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Constraint summation in phonological theory

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Introduction. *Classical* markedness and faithfulness constraints apply to individual candidates. Yet, the literature has also advocated constraints that instead apply to sets of candidates, such as *Distinctiveness constraints* (DCs; Flemming 2008) and *Optimal Paradigm faithfulness constraints* (OPFCs; McCarthy 2005). These approaches thus need to "lift" the classical constraints to sets of candidates by summing them across a set. Is this assumption of constraint summation typologically innocuous? We formalize this question and establish a positive answer for an *additive* model of constraint interaction.

Constraint summation. DCs embody a preference for more distinct contrasts among SRs: they penalize pairs of SRs whose perceptual distance is below a given threshold. For instance, the DC MinDist in tableau (1) penalizes the pairs of SRs ([ata], [ada]) and ([ada], [ⁿda]) but not the pair ([ata], [ⁿda]) because segment pairs ([t], [d]) and ([d], [ⁿd]) are less distinct perceptually than ([t], [ⁿd]), where prenasalization enhances the voicing contrast with the voiceless stop (Flemming 2004).

OPFCs embody a preference for greater similarity among SRs in the same morphological paradigm: they penalize pairs of paradigm members that differ along some relevant phonological dimension. For instance, the OPFC IdentOP(length) in tableau (2) penalizes paradigms (b) and (c) because the length of the stem-final vowel is not identical in the two SRs. It does not penalize paradigms (a) and (d), where all the vowels standing in correspondence in the two SRs have the same length (McCarthy 2005).

DCs and OPFCs are formally very different from classical faithfulness and markedness constraints. In fact, classical constraints assign a number of violations to each individual candidate mapping consisting of a UR and a corresponding SR. DCs and OPFCs instead compare the SRs of multiple candidate mappings. This difference has implications for the architecture of grammar. A "classical" constraint-based grammar evaluates the candidates of a single UR at a time. A grammar with DCs or OPFCs instead must evaluate sets of candidates corresponding to multiple URs, as illustrated in tableau (1) for the two URs /ata/ and /ada/ and in tableau (2) for the two URs /faʕaːl-a/ and /faʕaːl-tu/.

But what about the classical constraints that are now mixed up with DCs and OPFCs? Flemming and McCarthy make the natural suggestion that classical faithfulness and markedness constraints be "lifted" to sets of candidates by *summing* their violations across all candidates in a set. For instance, in (1), the candidate ([ata], [ata]) violates *VTV twice because the two SRs in this pair each violate it once. In (2), the paradigm ([faʕala], [faʕaltu]) violates the input-output faithfulness constraint IdentIO(length) twice because the two SRs in this pair each violate it once.

Typological innocuousness. Tableaux (1)/(2) have two novelties: they contain non-classical constraints such as DCs and OPFCs; and the classical constraints are summed over. Do both novelties contribute to the typological predictions of Flemming's and McCarthy's proposals? In other words, if DCs and OPFCs are left at the bottom,

(1) /ata/, /ada/	MinDist	Id(vce)	Id(nas)	$*^nD$	$*D$	$*VTV$
[ata], [ata]	*	*				**
[ata], [ada]	*				*	*
[ata], [ⁿda]			*	*	*	*
[ada], [ata]	*	**			*	*
[ada], [ada]	*	*				**
[ada], [ⁿda]	*	*	*	*		**
[ⁿda], [ata]		**	*	*	*	*
[ⁿda], [ada]	*	*	*	*		**
[ⁿda], [ⁿda]	*	*	**	**		**

(2) /faʕaːl-a/, /faʕaːl-tu/	$*V:CCV$	IdOP(length)	IdIO(length)
(a) [faʕaːl-a], [faʕaːl-tu]	*		
(b) [faʕaːl-a], [faʕal-tu]		*	*
(c) [faʕal-a], [faʕaːl-tu]	*	*	*
(d) [faʕal-a], [faʕal-tu]			**

do the classical constraints yield the same winners when they are summed over as when they are used classically for a single UR at the time? Or do the classical constraints make different typological predictions when they are summed as in (1)/(2)?

To formalize this question, we consider two URs (the extension to more than two URs is straightforward). Let A and B be their individual candidate sets, namely the classical tableaux where classical constraints work as usual. Let $<$ be an order over tuples of constraint violations which extends the notion "smaller" from numbers to tuples. We denote by $opt_< A$ and $opt_< B$ the sets of winner candidates in tableaux A and B, namely the sets of those candidates with the "smallest" tuples of violations. We allow $<$ to be a *partial* order, (as needed for HG; see below) whereby $opt_< A$ and $opt_< B$ can contain multiple winners.

Let $A \times B$ be the set of pairs (α, β) of a candidate α in A and a candidate β in B. By Flemming's and McCarthy's constraint summation assumption, a candidate pair (α, β) is represented by the sum $\mathbf{a} + \mathbf{b} = (a_1 + b_1, \ldots, a_n + b_n)$ of the tuples of constraint violations $\mathbf{a} = (a_1, \ldots, a_n)$ and $\mathbf{b} = (b_1, \ldots, b_n)$ of the two candidates α and β. Tableaux (1)/(2) (without MinDist and IdentOP) illustrate $A \times B$. We denote by $opt_< (A \times B)$ the set of winner pairs in $A \times B$, namely pairs with the smallest summed tuple of violations.

The typological innocuousness of constraint summation relative to a mode of constraint interaction $<$ can be formalized as the identity (3): the two URs considered end up with the same winner candidates if we optimize the product candidate set $A \times B$ relative to the summed constraints (left hand side) or if we optimize the two candidate sets A and B separately (right hand side).

$$
\textbf{(3)} \qquad \underbrace{opt_< (A \times B)}_{\text{with constraint summation}} = \underbrace{opt_< A \times opt_< B}_{\substack{\text{classical approach} \\ \text{without summation}}}
$$

Typological innocuousness in OT. The sum $\mathbf{a} + \mathbf{b}$ carries less information than the two individual tuples of constraint violations $\mathbf{a}$ and $\mathbf{b}$: the individual tuples cannot be reconstructed from their sum. One might thus expect (3) to fail because constraint summation wipes away crucial information. This pessimism is dispelled by an independent result due to Prince (2015): he effectively establishes (3) for the special case where $<$ is OT's lexicographic order. Yet, Prince's reasoning relies on ERCs, a piece of notation tailored to OT. His

proof is thus involved because constraint summation does not admit a simple counterpart in ERCs. We show that Prince's result admits the following elementary explanation without ERCs.

Suppose by contradiction that the candidate pair $(\widehat{\alpha}, \widehat{\beta})$ is OT optimal in $A \times B$ but that, say, the candidate $\widehat{\alpha}$ is not OT optimal in A. Hence, there exists another candidate α in A that beats $\widehat{\alpha}$: the tuple $\mathbf{a} = (a_1, \ldots, a_n)$ of constraint violations of α is smaller than the tuple $\widehat{\mathbf{a}} = (\widehat{a}_1, \ldots, \widehat{a}_n)$ of $\widehat{\alpha}$, namely $\mathbf{a} < \widehat{\mathbf{a}}$. Suppose (without loss of generality) that OT's order $<$ is relative to the ranking $C_1 \gg C_2 \gg \ldots \gg C_n$. Thus, $\mathbf{a} < \widehat{\mathbf{a}}$ means (4) holds for some k: the $k-1$ top constraints do not distinguish between the two candidates while the kth constraint decisively assigns less violations to α than to $\widehat{\alpha}$. By adding the corresponding components $\widehat{b}_1, \ldots, \widehat{b}_{k-1}, \widehat{b}_k$ of the tuple $\widehat{\mathbf{b}}$ of constraint violations of candidate $\widehat{\beta}$ to both sides of (4), we obtain (5), which says that $\mathbf{a} + \widehat{\mathbf{b}} < \widehat{\mathbf{a}} + \widehat{\mathbf{b}}$. The candidate pair $(\alpha, \widehat{\beta})$ thus beats the candidate pair $(\widehat{\alpha}, \widehat{\beta})$, contradicting the assumption that the candidate pair $(\widehat{\alpha}, \widehat{\beta})$ is OT optimal in $A \times B$. The proof of the reverse implication is analogous.

$$
\textbf{(4)} \quad \begin{aligned} a_1 &= \widehat{a}_1 \\ &\vdots \\ a_{k-1} &= \widehat{a}_{k-1} \\ a_k &< \widehat{a}_k \end{aligned} \qquad \textbf{(5)} \quad \begin{aligned} a_1 + \widehat{b}_1 &= \widehat{a}_1 + \widehat{b}_1 \\ &\vdots \\ a_{k-1} + \widehat{b}_{k-1} &= \widehat{a}_{k-1} + \widehat{b}_{k-1} \\ a_k + \widehat{b}_k &< \widehat{a}_k + \widehat{b}_k \end{aligned}
$$

Typological innocuousness beyond OT. Does the typological innocuousness of the constraint summation assumption extend beyond OT? In other words, besides OT's lexicographic order, which other ways $<$ of ordering tuples of constraint violations satisfy the identity (3)? The crucial property of OT's lexicographic order used in our analysis above is that (4) entails (5): if we add the same quantity to both sides of the inequality, the inequality is not affected. Thus, let us say that an arbitrary order $<$ over tuples of constraint violations is *additive* (Anderson & Feil 1988) provided, whenever a tuple $\mathbf{a}$ is $<$-smaller than a tuple $\widehat{\mathbf{a}}$ and the same tuple $\mathbf{b}$ is added to both, the sum $\mathbf{a} + \mathbf{b}$ is $<$-smaller than the sum $\widehat{\mathbf{a}} + \mathbf{b}$. Hence, (4)/(5) say that OT's lexicographic order is additive. Our main contribution is that **the identity (3) holds if and only if $<$ is an additive order**. In other words, additive orders provide necessary and sufficient structure for the typological innocuousness of the constraint summation assumption. As a corollary, we can extend typological innocuousness of constraint summation from OT to HG.