $^{**}p-value < 0.01$;$^{*}p-value < 0.05$

Treatment		**Intercept** $\alpha_{\mathbf{C}}^{\mathbf{t}}$	**Treatment Effect** $\beta_{\mathbf{C}}^{\mathbf{t}}$
LOWR (t=2)	Coef. α_C^2	-0.216* (0.1)	1.021*** (0.16)
	Wald χ^2	4.28	43.351
	$\exp(\beta_C^2)$	–	2.775
HPCO (t=3)	Coef. α_C^3	-0.216* (0.1)	0.404***(0.13)
	Wald χ^2	4.28	9.44
	$\exp(\beta_C^3)$	–	1.498
HDVAR (t=4)	Coef. α_C^4	-0.216* (0.1)	1.090*** (0.17)
	Wald χ^2	4.28	44.067
	$\exp(\beta_C^4)$	–	2.975

price contracts rather than buyback contract, the impact of rationality coefficient and demand variation play more significant roles than the production cost.

In Hypothesis 2, we focus on subjects' choices within each treatment. To test the first part of Hypothesis 2, we aggregate the two choices of wholesale price contracts (i.e. the contract A and C) and buyback contracts (i.e. the contract B and D) and check which contract type (wholesale or buyback) is chosen more frequently using *Chi-square test* (χ^2) (see Table B4 in Appendix B for *p-values* and the estimations).

In the BASE treatment, we find that the proportion of subjects who chose a wholesale price contract is significantly higher than those who chose a buyback contract . The null hypothesis that two types of contracts are chosen evenly is rejected with $\chi^2 = 8.54$ and *p-value*< 0.01. Since the buyback contract C and the wholesale price contract D contribute to 83% of choices, we also test the null hypothesis that D is chosen as often as C. The null hypothesis is rejected under a chi-square test with $\chi^2 = 4.1$ and *p-value*< 0.05.

In the LOWR treatment, 67% of subjects offered a wholesale price contract where the wholesale price contract C were offered more than 50%. The statistical tests provide strong support to reject the null hypothesis that the two types of contracts were chosen evenly. Wholesale price contracts were chosen significantly more than buyback contracts ($\chi^2 = 57.4$ and *p-value*< 0.001). In addition, comparing the two most popular contracts C and D, the wholesale price contract C was chosen significantly more often than the buyback contract D ($\chi^2 = 51.95$ and *p-value*< 0.001).

In the HPCO treatment, 52% of subjects offered a wholesale price contract whereas 48% offered a buyback contract (see Figure 1.4c). Despite that the dominance of adopting wholesale price contracts over buyback is not significant ($\chi^2 = 1.42$ and *p-value*> 0.1), the wholesale price contract C is chosen significantly more often than any other contract ($\chi^2 = 5.36$ and *p-value*< 0.05).

In the HDVAR treatment, 64% of subjects offered wholesale price contract, whereas 36% offered buyback contracts (see Figure 1.4d). This difference is also statistically significant ($\chi^2 = 35.2$ and *p-value*< 0.01). In addition, the wholesale price contract C was chosen more frequently than the buyback contract D ($\chi^2 = 50.6$ and *p-value*$<$ 0.001). These together provide strong supports for Hypothesis 2.

1.5.4 Robustness Checks

We conduct two additional treatments to check whether the results observed in this section hold in more complicated settings. In both treatments, we keep the parameters the same as LOWR treatment , i.e. $\lambda = 0.75$, $c = 9$, and $\Delta = 0.5$ where more human subjects chose a wholesale price contract. We extend the LOWR treatment to (1) the cases with 6 contracts (SIXC) and (2) the case without providing the retailer's optimal quantity (NOOPT). To examine whether there is a significant difference between treatments SIX($t = 5$) and NOOPT($t = 6$) with LOWR treatment, we estimate the multinomial logistic regression $\log\left(\frac{\pi_w^t}{\pi_{bb}^t}\right) = \alpha_w^t + \beta_w^t \, \mathrm{I}^t$ with the treatment dummy I^5 (I^6

Table 1.6: Summary of descriptive statistics for the Extra Experiments

Contracts	LOWR (n=16)		SIXC (n=14)		NOOPT (n=17)	
	Mean	Std.	Mean	Std.	Mean	Std.
A : (10.75, 0)	0.16	0.065	0.13	0.05	0.18	0.091
B : (11, 7.75)	0.10	0.083	0.08	0.064	0.08	0.099
C : (11.25, 0)	0.51	0.123	0.41	0.102	0.61	0.089
D : (11.5, 8.25)	0.23	0.081	0.23	0.085	0.13	0.102
E : (11, 0)	–	–	0.09	0.085	–	–
F : (11.25, 8)	–	–	0.06	0.085	–	–
Wholesale: {A, C, E}	0.67	–	0.63	–	0.79	–
Buyback: {B, D, F}	0.33	–	0.37	–	0.21	–

) taking value 1 when the data belongs to the treatment $t = 5$ (or 6) and zero when the data belongs to LOWR treatment.

In the treatment SIX, we find that the extra complexity from adding extra contracts to the menu does not change human subjects' contractual preferences. The null hypothesis of observing no treatment effect in the likelihood of offering a wholesale price contract between SIXC and LOWR, i.e. $H_0 : \beta_5 = 0$, cannot be rejected ($p\text{−}value > 0.1$; see Table B6 in Appendix B). This implies that human suppliers facing retailers with low rationality still prefer a wholesale price contract to a buyback contract when more choices of contracts are presented.

In the treatment NOOPT, we keep the same four contracts as presented in LOWR treatment, but we do not provide subjects with the optimal order quantity from a rational retailer (see Table 1.6 for summary of results). The coefficient β^6 is significant and positive ($p\text{−}value < 0.001$; see Table B6 in Appendix B). This implies that subjects in NOOPT treatment prefer the wholesale price contract even more than subjects in LOWR treatment.

In sum, the results obtained from comparing treatments SIXC and NOOPT with LOWR treatment indicate that the observations in Section 1.5.3 are not driven by the simplifications made in the experimental design and similar results can be observed once we introduce more complexity to the experiments.

1.5.5 Discussion: Supplier/Retailer's Bounded Rationality and Supply Chain Profit

While the retailer's bounded rationality would affect the supplier's choice of contract, the supplier's bounded rationality in choosing contracts would also affect the supply chain profit. In this section, we examine the impact of both the retailer's and the supplier's bounded rationality on the supply chain profit.

Assume that both the retailer and the supplier are boundedly rational. The retailer's bounded rationality is as described in Section 1.3 and the supplier chooses a contract according to a Multinomial Logit model (McKelvey and Palfrey, 1995; Su, 2008).We set $p = 12$, $\mu = 200$, $\lambda \in \{55\%\,, 75\%\}$, $c \in \{9,\, 10.5\}$, and $D \sim \mathcal{U}(100, 300)$, the same setting as used in the experiments. In addition, the supplier chooses from the same four contracts as in the experiments, i.e. $(w_1, b_1) = (10.75, 0)$, $(w_2, b_2) = (11, 7.75)$, $(w_3, b_3) = (11.25, 0)$, and $(w_4, b_4) = (11.5, 8.25)$. According to the Multinomial Logit model, a supplier chooses the contract (w_i, b_i), $i \in \{1, 2, 3, 4\}$, with probability

$$\mathbb{P}[(w_i, b_i)] = \frac{e^{\theta \Pi_s(w_i, b_i)}}{\sum_{j \in \{1,2,3,4\}} e^{\theta \Pi_s(w_j, b_j)}}$$

where θ denotes the supplier's rationality coefficient and a larger value of θ represents a more rational supplier. Therefore, when the supplier is boundedly rational, the expected supply chain profit can be represented by $\Psi_c = \sum_{j \in \{1,2,3,4\}} \mathbb{P}[(w_j, b_j)] \times \Pi_c(w_j, b_j)$ and, similarly, the retailer's expected profit can be represented by $\Psi_r = \sum_{j \in \{1,2,3,4\}} \mathbb{P}[(w_j, b_j)] \times \Pi_r(w_j, b_j)$.

As shown in Table 1.7, while the increase of a retailer's rationality and a supplier's would both increase the supply chain profit, the impact of the former is more salient

Table 1.7: Channel and retailer Profits (numbers rounded to the closest integer)

	c	λ θ	0.0025	0.005	0.0075	0.01	0.02	0.03	0.04	0.05
Channel	9	0.55	353	354	354	355	355	355	355	355
		0.75	365	366	366	367	368	368	369	369
	10.5	0.55	122	125	126	127	127	127	127	127
		0.75	154	155	155	156	156	156	157	157
Retailer	9	0.55	31	20	16	14	13	13	13	13
		0.75	66	56	54	53	52	527	52	512
	10.5	0.55	28	17	14	13	13	13	13	13
		0.75	63	55	56	56	56	57	57	57

than the latter. In particular, the increase of the supplier's rationality is ignorable when $\theta \geq 0.05$. In other words, the retailer's bounded rationality play a more important role in supply chain efficiency. In addition, when the supplier is more rational, the marginal increase of the retailer's rationality would result in more increase in supply chain profit.

1.6 Extensions

In this section, we pursue three extensions of the base model. Section 1.6.1 discusses how the base model can be modified to incorporate other behavioral ordering heuristics. In Section 1.6.2, we experimentally test whether the rationality coefficient λ is affected by the contract's parameters. Section 1.5.5 considers the case in which the supplier is also boundedly rational and investigates how it affects the supply chain profit.

1.6.1 Other Behavioral Order Quantity Models

Our base model in Section 1.3 is developed based on the assumption that errors follow the same distribution as demands. Under this assumption, the behavioral order quantity model in Equation (1.1) mimics the retailer's demand chasing heuristic as seen from the supplier's point of view. Here, we check the robustness of our results under two other behavioral models proposed in the literature, namely, mean-anchoring and Logit models.

Mean-anchoring Heurestic

Mean-anchoring is a well-documented heuristic which is repeatedly cited by researchers to explain human retailer's ordering behavior (Schweitzer and Cachon, 2000; Bostian et al., 2008). We adopt the same mean-anchoring model as in Bostian et al. (2008) which assumes that behavioral order quantity is the weighted average of the optimal order q^* (with weight λ) and demand mean (with weight $1-\lambda$). That is, the boundedly rational retailer's order quantity is $q_p = \lambda\, q^* + (1-\lambda)\mu$. This is a special case of the general behavioral model in (1.1) where $\tilde{q} = \mu$.

Our finding that a wholesale price contract may perform better than a nonlinear contracts still holds under this behavioral model. Under the mean-anchoring order quantity model, the behavioral order quantity is the same as the expected order quantity in our base model in Section 1.4 and the probability of overstocking stays unchanged as well. However, the expected leftover changes to $\mathbb{E}[q_p - \mathbf{D}]^+ = \frac{1}{4}\Delta\mu(\lambda\xi+1)^2$. Despite this difference, we show that still the wholesale price contract can perform better than nonlinear contracts (see Proposition 18 in the Appendix A).

The intuition behind the result is that when a retailer is boundedly rational and the profit margin is low, her order quantity is already higher than the order quantity which is optimal for the supplier. Hence, the supplier prefers to use a wholesale price contract. The necessary condition (Corollary 1) and the sufficient condition (Corollary 2) remains unchanged in the case of mean-anchoring heuristic and we only need to replace the wholesale price with w in Proposition 18.

Logit Model

Another widely acknowledged behavioral order quantity model in the literature is the Logit model proposed by Su (2008). We first show that the generic behavioral order quantity model in (1.1) can incorporate the Logit model in Su (2008) once being adapted for our setting. In particular, Lemma 3 shows that, when the demands follow a uniform distribution, the Logit model results in the behavioral order quantity following a truncated Normal distribution.

Lemma 3 *Let* $\mathbf{D} \sim \mathcal{U}(\mu(1-\Delta),\, \mu(1+\Delta))$ *and consider a contract* (w, u, v). *The behavioral order quantity predicated by Logit model follows a truncated normal distribution over the interval* $[\mu(1-\Delta),\, \mu(1+\Delta)]$ *with* $\mu_{\mathcal{N}} = \mu(1+\xi\,\Delta)$ *and* $\sigma^2_{\mathcal{N}} = \frac{2\beta\,\Delta\,\mu}{p-(v-u)}$.

Table 1.8 presents the supplier's choice of contract when the retailer's order quantities follow the Logit model. We vary the retailer's rationality coefficient over $\lambda \in \{0.45,\, 0.75\}$ and the supplier's production cost over $c \in \{8,\, 9,\, 10\}$. We assume that the retailer's random error are distributed according to $\tilde{q} \sim \text{Truncated}\mathcal{N}(0, \sigma^2_{\mathcal{N}})$ where the variation is either high ($\sigma^2_{\mathcal{N}} = 80$) or low ($\sigma^2_{\mathcal{N}} = 40$), and the truncated Normal distribution is distributed over the interval $[\mu(1-\Delta), \mu(1+\Delta)]$. All other parameters are fixed and similar to our experimental design in Section 1.5.2, i.e. $\mu = 200$, $\Delta = 0.5$, and $\alpha = 100$.

When the rationality is high ($\lambda = 0.75$), the optimal choice of contract is a wholesale contract only when the production cost and the variance are high ($c = 8$ and $\sigma^2_{\mathcal{N}} = 80$). We observe a similar trend when $\lambda = 0.45$. For example, when the production cost increases from $c = 8$ to $c = 10$, the choice of contract changes from a buyback contract to a wholesale price contract. In addition, when the production cost is medium and the variation is high ($c = 9$ and $\sigma^2_{\mathcal{N}} = 80$), decreasing λ from 0.75 to 0.45 changes the contract preferences of the supplier from a buyback contract to a wholesale price

Table 1.8: Numerical Results for the Logit Behavioral Order Quantity Model

Parameters	Optimal Contract			Parameters	Optimal Contract		
$(c,\ \lambda,\ \sigma^2_{\mathcal{N}})$	Type	w^*	b^*	$(c,\ \lambda,\ \sigma^2_{\mathcal{N}})$	Type	w^*	b^*
(10, 0.75, 40)	BB	11.10	2.25	(10, 0.45, 40)	WS	11.22	0
(10, 0.75, 80)	WS	10.97	0	(10, 0.45, 80)	WS	10.90	0
(9, 0.75, 40)	BB	11.22	7.50	(9, 0.45, 40)	BB	11.25	4.30
(9, 0.75, 80)	BB	11.17	7.45	(9, 0.45, 80)	WS	10.9	0
(8, 0.75, 40)	BB	11.22	9.00	(8, 0.45, 40)	BB	11.27	6.65
(8, 0.75, 80)	BB	11.19	8.75	(8, 0.45, 80)	BB	10.92	1.30

contract. These observations are consistent with the results derived in the base model.

Contracting with Optimistic and Pessimistic Retailers

In this section, we explore the effects of the retailer's optimistic or pessimistic ordering behavior on the supplier's contract choice. While keeping other assumptions unchanged, we extend the retailer's random error from a uniform distribution $\mathcal{U}(\mu(1-\Delta), \mu(1+\Delta))$ to a beta distribution with parameters β_1 and β_2 which is scaled to the interval $[\mu(1-\Delta), \mu(1+\Delta)]$.[9]

Figure 1.5 shows how the skewness of beta distribution changes with β_1. As β_1 increases, the random errors are more positively skewed. For example, the retailer's errors are symmetric around the mean with $(\beta_1, \beta_2) = (3, 3)$, are positively skewed with $(\beta_1, \beta_2) = (6, 3)$, and are negatively skewed with $(\beta_1, \beta_2) = (1.5, 3)$, which represents a neutral, pessimistic and optimistic retailer, respectively.

[9]To scale a beta random variable defined over $[0, 1]$, to any arbitrary interval $[c, d]$, we may use the following transformation. Let q follows a standard beta distribution with parameters (β_1, β_2), then $\tilde{q}_\beta$ corresponding to (β_1, β_2) follows the probability density function $\frac{(x-c)^{\beta_1-1}\ (d-x)^{\beta_2-1}}{(d-c)^{\beta_1+\beta_2-1}\ \mathrm{B}(\beta_1,\beta_2)}$ for x that takes values over the interval $[c, d]$ where $\mathrm{B}(.,.)$ denotes the beta function.

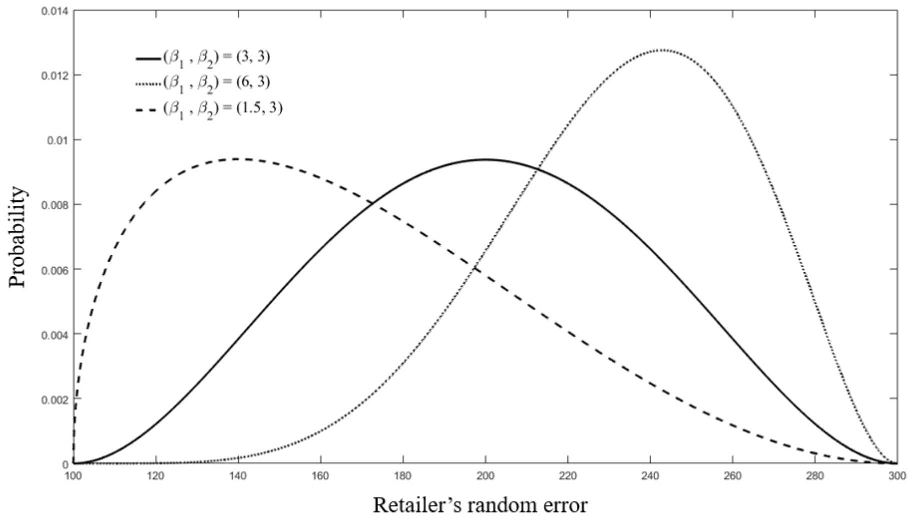


Figure 1.5: Modeling Retailer's Random Errors Using Beta Distribution

We consider the supplier's choice of contract when the retailer's random error $\tilde{q}$ follows a beta distribution. The retailer's rationality coefficient varies over $\lambda \in \{0.45, 0.75\}$ and the supplier's production cost takes values $c \in \{8, 9, 10\}$. All other parameters are fixed and similar to our experimental design in Section 1.5.2, i.e. $\mu = 200$, $\Delta = 0.5$, and $\alpha = 100$.

Table 1.9 illustrates how the changes in the skewness of the distribution of term $\tilde{q}$ can affect the supplier's contractual preferences. A wholesale price contract is more attractive for the supplier who sells the product to an optimistic retailer ($\beta_1 = 6$) than a pessimistic retailer ($\beta_1 = 3$ or 1.5). Intuitively, a more optimistic retailer is more likely to order more than that the supplier wishes. Hence, a buyback contract may reduce the supplier's profit by committing to purchase back the unsold items from the optimistic retailer. In addition, when c increases from 8 to 10, the preferences shift from a buyback contract to a wholesale price contract. Moreover, the supplier facing the retailer with lower rationality coefficient ($\lambda = 0.45$ compared to 0.75) earns a higher profit under a wholesale price contract. These results are consistent with our main findings in Section

Table 1.9: Numerical Results for the Case with Random Errors According to Beta Distribution

Parameters	Optimal Contract			Parameters	Optimal Contract		
$(c,\ \lambda,\ \beta_1)$	Type	w^*	b^*	$(c,\ \lambda,\ \beta_1)$	Type	w^*	b^*
(10, 0.75, 1.5)	BB	11.13	4.4	(10, 0.45, 1.5)	WS	10.93	0
(10, 0.75, 3)	WS	10.94	0	(10, 0.45, 3)	WS	10.53	0
(10, 0.75, 6)	WS	10.77	0	(10, 0.45, 6)	WS	10.11	0
(9, 0.75, 1.5)	BB	11.23	8.65	(9, 0.45, 1.5)	BB	11.17	6.75
(9, 0.75, 3)	BB	11.1	6.6	(9, 0.45, 3)	WS	10.53	0
(9, 0.75, 6)	BB	10.82	1.4	(9, 0.45, 6)	WS	9.98	0
(8, 0.75, 1.5)	BB	11.29	9.9	(8, 0.45, 1.5)	BB	11.31	9.75
(8, 0.75, 3)	BB	11.2	9.05	(8, 0.45, 3)	BB	10.68	2.25
(8, 0.75, 6)	BB	11.07	7.8	(8, 0.45, 6)	WS	9.99	0

1.5.3 that wholesale price contracts are preferred when λ is small, c is large, and Δ is large.

1.6.2 Retailer's Rationality Coefficient

In the base model, we assume that the retailer's rationality coefficient λ is independent of contracts' parameters. In this section, we conduct laboratory experiments on the retailer's ordering behavior to examine to what extent this assumption is valid. We explain how our findings from the laboratory experiments can be incorporated into our behavioral model and, then, discuss the implications to our main results.

We use the same basic parameters used in Section 1.5.2, i.e. $p = 12$, $\mu = 200$, and $\Delta = 0.5$. We consider four levels of buyback prices $b \in \{0, 3, 6, 9\}$ and four levels of critical ratios $cr \in \{0.42, 0.33, 0.25, 0.17\}$[10]. For each combination of buyback price and critical ratio, the corresponding wholesale price was calculated leading to 16 contracts

[10]We only focus on low critical ratio scenarios since, as suggested by Corollary 1, the wholesale contract superiority over nonlinear contracts only holds for low critical ratio scenarios.

Table 1.10: List of Contracts (w, b) in Treatment RET with $cr = \frac{p-w}{p-b}$

cr b	0	3	6	9	q^*
0.42	(7,0)	(8.25,3)	(9.5,6)	(10.75,9)	183
0.33	(8,0)	(9,3)	(10,6)	(11,9)	167
0.25	(9,0)	(9.75,3)	(10.5,6)	(11.25,9)	150
0.17	(10,0)	(10.5,3)	(11,6)	(11.5,9)	133

listed in Table 1.10. We recruited 19 human subjects playing as retailers to participate in the retailer experiments (referred to as RET). At each round, a subject was offered a randomly chosen contract, and then decided on the order quantity before the demand was realized. Each subject first played 3 practice rounds followed by 35 rounds of the same game. A subject was paid a performance-based bonus and $4 show-up fee. A session took 55 to 70 minutes and the payments were ranged from $12 to $20.

The goal of the retailer experiments is to examine whether human retailers' bounded rationality is affected by contracts' parameters. We used buyback contracts as examples of nonlinear contracts and investigated the impact of buyback price on the rationality coefficient λ under different. critical ratio. Our experimental results, summarized in Figure 1.6, find that the impact of the changes in the buyback price on the rationality coefficient is more pronounced when the critical ratio is low.confirm this intuition. For critical ratios 0.42 and 0.33, the human newsvendors order quantity was not affected significantly by the changes in the buyback price. However, for the critical ratios 0.25 and 0.17, as the buyback price increased, the human retailers' rationality dropped.

To rigorously test the above observations, at each level of critical ratio, we estimate the regression model $\lambda_b^{cr} = \lambda_\circ^{cr} + \tau^{cr}\, b + \epsilon^{cr}$ on the aggregate (over all subjects) rationality coefficient, where b is the buyback price and τ^{cr} is the coefficient of interest at the critical ratio cr. As indicated in Table 1.11, while the estimated coefficients $\tau^{0.42}$, $\tau^{0.33}$, and $\tau^{0.25}$ are not significantly different from zero, $\tau^{0.17}$ is significant with $p - value < 0.05$.

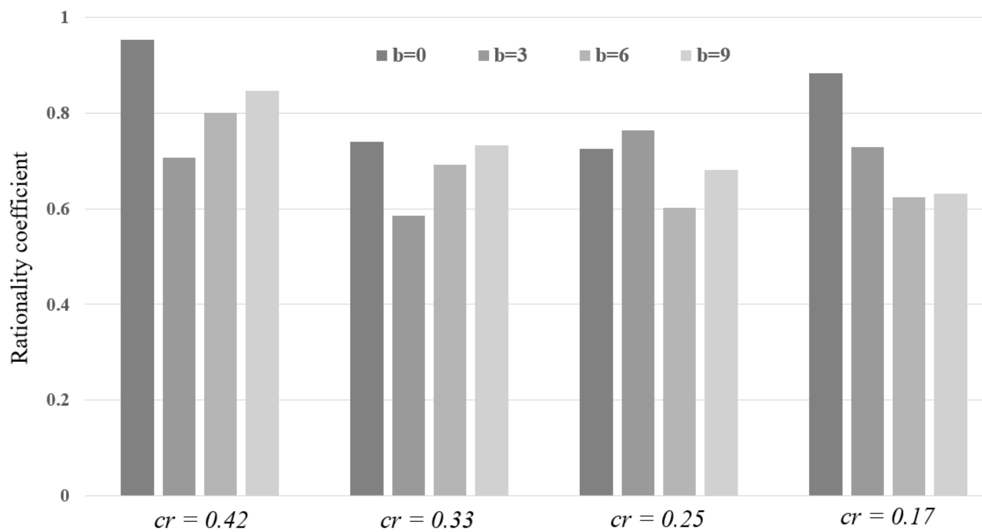


Figure 1.6: The Changes in Rationality Coefficient as the Buyback Price Changes

Table 1.11: Estimation Results for Treatment RET

		$cr = 0.42$	$cr = 0.33$	$cr = 0.25$	$cr = 0.17$
$\lambda_{\circ}^{cr}$	intercept	0.860 (0.110)	0.659 (0.072)	0.772 (0.040)	0.827 (0.021)
τ^{cr}	coefficient	-0.008 (0.020)	0.008 (0.013)	-0.021 (0.007)	-0.023 (0.004)
	p-value	0.734	0.586	0.100	0.026

Hence, we reject the null hypothesis of $H_0 : \tau^{0.17} = 0$ with 5% significant level[11]. We conclude that the retailer's bounded rationality coefficient is not significantly affected by the buyback price for the critical ratios higher than 0.25, but it decreases in the buyback price for the smaller values of critical ratio.

We now revisit the behavioral model in Section 1.3, and relax the assumption that λ is a constant and is not affected by contracts' parameters. Based on our experimental results, we propose that the relation between the rationality coefficient and buyback

[11] Hypothesis $H_0 : \tau^{0.25} = 0$ can be rejected at 10% significant level. However, this does not qualitatively change our results in this section.

price can be captured by

$$\lambda(b, cr) = \begin{cases} \lambda_\circ & \text{if } cr > cr_\ell, \\ \lambda_\circ - \tau^{cr} b & \text{if } cr \leq cr_\ell. \end{cases} \quad (1.13)$$

If $cr > cr_\ell$, the rationality coefficient is constant and it is not effected by changes in the buyback price[12]. If $cr \leq cr_\ell$, the retailer's bounded rationality decreases as the buyback price increases. The threshold cr_ℓ can be evaluated through the experiments. Our experiments indicate that $cr_\ell \leq 0.33$.

Our main result in Proposition 1 stays unchanged even if the rationality coefficient changes with buyback price according to (1.13). If $cr > cr_\ell$, the rationality coefficient is constant (as assumed in the base model) and our main result in Proposition 1 does not change. If $cr \leq cr_\ell$, the retailer's bounded rationality is less than its base value under positive buyback prices. This would make the buyback price less favourable since a retailer is more likely to make errors as the buyback price increases. In sum, since for any critical ratio cr we have $\lambda(b, cr) \leq \lambda_\circ$, the region in which a wholesale price contract is preferable expands.

1.7 Conclusion

Even though the advantages of adopting more complicated contracts, such as buyback and revenue sharing contracts, to reduce double marginalization have been well-documented in the supply chain literature, wholesale price contracts are still widely observed in the real-world practices. Our research aims to providing explanations for this puzzle by incorporating human retailers' bounded rationality into the traditional

[12]Theoretically, $\lambda_\circ$ can depend on the value of cr, however, our experimental results does not show a significant difference in $\lambda_\circ$ for different values of cr. See Appendix B for detailed analysis and estimations.

model of a supply chain. We propose a generic behavioral order quantity model which captures a retailer's deviation from normative theory's prediction. The behavioral order quantity model includes well-known newsvendor ordering heuristics as its special cases. We then examine the supplier's choice of contract between nonlinear contracts and wholesale price contracts in the presence of the retailer's bounded rationality. In contrast with the classic result in supply chain coordination literature which indicates that the supplier can always be better off by using a nonlinear contract, our results suggest that wholesale price contracts can actually be more profitable than many nonlinear contracts believed to achieve a higher profit for the supplier.

We identify the potential advantages of wholesale price contracts as compared to the more complicated nonlinear contracts when the retailer is boundedly rational. First, a boundedly rational retailer may place orders that already lead to a stockout probability that is lower than the rational best response under the wholesale price contract. Hence, a nonlinear contract does not help in moving the retailer's order quantity towards the supply chain optimal order quantity. This is in sharp contrast to the classic newsvendor coordination theory, which suggests that the supplier can always do better with an optimally designed nonlinear contract.

We provide the condition under which a wholesale price contract performs better than nonlinear contracts. We find that the supplier is more likely to implement a wholesale price contract when the profit margin is relatively low. This seems to be in line with most real-world practices, in which complicated contracts are usually adopted in the high profit margin industries such as DVD rentals, semiconductor, pharmaceutical, and software industries. On the other hand, low margin industries such as grocery are more likely to adopt a simple wholesale price contract. The advantage of wholesale price contracts also depends on the demand variance, the retailer's bounded rationality, and reservation value (or bargaining power). In particular, wholesale price contracts are more likely to be adopted when the retailers' demand variation is high, the retailer is less rational, and the retailer's reservation value (or bargaining power) is high. The results hold under different behavioral ordering heuristics such as demand chasing, mean-anchoring, and Logit model.

We test whether the results derived from the behavioral model sustain in a controlled laboratory environment with human subjects taking the role of the supplier. The laboratory experimental results show that human suppliers' contractual preference are consistent with the prediction from the behavioral model. That is, human suppliers are more likely to choose a wholesale price contract when the retailer's rationality is low, the production cost is high, and the demand variation is high.

We find that the retailer's bounded rationality plays an more important role than the supplier's in improving supply chain profit. We also find that the supplier's preference to wholesale price contracts would not be reduced when we vary the number of contracts presented to the supplier, or do not provide a rational retailer's optimal order quantity to the supplier as a reference. Moreover, the results are also robust when the retailer's rationality coefficient are dependent on the contract parameters.

Chapter 2

Peer-to-Peer Trading of Usage Allowances

2.1 Introduction

A growing number of businesses are being built around a model that provides customers access to a product or a service up to a specified amount (an allowance). Examples are many and include cloud-based data storage (for a specified fee, a customer is able to store data up to a certain amount), mobile phone service plans (a customer is able to use data up to a certain amount), fitness classes (a customers can attend up to a maximum number of classes), season tickets for sporting events (a ticket holder is able to attend up to a specified number of events), and fractional ownership of vacation homes (time shares), jets, and boats (an owner is entitled to a specified usage time per year). In each of these cases, the firm providing the access (the service provider) offers consumers a menu of prices and usage allowances. In most of these cases, the realized usage of consumers is uncertain with some consumers experiencing a need for usage (e.g., the need for phone data) that is below their allowance while others experiencing a need for usage that exceeds their allowance[1]. This possibility of either a shortfall or excess in realized usage offers an opportunity for a marketplace to emerge, in which consumers trade unused *capacity* among each other.

Service providers have tended to resist the emergence of such marketplaces, as they are perceived as potentially cannibalizing demand and resulting in downward pressure on prices. However, a growing number of service providers have recently allowed for these peer-to-peer trading markets to exist and, in some cases, facilitated their operation. For example, China Mobile Hong Kong (CMHK), a major mobile phone service provider in Hong Kong, has recently launched a platform (2CM) that allows its customers to trade unused data among each other. CMHK typically offers its customers two data plans: one with 1 GB and one with 5 GB. China Mobile Hong Kong chairman, Tiger Lin Zhenhui, explained the rationale for launching this service as follows : "*Many users who subscribe to a 5GB monthly plan may not use all the data. The 2CM platform would give them the chance to trade capacity*[2]." It is now also increasingly common for sporting event ticket sellers to allow (and in some cases facilitate through their sales portals) the peer-to-peer resale of unused tickets and passes. Similarly, operators of fractional vacation homes are increasingly enticing buyers with the promise of facilitating the rental of unused portions of their *time shares*. Note that the trading platform may not necessarily be owned or operated by the service provider, but rather by a third party entity.

There are several reasons why a service provider may allow or facilitate the peer-to-peer trading of usage allowances. Such trading could enhance the value of the service in the eyes of consumers. In turn, this could induce more demand for the product (e.g.,

[1]Note that the setting we describe does not preclude the case of a one time access to a product or a service, such as a single ticket to a sporting event. Tickets are typically bought before the day of the event. However, the circumstances of a consumer who bought a ticket could change on the day of the event and they may no longer wish to attend the event. Similarly, a consumer who did not buy a ticket before the day of the event may now have a desire to attend. Hence, there is an opportunity for a trading market for single use access.

[2]http://innovation-village.com/you-can-now-sell-unused-internet-data-china-mobile-customers

in the case of CMHK, more customers willing to buy the higher data plan). It may also allow the service provider to increase profits by increasing prices.

While there are clearly potential benefits to trading among consumers, it is less clear whether or not these benefits always outweigh the potential harm trading can cause a service provider because of cannibalization (e.g., in the case of CMHK, fewer customers deciding to purchase the high data plan and instead relying on the trading market to supplement the allowance that comes with the low data plan) or because of increased usage of the service and the associated cost. It is also not clear how trading impacts consumers. On one hand, consumers may be able to fulfill more of their usage and enjoy additional surplus if they sell their excess capacity. On the other hand, the service provider may increase the price or modify other aspects of the service so as to extract any additional surplus generated through trading. Similarly, it is not clear how trading affects social welfare (the sum of consumer surplus and firm's profit) given that any potential increase in usage is accompanied by an increase in usage cost. Given the heterogeneity of consumers in terms of their usage, it is also not clear if all consumers are similarly affected by trading or if some consumers benefit or are harmed more than others.

In this paper, we address these and other related questions. In particular, we consider an equilibrium model involving a service provider who sells access to a service or a product to risk neutral consumers. The service provider offers two contracts to heterogeneous consumers whose type determines their random usage. A contract specifies an upfront payment and, in exchange, grants access up to a predetermined allowance. In equilibrium, consumers self-select by signing up for one of the two contracts. We show that a unique equilibrium exists specified by thresholds on consumers' type.

We show that peer-to-peer trading affects equilibrium outcomes in two ways. First, it affects the choice of consumers between the contract with the higher usage allowance (the high plan) and the contract with the lower usage allowance (the low plan). The prospect of trading can lead more consumers (relative to the case of no trading) to choose the low plan, as they now also have an opportunity to purchase additional capacity on the trading market if they need it. We refer to this effect as *cannibalization*. It is also possible that for the prospect of trading to lead more consumers to purchase the high plan, as they now have an opportunity to sell excess capacity on the trading market if they need it. We refer to this effect as *market expansion*. Second, trading affects consumption (the aggregate amount of usage fulfilled). In particular, trading can result in more consumption if consumers are able to fulfill unmet demand by purchasing additional capacity on the trading market. We refer to this effect as *consumption enhancement*. However, it is also possible for trading to result in less consumption if more consumers choose the low plan and available capacity on the trading market is not sufficient to fulfill unmet demand. We refer to this effect as *demand throttling*.

We show that whether or not trading improves profit, consumer surplus, and social welfare depends on the relative strength of these four effects, which in turn depends on two factors: (1) the cost incurred by the service provider per unit of usage and (2) the prevailing trading price. In particular, trading is harmful to a service provider if either the trading price is sufficiently low (in this case cannibalization prevails with fewer consumers choosing the high plan) or if the usage cost is sufficiently high (in this case the consumption effect dominates, with consumption increasing without a sufficient increase in the number of consumers who choose the high plan). We show that these outcomes are determined by well-specified thresholds on cost and trading price. Assuming no changes are made to plans' prices and allowances, consumers of course always benefit from trading (a consumer can stick to the same plan but now benefits from buying and selling excess capacity). However, social welfare can be harmed if consumption is throttled. We show that this is the case when the trading price is sufficiently low, as determined by also a well-specified threshold. Put together, these results show that there are five distinct regions, dependent on service cost and trading price, that correspond to different combinations of positive and negative outcomes for profit, consumer surplus, and social welfare, with one of these regions corresponding to an improvement in all three (a region of low enough service cost and high enough trading price).

In settings where the service provider is able to modify the contract terms, we show that the service provider charges a higher price for the high plan. This comes at the expense of consumers who are no longer guaranteed to benefit. We show that, in this case, trading can hurt consumers if trading price is either sufficiently low (resulting in less consumption because more consumers opt for the low plan) or sufficiently high (resulting in consumers paying a higher price for the higher plan because of the adjustment to the price of the high plan made by the service provider). Again, depending on trading price and service cost, we can identify five regions corresponding to different combinations of positive and negative outcomes for service provider's profit, consumer surplus, and social welfare. An important difference is that in this case, it is possible for trading to be harmful to all three metrics. We also examine the impact of trading on individual consumers. We show that those who benefit the most are those with a moderate usage type, as they are more likely to experience a mismatch between the plan they purchased and their realized demand.

When the trading price is in the hands of consumers and is set via a market clearing mechanism, then perhaps surprisingly trading always improves outcomes for service provider's profit, consumer surplus, and social welfare. Moreover, in contrast to the case where the trading price is exogenously specified, the benefit from trading to the service provider is non-monotonic, first increasing and then decreasing. The implication is that when the trading price is set via a market clearing mechanism, trading is most beneficial to service providers with moderate service costs while when the trading price is exogenously specified (independently of service cost), trading is most beneficial to service providers with low service costs.

These results have several managerial implications. They provide guidance to service providers, consumers, and policy makers as to when peer-to-peer trading may or may not be beneficial, including when trading is beneficial to all or none. The results highlight the important interplay between trading price and cost of service in determining various outcomes. They also point to when there is an opportunity for the service provider to extract more of the surplus created by a trading marketplace through contract redesign. For policy makers and regulators, the results can be useful in pointing out when such trading improves outcomes for consumers or social welfare and to potential policy levers that could be deployed to affect outcomes. The results also point to the value of letting trading price be market-driven and determined by choices consumers make.

The rest of the paper is organized as follows. In Section 2.2, we provide a review of related literature. In Section 2.3, we describe the model. In Section 2.4, we characterize the equilibrium with and without trading. In Section 2.5, we examine the impact of trading on equilibrium outcomes. In Section 2.6, we consider the service provider's problem and characterize optimal contracts' prices. In Section 2.7, we consider variants and extensions to the basic model, including the case where the trading price is determined via a market clearing mechanism, the case of a single contract, and the case of a system with a general usage distribution. In Section 2.8, we offer some concluding comments.

2.2 Literature Review

There are two related streams of literature: the emerging literature on peer-to-peer product sharing and the literature on secondary markets (the peer-to-peer trading of usage allowances can be viewed as a secondary market for access). We start with reviewing the literature on peer-to-peer product sharing. This stream of literature aims to understand how consumer behavior (ownership and usage), firm's profit and decisions, and social welfare, among other metrics, may change when consumers who own a product are able to share it with those who do not own it via short term rentals.

Jiang and Tian (2016) examine how product sharing affects the firm's pricing and quality decisions. They find that, in the presence of sharing, the firm increases its quality allowing consumers with variable consumption to generate a higher rental income during low usage periods. Tian and Jiang (2018) consider a two-tier supply chain and show that when the cost of building the production capacity is high both the manufacturer and the retailer benefit from product sharing, with the retailer's share being larger. However, when the cost of capacity is low, the retailer benefits but not the manufacturer. Abhishek et al. (2016) study the effect of sharing on the manufacturer and identify consumer heterogeneity as a determining factor for the profitability of sharing. If the heterogeneity is moderate, both the firm and consumers can be better off in the presence of a peer-to-peer market. Our work is different from above studies in the following important aspects. First, sharing does not affect the production cost of a durable good whereas for a service good a firm's service cost depends on the actual usage which is affected by sharing. Hence, sharing only changes the sales (or revenue) of a manufacturer of durable goods whereas sharing changes both the revenue and cost of a service provider. Second, from the perspective of consumers, durable goods on the secondary markets are usually of lower quality, but service goods on the peer-to-peer market are of the same quality as goods on the primary market. Third, in the service good setting, the usage allowances expire after usage and there is no moral hazard or product depreciation as in the case of sharing of durable goods. Finally, different from Abhishek et al. (2016), our results are not driven by the level of consumer heterogeneity.

Benjaafar et al. (2018) study the ownership decision of heterogenous consumers and its implications on usage and social welfare. They show that peer-to-peer product sharing always benefits consumers, but ownership and usage levels may or may not increase. Cachon and Feldman (2018) examine how a firm's pricing strategy changes in the presence of a recourse strategy (e.g. reselling) which allows buyers or firms to transfer the ownership after the initial sale. They show that a recourse strategy can improve both the buyers' and the firm's welfare. Their setting is different from ours in that in their case each customer demands one unit of capacity. In ours, customers are heterogenous in their demands. Yang et al. (2016) show, in the context of a queueing system, that allowing consumers to trade their positions in a queue can improve both social welfare and the firm's profit.

There is also a growing number of empirical studies which attempt to assess the effect of peer-to-peer platforms on incumbent firms (e.g. taxies and hotels), social welfare, and consumers. Fraiberger and Sundararajan (2015) use US automobile industry data and study whether peer-to-peer rental is social welfare-improving. They report a 0.8 to 6.6% increase in consumer surplus attributed to peer-to-peer rentals. They show that increase in surplus is more pronounced for low to medium income consumers. Zervas et al. (2017) estimate that the entry of Airbnb in the city of Austin, Texas caused an 8-10% decrease in hotels revenue with economy and budget hotels effected the most. They argue that Airbnb not only provides a more affordable accommodation option, but also benefits those staying in hotels because of lower prices due to competition.

The second related stream of literature is that on secondary markets. This literature focuses on the tension that arises between value enhancement and cannibalization. The value of a product is enhanced because of the opportunity to resell the product on the secondary market. Cannibalization arises because of the possibility that some customers may forego purchasing a new product in favor of a used product in the secondary market. Zhu (2014) argues that a secondary market improves social welfare by increasing allocation efficiency while it negatively affects provider's profit. Lee and Whang (2002) study the implications of a secondary market in a supply chain setting. They show that a secondary market always improves retailers' profit. However, whether or not a secondary market improves the supplier's profit and supply chain welfare depends on the retailers' critical ratio. Desai and Purohit (1998) study a monopolist's tradeoff between leasing and selling durable goods and find that if the depreciation rate for the sold items is high, then the monopolist is better off by selling its products. Lim et al. (2014) use a similar setting as in Desai and Purohit (1998) to analyze the adoption of electric vehicles and show that when the firm adopts the battery leasing business model, a secondary market can improve its profit.

Our work is different from these papers in that we consider a service good which makes the firm's profit dependent on the actual usage not the units sold. Also, in contrast to durable goods, the usage allowances on the secondary market are of the same quality, i.e. there is no depreciation, and there is no moral hazard. The setting used by Lee and Whang (2002) is closest to ours with some notable differences. In our model consumers are heterogenous whereas the retailers in their setting are homogenous. In addition, we study the firm's pricing reaction to the secondary market whereas in Lee and Whang (2002) the firm's price is exogenous. The latter leads to a different result with respect to consumer surplus (individual and aggregate) which in our case may decrease when there is a secondary market. Other related research streams include dynamic models of selling durable goods in the presence of a secondary market (Rust, 1985; Huang et al., 2001), secondary market as a device for price discrimination (Anderson and Ginsburgh, 1994), adverse selection (Hendel et al., 2005), the effect of preference inconsistency (Johnson, 2011), and durable good oligopoly (Esteban and Shum, 2007).

There is extensive literature that studies the impact of a secondary market in the specific context of ticket resale. Swofford (1999) is the first to propose a theoretical framework to study ticket resale. He explains that brokers' different risk attitudes allows them to exist without hurting the provider's profit. Courty (2003b) considers a setting in which a monopolist sells tickets to customers who learn their valuations over time. He shows that the monopolist cannot do strictly better by allowing resale. This is different from our results that peer-to-peer trading can be profitable for the service provider. Courty (2003a) argues that the existence of brokers does not affect the social welfare and if they do not exist in the market their surplus will be captured by either the service providers or customers. In our setting, the secondary market is peer-to-peer and there is no broker. Also, we show that a secondary market can improve the social welfare by changing the behavior of consumers and enabling more consumption. Cui et al. (2014) consider an event provider selling capacity to two classes of customers: diehard and busy professionals. They argue that a provider with small capacity and fixed pricing is more likely to benefit from ticket resale. Similarly, Geng et al. (2007) show that, when the event provider's capacity is limited and the number of high valuation customers is large enough, a partial resale can improve both customers' surplus and the event provider's profit (relative to no resale or unrestricted resale). The service provider in our setting is uncapacitated and there are no restrictions on resale; a setting in which Cui et al. (2014) show that ticket resale hurts the event provider. Moreover, in our setting the service provider's cost depends on the actual usage (not the sold allowances) and we allow the service provider to set the prices.

Finally, two additional streams of literature are worth mentioning. First, there is a body of literature which focuses on information goods. Examples include Novos and Waldman (1984), Kim et al. (2018), and Galbreth et al. (2012) who argue that allowing some sharing (piracy) of information goods, such as software licenses, can improve social welfare and firm's profit. Second, there is a body of literature that examines the effect of secondary markets empirically. Leslie and Sorensen (2013) empirically study a major rock concert and report that the secondary market increased allocation efficiency by 5%. Chen et al. (2013) use US automobile industry data and estimate that the net effect of the secondary market could be up to a 35% decrease in the new car manufacturer's profits.

2.3 Model Setup

We consider a risk neutral monopolist selling a service good to a unit mass of risk neutral consumers. We refer to the monopolist seller as the *service provider*. The service provider offers two contracts (or plans): a low usage plan with a usage allowance q_1 and a price p_1 and a high usage plan with a usage allowance q_2 and a price p_2. For simplicity, we consider a setting with two plans. It is possible to study settings with multiple plans. However, the analysis is less tractable while most of the qualitative insights remain the same. The use of two plans is common in practice, including some of the examples mentioned in the introduction. Without loss of generality, we assume $q_2 > q_1$ and $p_2 > p_1$ and refer to q_1, q_2, p_1 and p_2 as contract parameters. The service provider incurs a service cost c for each unit of capacity that is used by the consumers. In other words, the service provider's service cost depends on the actual consumption, not the usage allowance she sells.

The consumption (demand) of a consumer is a random variable and consumers are heterogeneous in their distribution of consumption. Each consumer's type, denoted by θ, is his private information and the service provider only knows the distribution of types across the population which is distributed uniformly on the unit interval $[0, 1]$. We denote the consumption of a consumer of type θ by $\mathbf{X}_\theta$ (a random variable) and we assume it is distributed according to the probability distribution $F_\theta(x)$. The type θ distinguishes consumers in their consumption; a consumer of a higher type is more likely to have higher consumption. We capture this heterogeneity by assuming that each type's consumption stochastically dominates that of any lower type. More formally, we make the following assumption.

Assumption 1 (Stochastic Dominance) *Let θ_1 and θ_2 be two arbitrary types in $[0, 1]$ and, without loss of generality, assume $\theta_2 > \theta_1$. For any $x \in \mathbb{R}_+$ we have $\bar{F}_{\theta_2}(x) > \bar{F}_{\theta_1}(x)$ where $\bar{F}$ denotes the survival function of the distribution f.*

This assumption implies that a consumer of a higher type (say θ_2) is more likely to experience a higher demand than a consumer whose type is lower (say θ_1).

In the absence of peer-to-peer trading of usage allowance, the only interaction that takes place is between the service provider and the consumers. Each consumer chooses a contract under which she pays price p_i to the service provider for a usage allowance in the amount q_i with $i \in \{1, 2\}$. Once demands are realized, a consumer can consume up to his allowance which is either q_1 or q_2 depending on the contract he chose. In the presence of peer-to-peer trading, consumers privately learn their realized demands and they can trade the unused part of their quota at the prevailing trading price which we denote by π. Depending on the setting, the trading price may be set by the owner of the trading platform or through a market clearing mechanism if pricing is in the hands of individual sellers. In this chapter, for much of the analysis, we take trading price as being exogenously specified (this would arise naturally if the trading platform belongs to a third party who chooses trading prices to maximize profit or some other objective.). In Section 2.7.1, we consider the case where price is determined through a market clearing mechanism. Figure 2.1 summarizes the sequence of events that happens with or without peer-to-peer trading.

We assume consumers are risk neutral and have valuation r for each unit of usage. The utility of a consumer of type θ, who elects a plan (q, p), in the absence of trading

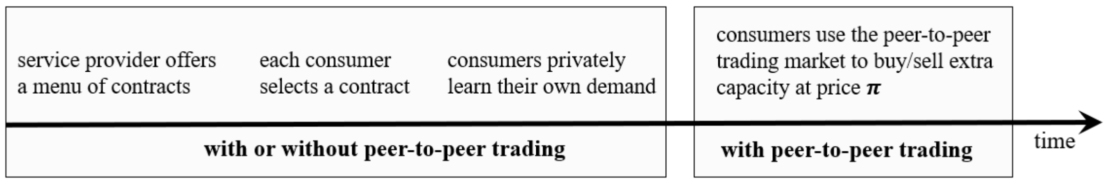


Figure 2.1: Sequence of events with and without peer-to-peer trading

is given by

$$u_n(q, \theta) = r\,\mathbb{E}[\min\{X_\theta, q\}] - p, \tag{2.1}$$

where $\mathbb{E}$ is the expectation operator over the type θ's random consumption.

When peer-to-peer trading is allowed, consumers can buy and sell excess capacity. In particular, when a consumer's realized demand is smaller than his contract's usage allowance (i.e. $X_\theta < q$), a situation we call *overage*, the consumer puts up his surplus capacity on the trading market. On the other hand, when a consumer's realized demand is larger than his allowance, a situation we call *underage*, he can purchase additional capacity from the trading market.

We denote the aggregate demand (supply) of capacity on the trading market by ψ_d (ψ_s). If the aggregate supply exceeds the aggregate demand, the available demand is uniformly rationed among all the sellers (we refer to this situation as *demand rationing*) such that a seller can sell a fraction $\alpha = \min\left\{\frac{\psi_d}{\psi_s}\,, 1\right\}$ of their excess capacity on the trading market. Similarly, when the aggregate supply exceeds the aggregate demand, the aggregate demand is uniformly assigned to individual sellers (we refer to this situation as *supply rationing*) and each buyer can acquire a fraction $\beta = \min\left\{\frac{\psi_s}{\psi_d}\,, 1\right\}$ of his need for extra capacity from the trading market. The functions α and β are endogenously determined in equilibrium. Note that the rationing functions α and β are complementary, i.e. $\alpha < 1$ implies $\beta = 1$ and, similarly, $\beta < 1$ implies $\alpha = 1$.

In the presence of trading, the expected utility of a consumer of type θ can be expressed as

$$
\begin{aligned}
u_t(q,\theta) &= r\mathbb{E}[\min\{X_\theta, q\}] + \alpha\,\pi\,\mathbb{E}[q - X_\theta]^+ + \beta\,(r-\pi)\,\mathbb{E}[X_\theta - q]^+ - p \\
&= u_n(q,\theta) + \alpha\,\pi\,\mathbb{E}[q - X_\theta]^+ + \beta\,(r-\pi)\,\mathbb{E}[X_\theta - q]^+. \qquad (2.2)
\end{aligned}
$$

If it turns out that $X_\theta < q$ (overage), a type θ individual can sell a fraction α of her overage at price π on the trading market. This gives her an additional utility which is captured by the amount $\alpha\,\pi\,\mathbb{E}[q - X_\theta]^+$. Similarly if $X_\theta > q$ (underage), the consumer can purchase additional capacity on the trading market. Her utility from each extra unit of capacity that she purchases and consumes is $r - \pi$. She can satisfy a fraction β of her underage through the trading market which gives her an additional utility equal to $\beta\,(r-\pi)\,\mathbb{E}[X_\theta - q]^+$.

To avoid trivial cases in which a portion of consumers leaves the market without purchasing a plan or there is a plan that does not attract a positive mass of consumers, we make the following two assumptions.

Assumption 2 (Full Market Coverage) *For the lowest type (i.e. $\theta = 0$), there exists a plan $\mathbf{m}_i$, $i \in \{1,2\}$ such that $u_\ell(q_i, 0) \geq 0$ for $\ell \in \{t\,,\,s\}$.*

Assumption 3 (Full Plan Coverage) *There exists a consumer of type θ_1 who prefers plan $\mathbf{m}_1$ over $\mathbf{m}_2$, i.e. $u_\ell(q_1,\theta_1) \geq u_\ell(q_2,\theta_1)$ with $\ell \in \{t\,,\,s\}$. Similarly, there exists a consumer of type θ_2 who prefers plan $\mathbf{m}_2$ over $\mathbf{m}_1$, i.e. $u_\ell(q_2,\theta_2) \geq u_\ell(q_2,\theta_2)$.*

In words, we avoid cases in which a portion of consumers leaves the market without purchasing a plan (Assumption 2) or there is a plan that attracts no consumers (Assumption 3). The full market coverage implies that the service provider wants to cover the whole market regardless of the market mechanism he implements. These two assumptions allow us to focus only on the effect of peer-to-peer trading contorting for market size.

Finally, and mostly for expositional simplicity, we assume that demand is distributed according to a two-point discrete distribution as specified in Assumption 4 below. In Section 2.7.3, we revisit this assumption and show that our main results hold under a more general distribution of demand.

Assumption 4 (Discrete Type-Dependent Demand) *A consumer of type θ has an uncertain demand which can be either high (H) or low (L) and follows the probability mass function*

$$\mathbb{P}_\theta(X_\theta = H) = \theta \quad \textit{and} \quad \mathbb{P}_\theta(X_\theta = L) = 1 - \theta. \tag{2.3}$$

Assumption 4 implies that as a consumer's type increases, she is more likely to have a higher demand. It is straightforward to verify that the distribution in Assumption 4 satisfies the stochastic ordering we specified earlier in Assumption 1. For expositional simplicity we also make the following assumption.

Assumption 5 $q_1 = L$ *and* $q_2 = H$.

The results we show here do not depend on the above assumption. However, it allows us to provide explicit expressions for various equilibrium outcomes, making our analysis in subsequent sections more tractable. Moreover, if we consider the case without trading as our status quo, choosing $q_1 = L$ and $q_2 = H$ is optimal for the service provider.

2.4 Equilibrium Analysis

In this section, we first analyze the equilibrium in the absence of peer-to-peer trading which, for our analysis, serves as a benchmark and may be viewed as the status quo. Then we do the same in the presence of trading.

2.4.1 Systems without Trading

In the following theorem, we characterize the equilibrium for a system without trading.

Theorem 1 *Let* $\rho = \frac{p_2 - p_1}{H-L}$. *Without trading, there exists an equilibrium specified by a threshold* $\theta_n = \frac{\rho}{r}$ *such that a consumer of type* $\theta < \theta_n$ *prefers plan* $\mathbf{m}_1$ *and a consumer of type* $\theta \geq \theta_n$ *prefers plan* $\mathbf{m}_2$. *The threshold* θ_n *decreases in* p_1 *and increases in* p_2.

Theorem 1 shows that, perhaps consistent with intuition, consumers with sufficiently low θ, as specified by the threshold θ_n, choose plan $\mathbf{m}_1$ and the rest choose plan $\mathbf{m}_2$. Moreover, a higher value for p_2 makes plan $\mathbf{m}_1$ a more attractive option and leads to higher θ_n while a higher value of p_1 leads to lower θ_n and more consumers preferring contract $\mathbf{m}_2$.

It follows that the service provider's profit is given by

$$\Pi_n = p_1\,\theta_n + p_2\,(1-\theta_n) - c\,P_n, \tag{2.4}$$

where P_n denotes the total consumption (aggregate amount of demand fulfilled) when no trading takes place and is given by

$$P_n = \int_0^{\theta_n} \{\theta\,L + (1-\theta)\,L\}\,\mathrm{d}\theta + \int_{\theta_n}^{1} \{\theta\,H + (1-\theta)\,L\}\,\mathrm{d}\theta,$$

or equivalently

$$P_n = L + \frac{H-L}{2}(1-\theta_n^2). \tag{2.5}$$

Finally, it is easy to verify that Assumptions 2 and 3 reduce to $r \geq \frac{p_1}{L}$, and $r \geq \rho$. From these two equations, it is easy to confirm that the consumer who has the highest type, i.e. $\theta = 1$, prefers high plan $\mathbf{m}_2$, and the consumer who has the lowest type, i.e.

$\theta = 0$, prefers low plan $\mathbf{m}_1$.

2.4.2 Systems with Peer-to-Peer Trading

First note that if a consumer of type θ purchases a plan with usage quota q, his expected overage and underage are given by (2.6) and (2.7), respectively:

$$\mathcal{O}(q,\, \theta) = (1-\theta)(q-L), \tag{2.6}$$

and

$$\mathcal{L}(q,\, \theta) = \theta(H-q). \tag{2.7}$$

We denote the set of types who opt for plan $\mathbf{m}_i$ by Θ_i where $i \in \{1, 2\}$. Using this notation, the aggregate demand on the trading market, denoted by ψ_d, and the aggregate supply, denoted by ψ_s, are given by

$$\psi_s = \int_{\theta \in \Theta_2} \mathcal{O}(H,\, \theta)\, \mathrm{d}\theta, \tag{2.8}$$

and

$$\psi_d = \int_{\theta \in \Theta_1} \mathcal{L}(L,\, \theta)\, \mathrm{d}\theta. \tag{2.9}$$

In the following theorem, we characterize the corresponding equilibrium.

Theorem 2 *In the presence of peer-to-peer trading, there exists a unique equilibrium characterized by the threshold*

$$\theta_t = \begin{cases} \dfrac{r-\pi}{2r-(\rho+\pi)} & \textit{if} \quad \pi \le \rho, \\ \dfrac{\rho}{\rho+\pi} & \textit{if} \quad \rho < \pi, \end{cases} \tag{2.10}$$

with $\rho = \frac{p_2 - p_1}{H-L}$, such that a consumer of type $\theta < \theta_t$ prefers plan $\mathbf{m}_1$ and a consumer of type $\theta \geq \theta_t$ prefers plan $\mathbf{m}_2$. The threshold θ_t increases in p_2 and decreases in p_1 and π.

Given θ_t, we can simplify (2.8) and (2.9) as $\psi_s = \frac{H-L}{2}(1-\theta_t)^2$ and $\psi_d = \frac{H-L}{2}\theta_t^2$, respectively. The rationing coefficients can now be explicitly expressed as

$$\alpha = \min\left\{\frac{\theta_t^2}{(1-\theta_t)^2}\,,\,1\right\}, \tag{2.11}$$

and

$$\beta = \min\left\{\frac{(1-\theta_t)^2}{\theta_t^2}\,,\,1\right\}. \tag{2.12}$$

Using the above equations, the service provider's profit can be written as

$$\Pi_t = p_1\,\theta_t + p_2\,(1-\theta_t) - cP_t$$

or, equivalently, as

$$\Pi_t \;=\; \Pi_n + (p_2 - p_1)(\theta_n - \theta_t) - c\,(P_t - P_n), \tag{2.13}$$

where Π_n is the service provider's profit without trading. The total consumption P_t is given by $\frac{H+L}{2} - \max\{\psi_d - \psi_s,\ 0\}$ or equivalently as

$$P_t = \frac{H+L}{2} - \max\left\{\frac{H-L}{2}(2\,\theta_t - 1),\ 0\right\}. \tag{2.14}$$

The first term in (2.14), i.e. $\frac{H+L}{2}$, captures the amount of capacity needed to fully satisfy all consumers' demand. If supply exceeds demand on the trading platform, i.e. $\psi_s \geq \psi_d$, all consumers consumption is fully satisfied and the total consumption is $P_t = \frac{H+L}{2}$. However, if $\psi_s < \psi_d$, not all consumers can satisfy all of their consumption and the total consumption is less than its maximum value of $\frac{H+L}{2}$ by an amount of $\frac{H-L}{2}(2\,\theta_t - 1)$.

Finally, in the presence of peer-to-peer trading, Assumptions 2 and 3 can be restated as follows:

$$r \geq \frac{p_1 - \beta\pi}{L - \beta} \tag{2.15}$$

$$\frac{\rho - \alpha\,\pi}{r - \alpha\,\pi - \beta(r - \pi)} \geq 0, \tag{2.16}$$

and

$$\frac{\rho - \alpha\,\pi}{r - \alpha\,\pi\, - \beta(r - \pi)} \leq 1. \tag{2.17}$$

2.5 Impact of Peer-to-Peer Trading

In this section, we examine the impact of introducing peer-to-peer trading assuming no changes are made to plan prices and usage allowances.

2.5.1 Impact on Plan Selection

As a preamble, we first characterize the impact of trading on consumer decisions regarding plan selection. The following proposition characterizes the impact of trading on consumer decisions regarding plan selection.

Proposition 6 (Plan Selection) *In the presence of trading, $\theta_t \leq \theta_n$ if and only if $\pi \geq \pi_h$ where $\pi_h = r - \frac{p_2 - p_1}{H-L}$.*

Proposition 6 shows that the higher the trading price, the more consumers purchase the high plan. This makes intuitive sense. A higher trading price enhances the perceived value of the high plan. The prospect of earning income from excess capacity reduces the effective cost of the high plan to a consumer. Proposition 6 also shows that there is a threshold on the trading price, π, above which more consumers (relative to the case without trading) opt for the high plan and below which more consumers opt for the low plan. In other words, if the trading price is sufficiently low, more consumers opt for the low plan.

2.5.2 Impact on the Service Provider

The following proposition describes the impact of trading on the service provider.

Proposition 7 *If $c > \frac{2r\rho}{r+\rho}$, trading (relative to no trading) leads to lower profit for the service provider. If $c \leq \frac{2r\rho}{r+\rho}$, there exists a unique threshold π_p such that trading leads to higher profit for the service provider if and only if $\pi > \pi_p$ where*

$$\pi_p = \begin{cases} \dfrac{\rho(2r-c)(r-\rho)}{\rho(2r-c)-rc} & \text{if} \quad c \leq 2r\left(1-\dfrac{r}{\rho}\right), \\ \dfrac{2r^2}{2r-c} - \rho & \text{if} \quad c > 2r\left(1-\dfrac{r}{\rho}\right). \end{cases}$$

Moreover, the threshold π_p increases in c.

Proposition 7 shows that whether or not trading is beneficial to the service provider depends on both the cost of usage and the trading price. If the cost of usage is sufficiently high, then trading will always harm the platform. This can be explained as follows. First, note that for the service provider to benefit from trading, more consumers need to opt for the high plan. However, more consumers opting for the high plan also means higher consumption and, therefore, higher usage cost. If the unit usage cost is sufficiently high, the increase in usage cost more than offsets the increase in revenue, resulting in a net decrease in profit. When the unit cost is sufficiently low, then it is possible for the firm to profit. However, this requires that the number of consumers who choose the high

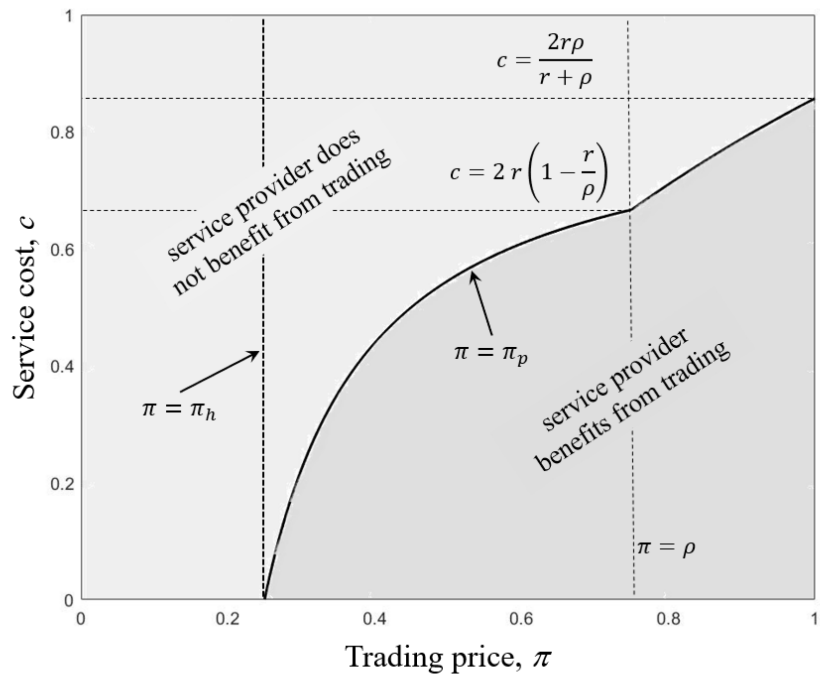
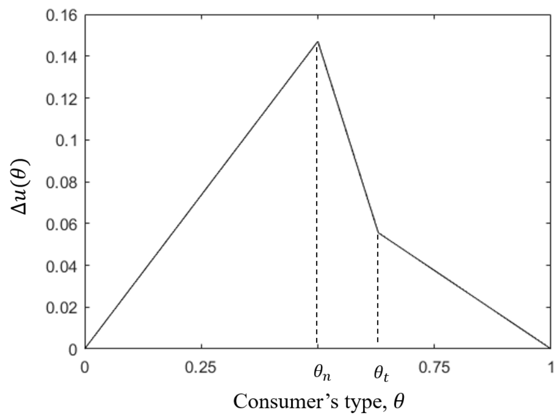


Figure 2.2: The service provider's profit with and without trading; $r = 1$, $H = 6$, $L = 2$, $p_2 = 5$, $p_1 = 2$

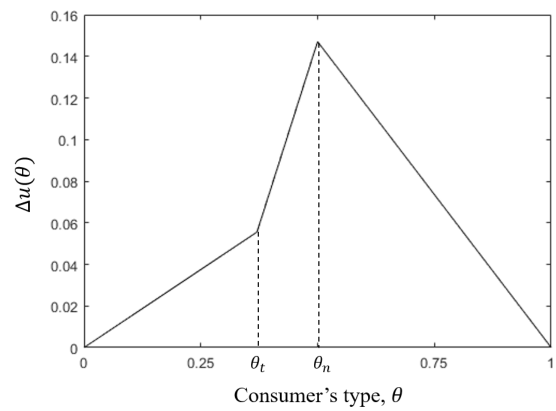
plan is sufficiently large. This is the case only if the trading price is sufficiently large (above threshold π_p). When the trading price is in the interval $[\pi_h, \pi_p]$, even though more consumers choose the high plan, the resulting increase in revenue is not enough to offset the increase in usage cost due to higher consumption. Figure 2.2 illustrates the combination of cost and trading prices under which trading is profitable for the service provider.

2.5.3 Impact on Consumers

As mentioned already, consumers always benefit from trading. However, consumers, depending on their type, may benefit differently. In this section, we characterize how consumers of different types are affected by trading. We first define the difference in the consumers' utility with and without trading as $\Delta u(\theta) = u_t(\theta, q) - u_n(\theta, q)$ where u_n



(a) $\pi = 0.15$, $r = 1$, $H = 2$, $L = 1$, $p_2 = 1.5$, $p_1 = 1$



(b) $\pi = 0.85$, $r = 1$, $H = 2$, $L = 1$, $p_2 = 1.5$, $p_1 = 1$

Figure 2.3: Improvement in consumer surplus due to trading

and u_t are defined in (2.1) and (2.2), respectively. The following proposition provides an explicit formula for $\Delta u(\theta)$ and characterizes how consumers of different type benefit from trading.

Proposition 8 *The function $\Delta u(\theta)$ is piece-wise linear and given by*

$$\Delta u(\theta) = \begin{cases} (H-L)\frac{(r-\rho)^2}{r-\pi}\theta & \text{if} \quad \theta < \min\{\theta_n, \theta_t\}, \\ (H-L)\rho - (H-L)\left(r - \frac{(r-\rho)^2}{r-\pi}\right)\theta & \text{if} \quad \theta_n < \theta_t \quad \text{and} \quad \theta_n < \theta < \theta_t, \\ (H-L)\frac{\rho(\rho-\pi)}{\pi} + (H-L)\left(r - \frac{\rho^2}{\pi}\right)\theta & \text{if} \quad \theta_n \geq \theta_t \quad \text{and} \quad \theta_t < \theta < \theta_n, \\ (H-L)\frac{\rho^2}{\pi}(1-\theta) & \text{if} \quad \theta > \max\{\theta_n, \theta_t\}, \end{cases} \tag{2.18}$$

Moreover, the consumer of type $\theta = \theta_n$ benefits the most from trading.

Proposition 8 shows that consumers fall in three segments. Consumers in the first segment with $\theta \geq max\{\theta_n, \theta_t\}$ purchase the high plan with or without trading. We refer to this segment as *high* type. Consumers in the second segment with $\theta \leq \min\{\theta_t\,, \theta_n\}$ purchase the low plan with or without trading. We refer to this segment as the *low* type. Lastly, consumers in the third segment, and perhaps the more interesting of the three, are *switchers* who change their purchasing behavior in the presence of trading. In one case, if $\theta_t = \max\{\theta_t\,,\,\theta_n\}$, switchers prefer to purchase the low plan and they rely on the trading market to purchase extra capacity they may need. In the other case, when $\theta_t = \max\{\theta_t\,,\,\theta_n\}$, the opposite happens. Switchers change from the low plan to the high plan and sell any leftover on the trading market.

As shown in Figure 2.3, the utility difference $\Delta u(\theta)$ increases in θ for low type consumers (i.e. $\theta < \min\{\theta_n\,,\,\theta_t\}$) and decreases for high type consumers (i.e. $\theta > \max\{\theta_n\,,\,\theta_t\}$). Therefore, the consumer who benefits the most should be a switcher. Intuitively, when the trading price is low, the consumer who benefits the most is the consumer with the highest supply of capacity on the trading market which happens to be $\theta = \theta_n$(see Figure 2.3a). On the other hand, when the trading price is high, the consumer who benefits the most is the consumer with the highest demand for capacity on the trading market which again is $\theta = \theta_n$(see Figure 2.3b).

2.5.4 Impact on Social Welfare

We define social welfare as the sum of aggregate consumer surplus and the service provider's profit. Let S_n and S_t denote, respectively, the social welfare in the case without and with trading. They can be obtained as $S_t = (r-c)P_t$ and $S_n = (r-c)P_n$ where P_t (as in (2.14)) and P_n (as in (2.5)) are, respectively, the total consumption with and without trading.

The following proposition characterizes the impact of trading on social welfare.

Proposition 9 *There exists a unique threshold* $\pi_s = \frac{\rho(r-\rho)(\rho+r)}{\rho^2+r^2}$, *with* $\rho = \frac{p_2-p_1}{H-L}$, *such that* $S_t(\pi) < S_n$ *if and only if* $\pi < \pi_s$. *Moreover, the threshold* π_s *is less than* π_p.

As mentioned earlier, the social welfare is higher as long as the total consumption is higher. Therefore, obviously, when more consumers opt for the higher plan, which

Trading price	$(0, \pi_s)$	$[\pi_s, \pi_h)$	$[\pi_h, \pi_p)$	$[\pi_p, r)$
Consumer surplus	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
Social welfare	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Portion choosing high plan	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$
Provider's profit	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$

Table 2.1: The impact of trading on consumer surplus, social welfare, portion of consumers choosing high plan, and the service provider profit

happens when $\pi \geq \pi_h$, social welfare increases. However, since trading improves efficiency by reallocating unused capacity, social welfare can still increase even if the total allowances sold shrinks. When the trading price is in the interval $[\pi_s, \pi_p)$, the total allowances sold decreases with trading. However, the social welfare increases by allowing consumers to trade.

2.5.5 Summary of Results

Table 2.1 summarizes the results regarding the impact of trading on the service provider, consumers, and social welfare when $c \leq \frac{2\,r\,\rho}{r+\rho}$ and shows that there exists a triple-win region. The outcomes can be partitioned, based on the trading price, into three regions: region I where only consumers benefit, region II where consumers benefit and social welfare improves but the service provider is hurt, and regions III where consumers and the service provider benefit and social welfare is improved (a triple win region). Finally, if $c > \frac{2\,r\,\rho}{r+\rho}$, the service provider's profit deceases by trading regardless of the trading price (i.e. $\pi_p \to r$). In this case, there does not exists a triple-win region, but both the consumer surplus and social welfare are improved by trading for $\pi \geq \pi_s$.

2.6 The Service Provider's Problem

So far, we treated the prices, p_1 and p_2, as being exogenously specified. In this section, we allow for these to be decisions that the service provider makes and that may differ with and without trading. The ability to modify prices in response to trading allows the service provider to extract surplus that may otherwise accrue to consumers. In this section, we examine the extent to which pricing flexibility affects equilibrium outcomes. We are particularly interested in investigating whether trading continues to be beneficial to consumers or if it is possible for consumers to be worse off with than without trading. Note that, for tractability, we leave the allowances unchanged and set at values that are optimal for the service provider in the absence of trading, i.e. $q_1 = L$ and $q_2 = H$. Allowing the service provider to modify these quantities would allow the service provider to extract additional surplus. In this sense, the results in this section provide a lower (upper) bound on service provider (consumer) surplus.

Without trading, the service provider's pricing problem can be stated as follows:

$$\begin{aligned} \max_{p_1,p_2} \quad & p_1\,\theta_n + p_2\,(1-\theta_n) - c\,P_n \qquad (2.19)\\ \text{subject to} \quad & p_1 \le r\,L,\\ & \rho \le r, \text{ and}\\ & \theta_n = \frac{\rho}{r}. \end{aligned}$$

Similarly, in the presence of trading, the service provider's problem can be stated as:

$$\begin{aligned} \max_{p_1,p_2} \quad & p_1\,\theta_t + p_2\,(1-\theta_t) - c\,P_t \qquad (2.20)\\ \text{subject to} \quad & p_1 \le r\,L - \beta(r-\pi)\,L, \qquad (2.21)\\ & 0 \le \frac{\rho - \alpha\,\pi}{r - \alpha\,\pi - \beta(r-\pi)} \le 1, \text{ and} \qquad (2.22)\\ & (2.10) - (2.12). \end{aligned}$$

Constraint (2.21) ensures that the *full plan coverage* requirement is satisfied and constraint (2.22) guarantees that the *full market* is covered by plans $\mathbf{m}_1^t$ and $\mathbf{m}_2^t$, i.e. all consumers participate. The last constraint ensures that the switching threshold θ_t is the equilibrium of the consumers purchasing game. The following Theorem characterizes the optimal prices with and without trading.

Theorem 3 (Optimal Prices) *Without trading, the service provider's optimal prices are* $p_1^n = r\,L$ *and* $p_2^n = r\,L + \frac{r^2}{2\,r-c}\,\rho$. *With trading, the optimal prices are* $p_1^t = r\,L$ *and*

$$p_2^t = \begin{cases} r\,L + \rho\,\pi & \text{if} \quad c \leq \pi - 2(r-\pi), \\ r\,L + \rho\,\left(2r - \pi - \sqrt{(2r-\pi-c)(r-\pi)}\right) & \text{if} \quad c > \pi - 2(r-\pi). \end{cases} \tag{2.23}$$

Moreover, $p_2^t \geq p_2^n$ *with both* p_2^n *and* p_2^t *increase in* c.

Theorem 3 shows that the price for the low plan is the same with or without trading and is set at rL, which corresponds to the maximum price at which the consumer with the lowest type ($\theta = 0$) is willing to purchase. This is not the case for the price of the high plan. Since trading enhances the value of the high plan, the service provider charges a higher price with trading. As illustrated in Figure 2.4, the difference in the two prices can be significant. It is affected by both the trading price and the service cost, with the difference being most significant when the service cost is low and the trading price is high. This suggests (as we will formally show next) that this is also the region where trading is most profitable for the service provider. With trading, prices are increasing in the trading price which makes intuitive sense (since the value of the high plan to a consumer increases in the trading price). The more interesting feature is perhaps the fact that, when the trading price is sufficiently high and the service cost is sufficiently low ($c < \pi - 2(r-\pi)$), the high plan's price is invariant to the service cost. In this case, the total consumption is at its maximum value $\bar{P}$ and if the service provider increases the high plan's price, some consumers would choose the low plan causing revenue loss while service cost is unaffected. Therefore, when $c < \pi - 2(r-\pi)$ holds, p_2^t does not

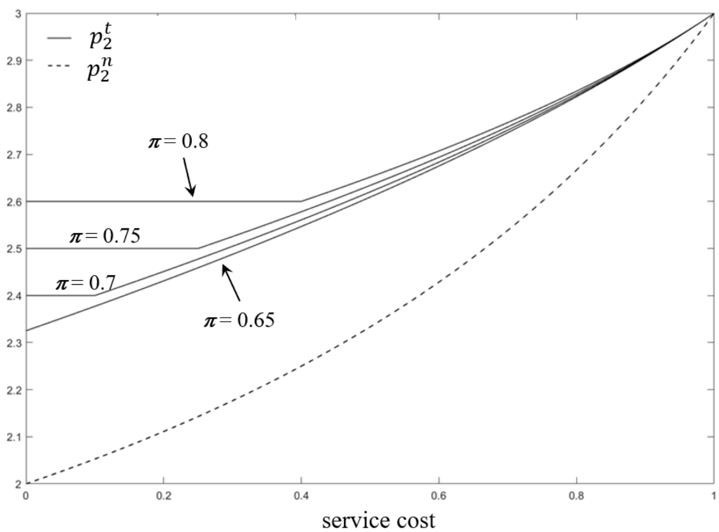


Figure 2.4: The effect of changes in the service cost on the high plan price; $r = 1$, $L = 1$, $H = 3$

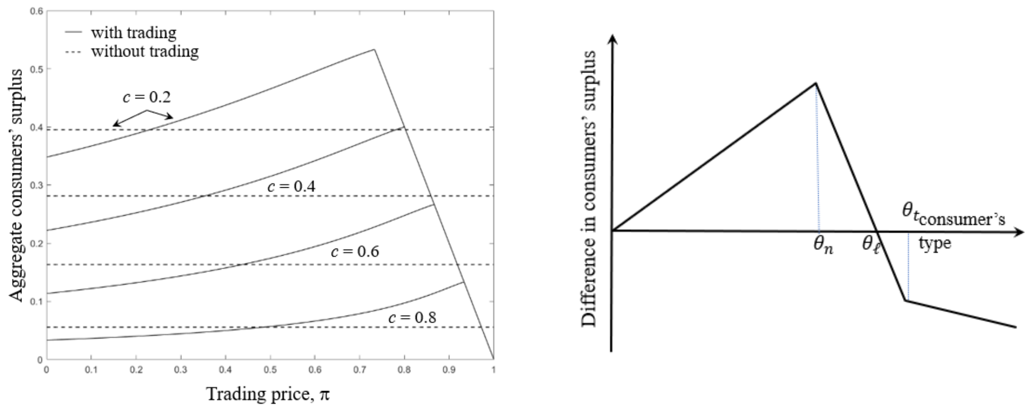
change as the service cost increases by a small amount. The flat part of the p_2^t curve in Figure 2.4 corresponds to this scenario.

Next, we examine how the pricing decision made by the service provider affects equilibrium outcomes. Of particular interest is the impact on consumers. As we show in the following proposition, it is now possible that consumers are actually worse off with trading than without it. The following proposition characterizes the impact of trading (under optimized prices) on consumers.

Proposition 10 (Consumer Surplus) *When prices are set optimally, the following statements hold.*

1. *Aggregate consumer surplus is higher with than without trading if and only $\pi_{c1}^* \leq \pi \leq \pi_{c2}^*$ with both π_{c1}^* and π_{c2}^* increasing in c.*

2. *There exists a type $\theta_\ell \in (\theta_n, \theta_t)$ such that all consumers with type $\theta \leq \theta_\ell$ benefit from trading and all consumers with type $\theta > \theta_\ell$ are worse off. Moreover, among*



(a) Aggregate consumer surplus

(b) Individual consumer surplus

Figure 2.5: The effect of trading on consumer surplus

all consumers, the consumer of type θ_n benefits the most.

Proposition 10 shows that, in contrast to the case where prices are exogenously specified, consumers may not necessarily benefit from trading. This can be explained as follows. Consumer surplus is determined by the difference in the value derived from consumption and the price paid to service provider. When service provider is able to respond to the presence of trading by modifying prices, it chooses a higher price for the high plan. This could result in more consumers choosing the lower plan which is more likely to occur when the trading price is low. In this case, we witness both a decrease in aggregate consumption and an increase in prices which causes the aggregate consumer surplus to drop below its value without trading.

A higher price for the high plan may not necessarily lead to more consumers choosing the low plan. When the trading price is high, more consumers choose the high plan despite the increase in its price. However, even in this case, it is possible for consumer surplus to decrease if the trading price is sufficiently high. This can be explained by the following. The increase in the trading price leads to more consumers choosing the higher plan. This increases the total consumption up to the point that the total consumption reaches its maximum value where all consumers demand is satisfied. If the trading price increases further, the total consumption stays unchanged while the high plan's price increases. This leads to a decrease in aggregate consumer surplus.

In summary, as indicated by Proposition 10, trading can hurt consumers if the trading price is either sufficiently low (resulting in less consumption) or sufficiently high (resulting in consumers paying a higher price for the higher plan). These results are depicted in Figure 2.5a. Note that, initially, consumer surplus increases with the trading price because the increase in the trading price leads to more consumers choosing the higher plan and an increase in the total consumption. As the trading price increases, eventually, the total consumption reaches its maximum value. An increase in trading price beyond this point only increases the high plan's price without changing the total consumption. Therefore, consumer surplus decreases in π for high trading prices.

At an individual level, consumers who decide to purchase the low plan benefit from trading (since they pay the same price with or without trading), with the benefit increasing in the type θ. The individual who benefits the most is the one who is indifferent between the low and high plan without trading. Individuals who decide to purchase the high plan may not necessarily benefit. This is because the high plan is now more expensive. Consumers who can be hurt by trading are those with the highest type since they are least likely to have excess capacity to sell. These results are depicted in Figure 2.5b.

The effect of trading on service provider and social welfare does not qualitatively change from the case with exogenous prices. In particular, we can show again that service provider benefits from trading if and only if the trading price is sufficiently high. Similarly, for social welfare, it increases by trading if and only if the trading price is sufficiently high. These results are formally stated in the following proposition.

Proposition 11 (Service Provider's Profit & Social Welfare) *The following* *statements hold.*

1. *There exists a threshold* π_p^* *such that trading leads to a higher service provider's profit if and only if* $\pi \geq \pi_p^*$.

2. *There exists a threshold* π_s^* *such that trading leads to higher social welfare if and only if* $\pi \geq \pi_s^*$.

3. *The thresholds* π_p^* *and* π_s^* *increase in* c *and* $\pi_s^* \leq \pi_p^*$

We conclude this section by highlighting thee main differences between the results here and those in Section 2.5 where prices are exogenous. First note that the *triple-win* region is characterized by both a lower and an upper bound on the trading price. This is different from the results in Section 2.5 in which the triple-win region is characterized by only a lower bound on the trading price. Secondly, when prices are exogenous, the triple-win region exists when the service cost is not too high, but when the prices are optimized, the triple-win region exists for any $c \in [0, r)$. Lastly, when prices are exogenous then, under any trading price, at least one of the performance metrics is improved by trading (see Table 2.1). However, when prices are set optimally, there exists a *no-win* region where none of the performance metrics is improved (see Figure 2.6).

2.7 Other Settings and Extensions

In this section, we consider variants and extensions to our basic setting. In Section 2.7.1, we consider the case where the trading price is determined via a market clearing mechanism. In Section 2.7.2, we consider the case of a single contract and study the effect of having more than one contract on the profitability of trading. Finally, in Section 2.7.3, we consider the case in which consumers' demands follow a general distribution.

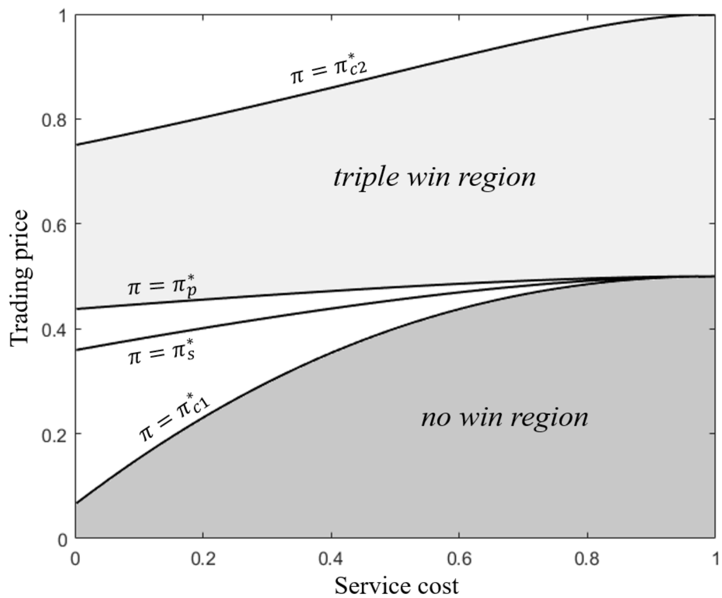


Figure 2.6: The impact of trading under optimal prices

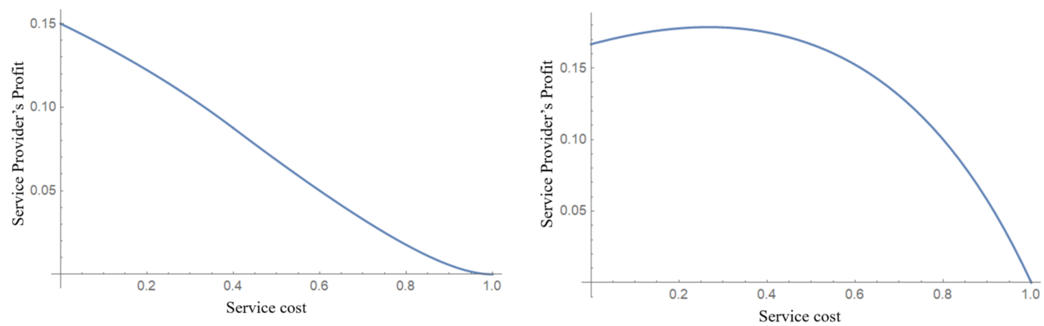
2.7.1 Market Clearing-Determined Trading Price

In this section, we study the case in which the trading price is set via a market clearing mechanism, which would arise if pricing is in the hands of individual sellers of excess capacity. The market clearing price is the price at which aggregate supply(ψ_s) equals aggregate demand(ψ_d). We maintain all other assumptions as in the base model described in Section 2.4 and Section 2.6.

Let π_m denote the market clearing price. Then π_m is the price π that solves $\psi_s = \psi_d$. In the following proposition, we characterize the market clearing price and its impact on various equilibrium outcomes.

Proposition 12 *The market clearing price is unique and given by $\pi_m = \frac{1}{3}(2r + c)$. Moreover, under this price, the service provider's profit, social welfare, and aggregate consumer surplus are all improved relative to the no trading scenario.*

Proposition 12 shows that when the trading price is set via a market clearing mechanism,



(a) Fixed trading price ($\pi = 0.8$) (b) The market clearing trading price

Figure 2.7: The effect of the service cost on the service provider's gain from trading

trading benefits everyone: the service provider, consumers, and social welfare. Recalling from Section 2.6 that the "*triple win*" region is characterized by lower and upper bounds π_p^* and π_{c2}^* on the trading price, Proposition 12 implies that the market clearing price π_m is always between these two thresholds, regardless of the value of c.

Note that the trading price now depends on service cost, c. This leads to an additional difference in outcomes. As the next proposition shows, the service cost impacts the gain from trading by the service provider differently from the case where the trading price is exogenously specified.

Proposition 13 *For a fixed trading price π, the service provider's profit gain from trading, $\Pi_t - \Pi_n$, decreases in c. However, under the market clearing trading price π_m, the profit gain is concave in c, first increasing and then decreasing, and attains its maximum at $c_m = (2 - \sqrt{3})r$.*

As the first part of Proposition 13 states, when the trading price is exogenous, the service provider's gain from trading, $\Pi_t - \Pi_n$, decreases in the service cost (see Figure 2.7a). The intuition is straightforward. Trading induces more consumption. So, if c increases, the cost to the service provider also increases. However, this is not the case when the trading price is set via a market clearing mechanism. In particular, the profit gain $\Pi_t - \Pi_n$ first increases and then decreases in c (see Figure 2.7b). This is because, π_m is now an increasing function of c. Hence, an increase in c is compensated by an increase in the trading price. The increase in the trading price enhances the value of the high plan (and hence the fraction of consumers who choose it), increasing the service provider's revenue. When c is sufficiently small, the increase in the service providers revenue is larger than the increase in the resulting service cost (due to more consumption). Therefore, the profit gain increases in c. However, when c is sufficiently large, the increase in the service cost surpasses the increase in revenue and the profit gain decreases.

The implication from this result is that, if the trading price is set independently of the service cost, then service providers with low costs benefit the most from trading. In contrast, when the trading price is set via a market clearing mechanism, then service providers with moderate service costs benefit the most. The behavior of other performance metrics (social welfare and consumer surplus) with respect to c is not qualitatively affected by the market clearing trading price and is similar to the case with the exogenously specified trading price.

2.7.2 The Case of a Single Contract

In this section, we consider a setting with a single contract (instead of two). In some applications, such as gym membership and residential internet, the service provider may offer a single contract. Our first goal here is to investigate if our main results continue to hold for the case with one contract. Second, we compare outcomes with one versus two contracts and assess the impact of having more than one contract on the various parties. The following theorem characterizes the service provider's optimal single contract.

Proposition 14 *For a service provider who offers a single contract, the contract that maximizes profit in the absence of trading is given by* $\mathbf{m}_\circ^* = (L, rL)$*. A contract that* *maximizes profit in the presence of trading is given by*

$$\mathbf{m}_{\circ}^{*} = \begin{cases} (L,\, rL) & \text{if} \quad \pi \le c, \\ \Big(\dfrac{H+L}{2},\ r\,L + \pi\dfrac{H-L}{2}\Big) & \text{if} \quad \pi > c. \end{cases} \tag{2.24}$$

Moreover, if $\pi \ge c$, trading increases the service provider's profit and social welfare relative to the case without trading while the aggregate consumer surplus decreases with trading. If $\pi < c$, the service provider profit, consumer surplus, and social are the same with and without trading.

The contract $\mathbf{m}_{\circ}^{*}$ has a few interesting features. When the trading price is smaller than the unit service cost ($\pi < c$), the service provider implements the same contract with trading as without trading. The reason is intuitive. When $\pi < c$, demand exceeds the supply on the trading market and, hence, any unit of capacity sold gets consumed and increases the service provider's cost by c. This decreases the service provider's profit since he can charge at most π for those units. Therefore, by offering the contract $(L,\, rL)$, the service provider effectively shuts down the trading market. As a result, when $\pi < c$, trading does not change equilibrium outcomes. In contrast, when $\pi \ge c$, there is an opportunity for the service provider to extract more surplus from consumers by offering a higher capacity and charging a higher price. Another notable feature of $\mathbf{m}_{\circ}^{*}$ (compared to the case with two plans) is that the *triple win* region does not exist. For $\pi \ge c$, both social welfare and the service provider's profit increase, but this comes at the expense of consumers who see their surplus decrease. When $\pi \ge c$, consumer surplus decreases since trading enables the service provider to extract more surplus from the high type consumers by charging a higher price.

Next, we study the effect of having more than one contract on equilibrium outcomes. In particular, we compare the outcomes with one versus two contracts and answer the

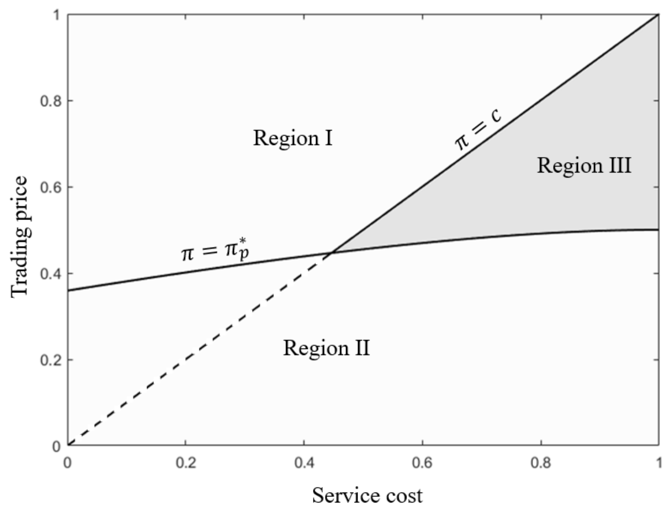


Figure 2.8: The effect of the number of contracts on the service provider's gain from trading

following question. Is trading more valuable to service providers when they offer two plans or when they offer only one? For the sake of brevity, we relegate the formal statements of results to Proposition 19 in the Appendix D.

We summarize the results below and illustrate them graphically in Figures 2.8.

Region I ($\pi \geq \max\{\pi_p^*, c\}$)**.** In this region, the gain from trading is positive regardless of the number of contracts. However, the gain is larger with one plan. The intuition is as follows. With two plans service providers can price-discriminate even without trading and, hence, the gain from trading is lower.

Region II ($\pi < \pi_p^*$). Trading decreases the service provider's profit when there are two plans and trading weakly increases the service provider's profit with one plan. Hence, the gain from trading is larger with a single plan.

Region III ($\pi_p^* \leq \pi \leq c$). The service provider's profit is not affected by trading when there is only a single plan. In contrast, the service provider's gain from trading is positive when there are two plans.

In summary, in Region III, the gain from trading is larger when there are two plans and elsewhere it is lager when there is a single plan.

2.7.3 General Demand Distribution

In this section, we consider a more general demand distribution $F_\theta(x)$ (with density $f_\theta(x)$ and survival function $\bar{F}_\theta(x)$). The utility of a consumer of type θ with and without trading can be, respectively, written as:

$$u_n(q,\theta) = r\int_0^q \bar{F}_\theta(x)\,\mathrm{d}\,x - p,$$

and

$$u_t(q,\theta) = r\int_0^q \bar{F}_\theta(x)\,\mathrm{d}x + \alpha\,\pi\int_0^q (q-x)\,\mathrm{d}\,F_\theta(x) + \beta\,(r-\pi)\int_q^\infty (x-q)\,\mathrm{d}\,F_\theta(x) - p,$$

where $\alpha = \min\left\{\frac{\psi_d}{\psi_s}, 1\right\}$ and $\beta = \min\left\{\frac{\psi_s}{\psi_d}, 1\right\}$ are defined (as in Section 2.3) in terms of aggregate supply (ψ_s) and aggregate demand (ψ_d) on the trading market. First, note that the expected overage and underage of a consumer of type θ who selects a plan with allowance q are

$$\mathcal{O}(q,\,\theta) = \int_0^q (q-x)\,\mathrm{d}F_\theta(x) \tag{2.25}$$

and

$$\mathcal{L}(q,\,\theta) = \int_q^\infty (x-q)\,\mathrm{d}F_\theta(x), \tag{2.26}$$

respectively. Denote the set of consumers who opt for contract $\mathbf{m}_i$ by Θ_i with $i = 1, 2$ and define

$$\psi_s = \int_{\theta\in\Theta_1} \mathcal{O}(q_1,\,\theta)\,\mathrm{d}\theta + \int_{\theta\in\Theta_2} \mathcal{O}(q_2,\,\theta)\,\mathrm{d}\theta \tag{2.27}$$

and

$$\psi_d = \int_{\theta\in\Theta_1} \mathcal{L}(q_1,\, \theta)\, \mathrm{d}\theta + \int_{\theta\in\Theta_2} \mathcal{L}(q_2,\, \theta)\, \mathrm{d}\theta. \tag{2.28}$$

In Theorem 4 below, we show that there are again unique thresholds characterizing the equilibrium behavior of consumers with and without trading.

Theorem 4 *There is a unique threshold $\theta_t(\theta_n)$ for the cases with (without) trading such that a consumer of type $\theta \geq \theta_t(\theta_n)$ prefers contract $\mathbf{m}_1 = (q_1\,,\, p_1)$ and otherwise prefers contract $\mathbf{m}_2 = (q_2\,,\, p_2)$. The threshold θ_t solves*

$$\int_{q_1}^{q_2} \bar{F}_{\theta_t}(x)\, \mathrm{d}x = \frac{p_2 - p_1 - \alpha\,\pi\,(q_2 - q_1)}{r - \alpha\,\pi - \beta(r-\pi)}, \tag{2.29}$$

and the threshold θ_n solves

$$\int_{q_1}^{q_2} \bar{F}_{\theta_n}(x)\, \mathrm{d}x = \frac{p_2 - p_1}{r}. \tag{2.30}$$

Moreover, thresholds θ_t and θ_n increase in p_2 and decrease in p_1. The threshold θ_t decreases in π.

Given the equilibrium thresholds obtained in Theorem 4, we can obtain the total consumption without and with trading as

$$\begin{aligned} P_n &= \int_0^{\theta_n} \int_0^{\infty} \min(d, q_1)\ \mathrm{d}F_\theta(d)\ \mathrm{d}\theta + \int_{\theta_n}^1 \int_0^{\infty} \min(d, q_2)\ \mathrm{d}F_\theta(d)\ \mathrm{d}\theta && (2.31) \\ &= \bar{P} - \int_0^{\theta_n} \mathcal{L}(q_1,\theta)\mathrm{d}G(\theta) - \int_{\theta_n}^1 \mathcal{L}(q_2,\theta)\mathrm{d}G(\theta), && (2.32) \end{aligned}$$

and

$$P_t = \bar{P} - \max\left\{ \int_0^{\theta_t} \mathcal{L}(q_1,\theta)\,\mathrm{d}\theta + \int_{\theta_t}^1 \mathcal{L}(q_2,\theta)\,\mathrm{d}\theta - \int_0^{\theta_t} \mathcal{O}(q_1,\theta)\,\mathrm{d}\theta - \int_{\theta_t}^1 \mathcal{O}(q_2,\theta)\,\mathrm{d}\theta,\ 0 \right\},$$

respectively, where $\bar{P}$ denotes the consumption level which satisfies all demands and is defined as $\bar{P} = \int_0^1 \int_0^\infty d \, \mathrm{d}F_\theta(d) \, \mathrm{d}\theta$. Finally, for the case without $(i = n)$ and with $(i = t)$ trading, social welfare $(S_i = (r - c)P_i)$ and the service provider's profit $(\Pi_i = p_1\theta_i + p_2(1 - \theta_i) - cP_i)$ are defined as before.

In the following proposition, we show that our main results (summarized in Table 2.1) does not change qualitatively and the existence and uniqueness of thresholds can be established when demands are distributed according to the distribution function $F_\theta(x)$[3].

Proposition 15 *There exists a well-defined $\bar{c}$ such that, if $c < \bar{c}$, the triple-win region is characterized by a lower bound π_p on trading price, i.e. $\pi \in [\pi_p, r)$ and if $c \geq \bar{c}$, under any π, service provider's profit decreases by trading and the triple-win region does not exist. Moreover, there exists thresholds π_s and π_h such that $S_t(\pi) \geq S_n$ if and only if $\pi \geq \pi_s$ and $\theta_t(\pi) \leq \theta_n$ if and only if $\pi \geq \pi_h$.*

Table 2.2 numerically illustrates the results in Proposition 15 by comparing equilibrium outcomes with and without trading for different trading prices. In these numerical illustrations, we assume that demands follow a truncated normal distribution on the interval $[0, 1]$ with demand mean $\theta \in [0, 1]$[4]. As in Table 2.2, trading improves social welfare if $\pi \geq 0.2$; more consumers opt for the high plan if $\pi \geq 0.5$; and the service provider's profit increases with trading if $\pi \geq 0.7$.

---

[3] We assume $F_\theta(x)$ is atomless and continuous in θ.

[4] Truncated normal distribution with mean μ and standard deviation σ on the interval $[a, b]$ is defined as

$$f(x) = \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\sigma\left(\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)\right)}$$

where $\phi(.)$ is the standard normal distribution and $\Phi(.)$ is its cumulative distribution function.

Table 2.2: Changes in equilibrium outcomes;
$f_\theta \sim \text{Truncated}\mathcal{N}(\theta, \frac{1}{2})$, $q_1 = 0.35$, $p_1 = 0.28$, $q_2 = 0.6$, $p_2 = 0.42$, $r = 1$, $c = 0.3$

Trading price	social welfare without trading	social welfare with trading	portion choosing high plan without trading	portion choosing high plan with trading	service provider's profit without trading	service provider's profit with trading
0.1	**0.255**	0.253	**0.435**	0.048	**0.231**	0.178
0.2	0.255	**0.267**	**0.435**	0.125	**0.231**	0.183
0.3	0.255	**0.283**	**0.435**	0.217	**0.231**	0.189
0.4	0.255	**0.304**	**0.435**	0.339	**0.231**	0.197
0.5	0.255	**0.331**	0.435	**0.492**	**0.231**	0.207
0.6	0.255	**0.350**	0.435	**0.661**	**0.231**	0.223
0.7	0.255	**0.350**	0.435	**0.783**	0.231	**0.240**

2.8 Conclusion

In this chapter, we showed that whether or not trading improves profit, consumer surplus, and social welfare depends on the relative strength of four effects: cannibalization, market expansion, consumption enhancement, and demand throttling, which in turn depends on two factors: (1) the cost incurred by the service provider per unit of usage and (2) the prevailing trading price. We found that trading is harmful to a service provider if either the trading price is sufficiently low (in this case cannibalization prevails with fewer consumers choosing the high plan) or if the usage cost is sufficiently high (in this case the consumption effect dominates, with consumption increasing without a sufficient increase in the number of consumers who choose the high plan). Assuming no changes are made to plans' prices and allowances, consumers of course always benefit from trading. However, social welfare can be harmed if consumption is throttled. We showed that this is the case when the trading price is sufficiently low. Putting these results together, we showed that there are five distinct regions, dependent on the service cost and the trading price, that correspond to different combinations of positive and negative outcomes for profit, consumer surplus, and social welfare, with one of these regions corresponding to an improvement in all three (a region of low enough service cost and high enough trading price).

In settings where the service provider is able to modify the contract terms, we showed that the service provider charges a higher price for the high plan. This comes at the expense of consumers who are no longer guaranteed to benefit. We show that, in this case, trading can hurt consumers if the trading price is either sufficiently low (resulting in less consumption because more consumers opt for the low plan) or sufficiently high (resulting in consumers paying a higher price for the higher plan because of the adjustment to the price of the high plan made by the service provider). Again, depending on trading price and service cost, we identified five regions corresponding to different combinations of positive and negative outcomes for service provider's profit, consumer surplus, and social welfare. An important difference is that in this case, it is possible for trading to be harmful to all three metrics.

Chapter 3

The Role of Information and Reciprocity in Revenue-sharing Crowdfunding

3.1 Introduction

For many years entrepreneurs and small businesses have been struggling to access financial resources to start or scale up their businesses. Many researchers reported the lack of capital as an important factor hindering the growth of entrepreneurship and innovation (Schumpeter, 2017; King and Levine, 1993; Brown et al., 2009). The traditional sources of capital for entrepreneurs are bank loans and venture capitals which can be difficult to access for many entrepreneurs. In early 2000s, the advent of crowdfunding platforms offered a substitute for traditional funding methods which quickly emerged as an innovative financing option for entrepreneurs. Ever since crowdfunding market keeps growing. In 2019, the crowdfunding market transactions valued at $6.9 billion and it is estimated to reach $11.2 billion by 2023 (Statista, 2019).

In recent years, crowdfunding grew both in size and diversity of models it implements. Crowdfunding platforms implement two main business models: non-investment-based and investment-based. In a non-investment-based platform, backers (or investors) receive a nonmonetary reward for their contributions. The rewards in non-investment-based platform could be in two forms. In the donation-based crowdfunding, rewards are simply the honor of donation or being listed as the contributor to a good cause. Examples of such platforms are *Gofundme*, *JustGiving*, and *Kiva*. Alternatively, in platforms such as *Indiegogo*, *Kickstarter*, and *Rockethub*, rewards are usually an early version of an actual product (e.g. a new smart watch, a creative backpack, a smart cooler, or a book signed by the author).

In an investment-based (or financial) crowdfunding, backers (or investors) enter into a security contract with the entrepreneur through the crowdfunding platform and they will receive financial returns for their investments. Financial crowdfunding platforms implement a variety of financial securities. The common financial contracts used by crowdfunding platforms are debt (e.g. *LendingClub*), equity (e.g.*crowdfunder*), and revenue-sharing (e.g.*Bolster*). Comparing to the traditional financing methods such as bank loans (a debt contract) and Venture Capital(VC) investments (usually an equity contract), the most distinct contract which is used in financial crowdfunding is the revenue-sharing contract. Under this contract, the entrepreneur commits to return a portion of her future revenue to the campaign. Then, the crowdfunding platform distributes this return among investors proportional to their investments. This contract is an appealing option for the entrepreneurs and small businesses for two main reasons. First, they do not need to forgive the ownership of their idea or firm (as in an equity crowdfunding or VC investment). Second, the revenue sharing contract does not create an obligation for entrepreneurs to return a pre-determined payoff (as in a debt crowdfunding or bank loans). Hence, the revenue sharing contract has usually a lower level of liability for entrepreneurs. Platforms such as *Bolster*, *Fig*, *Localstake*, and *Micro-finance* are a few examples of crowdfunding platforms which implement a revenue sharing contract.

Despite its flexibility and ease of raising money, crowdfunding comes with its own challenges. Managing a crowdfunding campaign can be a daunting task. One fundamental challenge that entrepreneurs are facing is creating the initial investment momentum. The early investments appeared to be crucial in the success of a crowdfunding campaign. Kickstarter reports that once a campaign raises over 20% of the initial funding goal, the project has an 80% chance of success. Similarly, Seedrs reports that once a campaign hits 30% of its funding goal the success rate climbs to 90% (compared to only 50% after a campaign reaches the 5% mark). This is also empirically validated by some studies in the entrepreneurship literature (Kuppuswamy and Bayus, 2018). Hence, most crowdfunding platforms emphasize the importance of early investors and advice entrepreneurs to tap into their social networks to bring the early investors to their campaign.

The common practice of attracting early stage investors, often from her network and with social tie with the entrepreneur, can create an additional layer of complexity. The investment motives of these investors are different from other investors who only invest for financial return. Polzin et al. (2017) argue that the investors with strong tie with the entrepreneur care more about her success than their own financial returns. On the other hand, those investors without a social tie with entrepreneur are attracted to the campaign solely for its financial returns. These different motives among investors can distort the information flow from informed to uninformed investors. In this paper, we address questions around the information and reciprocity in the context of the revenue sharing crowdfunding. We are interested in the interaction between the reciprocity and information. In particular, whether the reciprocity between the entrepreneur and investors can influence the existence and types of outcome (separating and pooling) which arise in the equilibrium.

In this work, we develop a signaling game to address the questions possed above. The type of the entrepreneur represents the future revenue of his project which can be either high or low. The entrepreneur sets up a revenue-sharing crowdfunding campaign and seeks funding from two investors. There is one inside investor (*insider*) and one outside investor (*outsider*). The main difference between the insider and the outsider is that the insider has social tie with the entrepreneur. This has two implications to our model. First, the insider is informed about the quality of the project and its future revenue. Second, the existence of reciprocity implies that the insiders investment motives are not solely financial. The insider cares about the entrepreneurs payoff as well. On the other hands, the outsider is neither informed about the quality of project nor has social ties with the entrepreneur. The outsider holds some prior beliefs about the quality of project and, upon her arrival and observing the insiders investment, she updates her beliefs and then makes an investment.

Our analyses reveal a few interesting findings. In our signaling game, we find that the reciprocity which exists between the entrepreneur and the insider may hinder the information transmission from the insider to the outsider. In particular, we specified a well-defined parameters region with relatively high reciprocity and uncertainty in which a separating equilibrium does not exist which implies that in this region there is no information transmission from informed agent (insider) to the uninformed agent (outsider).

The rest of this chapter is organized as follows. First, we review related literature in Section 3.2. Section 3.3 introduces our model setup and in Section 3.4 we define and derive the equilibria as well as our off-equilibrium beliefs refinement. Finally, we explore the effects of reciprocity an information asymmetry in Section 3.5 before we conclude this chapter in Section 3.6.

3.2 Literature Review

The work in this chapter contributes to two main streams of Operations Management (OM) literature, namely, the emerging literature on crowdfunding and the literature on signaling. This work also relates to the informal finance literature.

The OM literature on crowdfunding mostly focuses on reward-based crowdfunding where papers study the issues related to the pricing, signaling, revenue management, and predicting the dynamics or outcomes of a reward-based crowdfunding campaign. Examples are signaling the quality (Chakraborty and Swinney, 2019), revenue management (Zhang et al., 2017), pledging dynamics (Alaei et al., 2016; Kuppuswamy and Bayus, 2018), predicting a campaigns success (Mollick, 2014), risk of fraud and misconduct (Belavina et al., 2018), and coordinating the crowdfunding and Venture Capitalist (VC) funding (Roma et al., 2018; Babich et al., 2018). Among these studies those who consider signaling and herding in reward-based crowdfunding are closer to our work. Babich et al. (2018) investigate the role of reward-base crowdfunding as a signaling device to VC and banks. Surprisingly, they find that a successful reward-based crowdfunding may induce competition among investors and may reduce VCs' propensity to fiance the entrepreneur's project. This eventually may hurt the entrepreneur who loses the value of VC's experience and expertise. Roma et al. (2018) study how a reward-base crowdfunding campaign can serve as a signaling devise to VCs. They show that the entrepreneur has a stronger preference, comparing to a VC, to run a crowdfunding campaign. Also, it can be possible that the entrepreneur prefers to approach VC directly. Jiang et al. (2018) empirically study the effects of project updates by the entrepreneur on the investment and herding behavior of investors. They conclude that the platform's market share and regulations are among the most important factors affecting the herding behavior. There are two fundamental differences between our work and these studies. First, in the financial crowdfunding, investors seek financial return from their investment and, second, in a revenue-sharing crowdfunding the return to each investor depends on the total investment on the campaign which, in turn, may affect how informed investors reveal their information.

The literature on financial crowdfunding is relatively small. Chen (2018) studies the effects of information asymmetry on an equity-based crowdfunding. He shows that information aggregation may fail and higher quality projects may not have a higher chance of getting founded. Fatehi and Wagner (2019) consider a revenue-sharing crowdfunding setting and study the dynamic stochastic control problem arising in this setting. They argue that the revenue-sharing crowdfunding could be superior to other methods of entrepreneurial financing such as bank loans and equity securities. Li (2018) shows that a simple profit-sharing contract could often coordinate investments of individuals with private information and achieve the first best (harness the wisdom of crowd). In our model we assume investors are risk-neutral and we focus on the interaction between information asymmetry and reciprocity. These two features distinguishing our work from Fatehi and Wagner (2019) and Li (2018). Our setting is different from that of Chen (2018) in that investors are different from each other in two dimensions, namely information and reciprocity, and we consider a revenue-sharing contract instead of an equity agreement.

The model we have in this chapter also relates to the literature on informal finance. Different from a regular financial contracts (e.g. bank loans and equity investments), in an informal finance agreement there is no legal commitment. A browser receives cash from lenders (in most cases family members and friend) and promises to return cash in future. However, the commitment (or promise) is not legally binding. The main focus of informal finance literature is to understand how informal finance resources can substitute or complement the other types of securities (for a review of alternative investments literature see Cumming and Zhang, 2016). In particular, how the extra information that informal lenders have affects their investment (or lending) decisions (Lee and Persson, 2016). Since informal finance resources are a significant source of finance for small companies and entrepreneurs, especially those with limited access to formal financial resources, understanding their interaction with formal financial resources is the key to developing better understanding of entrepreneurship growth (Allen et al., 2018; Karaivanov et al., 2013). Polzin et al. (2018) study investment motives in financial crowdfunding by using a data from a large-scale survey. They find that the level of social tie with the entrepreneur significantly affects investment and information seeking behavior. Results in Polzin et al. (2018) are consistent with other studies reporting that friends and family may accept below market or even negative return (Karaivanov et al., 2013; Banerjee et al., 2017). To the best of our knowledge, we are the first to study the role of informal financial resources and their abilities to convey private information in a crowdfunding setting. Our work complements the empirical works in this literature by providing a rigorous game-theoretical framework to study the effects of social ties on the information flow in a revenue-sharing crowdfunding where formal and informal resources coexists.

3.3 Model Setup

We present our model setup and assumptions in this section. First, we explain the characteristics of the entrepreneur's project and crowdfunding campaign. Then, we introduce investors and entrepreneur's utility functions. Lastly, we outline the chronology of events and actions.

Crowdfunding Campaign. We consider a penniless entrepreneur who would like to initiate a project (or an idea) and needs an initial investment of size F to execute his project. The revenue that the project generates depends on the entrepreneur's type

θ which could be either high(h) or low(ℓ). If the project is successfully executed, it generates non-negative revenue μ_θ with $\mu_h > \mu_\ell$. It is commonly believed that the state of economy is high with probability p and low with probability $1 - p$.

In order to raise capital, the entrepreneur sets up a crowdfunding campaign. We consider a crowdfunding platform with *All-or-Nothing* crowdfunding format. All-or-Nothing crowdfunding is widely used by many financial crowdfunding platforms such as *Fig*, *SeedInvest* and *EquityNet*. In an All-or-Nothing crowdfunding campaign, the entrepreneur receives the campaign's investment if and only if the total investment is larger than a predetermined threshold. We denote the crowdfunding goal by F. If the campaign is successful, i.e. the campaign attracts at least F of investment, the entrepreneur executes the project and shares his revenue with investors according to a revenue sharing contract. Under a revenue sharing contract with parameter α, the entrepreneur shares $\alpha \times 100\%$ of his revenue with investors in the campaign. We assume that the platform has enough monitoring power that can verify the entrepreneur's revenue. After receiving $\alpha \times 100\%$ of the entrepreneur's revenue, the platform distributes the return among the investors proportional to their investments. We assume that α is exogenously given and $F \leq \alpha\mu_\ell$. [1]

Utilities. There are two (type of) potential investors: the inside investor or the *insider* (indexed by i) and the outside investor or the *outsider* (indexed by t). The main distinguishing differences between an insider and an outsider are the information and the social tie. The insider belongs to the entrepreneur's social network, e.g. friends and family[2], and is informed about the quality of the entrepreneur's project. Therefore, her investment can serve as a signaling device. In contrast, the outside investor is uninformed about the quality of the project and do not have a social tie with the

[1] Our results could be generalized, without much of difficulties, to the case in which $\alpha\mu_\ell < F \leq \alpha\mu_h$.

[2] Alternatively, one can interpret insiders as investors who have been attracted from the entrepreneur's refunding activities. There are recent examples of pre-crowdfunding platforms who try to faceplate the interaction of the entrepreneur and some investors before the official lunch of a crowdfunding campaign.

entrepreneur.

In our model, there is one insider whose investment is denoted by x_i and one outsider who invests x_t. Therefore, the crowdfunding campaign receives the total investment of $x_i + x_t$. The crowdfunding campaign is successful if and only if $x_i + x_t > F$ in which case the entrepreneur executes the project and returns an $\alpha \times 100\%$ of his revenue to the crowdfunding platform. The investors' return is proportional to their investments, i.e. the insider receives $\frac{x_i}{x_i+x_t} \times 100\%$ of the return and the outsider receives $\frac{x_t}{x_i+x_t} \times 100\%$ of the return to the campaign.

The outsider's investment decision is only driven by financial returns and there is no social tie between the entrepreneur and the outsider. Therefore, the outsider's utility is defined as follows:

$$u_t(x, x_i, \mu) = \begin{cases} \frac{x}{x_i+x}\,\alpha\mu - x & \text{if } \; x_i + x \geq F, \\ 0 & \text{otherwise,} \end{cases} \tag{3.1}$$

where the outsider believes project is of high type with probability β and $\mu = \beta\mu_h + (1-\beta)\mu_\ell$.

The insider's return from the campaign, similar to that of the outsider, is proportional to her investment and is equal to $\frac{x_i}{x_i+x_t}\alpha\mu_\theta$. However, the insider not only cares about the financial return, but also she cares about the entrepreneur's success and payoff. We capture this altruistic utility by introducing the parameter $\phi \in [0, 0.5]$ which we refer to it as *reciprocity coefficient*. The reciprocity coefficient ϕ denotes the degree to which the insider cares about the entrepreneur's payoff. We express the insider's altruistic utility as a weighted average of her payoff and the entrepreneur's payoff. That is, the insider's utility is

$$u_i(x, \mu_\theta, \mu) = \begin{cases} \phi\, \pi_e(x, \mu_\theta, \mu) + (1-\phi)\left(\frac{x}{x+x_t}\, \alpha\mu_\theta - x\right) & \text{if } \; x + x_t \geq F, \\ 0 & \text{otherwise,} \end{cases} \tag{3.2}$$

where π_e is the entrepreneur's payoff and $\frac{x}{x+x_t}$ is the outsider's own financial return.

When $\phi = 0$, the insider does not have reciprocity with the entrepreneur. When $\phi = 0.5$, the insider's utility is fully aligned with that of the entrepreneur and she equally cares about her payoff as well as the entrepreneur's payoff. The way we model reciprocity in (3.2) is commonly used in the informal finance literature (see for example Lee and Persson (2016)).

In the case that campaign is successful, the entrepreneur receives the total investment of $x_i + x_t$. After spending a fixed cost of F, the entrepreneur executes the project and return $\alpha\mu_\theta$ to the crowdfunding campaign keeping $(1-\alpha)\mu_\theta$ for himself. Therefore, the insider's utility in (3.2) can be written as

$$u_i(x_i, \mu_\theta, \mu) = \begin{cases} \phi\big((1-\alpha)\mu_\theta - F + x_i + x_t\big) & \\ \qquad + (1-\phi)\left(\frac{x_i}{x_i+x_t}\, \alpha\mu_\theta - x_i\right) & \text{if } \; x_i + x_t \geq F, \\ 0 & \text{otherwise.} \end{cases} \tag{3.3}$$

Timeline. Figure 3.1 summarizes the sequence of events. At the beginning, the entrepreneur sets up his crowdfunding campaign which is characterized by two parameters α and F (exogenously given). The insider arrives and invests x_i. Next, the outsider arrives and observes the insider's investment. The outsider updates her beliefs to β and invests x_t. At the end, if $x_i + x_t < F$, the campaign fails and investments are returned and the entrepreneur does not receive funding from the crowdfunding campaign. If $x_i + x_t \geq F$, the campaign is successful and the platform charges investors accounts and transfers the total investment amount of $x_i + x_t$ to the entrepreneur. The entrepreneur executes the project at fixed cost F, the project's revenue is realized, and the entrepreneur transfers $\alpha\mu_\theta$ to the campaign. The platform distributes the return from the entrepreneur among the investors proportional to their investments.

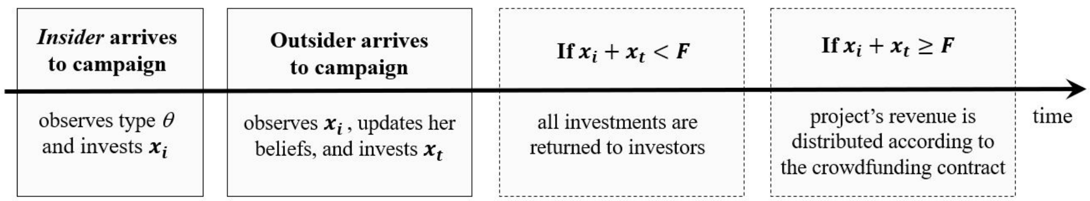


Figure 3.1: Sequence of events

The sequence of events that we follow is similar to Chen (2018) who studies information asymmetry in an equity crowdfunding market. Our timeline is also similar to papers in the observational learning literature which assume that the informed agent moves first and, then, the uninformed agents moves after observing the informed agent's action.

3.4 Equilibrium

In this section, we describe our solution concept which is a *Perfect Bayesian Nash equilibrium* and introduce the method which we use to refine the off-equilibrium path beliefs. A *Perfect Bayesian Nash equilibrium* of the game described in Section 3.3 should specify the insider's investment for each type of insider(denoted by x_θ, $\theta \in \{h, \ell\}$), the corresponding outsider's belief (denoted by β), and investment for each type of insider (denoted by x_t). Formally, we denote a pure strategy perfect Bayesian Nash equilibrium by tuple $\langle x_h,\, x_\ell,\, \beta \rangle$ and specify it as follows.

Definition 1 *A pure strategy equilibrium $\langle x_h,\, x_\ell,\, \beta \rangle$ consists of (1) an insider's investment for each type, $x_\theta \in \mathbb{R}_+$ with $\theta \in \{h, \ell\}$, (2) an outsider's posterior belief that the* *entrepreneur is of high type, $\beta : \mathbb{R}_+ \to [0,1]$, and (3) an outsider's investment strategy $x_t : \mathbb{R}_+ \times [0,1] \to \mathbb{R}_+$ such that the subgame perfect Bayesian Nash equilibrium of the crowdfunding game satisfies:*

1. *For each level of insider's investment, the outsider's belief β is update from prior $\beta_\circ$ according to Bayes' rule.*

2. *For given x_θ and β with $\mu = \beta\mu_g + (1-\beta)\mu_\ell$, the outsider's investment x_t solves*

$$x_t = \arg\max_{z \geq 0} \left\{ I_{\{z+x_i \geq F\}} \left(\frac{z}{x_i + z} \, \alpha \, \mu - z \right) \right\} \tag{3.4}$$

3. *The insider's investment x_θ solves*

$$x_\theta = \arg\max_{z \geq 0} \left\{ I_{\{z+x_t \geq F\}} \left(\phi\big((1-\alpha)\mu_\theta - F + z + x_t\big) + (1-\phi) \left(\frac{z}{z + x_t} \, \alpha\mu_\theta - z \right) \right) \right\} \tag{3.5}$$

3.4.1 Full Information Benchmark

In this section, we derive the equilibrium for a benchmark case without information asymmetry. We denote the future return of the project by μ and assume that $\frac{\alpha\mu}{F} \geq 2(1-\phi)$. This assumption ensures interior solutions for the investments. In other words, we avoid cases in which campaign fails, i.e. $x_i + x_t < F$, or the total investment on the campaign is more than the future return, i.e. $x_i + x_t > \alpha\mu$.

Next proposition presents the equilibrium outcome for a simple benchmark in which there is no reciprocity and no information asymmetry.

Proposition 16 *Assume $\phi = 0$. In the equilibrium, investors equally split the profit of the campaign by investing $\frac{\alpha\mu}{4}$.*

In a setting described in Proposition 16, the only difference between the insider and the outsider is that the insider has the first mover's advantage. However, interestingly, the insider could not use this to her advantage to earn a higher utility. The intuition is as follows. If the insider wants to earn a higher return, she needs to invest more. In such a case, the outsider finds her share damped. To compensate and earn a larger share, the outsider also increases her investment which creates competition between the investors for a larger share.

Next we consider the benchmark in which there is no information asymmetry, but $\phi > 0$. Proposition 17 characterizes the investors equilibrium investments for this case.

Proposition 17 *Assume $\phi > 0$. In the equilibrium, $x_i = \frac{\alpha\mu}{4(1-\phi)^2}$ and $x_t = \frac{(1-2\phi)\alpha\mu}{4(1-\phi)^2}$. Moreover, the outsider's utility decreases in ϕ while the utility of the insider and the entrepreneur increases in ϕ.*

When $\phi > 0$, the insider has incentives to invest more. As a result, the outsider responds by reducing her investment. This makes the outsider's share, and hence her payoff, smaller. However, since the increase in the insider's investment is larger than the decrease in the outsider's investment, the total investment on the campaign and the entrepreneur's utility increase in ϕ. As ϕ increases the insider's payoff decreases. However, her utility increases in ϕ due to earning more reciprocal utility through her investment.

3.4.2 Off-equilibrium Beliefs Refinement

Now, we turn to the solution for the case with asymmetric information. In order to restrict off-equilibrium path beliefs, we use the widely-used *Intuitive Criterion* (Cho and Kreps, 1987). We also make the following assumption.

Assumption 6 $\phi \leq \frac{\rho}{1+\rho}$ *where* $\rho = \sqrt{\frac{\mu_\ell}{\mu_h}}$.

Assumption 6 makes the exposition easier. It is also necessary condition for having a pure strategy separating equilibrium which we study next in Section 3.4.3. Moreover, the parameter region characterized by Assumption 6 is a more realistic setting to consider. In particular, in most of relevant cases the difference between μ_h and μ_ℓ is not very large, i.e. the value of $\sqrt{\frac{\mu_\ell}{\mu_h}}$ closer to 1 are more realistic to consider in a revenue sharing crowdfunding. Assumption 6 rules out a rather small parameter region in which both the reciprocity and uncertainty regarding the revenue of project are high.

Fix an equilibrium $\langle x_h,\, x_\ell,\, \beta\rangle$[3]. For an off-equilibrium path investment $\tilde{x} \notin \{x_h, x_\ell\}$ note that $0 = \arg\max_{\beta\in[0,1]} u_i(\tilde{x}, \mu_\theta, \mu)$. Hence, $u_i(\tilde{x}, \mu_\theta, \mu_\ell)$ represents the maximum profit an insider of type θ earns from deviation to an off-equilibrium investment $\tilde{x}$. An equilibrium is said to *fail the Intuitive Criterion* if there exists some off-equilibrium action $\tilde{x} \notin \{x_h, x_\ell\}$ such that the inequality $u_i(x_\theta, \mu_\theta, \mu) < u_i(\tilde{x}, \mu_\theta, \mu_\ell)$ holds only for one type. We formalize this idea in Definition 2.

Definition 2 (Intuitive Criterion) *Consider an equilibrium $\langle x_h,\, x_\ell,\, \beta\rangle$ with $\mu = (1-\beta)\mu_\ell + \beta\mu_h$. The equilibrium $\langle x_h,\, x_\ell,\, \beta\rangle$ fails Intuitive Criterion otherwise the conditions*

1. *If $u_i(x_h, \mu_h, \mu) < u_i(\tilde{x}, \mu_h, \mu_\ell)$ and $u_i(x_\ell, \mu_\ell, \mu) \geq u_i(\tilde{x}, \mu_\ell, \mu_\ell)$, then $\beta(\tilde{x}) = 1$,*
2. *If $u_i(x_h, \mu_h, \mu) \geq u_i(\tilde{x}, \mu_h, \mu_\ell)$ and $u_i(x_\ell, \mu_\ell, \mu) < u_i(\tilde{x}, \mu_\ell, \mu_\ell)$, then $\beta(\tilde{x}) = 0$,*
3. *In all other cases, no restriction on $\beta(\tilde{x}) = \beta$,*

hold for all off-equilibrium path investments $\tilde{x} \neq \{x_h, x_\ell\}$.

The first condition in the Definition 2 indicates that if an off-equilibrium path investment $\tilde{x}$ is such that the low-type insider's utility is less than her equilibrium utility and the high-type insider's utility is more than her equilibrium utility, then the off-equilibrium

[3]In a separating equilibrium we have $x_h \neq x_\ell$ and in a pooling equilibrium we have $x_h = x_\ell$

investment $\tilde{x}$ should come from a high-type insider, i.e. $\beta(\tilde{x}) = 1$. The intuition behind the second condition is also similar. Finally, the last condition indicates that if there exist an off-equilibrium investment under which both types earn a higher (or lower) utility than the equilibrium utility, then the intuitive criterion does not impose a restriction.

3.4.3 Separating Equilibrium

We first start with introducing some notations. Denote the investment level of an investor of type θ in a separating equilibrium by x_θ^{SP} and let $\tilde{\beta}$ denote off-equilibrium beliefs and define $\tilde{\mu} = \tilde{\beta}\mu_h + (1-\tilde{\beta})\mu_\ell$. More precisely, a separating equilibrium is defined by a tuple $\langle x_h^{SP}, x_\ell^{SP}, \tilde{\beta}\rangle$ where $x_\ell^{SP} \neq x_h^{SP}$, $\beta(x_h^{SP}) = \mathbb{P}(\mu = \mu_h | x_i = x_h^{SP}) = 1$, $\beta(x_h^{SP}) = \mathbb{P}(\mu = \mu_\ell | x_i = x_\ell^{SP}) = 0$, and for any off-equilibrium investment $\tilde{x} \notin \{x_h^{SP}, x_\ell^{SP}\}$ off-equilibrium beliefs are $\beta(\tilde{x}) = \mathbb{P}(\mu = \mu_h | x_i = \tilde{x}) = \tilde{\beta}$. As in Section 3.4.1, we focus on the cases in which the project is feasible and can generate non-negative utility for investors and the entrepreneur. Also, since the constraint $x_i + x_t > F$ is already incorporated in the the mathematical programs (3.4) and (3.5), here we focus on the incentive compatibility constraints in the equilibrium.

Fix an off-equilibrium path beliefs $\tilde{\beta}$. A separating equilibrium satisfies

$$u_i(x_\ell^{SP}, \mu_\ell, \mu_\ell) \geq u_i(x_h^{SP}, \mu_\ell, \mu_h), \tag{IC$_\ell^{\mathrm{s}}$}$$

and

$$u_i(x_h^{SP}, \mu_h, \mu_h) \geq u_i(x_\ell^{SP}, \mu_h, \mu_\ell). \tag{IC$_{\mathrm{h}}^{\mathrm{s}}$}$$

where x_t^θ is defined in (3.4). In words, to sustain a separating equilibrium, we need to ensure that neither type deviates from its separating equilibrium investment. As we formalize in next lemma, the set of separating equilibrium investments can be conveniently represented by two sets.

Lemma 4 *Assume there exists a separating equilibrium $\langle x_h^{SP},\, x_\ell^{SP},\, \tilde{\beta}\rangle$ with off-equilibrium path belief $\tilde{\beta}$ and $\tilde{\mu} = \tilde{\beta}\mu_h + (1-\tilde{\beta})\mu_\ell$ which passes the intuitive criterion as defined in Definition 2. Define sets*

$$\mathcal{N}_\theta(\tilde{\mu}) = \big\{x \mid u_i(x, \mu_\theta, \mu_\theta) \geq \max_z u_i(z, \mu_\theta, \tilde{\mu})\big\}, \tag{3.6}$$

and

$$\mathcal{M}_\theta = \big\{x \mid u_i(x, \mu_\theta, \mu_{-\theta}) \leq u_i(x_\theta^{SP}, \mu_\theta, \mu_\theta)\big\}. \tag{3.7}$$

Then the equilibrium investment satisfies $x_\theta^{SP} \in \mathcal{N}_\theta(\tilde{\mu}) \cap \mathcal{M}_{-\theta}$, $\theta \in \{h, \ell\}$.

The set $\mathcal{N}_\theta(\tilde{\mu})$ denotes all investment levels by an insider of type θ for which she earns a higher utility from revealing her true type than the maximum utility she can earn if she deviates to an off-equilibrium path action when off-equilibrium belief is $\tilde{\mu}$. The set $\mathcal{M}_\theta$ includes all investments by an insider of type θ for which she does not wish to pretend to be the other type $(-\theta)$ if she can signal her type by investing x_θ^{SP}.

In other words, $\langle x_h^{SP},\, x_\ell^{SP},\, \tilde{\beta}\rangle$ forms a separating equilibrium if the insider does not wish to deviate from her type to an off-equilibrium path investment, i.e. $x_\theta^{SP} \in \mathcal{N}_\theta(\tilde{\mu})$, or the other type's investment, i.e. $x_\theta^{SP} \in \mathcal{M}_{-\theta}$.

In the next theorem, we characterize the the separating equilibrium and the condition for its existence.

Theorem 5 *Assume the set $\mathcal{N}_\ell(\mu_h) \cap \mathcal{M}_h$ is non-empty and let $x_h^{FI} = \frac{\alpha\mu_h}{4(1-\phi)^2}$ and $x_\ell^{SP} = \{\min x \quad s.t. \quad x \in \mathcal{M}_h\}$. There exists a separating equilibrium in the form of $\langle x_h^{FI},\, x_\ell^{SP},\, 1\rangle$ which satisfies the intuitive criterion.*

Figure 3.2 illustrates the equilibrium described in Theorem 5. For any fixed level of investment, a high type insider's utility is higher if she is believed to be low and a low

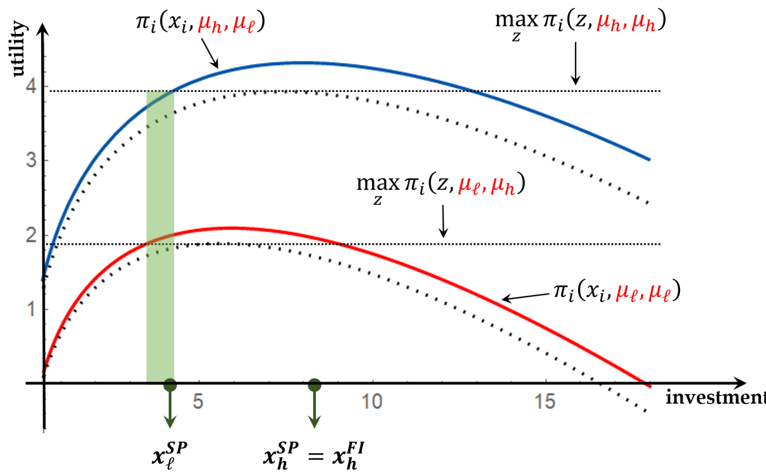


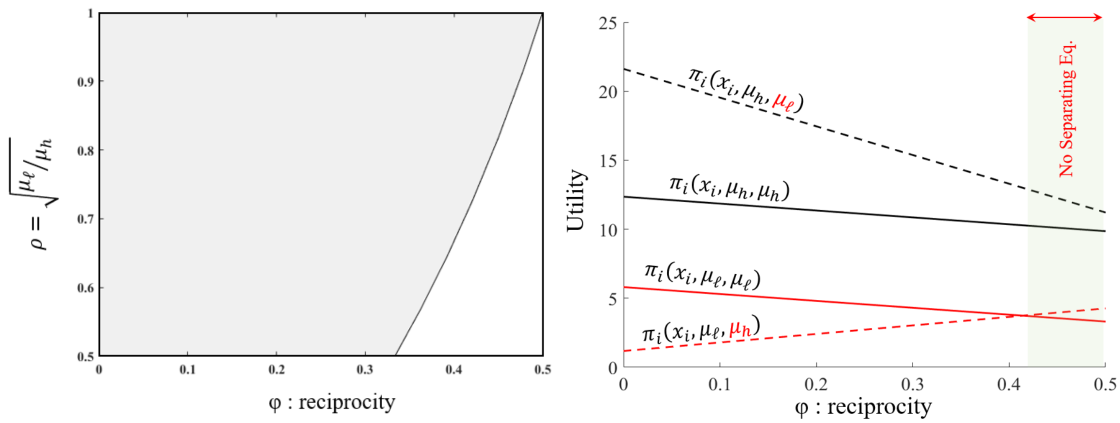
Figure 3.2: Illustrating the separating equilibrium ($\alpha = 0.5$, $\mu_h = 25$, $\mu_\ell = 20$, $F = 15$, and $\phi = 0.35$)

type insider prefers to reveal her true type. Therefore, it is intuitive that we set the off-equilibrium path believes to be high. This implies that, in a separating equilibrium, the high type insider invests her full information investment x_h^{FI}. In order to separate herself from a high type, a low type should chose an investment level from the shaded area in Figure 3.2. Since her utility is increasing in this area, the low type chooses the largest possible investment which is denoted by x_ℓ^{SP} in Figure 3.2.

3.5 Effects of Reciprocity and Information Asymmetry

In this section, we explore how the reciprocity and information asymmetry affect the outcomes in the separating equilibrium derived in Section 3.4.3. As described in Theorem 5, in a separating equilibrium, the high-type insider invests her full information investment. Therefore, we focus on the effects of reciprocity on the low-type insider who needs to adjust her investment in order to separate herself from a high-type insider.

Effects of Reciprocity on the Information Flow. Interestingly, the reciprocity may create a condition under which there is no separating equilibrium. Figure 3.3a shows



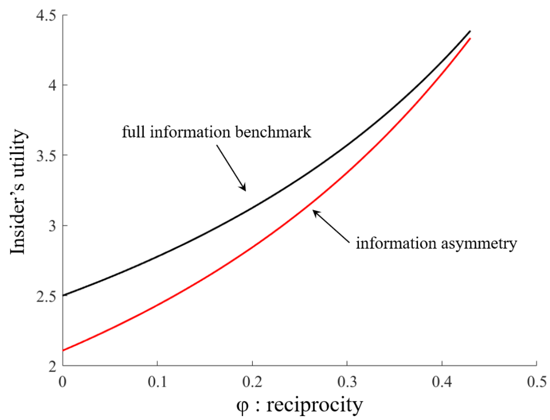
(a) Existence of a separating equilibrium

(b) Changes in the insider's incentives

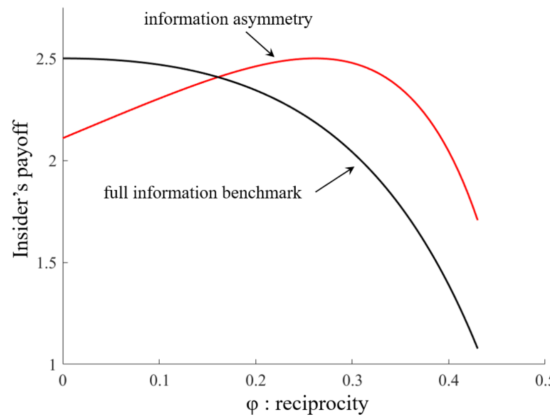
Figure 3.3: Effects of reciprocity on the information and incentives

how the existence of a separating equilibrium will be affected by ϕ. For any value of $\rho = \sqrt{\frac{\mu_\ell}{\mu_h}}$, if ϕ is large enough we reach the parameter region in which there is no separating equilibrium. This suggests that, if the reciprocity is high, there could be a situation in which signaling fails and the insider (the informed agent) cannot signal her type to the outsider (the uninformed agent) via distorting her investment. This result is in contrast to the common wisdom that entrepreneurs should bring investors from their social network to the crowdfunding campaign to create the first momentum.

The above result is easiest explained by analyzing how the insider's incentives to deviate change as ϕ increases(see Figure 3.3b). Regardless of ϕ, the high-type insider always prefers to be believed to be of low-type in which case she enjoys a larger share of crowdfunding campaign. If the reciprocity is not too high, a similar intuition applies to low-type insider as well. However, if the reciprocity is larger than a threshold, the low-type insider would like to signal a high revenue project in order to induce a larger investment by the outsider. Although this would dampen her payoff, it increases her reciprocal utility. This dynamic creates a situation in which each type wants to pretend to be the other type. Hence, a separating equilibrium is not sustainable.



(a) Equilibrium investment



(b) Share of crowdfunding campaign

Figure 3.4: Effects of changes in reciprocity on the equilibrium outcomes for the insider

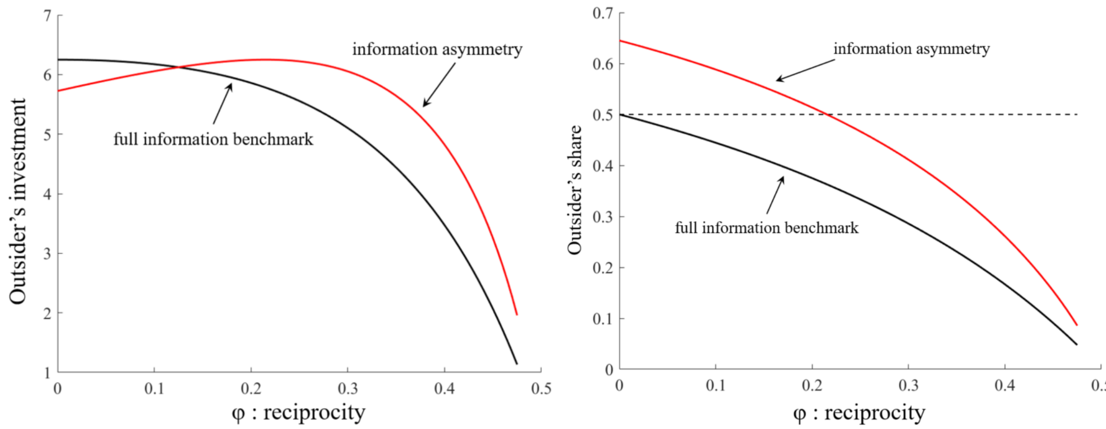
Cost of Separation. Sustaining a separating equilibrium is costly for the insider (comparing to the full information benchmark) and, in the equilibrium, the insider always under-invests (see Figure 3.4a). For higher ϕ, the gap between the investments decreases and the insider's investment is closer to her full information benchmark which means separating is less costly.

Although the insider's utility is negatively affected by information asymmetry, interestingly, her payoff (i.e. $\frac{x_i}{x_i+x_t}\alpha\mu_\ell - x_i$) can be larger under information asymmetry if the reciprocity is large enough(see Figure 3.4b). The intuition is as follows. When there is no information asymmetry, as the reciprocity increases, the insider invests more aggressively which improves her reciprocal utility but reduces her payoff. This trend continues to hold when there is information asymmetry as well. However, its magnitude is moderated by the fact that the insider under-invests in the separating equilibrium (see Figure 3.4a). This is especially significant when ϕ is high. Therefore, when there is reciprocity, the existence of information asymmetry can improve the insider's payoff share by moderating her investment.

Impacts on the Outsider. Figure 3.5a shows how the outsider's equilibrium investment changes in reciprocity with and without information asymmetry. When there is no information asymmetry, the outsider's equilibrium investment decreases with reciprocity. This is intuitive because as ϕ increases, the insider invests more and the outsider's share of campaign decreases. Hence, she decreases her investment. However, when there is information asymmetry the outsider's investment is non-monotonic in ϕ. As we mentioned above, the insider under-invests in the equilibrium and, therefore, when the reciprocity is relatively small the outsider can earn the same share of crowdfunding campaign by a smaller investment (comparing to full information benchmark). As reciprocity increases, the insider increases her investment and the outsider does the same to compete with the insider for a larger share of crowdfunding campaign. However, as the reciprocity increases, the difference $x_\ell^{FI} - x_\ell^{SP}$ decreases (i.e. the insider under-invests less) and eventually the outsider finds it unprofitable to invest more and she decreases her investment level.

This dynamic of equilibrium investments also determines how investors share the project's revenue. Recall from Proposition 16 that, when there is no reciprocity and no information asymmetry, investors split the crowdfunding share equally. When there is reciprocity, since the insider under-invests, initially the outsider share is larger than 50% (see Figure 3.5b). As the reciprocity continues to increase, the insider increases her investment level and eventually the outsider's share of crowdfunding campaign drops below 50% benchmark, but it is always more than her share when there is no information asymmetry. Finally, it is interesting to observe that the information asymmetry benefits the outsider who is uninformed. This is because the insider under-invests and the outsider gets an opportunity to earn a larger share of the revenue with the same or smaller amount of investment.

Impacts of Information Asymmetry on Social Welfare. We define social welfare as



(a) Equilibrium investment

(b) Share of crowdfunding campaign

Figure 3.5: Effects of changes in reciprocity on the equilibrium outcomes for the outsider

the sum of investors' payoffs [4]. Since the project's revenue (i.e. μ_h or μ_ℓ), and hence the return to the campaign, is not affected by the investments as far as $x_i + x_t \geq F$, social welfare is reversely related to the total investment. For a fixed level of reciprocity, the total investment is always higher and, hence, the social welfare is lower without information asymmetry. The reason is that the information asymmetry induces a lower investment by the insider which, in turn, reduces the outsider's investment. In short, the information asymmetry improves the social welfare by moderating the competition between investors for earning a larger shares of crowdfunding campaign.

3.6 Conclusion

This chapter explored the effects of reciprocity and information asymmetry in a revenue sharing crowdfunding context. We built a signaling game to study how reciprocity may affect the informed agent's ability to signal the quality of the project via distorting her investment. We showed that the reciprocity between informed investor and the

[4] Alternatively, one can define the social welfare as the sum of investors' utility. However, this does not change the main result here saying the information asymmetry increases social welfare

entrepreneur may negatively affect her ability to signal the project's quality. When a separating equilibrium exists, surprisingly, the outside investor who is uninformed benefits from the information asymmetry. This is because information asymmetry moderates insider's investment and alleviate the competition between the investors.

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Appendix A

Omitted Theoretical Proofs for Chapter 1

Lemma 5 *The expected leftover inventory and the probability of overstocking are as follows.*

$$\mathbb{E}[Q-\mathbf{D}]^{+} = \frac{\mu\Delta}{12}\Big(3+6\lambda\xi+\Big(1+\lambda\left(-2+\lambda+3\lambda\xi^{2}\right)\Big)\Big) \tag{A.1}$$

$$\mathbb{P}(Q\geq\mathbf{D}) = \frac{1+\lambda\xi}{2} \tag{A.2}$$

Proof of Lemma 5: In order to calculate the $\mathbb{E}[q_b-\mathbf{D}]^{+}$ and $\mathbb{P}(q_b\geq\mathbf{D})$, we define d_L and d_H as follows.

$$d_L = \lambda\,\mu\,(1+\xi\,\Delta\,)+(1-\lambda)\,\mu\,(1-\Delta)=\mu\Big(1+(\lambda\,\xi-(1-\lambda))\Delta\Big),$$

$$d_H = \lambda\,\mu\,(1+\xi\,\Delta\,)+(1-\lambda)\,\mu\,(1+\Delta)=\mu\Big(1+(\lambda\,\xi+(1-\lambda))\Delta\Big),$$

where d_L and d_H are the lowest and highest possible order quantity from a boundedly rational retailer, respectively. Thus, the retailer's order quantity q_b follows a uniform distribution $\mathcal{U}[\mu\,\big(1+(\lambda\,\xi-(1-\lambda))\Delta\big), \mu\,\big(1+(\lambda\,\xi+(1-\lambda))\Delta\big)]$, and the density function is $\frac{1}{2\,\mu\,\Delta\,(1-\lambda)}$. Since the demand is independent with the retailer's order quantity, the joint density function of q_b and $\mathbf{D}$ is $\frac{1}{4\,\mu^2\,\Delta^2\,(1-\lambda)}$. Then, we have

$$
\begin{aligned}
\mathbb{E}[q_b - \mathbf{D}]^+ &= \int_{d_L}^{d_H} \int_{\mu\,(1-\Delta)}^{q_b} \frac{q_b - D}{4\,\mu^2\,\Delta^2\,(1-\lambda)} dD\; dq_b \\
&= \frac{\mu\,\Delta}{12}\Big(3 + 6\lambda\xi + \Big(1 + \lambda\left(-2 + \lambda + 3\lambda\xi^2\right)\Big)\Big) \\
\mathbb{P}(q_b \geq \mathbf{D}) &= \int_{d_L}^{d_H} \int_{\mu\,(1-\Delta)}^{q_b} \frac{1}{4\,\mu^2\,\Delta^2\,(1-\lambda)} dD\; dq_b = \frac{1+\lambda\,\xi}{2}
\end{aligned}
$$

This completes the proof.

Before proceeding with other theoretical proofs, we present an intermediate results which we use later in the proof of Lemma 1.

Lemma 6 *For any contract* $(\hat{w}, \hat{u}, \hat{v}) \in \mathcal{F}$*, there exists an equivalent contract*

$$
(w, u, v) = \begin{cases} (\hat{w} - \hat{u}, 0, \hat{v} - \hat{u}) & \textit{if} \quad \hat{v} - \hat{u} \geq 0, \\ (\hat{w} - \hat{v}, -(\hat{v} - \hat{u}), 0) & \textit{if} \quad \hat{v} - \hat{u} < 0, \end{cases}
$$

with $0 \leq w \leq p$*,* $u \geq 0$*, and* $v \geq 0$*, under which the retailer's order quantity and all profits are the same.*

Proof of Lemma 6: First note that two contracts (w, u, v) and $(\hat{w}, \hat{u}, \hat{v})$ are equivalent if both induce the same critical ratio (and the probability of stockout) and profits for the retailer, the supplier, and the supply chain. Since all profits and the critical ratio can be presented in the term of retailer's underage and overage costs, i.e.

$$
\begin{aligned}
\Pi_c &= (p-c)\,\mathbb{E}\big[Q\big] - p\,\mathbb{E}\big[Q-\mathbf{D}\big]^+ \\
\Pi_r &= c_u^r\,\mathbb{E}\big[Q\big] - (c_u^r + c_o^r)\,\mathbb{E}\big[Q-\mathbf{D}\big]^+, \ \text{ and} \\
\Pi_s &= (p-c-c_u^r)\,\mathbb{E}\big[Q\big] - (p-c_u^r-c_o^r)\,\mathbb{E}\big[Q-\mathbf{D}\big]^+.
\end{aligned}
$$

To prove that contracts (w,u,v) and $(\hat{w},\hat{u},\hat{v})$ are equivalent, it is enough to show that both contracts have the same underage and overage costs. It is now easy to check that both contracts (w,u,v) and $(\hat{w},\hat{u},\hat{v})$ correspond to the same underage and overage costs.

Assume $\hat{v}-\hat{u}<0$, then we have

$$
c_u^r(\hat{w},\hat{u},\hat{v}) = p-\hat{w}+\hat{u} = p-(\hat{w}-\hat{v})-(\hat{v}-\hat{u}) = p-w+u = c_u^r(w,u,v),
$$

and

$$
c_o^r(\hat{w},\hat{u},\hat{v}) = \hat{w}-\hat{v} = (\hat{w}-\hat{v})-0 = w-v = c_o^r(w,u,v),
$$

which implies that both contracts (w,u,v) and $(\hat{w},\hat{u},\hat{v})$ induce the same overage and underage costs. Also, note that the contract (w,u,v) is a feasible contract. Since $\hat{u}\geq\hat{v}$, we have $0\leq\hat{w}-\hat{v}\leq p$ and $0\leq-(\hat{v}-\hat{u})\leq w$, therefore $(w,u,v)\in\mathcal{F}$. This completes the proof for the first case. The case of $\hat{v}-\hat{u}\geq 0$ can be proven by following a similar approach.

Proof of Lemma 1: Let $d\equiv u-v$ and $w_u\equiv w-u$ and applying Lemma 6, without lose of generality, the contract (w,u,v) can be written as (w_u,d). In what follows we prove the results in three steps.

Step (i). We first show that $\Pi_s(w_u, d)$ and $\Pi_c(w_u, d)$ are concave in w_u, and $\Pi_r(w_u, d)$ is convex in w_u for a given d. By Lemma 5, substituting the values of $\mathbb{E}[q_b - \mathbf{D}]^+$ and $\mathbb{P}(q_b \geq \mathbf{D})$ into the profit functions, we obtain

$$\Pi_s(w_u, d) = \mu(-(c-w))(\Delta\lambda\psi + 1) - \frac{1}{12} d\Delta\mu(\lambda(\lambda + 3\psi(\lambda X + 2) - 2) + 4), \tag{A.3}$$

$$\Pi_r(w_u, d) = \frac{1}{12}\Delta\mu(d-p)(\lambda(\lambda + 3\psi(\lambda\psi + 2) - 2) + 4) + \mu(p-w)(\Delta\lambda\psi + 1), \tag{A.4}$$

$$\Pi_c(w_u, d) = \frac{\mu}{12}\Big(12 - 12\,c\,(\Delta\,\lambda\,\xi + 1) - p\,\big(\Delta\,(\lambda\,(\lambda + 3\,\xi\,(\lambda\,\xi - 2) - 2) + 4)\big)\Big) \tag{A.5}$$

Taking the second derivatives of profit functions with respect to w, we get

$$\frac{\partial^2}{\partial\, w^2}\,\Pi_s(w_u, d) = -\frac{2\,\Delta\,\lambda\,\mu\,(d\,(\lambda - 2) + 2\,p)}{(p-d)^2} < 0, \tag{A.6}$$

$$\frac{\partial^2}{\partial\, w^2}\,\Pi_r(w_u, d) = \frac{2\,\Delta\,(2-\lambda)\,\lambda\,\mu}{p-d} > 0, \tag{A.7}$$

$$\frac{\partial^2}{\partial\, w^2}\,\Pi_c(w_u, d) = -\frac{2\,\Delta\,\lambda^2\,\mu\,p}{(p-d)^2} < 0. \tag{A.8}$$

Hence we proved $\Pi_s(w_u, d)$ and $\Pi_c(w_u, d)$ are concave and $\Pi_r(w_u, d)$ is convex in w_u for fixed d.

Step (ii). We show that there is a unique $w_\alpha(d)$ that solves $\Pi_r(w_\alpha(d), d) = \alpha$ for a fixed $\alpha > 0$ and d. This also implies that the set of wholesale price w that satisfies $\Pi_r(w_\alpha(d), d) \geq \alpha$ is $[0, w_\alpha(d)]$. Without loss of generality, we assume that the maximum supply chain profit is no less than than the retailer's outside option, i.e. $\mu\left(-12c^2\Delta + 12c(\Delta+1)p + p^2\left(\Delta(\lambda-1)^2 - 12\right)\right)/12p \geq \alpha$; otherwise, the problem $\mathcal{P}_s$ is infeasible.

The proof uses *Intermediate Value Theorem* and the fact that $\Pi_r(w_u, d)$ is strictly convex with respect to w_u. On the one hand, for any $d > 0$, we have $\lim_{(w_u,d)\to(0,d)} \Pi_r(w_u, d) \geq \alpha$ and, on the other hand, $\lim_{(w_u,d)\to(p,d)} \Pi_r(w_u, d) \leq \lim_{(w_u,d)\to(p,p)} \Pi_r(w_u, d) = 0 < \alpha$.

In addition, as will be shown in the proof of Lemma 2 (i), $\frac{\partial}{\partial w}\Pi_r(w_u, d) < 0$. Hence, there exists a unique solution to the equation $\Pi_r(w_u, d) = \alpha$.

Step (iii). Lastly, we prove that the constraint $\Pi_r(w_u, d) \geq \alpha$ is binding at the optimal solution of $\mathcal{P}_{sw}$. Since $\Pi_s(w_u, d)$ is strictly concave with respect to w, the optimal solution of the problem $\mathcal{P}_{sw}$ is either $w^* = w_\alpha(d)$ (a corner solution), or the interior solution $w^* = \hat{w}(d)$ that satisfies the first order condition $\frac{\partial}{\partial w}\Pi_s(w_u, d) = 0$, i.e.

$$\hat{w}(d) = \frac{\Delta\lambda(p-d)(2(c+d)+p) + d\Delta\lambda^2(d+p) + (p-d)^2}{2\Delta\lambda(2p - d(2-\lambda))}. \tag{A.9}$$

Next, we show that the solutions of the problem $\mathcal{P}_{sw}$ cannot be $w = \hat{w}(d)$. Without loss of generality, we assume that $\Pi_r(\hat{w}(d), d) > \alpha$. Otherwise, if $\Pi_r(\hat{w}(d), d) \leq \alpha$, then $w^* = w_\alpha(d)$. Substituting $w = \hat{w}(d)$ into the supplier's profit function, we get

$$\begin{aligned}\Pi_s(\hat{w}(d), d) \quad = \quad & \frac{1}{12\Delta\lambda(2p - d(2-\lambda))}\Big[\mu\left(d^2\left(3 - \Delta\left(\lambda-2\right)\lambda\left(\Delta\left(\left(\lambda-2\right)\lambda+4\right)-6\right)\right) + \right.\\ & 2\,d\left(p\left(\Delta\,\lambda\left(3(\lambda+1) - \Delta\left(\left(\lambda-2\right)\lambda+4\right)\right)-3\right) - 6\,c\,\Delta\left(\lambda-1\right)\lambda\right) + \\ & \left. 3\left(\Delta\,\lambda\left(p-2\,c\right)+p\right)^2\right)\Big].\end{aligned}$$

It follows that

$$\frac{\partial^2}{\partial d^2}\Pi_s(\hat{w}(d), d) = \frac{\lambda\,\mu\left(p\left(\Delta\left(\lambda-2\right)-1\right) - 2\,c\,\Delta\left(\lambda-2\right)\right)^2}{2\,\Delta\left(2\,p - d(2-\lambda)\right)^3} \geq 0. \tag{A.10}$$

If the constraint $\Pi_r(w, d) \geq \alpha$ is not binding, since $\Pi_s(\hat{w}(d), d)$ is convex with respect to d, either $d = p$ or $d = 0$ is the optimal solution.
If $d = p$, we get $\hat{w}(d) = p$ which also yields in an infeasible solution because $\Pi_r(p, p) = 0 < \alpha$. Next we show that the contract $(\hat{w}(0), 0)$ cannot be optimal. If $\frac{\partial}{\partial d}\Pi_s(\hat{w}(d), d)|_{d=0} >$

0, the statement is trivially true. Now consider the case $\frac{\partial}{\partial d}\Pi_s(\hat{w}(d),d)|_{d=0} < 0$. Since

$$\lim_{d\to p}\Pi_s(\hat{w}(d),d) = \Pi_c(p,p) = max_\xi\Pi_c(\xi) > max_\xi\Pi_c(\xi) - \alpha > \Pi_s(\hat{w}(0),0),$$

there exists $0 < d' < p$ such that $\Pi_s(\hat{w}(0),0) = \Pi_s\hat{w}(d'),d')$. Since

$$\frac{\partial}{\partial d}\Pi_c(\hat{w}(d),d) = \frac{\lambda\,\mu\,(p-d)\left(p\,(\Delta\,(\lambda-2)-1)-2\,c\,\Delta\,(\lambda-2)\right)^2}{2\Delta\left(d\,(\lambda-2)+2\,p\right)^3} > 0,$$

the supply chain profit increases in d. We have

$$\begin{aligned}\Pi_r(\hat{w}(0),0)+\Pi_s(\hat{w}(0),0) &= \Pi_c(\hat{w}(0),0)\\ &< \Pi_c(\hat{w}(d'),d')\\ &= \Pi_r(\hat{w}(d'),d')+\Pi_s(\hat{w}(d'),d').\end{aligned}$$

Since $\Pi_s(\hat{w}(0),0) = \Pi_s(\hat{w}(d'),d')$, it follows that $\Pi_r(\hat{w}(d'),d') > \Pi_r(\hat{w}(0),0) \geq \alpha$ (See Figure A.1 for illustration). Therefore, the supplier can improve his profit by increasing d higher than d' while keeping the retailer's participation constraint satisfied. This suggests that $(\hat{w}(0),0)$ is not the optimal solution and the proof is complete.

Proof of Lemma 2: Similar to the proof of Lemma 1, we use change of variable $d \equiv u - v$ and $w_u \equiv w - u$. The proof consists of three steps. Step (i) we show that $\frac{\partial}{\partial w}\Pi_r(w_u,\, d) < 0$ for any given d; in Step (ii) we study how $\frac{\mathrm{d}}{\mathrm{d}\,d}\,\xi(w_\alpha(d),d) > 0$ behaves when $\lambda < 1$; and finally in Step (iii) we show that $\frac{\mathrm{d}}{\mathrm{d}\,d}\,\xi(w_\alpha(d),d) = 0$ for $\lambda = 1$.

Step (i). Note that the values of $\mathbb{E}[q_b - \mathbf{D}]^+$ and $\mathbb{E}[q_b]$ depend on (w,d) only through the intermediate variable $\xi = \frac{p-2w_u+d}{p-d}$. Let $L(\xi) \equiv \mathbb{E}[q_b - \mathbf{D}]^+$ and $Q(\xi) \equiv \mathbb{E}[q_b]$. Hence, $\Pi_r(w_u,d) = (p-w_u)Q(\xi) - (p-d)L(\xi)$, and

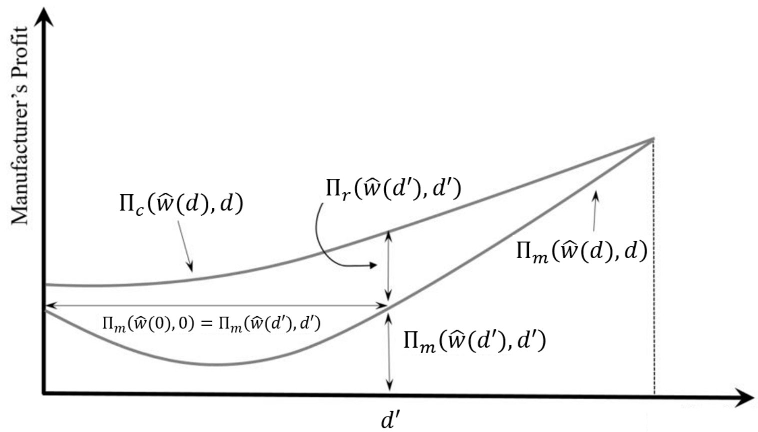


Figure A.1: The Solution $(\hat{w}(0)\,, 0)$ Cannot Be Optimal

$$\begin{aligned}\frac{\partial}{\partial\, w_u}\Pi_r(w_u, d) &= \frac{\partial\,\xi}{\partial\, w}\cdot\frac{\partial\, Q}{\partial\,\xi}\,(p - w_u) - \frac{\partial\,\xi}{\partial\, w_u}\cdot\frac{\partial\, L}{\partial\,\xi}\,(p-d) - Q \\ &= \frac{\mu\Big(d\big(\Delta\,\lambda\,(\lambda-2)+1\big)+\Delta\,\lambda\,(\lambda-2)(p-2w_u)-p\Big)}{p-d}.\end{aligned}$$

Hence, $\frac{\partial}{\partial\, w_u}\Pi_r(w_u, d) < 0$ if and only if $f(w_u, d) = d\big(\Delta\,\lambda\,(\lambda-2)+1\big)+\Delta\,\lambda\,(\lambda-2)(p-2w_u)-p < 0$. Since $\frac{\partial}{\partial\, w_u}f(w_u, d) = 2\Delta\,\lambda(2-\lambda) > 0$, the maximum of $f(w_u, d)$ is archived at $w_u = p$. Observing that $f(p, d) = -(p-d)(1-\Delta\,\lambda(2-\lambda)) < 0$. We obtain that $\frac{\partial}{\partial\, w_u}\Pi_r(w_u, d) < 0$.

Step (ii). From Lemma 1, the constraint $\Pi_r(w_u, d) \geq \alpha$ is binding, i.e. $\Pi_r(w_u, d) = \alpha$. Applying the *Implicit Function Theorem* to $\Pi_r(w_u, d)$, we get

$$\frac{\partial}{\partial\, d}w_\alpha(d) = -\frac{\frac{\partial}{\partial\, d}\Pi_r(w_u, d)}{\frac{\partial}{\partial\, w_u}\Pi_r(w_u, d)}. \tag{A.11}$$

where $\frac{\partial\,\xi}{\partial\, d} = \frac{2(p-w)}{(p-d)^2} > 0$ and $\frac{\partial\,\xi}{\partial\, w} = -\frac{2}{p-d} < 0$. Now we consider two cases when $d \geq 0$ and $d > 0$ separately.

Case 1. Assume $\hat{v} - \hat{u} \geq 0$. From Lemma 6 we know that when $d = v - u \geq 0$, we can consider the equivalent contract $(w_u, d) = (w - u, v - u)$ which gives us

$$\Pi_r(w_u, d) = (p - w_u)Q(\xi) - (p - d)L(\xi)$$

with the partial derivatives

$$\begin{aligned} \frac{\partial}{\partial w_u}\Pi_r(w_u, d) &= \frac{\partial \xi}{\partial w_u} \cdot \frac{\partial Q}{\partial \xi}(p - w_u) - \frac{\partial \xi}{\partial w_u} \cdot \frac{\partial L}{\partial \xi}(p - d) - Q, \text{ and} &\text{(A.12)} \\ \frac{\partial}{\partial d}\Pi_r(w_u, d) &= \frac{\partial \xi}{\partial d} \cdot \frac{\partial Q}{\partial \xi}(p - w_u) - \frac{\partial \xi}{\partial d} \cdot \frac{\partial L}{\partial \xi}(p - d) + L. &\text{(A.13)} \end{aligned}$$

As a result, we have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\,u}\,\xi(w_\alpha(d), d) &= \frac{\partial \xi}{\partial u} + \frac{\partial}{\partial d} w_\alpha(d) \cdot \frac{\partial \xi}{\partial w_u} \\ &= \frac{\partial \xi}{\partial d} - \frac{\frac{\partial}{\partial d}\Pi_r(w_u, d)}{\frac{\partial}{\partial w_u}\Pi_r(w_u, d)} \cdot \frac{\partial \xi}{\partial w_u} \\ &= \frac{\partial \xi}{\partial d} - \frac{\frac{\partial \xi}{\partial d} \cdot \frac{\partial Q}{\partial \xi}(p - w_u) - \frac{\partial \xi}{\partial d} \cdot \frac{\partial L}{\partial \xi}(p - d) + L}{\frac{\partial \xi}{\partial w_u} \cdot \frac{\partial Q}{\partial \xi}(p - w_u) - \frac{\partial \xi}{\partial w_u} \cdot \frac{\partial L}{\partial \xi}(p - d) - Q} \cdot \frac{\partial \xi}{\partial w_u} \\ &= \frac{\partial \xi}{\partial d}\left(1 - \frac{\frac{\partial Q}{\partial \xi}(p - w_u) - \frac{\partial L}{\partial \xi}(p - d) + L(\frac{\partial \xi}{\partial w_u}/\frac{\partial \xi}{\partial d})}{\frac{\partial Q}{\partial \xi}(p - w_u) - \frac{\partial L}{\partial \xi}(p - d) - Q}\right) \\ &= \frac{\partial \xi}{\partial d}\left(\frac{-Q - L(\frac{\partial \xi}{\partial w_u}/\frac{\partial \xi}{\partial d})}{\frac{\partial}{\partial d}\Pi_r(w_u, d)}\right) \\ \text{(equal in sign)} \quad &\overset{\text{sgn}}{=} Q + L\left(\frac{\partial \xi}{\partial w_u}/\frac{\partial \xi}{\partial d}\right) \\ &= Q + L\left(\frac{-(p - d)}{p - w_u}\right) \\ \text{(equal in sign)} \quad &\overset{\text{sgn}}{=} (p - w_u)Q - (p - d)L \\ &= \Pi_r(w_u, d) \geq \alpha > 0. \end{aligned}$$

The first equal in sign holds because, as shown in step (i), we have $\frac{\partial}{\partial w}\Pi_r(w_u, d) < 0$

and $\frac{\partial \xi}{\partial d} = \frac{2(p-w_u)}{(p-d)^2} > 0$. The second equal in sign holds because $p - w_u = c_u^r > 0$. This completes the proof of the claim $\frac{\mathrm{d}}{\mathrm{d}\, d}\, \xi(w_\alpha(d), d) > 0$.

Case 2. Assume $d = v - u < 0$. From Lemma 6, we can consider the equivalent contract $(w_u, d) = (w - v, -(v - u))$. We now write the retailer's profit as

$$\Pi_r(w_u, d) = (p - w_u)Q(\xi) - (p + d)L(\xi)$$

with the partial derivatives

$$\begin{aligned}
\frac{\partial}{\partial\, w_u}\Pi_r(w_u, d) &= \frac{\partial\, \xi}{\partial\, w_u} \cdot \frac{\partial\, Q}{\partial\, \xi}\,(p - w_u) - \frac{\partial\, \xi}{\partial\, w_u} \cdot \frac{\partial\, L}{\partial\, \xi}\,(p + d) - Q, \text{ and} && \text{(A.14)}\\
\frac{\partial}{\partial\, d}\Pi_r(w_u, d) &= \frac{\partial\, \xi}{\partial\, d} \cdot \frac{\partial\, Q}{\partial\, \xi}\,(p - w_u) - \frac{\partial\, \xi}{\partial\, d} \cdot \frac{\partial\, L}{\partial\, \xi}\,(p + d) - L. && \text{(A.15)}
\end{aligned}$$

As a result, we have

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\, d}\, \xi(w_\alpha(d), d) &= \frac{\partial\, \xi}{\partial\, d} + \frac{\partial}{\partial\, d} w_\alpha(d) \cdot \frac{\partial\, \xi}{\partial\, w_u} \\
&= \frac{\partial\, \xi}{\partial\, d}\left(\frac{-Q + L(\frac{\partial\, \xi}{\partial\, w_u}/\frac{\partial\, \xi}{\partial\, d})}{\frac{\partial}{\partial\, d}\Pi_r(w_u, d)}\right) \\
\text{(equal in sign)} &\overset{\text{sgn}}{=} -Q + L\left(\frac{\partial\, \xi}{\partial\, w_u}/\frac{\partial\, \xi}{\partial\, d}\right) \\
&= -Q + L\left(\frac{p + d}{p - w_u}\right) \\
\text{(equal in sign)} &\overset{\text{sgn}}{=} -(p - w_u)Q + (p + d)L \\
&= -\Pi_r(w_u, d) \leq -\alpha < 0.
\end{aligned}$$

The first equal in sign holds because, as shown in step (i), we have $\frac{\partial}{\partial\, w}\Pi_r(w_u, d) < 0$ and $\frac{\partial \xi}{\partial d} = -\frac{p-w_u}{(p+d)^2} > 0$. The second equal in sign holds because $p - w_u = c_u^r > 0$. This completes the proof of the claim $\frac{\mathrm{d}}{\mathrm{d}\, d}\, \xi(w_\alpha(d), d) > 0$.

Step (iii). Lastly, we show that $\xi(w_\alpha(d), d)$ is a constant when the retailer is fully rational, i.e. $\lambda = 1$. The profit functions in this case can be simplified to

$$\begin{aligned}
\pi_s(w_u, d) &= \mu(w_u - c)\left(1 - \frac{\Delta(d + p - 2w_u)}{d - p}\right) - \frac{d\Delta\mu(p - w_u)^2}{(d - p)^2}, \\
\pi_r(w_u, d) &= -\frac{\mu(p - w_u)(d(\Delta - 1) + p - \Delta w_u)}{d - p}, \text{ and} \\
\pi_c(w_u, d) &= \frac{\mu\left(c(d - p)(\Delta(d + p - 2w_u) - d + p) + p\left((d - p)^2 - \Delta(d - w_u)^2\right)\right)}{(d - p)^2},
\end{aligned}$$

where $w_u = w - u$ and $d = v - u$. From Lemma 1, the constraint in problem $\mathcal{P}_s$ is binding and, hence, the supplier maximizes his own profit by maximizing the supply chain profit. For fixed u and d, the optimal wholesale price for the supply chain is $w_u(d) = d + c - \frac{c\,d}{p}$. Different values of d corresponds to different share of supply chain profit between the supplier and the retailer. Since

$$\begin{aligned}
\frac{\partial}{\partial d}\pi_s(w_u, d) &= \frac{\mu\,(p - c)(p - c\,\Delta)}{p^2} > 0 \\
\frac{\partial}{\partial d}\pi_r(w_u, d) &= -\frac{\mu\,(p - c)(p - c\,\Delta)}{p^2} < 0,
\end{aligned}$$

the supplier's profit is increasing in d and the retailer's profit is decreasing in d. Therefore, the optimal contract that the supplier chooses is a nonlinear contract with the largest possible d which makes the retailer's participation constraint binding. That is, the contract that the supplier offers to the retailer with outside option α is

$$(w_u, d) = \left(p + u - \frac{\alpha\,p}{\mu(p - c\,\Delta)},\ p - \frac{\alpha\,p^2}{\mu(p - c)(p - c\,\Delta)}\right).$$

Hence, $\xi(w_\alpha(d), d) = 1 - \frac{2c}{p}$ which is constant.

Lemma 7 *Let* $\underline{\xi} = 1 - 2\frac{w_\alpha(0,0)}{p}$ *and* $\bar{\xi} = 1$*. The optimal solution* $\xi^* \in [\underline{\xi}, \bar{\xi}]$ *of the problem* (1.4.2) *satisfies*

$$\xi^* = \begin{cases} \bar{\xi} & \text{if} \quad \bar{\xi} \leq \frac{1}{\lambda}\left(\frac{p-2c}{p}\right), \\ \frac{1}{\lambda}\left(\frac{p-2c}{p}\right) & \text{if} \quad \underline{\xi} < \frac{1}{\lambda}\left(\frac{p-2c}{p}\right) < \bar{\xi}, \\ \underline{\xi} & \text{if} \quad \frac{1}{\lambda}\left(\frac{p-2c}{p}\right) \leq \underline{\xi}. \end{cases} \tag{A.16}$$

Proof of Lemma 7: As suggested by (1.10),

$$\Pi_c(\xi) = -\frac{\mu}{12}\Big(12c(\Delta\lambda\xi + 1) + p\Big(\Delta\big(\lambda(\lambda + 3\,\xi(\lambda\xi - 2) - 2) + 4\big) - 12\Big)\Big). \tag{A.17}$$

Since $\frac{\partial^2}{\partial\xi^2}\Pi_c(\xi) = -\frac{\mu\lambda p}{4} < 0$, $\Pi_c(\xi)$ is concave with respect to ξ. Solving the first order condition $\frac{\partial}{\partial\xi}\Pi_c(\xi) = 0$, we get $\xi = \frac{1}{\lambda}\left(\frac{p-2c}{p}\right)$.

The optimal solution of (1.4.2) is either the interior solution that $\xi^* = \frac{1}{\lambda}\left(\frac{p-2c}{p}\right)$ or the boundary solution $\xi^* = \bar{\xi}$ or $\underline{\xi}$. Specifically, the optimal solution ξ^* is $\frac{1}{\lambda}\left(\frac{p-2c}{p}\right)$ if $\frac{1}{\lambda}\left(\frac{p-2c}{p}\right) \in [\underline{\xi}, \bar{\xi}]$. Otherwise, if $\frac{1}{\lambda}\left(\frac{p-2c}{p}\right) > \bar{\xi}$, the profit function $\Pi_c(\xi)$ is increasing on $[\underline{\xi}, \bar{\xi}]$, and the optimal solution is $\xi^* = \bar{\xi}$. Similarly, if $\underline{\xi} > \frac{1}{\lambda}\left(\frac{p-2c}{p}\right)$, then $\Pi_c(\xi)$ is decreasing on $[\underline{\xi}, \bar{\xi}]$, and the optimal solution is $\xi^* = \underline{\xi}$.

Proof of Proposition 1: From Lemma 1, the constraint $\Pi_r(w_u, d) \geq \alpha$ is binding. Hence, the problem $\mathcal{P}_{sw}$ is equivalent to

$$\max_d\{\Pi_c(w_\alpha(d), d) \quad \text{s.t.} \quad (w_\alpha(d), d) \in \mathcal{F}\}.$$

Note that $\Pi_c(w_\alpha(d), d)$ can be written as $\Pi_c(\xi(w_\alpha(d), d))$, where

$$\xi(w_\alpha(d), d) = \frac{p - 2w_\alpha(d) + d}{p - d}$$

. By Lemma 7, if

$$\underline{\xi} > \frac{1}{\lambda}\left(\frac{p - 2\,c}{p}\right), \tag{A.18}$$

the optimal solution is $\xi^* = \underline{\xi}$. Since from Lemma 1 we know that $\xi(w_\alpha(d), d)$ is increasing in d when $d = v - u \geq 0$ and is decreasing when $d = v - u < 0$, the minimum value of $\underline{\xi} = \xi(w_\alpha(d), d)$ is attainted at $d = 0$. That is, the optimal choice of contract is a wholesale contract.

Now substituting $\underline{\xi} = \xi(w_\alpha(0), 0) = 1 - 2\,\frac{w_\alpha(0)}{p}$ into (A.18), we get the condition

$$w_\alpha(0) \leq \frac{1}{2}\left(p - \frac{1}{\lambda}\,(p - 2\,c)\right)$$

under which the no protection contract $(w_u, d) = (w_\alpha(0), 0)$ dominates all other nonlinear contracts with $d > 0$.

Proof of Corollary 1: The proof is shown by contradiction. Assume that $p - 2c > 0$. By Theorem 1, a no-protection contract is optimal if the inequality (1.11) is satisfied. Therefore, for any λ, we have

$$0 \quad \leq \quad w_\alpha(0) \leq \frac{1}{2}\left(p - \frac{1}{\lambda}\,(p - 2\,c)\right),$$

which can be simplified to $p - 2c \leq 0$, a contradiction to the assumption that $p - 2c > 0$.

Proof of Corollary 2: As a wholesale price cannot exceed a market price p, the inequality (1.11) is always satisfied if $\frac{1}{2}\left(p-\frac{1}{\lambda}\left(p-2\,c\right)\right)\geq p$. This condition can be simplified to $\frac{2\,c}{p}>1+\lambda$.

Proof of Proposition 2: We first, in step (i), show the existence of $\bar{\lambda}$ and then, in step (ii), we show that $\bar{\lambda}$ is non-decreasing in c.

Step (i). We first show that there exist $\bar{\lambda}$ such that $w_\alpha(0)=\frac{1}{2}\left(p-\frac{1}{\lambda}(p-2\,c)\right)$. Note that $w_\alpha(d)$ is also a function of other parameters including λ. We show that $w_\alpha(0)$ is increasing in λ. By the *implicit function theorem* we have $\frac{\partial}{\partial\lambda}w_\alpha(d)=-\frac{\frac{\partial}{\partial\lambda}\Pi_r(w_u,d)}{\frac{\partial}{\partial w_u}\Pi_r(w_u,d)}$ and $\frac{\partial}{\partial w_u}\Pi_r(w_u,d)<0$ by the proof of Lemma 2. In order to show that $\frac{\partial}{\partial\lambda}w_\alpha(0)\geq 0$, we only need to show that $\frac{\partial}{\partial\lambda}\Pi_r(w_u,0)\geq 0$. Since

$$\frac{\partial}{\partial\lambda}\Pi_r(w_u,d)=\frac{\mu\,\Delta\left(3\,\xi(p-2w_u)-p\left(\lambda+3\lambda\,\xi^2-1\right)\right)}{6},$$

expressions $\frac{\partial}{\partial\lambda}\Pi_r(w_u,d)$ and $3\,\xi\,(p-2w_u)+p\left(1-\lambda-3\,\lambda\,\xi^2\right)$ are equal in sign. By substituting $\xi(w,0)=\frac{p-2w_u}{p}$ into $3\,\xi\,(p-2w_u)+p\left(1-\lambda-3\,\lambda\,\xi^2\right)$, we have

$$\frac{4(1-\lambda)}{p}\left(p^2-3pw_u+3w_u^2\right)\geq 0.$$

The above inequality holds because $\min_{0\leq w_u\leq p}\{p^2-3\,p\,w_u+3w_u^2\}=\frac{1}{4}p^2\geq 0$ and, as a result, $\frac{\partial}{\partial\lambda}w_\alpha(0)$, is increasing in λ with $0\leq w_\alpha(0)\leq p$.

On the other hand, since the necessary condition for the no-protection contracts to be optimal is $p-2c<0$, the right hand side of (1.11), i.e., $\frac{1}{2}\left(p-\frac{1}{\lambda}\left(p-2\,c\right)\right)$, is decreasing in λ. When $\lambda\to 0$, we have $\lim_{\lambda\to 0}\frac{1}{2}\left(p-\frac{1}{\lambda}\left(p-2\,c\right)\right)\to\infty>w_\alpha(0)$, and when $\lambda=1$, $\frac{1}{2}\left(p-\frac{1}{\lambda}\left(p-2\,c\right)\right)=c\leq w_\alpha(0)$. Therefore there exists $\bar{\lambda}\in(0,1]$ such that $w_\alpha(0)=\frac{1}{2}\left(p-\frac{1}{\lambda}\left(p-2\,c\right)\right)$. In addition, the inequality (1.11) is satisfied if and only if

$\lambda < \bar{\lambda}$.

Step (ii). Next, we show $\bar{\lambda}$ is non-decreasing in c. First, note that $w_\alpha(d)$ is the wholesale price that makes the retailer's participation constraint binding and does not depend on c. Hence, $w_\alpha(0)$ is constant with respect to c. Taking the derivative of both sides of (1.11) with respect to c, we have

$$\frac{\mathrm{d}w_\alpha(0)}{\mathrm{d}\bar{\lambda}}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}c} = \frac{1}{2}(\frac{1}{\bar{\lambda}^2}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}c}(p-2c)+\frac{2}{\lambda}). \tag{A.19}$$

Since $\frac{\mathrm{d}w_\alpha(0)}{\mathrm{d}\bar{\lambda}} \geq 0$ and $p-2c \leq 0$, we have $\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}c} \geq 0$. That is, $\bar{\lambda}$ increases in c.

Proof of Proposition 3: The proof consists of two steps.

Step (i). We first show the existence of $\bar{c}$. We observe that $w_\alpha(0)$ does not change with c and $c \leq w_\alpha(0) \leq p$. In addition, the right hand side of the inequality (1.11) increases in c because $\frac{\partial}{\partial c}\left[\frac{1}{2}\left(p-\frac{1}{\lambda}(p-2\,c)\right)\right] = \frac{1}{\lambda} > 0$. Since $\lim_{c\to p}\frac{1}{2}\left(p-\frac{1}{\lambda}(p-2\,c)\right) > p$ and $\lim_{c\to 0}\frac{1}{2}\left(p-\frac{1}{\lambda}(p-2\,c)\right) < 0$, there exists $\bar{c}$ such that it solves the equation $w_\alpha(0) = \frac{1}{2}\left(p-\frac{1}{\lambda}(p-2\,\bar{c})\right)$.

Step (ii). Taking the derivative of both sides of $w_\alpha(0) = \frac{1}{2}\left(p-\frac{1}{\lambda}(p-2\,\bar{c})\right)$

$$\frac{\mathrm{d}w_\alpha(0)}{\mathrm{d}\bar{c}} = \frac{1}{2}\left(\frac{1}{\lambda^2}(p-2\,c)+\frac{2}{\lambda}\frac{\mathrm{d}\bar{c}}{\mathrm{d}\lambda}\right)$$

Since $\frac{\mathrm{d}w_\alpha(0)}{\mathrm{d}\bar{c}} = 0$ and $p-2c < 0$, we have $\frac{\mathrm{d}\bar{c}}{\mathrm{d}\lambda} \geq 0$.

Proof of Proposition 4: The proof is similar to Proposition 3. First, applying the implicit function theorem to the equation $\Pi_r(w_\alpha(0),0)-\alpha=0$, we obtain

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\,\Delta} w_\alpha(0) &= -\frac{\frac{\partial}{\partial\Delta}\Pi_r(w_u,d)}{\frac{\partial}{\partial w_u}\Pi_r(w_u,d)} \\
&= -\frac{-\frac{\mu}{3\,p}\Big(p^2\,(1-\lambda)^2-3(\lambda-2)\lambda p w_u+3(\lambda-2)\lambda w_u^2\Big)}{\frac{\partial}{\partial, w_u}\Pi_r(w_u,d)} \\
&= \frac{\frac{\mu}{3\,p}\Big(p^2\,(1-\lambda)^2+3\,w_u\,\lambda(2-\lambda)\,(p-w_u)\Big)}{\frac{\partial}{\partial w_u}\Pi_r(w_u,d)} \\
&< 0.
\end{aligned}$$

Taking the derivative of both sides of $w_\alpha(0)=\frac{1}{2}\left(p-\frac{1}{\lambda}\,(p-2\,\bar{c})\right)$ with respect to λ, we have,

$$\frac{\mathrm{d}w_\alpha(0)}{\mathrm{d}\bar{\Delta}}\,\frac{\mathrm{d}\bar{\Delta}}{\mathrm{d}\lambda}=\frac{1}{2\,\lambda^2}(p-2c).$$

Since $\frac{\mathrm{d}w_\alpha(0,0)}{\mathrm{d}\bar{\Delta}}<0$ and $p-2c\le 0$, we have $\frac{\mathrm{d}\bar{\Delta}}{\mathrm{d}\lambda}\ge 0$. That is, $\bar{\Delta}$ increases in λ.

Proof of Proposition 5: Applying the implicit function theorem to the equation $\Pi_r(w_\alpha(0),0)-\alpha=0$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}\,\alpha} w_\alpha(0)=\frac{1}{\frac{\mathrm{d}}{\mathrm{d}\,w_u}\Pi_r(w_u,d)}<0.$$

Taking the derivative of both sides of $w_\alpha(0)=\frac{1}{2}\left(p-\frac{1}{\lambda}\,(p-2\,\bar{c})\right)$ with respect to λ, we have

$$\frac{\mathrm{d}w_\alpha(0)}{\mathrm{d}\alpha}\,\frac{\mathrm{d}\bar{\alpha}}{\mathrm{d}\lambda}=\frac{1}{2\,\lambda^2}(p-2c).$$

Since $\frac{\mathrm{d}w_\alpha(0)}{\mathrm{d}\alpha}<0$ and $p-2c\le 0$, we have $\frac{\mathrm{d}\bar{\alpha}}{\mathrm{d}\lambda}\ge 0$. That is, $\bar{\alpha}$ increases in λ.

Proposition 18 *Suppose the boundedly rational retailer follows the mean-anchoring heuristic and places orders according to* $q_p = \lambda\, q^* + (1-\lambda)\mu$ *where* λ *is the retailer's rationality coefficient. If the condition*

$$p + (p - 2\,c)\,\Delta\,(2-\lambda) \leq \sqrt{4\alpha\Delta(2-\lambda)\lambda p\mu^{-1} + p^2(1 - \Delta(2-\Delta)(2-\lambda)\lambda)} \qquad \text{(A.20)}$$

holds, the optimal contract for the supplier among all nonlinear contracts is a wholesale price contract with wholesale price

$$w = \frac{p(1 + \Delta(2-\lambda)\lambda) - \sqrt{4\alpha\Delta(2-\lambda)\lambda p\mu^{-1} + p^2(1 - \Delta(2-\Delta)(2-\lambda)\lambda)}}{2\Delta(2-\lambda)\lambda}. \qquad \text{(A.21)}$$

Proof of Proposition 18: Under the mean-anchoring heuristic, the retailer's profit function is according to

$$\Pi_r^p(w,b) = \frac{\mu\left(\Delta(\lambda(d+p-2w_u) - d + p)^2 + 4(p - w_u)(-\Delta\,\lambda(d+p-2w_u) + d - p)\right)}{4(d-p)}.$$

Similar to the main model in §1.3.1, a wholesale price contract is optimal if $\underline{\xi} > \frac{1}{\lambda}\left(\frac{p-2\,c}{p}\right)$, where $\underline{\xi} = \xi(w_\alpha^p(0), 0)$. This condition can be simplified to

$$w_\alpha^p(0) \leq \frac{1}{2}\left(p - \frac{1}{\lambda}\,(p - 2c)\right),$$

where $w_\alpha^p(0)$ solves $\Pi_r^p(w,0) = \alpha$. The $\Pi_r^p(w,0) = \alpha$ has two solutions:

$$w_{\alpha,1}^p(0) \quad = \quad \frac{\mu p(\delta(\lambda-2)\lambda - 1) - \sqrt{p}\sqrt{\mu(\mu p - \delta(\lambda-2)\lambda(4a + (\delta-2)\mu p))}}{2\delta(\lambda-2)\lambda\mu},$$

and

$$w_{\alpha,2}^p(0) \quad = \quad \frac{\sqrt{p}\sqrt{\mu(\mu p - \delta(\lambda-2)\lambda(4a + (\delta-2)\mu p))} + \mu p(\delta(\lambda-2)\lambda - 1)}{2\delta(\lambda-2)\lambda\mu}.$$

Since

$$\begin{aligned} w^p_{\alpha,2}(0) &= \frac{\sqrt{p}\sqrt{\mu(\mu p-\delta(\lambda-2)\lambda(4a+(\delta-2)\mu p))}+\mu p(\delta(\lambda-2)\lambda-1)}{2\delta(\lambda-2)\lambda\mu} \\ &\geq \frac{p(1+\Delta\,\lambda)}{2\,\Delta\,\lambda} > p, \end{aligned}$$

$w^p_{\alpha,2}(0)$ is not feasible. Thus, the solution is $w^p_{\alpha}(0)$. Now, Substituting $w^p_{\alpha}(0)$ into the condition $w^p_{\alpha}(0) \leq \frac{1}{2}\left(p - \frac{1}{\lambda}\left(p-2c\right)\right)$, we get

$$p+\Delta\,(p-2\,c) \leq \sqrt{p^2(1-\Delta\,\lambda\,(2-\Delta))+4\,\Delta\,\lambda\,\alpha\,p\,\mu^{-1}}.$$

Proof of Lemma 3: We adapt the Logit model in Su (2008) under a wholesale price contract for a model under a general contract (w,u,v). By Lemma 6, a contract (w,u,v) can be written in the form of a new contract with two parameters, i.e., (w_u,d), where $d=v-u$. Under a given contract (w_u,d), if the retailer orders q units, her expected profit could be written as

$$\begin{aligned} \Pi_r(q) &= (p-w_u)q-(p+d)\mathbb{E}[q-\mathbf{D}]^+ \\ &= q(p-w_u)-\frac{(p+d)((\Delta-1)\mu+q)^2}{4\Delta\mu} \end{aligned} \tag{A.22}$$

The retailer's profit in (A.22) could be written as a quadratic function of q as $\Pi_r(q) = Aq^2+Bq+C$, with coefficients $A=-\frac{p-d}{4\Delta\mu}$, $B=\frac{(1+\xi\Delta)(p-d)}{2\Delta}$, and $C=-\frac{\mu\,(1+\xi\,\Delta)^2(p-d)}{4\Delta}$, where $\xi=\frac{p+w_u}{p-d}$.

When the retailer places an order according to the Logit model, the density function of the order quantity Q is given by

$$\frac{e^{(Aq^2+Bq+C)/\beta}}{\int_{\mu(1-\Delta)}^{\mu(1+\Delta)} e^{(Az^2+Bz+C)/\beta}\,\mathrm{d}z}.$$

Denote the probability density function of the standard normal distribution by ϕ and its cumulative distribution function by Φ. The probability density function of a truncated Normal distribution with parameters $\mu_{\mathcal{N}}$ and $\sigma^2_{\mathcal{N}}$ over the interval $[\mu(1-\Delta), \mu(1+\Delta)]$ is

$$
\begin{aligned}
f_{\mathcal{N}}(q) &= \frac{\phi\left(\frac{q-\mu_{\mathcal{N}}}{\sigma_{\mathcal{N}}}\right)}{\Phi\left(\frac{\mu(1+\Delta)-\mu_{\mathcal{N}}}{\sigma_{\mathcal{N}}}\right) - \Phi\left(\frac{\mu(1-\Delta)-\mu_{\mathcal{N}}}{\sigma_{\mathcal{N}}}\right)} \\
&= \frac{e^{-(q-\mu_{\mathcal{N}})^2/2\sigma^2_{\mathcal{N}}}}{\int_{\mu(1-\Delta)}^{\mu(1+\Delta)} e^{-(z-\mu_{\mathcal{N}})^2/2\sigma^2_{\mathcal{N}}}\,\mathrm{d}z}. \qquad \text{(A.23)}
\end{aligned}
$$

Let $\mu_{\mathcal{N}} = \mu(1+\xi\,\Delta)$ and $\sigma^2_{\mathcal{N}} = \frac{2\beta\,\Delta\,\mu}{p-d}$, we obtain:

$$
\begin{aligned}
-(q-\mu_{\mathcal{N}})^2/2\sigma^2_{\mathcal{N}} &= -\frac{p-d}{4\beta\,\Delta\,\mu}\big(q-\mu(1+\xi\,\Delta)\big)^2 \\
&= -\frac{p-d}{4\beta\,\Delta\,\mu}\left(q^2 - 2\,\mu\,(1+\xi\,\Delta)\,q + \mu^2(1+\xi\,\Delta)^2\right) \\
&= \frac{Aq^2+Bq+C}{\beta}.
\end{aligned}
$$

The comparison of (A.22) and (A.23) suggests that the behavioral order quantities Q follows a truncated Normal distribution with $\mu_{\mathcal{N}} = \mu(1+\xi\,\Delta)$ and $\sigma^2_{\mathcal{N}} = \frac{2\beta\,\Delta\,\mu}{p-(v-u)}$. If we have $\tilde{q} \sim \text{Truncated}\mathcal{N}(0, \frac{2\,\beta\,\Delta\,\mu}{(p-d)(1-\lambda)^2})$ on the interval $[-\mu\,\Delta(1+\xi), \mu\,\Delta(1-\xi)]$, the behavioral model in (1.1) induces the same distribution as the Logit model by Su (2008).

Appendix B

Extra Experimental Results for Chapter 1

Extra Experimental Results

Statistical Results of Hypothesis 1

In the between treatment analysis, we first compare LOWR treatment, HPCO and HDVAR to BASE for contracts A, B and C. In the experiments, subjects make 30 consecutive decisions (denoted by $n_t \in \{1, 2, \cdots, 30\}$) in each treatment. In order to evaluate the effect of time periods, we divide the 30 periods into two halves. Let the indicator function $\mathrm{J}^t(n_t)$ takes value zero if the time period $n_t \leq 15$ and takes value one otherwise. We extend the model in Equation (1.12) by including time period effect:

$$\log\left(\frac{\pi_j^t}{\pi_D^t}\right) = \alpha_j^t + \sum_{k=2}^{4} \mathrm{I}^k(t)\left(\beta_j^t + \tau^t\,\mathrm{J}^t(n_t)\right) \qquad j \in \{A, B, C\}.$$

Table B1 shows the statistical results with and without the timer period effect. It shows that the time period effect is not significant in most of the cases. In particular, the results inferences in Section 1.5.3 does not change when taking into account the time period effect. The only case in which the time period effect is significant (at 95% level) is the comparison between LOWR with BASE for contract B. In addition, although the two segments division is the simplest way to evaluate time period effect, more granulated divisions of the time periods do not change the results.

We next analyze the choice of different type of contracts (i.e. wholesale price vs. buyback) by aggregating the data for individual contracts in the between treatment analysis. Let π_{WS} and π_{BB} be the probability of choosing a wholesale price contract and that of of choosing a buyback contract, respectively. We run a multiracial logistic regression to estimates the equation

$$\log\left(\frac{\pi_{WS}}{\pi_{BB}}\right) = \alpha^t + \sum_{k=2}^{4} \mathrm{I}^k(t)\left(\beta^t + \tau^t\,\mathrm{J}^t(n_t)\right),$$

where the superscripts $t \in \{2(\text{LOWR}), 3(\text{HPCO}), 4(\text{HDVAR})\}$ indicate the treatments compared to BASE treatment. The results of estimation is presented in Table B2. It suggests that subjects are more likely to choose wholesale price contracts in all the three treatments as deviated from BASE treatment. The effects are all significant with p value at least ≤ 0.01.

Statistical Results of Hypotheses 2

Tables B3 and Table B4 show the results of statistical tests for Hypotheses 2. Let f_j^t denote the proportion of the contract $j \in \{A, B, C\}$ is chosen in the treatment $t \in \{1(\text{BASE}), 2(\text{LOWR}), 3(\text{HPCO}), 4(\text{HDVAR})\}$. Similarly, $f_{WS}^t = f_A^t + f_C^t$ denotes the fraction of a wholesale price contract is chosen and $f_{BB}^t = f_B^t + f_D^t$ the fraction of a buyback contract is chosen in treatment t. In BASE, since a subject has the highest expected payoff under contract D, we compare contracts A, B and C with contract D. In all other treatments, we compare contracts A, B, and D with contract C as a subject has the highest expected payoff under this contract in treatments LOWR, HPCO, HDVAR.

More Statistical Results and Estimations for Section 1.6.2

In our analysis in Section 1.6.2, we consider different values of critical ration which serves as our treatments. Here we investigate if there is any treatment effect. Formally, we use a difference in difference regression model as follows.

$$\lambda_b^{cr} = \lambda_\circ + \sum_{cr \in \{.33,.25,.17\}} \tau^{cr} I^{cr} + \sum_{cr \in \{.33,.25,.17\}} \gamma^{cr} I^{cr} b + \eta b + \epsilon_b^{cr} \tag{B1}$$

where I^{cr} is a dummy variable set to 1 when the observation is from the treatment group with critical ratio equal to cr. Table B5 presents the estimation results. From Table B5, we can see that the coefficients of treatment effect are not significant and, hence, the only significant part of regression model in (B1) is its constant term $\lambda_\circ$.

Table B1: Logistic Regression Estimation for Models with and without Time Period Effect
$^{***}p-value < 0.001$; $^{**}p-value < 0.01$; $^{*}p-value < 0.05$.

	Contract		**Without τ^t**		**With τ^t**	
			Coef.	**exp(.)**	**Coef.**	**exp(.)**
LOWR vs. BASE	A	Intercept	-2.070***	–	-2.066***	–
		Treatment	1.713***	5.546	1.713***	5.546
		Time period	–	–	0.009	0.991
	B	Intercept	-1.377***	–	-1.684***	–
		Treatment	0.526*	1.693	0.526*	1.692
		Time period	–	–	0.562*	1.754
	C	Intercept	-0.216*	–	-0.249	–
		Treatment	1.021***	2.775	1.021***	2.775
		Time period	–	–	-0.069	0.933
HPCO vs. BASE	A	Intercept	-2.070***	–	-2.112***	–
		Treatment	-0.117	0.890	-0.117	0.89
		Time period	–	–	0.086	1.089
	B	Intercept	-1.377***	–	-1.585***	–
		Treatment	-0.198***	0.820	-0.199	0.820
		Time period	–	–	0.394	1.484
	C	Intercept	-0.216*	–	-0.265*	–
		Treatment	0.404**	1.498	0.404***	1.498
		Time period	–	–	0.101	1.106
HDVAR vs. BASE	A	Intercept	-2.065***	–	-2.160***	–
		Treatment	2.136***	8.463	2.137***	8.478
		Time period	–	–	0.180	1.198
	B	Intercept	-1.377***	–	-1.366***	–
		Treatment	1.365***	3.917	1.365***	3.916
		Time period	–	–	-0.022	0.978
	C	Intercept	-0.216*	–	-0.320*	–
		Treatment	1.090***	2.975	1.092***	2.981
		Time period	–	–	0.206	1.229

Table B2: Estimation Results for Multinomial Logistic Regression when Comparing with Baseline Treatment for the Type of Contract
$^{***}p-value<0.001$; $^{**}p-value<0.01$; $^{*}p-value<0.05$

Treatment		**Intercept** $\alpha^{\mathbf{t}}$	$\beta^{\mathbf{t}}$	$\tau^{\mathbf{t}}$
LOWR (t=2)	Coef.	-0.236* (0.12)	0.999*** (0.14)	-0.083 (0.14)
	Wald χ^2	4.28	53.858	0.375
	exp(.)	–	2.716	0.92
HPCO (t=3)	Coef.	-0.291** (0.11)	0.366**(0.12)	0.028 (0.12)
	Wald χ^2	6.768	9.172	0.055
	exp(.)	–	1.442	1.028
HDVAR (t=4)	Coef.	-0.382** (0.12)	0.835*** (0.14)	0.208 (0.14)
	Wald χ^2	10.617	38.507	2.385
	exp(.)	–	2.305	1.231

Table B3: P-values for Hypothesis Testing of within Treatment Analysis for BASE

	χ^2	Exact Sig.
$H_0 : f^t_{WS} = f^t_{BB}$	8.542	0.004
$H_0 : f^t_D = f^t_A$	137.755	<0.001
$H_0 : f^t_D = f^t_B$	90.25	<0.001
$H_0 : f^t_D = f^t_C$	4.301	0.043

Table B4: P-values for Hypothesis Testing of within Treatment Analysis for LOW, HPCO, and HDVAR

	LOWR ($t=2$)		**HPCO** ($t=3$)		**HDVAR**($t=4$)	
	χ^2	Sig.	χ^2	Sig.	χ^2	Sig.
$H_0 : f^t_{WS} = f^t_{BB}$	57.408	$< 10^{-3}$	1.422	0.248	50.599	$< 10^{-3}$
$H_0 : f^t_C = f^t_A$	88.424	$< 10^{-3}$	41.882	<0.001	51.955	$< 10^{-3}$
$H_0 : f^t_C = f^t_B$	135.157	$< 10^{-3}$	201.551	<0.001	51.597	$< 10^{-3}$
$H_0 : f^t_C = f^t_D$	51.955	$< 10^{-3}$	5.357	0.023	35.208	$< 10^{-3}$

Table B5: Estimation Results for Difference in Difference Regression model

	coefficient	t-test	p-value
$\lambda_\circ$	0.826 (0.040)	20.632	0.000
$\tau^{0.33}$	-0.168 (0.075)	-2.222	0.053
$\tau^{0.25}$	-0.054 (0.082)	-0.667	0.521
$\gamma^{0.33}$	0.008 (0.012)	0.696	0.504
$\gamma^{0.25}$	-0.021 (0.012)	-1.729	0.118
$\gamma^{0.17}$	-0.023 (0.012)	-1.949	0.083
η	0.860 (0.110)	0.023	0.982

Table B6: Estimation Results for Multinomial Logistic Regression when Comparing with LOWR Treatment for the Type of Contract
$^{***}p - value < 0.001$; $^{**}p - value < 0.01$; $^{*}p - value < 0.05$

Treatment		**Intercept $\alpha_{\mathbf{w}}^{\mathbf{t}}$**	**Treatment Effect $\beta_{\mathbf{w}}^{\mathbf{t}}$**
SIXC (t=5)	Coef.	-0.536*** (0.10)	-0.185(0.14)
	Wald χ^2	28.13	1.74
	exp(.)	–	0.831
NOOPT (t=6)	Coef.	-1.285*** (0.11)	1.734***(0.15)
	Wald χ^2	134.56	142.98
	exp(.)	–	5.665

Appendix C

Sample Experimental Instructions for Chapter 1

Experimental Instructions: HPCO Treatment

Supply Chain Contracting Game

Payment Method

You will be paid $4 of show-off fee and a performance-based reward which depends on your performance during the experiment. The higher profit you earn, the higher your payment would be. The experimenter will calculate your total performance-based reward at the end of the session.

Experimental Procedure and Setting

The order of events in the experiment is as follows

1. You (as a supplier) choose a contract to offer to a computerized retailer.

2. Retailer places and order with you and receives his order instantaneously and pays

you based on the wholesale price specified in the contract.

3. A random number of customers visit the retailer's store and purchase the item at market price which is known to you.

4. The retailer only can sell to the customers up to the number of items he ordered. If retailer has any leftover, you need to pay the retailer b for each leftover item.

Number of Customers or Demand

Demand is a random number at each period and independent of the realized demand in the previous periods. In the experiment, the number of customers that visit the retailer's store can be any integer number between 100 to 300 with equal probability.

Retailer's Decision and Profit

The retailer makes decisions to maximizes his own profit, however he is prone to mistakes. The retailer's order, with 75% of chance, is such that it leads to the maximum profit for him. However, with (100-75)% = 25% of chance, he makes an error and places a random order which can be any integer number between 100 to 300.

Retailer purchases each item at wholesale price specified in the contract you (as a supplier) offered to him and sells it at market price at his store. He will return the leftover items at the end of the period for the reimbursement. According to the buyback price specified in the contract, you pay him for returned items. for each item you sell to the retailer your net profit is the difference between the wholesale price and production cost. If the demand realized to be less than retailer's orders quantity, for leftover items you pay the retailer according to the buyback price. Your profit is going to be calculated as follows. Your cost of production is 10.5 per unit produced.

$$\textbf{Your Profit} = \textbf{(wholesale price} - \textbf{cost of production)} \times \textbf{order quantity} - \textbf{buyback price} \times \textbf{Maximum} \{\textbf{order quantity} - \textbf{demand, 0}\}$$

You will be given a table similar to the one in the Figure C1 below to help you make your decision. The first column is the contract code, the second and third are the wholesale and buyback prices. The retailer's order, for each contract, will be the weighted average of the number in the fourth column and a random number between 100 and 300. For example, if you choose contract C, the retailer's order is according to $0.75 \times 127 + 0.25 \times$ **random number**.

Numerical Examples

Let set market price to be 12 and production cost to be 10.5 in these examples. Please get sure you can follow the calculations.

Example 1. Suppose you offered contract D, retailer's order is 100 units, and the demand is 114, then **Your Profit** $= (\mathbf{11.25} - 10.5) \times 100 - \mathbf{0} \times 0 = \cdots$.

Example 2. Example 2. Suppose you offered contract C, retailer's order is 137 units, and the demand is 144, then **Your Profit** $= (\mathbf{11.5} - 10.5) \times 137 - \mathbf{8.25} \times 0 = \cdots$.

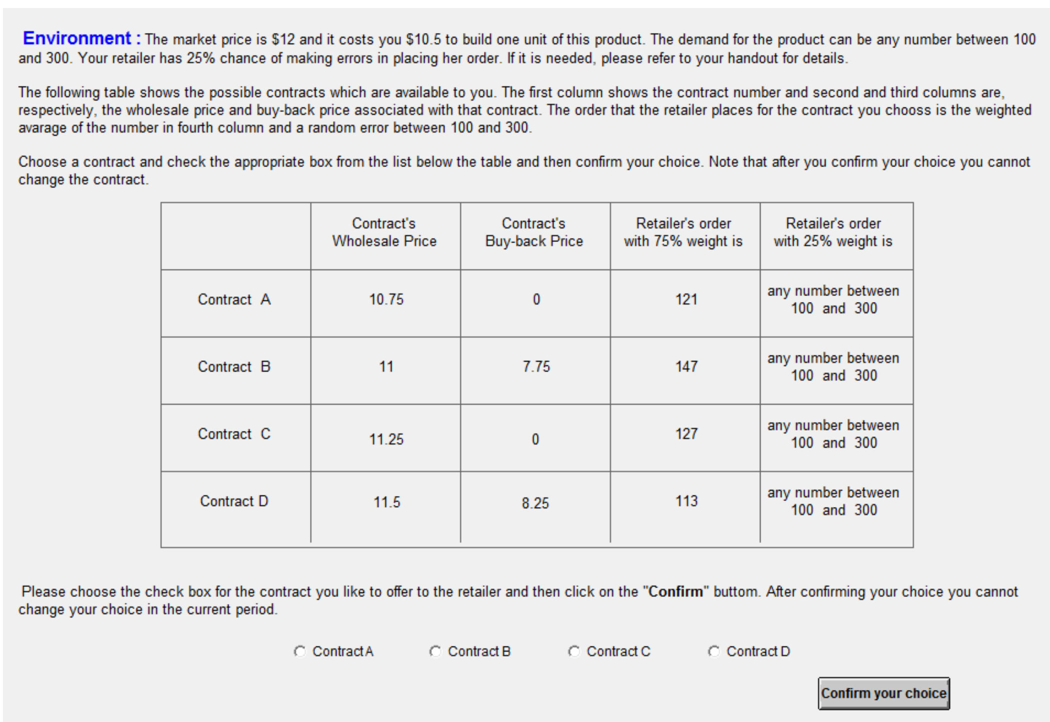
Example 3. Suppose you offered contract C, retailer's order is 210 units, and the demand is 200, then **Your Profit** $= (\mathbf{11.5} - 10.5) \times 210 - \mathbf{8.25} \times 10 = \cdots$.

Example 4. Suppose you offered contract B, retailer's order is 184 units, and the demand is 200, then **Your Profit** $= (\mathbf{11} - 10.5) \times 184 - \mathbf{7.75} \times 0 = \cdots$.

Example 5. Suppose you offered contract B, retailer's order is 230 units, and the demand is 200, then **Your Profit** $= (\mathbf{11} - 10.5) \times 230 - \mathbf{7.75} \times 30 = \cdots$.

Sample Screenshots: HPCO Treatment

The experimental game is simple and consists of one choice stage and one summary of results stage. In the first stage, subjects observe a page similar to one in Figure C1 which show the environment and parameters and ask them to choose a contract among four possible contracts. As soon as a subject chooses a contract and confirms his/her choice, he/she moves to the second stage in which he/she observes the realized demand and retailer's order as well as the subsequent reward in the current period (see Figure C2).



Environment : The market price is $12 and it costs you $10.5 to build one unit of this product. The demand for the product can be any number between 100 and 300. Your retailer has 25% chance of making errors in placing her order. If it is needed, please refer to your handout for details.

The following table shows the possible contracts which are available to you. The first column shows the contract number and second and third columns are, respectively, the wholesale price and buy-back price associated with that contract. The order that the retailer places for the contract you choos is the weighted avarage of the number in fourth column and a random error between 100 and 300.

Choose a contract and check the appropriate box from the list below the table and then confirm your choice. Note that after you confirm your choice you cannot change the contract.

	Contract's Wholesale Price	Contract's Buy-back Price	Retailer's order with 75% weight is	Retailer's order with 25% weight is
Contract A	10.75	0	121	any number between 100 and 300
Contract B	11	7.75	147	any number between 100 and 300
Contract C	11.25	0	127	any number between 100 and 300
Contract D	11.5	8.25	113	any number between 100 and 300

Please choose the check box for the contract you like to offer to the retailer and then click on the "**Confirm**" buttom. After confirming your choice you cannot change your choice in the current period.

Contract A　Contract B　Contract C　Contract D

Confirm your choice

Figure C1: A Screenshot from Choice Stage in the Laboratory Experiments

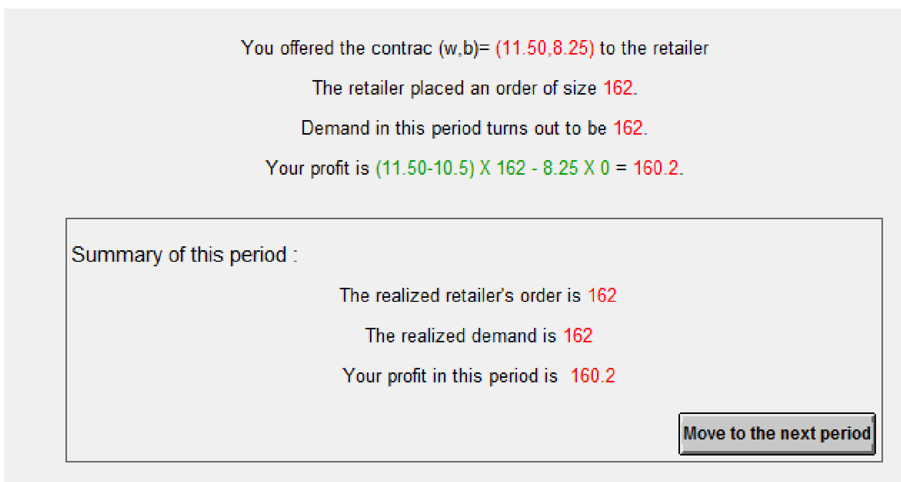


Figure C2: A Screenshot from Summary Stage

Appendix D

Omitted Theoretical Proofs for Chapter 2

Proof of Theorem 1: Consider the case without trading. By Equation (2.1), utility of a consumer of type θ from choosing plans $\mathbf{m}_1$ and $\mathbf{m}_2$ are, respectively, $u_n(L,\theta) = rL - p_1$ and $u_n(H,\theta) = r(\theta H + (1-\theta)L) - p_2$. Because $u_n(H,1) > u_n(L,1)$ and $u_n(H,0) < u_n(L,0)$, there exists a threshold θ_n such that $u_n(H,\theta_n) = u_n(L,\theta_n)$ and $u_n(H,\theta) > u_n(L,\theta)$ if and only if $\theta > \theta_n$. Moreover, since $\frac{\mathrm{d}}{\mathrm{d}\theta}u_n(H,\theta) > \frac{\mathrm{d}}{\mathrm{d}\theta}u_n(L,\theta)$, the threshold θ_n is unique. Therefore, the equilibrium is in the form of a switching strategy characterized by the threshold θ_n which solves $u_n(L,\theta_n) = u_n(H,\theta_n)$ or, equivalently, $\theta_n = \frac{\rho}{r}$ with $\rho = \frac{p_2 - p_1}{H-L}$.

Proof of Theorem 2: Consider the case with trading. By Equation (2.2), utility of a consumer of type θ from choosing plans $\mathbf{m}_1$ and $\mathbf{m}_2$ are, respectively, $u_t(L,\theta) = u_n(L,\theta) + \beta(r-\pi)(H-L)\theta$ and $u_t(H,\theta) = u_n(H,\theta) + \alpha\pi(H-L)(1-\theta)$. By a similar argument as in the proof of Theorem 1, since $\frac{\mathrm{d}}{\mathrm{d}\theta}u_t(H,\theta) > \frac{\mathrm{d}}{\mathrm{d}\theta}u_t(L,\theta)$, $u_t(H,1) >$

$u_t(L,1)$, and $u_t(H,0) < u_t(L,0)$, there exists a unique threshold θ_t such that $u_t(H,\theta) > u_t(L,\theta)$ if and only if $\theta > \theta_t$. Moreover, θ_t solves $u_t(L,\theta_t) = u_t(H,\theta_t)$ and can be expressed as

$$\theta_t = \frac{\rho - \alpha\,\pi}{r - \alpha\,\pi\, - \beta(r - \pi)} \tag{C1}$$

where $\rho = \frac{p_2 - p_1}{H - L}$, $\alpha = \min\left\{\frac{\psi_d}{\psi_s}\,,\,1\right\}$, and $\beta = \min\left\{\frac{\psi_s}{\psi_d}\,,\,1\right\}$.

The total supply and demand on the trading market can be calculated as

$$\psi_t = \int_{\theta_t}^{1} U(q_2,\,\theta)\mathrm{d}G(\theta) = \frac{1}{2}(1-\theta_t)^2(H-L),$$

and

$$\psi_d = \int_{0}^{\theta_t} L(q_1,\,\theta)\mathrm{d}G(\theta) = \frac{1}{2}\theta_t^2(H-L).$$

Assume $\theta_t \geq \frac{1}{2}$. Since $\frac{\psi_s}{\psi_d} = \frac{(1-\theta_t)^2}{\theta_t^2}$, we have $\alpha = 1$ and $\beta = \frac{(1-\theta_t)^2}{\theta_t^2}$. Plugging back these values of α and β into (C1) and solving for θ_t, we get $\theta_t = \frac{r-\pi}{2r-\pi-\rho}$. Note that $\theta_t \geq \frac{1}{2}$ holds only if $\pi \leq \rho$. Thus, we have the first expression in Equation (2.10). Assume $\theta < \frac{1}{2}$. We have $\alpha = \frac{\theta_t^2}{(1-\theta_t)^2}$ and $\beta = 1$. Plugging back these values of α and β into (C1) and solving for θ_t, we get $\theta_t = \frac{\rho}{\rho+\pi}$. Therefore, we have shown

$$\theta_t = \begin{cases} \dfrac{r-\pi}{2r-(\rho+\pi)} & \text{if} \quad \pi \leq \rho, \\ \dfrac{\rho}{\rho+\pi} & \text{if} \quad \pi > \rho. \end{cases}$$

Using θ_t we obtained above, we get

$$\alpha = \begin{cases} 1 & \text{if} \quad \pi \leq \rho, \\ \dfrac{\rho^2}{\pi^2} & \text{if} \quad \pi > \rho, \end{cases} \tag{C2}$$

and

$$\beta = \begin{cases} \dfrac{(r-\rho)^2}{(r-\pi)^2} & \text{if} \quad \pi \leq \rho, \\ 1 & \text{if} \quad \pi > \rho. \end{cases} \tag{C3}$$

This completes the proof.

Proof of Proposition 6: Assume $c = 0$. Note that, since the unit service cost is zero, service provider's profit increases if and only if more consumers opt for the high plan. Therefore, the threshold π_h can be viewed as a special case of the threshold π_p (which we derive in Proposition 7) where $c = 0$. Hence, the proof follows similarly.

Proof of Proposition 7: Define $\Delta_p = \frac{\Pi_t - \Pi_n}{H-L}$ where $\Pi_n(\Pi_t)$ denotes the service provider's profit without (with) trading[1]. After some algebra, Δ_p can be written as

$$\Delta_p(\pi, c) = \begin{cases} \dfrac{(r-\rho)\big(2\rho r(\rho - r + \pi) - c(r(\pi-\rho) + \rho(\rho+\pi))\big)}{2r^2(2r-\rho-\pi)} & \text{if} \quad \pi \leq \rho, \\ \dfrac{\rho^2\left(2r(\rho+\pi-r) - c(\rho+\pi)\right)}{2r^2(\rho+\pi)} & \text{if} \quad \pi > \rho. \end{cases}$$

Before proceeding, we verify that

$$\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_p = \begin{cases} \dfrac{(\rho-c)(r-\rho)}{(2r-\rho-\pi)^2} & \text{if} \quad \pi \leq \rho, \\ \dfrac{\rho^2}{(\rho+\pi)^2} & \text{if} \quad \pi > \rho, \end{cases}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}c}\Delta_p = \begin{cases} -\dfrac{(r-\rho)(r(\pi-\rho)+\rho(\rho+\pi))}{2r^2(2r-\rho-\pi)} & \text{if} \quad \pi \leq \rho, \\ -\dfrac{\rho^2}{2r^2} & \text{if} \quad \pi > \rho. \end{cases}$$

[1] Scaling the profit difference by $H - L$ does not change the sign of $\Pi_t - \Pi_n$.

Based on value of the service cost c, consider the following three mutually exclusive cases.

1. **Case I**: Assume $c \leq 2r\left(1 - \frac{r}{\rho}\right)$.

   (a) Assume $\pi \leq \rho$. Then $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_p > 0$ and, because $\Delta_p(0, c) \leq 0$ and $\Delta_p(\rho, c) \geq 0$, there exists a threshold π_p such that $\Delta_p \geq 0$ if and only if $\pi \geq \pi_p$. To obtain π_p, we solve $\Delta_p = 0$ which gives us $\pi_p = \frac{\rho(2r-c)(r-\rho)}{\rho(2r-c)-rc}$.

   (b) Assume $\pi > \rho$. Then, $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_p > 0$, $\frac{\mathrm{d}}{\mathrm{d}c}\Delta_p < 0$, and $\Delta_p(\rho, 2r(1 - r/\rho)) = 0$ which implies $\Delta_p \geq 0$.

2. **Case II**: Assume $2r\left(1 - \frac{r}{\rho}\right) < c \leq \frac{2r\rho}{r+\rho}$.

   (a) If $\pi \leq \rho$, $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_p < 0$ and $\max_\pi \Delta_p(\pi, c) = \Delta_p(0, c) = -\frac{\rho(2r-c)(r-\rho)^2}{2r^2(2r-\rho)} < 0$. Therefore, $\Delta_p < 0$.

   (b) If $\pi > \rho$, $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_p > 0$. Therefore, since $\Delta_p(\rho, c) \leq 0$ and $\Delta_p(r, c) \geq 0$, there exists a threshold π_p such that $\Delta_p \geq 0$ if and only if $\pi \geq \pi_p$. To obtain π_p, we solve $\Delta_p = 0$ which gives us $\pi_p = \frac{2r^2}{2r-c} - \rho$.

3. **Case III**: Assume $c > \frac{2r\rho}{r+\rho}$.

   (a) If $\pi \leq \rho$, using a similar argument as in Case II part (b), we have $\Delta_p < 0$.

   (b) If $\pi > \rho$, since $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_p > 0$ and $\max_\pi \Delta_p(\pi, c) = \Delta_p(r, c) = \frac{\rho^2(2\rho r - c(\rho+r))}{2r^2(\rho+r)} < 0$, we have $\Delta_p < 0$.

To summarize, we showed that when $c > \frac{2r\rho}{r+\rho}$, regardless of the value of π, we have $\Delta_p < 0$. When $c \leq \frac{2r\rho}{r+\rho}$, $\Delta_p \geq 0$ if and only if $\pi \geq \pi_p$. Moreover, $\pi_p = \frac{2r^2}{2r-c} - \rho$ if $\pi > \rho$ and $\pi_p = \frac{\rho(2r-c)(r-\rho)}{\rho(2r-c)-rc}$ if $\pi \leq \rho$.

Proof of Proposition 8: Define the difference in consumer utility with and without trading (scaled by $H-L$) as $\Delta_u(\theta) = \frac{u_t - u_n}{H-L}$. Let $\rho = \frac{p_2 - p_1}{H-L}$ and consider the following three cases.

1. Consider high type consumers, i.e. $\theta > \max\{\theta_t, \theta_n\}$. The utility difference $\Delta_u(\theta)$ captures the extra utility that consumers earn due to trading which is equal to $\alpha\pi(1-\theta)$. Plugging back α from (C2), we get $\frac{\rho^2}{\pi}(1-\theta)$.

2. Consider switchers, i.e. $\min\{\theta_t, \theta_n\} < \theta \leq \max\{\theta_t, \theta_n\}$. The utility difference $\Delta_u(\theta)$ captures the extra surplus due to both trading and plan adjustment. Two cases are possible.

   (a) Assume $\theta_t \leq \theta_n$. The utility difference is $\alpha\pi(1-\theta) + r\theta - (p_2 - p_1)$ which, using (C2), can be simplified to $(1-\theta)\frac{\rho^2}{\pi} + r\theta - \rho$.

   (b) Assume $\theta_t > \theta_n$. The utility difference is $\beta(r-\pi)\theta - r\theta + (p_2 - p_1)$ which, using (C3), can be simplified to $\rho - \theta\left(r - \frac{(r-\rho)^2}{r-\pi}\right)$.

3. Consider low type consumers, i.e. $\theta \leq \min\{\theta_t, \theta_n\}$. The utility difference $\Delta_u(\theta)$ captures the extra utility that consumers earn due to trading which is equal to $\beta(r-\pi)\theta$. Plugging back β from (C3), we get $\frac{(r-\rho)^2}{r-\pi}\theta$.

Next, we show that $\theta_n = \arg\max_\theta \Delta_u(\theta)$. Assume $\theta_t = \min\{\theta_t, \theta_n\}$ and observe that, from derivations above, $\Delta_u(\theta)$ can be written as

$$\Delta_u(\theta) = \begin{cases} \frac{\rho^2}{\pi}(1-\theta) & \text{if} \quad \theta \geq \theta_n, \\ \left(r - \frac{\rho^2}{\pi}\right)\theta + \frac{\rho^2}{\pi} - \rho & \text{if} \quad \theta_t \leq \theta < \theta_n, \\ \frac{(r-\rho)^2}{r-\pi}\theta & \text{if} \quad \theta < \theta_t. \end{cases}$$

It is straight forward to show that $\Delta_u(\theta)$ is increasing in θ for $\theta < \theta_n$ and decreasing in θ for $\theta \geq \theta_n$. Therefore, the consumer that benefits the most is of type $\theta = \theta_n$. The proof follows similarly for the case of $\theta_n = \min\{\theta_t, \theta_n\}$.

Proof of Proposition 9: Recall that $S_i = (r-c)P_i$ with $i \in \{n, t\}$. Define $\Delta_s(\pi) = \frac{S_t(\pi)-S_n}{(r-c)(H-L)} = \frac{P_t(\pi)-P_n}{H-L}$. Consider the following two cases.

Case I. Assume $\pi \leq \rho$. We have $\Delta_p = \frac{(\pi-\rho)r^2+(\rho+\pi)\rho^2}{2r^2(\rho+\pi)}$. Since $\Delta_p(0) = -\frac{1}{2}\left(1 - \frac{\rho^2}{r^2}\right) < 0$, $\Delta_p(\rho) = \frac{1}{2}\left(\frac{\rho}{r}\right)^2 > 0$, and $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_p(\pi) = \rho\left(\frac{\rho}{\rho+\pi}\right)^2 > 0$, there exists a unique threshold π_s such that $P_t(\pi) \geq P_n$ if and only if $\pi \geq \pi_s$ and π_s solves $P_t(\pi_s) = P_n$.

Case II. Assume $\pi > \rho$. We have $\Delta_p = \frac{\rho^2}{2r^2} > 0$ implying $P_t(\pi) > P_n$.

Proof of Theorem 3: We follow three steps to complete this proof. We first solve for the optimal prices for the case without trading following by the case with trading. Then, we show that $p_2^n < p_2^t$.

Step 1: Optimal prices without trading. From Theorem 1, we have $\theta_n = \frac{\rho}{r}$ with $\rho = \frac{p_2-p_1}{H-L}$ which implies $p_2 = (H-L)\rho + p_1$. The full market coverage constraint implies $u_n(L,0) = rL - p_1 \geq 0$. At the optimal solution, this constraint binds. To see why, assume by contradiction that it does not bind, i.e. $rL - p_1 > 0$. Increase p_1 and p_2 by a small amount $\epsilon > 0$. This increases the service provider's profit by ϵ without violating the constraint. Therefore, the constraint $rL - p_1 > 0$ binds and $p_n^1 = rL$. We can now write p_2 as a function of ρ only, i.e. $p_2 = rL + (H-L)\rho$.

Denote the service provider's revenue without trading by R_n which can be calculated as

$$\begin{aligned} R_n &= p_1\theta_n + p_2(1-\theta_n) \\ &= p_2 - (p_2 - p_1)\theta_n \\ &= p_2 - \frac{\rho^2}{r}(H-L) \\ &= rL + \left(\rho - \frac{\rho^2}{r}\right)(H-L). \end{aligned}$$

Total consumption without trading, denote by P_n, can be calculated as

$$\begin{aligned} P_n &= \theta_n L + \int_{\theta_n}^{1} \{L + \theta(H-L)\}\,\mathrm{d}\theta \\ &= L + \frac{H-L}{2}(1-\theta_n^2) \\ &= L + \frac{H-L}{2}\left(1 - \frac{\rho^2}{r^2}\right). \end{aligned}$$

The service provider's profit without trading is $\Pi_n = R_n - c\,P_n$. Plugging back the values of R_n and P_n obtained above, we get

$$\Pi_n = (r-c)L + (H-L)\left(-\left(\frac{1}{r} - \frac{c}{2r^2}\right)\rho^2 + \rho - \frac{c}{2}\right).$$

Since $\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}\Pi_n = -2\left(\frac{1}{r} - \frac{c}{2r^2}\right) < 0$, the optimal ρ can be obtained by solving the first order condition which gives us

$$\rho_n^* = \frac{r^2}{2r-c}. \tag{C4}$$

Therefore, $p_2^n = rL + (H-L)\rho_n^* = rL + (H-L)\frac{r^2}{2r-c}$. Finally, for our future use, the equilibrium threshold with the optimal prices is $\theta_n = \frac{r}{2r-c}$.

Step 2: Optimal prices with trading. Similar to step 1, the participation constraint binds and $p_t^1 = r\,L$. The service provider's revenue with trading is

$$\begin{aligned} R_t &= p_1 + (p_2 - p_1)(1-\theta_t) \\ &= rL + (p_2 - rL)(1-\theta_t) \\ &= rL + (H-L)\,\rho\,(1-\theta_t) \end{aligned}$$

and the total consumption with trading (denoted by P_t) is

$$P_t = \frac{H+L}{2} - \max\left\{\frac{H-L}{2}(2\,\theta_t - 1),\ 0\right\}.$$

If $\theta_t \geq \frac{1}{2}$, or equivalently when $\pi \leq \pi$, we have $P_t = \frac{H+L}{2} - (\psi_d - \psi_s)$. If $\theta_t < \frac{1}{2}$, or equivalently when $\pi > \rho$, demands are fully satisfied and $P_t = \frac{H+L}{2}$. Putting these together, total consumption can be written as

$$P_t = \begin{cases} \dfrac{H+L}{2} - \dfrac{H-L}{2}(2\theta_t - 1) & \text{if} \quad \pi \leq \rho, \\ \dfrac{H+L}{2} & \text{if} \quad \pi > \rho. \end{cases} \tag{C5}$$

Subtracting service costs from revenue, and after some algebra, the service provider's profit with trading is

$$\Pi_t = \begin{cases} (rL - c\,H) + (H-L)\left(\dfrac{-\rho^2 + r\rho + c(r-\pi)}{2r - (\rho+\pi)}\right) & \text{if} \quad \pi \leq \rho, \\ \left(rL - c\,\dfrac{H+L}{2}\right) + (H-L)\,\dfrac{\rho\,\pi}{\rho+\pi} & \text{if} \quad \pi > \rho. \end{cases} \tag{C6}$$

If $\pi \leq \rho$, since $\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}\Pi_t = -\frac{2(H-L)(r-\pi)}{(2r-\pi-c)^2} < 0$, the service provider's profit is concave with respect to ρ and there exists a unique optimal maximizer of Π_t which can be obtained by solving the first order condition. If $\pi > \rho$, since $\frac{\mathrm{d}}{\mathrm{d}\rho}\Pi_t = -\frac{\pi\,(H-L)}{(\rho+\pi)^2} < 0$, the service provider's revenue increases in ρ and $\rho^* = \pi$. This gives us

$$\rho_t^* = \begin{cases} 2r - \pi - \sqrt{(2r-\pi-c)(r-\pi)} & \text{if} \quad \pi \leq \dfrac{1}{3}(2r+c), \\ \pi & \text{if} \quad \pi > \dfrac{1}{3}(2r+c). \end{cases} \tag{C7}$$

Given ρ_t^* above, $p_2^s = (H-L)\rho_t^* + Lr$. For our future reference, we calculate θ_t under the optimal prices as follows:

$$\theta_t = \begin{cases} \sqrt{\dfrac{r-\pi}{2r-\pi-c}} & \text{if} \quad \pi \leq \dfrac{1}{3}(2r+c), \\ \dfrac{1}{2} & \text{if} \quad \pi > \dfrac{1}{3}(2r+c). \end{cases} \tag{C8}$$

Step 3: Proving $p_2^n < p_2^t$. In order to prove $p_2^n < p_2^t$, it is sufficient to show $\rho_n^* < \rho_t^*$. After some algebra, we have

$$\rho_t^* - \rho_n^* = \begin{cases} 2r - \pi - \sqrt{(2r-\pi-c)(r-\pi)} - \dfrac{r^2}{2r-c} & \text{if} \quad \pi \leq \dfrac{1}{3}(2r+c), \\ \dfrac{r^2-c^2}{6r-3c} & \text{if} \quad \pi > \dfrac{1}{3}(2r+c). \end{cases} \tag{C9}$$

The following two steps complete the proof.

1. Assume $\pi \leq \frac{1}{3}(2r+c)$. In this case, since $\frac{\mathrm{d}}{\mathrm{d}\pi}[\rho_t^* - \rho_n^*] < 0$, the minimum value of $\rho_t^* - \rho_n^*$ is attained at $\pi = \frac{1}{3}(c+2r)$. The value of $\rho_t^* - \rho_n^*$ at this point is equal to $\frac{3(3r-c)(r-c)}{2r-c}$ which is positive, hence $\rho_t^* > \rho_n^*$ for $\pi \leq \frac{1}{3}(2r+c)$.

2. Assume $\pi > \frac{1}{3}(2r+c)$. Since $r > c$, we have $\rho_t^* > \rho_n^*$.

Proof of Proposition 10: Part 1: Aggregate consumer surplus. When prices are set optimally, denote aggregate consumer surplus without trading by U_n^*. It can be calculated as

$$\begin{aligned}
U_n^* &= r\,L + \frac{r}{2}\left(1 - \left(\frac{\rho_n^*}{r}\right)^2\right)(H-L) - rL - \left(\rho_n^* - r\left(\frac{\rho_n^*}{r}\right)^2\right)(H-L) \\
&= \left(\frac{r}{2}\left(1 + \left(\frac{\rho_n^*}{r}\right)^2\right) - \rho_n^*\right)(H-L) \\
&= \frac{r}{2}\left(\frac{r-c}{2\,r-c}\right)^2 (H-L).
\end{aligned}$$

When prices are set optimally, denote aggregate consumer surplus with trading by U_t^*. Also define $\Delta_c^* = \frac{U_t^* - U_n^*}{H-L}$. Now, consider two cases as follows.

Case I: Assume $\pi \le \frac{1}{3}(2r+c)$. In this case, aggregate consumer surplus with trading is

$$\begin{aligned}
U_t(\pi) &= (r-c)\left(H - (H-L)\sqrt{\frac{r-\pi}{2r-\pi-c}}\right) - \\
&\qquad \left((r-c)L + (H-L)\left(-2\sqrt{(r-\pi)(2r-c-\pi)} + 3r - 2\pi - c\right)\right) \\
&= (H-L)\left(2\left(\sqrt{(r-\pi)(2r-c-\pi)} - r\right) - (r-c)\sqrt{\frac{r-\pi}{2r-c-\pi}} + 2\pi\right),
\end{aligned}$$

and

$$\begin{aligned}
\Delta_c^*(\pi) &= \frac{1}{2}\left(2\left(2\left(\sqrt{(r-\pi)(2r-c-\pi)} - r\right)\right.\right. \\
&\qquad \left.\left. -(r-c)\sqrt{\frac{r-\pi}{2r-c-\pi}} + 2\pi\right) - \frac{r(r-c)^2}{(2r-c)^2}\right).
\end{aligned} \tag{C10}$$

The function Δ_c^* satisfies the following three properties.

1. $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_c^* > 0$. Let $H(c) = \frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_c^*$. Function $H(c)$ attains its minimum at $c = r$. Because $H(r) = 0$, we have $H(c) < 0$.

2. $\Delta_c^*(0) \le 0$. The first order condition $\frac{\mathrm{d}}{\mathrm{d}c}\Delta_c^*(0)$ has two real solutions, $c_1 = (2 - \sqrt[3]{2^2})r$

and $c_2 = r$. Moreover, $\frac{\mathrm{d}^2}{\mathrm{d}c^2}\Delta_c^*(0)|_{c=(2-\sqrt[3]{2^2})r} = \frac{3(2-\sqrt[3]{2})}{8r} > 0$ and $\frac{\mathrm{d}^2}{\mathrm{d}c^2}\Delta_c^*(0)|_{c=r} = -\frac{1}{r}$. Therefore, $\Delta_c^*(0)$ attains its maximum at $c = r$. Since $\Delta_c^*(r) = 0$, we have $\Delta_c^*(0) \leq 0$.

3. $\Delta_c^*(\frac{1}{3}(2r + c)) > 0$**.** After some algebra, we have $\Delta_c^*(\frac{1}{3}(2r+c)) = \frac{(r-c)(r^2+c^2-rc)}{3(2r-c)^2} > 0$.

These three properties together imply that there is a unique threshold π_{c1}^* such that $\Delta_c^*(\pi) \geq 0$ if and only if $\pi \geq \pi_{c1}^*$. Moreover, since $-\frac{\mathrm{d}\Delta_c^*/\,\mathrm{d}\pi}{\mathrm{d}\Delta_c^*/\,\mathrm{d}c} > 0$, π_{c1}^* increases in c.

Case II: Assume $\pi > \frac{1}{3}(2r + c)$ and observe that

$$\begin{aligned} U_t^*(\pi) &= \frac{H+L}{2}(r-c) - \left(\left(rL - \frac{H+L}{2}c\right) + \frac{\pi}{2}(H-L)\right) \\ &= \frac{H-L}{2}(r-\pi), \end{aligned}$$

and

$$\begin{aligned} \Delta_c^*(\pi) &= \frac{1}{2}(r-\pi) - \left(\frac{r}{2}\left(\frac{r-c}{2r-c}\right)^2\right) \\ &= \frac{1}{2}\left(r - \pi - \frac{r(r-c)^2}{(2r-c)^2}\right). \end{aligned}$$

Since $\frac{\mathrm{d}\Delta_c^*}{\mathrm{d}\pi} = -\frac{H-L}{2} < 0$, $\Delta_c^*(r) < 0$, and $\Delta_c^*(\frac{1}{3}(2r + c)) > 0$, there exists a threshold $\pi_{c2}^* = \{\pi \mid \Delta_c^*(\pi) = 0\}$ such that $\Delta_c^*(\pi) \geq 0$ if and only if $\pi \leq \pi_{c2}^*$. Moreover, since we have $-\frac{\mathrm{d}\Delta_c^*/\,\mathrm{d}\pi}{\mathrm{d}\Delta_c^*/\,\mathrm{d}c} = \frac{2r^2(r-c)}{(2r-c)^2} > 0$, the threshold π_{c2}^* increases in c.

Part 2: Individual consumer surplus When prices are set optimally and without trading, consumer's utility is

$$u_n^*(\theta) = \begin{cases} 0 & \text{if } \theta \leq \frac{r}{2r-c}, \\ (\theta - \frac{r}{2r-c})\, r\,(H-L) & \text{if } \theta > \frac{r}{2r-c}. \end{cases}$$

When prices are set optimally and with trading, consumer's utility is

$$u_t^*(\theta,\pi) = \begin{cases} \theta\beta(r-\pi)\,(H-L) & \text{if } \ \theta \le \theta_t, \\ r\theta\,(H-L) + \alpha\,\pi\,(1-\theta)\,(H-L) + r\,L - p_2^t & \text{if } \ \theta > \theta_t, \end{cases}$$

where p_2^t is according to (2.23). Define $\Delta_u^*(\theta,\pi) = \frac{u_t^*(\theta,\pi) - u_n^*(\theta)}{H-L}$ and consider the following two cases.

Case I. If $\pi \le \frac{1}{3}(c+2r)$, we have $\theta_t = \sqrt{\frac{1-\pi/r}{2-\pi/r}}$, $\alpha = 1$, and $\beta = \frac{(r-\rho)^2}{(r-\pi)^2} = \frac{\left(r-\pi-\sqrt{(2r-\pi-c)(r-\pi)}\right)^2}{(r-\pi)^2}$. After some algebra, one can show that

$$u_t^*(\theta,\pi) = \begin{cases} \frac{\left(r-\pi-\sqrt{(2r-\pi-c)(r-\pi)}\right)^2}{(r-\pi)^2}\,\theta\,(H-L) & \text{if } \ \theta \le \sqrt{\frac{1-\pi/r}{2-\pi/r}}, \\ \left(\sqrt{(r-\pi)(2r-\pi-c)} - (2-\theta)(r-\pi)\right)\,(H-L) & \text{if } \ \theta > \sqrt{\frac{1-\pi/r}{2-\pi/r}}. \end{cases}$$

Noting that $\theta_t = \sqrt{\frac{r-\pi}{2r-\pi-c}} > \frac{r}{2r-c}$ when $\pi \le \frac{1}{3}(c+2r)$, we have

$$\Delta_u^*(\theta,\pi) = \begin{cases} \frac{\left(r-\pi-\sqrt{(2r-\pi-c)(r-\pi)}\right)^2}{(r-\pi)^2}\,\theta & \text{if } \ \theta < \frac{r}{2r-c}, \\ \frac{r^2}{2r-c} - \left(r - \left(1 - \sqrt{\frac{r-c}{r-\pi}}\right)^2\right)\theta & \text{if } \ \frac{r}{2r-c} \le \theta < \sqrt{\frac{r-\pi}{2r-\pi-c}}, \\ \sqrt{(2r-\pi-c)(r-\pi)} + \frac{r^2}{2r-c} - 2(r-\pi) - \pi\theta & \text{if } \ \theta \ge \sqrt{\frac{r-\pi}{2r-\pi-c}}. \end{cases} \tag{C11}$$

The function Δ_u^* increases in θ when $\theta < \frac{r}{2r-c} = \theta_t$ and decreases when $\theta > \frac{r}{2r-c} = \theta_t$. Therefore, a consumer of type $\theta = \theta_t$ benefits the most from trading. In addition, there exists some $\theta_\ell > \frac{r}{2r-c}$ such that a consumer of type $\theta < \theta_\ell$ benefits from trading and consumers with type $\theta > \theta_\ell$ are worse off.

Case II. Assume $\pi > \frac{1}{3}(2r+c)$. It follows that $\theta_t = \frac{1}{2}$ and $\beta = 1$. Therefore, we have $u_t^*(\theta,\pi) = \theta\beta(r-\pi)$. In this case, we have

$$\Delta_u^*(\theta,\pi) = \begin{cases} (r-\pi)\,\theta & \text{if } \theta \le \frac{r}{2r-c}, \\ \left(\frac{r^2}{2r-c} - \pi\theta\right) & \text{if } \theta > \frac{r}{2r-c}. \end{cases}$$

The function Δ_u^* increases in θ when $\theta \le \theta_t = \frac{r}{2r-c}$ and decreases when $\theta > \frac{r}{2r-c}$. Therefore, the consumer of type θ_t benefits the most from trading. Finally, all consumers of type $\theta > \frac{r^2}{\pi(2r-c)}$ are worse off and all other consumers are better off by trading.

To sum up, a consumer of type θ_n benefits the most from trading. In addition, there exist a threshold θ_ℓ such that a consumer of type θ benefits from trading if and only if $\theta < \theta_\ell$.

Proof of Proposition 11: Part 1: Service provider's profit. When prices are optimal, the service provider's profit without trading (denoted by Π_n^*) and with trading (denoted by Π_t^*) are, respectively,

$$\Pi_n^* = (r-c)L + \left(\frac{(r-c)^2}{2r-c}\right)\frac{H-L}{2}, \tag{C12}$$

and

$$\Pi_t^*(\pi) = \begin{cases} (r-c)L + \Big(3r - 2\sqrt{(r-\pi)(2r-c-\pi)} \\ \qquad - 2\pi - c\Big)(H-L) & \text{if } \pi \le \dfrac{1}{3}(c+2r), \\ (r-c)L + (\pi - c)\dfrac{H-L}{2} & \text{if } \pi > \dfrac{1}{3}(c+2r). \end{cases} \tag{C13}$$

Define $\Delta_p^*(\pi) = \frac{\Pi_t^*(\pi) - \Pi_n^*}{H-L}$. After some algebra, Δ_p^* can be written as

$$\Delta_p^*(\pi) = \begin{cases} 3r - \left(2\pi + c + 2\sqrt{(r-\pi)(2r-c-\pi)}\,\right) - \dfrac{(r-c)^2}{2(2r-c)} & \text{if} \quad \pi \le \dfrac{1}{3}(c+2r), \\ \dfrac{1}{2}\left(\pi - c - \dfrac{(r-c)^2}{2r-c}\right) & \text{if} \quad \pi > \dfrac{1}{3}(c+2r). \end{cases}$$

The following steps show that there exists a unique threshold π_p^* such that $\Delta_p^*(\pi) \ge 0$ if and only if $\pi \ge \pi_p^*$.

1. Δ_p^* is increasing in π because, if $\pi \le \frac{1}{3}(c+2r)$, we have $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_p^* = \frac{3r-c-2\pi}{\sqrt{(r-\pi)(2r-\pi-c)}} - 2 > 0$ and, if $\pi > \frac{1}{3}(c+2r)$, we have $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_p^* = \frac{1}{2} > 0$.
2. At $\pi = \frac{1}{3}(2r+c)$, we have $\Delta_p^*(\frac{1}{3}(2r+c)) = \frac{1}{2}\left(\frac{r^2-c^2}{6r-3c}\right) > 0$.
3. At $\pi = 0$, we have $\Delta_p^*(0) = \frac{(r-c)^2}{2(2r-c)} + 3r - \left(c + 2\sqrt{r(2r-c)}\right) < (3-2\sqrt{2})r - \frac{(r-c)^2}{2(2r-c)} < 0$.

Part 2: Social welfare. Social welfare (with optimal prices) is given by $S_i^* = (r-c)\,P_i^*$ with $i \in \{t, s\}$. Define $\Delta_s^*(\pi) = \frac{2(S_t^*(\pi) - S_n^*)}{(r-c)(H-L)}$ where $S_t^*(S_n^*)$ denotes social welfare with(without) trading when prices are set optimally. $\Delta_s^*(\pi)$ can be simplified to

$$\Delta_s^*(\pi) = \begin{cases} 1 + \left(\dfrac{r}{2r-c}\right)^2 - 2\sqrt{\dfrac{r-\pi}{2r-\pi-c}} & \text{if} \quad \pi \le \dfrac{1}{3}(2r+c), \\ \left(\dfrac{r}{2r-c}\right)^2 & \text{if} \quad \pi > \dfrac{1}{3}(2r+c). \end{cases} \tag{C14}$$

If $\pi \le \frac{1}{3}(2r+c)$, observe that $\frac{\mathrm{d}}{\mathrm{d}\pi}\Delta_s^* > 0$, $\Delta_s^*(0) < 0$, and $\Delta_s^*(\frac{1}{3}(2r+c) > 0$. If $\pi > \frac{1}{3}(2r+c)$, we have $\Delta_s^* = \left(\frac{r}{2r-c}\right)^2 > 0$. These together imply that there is a unique threshold π_s^* such that $\Delta_s^*(\pi) \ge 0$ if and only if $\pi \ge \pi_s^*$.

Part 3: Order of thresholds. Thresholds π_p^* and π_s^* can be calculated as $\pi_p^* = \frac{7r^2-c^2-2rc}{16r-8c}$ and $\pi_s^* = \frac{(c-2r)(c^3-3c^2r-cr^2+7r^3)}{(c-3r)(3c^2-12cr+13r^2)}$. It is now straightforward to show that

$\frac{\mathrm{d}}{\mathrm{dc}}\pi_p^* > 0$ and $\frac{\mathrm{d}}{\mathrm{dc}}\pi_s^* > 0$. Next, observe that

$$\begin{aligned}\pi_p^* - \pi_s^* &= \frac{(r-c)^2\left(49r^3 - 67cr^2 + 31c^2r - 5c^3\right)}{8(3r-c)(2r-c)\left(13r^2 - 12cr + 3c^2\right)} \\ \text{(equal in sigh)} &= \frac{49r^3 - 67cr^2 + 31c^2r - 5c^3}{13r^2 - 12cr + 3c^2} \\ &> 0\end{aligned}$$

where the last inequality holds because $\min_c\{13r^2 - 12cr + 3c^2\} = 4r^2 > 0$ and $\min_c\{49r^3 - 67cr^2 + 31c^2r - 5c^3\} = 8r^3 > 0$.

Proof of Proposition 12: The market clearings price π_m should equalize supply and demand on trading market, i.e. $\psi_s = \psi_d$. This holds when $\alpha = \beta = 1$ or, equivalently, $\pi_m = \frac{1}{3}(2r + c)$.

In order to show that the market clearing price π_m leads to win-win, we need to show that $\pi_p^* \le \pi_m \le \pi_{c2}^*$. First, note that $\pi_m - \pi_p^* = \frac{(11r-5c)(c+r)}{24(2r-c)}$. Since $c \le r$, we have $\pi_m > \pi_p^*$. Next note that, since $\pi_{c2}^* = \frac{r^2(3r-2c)}{(2r-v)^2}$, we have $\pi_{c2}^* - \pi_m = \frac{(r-c)(r^2-cr+c^2)}{3(2r-c)^2} > 0$ which implies $\pi_m \le \pi_{c2}^*$. This completes the proof.

Proof of Proposition 13: First, we show that Δ_p^* (as defined in part 1 of Proposition 10) is decreasing in c. It follows that

$$\frac{\mathrm{d}}{\mathrm{dc}}\Delta_p^* = \begin{cases} -\dfrac{1}{2}\left(1 + \dfrac{(3r-c)(r-c)}{(2r-c)^2}\right) & \text{if } \ \pi \le \dfrac{1}{3}(2r+c), \\ -1 + \dfrac{r-\pi}{\sqrt{(r-\pi)(2r-\pi-c)}} - \dfrac{(3r-c)(r-c)}{2(2r-c)^2} & \text{if } \ \pi > \dfrac{1}{3}(2r+c). \end{cases} \tag{C15}$$

It is easy to verify that for both cases $\frac{\mathrm{d}}{\mathrm{dc}}\Delta_p^* < 0$.

Second, we show that $\Delta_p^*(\pi_m)$ is concave in c; first increasing and then decreasing.

The profit difference $\Delta_p^*(\pi_m)$ can be simplified to $\Delta_p^*(\pi_m) = \frac{1}{6}\left(\frac{r^2-c^2}{2r-c}\right)$. Hence, we have $\frac{\mathrm{d}}{\mathrm{d}c}\Delta_p^* = \frac{(r-c)^2-2cr}{(2r-c)^2}$ and $\frac{\mathrm{d}^2}{\mathrm{d}c^2}\Delta_p^* = -\frac{6\,r^2}{(2r-c)^3}$. The second equation verifies the concavity and solving the first order condition leads to $c_m = (2-\sqrt{3})r$ which maximizes Δ_p^*.

Proof of Theorem 14: Without trading and when there is a single plan, the service provider solves

$$\max_{q_\circ, p_\circ} \left\{p_\circ - c\,P_\circ^n \;\text{ s.t. }\; r\,\theta \min\{H,\, q_\circ\} + r\,(1-\theta)\min\{L,\, q_\circ\} - p_\circ \geq 0 \text{ for all } \theta \in [0,1]\right\} \tag{C16}$$

where $P_\circ^n$ denotes total consumption without trading. Since the left hand side of the constraint in (C16) is monotone increasing in θ, it is sufficient to consider the constraint for $\theta = 0$, i.e. $r\ \min\{L,\, q_\circ\} \geq p_\circ$. Because the objective function in (C16) is monotone increasing in $p_\circ$, the constraint $r\ \min\{L,\, q_\circ\} \geq p_\circ$ binds. Moreover, for any $q_\circ > L$, the objective function decreases in $q_\circ$. Hence, we only need to consider $q_\circ \leq L$ and solve $\max_{\,q_\circ \leq L}(r-c)q_\circ$ which gives us $\mathbf{m}_\circ^n = (L,\, rL)$ as the optimal contract without trading.

Now consider the case with trading. The total supply and demand on trading market is given by

$$\psi_s = \int_0^1 U(q_\circ, \theta)\,\mathrm{d}G(\theta) = \frac{1}{2}(H - q_\circ)$$

and

$$\psi_d = \int_0^1 L(q_\circ, \theta)\,\mathrm{d}G(\theta) = \frac{1}{2}(q_\circ - L).$$

Therefore, $\frac{\psi_d}{\psi_s} = \frac{H-q_\circ}{q_\circ-L}$ and we have

$$\alpha = \begin{cases} 1 & \text{if } \; q_\circ \leq \dfrac{H+L}{2}, \\ \dfrac{H-q_\circ}{q_\circ-L} & \text{if } \; q_\circ > \dfrac{H+L}{2}, \end{cases}$$

and

$$\beta = \begin{cases} \dfrac{q_\circ - L}{H - q_\circ} & \text{if } \; q_\circ \le \dfrac{H+L}{2}, \\ 1 & \text{if } \; q_\circ > \dfrac{H+L}{2}. \end{cases}$$

Given α and β above, we can calculate total consumption (denoted by $P_\circ^t$). If $q_\circ > \frac{H+L}{2}$, total consumption reaches its maximum value $\frac{H+L}{2}$ and, if $q_\circ \le \frac{H+L}{2}$, trading market reallocates unused capacities and aggregate consumption is equal to the total allowances sold. Denotes aggregate consumption with trading by $P_\circ^t$ and observe that

$$P_\circ^t = \begin{cases} q_\circ & \text{if } \; q_\circ \le \dfrac{H+L}{2}, \\ \dfrac{H+L}{2} & \text{if } \; q_\circ > \dfrac{H+L}{2}. \end{cases}$$

Consider the case with trading. Since participation constraint is monotone in types, the service provider's problem can be written as

$$\max_{\;q_\circ, p_\circ} \left\{ \; p_\circ - c\, P_\circ^t \quad \text{s.t.} \quad r\,L - p_\circ + \alpha\,\pi\,(q_\circ - L) \ge 0 \; \right\}. \tag{C17}$$

Since $\frac{\mathrm{d}}{\mathrm{d}p_\circ}[rL - p_\circ + \alpha\,\pi\,(q_\circ - L)] < 0$ and $\frac{\mathrm{d}}{\mathrm{d}q_\circ}[rL - p_\circ + \alpha\,\pi\,(q_\circ - L)] > 0$, the constraint in (C17) binds and we have $p_\circ = r\,L + \alpha\,\pi\,(q_\circ - L)$. Then we can write the objective function in (C17) as

$$\Pi_\circ^t = \begin{cases} (\pi - c)q_\circ + (r - \pi)\,L & \text{if } \; q_\circ \le \dfrac{H+L}{2}, \\ -\pi\,q_\circ + r\,L + \pi\,H - c\,\dfrac{H+L}{2} & \text{if } \; q_\circ > \dfrac{H+L}{2}. \end{cases}$$

We now consider two cases.

Case I. Assume $\pi \le c$. Because $\frac{\mathrm{d}}{\mathrm{d}q_\circ}\Pi_\circ^t < 0$ for all $q_\circ$, the the optimal contract is $\mathbf{m}_\circ^t = (L,\, rL)$.

Case II. Assume $\pi > c$. Because $\frac{\mathrm{d}}{\mathrm{d}q_\circ}\Pi_\circ^t < 0$ for all $q_\circ \ge \frac{H+L}{2}$ and $\frac{\mathrm{d}}{\mathrm{d}q_\circ}\Pi_\circ^t > 0$ for all

Table C1: Summary of notations for Proposition 19

Performance metric	# of contracts	Value added by trading
Service provider's profit	one	$\Delta_1^p = \Pi_\circ^t - \Pi_\circ^n$
	two	$\Delta_2^p = \Pi_t - \Pi_n$
Social welfare	one	$\Delta_1^s = S_\circ^t - S_\circ^n$
	two	$\Delta_2^s = S_t - S_n$

$q_\circ < \frac{H+L}{2}$, the optimal quantity is $q_\circ^t = \frac{H+L}{2}$ and $p_\circ^t = r\,L + \alpha\,\pi\,(q_\circ - L) = r\,L + \pi\,\frac{H-L}{2}$.

Before proceeding, in Table C1, we summarize the notations that we use. The superscript "p" refers to the service provider's profit; the superscript "s" refers to social welfare; subscript "1" represents the case with single contract; and the subscript "2" represents the case with two contracts we study in Section 2.6.

Proposition 19 (Number of Contracts & Value of Trading) *The following statements hold.*

1. *$\Delta_2^p > \Delta_1^p$, if and only if $(c,\pi) \in \{(c,\pi)$ s.t. $\pi < c$ and $\pi \geq \pi_p^*\}$,*

2. *$\Delta_2^s > \Delta_1^s$, if and only if $(c,\pi) \in \{(c,\pi)$ s.t. $\pi < c$ and $\pi \geq \pi_s^*\}$,*

where π_p^* and π_s^*, are defined in Proposition 11.

Proof of Proposition 19: Service provider's profit. When there is one contract the service provider's profit difference with and without trading is

$$\Delta_1^p(\pi) = \begin{cases} 0 & \text{if } \pi \leq c, \\ \dfrac{H-L}{2}(\pi - c) & \text{if } \pi > c. \end{cases}$$

When there are two contracts, we already calculated the service provider's profit difference which we denote by $\Delta_2^p(\pi)$ here. After scaling by $\frac{H-L}{2}$, we can write the difference of differences in the service provider's profit as

$$\frac{\Delta_1^p(\pi) - \Delta_2^p(\pi)}{(H-L)/2} = \begin{cases} 4\sqrt{(r-\pi)(2r-c-\pi)} - \\ \qquad 4(r-\pi) - \dfrac{(3r-c)(r-c)}{2r-c} & \text{if } \pi \le c, \\ 4\sqrt{(r-\pi)(2r-c-\pi)} - \\ \qquad r\left(\dfrac{r}{c-2r} + 6\right) + 5\pi & \text{if } c < \pi \le \dfrac{1}{3}(2r+c), \\ \dfrac{(r-c)^2}{(2r-c)} & \text{if } \pi > \dfrac{1}{3}(2r+c). \end{cases}$$

As the above equation suggests, we consider the following three cases.

Case I: Assume $\pi \le c$. Trading does not add an extra value for the service provider when there is only one contract, i.e. $\Delta_1^p = 0$. When there are two contracts, $\Delta_2^p > 0$ if and only if $\pi > \pi_1^s$. Hence, $\Delta_1^p > \Delta_2^p$ if and only if $\pi < \pi_1^s$.

Case II: Assume $c < \pi \le \frac{1}{3}(2r+c)$. Since $\frac{d}{dc}[\Delta_1^p(\pi) - \Delta_2^p(\pi)] < 0$, $\frac{d}{d\pi}[\Delta_1^p(\pi) - \Delta_2^p(\pi)] < 0$, and at point $(c, \pi) = (r, r)$ we have $\Delta_1^p - \Delta_2^p = 0$, it follows that $\Delta_1^p > \Delta_2^p$ for all c and π satisfying $c \le \pi < \frac{1}{3}(2r+c)$.

Case III: Assume $\pi > \frac{1}{3}(2r+c)$. Since $2r - c > 0$, we always have $\Delta_1^p > \Delta_2^p$.

This completes the proof for the service provider's profit.

Social Welfare. After scaling by $\frac{(r-c)(H-L)}{2}$, we can write the difference of differences in social welfare as

$$\frac{\Delta_1^s(\pi)-\Delta_2^s(\pi)}{\frac{(r-c)(H-L)}{2}} = \begin{cases} 2\sqrt{\dfrac{r-\pi}{2r-\pi-c}} - \left(\dfrac{r}{2r-c}\right)^2 - 1 & \text{if} \quad \pi \le c, \\ 2\sqrt{\dfrac{r-\pi}{2r-\pi-c}} - \left(\dfrac{r}{2r-c}\right)^2 & \text{if} \quad c < \pi \le \dfrac{1}{3}(2r-c), \\ 1 - \left(\dfrac{r}{2r-c}\right)^2 & \text{if} \quad \pi > \dfrac{1}{3}(2r-c). \end{cases}$$

We consider three cases separately.

Case I: $\pi \le c$**.** Since the optimal contract with or without trading is (L, rL), $\Delta_1^s(\pi) = 0$. Hence, $\Delta_1^s > \Delta_2^s$ if and only $\pi < \pi_2^s$.

Case II: $c < \pi \le \frac{1}{3}(2r+c)$**.** Since $\frac{d}{dc}[\Delta_1^s(\pi) - \Delta_2^s(\pi)] < 0$, it attains its minimum value $\sqrt{2} - \frac{1}{(2-c)^2} > 0$ at $\pi = c$. Hence, $\Delta_1^s > \Delta_2^s$.

Case III: $\pi > \frac{1}{3}(2r+c)$**.** Since $\frac{r}{2r-c}$ increases in c, we have $\frac{r}{2r-c} \le \frac{1}{2}$. This implies $1 - \left(\frac{r}{2r-c}\right)^2 \ge \frac{3}{4}$. Hence, $\Delta_1^s > \Delta_2^s$.

Proof of Proposition 15: Equilibrium without trading. For a consumer of type θ, define $\Delta_n(\theta) = u_n(q_2, \theta) - u_n(q_1, \theta)$ which can be calculated as

$$\begin{aligned} \Delta_n(\theta) &= u_n(q_2,\theta) - u_n(q_1,\theta) \\ &= \left(r\int_0^{q_2} x f_\theta(x)\,\mathrm{d}x + rq_2\int_{q_2}^{\infty} x f_\theta(x)\,\mathrm{d}x - p_2\right) \\ &\qquad\qquad - \left(r\int_0^{q_1} x f_\theta(x)\,\mathrm{d}x + rq_1\int_{q_1}^{\infty} x f_\theta(x)\,\mathrm{d}x - p_1\right) \\ &= r\int_{q_1}^{q_2} \bar{F}_\theta(x)\,\mathrm{d}x - (p_2 - p_1). \end{aligned}$$

Now consider two types θ_1 and θ_2 such that $\theta_1 < \theta_2$ and observe that

$$\Delta_n(\theta_2) - \Delta_n(\theta_1) = r \int_{q_1}^{q_2} \left(\bar{F}_{\theta_2}(x) - \bar{F}_{\theta_1}(x)\right) \mathrm{d}x > 0,$$

where the inequality holds by Assumption 1. Therefore, $\Delta_n(\theta)$ is continuous and increasing in θ. Assumption 3 and the monotonicity of $\Delta_n(\theta)$ together imply that $\Delta_n(0) < 0$ and $\Delta_n(1) > 0$. Therefore, there exists a unique $\theta_n \in (0, 1)$ which solves $\Delta_n(\theta_n) = 0$ and a consumer of type θ opts for the high plan if and only if $\theta \geq \theta_n$. The threshold θ_n is the solution to $\Delta u_n(\theta) = r \int_{q_1}^{q_2} \bar{F}_\theta(x) \,\mathrm{d}x - (p_2 - p_1) = 0$ or, equivalently,

$$\int_{q_1}^{q_2} \bar{F}_\theta(x) \,\mathrm{d}x = \frac{p_2 - p_1}{r}. \tag{C18}$$

Equilibrium with trading. The utility of a consumer of type θ from opting for the high plan is

$$\begin{aligned}
u_t(q_2, \theta) &= r \int_0^{q_2} x f_\theta(x) \,\mathrm{d}x + r q_2 \int_{q_2}^{\infty} x f_\theta(x) \,\mathrm{d}x \\
&\qquad + \alpha p_s \int_0^{q_2} (q_2 - x) f_\theta(x) \,\mathrm{d}x + \beta(r - \pi) \int_{q_2}^{\infty} (x - q_2) f_\theta(x) \,\mathrm{d}x - p_2 \\
&= r \int_0^{q_2} \bar{F}_\theta(x) \,\mathrm{d}x + \alpha\pi \int_0^{q_2} F_\theta(x) \,\mathrm{d}x + \\
&\qquad \beta(r - \pi) \left[\int_0^{\infty} x f_\theta(x) \,\mathrm{d}x - \int_0^{q_2} \bar{F}_\theta(x) \,\mathrm{d}x\right] \\
&= [r - \alpha\pi - \beta(r - \pi)] \int_0^{q_2} \bar{F}_\theta(x) \,\mathrm{d}x + \alpha\pi q_2 + \beta_0(r - \pi) \int_0^{\infty} x f_\theta(x) \,\mathrm{d}x
\end{aligned}$$

and, similarly, the utility of a consumer of type θ from opting for the low plan is

$$u_t(q_1, \theta) = [r - \alpha\pi - \beta_0(r - \pi)] \int_0^{q_1} \bar{F}_\theta(x) \,\mathrm{d}x + \alpha\pi q_1 + \beta(r - \pi) \int_0^{\infty} x f_\theta(x) \,\mathrm{d}x.$$

Define $\Delta u_t(\theta) = u_t(q_2, \theta) - u_t(q_1, \theta)$ and observe that

$$\Delta u_t(\theta) = \Big(r - \alpha_0 \pi - \beta_0 (r - \pi)\Big) \int_{q_1}^{q_2} \bar{F}_\theta(x) dx + \alpha_0 \pi (q_2 - q_1) - (p_2 - p_1).$$

Next, we show that $\Delta u_t(\theta)$ increases in θ. Consider two types θ_1 and θ_2 such that $\theta_1 < \theta_2$ and observe that

$$\begin{aligned} \Delta u_t(\theta_1) - \Delta u_t(\theta_2) &= \Big(r - \alpha\pi - \beta(r - \pi)\Big) \int_{q_1}^{q_2} \left(\bar{F}_{\theta_1}(x) - \bar{F}_{\theta_2}(x)\right) \mathrm{d}x \\ &= \Big((1-\beta)(r-\pi) + (1-\alpha)\pi\Big) \int_{q_1}^{q_2} \left(\bar{F}_{\theta_1}(x) - \bar{F}_{\theta_2}(x)\right) \mathrm{d}x. \end{aligned}$$

Since $\alpha \leq 1$ and $\beta \leq 1$, $(1-\beta)(r-\pi) + (1-\alpha)\pi > 0$ and $\Delta u_t(\theta)$ increases in θ. Because $\Delta u_t(0) < 0$ and $\Delta u_t(1) > 0$, there exists a threshold θ_t such that a consumer of type θ selects the high plan if and only if $\theta \geq \theta_t$ where θ_t solves

$$\int_{q_1}^{q_2} \bar{F}_{\theta_t}(x)\, \mathrm{d}x = \frac{p_2 - p_1 - \alpha\,\pi(q_2 - q_1)}{r - \alpha\,\pi - \beta(r - \pi)}. \tag{C19}$$

Changes in prices. First consider the threshold θ_n which is the solution to (C18). Taking derivative with respect to p_1 and p_2, respectively, gives us

$$\frac{\mathrm{d}\,\theta_n}{\mathrm{d}\,p_1} = -\frac{1}{r \int_{q_1}^{q_2} \frac{\partial \bar{F}_\theta(x)}{\partial \theta}\, \mathrm{d}x} < 0$$

and

$$\frac{\mathrm{d}\,\theta_n}{\mathrm{d}\,p_2} = \frac{1}{r \int_{q_1}^{q_2} \frac{\partial \bar{F}_\theta(x)}{\partial \theta}\, \mathrm{d}x} > 0.$$

The inequalities hold because, by Assumption 1, we have $\frac{\partial \bar{F}_\theta(x)}{\partial \theta}\, \mathrm{d}x > 0$.

Define $L(\theta_t) = \int_{q_1}^{q_2} \bar{F}_{\theta_t}(x)\, \mathrm{d}x$ and verify that the following properties of $L(\theta_t)$.

I) $q_2 - q_1 > L(\theta_t)$ because $q_2 - q_1 = \int_{q_1}^{q_2} \mathrm{d}x > \int_{q_1}^{q_2} \bar{F}_{\theta_t}(x)\, \mathrm{d}x$.

II) $\frac{\partial L(\theta_t)}{\partial \theta_t} \geq 0$ because, by assumption 1, $\bar{F}_\theta(x)$ increases in θ_t.

III) $\frac{\partial \alpha}{\partial \theta_t} \geq 0$ and $\frac{\partial \beta}{\partial \theta_t} \leq 0$ because ψ_s (ψ_d) decreases (increases) in θ_t.

IV) $r - \alpha\,\pi - \beta(r-\pi) \geq 0$ because $\min_{\alpha,\beta}\{r - \alpha\,\pi - \beta(r-\pi)\} = r > 0$.

Now we can rewrite the equilibrium condition in (C19) as $\Big(r - \alpha\,\pi - \beta(r-\pi)\Big)L(\theta_t) = p_2 - p_1 - \alpha\,\pi\,(q_2 - q_1)$. Taking derivative with respect to p_1, p_2, and π, respectively, gives us

$$\frac{\partial \theta_t}{\partial\,\pi} = \frac{-\left(\beta L(\theta_t) + \alpha(q_2 - q_1 - L(\theta_t))\right)}{(r - \alpha\,\pi - \beta(r-\pi))\frac{\partial L(\theta_s)}{\partial \theta_t} + \pi(q_2 - q_1 - l(\theta_t))\frac{\partial \alpha}{\partial \theta_t} - (r-\pi)L(\theta_t)\frac{\partial \beta}{\partial \theta_t}} < 0,$$

$$\frac{\partial \theta_t}{\partial\,p_2} = \frac{1}{(r - \alpha\,\pi - \beta(r-\pi))\frac{\partial L(\theta_s)}{\partial \theta_t} + \pi)(q_2 - q_1 - L(\theta_t))\frac{\partial \alpha}{\partial \theta_t} - (r-\pi)L(\theta_t)\frac{\partial \beta}{\partial \theta_t}} < 0,$$

and

$$\frac{\partial \theta_t}{\partial\,p_1} = -\frac{\alpha\pi}{(r - \alpha\,\pi) - \beta(r-\pi))\frac{\partial L(\theta)}{\partial \theta_t} + \pi(q_2 - q_1 - L(\theta_s))\frac{\partial \alpha}{\partial \theta_t} - (r-\pi)L(\theta_t)\frac{\partial \beta}{\partial \theta_t}} > 0.$$

The first inequality holds because the nominator is negative (by I) and the denominator is positive (by II-IV). Similarly, the last two inequalities hold by properties II-IV.

Proof of Proposition 15: We first establish an intermediate result. We show that there exists a threshold ρ such that $\alpha < 1$ and $\beta = 1$ if $\pi > \rho$ and $\alpha = 1$ and $\beta \leq 1$ if $\pi \leq \rho$. The following steps prove this claim.

1. $\frac{\mathrm{d}}{\mathrm{d}q}\mathcal{O}(q,\,\theta) = F_\theta(q) > 0$ and $\frac{\mathrm{d}}{\mathrm{d}q}\mathcal{L}(q,\,\theta) = -(1 - F_\theta(q)) < 0$, therefore $\mathcal{O}(q_1,\,\theta) < \mathcal{O}(q_2,\,\theta)$ and $\mathcal{L}(q_2,\,\theta) < \mathcal{L}(q_1,\,\theta)$ for some q_1 and q_2 such that $q_1 < q_2$.

2. The above step, together with the fact that θ_t decreases in π, imply that ψ_s increases in π and ψ_d decreases in π.

3. The above step implies that $\frac{\psi_s}{\psi_d}$ increases in π. Hence, $\alpha(\beta)$ decreases(increases) in π and there is a threshold ρ such that $\alpha = 1$ and $\beta \leq 1$ if and only if $\pi \leq \rho$.

We now continue with the proof of Proposition 15.

Portion choosing high plan. Define the right hand sides of (2.29) and (2.30) as $H_n = \frac{p_2 - p_1}{r}$ and $H_t(\pi) = \frac{p_2 - p_1 - \alpha\pi(q_2 - q_1)}{r - \alpha\pi - \beta(r-\pi)}$, respectively. Moreover, $H_t(\pi)$ is continuous and increasing in π. Because the left hand sides of (2.30) and (2.29) are identical, $\theta_t(\pi) \leq \theta_n$ if and only if $H_t(\pi) \leq H_n$. Next, we show that there exists a unique threshold π_h such that $H_t(\pi) \leq H_n$ if and only if $\pi \geq \pi_h$.

Observe that $H_t(0) = \frac{p_2-p_1}{(1-\beta)r} > \frac{p_2-p_1}{r} = H_n$ and $H_t(r) = \frac{p_2-p_1-\alpha r(q_2-q_1)}{(1-\alpha)r} = \frac{p_2-p_1}{r} - \frac{\frac{\alpha(q_2-q_1)}{1-\alpha}\left(r - \frac{p_2-p_1}{q_2-q_1}\right)}{r} < \frac{p_2-p_1}{r} = H_n$. Hence, there exists a threshold π_h such that $\theta_t(\pi) \leq \theta_n$ if and only if $\pi \geq \pi_h$ and π_h solves $H_t(\pi_h) = H_n$. The uniqueness is due to monotonicity of $H_t(\pi)$ with respect to π.

Social Welfare. We consider the following two cases separately.

Case I: Assume $\pi \leq \rho$ which implies $\alpha = 1$ and $\beta < 1$. The proof relies on two facts: (1) $S_t(\pi)$ increases in π (because $\theta_t(\pi)$ decreases in π) and (2) $S_t(\rho) > S_t$ (as argued in Case I). If $S_t(0) \geq S_n$, social welfare is always higher and $\pi_h = 0$. If $S_t(0) < S_n$, there exists a threshold π_s (which solves $S_t(\pi_s) = S_n$) such that $S_t(\pi) \geq S_n$ holds if and only if $\pi \geq \pi_s$.

Case II: Assume $\pi > \rho$ which implies $\alpha < 1$ and $\beta = 1$. Define $\Delta_s(\pi) = S_t(\pi) - S_n = (r-c)(P_t(\pi) - P_n)$ and simplify it to

$$\Delta_s(\pi) = (r-c)\int_0^{\theta_n} \mathcal{L}(q_1,\theta)\mathrm{d}G(\theta) + (r-c)\int_{\theta_n}^1 \mathcal{L}(q_2,\theta)\mathrm{d}G(\theta) > 0.$$

Therefore, for any $\pi > \rho$, we have $S_t(\pi) > S_n$.

Service Provider. We first show that Π_t increases in π. Assume $\pi > \rho$. since θ_t

decreases in π and $p_1 < p_2$, $\Pi_t = p_1\,\theta_t + p_2\,(1-\theta_t) - c\bar{P}$ increases in π. Assume $\pi \le \rho$ and observe that

$$\begin{aligned}\Pi_t &= p_1\,\theta_t + p_2\,(1-\theta_t) - \\ &\quad c\left(\int_0^{\theta_t} \mathcal{L}(q_1,\theta)\,\mathrm{d}\theta - \int_{\theta_t}^1 \mathcal{L}(q_2,\theta)\,\mathrm{d}\theta + \int_0^{\theta_t} \mathcal{O}(q_1,\theta)\,\mathrm{d}\theta + \int_{\theta_t}^1 \mathcal{O}(q_2,\theta)\,\mathrm{d}\theta\right).\end{aligned}$$

Consider π_1 and π_2 such that $\pi_1 < \pi_2$ and define $\omega = \theta_t(\pi_1) - \theta_t(\pi_2)$. Observe that

$$\begin{aligned}\Pi_t(\pi_2) - \Pi_t(\pi_1) &= (p_2 - p_1)\omega + \\ &= c\int_{\theta_t(\pi_1)-\omega}^{\theta_t(\pi_1)} \Big(\mathcal{O}(q_2,\theta) - \mathcal{O}(q_1,\theta) - \big(\mathcal{L}(q_2,\theta) - \mathcal{L}(q_1,\theta)\big)\Big)\,\mathrm{d}\theta.\end{aligned}$$

Since $\mathcal{O}(q_2,\theta) > \mathcal{O}(q_1,\theta)$ and $\mathcal{L}(q_2,\theta) < \mathcal{L}(q_1,\theta)$, we have $\Pi_t(\pi_2) > \Pi_t(\pi_1)$. Therefore, Π_t increases in π. Moreover, it is easy to verify that Π_t decreases in c.

Next, we establish the existence of threshold π_p. Define $\Delta_p(\pi, c) = \Pi_t(\pi, c) - \Pi_n(c)$ and notice that, because Π_t increases in π, Δ_p increases in π as well. Assume $\Delta_p(r, r) > 0$ (otherwise we have $\bar{c} = r$) and define $\bar{c} = \{c \mid \Delta_p(r, c) = 0\}$. Consider the following two cases.

Case I: Assume $c \le \bar{c}$ which implies $\Delta_p(r, c) \ge 0$. Since Δ_p decreases in c, we have $\Delta_p(0, c) < \Delta_p(0, 0) = -(p_2 - p_1)(\theta_t(0) - \theta_n) \le 0$. Hence, there exists a threshold π_p such that $\Pi_t(\pi, c) \ge \Pi_n(c)$ if and only if $\pi \ge \pi_p$.

Case II: Assume $c > \bar{c}$. Because Δ_p is increasing in π, we have $\Delta_p(\pi, c) < \Delta_p(r, c) < 0$. Therefore, $\Pi_t(\pi, c) < \Pi_n(c)$.

This completes the proof.

Appendix E

Omitted Theoretical Proofs for Chapter 3

Proof of Proposition 16. Set $\phi = 0$ and follow the steps in the proof of Proposition 17. The results follow.

Proof of Proposition 17. Assume the expected return of the project is μ and its known to both the insider and the outsider. In this case, the insider solves the following problem.

$$\max_{x\geq 0} \quad \phi\Big((1-\alpha)\mu - F + x + x_t(x,\mu)\Big) + (1-\phi)\left(\frac{x}{x + x_t(x,\mu)}\alpha\mu - x\right), \tag{C1}$$

$$\text{s.t.} \quad x + x_t(x,\mu) \geq F, \tag{C2}$$

$$x_t \text{ solves (3.4)} \tag{C3}$$

Since we assume $F \leq \alpha\mu$ and the constraint (C2) is already included in the outsider's problem in (3.4), it is sufficient to use backward induction by first characterizing the outsider's best response and then plugging back its best response into (C2).

Upon her arrival, the outsider observes x_i and invests x_t which solves (3.4). It is easy to verify that the objective function in (3.4) is concave and the unconstrained problem has a unique solution, i.e. $x_t^*(x_i) = \sqrt{\alpha\mu\, x_i} - x_i$. If $x_t^*(x_i) + x_i < F$ then the outsider invests $F - x_i$ which is just enough investment such that the entrepreneur has enough capital to execute the project. Putting these together the outsider's best response is as follows.

$$x_t(x_i) = \begin{cases} F - x_i, & \text{if } x_i \le \frac{F^2}{\alpha\mu} \\ \sqrt{\alpha\mu\, x_i} - x_i, & \text{if } \frac{F^2}{\alpha\mu} < x_i \le \alpha\mu \\ 0. & \text{if } x_i > \alpha\mu \end{cases} \tag{C4}$$

If $\alpha\mu$ is small, the best response function $x_t(x_i)$ is monotone decreasing in x_i. However, if $\alpha\mu$ is sufficiently high then the function is not monotone. It decreases for $x_i \le \frac{F^2}{\alpha\mu}$ and then it is concave for $x_i > \frac{F^2}{\alpha\mu}$ with increasing slope at $x_i = \frac{F^2}{\alpha\mu}$. Plugging back $x_t(x_i)$ from (C4) into the insider's utility we have

$$u_i(x_i, x_t(x_i,\mu), \mu) = \begin{cases} \phi(1-\alpha)\mu + (1-\phi)\, x_i \left(\frac{\alpha\mu}{F} - 1\right), & \text{if } x_i \le \frac{F^2}{\alpha\mu} \\ \phi((1-\alpha)\mu - F) - x_i(1-\phi) + \sqrt{\alpha\mu\, x_i}, & \text{if } \frac{F^2}{\alpha\mu} < x_i \le \alpha\mu \\ (1-\phi)\,(\alpha\mu - x_i) + \phi\big((1-\alpha)\mu - F + x_i\big). & \text{if } x_i > \alpha\mu \end{cases} \tag{C5}$$

We exclude the case of $x_i > \alpha\mu$ since it implies that the investors gives free money to the entrepreneur which is even larger than his project's value. First, assume $x_i \le \frac{F^2}{\alpha\mu}$ and observe that $\frac{\partial}{\partial x_i} u_i(x_i, x_t, \mu) = (1-\phi)\left(\frac{\alpha\mu}{F} - 1\right) > 0$. Next, assume $x_i > \frac{F^2}{\alpha\mu}$. Since $\frac{\partial^2}{\partial x_i^2} u_i(x_i, x_t, \mu) = -\frac{1}{4x_i}\sqrt{\frac{\alpha\mu}{x_i}} < 0$, the utility function is concave with $\frac{\partial}{\partial x_i} u_i(x_i, x_t, \mu) = -(1-\phi) + \frac{1}{2}\sqrt{\frac{\alpha\mu}{x_i}}$ and $\lim_{x_i \downarrow \frac{F^2}{\alpha\mu}} \frac{\partial}{\partial x_i} u_i(x_i, x_t, \mu) = -(1-\phi) + \frac{\alpha\mu}{2F}$. Therefore, if $-(1-\phi) + \frac{\alpha\mu}{2F} < 0$, then the insider's optimal investment is $x_i^{FI} = \frac{F^2}{\alpha\mu}$. On the other hand, if $-(1-\phi) + \frac{\alpha\mu}{2F} \ge 0$, the insider's optimal investment is an interior solution $x_i^{FI} = \frac{\alpha\mu}{4(1-\phi)^2}$. The condition for always having an interior solution is $\frac{\alpha\mu}{4(1-\phi)^2} \ge \frac{F^2}{\alpha\mu}$ which can be simplified to $\frac{\alpha\mu}{F} \geq 2(1-\phi)$. Putting everything together, the insider's full information investment is

$$x_i^{FI}(\mu) = \begin{cases} \frac{\alpha\mu}{4(1-\phi)^2}, & \text{if } \frac{\alpha\mu}{F} \geq 2(1-\phi), \\ \frac{F^2}{\alpha\mu}, & \text{otherwise.} \end{cases} \tag{C6}$$

This completes the proof.

Before proceeding with the proof of Theorem 5, we establish an intermediate result in the next lemma.

Lemma 8 *For a fixed investment level x, define the utility difference between being truthful and imitating the other type by $\Delta u_\theta(x) = u_i(x, \mu_\theta, \mu_\theta) - u_i(x, \mu_\theta, \mu_{-\theta})$, $\theta \in \{h, \ell\}$. Then, $\Delta u_\ell \geq 0$ if and only if $\sqrt{\frac{\mu_\ell}{\mu_h}} \geq \frac{\phi}{1-\phi}$ and $\Delta u_h < 0$ for all $\phi \in [0, 0.5]$.*

Proof of Lemma 8. Consider first Δu_h. After some algebra, we can write as

$$\Delta u_h(x) = \left(\sqrt{\mu_\ell\,\mu_h} - (1-\phi)\mu_h - \phi\,\mu_\ell\right)\sqrt{\frac{\alpha x}{\mu_\ell}}.$$

It is straight forward to show that $\Delta u_h < 0$ if and only if $\sqrt{\frac{\mu_\ell}{\mu_h}} < \frac{1-\phi}{\phi}$ holds. Since $\mu_\ell < \mu_h$ and $\phi \leq 0$, we always have $\Delta u_h < 0$. In a similar way, we can show that

$$\Delta u_\ell(x) = \left(\sqrt{\mu_\ell\,\mu_h} - (1-\phi)\mu_\ell - \phi\,\mu_h\right)\sqrt{\frac{\alpha x}{\mu_h}}$$

is positive if and only if $\sqrt{\frac{\mu_\ell}{\mu_h}} \geq \frac{\phi}{1-\phi}$ or $\phi \geq \Phi_\ell$ with $\Phi_\ell = \frac{\rho}{1-\rho}$.

Proof of Lemma 4. Consider a separating equilibrium $\langle x_h^{SP}, x_\ell^{SP}, \tilde{\mu}\rangle$. Consider the type θ and denote the other type by $-\theta$. To sustain the separating equilibrium, the type θ should not deviate to an off-equilibrium path investment $\tilde{x}$. Therefore, we have the condition $u_i(x_\theta^{SP}, \mu_\theta, \mu_\theta) \geq u_i(\tilde{x}, \mu_\theta, \tilde{\mu})$. Since this should holds for all possible

$\tilde{x} \neq x^{SP}$, we have $u_i(x_\theta^{SP}, \mu_h, \mu_h) \geq \max_z u_i(z, \mu_\theta, \tilde{\mu})$. This condition is captured by the set $\mathcal{N}_h(\tilde{\mu})$.

Also, the repeating investment by type $-\theta$ should not be such that the type θ deviates to that, hence $x_\theta^{SP} \notin \mathcal{M}_\theta = \{x \mid u_i(x, \mu_\theta, \mu_{-\theta}) \leq u_i(x_\theta^{SP}, \mu_\theta, \mu_\theta)\}$. These two conditions ensure us that there does not exists an off-equilibrium path investment $\tilde{x}$ such that type $\theta \in \{h, \ell\}$ wants to deviate to that investment from her equilibrium investment x_θ^{SP}. This implies that conditions (1) and (2) in the Definition 2 are not restricting the off-equilibrium path beliefs and the intuitive criterion does not impose a restriction on the off-equilibrium path beliefs.

Proof of Theorem 5. Based on the results in Lemma 8, we consider two cases separately. In the first case we assume $\sqrt{\frac{\mu_\ell}{\mu_h}} \geq \frac{\phi}{1-\phi}$ and show that there exists a separating equilibrium and specify it. In the second case, we assume $\sqrt{\frac{\mu_\ell}{\mu_h}} < \frac{\phi}{1-\phi}$ and show there does not exists a separating equilibrium.

Case 1. Assume $\sqrt{\frac{\mu_\ell}{\mu_h}} \geq \frac{\phi}{1-\phi}$. We show that there exists a separating equilibrium such that $x_h^{SP} = x_h^{FI}$, $x_\ell^{SP} = x_\ell^{SP}$, $\tilde{\beta} = 1$ and it satisfies the initiative criterion as defined in Definition 2.

First consider the high type insider. By Lemma 4, the high type's investment in the equilibrium needs to satisfy $x_h^{SP} \in \mathcal{N}_h(1) \cap \mathcal{M}_\ell$. Since in the conjectured separating equilibrium the off-equilibrium path beliefs are set to be high, we have $\mathcal{N}_h(1) = \{x_h^{FI}\}$. Next, consider the set $\mathcal{M}_\ell = \{x \mid u_i(x, \mu_\ell, \mu_h) \leq \max_z u_i(z, \mu_\ell, \mu_\ell)\}$. Assumption $\sqrt{\frac{\mu_\ell}{\mu_h}} \geq \frac{\phi}{1-\phi}$ implies $u_i(x, \mu_\ell, \mu_h) \leq u_i(z, \mu_\ell, \mu_\ell)$ for any $x \in \mathbb{R}_+$ which implies $\mathcal{M}_\ell = \mathbb{R}_+$. By Lemma 4, we should have $x_h^{SP} \in \{x_h^{FI}\} \cap \mathbb{R}_+ = \{x_h^{FI}\}$.

Consider now the low type insider. To characterize a low type insider's investment in the equilibrium, we need to specify sets $\mathcal{N}_\ell$ and $\mathcal{M}_h$. First consider the set $\mathcal{N}_\ell$ and define

$$\begin{aligned}
\Delta_{\ell\,h}(x) &= u_i(x,\mu_\ell,\mu_\ell) - \max_z u_i(z,\mu_\ell,\mu_h) \\
&= \phi((1-\alpha)\mu_\ell - F) - x(1-\phi) + \sqrt{\alpha\,\mu_\ell\,x} \\
&\qquad - \frac{1}{4}\left(\frac{\phi(\alpha\mu_h\phi - 4F(1-\phi))}{(1-\phi)} + \frac{\alpha\mu_\ell^2(1-\phi)}{\mu_h} + 2(2-\alpha)\mu_\ell\phi\right) \\
&= \sqrt{\alpha\mu_\ell x} - x(1-\phi) - \frac{1}{4}\left(\frac{\alpha\mu_\ell^2(1-\phi)}{\mu_h} + \frac{\alpha\mu_h\phi^2}{1-\phi} + 2\alpha\mu_\ell\phi\right).
\end{aligned}$$

Note that $\mathcal{N}_\ell$ is non-empty (because $u_i(x,\mu_\ell,\mu_\ell) > u_i(x,\mu_\ell,\mu_h)$ for $x \in \mathbb{R}_+$) and the function $\Delta_{\ell\,h}$ is concave (because $\frac{\mathrm{d}^2}{\mathrm{d}x^2}\Delta_{\ell\,h} = -2(1-\phi) < 0$). Therefore, the set $\mathcal{N}_\ell$ is a well-defined convex set which can be written as $[n_h^1, n_h^2]$ where

$$n_h^1 = \frac{\sqrt{\alpha(-\mu_h)(\mu_h-\mu_\ell)\left(\mu_h\phi^2 - \mu_\ell(\phi-1)^2\right)} - \mu_h\sqrt{\alpha\mu_\ell}}{2\mu_h(\phi-1)}$$

and

$$n_h^2 = \frac{-\sqrt{\alpha(-(\mu_\ell-\mu_h))\left(\mu_\ell(\phi-1)^2 - \mu_h\phi^2\right)} - \sqrt{\alpha\mu_\ell\mu_h}}{2\sqrt{\mu_h}(\phi-1)}$$

are obtained by solving $\Delta_{\ell\,h}(x) = 0$.

Using a similar argument, we can write $\mathcal{M}_h = [m_h^1, m_h^2]$ with

$$m_h^1 = \frac{-\sqrt{\alpha}\mu_\ell\phi + \sqrt{\alpha(\mu_\ell-\mu_h)\left(\mu_\ell\phi^2 - \mu_h(\phi-1)^2\right)} + \sqrt{\alpha}\mu_h\phi + \sqrt{\alpha}(-\mu_h)}{2\sqrt{\mu_\ell}(\phi-1)}$$

and

$$m_h^2 = -\frac{\sqrt{(\mu_\ell-\mu_h)\left(\mu_\ell\phi^2 - \mu_h(\phi-1)^2\right)} + \mu_\ell\phi + \mu_h(-\phi) + \mu_h}{2(\phi-1)\sqrt{\frac{\mu_\ell}{\alpha}}}.$$

Next we show that $\mathcal{N}_\ell(\mu_h) \cap \mathcal{M}_h = [m_h^1, n_h^2]$ and nonempty, i.e. $m_h^1 \leq n_h^2$. To show this observe that

$$n_h^2 - m_h^1 = \frac{(\mu_h-\mu_\ell)(\mu_\ell(-\phi)+\mu_\ell+\mu_h\phi)}{\sqrt{\mu_h(\mu_h-\mu_\ell)\left(\mu_\ell(\phi-1)^2-\mu_h\phi^2\right)}} + \frac{-\frac{(\mu_\ell-\mu_h)(\mu_\ell\phi+\mu_h(-\phi)+\mu_h)}{\sqrt{(\mu_h-\mu_\ell)(\mu_h(\phi-1)^2-\mu_\ell\phi^2)}}+\mu_\ell-\mu_h}{\sqrt{\mu_\ell}}. \tag{C7}$$

The first term is trivially positive. We show that the second term is positive if and only if

$$\begin{aligned}
& -\frac{(\mu_\ell-\mu_h)(\mu_\ell\phi+\mu_h(-\phi)+\mu_h)}{\sqrt{(\mu_h-\mu_\ell)\left(\mu_h(\phi-1)^2-\mu_\ell\phi^2\right)}}+\mu_\ell-\mu_h>0 \\
(iff) \iff \quad & ((\mu_h-\mu_\ell)(\mu_\ell\phi+\mu_h(1-\phi)))^2 \\
& \quad -(\mu_h-\mu_\ell)^2(\mu_h-\mu_\ell)\left(\mu_h(1-\phi)^2-\mu_\ell\phi^2\right)>0 \\
(iff) \iff \quad & \mu_\ell\mu_h(\mu_h-\mu_\ell)^2>0
\end{aligned}$$

This completes the proof.

Case 2. Assume $\sqrt{\frac{\mu_\ell}{\mu_h}} < \frac{\phi}{1-\phi}$ and recall that, by Lemma 8, we have $\Delta u_\ell < 0$ and $\Delta u_h < 0$. Under this assumption, we show that there is no off-equilibrium path beliefs $\tilde{\beta}$ which can be supported as a separating equilibrium. By contradiction, assume that $\langle x_h^{SP}, x_\ell^{SP}, \tilde{\beta}\rangle$ is a separating equilibrium. When off-equilibrium beliefs are $\tilde{\beta}$, define the deviation set for the high type by $\mathcal{D}_h(\tilde{\mu}) = \left\{x \mid u_i(x,\mu_h,\tilde{\mu}) \geq \max_z u_i(z,\mu_h,\mu_h)\right\}$. Using a similar argument as in Case 1, we can show that the set $\mathcal{D}_h(\mu_\ell)$ is a convex set and can be written as $\mathcal{D}_h = [d_h^1, d_h^2]$. For any investment $x \in \mathcal{D}_h$ the $h-$type insider deviates from her separating equilibrium because $u_i(x_h^{SP},\mu_h,\mu_h) \leq \max_z u_i(z,\mu_h,\mu_h)$. Hence, to sustain a separating equilibrium, we should ensure that $x_\ell^{SP} \notin \mathcal{D}_h(\mu_\ell)$ and, for any $x \in \mathcal{D}_h(\mu_\ell)$, $\beta(x) > 0$.

Since $u_i(x, \mu_\ell, \mu_\ell)$ is concave in x and $x_\ell^{SP} \notin [d_h^1, d_h^2]$, the separating investment level which gives the highest utility to the ℓ-type insider is either $x_\ell^{SP} = d_h^1$ or $x_\ell^{SP} = d_h^2$. We show that neither $x_\ell^{SP} = d_h^1$ nor $x_\ell^{SP} = d_h^2$ can be supported as an equilibrium leading to a contradiction. Assume $x_\ell^{SP} = d_h^1$. Since by assumption $\sqrt{\frac{\mu_\ell}{\mu_h}} < \frac{\phi}{1-\phi}$ we have $u_i(x, \mu_\ell, \mu_\ell) < u_i(x, \mu_\ell, \mu)$ for $\mu > \mu_\ell$, we can always find a small positive number ϵ such that $u_i(d_h^1, \mu_\ell, \mu_\ell) < u_i(d_h^1 + \epsilon, \mu_\ell, \mu)$ which implies that $x = d_h^1$ cannot be supported as a separating equilibrium. The proof follows similarly for the case of $x_\ell^{SP} = d_h^2$.