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Simple Basic and Convex Processing Time and Power Models for Common Subtractive Manufacturing Processes

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Abstract

This technical report provides formulations of processing time and power requirement models for several common subtractive manufacturing operations, including milling, turning, facing, boring, drilling, reaming, tapping, threading, grinding, and polishing processes. Basic formulations are given using a consistent set of variables and units, as well as convex model formulations for each (if different from the basic formulation). When convex models are given, proof of convexity is provided or discussed. The purpose of these models is to provide a set of potential objective functions for various types of optimization problems in manufacturing and materials processing.

Keywords: Optimization models; manufacturing processes; manufacturing systems; convex model

Software and Code: No code or software was used in the completion of this technical report

1 Introduction

This technical report presents the derivation of processing time and power requirement models for a number of common subtractive manufacturing processes. In this context, the processing time is defined as the time required to complete one planned manufacturing operation (in minutes), while the processing power requirement is the amount of electrical or mechanical power needed to run the process for the needed operation (in kW). Total power needed for an operation (in $kW \cdot hr$) can be calculated by combining the processing time and power requirement values. These models are relatively simple, having a small number of basic variables, and are formulated for use in the construction of objective functions for optimization problems involving manufacturing processes; these will be particularly useful in manufacturing layout problems where a model for each process or machine is needed. In most cases, the processes genuinely are based only on a small number of independent variables, so these models are appropriate to predict and optimize their behavior. Some of these models are natural convex functions, while the others can be converted into convex functions via change-of-variable operations. Depending on the needs of the problem, they can be used as-derived or in their convex form. In each case, the convexity of the function is shown or explained. The manufacturing processes covered here are the most common subtractive processes, a wide, but not a complete, set of the available processes. Several more obscure processes, such as broaching, subtractive sheet metal operations, and similar are not included here and will be the subject of future work. In addition, no additive or formative processes are discussed here.

2 Production Time and Power Models

2.1 Milling Machine Time and Power Models

The most common process typically encountered in a manufacturing cell is the milling process, which processes parts and raw materials by cutting away material using a rotating tool. The work-piece is clamped into a fixed or axis-based clamp or jig during the operation. A milling machine and its basic operation are shown in Figure 1. The behavior of the milling process depends on five basic parameters:

- **Cutting feed**: (CF) The speed at which the cutting tool or work-piece is moved forward through the material during one rotation of the cutting tool. The typical units of measure for this parameter are $mm/$revolution. In the case where the tool has more than one tooth, the cutting feed is the feed per tooth (FPT) (a common parameter in handbooks) multiplied by the number of teeth ζ on the tool. Formally, this can be expressed as:

$$CF = \zeta \times FPT \quad [mm/rev] \tag{1}$$

- **Cutting speed**: (CS) The speed of the tool relative to the surface of the workpiece in $m/$min
- **Tool speed**: (TS) The speed that the spindle rotates during the cutting process, which is equal to the cutting speed CS divided by the tool circumference c with units rotations/min. Formally,

$$TS = \frac{CS}{c} \quad [RPM] \tag{2}$$

- **Feed rate**: (FR) Speed of the tool movement relative to the work-piece during cutting, measured in $mm/$min. This is the combination of the cutting feed and the spindle speed:

$$FR = CF \times TS \quad [mm/min] \tag{3}$$

- **Depth of cut**: (DOC) The depth the tool cuts into the material in a single pass in units of mm

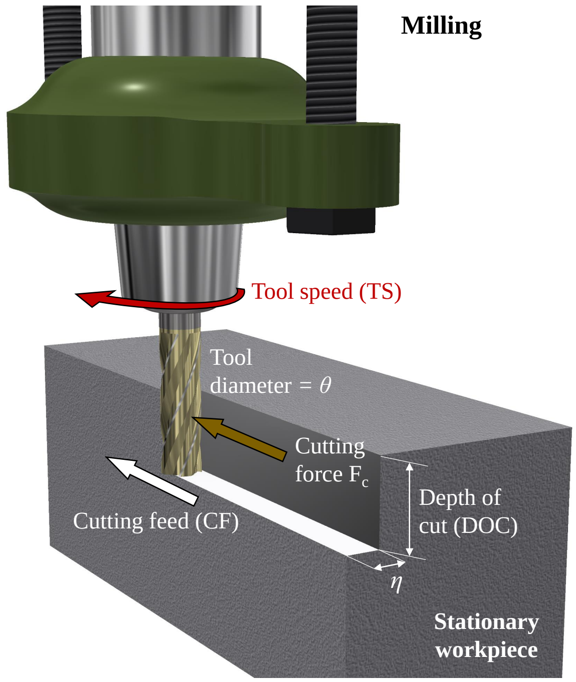

Figure 1: *Basic milling process (both manual and CNC) with important parameters and variables shown*

In terms of machining time, the most important consideration is the material removal rate for the tool. A *fully-engaged* tool will use 100% of its cutting area to cut material, but this case is rare in practice, so a parameter $\eta \in [0,1]$ can be defined which describes the fraction of the tool that is engaged for cutting. Given a tool diameter θ (Figure 1), the material removal rate MRR [1] can be expressed as:

$$MRR = \eta \times \theta \times DOC \times FR = \eta \times \theta \times DOC \times CF \times TS \quad [mm^3/min] \tag{4}$$

Therefore, the manufacturing time for a material volume removal of V_R $[mm^3]$ can be expressed as:

$$t_m = \frac{V_R}{MRR} \quad [min] \tag{5}$$

Suppose now that that parameter set $\mathbf{x} = [x_1, x_2, x_3]$ describes the variables here where $x_1 = CF$, $x_2 = TS$, $x_3 = DOC$. Assuming that the total material removal amount is a specified constant and the diameter of the tool is fixed, the time can be expressed as:

$$t_m(x_1, x_2, x_3) = \frac{V_R}{\eta \theta x_1 x_2 x_3} \quad [min] \tag{6}$$

where the constraints on the domain of each variable are determined by the physical characteristics of the machine. The gradient function for t_m can be calculated as:

$$\Delta t_m(x_1,x_2,x_3) = \begin{bmatrix} \frac{\partial t_{mill}}{\partial x_1} \\ \frac{\partial t_{mill}}{\partial x_2} \\ \frac{\partial t_{mill}}{\partial x_3} \end{bmatrix} = \begin{bmatrix} -\frac{V_R}{\eta\theta x_1^2 x_2 x_3} \\ -\frac{V_R}{\eta\theta x_1 x_2^2 x_3} \\ -\frac{V_R}{\eta\theta x_1 x_2 x_3^2} \end{bmatrix} \tag{7}$$

The equivalent Hessian for this equation is:

$$\Delta^2 t_m(x_1,x_2,x_3) = \begin{bmatrix} \frac{\partial^2 t_m}{\partial x_1^2} & \frac{\partial^2 t_m}{\partial x_1 x_2} & \frac{\partial^2 t_m}{\partial x_1 x_3} \\ \frac{\partial^2 t_m}{\partial x_2 x_1} & \frac{\partial^2 t_m}{\partial x_2^2} & \frac{\partial^2 t_m}{\partial x_2 x_3} \\ \frac{\partial^2 t_m}{\partial x_3 x_1} & \frac{\partial^2 t_m}{\partial x_3 x_2} & \frac{\partial^2 t_m}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} \frac{2V_R}{\eta\theta x_1^3 x_2 x_3} & \frac{V_R}{\eta\theta x_1^2 x_2^2 x_3} & \frac{V_R}{\eta\theta x_1^2 x_2 x_3^2} \\ \frac{V_R}{\eta\theta x_1^2 x_2^2 x_3} & \frac{2V_R}{\eta\theta x_1 x_2^3 x_3} & \frac{V_R}{\eta\theta x_1 x_2^2 x_3^2} \\ \frac{V_R}{\eta\theta x_1^2 x_2 x_3^2} & \frac{V_R}{\eta\theta x_1 x_2^2 x_3^2} & \frac{2V_R}{\eta\theta x_1 x_2 x_3^3} \end{bmatrix} \tag{8}$$

Given that $\mathbf{x} = x_1,x_2,x_3 > 0$ and V_R,η,θ are non-negative constant values, the Hessian is positive semi-definite. Therefore, t_m is a convex function which describes the processing time of a milling process.

The power required to run the machine is based on the same basic parameters as the manufacturing time, in addition to a specific cutting force factor F_c, and can be calculated as:

$$P_m(x_1,x_2,x_3) = \frac{1}{u_p} DOC \times \eta \times \theta \times FR \times F_c = \frac{1}{u_p} DOC \times \eta \times \theta \times CF \times TS \times F_c \ \ [kW] \tag{9}$$

where $u_p = 60 \times 10^6$ is a unit conversion factor used to obtain the power consumption in terms of kilowatts. The ideal value of F_c $[N/mm^2]$ is dependent upon the material being cut, and is assumed to be a constant that depends on material choice (from a machining handbook) for the purposes of this study. Therefore, the power consumption [2] is:

$$P_m(x_1,x_2,x_3) = \frac{1}{u_p} \eta\theta x_1 x_2 x_3 F_c \tag{10}$$

This is not a convex function, but it can be re-formulated as a convex function by change of variables. Let $x_4 = 1/CF$ [rev/*mm*], $x_5 = 1/TS$ [min/rev], and $x_6 = 1/DOC$ [1/*mm*], and x_4, x_5, $x_6 > 0$. The convex function in terms of the substitute variables will then be:

$$P_m(x_5,x_5,x_6) = \frac{\eta\theta F_c}{u_p x_4 x_5 x_6} \tag{11}$$

Since the machine parameters can easily be measured in terms of $x_4 = 1/x_1$, etc., this formulation is reasonable and allows for easy calculation of realistic constraints, as shown above. It is in the same basic form as the time function, which was already shown to be convex.

The total power cost will be a function of the machining time, the power needed for machining, and the power needed for machine idling such that:

$$P_{\text{cost}} = t_m(P_m + P_{\text{idle}}) \tag{12}$$

where the idle power P_{idle} is an input and assumed to be constant.

2.2 Lathe Time and Power Models

A lathe (Figures 2-3) is a machine tool commonly used in subtractive manufacturing, one that is very flexible and can perform several different jobs; unlike in a milling process where the tool rotates to cut material from the part, the lathe rotates the work-piece itself and the tooling is fixed. A lathe can perform several basic machining operations, mainly:

- **Turning**: Reducing the external diameter of a circular work-piece
- **Facing/parting**: Cutting a flat face, usually perpendicular to the rotational axis of the work-piece. This operation is typically used for creating smooth faces perpendicular to the rotational axis and to cut the part into sections ("parting").
- **Drilling/Reaming/Tapping**: The production of a circular hole (rough (drilling), precise (reaming), or threaded (tapping)) in the part
- **Boring**: Cutting away material from the inside of the work-piece
- **Threading**: Cutting external threads on a work-piece

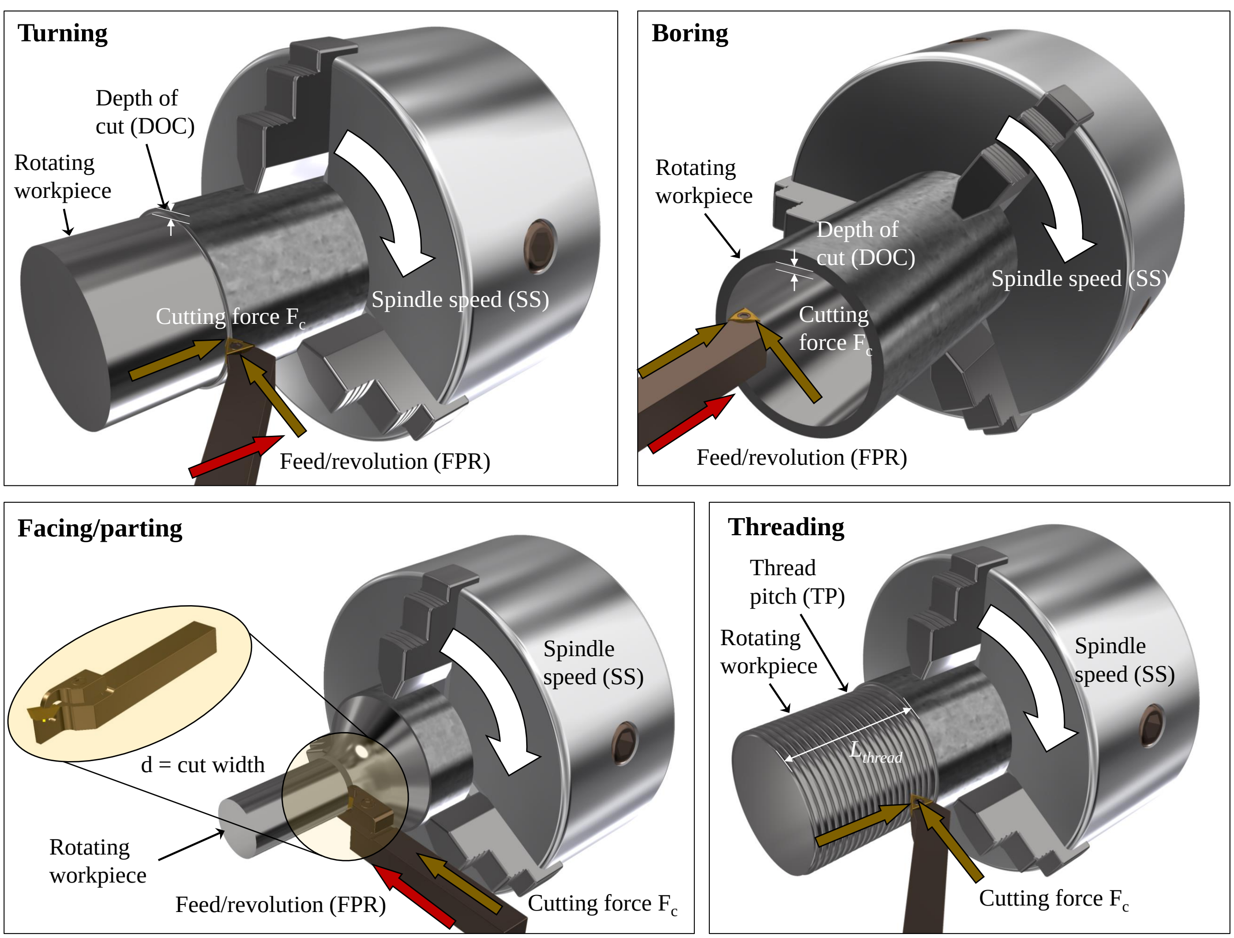

Figure 2: Basic lathe processes: Turning, boring, threading, and facing/parting

Unlike the milling process, where the time is calculated based on the total material removal volume, lathe

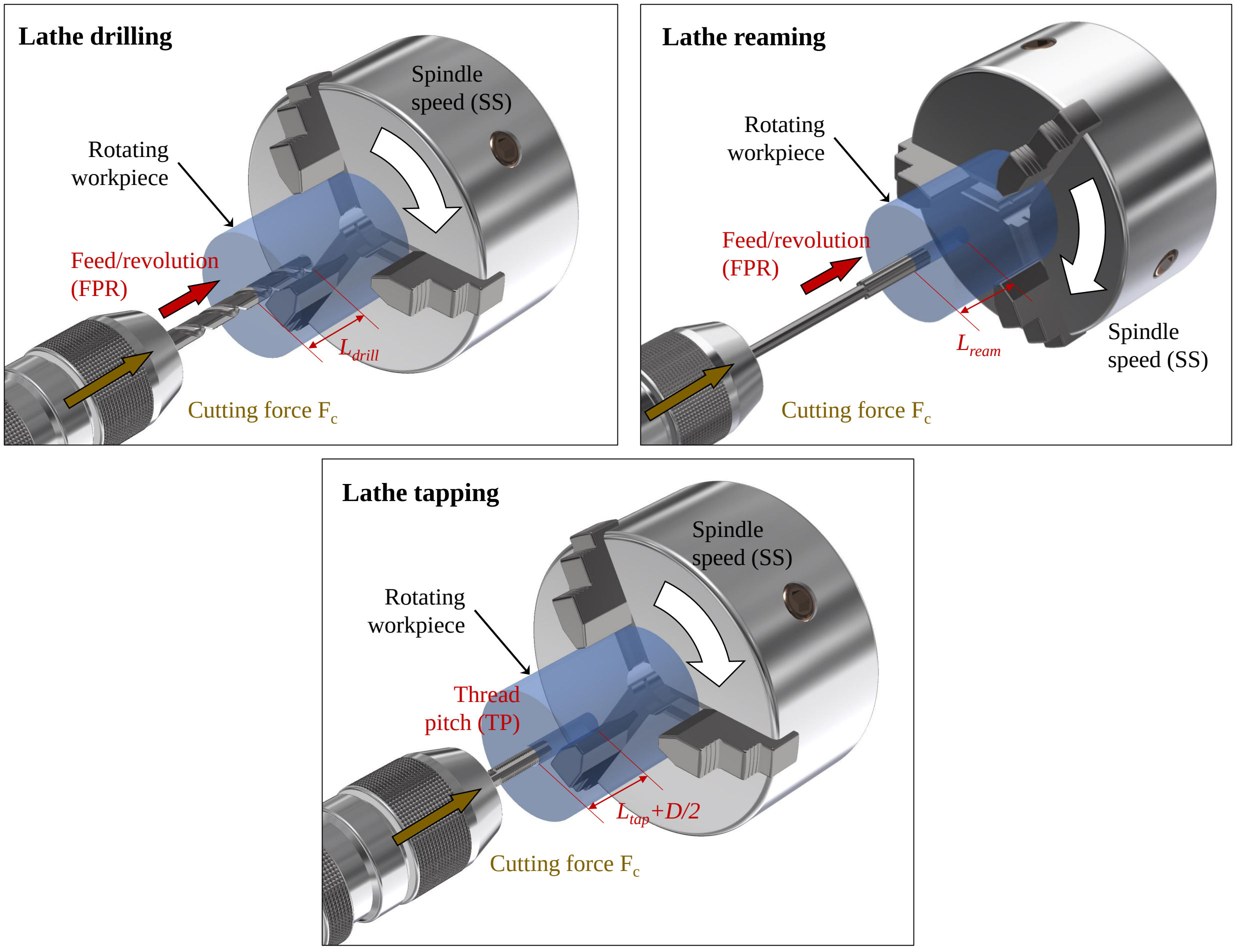

Figure 3: Basic lathe processes: Drilling, reaming, and tapping

operations are done in a series of "cuts" of regular depth and the basic machining time equations in handbooks are typically given in time per cut. This is due to the fact that the work-piece is moving and needs to be under uniform machining force in order to preserve the dimensional accuracy of the final product. Therefore, this model will use a dimensionless factor ω:

$$\omega = \frac{V_R}{V_C} \tag{13}$$

where V_R is the volume of material to be removed, V_C is the volume removed in a single cut, and $\omega \geq 1$. The only exceptions to the multiple-cut rule is the lathe drilling/reaming/tapping and facing processes, but these exceptions are accounted for by setting $\omega = 1$ in the model. Threading uses a specific number of cuts as a function of the thread pitch, so the factor ω will not be used.

For all lathe processes, three basic parameters determine the cutting behavior:

- **Spindle speed**: (SS) Unlike with the milling process, the work-piece itself is turning, so the spindle speed is a process input that is not dependent on the tooling used.
- **Feed per revolution**: (FPR) this is the feed rate of the material onto the fixed tool and is typically an input into the problem. It is sometimes approximated as the cutting length per minute divided by

the spindle speed, but this approximation will not be used in the current project. However, the spindle speed can be used to calculate the constraints on the value of FPR.

- **Depth of cut**: (DOC) Similar to that used in the milling process, except that it is more carefully defined for a lathe process since the work-piece itself is turning during processing. The DOC may be based on a set number of cuts (as in threading), or may be a function of the total material volume to be removed and the number of cuts needed. It should be further noted that the DOC may be a much smaller value for lathe-based processes that for milling, perhaps by an order of magnitude. The factor ω can be considered as a function of this DOC value:

$$\omega(\mathbf{x}) = \frac{V_R}{V_C(\mathbf{x})} \tag{14}$$

where, for a lathe-based process, the value can be formulated as:

$$\omega(DOC) = \frac{V_R}{\pi L(DOC)^2} \tag{15}$$

where L is the total length of the cut (mm).

Each of the major process types performed using a lathe involve a different function for calculating the processing time, so each should be calculated individually.

- The time to complete a *turning* or *boring* process (external and internal versions of the same process), in terms of the variables already defined, is:

$$t_{\text{turn}} = \omega(DOC)\frac{L_{\text{turn}}}{FPR \times SS} = \frac{V_R}{\pi(DOC)^2 \times FPR \times SS} \quad [min] \tag{16}$$

where L_{turn} is a problem input, is a constant value, and is determined by the geometry of the final product and the planning of the production process. In terms of optimization variables: $x_1 = DOC$, $x_2 = FPR$, and $x_3 = SS$, the time can be expressed as:

$$t_{\text{turn}}(x_1, x_2, x_3) = \frac{V_R}{\pi x_1^2 x_2 x_3} \quad [min] \tag{17}$$

The gradient function for t_{turn} can be calculated as:

$$\Delta t_{\text{turn}}(x_1, x_2, x_3) = \begin{bmatrix} \frac{\partial t_{\text{turn}}}{\partial x_1} \\ \frac{\partial t_{\text{turn}}}{\partial x_2} \\ \frac{\partial t_{\text{turn}}}{\partial x_3} \end{bmatrix} = \begin{bmatrix} -\frac{2V_R}{\pi x_1^3 x_2 x_3} \\ -\frac{V_R}{\pi x_1^2 x_2^2 x_3} \\ -\frac{V_R}{\pi x_1^2 x_2 x_3^2} \end{bmatrix} \tag{18}$$

The equivalent Hessian for this equation is:

$$\Delta^2 t_m(x_1, x_2, x_3) = \begin{bmatrix} \frac{\partial^2 t_{\text{turn}}}{\partial x_1^2} & \frac{\partial^2 t_{\text{turn}}}{\partial x_1 x_2} & \frac{\partial^2 t_{turn}}{\partial x_1 x_3} \\ \frac{\partial^2 t_{\text{turn}}}{\partial x_2 x_1} & \frac{\partial^2 t_{\text{turn}}}{\partial x_2^2} & \frac{\partial^2 t_{\text{turn}}}{\partial x_2 x_3} \\ \frac{\partial^2 t_{\text{turn}}}{\partial x_3 x_1} & \frac{\partial^2 t_{\text{turn}}}{\partial x_3 x_2} & \frac{\partial^2 t_{\text{turn}}}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} \frac{6V_R}{\pi x_1^4 x_2 x_3} & \frac{2V_R}{\pi x_1^3 x_2^2 x_3} & \frac{2V_R}{\pi x_1^3 x_2 x_3^2} \\ \frac{2V_R}{\pi x_1^3 x_2^2 x_3} & \frac{2V_R}{\pi x_1^2 x_2^3 x_3} & \frac{V_R}{\pi x_1^2 x_2^2 x_3^2} \\ \frac{2V_R}{\pi x_1^3 x_2 x_3^2} & \frac{V_R}{\pi x_1^2 x_2^2 x_3^2} & \frac{2V_R}{\pi x_1^2 x_2 x_3^3} \end{bmatrix} \tag{19}$$

Given that $\mathbf{x} = x_1,\ x_2,\ x_3 > 0$ and π is a non-negative constant value, the Hessian is positive semi-definite. Therefore, t_{turn} is a convex function which describes the processing time of a lathe process.

- For the *facing/parting* process, the time of cutting is

$$t_{\text{face}} = \frac{L_{\text{cut}}}{FPR \times SS} = \frac{D}{2 \times FPR \times SS} \tag{20}$$

where D is the diameter of the work-piece at the start of the operation. Note that facing operations usually take just one cut which is only deep enough to square the end of the material. Assuming that the provided raw material is in reasonably good condition and does not require extensive squaring, the depth of cut is not an important parameter to be considered in this model of manufacturing time. Therefore,

$$t_{\text{face/part}}(x_2, x_3) = \frac{D}{2x_2x_3} \quad [min] \tag{21}$$

This is in the same form as the milling time formula (Eqn. 6), so it has already been established as convex and no further proof is needed.

- The time to complete a *drilling*, *tapping*, or *reaming* operations is very simple to calculate, as it does not need to consider the diameter of the work-piece, the DOC, or the number of passes per cut. Since, in all three processes, the parameters are simply based on the depth of the hole and the speed of processing, the processing time can be calculated as:

$$t_{\text{drill}} = \frac{L_{\text{drill}}}{FPR \times SS} \quad [min] \tag{22}$$

$$t_{\text{ream}} = \frac{L_{\text{ream}}}{FPR \times SS} \quad [min] \tag{23}$$

$$t_{\text{tap}} = \frac{\frac{3}{2}(L_{\text{tap}} + D/2)}{TP \times SS} \quad [min] \tag{24}$$

where L_{drill}, L_{ream}, and L_{tap} are the desired depth of hole needed, the factor $L_{\text{tap}} + D/2$ accounts for the needed depth of the hole to allow the tap to cut the threads, and TP is the thread pitch (constant input for the problem). The time for the tapping includes the need for cutting the threads and removing the tap from the hole without damaging it. Note further that for these processes, the standard operation (with a drill press) time will be dependent on the number of holes and the time for positioning the tool. However, with a lathe, a single hole is produced about the rotational axis. In terms of the optimization variables, these machining times can be expressed as:

$$t_{\text{drill-lathe}}(x_2, x_3) = \frac{L_{\text{drill}}}{x_2x_3} \quad [min] \tag{25}$$

$$t_{\text{ream-lathe}}(x_2, x_3) = \frac{L_{\text{ream}}}{x_2x_3} \quad [min] \tag{26}$$

$$t_{\text{tap-lathe}}(x_3) = \frac{\frac{3}{2}(L_{\text{tap}} + D/2)}{TPx_3} \quad [min] \tag{27}$$

These are all in the same form as functions previously shown to be convex, so no further convexity proof is needed here.

- Finally, the threading operation requires a simple calculation and it is dependent only on the spindle speed and the number of cuts to complete the threads. It is typical, however, to assign a number of cuts NC for the process (thereby automatically generating DOC values) where:

$$NC_{\text{fine thread}} = 32 \times TP$$
$$NC_{\text{rough thread}} = 25 \times TP$$

Therefore, the time required to do threading is:

$$t_{\text{thread}} = \frac{NC \times L_{\text{thread}}}{TP \times SS} \quad [min] \tag{28}$$

In terms of the optimization variables, this will be:

$$t_{\text{thread}} = \frac{NC \times L_{\text{thread}}}{TPx_3} \quad [min] \tag{29}$$

Calculating the power required for the lathe process is much more simple than calculating the manufacturing time, as the only driver in the lathe is the main motor turning the spindle. The amount of power required to run the lathe is primarily dependent on the force generated by the tool cutting into the material as it rotates. The cutting speed CS $[m/min]$ for lathe-based processes is the speed of the point contacting the tool and is calculated as:

$$CS = \frac{\pi \times D \times SS}{1000} \quad [m/min] \tag{30}$$

For the turning, facing, boring, and threading processes, this can be expressed as:

$$\begin{aligned} P_{t,b,th} &= \frac{1}{u_p} \times CS \times DOC \times FPR \times F_c = \frac{1}{u_p} \times \frac{\pi \times D \times SS}{1000} \times DOC \times FPR \times F_c \\ &= \frac{\pi \times D \times SS \times DOC \times FPR \times F_c}{60 \times 10^6} \quad [kW] \end{aligned} \tag{31}$$

where the factor 60×10^6 is a conversion factor to obtain units in terms of kW. Note that for the facing process, the DOC value is a constant value d, not a variable. The drilling, reaming, and tapping process will require a different perspective on the power consumption, as the diameter is fixed and it is cutting along the rotational axis of the work-piece. There are several ways to calculate it, but one of the most common is:

$$\begin{aligned} P_{d,r,ta} &= \frac{1}{u_p} \times CS_{\text{drill}} \times \theta \times FPR \times F_c = \frac{1}{u_p} \times \frac{\pi \times \theta \times SS}{1000} \times \theta \times FPR \times F_c \\ &= \frac{\pi \times SS \times \theta^2 \times FPR \times F_c}{240 \times 10^6} \quad [kW] \end{aligned} \tag{32}$$

where θ is the diameter of the tool, F_c is the force applied to the drill (assumed to be a constant input), and the factor $1/(240 \times 10^6)$ is a conversion factor to produce units in $[kW]$. Note that this is not a convex function as stated, so it is necessary to reformulate it as a convex function for this study. Since the variables can easily be measured as stated or as inverses, it is reasonable to use $x_4 = 1/FPR$ $[rev/mm]$, $x_5 = 1/SS$ $[min/rev]$, and $x_6 = 1/DOC$ $[1/mm]$ as optimization variables. The power functions then become:

$$P_{t,b,th}(x_4,x_5,x_6) = \frac{\pi F_c}{60\times 10^6(x_4x_5x_6)} \quad \text{(turning, boring, threading)} \tag{33}$$

$$P_{t,b,th}(x_4,x_5) = \frac{\pi d^2 F_c}{60\times 10^6(x_4x_5)} \quad \text{(facing)} \tag{34}$$

$$P_{d,r,ta}(x_4,x_5) = \frac{\theta^2 F_c}{240\times 10^3(x_4x_5)} \quad \text{(drilling, reaming, tapping)} \tag{35}$$

2.3 Drilling Time and Power Models

Drilling processes perform the same basic operations as described previously in the lathe-based processes (drilling, reaming, and tapping). The manufacturing time is calculated in a way that is identical to the lathe processes, with the exception that the values of SS are replaced with TS values (as in milling), and the fact that the process typically produces a hole pattern and not just a single hole. Therefore, for number of holes NH and idle time t_{move} to move between holes, the manufacturing time is:

$$t_{\text{drill},dp} = \frac{NH\times L_{\text{drill}}}{FPR\times TS} + t_{\text{move}}(NH-1) \quad [min] \tag{36}$$

$$t_{\text{ream},dp} = \frac{NH\times L_{\text{ream}}}{FPR\times TS} + t_{\text{move}}(NH-1) \quad [min] \tag{37}$$

$$t_{\text{tap},dp} = \frac{\frac{3}{2}\times NH\times(L_{\text{tap}}+D/2)}{TP\times TS} + t_{\text{move}}(NH-1) \quad [min] \tag{38}$$

Letting $x_1 = FPR$ $[mm/rev]$, and $x_2 = TS$ $[RPM]$, the processing time for the basic drilling processes are:

$$t_{\text{drill},dp}(x_1,x_2) = \frac{(NH)L_{\text{drill}}}{x_1x_2} + t_{\text{move}}(NH-1) \quad [min] \tag{39}$$

$$t_{\text{ream},dp}(x_1,x_1) = \frac{(NH)L_{\text{ream}}}{x_2x_2} + t_{\text{move}}(NH-1) \quad [min] \tag{40}$$

$$t_{\text{tap},dp}(x_2) = \frac{\frac{3}{2}(NH)(L_{\text{tap}}+D/2)}{TPx_2} + t_{\text{move}}(NH-1) \quad [min] \tag{41}$$

These are in the same form as the lathe-based drilling formulas, which were already established as convex. Therefore, in the domain $x_1, x_2 > 0$, the processing time for the drilling processes is a convex function.

In a similar way, the power function for the drilling process can be formulated the same was as that of the lathe drilling/reaming/tapping, with the exception that there will likely be more than one hole to process and that the tool will be turning instead of the work-piece. Therefore,

$$P_{d,r,ta} = \frac{1}{u_p}\times NH\times CS_{\text{drill}}\times\theta\times FPR\times F_c = \frac{1}{u_p}\times NH\times\frac{\pi\times\theta\times TS}{1000}\times\theta\times FPR\times F_c \tag{42}$$

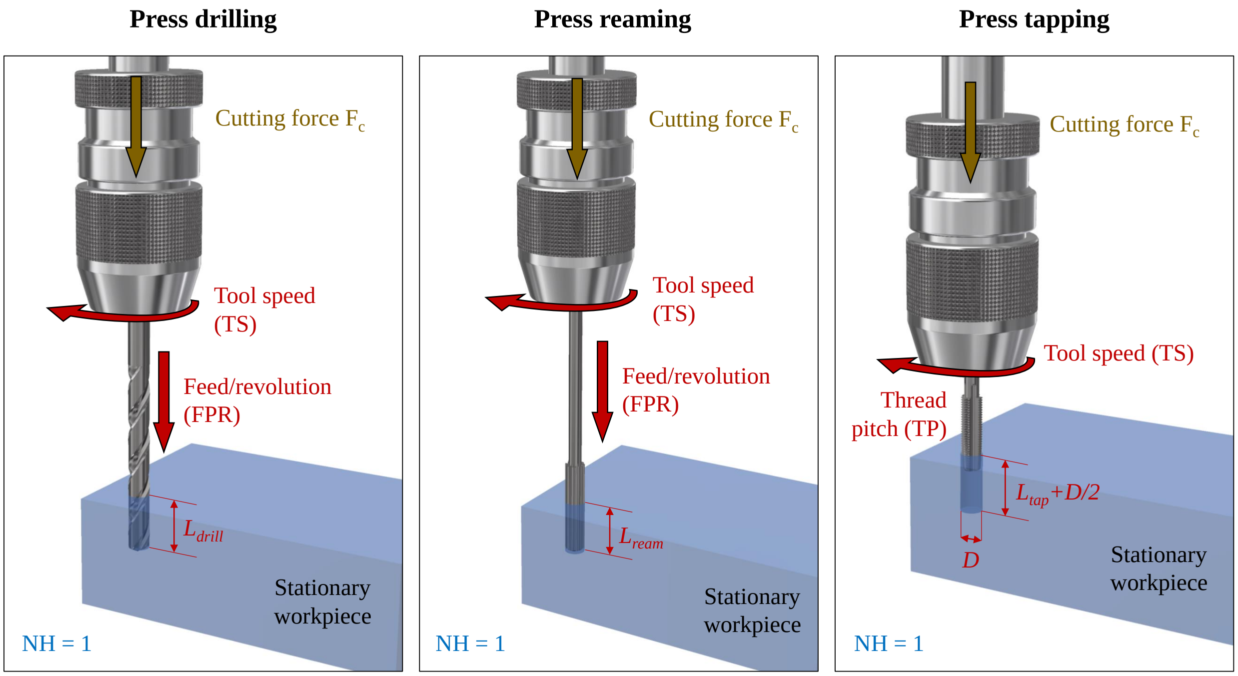

Figure 4: *Milling process (a) basic milling machine and (b) parameters*

$$= \frac{\pi \times NH \times TS \times \theta^2 \times FPR \times F_c}{240 \times 10^6} \quad [kW]$$

Putting this into the form of design variables $x_4 = 1/FPR$ $[min/mm]$ and $x_5 = 1/TS$ to ensure a convex function for NH total holes,

$$P_{d,r,ta}(x_4, x_5) = \times \frac{(NH)\theta^2 F_c}{240 \times 10^3 (x_4 x_5)} \quad [kW] \text{ (drilling, reaming, tapping)} \tag{43}$$

2.4 Grinding and Polishing Time and Power Models

A grinding and polishing process is usually a standard feature of manufacturing cells, as it is needed to ensure that tolerances are met and that the products are the final desired shape. Mathematically, the grinding process works in a similar way to the machining process except for the definitions of some parameters and the fact that the volume of material removed is usually very small compared to that of milling. The time for a grinding/polishing process can be described by

$$t_{\text{grind}} = \frac{(NP)V_R}{\eta \times \theta \times DOC \times CF \times TS} = \frac{w_p V_R}{\eta^2 \times \theta^2 \times DOC \times CF \times TS} \quad [min] \tag{44}$$

where V_R $[mm^3]$ is the volume of material to be removed (the value for polishing is very small compared to the value for grinding), θ $[mm]$ is the width of the tool surface in contact with the work-piece (typically a grind stone or polishing brush), DOC $[mm]$ is the depth of cut, CF $[mm/rev]$ is the cutting feed, and TS $[RPM]$ is the rotational speed of the tool. The value $\eta \in [0,1]$ describes the fraction of the tool in contact

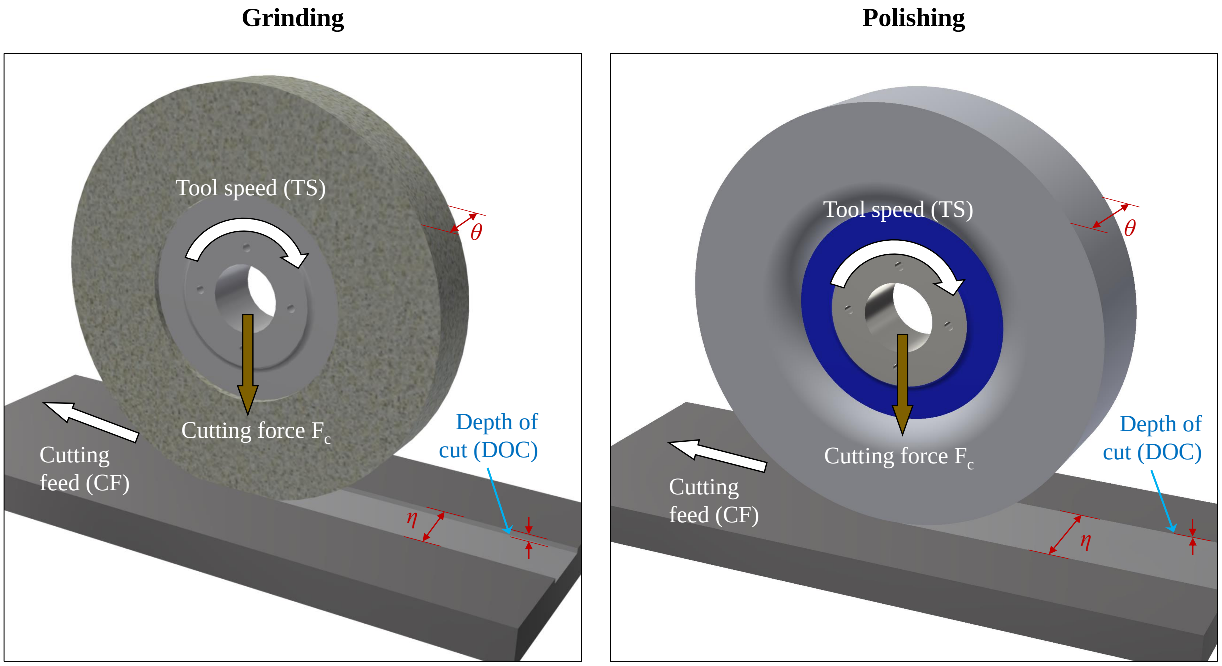

Figure 5: *Milling process (a) basic milling machine and (b) parameters*

with the surface on each pass. Finally, the factor NP is the number of passes required to complete the grinding, which can be described as the surface width divided by the tool engagement width $w_s/(\eta \times \theta)$. In terms of optimization variables $x_1 = DOC$, $x_2 = CF$, and $x_3 = TS$, this can be formulated as:

$$t_{\text{grind}}(x_1, x_2, x_3) = \frac{w_p V_R}{\eta^2 \theta^2 x_1 x_2 x_3} \tag{45}$$

This equation is the in the same form as the milling model, which was already shown to be convex so no further proof is needed.

The power requirement for the grinding process is similar to that of milling, with the same exceptions in the variable definitions as used for the grinding time. Since the power required is not dependent on time, the number of passes of the grinder is not a consideration here. Therefore,

$$P_{\text{grind}} = \frac{1}{u_p} DOC \times \eta \times \theta \times CF \times TS \times F_c \;\; [kW] \tag{46}$$

As in previous derivations, letting $x_4 = 1/DOC$ $[1/mm]$, $x_5 = 1/CF$ $[rev/mm]$, and $x_6 = 1/TS$ $[min/rev]$, the grinder power function is:

$$P_{\text{grind}}(x_4, x_5, x_6) = \frac{\eta \theta F_c}{u_p x_4 x_5 x_6} \tag{47}$$

where $u_p = 60 \times 10^6$ is the unit conversion to obtain power output in terms of kW, and F_c $[N/mm^2]$ is the force applied to the surface. For a polishing process, the time and power functions are the same, but the constraints on the variables will be different.

References

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