PROFIT MAXIMIZATION WITH CUSTOMER SATISFACTION CONTROL FOR ELECTRIC VEHICLE CHARGING IN SMART GRIDS

A Dissertation

by

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ABSTRACT

As the market of electric vehicles is gaining popularity, large-scale commercialized or privately-operated charging stations are expected to play a key role as a technology enabler. In this dissertation, we study the problem of charging electric vehicles at stations with limited charging machines and power resources. Our electric vehicle charging station is composed of a central controller, multiple charging machines, and a plurality of parking lots. Each parking lot has a plug connectable to an arbitrary charging machine through a *switching bar system*. The switching bar system allows the station owner to serve a larger number of customers at the same time by enabling dynamic connections, where the number of charging machines could be much less than the number of plugs. The central controller collects all the information provided by the customers in advance or on the fly and decides when to activate or de-activate a machine-to-plug connection, how fast the vehicles should be charged, and how much energy should be delivered to each vehicle.

The purpose of this study is to develop a novel profit maximization framework for charging station operation in both offline and online charging scenarios, under certain customer satisfaction constraints. The main goal is to maximize the profit obtained by the station owner and provide a satisfactory charging service to the customers. The framework includes not only the vehicle scheduling and charging power control, but also the managing of user satisfaction factors, which are defined

as the percentages of finished charging targets. The profit maximization problem is proved to be NP-complete in both scenarios, for which two-stage charging strategies are proposed to obtain efficient suboptimal solutions. Competitive analysis is also provided to analyze the performance of the proposed online two-stage charging algorithm against the offline counterpart under non-congested and congested charging scenarios.

Finally, the simulation results show that the proposed two-stage charging strategies have remarkable performance gains compared to the exhaustive search and other conventional charging strategies with respect to not only the unified profit, but also other practical interests, such as the computational time, the user satisfaction factor, the percentage of electric vehicles serviced, the power consumption, the competitive ratio, and the load factor.

DEDICATION

This dissertation is dedicated to my beloved family, for their support along the way.

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NOMENCLATURE

EV	Electric Vehicle
NP	Non-deterministic Polynomial-time
NP-Hard	Non-deterministic Polynomial-time Hard
NP-Complete	Non-deterministic Polynomial-time Complete
LP	Linear Programming
MILP	Mixed Integer Linear Programming
KKT	Karush-Kuhn-Tucker
$O(\ )$	Big O Notation
FIFO	First-In First-Out
QoS	Quality of Service
MATLAB	Matrix Laboratory Software
CVX	MATLAB Software for Convex Optimization
PC	Personal Computer
CPU	Central Processing Unit
RAM	Random Access Memory

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1. INTRODUCTION

1.1 Background

Electric Vehicle (EV) is a promising solution to future green transportation needs due to its economic and environmental benefits, such as fuel economy, reduction of petroleum consumption, and reduction of environmental pollution [1]-[2]. According to the US Environmental Protection Agency [3], a typical EV can transform about 59% to 62% of the electrical energy to power, while conventional gasoline vehicles can only transform about 17% to 21% of the energy stored in gasoline to power. Moreover, it is well known that fossil fuel energy sources are becoming more and more scarce. EVs help us mitigate this problem by utilizing other energy sources, such as wind power, solar energy, hydroelectric energy, ocean energy, etc. In addition, the adoption of EVs will help reduce the global emission of CO_2 by half by 2050 as predicted in [1], which will significantly reduce the environmental pollution.

EVs are mainly divided into three categories: Hybrid Electric Vehicles (HEV), Plug-in Hybrid Electric Vehicles (PHEV), and Full Electric Vehicles (FEV or EV) [4].

- The HEV model combines both gasoline and electric propulsion systems. In a typical HEV, the electric motor assists the internal combustion engine. An example of HEV is the Toyota Prius C, which provides about 1.5 miles of electric-only driving with a 1 kWh battery (charged every time the vehicle brakes) and a maximum speed of 104 mph.

- The PHEV model also combines both gasoline and electric propulsion systems, but the vehicle is powered mainly by the electric propulsion systems. This type of EV needs to be recharged from an external source of electricity. An example of PHEV is the Chevy Volt, which provides about 53 miles of electric-only driving with a fully charged 18.4 kWh battery and a maximum speed of 100 mph.

- The FEV (or EV) utilizes one or more electric motors powered by rechargeable battery packs, and thus have no internal combustion engine, fuel cells, or fuel tanks. This type of EV also needs to be recharged from an external source of electricity. Examples of FEV (or EV) are Tesla Model S and Nissan Leaf, which provide respectively about 253 and 107 miles of electric-only driving with a fully charged 85 kWh and 30 kWh batteries and 150 and 100 mph of maximum speed.

In this work, we study the charging problem for PHEV and EV models. Fig. 1.1 shows the expected number of models offered and sales per model for PHEV and EV through 2020, respectively [1].

Currently, there are mainly three methods of recharging EVs: battery swapping, residential charging, and public charging.

- In battery swapping, the EV owner can swap a discharged battery for a fully

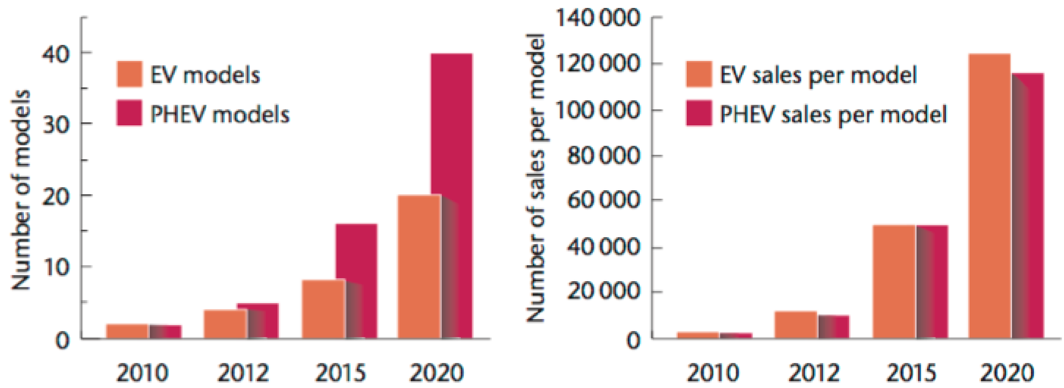

Figure 1.1: Number of models offered and sales per model for PHEV and EV through 2020 [1]

charged one, saving the waiting time in the charging process.

- In residential charging, the EV owner connects the vehicle to the power grid of the household. This method provides slow and regular charging speeds.
- In public charging, the EV owner connects the vehicle to an available public charging station. This method provides optional slow, regular, fast, and superfast charging speeds.

In this work, we focus on the public charging scenario, where the total power demand will cause an additional load on the power grid that might seriously affect the grid stability when each EV is charged at a fixed high charging speed and the number of EVs is large. Table 1.1 shows the fixed charging speed and time provided by a typical charging system. On the other hand, the current power grid infrastructure might not be able to support a large number of EVs being charged simultaneously

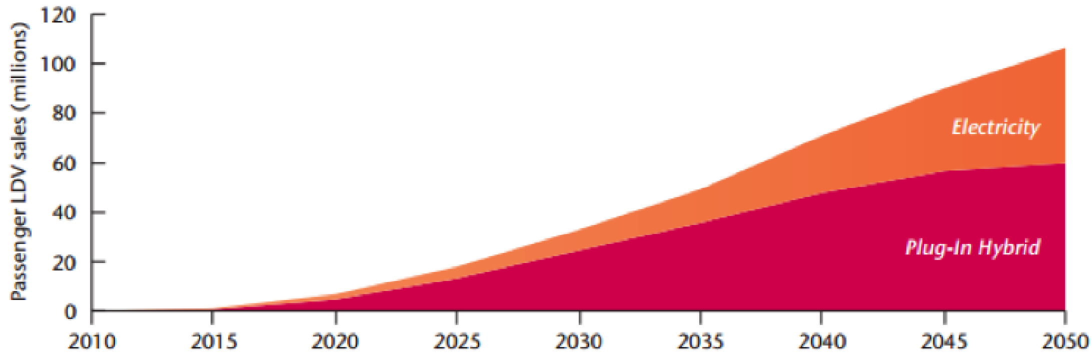

Figure 1.2: Global EV and PHEV sales from 2010 to 2050 [1]

[5]-[8]. According to the International Energy Agency [1], the adoption of electric vehicles will increase exponentially from 2010 to 2050, achieving an annual global sale of 106 million EVs, as shown in Fig. 1.2. In addition, the vehicle industry predicts a global adoption of 20 million EVs by the end of 2020 [7], which will increase the power load to approximately 60 GW when all EVs are charged at the same time at a slow charging speed (e.g., 3kW/h [9]). Therefore, it is important to take into account the power overloading issues when designing the EV charging strategies.

Table 1.1: Charging speed and time provided by a typical charging system

Charging time	Power supply
6-8 hrs	3.3 KW
3-4 hrs	7 KW
1-2 hrs	22 KW
20-30 mins	50 KW
10 mins	120 KW

1.2 Related Work

Two types of EV charging solutions with power control have been studied in past years based on the mode of charging station operation: offline EV charging (the station knows the present and future charging information, say through a reservation system) and online EV charging (the station knows only the present charging information).

Many studies have been conducted to analyze the EV charging problems in residential[1] environments for both offline [10]-[13] and online [14]-[18] solutions. In contrast to residential charging, public[2] charging can serve EV customers in more flexible places and provide faster charging services. The authors in [19]-[20] studied the offline EV charging problem in a public environment. Due to the difficulty of collecting charging information in advance in the public domain, several works have been conducted to study the online EV charging problem in a public environment [21]-[23].

The general scheduling problem in multiple-machine scenarios has been extensively studied during the past decades [24]-[26]. For the offline EV charging-scheduling problem, the authors in [27] presented a two-stage cost minimization framework that optimizes the power allocation, the energy price, and the EV scheduling. The presented framework minimizes the power losses and voltage deviations at the first stage and the cost of users at the second stage. In [28], an EV charging mechanism was

[1]Each household owns a private station to charge the owner's vehicle.
[2]Each public facility owns multiple charging machines to serve a large number of EVs.

proposed to optimize the EV scheduling in order to reduce the total charging time. They formulated this problem as an integer programming (IP) problem that is proved to be NP-complete. Two heuristic algorithms were proposed: the Earliest Start Time (EST) algorithm and the Earliest Finish Time (EFT) algorithm. Also, several approaches have been presented to study the online EV charging-scheduling problem [29]-[31]. The authors in [29]-[30] presented online charging scheduling mechanisms to maximize the unified profit of the system in single-machine and multiple-machine scenarios, respectively. In [31], an online charging strategy was proposed to schedule EV charging among multiple charging stations and allocate power to the EVs in order to minimize the time-averaged electricity cost.

1.3 Purpose

Note that all the works in [10]-[23] consider EV charging scenarios with a sufficient number of machines to satisfy the charging requests of all customers. However, as aforementioned, the number of EVs will increase drastically in the next few years, which shows the importance of developing scheduling strategies to accommodate more EVs and better utilize the charging station resources. We can claim that the EV charging industry will face two main problems in the future: the high power demand and the lack of sufficient charging machines to serve all customers.

It is also worth noticing that all the works in [10]-[31] aim to provide a full-charge service to their customers. However, under large-scale scenarios, if the station must fully charge each EV, the high power demand and charging facility requirements might negatively impact the profit of the operator. In this work, we introduce the concept of user satisfaction factor control. The main idea is to adjust the fulfilled percentage of the charging target for each user in order to strike a balance between the profit and the quality of service (QoS).

Our work focuses on maximizing a unified profit for the EV charging, while providing a satisfactory charging service for both offline and online scenarios. Based on our results, we claim that the proposed EV charging strategies not only maximize the station profit, but also address the issues of power overloading and charging station shortage.

The main contributions of this study are summarized as follows:

- A profit maximization framework for charging is proposed, which jointly schedules EVs, allocates power, and adjusts the user satisfaction factor, under peak power and charging facility constraints. It is shown that the profit maximization problem is NP-complete in both offline and online scenarios.

- An efficient two-stage charging strategy is proposed to solve the profit maximization problem for each charging scenario.

- The computational complexity is analyzed for both proposed offline and online algorithms, where it shows that the greedy scheduling algorithms outperform the LP relaxation scheduling algorithms in term of computational time by slightly sacrificing the overall profit.

- A competitive analysis for the online two-stage charging algorithm is provided. The lower bound of competitive ratio is derived in terms of the unified profit for a special case.

- Simulation results show that the proposed offline and online two-stage LP and greedy strategies provide remarkable results with respect to the profit, the user satisfaction factors, the percentage of EVs serviced, the power consumption, the load factor of the system, the competitive ratio, and the computational time.

The rest of the work is organized as follows. In Section 2, we describe the system model and present the profit maximization framework under both offline and online charging setups. In Section 3, we introduce an offline two-stage charging strategy and analyze its properties. Similarly, we introduce an online two-stage charging strategy and analyze its properties in Section 4. In addition, we provide a competitive ratio analysis to analyze the proposed online two-stage charging algorithms. In both Sections 3 and 4, we present simulation results to illustrate the performance of the proposed two-stage charging strategies. Finally, we provide the main results and contribution of our research work and discuss some promising future research problems in Section 5.

2. SYSTEM MODEL AND PROBLEM FORMULATION

2.1 System Model

Suppose that the EV charging operator owns C charging machines that operate in a time-slotted fashion. During the day, a total of N vehicles arrive at the facility, and are accommodated in the station's parking lots, where each lot has a plug connectable to an arbitrary charging machine through a *switching bar system*, as shown in Fig. 2.1.

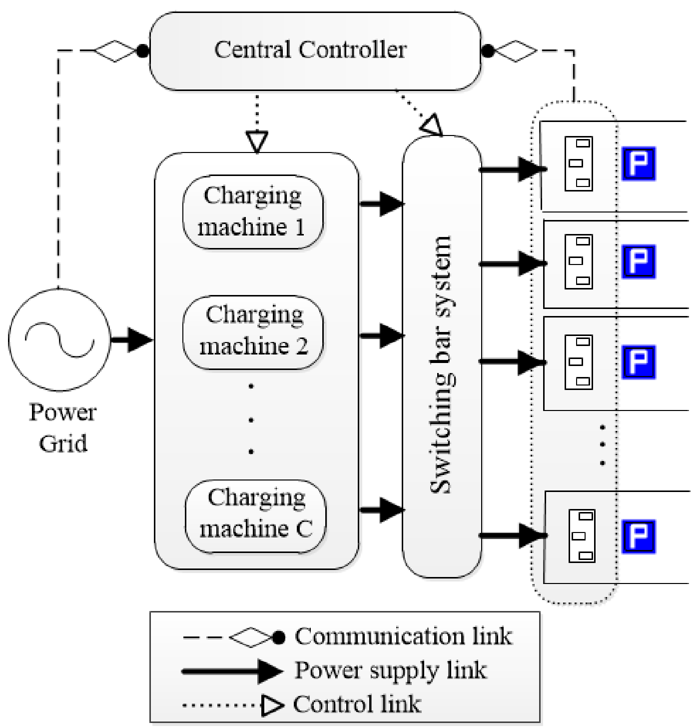

Figure 2.1: Charging station with multiple machines and plugs

The switching bar system allows the station owner to serve a large number of customers at the same time by enabling dynamic connections from an arbitrary plug to an arbitrary charging machine, where the number of charging machines could be much less than the number of plugs (i.e., the number of charging parking lots). The central controller collects all the information provided by the customers in advance or on the fly and decides when to activate or de-activate a machine-to-plug connection, how fast the EVs should be charged, and how much energy should be delivered to each vehicle. All the charging machines are assumed to be identical, and thus which parking lot is used does not affect the charging performance. As such, an EV $i \in \{1, 2, \dots, N\}$ can be charged at any parking lot by any charging machine $j \in \{1, 2, \dots, C\}$ to deliver a unified performance with $C << N$.

We consider a finite time horizon (e.g. 24 hours) that contains T time slots. For each EV i, let the charging job be described by the arrival time $r_i \in [1, T]$, the departure time $d_i \in [1, T]$, and the required energy w_i, where $r_i < d_i \leq T$. The charging period of each EV is denoted by $T_i = [r_i, d_i]$ and its length is given by $|T_i| = d_i - r_i + 1$.

In the offline charging scenario, we assume the station is equipped with an web-based reservation system such that every EV owner can book both the parking lot and charging time window in advance. The station utilizes the above information to design the charging strategy to obtain the maximum profit. In contrast, in the online charging scenario, the operator learns the charging information on the fly after the EVs are connected to the system. Therefore, the station can only maximize the instantaneous profit by optimizing the charging process based on the available information of the customers currently connected and just arrived. Due to the lack of information about future requests, the online charging strategies are forced to make real-time decisions that may later turn out to be suboptimal. Thus, it is clear that the online charging mechanisms will often perform worse than their offline counterparts.

Next, we first discuss the overall EV charging-scheduling problem for the offline scenario.

2.2 Offline EV Charging-Scheduling Problem

In the offline EV charging-scheduling problem the goal is to maximize the unified overall profit for the charging station by jointly optimizing over the EV scheduling, the charging power, and the user satisfaction factors that are defined as the percentages of charging given the desired target energy w_i. The scheduling of EVs at time t is represented by $X^t = \{x^t_{i,j}\}$, where $1 \leq i \leq N$, $1 \leq j \leq C$, and $x^t_{i,j}$ is given by

$$x^t_{ij} = \begin{cases} 1, & \text{if the } i^{th} \text{ EV is assigned to the } j^{th} \text{ charging machine at time } t, \\ 0, & \text{otherwise.} \end{cases}$$

Then, the overall scheduling is denoted by $X = \{X^1, X^2, \ldots, X^T\}$. Similarly, the charging power at time slot t is represented by $P^t = \{p^t_{i,j}\}$, where $p^t_{i,j}$ is the charging power level of the j^{th} machine for the i^{th} EV at time t. Here, we assume that the $p^t_{i,j}$ is limited by a safe maximum charging power p_{safe}, which is a system constant set by the operator in advance. The overall power allocation is denoted by

$P = \{P^1, P^2, \ldots, P^T\}$. Note that we normalize the slot length such that the power allocation vector is also the energy charging vector. Now, let $\gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_N\}$ denote the set of user satisfaction factors, at which each vehicle is serviced at the end of the schedule, where $\gamma_i \in [\gamma_{\min}, 1]$ for $1 \leq i \leq N$. Here, we assume that the minimum user satisfaction factor $\gamma_{\min}$ is also a system constant set by the operator in advance.

Next, we define the unified profit function for offline charging, which is formulated as the difference between a linear revenue function and a quadratic cost function in order to make the profit maximization problem economically plausible and computationally simple [32]. Let $\alpha > 0$ be the price per unit of electrical energy (e.g., \$/kWh) sold to the customers, and the revenue function is given as

$$U(X, P) = \alpha \sum_{t=1}^{T} \sum_{j=1}^{C} \sum_{i=1}^{N} p_{ij}^t x_{ij}^t.$$

The operation cost of the station includes two parts:

- The power consumption cost is given by

$$C_1(X, P) = \beta \sum_{t=1}^{T} \left(\sum_{j=1}^{C} \sum_{i=1}^{N} p_{ij}^t x_{ij}^t \right)^2,$$

  where $\beta > 0$ is the purchase cost weighting parameter. Note that the quadratic dependence reflects the fact that the per unit cost of power consumption for the operator increases at a higher rate than the revenue as the total demand increases, which is matching the differential pricing strategy in power market [33].

- The second part of operation cost is the satisfaction penalty, which is given by

$$C_2(X,\gamma) = \eta \sum_{j=1}^{C} \sum_{i=1}^{N} \left(w_i - \gamma_i w_i\right)^2 \mathbb{1}_{\left\{\sum_{t=1}^{T} x_{ij}^t \geq 1\right\}},$$

  where $\eta > 0$ is the penalty weighting parameter. The function $C_2(X,\gamma)$ was defined to be the residual sum of the squared discrepancy between the delivered and the desired values.

We assume that α, β, and η are constants over time and known by the system in advance. With the notations introduced above, the unified profit of the system is given by:

$$F(X,P,\gamma) = U(X,P) - \left[C_1(X,P) + C_2(X,\gamma)\right].$$

Thus, the overall offline EV charging-scheduling problem can be formulated as follows:

$$
\begin{aligned}
&\underset{X,P,\gamma}{\text{maximize}} && F(X,P,\gamma) && \text{(Problem 1)}\\
&\text{subject to} && \sum_{j=1}^{C}\sum_{i=1}^{N} p_{ij}^{t}x_{ij}^{t} \le p_{\max}, t = 1,\ldots,T; && (2.1)\\
& && \gamma_i w_i \le \sum_{t=r_i}^{d_i} p_{ij}^{t}x_{ij}^{t} \le w_i, i = 1,\ldots,N, j = 1,\ldots,C; && (2.2)\\
& && \sum_{j=1}^{C} x_{ij}^{t} \le 1, i = 1,\ldots,N, t = 1,\ldots,T; && (2.3)\\
& && \sum_{i=1}^{N} x_{ij}^{t} \le 1, j = 1,\ldots,C, t = 1,\ldots,T; && (2.4)\\
& && x_{ij}^{t} \in \{0,1\}, i = 1,\ldots,N, j = 1,\ldots,C, t = 1,\ldots,T; && (2.5)\\
& && 0 \le p_{ij}^{t} \le p_{\text{safe}}, i = 1,\ldots,N, j = 1,\ldots,C, t = 1,\ldots,T; && (2.6)\\
& && \gamma_{\min} \le \gamma_i \le 1, i = 1,\ldots,N. && (2.7)
\end{aligned}
$$

Here, (2.1) ensures that the total power allocation at each time slot does not exceed the power limit $p_{\max}$; (2.2) guarantees that every EV will be charged at or above the minimum user satisfaction factor provided by the station; (2.3) and (2.4) indicate that every single machine can only charge one vehicle at a time and each EV can only be charged by one machine at a time; (2.5) defines the charging machine assignment indicator; (2.6) defines the feasible range for $p_{i,j}^{t}$, which is limited by a safe maximum charging power p_{safe} for EVs; and (2.7) requires the user satisfaction factor to meet a minimum target.

Notice that problem (1) is a mixed integer linear programming (MILP) problem due to the EV scheduling constraint (2.5). Theorem 2.2.1 below establishes the complexity of solving problem (1) in an offline charging scenario.

Theorem 2.2.1 The offline EV charging-scheduling problem (1) is NP-complete.

Proof: To prove a problem is NP-complete, we need to show it is both NP and NP-hard. First, we prove that problem (1) is NP. Recall that a problem is considered to be NP if the verification process can be done in polynomial time. We assume that we are given some instances and S is our certificate. A deterministic algorithm verifies whether each EV $i \in \{1, 2, \ldots, N\}$ is assigned to any charging machine $j \in \{1, 2, \ldots, C\}$. Then, it checks if the total number of vehicles being charged is less than or equal to C and if the total power consumption is less than or equal to $p_{\max}$ at each time slot. Notice that the verification process can be completed in polynomial time $O(NC)$. Therefore, our problem is NP.

Next, consider the special case when the power is uniformly allocated and the user satisfaction factors are set to be equal to 1, which means all EVs must be fully charged. Problem (1) is then reformulated as:

$$\underset{X}{\text{maximize}} \quad \sum_{j=1}^{C}\sum_{i=1}^{N}\left(\alpha w_i - \beta\frac{w_i^2}{|T_i|} - 2\beta\frac{w_i}{|T_i|}\sum_{k=1,k\neq i}^{N}\frac{|T_{ik}|w_k}{|T_k|}\right)x_{ij} \qquad \text{(Problem 2)}$$

$$\text{subject to} \quad \sum_{j=1}^{C}\sum_{i=1}^{N}\frac{w_i}{|T_i|}x_{ij}^t \leq p_{\max}, t = 1,\ldots,T; \qquad (2.8)$$

$$\sum_{j=1}^{C} x_{ij}^t \leq 1, i = 1,\ldots,N, t = 1,\ldots,T; \qquad (2.9)$$

$$\sum_{i=1}^{N} x_{ij}^t \leq 1, j = 1,\ldots,C, t = 1,\ldots,T; \qquad (2.10)$$

$$x_{ij}^t \in \{0,1\}, i = 1,\ldots,N, j = 1,\ldots,C, t = 1,\ldots,T, \qquad (2.11)$$

where $x_{ij} = \sum_{t=1}^{T} x_{ij}^t$ and $|T_{ik}|$ is the number of time slots when job i and job k overlap.

Then, we prove that problem (2) is NP-hard. Notice that problem (2) can be reduced from the fixed time interval scheduling (IS) with parallel machines problem [34]-[36], or the resource allocation problem [37]-[39], which both have been largely studied and proven to be NP-hard. Thus, according to the reducibility principle, we can claim that problem (2) is at least as "hard" as those problems, which implies it is also NP-hard.

Since problem (1) contains the combinatorial optimization problem (2) for fixed power allocation and user satisfaction factor, we can claim that problem (1) is also NP-hard. Therefore, we conclude that problem (1) is both NP and NP-hard, which proves its NP-completeness. ■

The optimal solution to NP-complete problems can be obtained by exhaustive search, but the computational cost is far too high. In Section 3, we propose an offline two-stage charging strategy to find a suboptimal solution to problem (1).

Next, we discuss the EV charging-scheduling problem for the online scenario to address the issue when future EV request information is not available.

2.3 Online EV Charging-Scheduling Problem

In this scenario, the station only has knowledge about the present charging requests. In addition, statistical information about future charging requests is not considered, and therefore the station can only control the charging process of the customers currently connected and just arrived. We propose a deterministic and greedy online EV charging-scheduling approach that jointly optimizes the EV scheduling, the power allocation, the user satisfaction factors to maximize the instantaneous profit for the EVs connected to the station.

When a new EV n arrives at the station, let J_n be the set of EVs connected to the system and $L_n = [r_n, \max_{k \in J_n} d_k]$ be the charging period from the arrival time of EV n to the latest departure time of the EVs in J_n. Both J_n and L_n are updated at every arrival time. The remaining desired energy target is also updated for all EVs already connected. Since the system already knows the charging energy delivered to each EV before the arrival of the new EV n, the central controller can recalculate the remaining desired energy target by $w_i^{L_n} = w_i - v_i^{L_n}$, where $i \in J_n$ and $v_i^{L_n} = \sum_{t=1}^{r_n - 1} p_{ij}^t$ is the energy already delivered before the arrival time r_n of EV n.

Our main goal is to maximize the instantaneous profit of the operator during the period L_n by jointly optimizing over the EV scheduling, the charging power, and the user satisfaction factors that are modified as the percentages of charging given the updated desired energy target $w_i^{L_n}$. Similar to the offline section, the schedule of all EVs at time t is represented by $X^t = \{x_{i,j}^t\}$, where $i \in J_n$, $1 \leq j \leq C$, and $t \in L_n$.

Then, the overall scheduling during the charging period L_n is denoted by $X^{L_n} = \{X^t : t \in L_n\}$. Similarly, the charging power at time slot t is represented by $P^t = \{p_{i,j}^t\}$, where $p_{i,j}^t$ is the charging power level of the j^{th} machine for the i^{th} EV at $t \in L_n$. As such, the overall power allocation is denoted by $P^{L_n} = \{P^t : t \in L_n\}$. Now, let $\gamma^{L_n} = \{\gamma_1^{L_n}, \gamma_2^{L_n}, \ldots, \gamma_{|J_n|}^{L_n}\}$ denote the set of user satisfaction factors, at which each vehicle is serviced at the end of the period L_n, where $\gamma_i^{L_n} \in [\gamma_{\min}, 1]$ for $i \in J_n$.

Similarly, we define the profit function for the online charging as the difference between a linear revenue function and a quadratic cost function for a given charging period L_n. Let $\alpha > 0$ be the price per unit of electrical energy (e.g., \$/kWh) sold to the customers, and the revenue function is given as

$$U(X^{L_n}, P^{L_n}) = \alpha \sum_{t \in L_n} \sum_{j=1}^{C} \sum_{i \in J_n} p_{ij}^t x_{ij}^t.$$

The operation cost of the station includes two parts:

- The power consumption cost is given by

$$C_1(X^{L_n}, P^{L_n}) = \beta \sum_{t \in L_n} \left(\sum_{j=1}^{C} \sum_{i \in J_n} p_{ij}^t x_{ij}^t \right)^2 ,$$

  where $\beta > 0$ is the purchase cost weighting parameter.

- The second part of operation cost is the satisfaction penalty, which is given by

$$C_2(X^{L_n}, \gamma^{L_n}) = \eta \sum_{j=1}^{C} \sum_{i \in J_n} \left(w_i^{L_n} - \gamma_i^{L_n} w_i^{L_n} \right)^2 \mathbb{1}_{\left\{ \sum_{t \in L_n} x_{ij}^t \geq 1 \right\}} ,$$

  where $\eta > 0$ is the penalty weighting parameter.

Here, we also assume that α, β, and η are constants over time and known by the system in advance. With the notations introduced above, the instantaneous profit of the system for the online charging is given by:

$$F(X^{L_n}, P^{L_n}, \gamma^{L_n}) = U(X^{L_n}, P^{L_n}) - \left[C_1(X^{L_n}, P^{L_n}) + C_2(X^{L_n}, \gamma^{L_n}) \right] .$$

The overall online EV charging problem for a given charging period L_n is formulated as follows:

$$\underset{X^{L_n}, P^{L_n}, \gamma^{L_n}}{\text{maximize}} \quad F(X^{L_n}, P^{L_n}, \gamma^{L_n}) \qquad \text{(Problem 3)}$$

subject to

$$\sum_{j=1}^{C} \sum_{i \in J_n} p_{ij}^t x_{ij}^t \leq p_{\max}, t \in L_n; \tag{2.12}$$

$$\gamma_i^{L_n} w_i^{L_n} \leq \sum_{t=r_i}^{d_i} p_{ij}^t x_{ij}^t \leq w_i^{L_n}, i \in J_n, j = 1, \ldots, C; \tag{2.13}$$

$$\sum_{j=1}^{C} x_{ij}^t \leq 1, i \in J_n, t \in L_n; \tag{2.14}$$

$$\sum_{i \in J_n} x_{ij}^t \leq 1, j = 1, \ldots, C, t \in L_n; \tag{2.15}$$

$$x_{ij}^t \in \{0, 1\}, i \in J_n, j = 1, \ldots, C, t \in L_n; \tag{2.16}$$

$$0 \leq p_{ij}^t \leq p_{\text{safe}}, i \in J_n, j = 1, \ldots, C, t \in L_n; \tag{2.17}$$

$$\gamma_{\min} \leq \gamma_i^{L_n} \leq 1, i \in J_n. \tag{2.18}$$

Here, (2.12) ensures that the total power allocation does not exceed the power limit $p_{\max}$; (2.13) guarantees that every EV will be charged at or above the minimum user satisfaction factor provided by the station; (2.14) and (2.15) indicate that every single machine can only charge one vehicle at a time and each EV can only be charged by one machine at a time; (2.16) defines the charging machine assignment indicator; (2.17) defines the feasible range for p_{ij}^t, which is limited by a safe maximum charging power p_{safe} for EVs; and (2.18) requires the user satisfaction factor to meet the minimum target.

Notice that problem (3) has the same structure as problem (1), and therefore it is also an NP-complete problem. In Section 4, we introduce an online two-stage EV charging strategy as a suboptimal solution to problem (3) given the complexity issue.

In the next sections, we introduce the proposed offline and online two-stage EV charging strategies to find efficient suboptimal solutions to problems (1) and (3).

3. OFFLINE TWO-STAGE CHARGING STRATEGY

As aforementioned, the station might not be able to serve all EVs that require service at each time slot due to the limited number of charging machines. Therefore, the station needs to first determine “whom” it will charge (i.e., a subset of vehicles with a maximum size C) at each time slot and then decide “how much” it should charge. Thus, our offline two-stage charging strategy is to first find a schedule for the EVs and then optimize the charging power and user satisfaction factors. Then, it verifies if every EV is charged with at least the minimum user satisfaction factor. Figure 3.1 shows how this algorithm works. Note that these two stages could iterate between each other to further improve the performance locally. However, it is not the focus in this dissertation, as we target at a very simple and efficient charging strategy. Meanwhile, such heuristic iterations cannot lead to any optimality guarantee anyway.

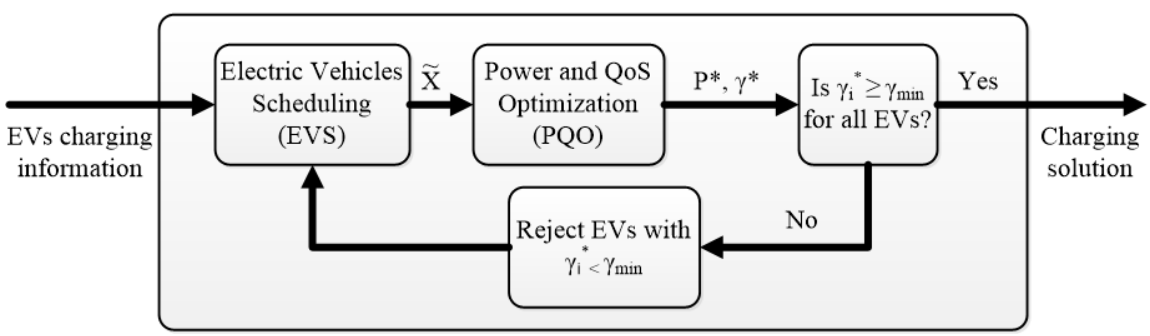

Figure 3.1: Offline two-stage charging algorithm with γ verification

Specifically, the first stage, called *Electric Vehicle Scheduling (EVS)*, is responsible for finding the feasible schedule for the EVs such that the unified profit is maximized given a fixed charging power solution and the desired user satisfaction factors. Based on the schedule obtained in the first stage, the second stage, called *Power and QoS Optimization (PQO)*, optimizes the power allocation and the user satisfaction factors under the peak power and charging level constraints. Then, the algorithm verifies if every EV is charged with at least $\gamma_{\min}$ of the desired energy target. If not, the EVs with invalid γ's are rejected and the overall algorithm is re-executed until a feasible solution is found. The final charging solution will be obtained after these steps.

3.1 Electric Vehicle Scheduling (EVS)

The goal here is to find a feasible schedule of EVs that maximizes the unified profit under a fixed power allocation and fixed user satisfaction factors. We introduce two algorithms: the offline LP relaxation and greedy scheduling algorithms. Here, we set $p_{ij}^t = \frac{w_i}{|T_i|}$ and $\gamma_i = 1$ for all $i \in \{1, 2, \ldots, N\}$, $j \in \{1, 2, \ldots, C\}$, and $t \in [r_i, d_i]$. This special problem can be formulated as:

$$\underset{X}{\text{maximize}} \quad \sum_{j=1}^{C}\sum_{i=1}^{N}\left(\alpha w_i - \beta\frac{w_i^2}{|T_i|} - 2\beta\frac{w_i}{|T_i|}\sum_{\substack{1\le k\le N\\ k\ne i}}\frac{|T_{ik}|w_k}{|T_k|}\right)x_{ij} \qquad \text{(Problem 4)}$$

$$\text{subject to} \quad \sum_{j=1}^{C}\sum_{i=1}^{N}\frac{w_i}{|T_i|}x_{ij}^t \le p_{\max}, t = 1,\dots,T; \qquad (3.1)$$

$$\sum_{j=1}^{C} x_{ij}^t \le 1, i = 1,\dots,N, t = 1,\dots,T; \qquad (3.2)$$

$$\sum_{i=1}^{N} x_{ij}^t \le 1, j = 1,\dots,C, t = 1,\dots,T; \qquad (3.3)$$

$$x_{ij}^t \in \{0,1\}, i = 1,\dots,N, j = 1,\dots,C, t = 1,\dots,T, \qquad (3.4)$$

where $x_{ij} = \sum_{t=r_i}^{d_i} x_{ij}^t$ and $|T_{ik}|$ is the number of time slots when job i and job k overlap.

3.1.1 Offline LP Relaxation Scheduling Algorithm

In this algorithm, the idea is to relax the EV scheduling constraints (3.4) by replacing $x_{ij}^t \in \{0,1\}$ with a weaker constraint $0 \le x_{ij}^t \le 1$. The obtained optimal fractional solution to the relaxed LP problem is then rounded using a derandomization algorithm [40]. In this work, we utilize a greedy rounding algorithm to obtain the desired integer solution $\widetilde{x}_{ij}^t \in \{0,1\}$. This approximation algorithm runs in polynomial time and determines a feasible solution, which is close to the optimal solution.

The relaxed LP problem can be formulated as:

$$\underset{X}{\text{maximize}} \quad \sum_{j=1}^{C}\sum_{i=1}^{N}\left(\alpha w_i - \beta\frac{w_i^2}{|T_i|} - 2\beta\frac{w_i}{|T_i|}\sum_{\substack{1\leq k\leq N\\ k\neq i}}\frac{|T_{ik}|w_k}{|T_k|}\right)x_{ij} \qquad \text{(Problem 5)}$$

$$\text{subject to} \quad \sum_{j=1}^{C}\sum_{i=1}^{N}\frac{w_i}{|T_i|}x_{ij}^t \leq p_{\max}, t=1,\ldots,T; \qquad (3.5)$$

$$\sum_{j=1}^{C}x_{ij}^t \leq 1, i=1,\ldots,N, t=1,\ldots,T; \qquad (3.6)$$

$$\sum_{i=1}^{N}x_{ij}^t \leq 1, j=1,\ldots,C, t=1,\ldots,T; \qquad (3.7)$$

$$0 \leq x_{ij}^t \leq 1, i=1,\ldots,N, j=1,\ldots,C, t=1,\ldots,T, \qquad (3.8)$$

It can be shown that the offline LP relaxation scheduling algorithm guarantees at least $(e-1)/e$ of the optimal solution in the worst-case scenarios [41]-[43]. In the following theorem, we analyze the complexity of the proposed scheduling algorithm.

Theorem 3.1.1 Given a set of N jobs and C machines, the offline LP relaxation scheduling algorithm finds a feasible schedule in $O(T \cdot \min\{N, C\} \cdot (2N+T+1))$ time.

Proof: The algorithm starts by solving the relaxed LP problem (5). This process depends on the algorithm utilized to solve this problem. A good complexity approximation is dependent on the product of the number of variables and the number of constraints. Specifically, it needs $O(\min\{N, C\} \cdot T \cdot (2N+T))$ computation times. Here, $\min\{N, C\} \cdot T$ is the number of variables and $(2N+T)$ is the number of constraints. After the relaxed LP problem (5) is solved, the algorithm needs to round the fractional solutions to obtain the desired integer solution. This process also depends on the rounding algorithm implemented. In this case, we utilize a greedy rounding algorithm, which takes $O\left(\min\{N, C\} \cdot T\right)$ computation times. Finally, the total computational time of the offline LP relaxation scheduling algorithm is $O\left(T \cdot \min\{N, C\} \cdot (2N + T + 1)\right)$. ■

As shown in Theorem 3.1.1, finding the exact EV schedule at each time slot to maximize the unified profit is a challenging task, specially in large-scale systems. In the next subsection, we address this problem by proposing an offline greedy scheduling algorithm that decides whether to schedule an EV based on its individual profit and charging time. We will show later that, in contrast to the LP relaxation approach, the computational time of the greedy scheduling algorithm does not increase rapidly with respect to the number of variables, at the cost of sacrificing certain optimality.

3.1.2 Offline Greedy Scheduling Algorithm

The offline greedy scheduling algorithm schedules the EVs to idle machines in a non-increasing order of their individual profits. If two or more EVs have the same individual profit, the algorithm chooses the one with the shortest charging time. Once all machines are occupied, the algorithm needs to decide whether to accept part of a new charging job or decline it. In [44]-[45], the authors proposed a similar algorithm based on individual profit maximization. Their approach schedules only the non-overlapping jobs with the highest individual profit. But our algorithm is able to schedule part of certain charging jobs, which improves the unified profit. The proposed greedy algorithm guarantees at least 1/2 of the optimal solution in the worst-case scenarios [44].

The offline greedy scheduling algorithm (see Algorithm 1) first calculates the individual profit of all users and sorts them into a non-increasing order. Then each job is scheduled based on this order until all the charging machines are occupied. After that, the system has to make the decision whether to accept or decline some charging requests. The station checks if part of the latest charging job can be processed and chooses the machine that provides the largest profit. If it is not possible, the considered charging job is declined. The following theorem derives the computational time of the offline greedy scheduling algorithm.

Theorem 3.1.2 Given a set of N jobs and C machines, the offline greedy scheduling algorithm finds a feasible schedule in $O\left(N(\log N + C)\right)$ time.

Proof: The algorithm starts by calculating the individual profit of each user and then sorting them into a non-increasing order. The process of sorting the N charging jobs takes $O\left(N \log N\right)$ time. Then, the algorithm schedules the sorted jobs one by one to the idle machines. Since the algorithm needs to check if the latest job can be scheduled to any machine, the process of selecting the machine takes $O\left(NC\right)$ time. Therefore, the total computational time of the offline greedy scheduling algorithm is $O\left(N(\log N + C)\right)$. ■

Table 3.1: Algorithm 1: Offline greedy scheduling algorithm

1. Let X be the total EV schedule and S be the set of accepted EVs. Initialize X and S.

2. Let $p_{it} = \frac{w_i}{|T_i|}$ and $\gamma_i = 1$ for all $i = 1, \ldots, N$, $t = 1, \ldots, T$, where T_i is the charging period of EV i.

3. Calculate the individual profit $f_i = \alpha w_i - \beta \frac{w_i^2}{|T_i|} - 2\beta \frac{w_i}{|T_i|} \sum_{\substack{1 \leq k \leq N \\ k \neq i}} \frac{|T_{ik}| w_k}{|T_k|}$ for all $i = 1, \ldots, N$, $t = 1, \ldots, T$, where $|T_{ik}|$ is the number of time slots when job i overlaps with job k.

4. Sort users in a non-increasing order of their individual profits (i.e. $f_1 \geq f_2 \geq \ldots \geq f_N$). If two or more EVs have the same individual profit, then choose the one with the shortest charging time first.

5. Run the following:

FOR $i := 1$ **TO** N **DO**

Let $H_i = \{j : j \text{ is idle between time } r_i \text{ and } d_i,\ j \in \{1, 2, \ldots, C\}\}$.

IF $|H_i| \geq 1$ **THEN**

$z_i^* = \min\{j : j \in H_i\}$.

Let $x_i^t = z_i^*$, for $r_i \leq t \leq d_i$.

ELSE

Calculate $|G_{ik}|$, the number of time slots when job i does not overlap with job k.

IF $|G_{ik}| > 0$ for any $k \in \{1, 2, \ldots, N\}$ **THEN**

Choose $k^* = \underset{k \in \{1,2,\ldots,N\}, k \neq i}{\text{argmax}} |G_{ik}|$.

Let $x_i^t = x_{k^*}^t$, for $t \in G_{ik}$.

ELSE

Reject EV i.

END IF

END IF

END FOR

6. Output X and S.

3.2 Power and QoS Optimization (PQO)

The goal in this step is to maximize the unified profit of the system based on the schedule obtained from *EVS*. Recall that given a schedule $\widetilde{X}$, the unified profit is given by

$$F(P,\gamma) = \sum_{t=1}^{T} \left[\alpha \sum_{j=1}^{C} \sum_{i=1}^{N} p_{ij}^{t} - \beta \left(\sum_{j=1}^{C} \sum_{i=1}^{N} p_{ij}^{t} \right)^2 \right] - \eta \sum_{j=1}^{C} \sum_{i=1}^{N} \big(w_i - \gamma_i w_i \big)^2.$$

Based on the knowledge of future charging requests, we can partition the schedule into multiple independent groups of EVs based on their arrival and departure times. Here, the EVs from different independent groups are not overlapping in term of their charging times. Those EVs will be in one group if each EV in this group is overlapping with at least another EV. For instance, as shown in Fig. 3.2, we can partition the set of scheduled users $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ into three independent groups $\{1, 2, 3\}$, $\{4, 5, 6\}$, and $\{7, 8, 9\}$.

Let M denote the number of independent groups. For $1 \leq m \leq M$, let I_m be the set of EVs included in the m^{th} group, and $|I_m|$ be the size of I_m. The charging period of the m^{th} group is denoted by $D_m = [\min_{i \in I_m} r_i, \max_{i \in I_m} d_i]$ and its length is given by $|D_m| = \max_{i \in I_m} d_i - \min_{i \in I_m} r_i + 1$.

After grouping, the original problem is divided into multiple subproblems, which can be solved independently by the same technique. Thus, the unified profit for the station is given by the sum of the profits from each group m. The optimization

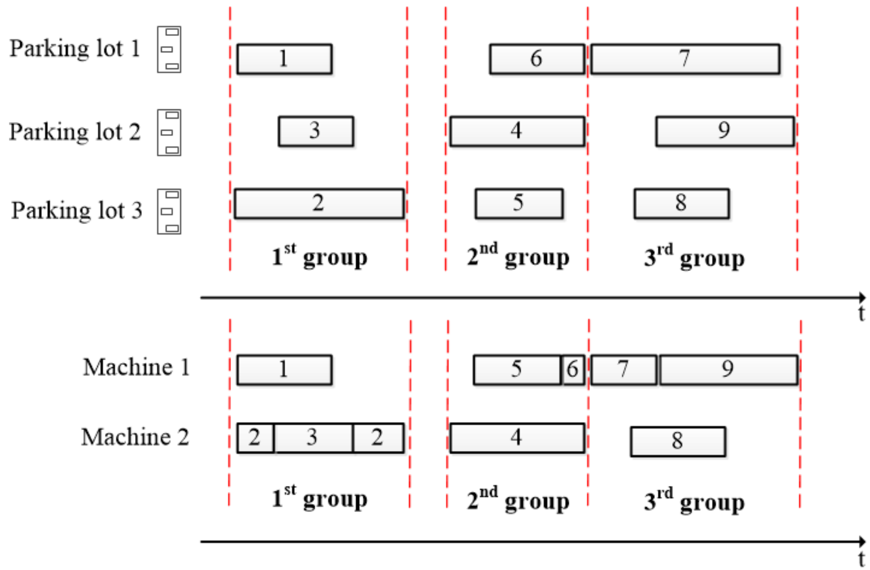

Figure 3.2: Partition the set of all users into independent groups

problem to find the profit for the m^{th} group can be formulated as:

$$\underset{P,\gamma}{\text{maximize}} \quad \sum_{t \in D_m} \left[\alpha \sum_{i \in I_m} p_{ij}^t - \beta \left(\sum_{i \in I_m} p_{ij}^t \right)^2 \right] - \eta \sum_{i \in I_m} (w_i - \gamma_i w_i)^2 \qquad \text{(Problem 6)}$$

$$\text{subject to} \quad \sum_{i \in I_m} p_{ij}^t \leq p_{max}, j = 1, \ldots, C, m = 1, \ldots, M, t \in D_m; \tag{3.9}$$

$$\gamma_i w_i \leq \sum_{t=r_i}^{d_i} p_{ij}^t \leq w_i, i \in I_m, j = 1, \ldots, C, m = 1, \ldots, M; \tag{3.10}$$

$$0 \leq p_{ij}^t \leq p_{\text{safe}}, i \in I_m, j = 1, \ldots, C, m = 1, \ldots, M, t \in D_m; \tag{3.11}$$

$$0 \leq \gamma_i \leq 1, i \in I_m. \tag{3.12}$$

The above problem is a convex quadratic problem, and thus its optimal solutions can be obtained by solving the Karush-Kuhn-Tucker (KKT) conditions [46]. For better exposition, let $W_m = \sum_{i \in I_m} w_i$ be the total energy demanded by the m^{th} group

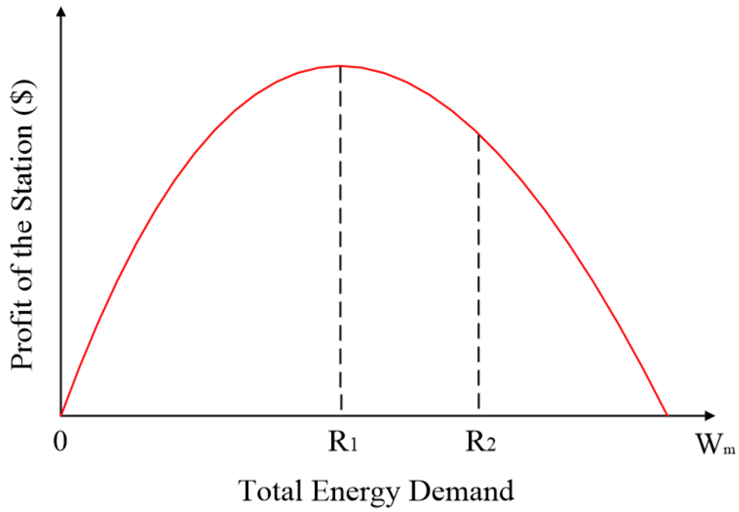

Figure 3.3: Impact of the energy demand on the unified profit of the system of EVs.

Remark 3.2.1 In order to better understand the structure of the optimal solution for problem (6), it is worth analyzing the relationship between the achievable profit and W_m, which is shown in Fig. 3.3. Let $R_1 = \frac{\alpha|D_m|}{2\beta}$ and $R_2 = \frac{1}{2\eta}\left\{2\min\left(|I_m|p_{\text{safe}},\ p_{max}\right)\left(|I_m|\beta + \eta|D_m|\right) - \alpha|I_m|\right\}$ for notation simplicity. Then, we observe the following operation regions:

- When $W_m \in [0, R_1)$, the profit increases as W_m increases until its maximum is reached. This region can be viewed as the "low demand" region, and it is anticipated that the station can fully satisfy all EVs in the m^{th} group.
- When $W_m \in [R_1, R_2]$, the profit starts decreasing but it remains acceptable.
- When $W_m \in (R_2, \infty)$, the profit decreases fast until it reaches 0. In this region, the energy demand is too high, which is beyond the capability of the charging

station. It will be shown later that in this region, no EV can be fully charged.

In Theorem 3.2.1 below, we provide the optimal solution to the sum of charging power and the user satisfaction factors for a given feasible schedule.

Theorem 3.2.1 The optimal solution to problem (6) is given as follows:

- *If* $0 \leq W_m < R_1$, *then* $\gamma_i^* = 1$ *and* $\sum_{i \in I_m} {p_{ij}^*}^t = \sum_{i \in I_m} \frac{w_i}{|D_m|}$.
- *If* $R_1 \leq W_m \leq R_2$, *then* $\gamma_i^* = 1 - \frac{2\beta \sum_{i \in I_m} w_i - \alpha |D_m|}{2w_i(|I_m|\beta + \eta|D_m|)}$

  and $\sum_{i \in I_m} {p_{ij}^*}^t = \frac{\alpha|I_m| + 2\eta \sum_{i \in I_m} w_i}{2(|I_m|\beta + \eta|D_m|)}$.
- *If* $W_m > R_2$, *then* $\gamma_i^* = 1 - \frac{\sum_{i \in I_m} w_i - |D_m| \min\left(|I_m| p_{safe},\ p_{max}\right)}{|I_m| w_i}$

  and $\sum_{i \in I_m} {p_{ij}^*}^t = \min\left(|I_m| p_{safe},\ p_{max}\right)$.

As aforementioned, this problem can be solved by standard convex optimization techniques. The detailed proof is presented in Appendix A.

Remark 3.2.2 We can obtain the following lower and upper bounds of γ_i^* from Theorem 3.2.1.

- If $0 \leq W_m < R_1$, then $\gamma_i^* = 1$.
- If $R_1 \leq W_m \leq R_2$, then $1 - \frac{2\beta \min(|I_m| p_{\text{safe}},\ p_{max}) - \alpha}{2\eta w_i} \leq \gamma_i^* \leq 1$.
- If $W_m > R_2$, then $0 \leq \gamma_i^* < 1 - \frac{2\beta \min(|I_m| p_{\text{safe}},\ p_{max}) - \alpha}{2\eta w_i}$.

From the station owner's point of view, the station is able to compute the expected range of user satisfaction factor guaranteed at a certain time based on the total energy demand.

Remark 3.2.3 The optimal sum power $\sum_{i \in I_m} p_{ij}^{*\,t}$ is constant over time for all $t \in D_m$. From the KKT conditions given in the Appendix A, we observe that $\sum_{i \in I_m} p_{ij}^{*\,t} = \sum_{i \in I_m} \frac{\gamma_i^* w_i}{|D_m|}$, where $\gamma_i^* \in [0, 1]$. Notice that the right-hand side of the above equation does not depend on t, and therefore the sum power at each time slot is constant over time.

Remark 3.2.4 The optimal power allocation P^* may not be unique. The system of equations to solve the power allocation consists of $|I_m| + (\max_{i \in I_m} d_i - \min_{i \in I_m} r_i + 1)$ equations and $|I_m| + \sum_{i \in I_m} (d_i - r_i)$ unknown variables. Since the arrival and departure times satisfy $r_i < d_i$, we have more unknown variables than equations in most cases. This implies our system of equations is underdetermined, and therefore the optimal power allocation P^* may not be unique.

As aforementioned, after P^* and γ^* are obtained, the algorithm verifies if every EV satisfies the condition $\gamma_i^* \geq \gamma_{\min}$. If not, the group of EVs with $\gamma_i^* < \gamma_{\min}$ are rejected and the algorithm re-executes the first and second stages until either a feasible solution is found or all EVs are rejected.

Next, we provide a numerical analysis to analyze the performance of our offline two-stage EV charging algorithm.

3.3 Numerical Analysis

This section presents some simulation results to illustrate the performance of our offline two-stage charging algorithms. The numerical analysis was conducted using the MATLAB-based optimization tool CVX [48] on a PC with Intel Core i7-4770, CPU speed 3.40 GHz, and 8 GB RAM.

We consider a public charging station with $C = 12$ charging machines and $T = 24$ time slots. We partition the entire frame $[0, 24]$ into multiple slots, each of which is of length $\Delta = 1$ hour. The total demand of the system and the individual charging speed are limited to $p_{\max} = 1MW$ and $p_{\text{safe}} = 20kW$, respectively. The number of EVs is $N = 30$, and the amount energy (in kWh) that each EV asks is a random real number over [10, 40]. All EV customers have different arrival and departure times. We randomly pick r_i from $[1, 16]$ and d_i from $[r_i + 2, r_i + 8]$. The charging time is restricted to be at least 2 hours since currently available charging stations take around 2-3 hours to charge their EVs [8]. We set $\gamma_{\min}$ to be 0, 0.5 or 0.7, respectively, $\alpha = 4\$/kWh$, $\beta = 0.10\$/(kWh)^2$, and $\eta = 0.20\$/(kWh)^2$.

We illustrate the performance of our offline two-stage charging algorithms in terms of the average unified profit, user satisfaction factor, percentage of EVs serviced, power consumption, load factor, and computational time, where the average results are taken over 100 random realizations. We compare our algorithms against an exhaustive search algorithm and some conventional charging strategies.

First, we show the efficiency of our offline two-stage charging algorithms by com-

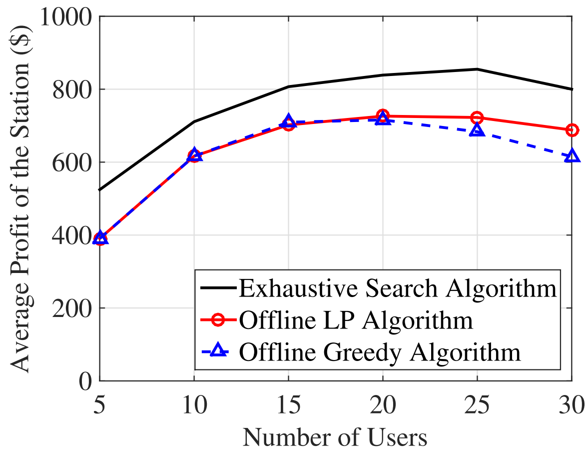

Figure 3.4: Average profit of the offline two-stage algorithms against the exhaustive search

paring it with the exhaustive search algorithm. Due to the high computational cost of exhaustive search, we only consider the case when $\gamma_{\min} = 0$.

Fig. 3.4 illustrates how the average profit of the station is affected by the number of vehicles N when $C = 10$. Due to its complexity, the profit of the exhaustive search algorithm is approximated by $f(N) = -0.035N^3 - 2.9N^2 + 74N + 220$ when $N > 20$. The function $f(N)$ is obtained using the MATLAB tool called *Basic Fitting* [49]. We can observe that the offline two-stage LP (vs. greedy) algorithm obtains its maximum profits when $N = 25$ (vs. $N = 20$). Notice that the profit starts to decrease when the number of EVs gets larger than $N = 25$ (vs. $N = 20$). Such a critical number can be viewed as the "service capacity" with each algorithm. It is worth mentioning that the profit will no longer increase even if more EVs can be charged due to the cost of power consumption for the operator, which increases at a higher rate than the revenue.

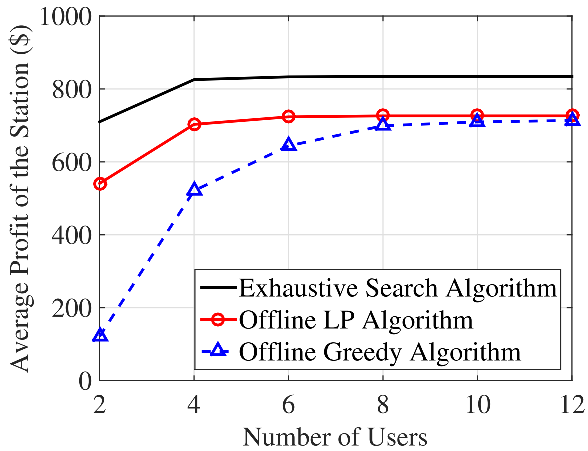

Figure 3.5: Influence of the number of charging machines on the average profit of the station

Table 3.2: Average computational time (sec)

Number of EVs	Exhaustive Search	Offline LP	Offline Greedy
5	18.9932	1.3686	1.2771
10	753.8080	1.9585	1.8642
15	5140.2536	5.3386	2.4304
20	567887.6684	6.6386	2.9947
25	10262000.0000	7.9902	3.5720
30	88354800.0000	9.3342	4.1654

Then, we analyze the influence of the number of charging machines C on the average profit of the station. Fig. 3.5 shows that when $N = 15$, the offline exhaustive search and the offline two-stage LP algorithms need approximately $C = 4$ machines to achieve their maximum profits. Meanwhile, the offline two-stage greedy algorithm needs about $C = 10$ machines to achieve its maximum profit. For the design of the station infrastructure, this information is useful with large number of charging stations.

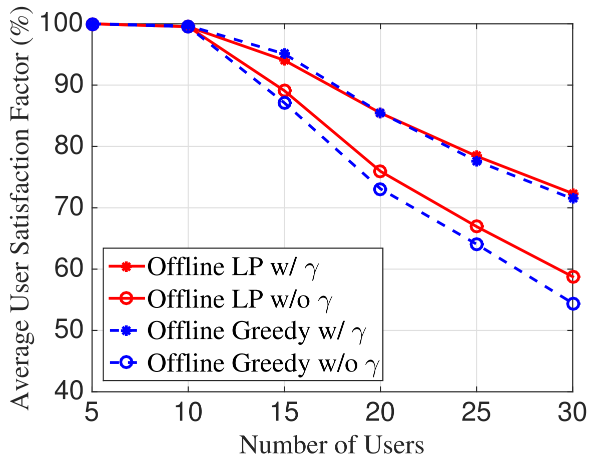

Figure 3.6: Benefit of controlling the user satisfaction factor

In Table 3.2, we show the average computational time when $C = 10$. Notice that both two-stage algorithms consume almost the same amount of computational time when $N < C$. After this point, the computational time of the offline two-stage LP algorithm increases at a higher rate. Due to its complexity, the computational time of the exhaustive search algorithm is approximated by $Q(N) = 4.5 \times 10^{-7} N^{10} - 1.1 \times 10^{-5} N^9 + 5.3 \times 10^{-5} N^8 + 2 \times 10^{-4} N^7$ when $N > 20$. Similarly, the function $Q(N)$ is obtained using Basic Fitting. Table 3.2 shows that the exhaustive search algorithm consumes much more time and resources compared to the proposed strategies. As expected, the offline two-stage algorithms utilize less system resources to find the solution at the cost of decreasing certain optimality. However, as shown in Fig. 3.4, both algorithms provide an acceptable unified profit compared to the exhaustive search algorithm.

We show the benefit of controlling the user satisfaction factor in Fig. 3.6, where the average is taken over both random realizations and different customers. When $N = 30$, the two-stage LP and greedy algorithms with control of the user satisfaction factor γ achieve a percentage of charging close to 72%. On the other hand, the two-stage LP and greedy algorithms without γ control can only guarantee about 60% and 55% percentages of charging, respectively. Therefore, the control of user satisfaction factors provides a better QoS.

Next, we compare our proposed charging strategies with two other practical charging algorithms. The first benchmark model is a greedy charging algorithm with fixed power allocation, where the power is delivered at a constant charging speed. To fully charge the vehicle, the customers have to stay connected until the expected charging time ends. The second benchmark model is a greedy charging algorithm with uniform power allocation, where the power is allocated uniformly based on the charging time and desired energy target of each user. Both charging mechanisms utilize a *First-In, First-Out (FIFO)* scheduling policy, where users are served in the order of their arrivals whenever a charging machine is idle. If all machines are occupied, the incoming EVs will be rejected. Due to the relatively small number of EVs nowadays and the simplicity of those algorithms, current public charging stations have implemented similar ideas to charge the EVs. In the following simulations, the number of charging machines is set as $C = 10$ and $\gamma_{\min}$ is set to be 0, 0.5 or 0.7, respectively.

In Fig. 3.7, we show the average profit attained by different charging strategies. Notice that the average profit provided by the two offline two-stage algorithms de-

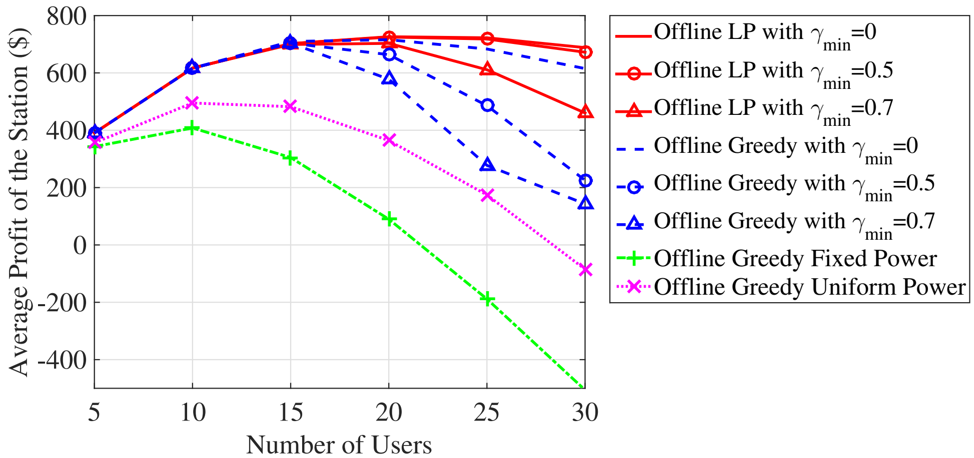

Figure 3.7: Average profit of the offline two-stage algorithms against other practical charging algorithms

creases as $\gamma_{\min}$ increases. This is an expected result since the greater the value of $\gamma_{\min}$, the larger the number of EVs rejected. However, we can observe that both offline two-stage algorithms outperform the benchmark charging approaches for any value of $\gamma_{\min}$. Also, notice that the greedy fixed and uniform power allocation algorithms have very poor performance since they provide negative profits when $N \geq 22$ and $N \geq 28$, respectively.

Furthermore, the average user satisfaction factor is shown in Fig. 3.8, where the average is taken over both random realizations and different customers. In contrast to the previous result, the average user satisfaction factor increases as $\gamma_{\min}$ increases. This is obvious since our algorithms reject all the EVs with γ smaller than $\gamma_{\min}$. Notice that both offline two-stage algorithms provide about 72% of charging when $\gamma_{\min} = 0$. Moreover, those algorithms outperform the conventional charging strategies when $\gamma_{\min} = 0.7$, achieving about 93% and 89% of charging. Thus, both

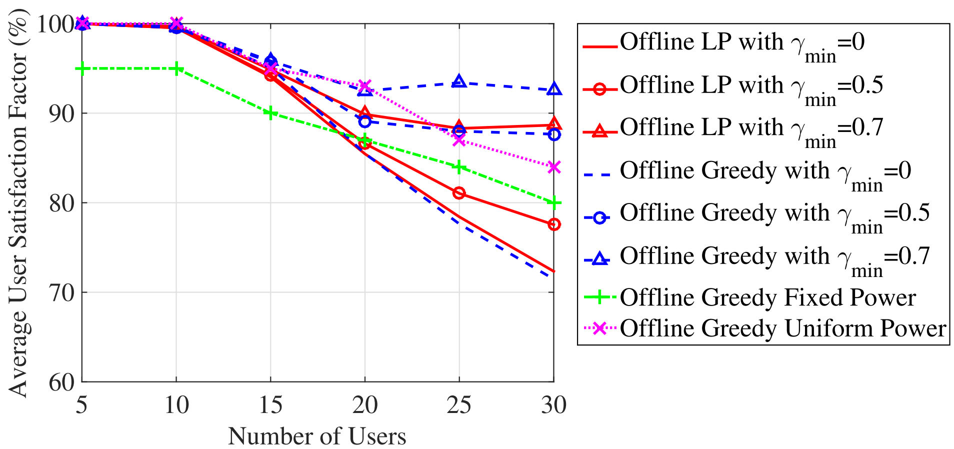

Figure 3.8: Average user satisfaction factor of the offline two-stage algorithms against other practical charging algorithms

algorithms provide satisfactory results in terms of the user satisfaction factor while providing a larger profit.

In Fig. 3.9, we show the percentage of EVs serviced. As aforementioned, the percentage of EVs serviced will decrease as $\gamma_{\min}$ increases. We can observe that both offline two-stage algorithms outperform the *First-In, First-Out (FIFO)* scheduling policy when $\gamma_{\min} = 0$. The offline two-stage LP and greedy algorithms provide about 100% and 95% of vehicles serviced, respectively. Moreover, when $\gamma_{\min} = 0.7$, those algorithms provide respectively 65% and 50% of EVs serviced. This result is still acceptable considering that both algorithms guarantee at least 93% and 89% of charging, respectively.

We present the results related to the average power consumption in Fig. 3.10, where the average is taken over both random realizations and time. Notice the proposed offline two-stage algorithms consume less power compared with the other

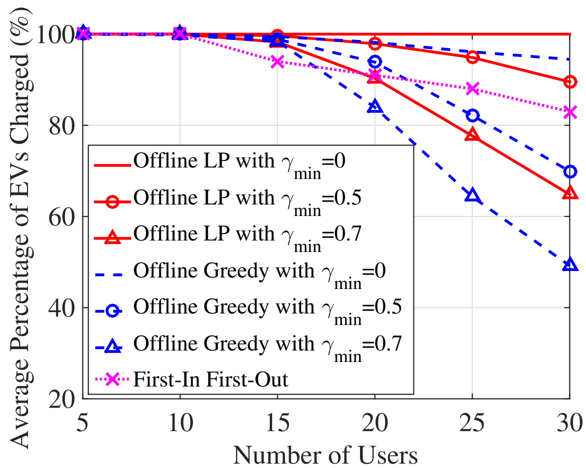

Figure 3.9: Average percentage of vehicles serviced of the offline two-stage algorithms against other practical charging algorithms

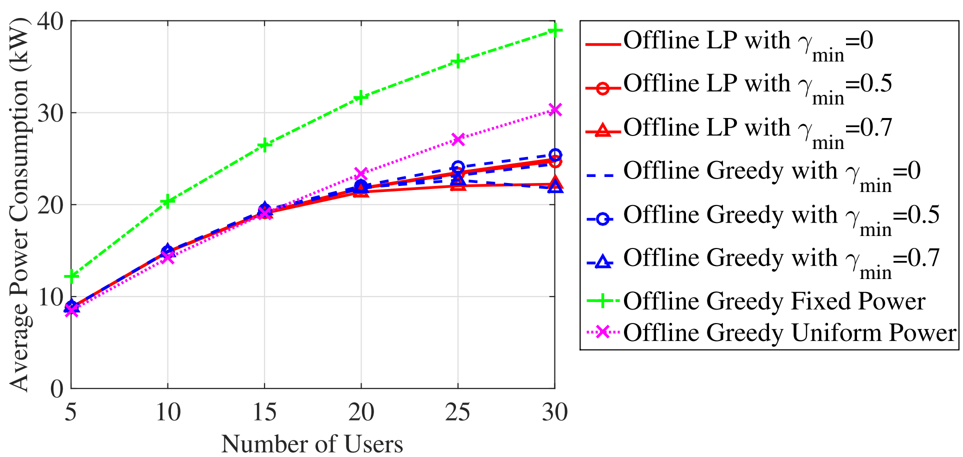

Figure 3.10: Average power consumption of the offline two-stage algorithms against other practical charging algorithms

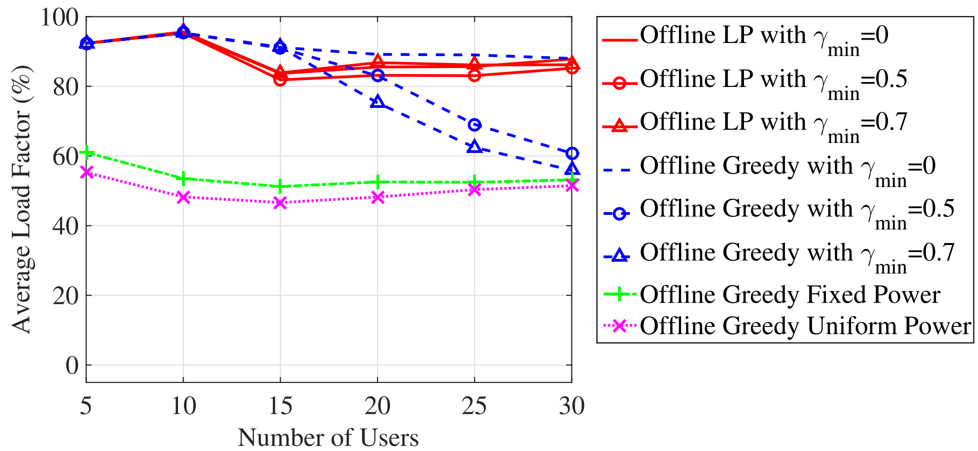

Figure 3.11: Average load factor of the offline two-stage algorithms against other practical charging algorithms

practical approaches for any value of $\gamma_{\min}$. This result is reflected on the profit shown in Fig. 3.7. As expected, the larger the power demand, the higher the consumption cost, which affects negatively the profit of the station.

We introduce the concept of load factor to measure the efficiency of the electrical energy usage. It is defined to be the average load divided by the peak load over a specified time period as:

$$\text{Load factor} = \frac{\text{Average load}}{\text{Maximum load in a given time period}}.$$

A high load factor implies that the power usage is relatively constant and efficient.

In Fig. 3.11, we show the average load factor of the system, where the average is taken over both random realizations and time. We can observe that both offline two-stage algorithms achieve at least 85% of load factor when $\gamma_{\min} = 0$. Notice that

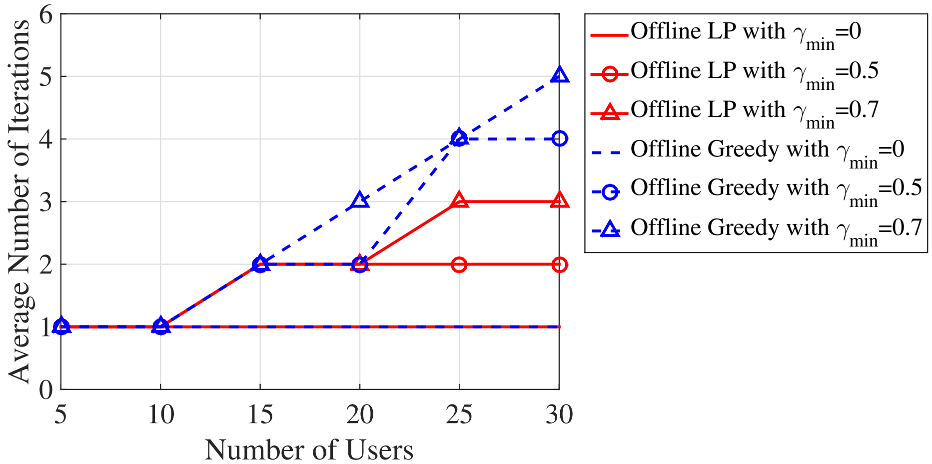

Figure 3.12: Average number of iterations of the offline two-stage algorithms

the load factor of the greedy algorithm decreases faster as $\gamma_{\min}$ increases. This result is expected since the greedy algorithm rejects more EVs than the LP algorithm, which causes more variations on the power consumption. On the other hand, both the fixed and uniform power allocation models provide a load factor close to 50%, which means that the power consumption is not steady enough. This result is also reflected in the achieved profit shown in Fig. 3.7.

Finally, we show the average number of iterations needed to find the solution in Fig. 3.12. Notice that the two offline two-stage charging algorithms find a feasible solution after a few iterations. As expected, the number of iterations needed increases as $\gamma_{\min}$ increases. As aforementioned, our offline charging algorithms are re-executed until either a feasible solution is found or all EVs are rejected. Therefore, our algorithm always finds a solution after a certain number of iterations.

In the next section, we introduce an online two-stage EV charging strategy as a suboptimal solution to problem (3) in Section 2 when the future customer arrival information is not available.

4. ONLINE TWO-STAGE CHARGING STRATEGY

In this scenario, the station also needs to first determine "whom" it will charge (i.e., a subset of vehicles with a maximum size C) and then decide "how much" it should charge at each time slot. In contrast to the offline case, the online EV charging strategy is executed every time when a new user arrives at the charging facility. Specifically, every time a new EV arrives at the station, our online two-stage charging strategy first finds a schedule for the EVs currently connected to the system and then optimizes the charging power and user satisfaction factors. Afterwards, the algorithm verifies if every EV could be charged with at least $\gamma_{\min}$. If not, the new EV is rejected immediately and the previous charging strategy is resumed. Figure 4.1 shows how this algorithm works. Here, the two stages could also iterate between each other to further improve the performance locally. However, the purpose of this work is to provide a very simple and efficient charging strategy. Meanwhile, such heuristic iterations cannot lead to any optimality guarantee anyway.

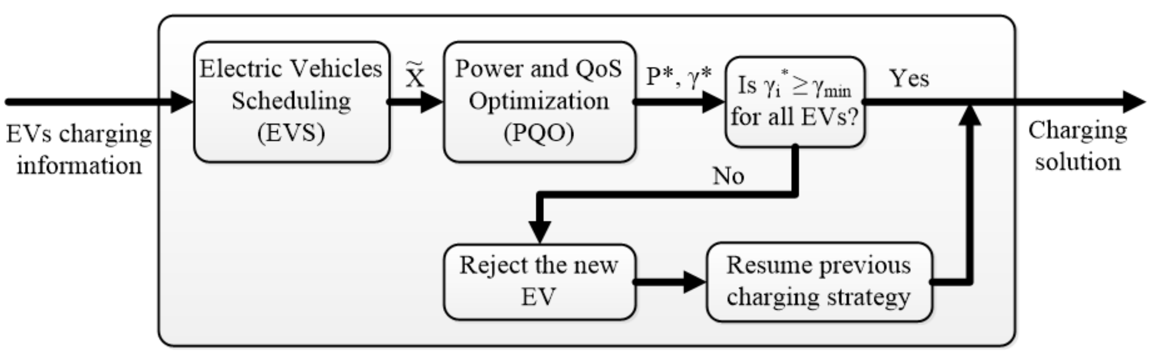

Figure 4.1: Online two-stage charging algorithm with γ verification

4.1 Electric Vehicle Scheduling (EVS)

The goal here is to find the feasible schedule of EVs that maximizes the instantaneous profit. Similar to the offline case, we introduce two algorithms: the online LP relaxation and greedy scheduling algorithms. We set $p_{ij}^t = \frac{w_i^{L_n}}{|T_i|}$ and $\gamma_i^{L_n} = 1$, where the individual charging period is denoted by $T_i = [r_n, d_i]$ and its length is given by $|T_i| = d_i - r_n + 1$ for all $i \in J_n,\ j \in \{1, 2, \ldots, C\}$. This problem can be formulated as follows:

$$\underset{X^{L_n}}{\text{maximize}} \qquad \sum_{j=1}^{C} \sum_{i \in J_n} \left(\alpha w_i^{L_n} - \beta \frac{\left(w_i^{L_n}\right)^2}{|T_i|} - 2\beta \frac{w_i^{L_n}}{|T_i|} \sum_{k \in J_n, k \neq i} \frac{|T_{ik}| w_k^{L_n}}{|T_k|} \right) x_{ij} \tag{Problem 7}$$

$$\text{subject to} \qquad \sum_{j=1}^{C} \sum_{i \in J_n} \frac{w_i^{L_n}}{|T_i|} x_{ij}^t \leq p_{\max}, t \in L_n; \tag{4.1}$$

$$\sum_{j=1}^{C} x_{ij}^t \leq 1, i \in J_n, t \in L_n; \tag{4.2}$$

$$\sum_{i \in J_n} x_{ij}^t \leq 1, j = 1, \ldots, C, t \in L_n; \tag{4.3}$$

$$x_{ij}^t \in \{0, 1\}, i \in J_n, j = 1, \ldots, C, t \in L_n. \tag{4.4}$$

where $x_{ij} = \sum_{t \in L_n} x_{ij}^t$ and $|T_{ik}|$ is the number of timeslots in L_n when job i and job k overlap.

4.1.1 Online LP Relaxation Scheduling Algorithm

Similar to the approach presented in the offline section, the idea is to replace $x_{ij}^t \in \{0,1\}$ with a weaker constraint $0 \leq x_{ij}^t \leq 1$, for all $t \in L_n$. The obtained optimal fractional solution to the relaxed LP problem is then rounded using a greedy rounding algorithm to obtain the desired integer solution $\widetilde{x}_{ij}^t \in \{0,1\}$. This online LP relaxation scheduling algorithm is executed every time when a new EV arrives at the station. It can be shown that this algorithm also runs in polynomial time and guarantees at least $(e-1)/e$ of the optimal solution in the worst-case scenarios. In the following theorem, we show the complexity of the online LP relaxation algorithm.

Theorem 4.1.1 Given a set of J_n jobs and C machines at the arrival time r_n, the online LP relaxation scheduling algorithm finds a feasible schedule in approximately $O\left(N \cdot T \cdot \min\{N, C\} \cdot (2N + T + 1)\right)$ time, where $N = \max_n |J_n|$.

Proof: The algorithm starts by solving the relaxed LP problem when a new EV arrives at the system. A good complexity approximation for the computational time is dependent on the product of the number of variables V_t and the number of constraints K_t at each time slot $t \in L_n$, where the total computation takes $O\left(L_n \cdot \min\{|J_n|, C\} \cdot (2|J_n| + L_n + 1)\right)$ times. In the worst case, the computational time is upper bounded by $N \cdot (\min\{N, C\} \cdot T \cdot (2N + T))$. After the relaxed LP relaxation problem is solved, we utilized a greedy rounding algorithm to obtain the desired integer solution. This process takes $O\left(N \cdot \min\{N, C\} \cdot T\right)$ computational times. Finally, the total computational time of the online LP relaxation scheduling algorithm considering all the possible arrivals is $O\left(N \cdot T \cdot \min\{N, C\} \cdot (2N + T + 1)\right)$. ∎

Similar to the offline charging scenario, we propose an online greedy scheduling algorithm to address the computational cost at the expense of decreasing the optimality.

4.1.2 Online Greedy Scheduling Algorithm

The online greedy scheduling algorithm schedules the EVs to idle machines in a non-decreasing order of their arrivals. If two or more EVs arrive at the same time, the algorithm chooses the one with the shortest charging time. Once all machines are occupied, the algorithm needs to decide whether to accept or decline the new EV.

The online greedy scheduling algorithm (see Algorithm 2) first checks if there is any charging machine idle to schedule the new EV n. If not, the algorithm calculates the individual profit f_k of all EVs already connected to the station $k < n$ and the individual profit f_n of EV n. Then, the algorithm needs to immediately make the decision whether to accept or decline EV n. If $f_n \geq f_k$ for any EV $k < n$, the station stops charging EV k and schedule EV n to the idle charging machine. Moreover, if $f_n < f_k$ and $d_n > d_k$ for any EV $k < n$, the station starts charging EV n after EV k is charged. Finally, if none of the above conditions are satisfied, the EV n is declined immediately. The following theorem derives the computational time of the online greedy scheduling algorithm.

Theorem 4.1.2 Given a set of J_n jobs and C machines at the arrival time r_n, the

Table 4.1: Algorithm 2: Online greedy scheduling algorithm

FOR each EV n arriving at the station **DO**

Let X^{L_n} be the total EV schedule and S^{L_n} be the set of accepted EVs in L_n.

Initialize X^{L_n} and S^{L_n}.

FOR $t := r_n$ **TO** $\max_{k \in J_n} d_k$ **DO**

Let $H_n = \{j : j \text{ is idle between time } r_n \text{ and } d_n, j \in \{1, 2, \dots, C\}\}$.

IF $|H_n| \geq 1$ **THEN**

$z_n^* = \min\{j : j \in H_n\}$.

Let $x_n^t = z_n^*$, for $r_n \leq t \leq d_n$.

ELSE

Let $p_n^t = \frac{w_n^{L_n}}{|T_n|}$ and $\gamma_n^{L_n} = 1$, for all $t \in L_n$.

Calculate $f_n = \alpha w_n^{L_n} - \beta \frac{\left(w_n^{L_n}\right)^2}{|T_n|} - 2\beta \frac{w_n^{L_n}}{|T_n|} \sum_{k<n} \frac{|T_{nk}| w_k^{L_n}}{|T_k|}$, where f_n is the individual profit and $|T_{nk}|$ is the number of time slots in L_n when job n and job k overlap.

IF $f_n \geq f_k$ **THEN**

Choose $k^* = \underset{k<n}{\text{argmax}} \frac{f_n}{f_k}$.

Let $x_n^t = x_{k^*}^t$, for $r_n \leq t \leq d_n$.

ELSE IF $f_n < f_k$ and $|G_{nk}| > 0$ for any $k < n$ **THEN**

Choose $k^* = \underset{k<n}{\text{argmax}} |G_{nk}|$.

Let $x_n^t = x_{k^*}^t$, for $t \in G_{nk^*}$.

ELSE

Reject EV n.

END IF

END IF

END FOR

Output X^{L_n} and S^{L_n}.

END FOR

online greedy scheduling algorithm finds a feasible schedule in $O\left(N^2\left(\log N + C\right)\right)$ *time, where* $N = \max_n |J_n|$.

Proof: The algorithm starts by calculating the individual profit of each user in J_n at time r_n when all charging machine are occupied. If two or more EVs arrive at the same time, the algorithm chooses the one with the shortest charging time. The process of sorting the $|J_n|$ charging jobs takes $O\left(|J_n| \log |J_n|\right)$ computational times. Since $N = \max_n |J_n|$, we say this process can be completed in at most $O\left(N \log N\right)$ time. Then, the algorithm schedules the sorted jobs one by one to the idle machines. The process of selecting the machine takes at most $O\left(NC\right)$ computational times. This algorithm runs every time a new EV arrives at the station, and therefore the total computational time of the online greedy scheduling algorithm is $O\left(N^2\left(\log N + C\right)\right)$ in the worst-case. ■

4.2 Power and QoS Optimization (PQO)

The goal in this step is to maximize the profit of the station operator based on the schedule obtained from the previous stage. The optimization problem to find the maximum instantaneous profit for the station can be formulated as:

$$\underset{P^{L_n},\gamma^{L_n}}{\text{maximize}} \quad \sum_{t\in L_n}\left\{\alpha\sum_{j=1}^{C}\sum_{i\in J_n}p_{ij}^t-\beta\left(\sum_{j=1}^{C}\sum_{i\in J_n}p_{ij}^t\right)^2\right\}-\eta\sum_{j=1}^{C}\sum_{i\in J_n}\left(w_i^{L_n}-\gamma_i^{L_n}w_i^{L_n}\right)^2$$

(Problem 8)

$$\text{subject to} \quad \sum_{j=1}^{C}\sum_{i\in J_n}p_{ij}^t\leq p_{\max},t\in L_n; \tag{4.5}$$

$$\gamma_i^{L_n}w_i^{L_n}\leq\sum_{t=r_i}^{d_i}p_{ij}^t\leq w_i^{L_n},i\in J_n,j=1,\ldots,C; \tag{4.6}$$

$$0\leq p_{ij}^t\leq p_{\text{safe}},i\in J_n,j=1,\ldots,C,t\in L_n; \tag{4.7}$$

$$0\leq\gamma_i^{L_n}\leq 1,i\in J_n,t\in L_n. \tag{4.8}$$

The above problem is a convex quadratic problem, and thus its optimal solutions can be obtained by solving the KKT conditions. Similar to the offline case, let the total energy demanded by the EVs $i\in J_n$ during the period L_n be defined by $W_{L_n}=\sum\limits_{i\in J_n}w_i^{L_n}$.

Let $R_1^{L_n}=\frac{\alpha|L_n|}{2\beta}$ and $R_2^{L_n}=\frac{1}{2\eta}\left[2\min\left(|J_n|p_{\text{safe}},\ p_{max}\right)\left(|J_n|\beta+\eta|L_n|\right)-\alpha|J_n|\right]$, where $R_1^{L_n}$ and $R_2^{L_n}$ are also updated at every arrival time based on the available information. In Figure 4.2, we provide the similar operation regions illustrated in the offline case for any given charging period L_n.

- When $W_{L_n}\in[0,R_1^{L_n})$, the profit increases as W_{L_n} increases until its maximum is reached. This region can be viewed as the "low demand" region, and it is anticipated that the station can fully satisfy all EVs in J_n.

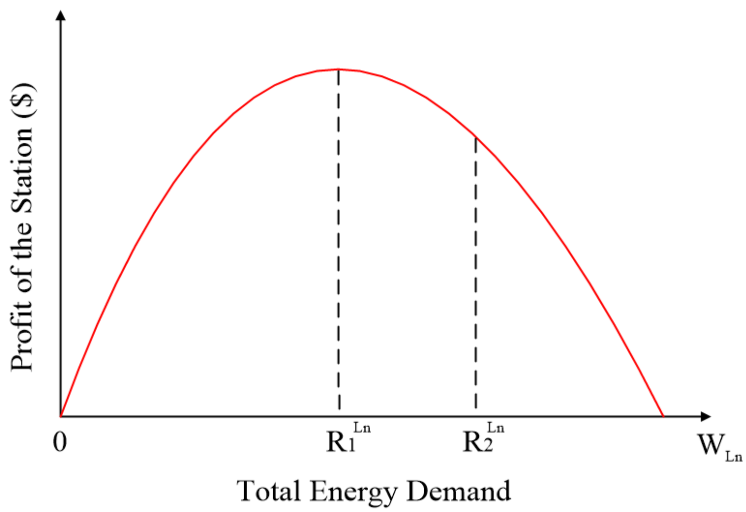

Figure 4.2: Impact of the energy demand on the instantaneous profit of the system

- When $W_{L_n} \in [R_1^{L_n}, R_2^{L_n}]$, the profit starts decreasing but it remains acceptable.
- When $W_{L_n} \in (R_2^{L_n}, \infty)$, the profit decreases fast until it reaches 0. In this region, the energy demand is too high, which is beyond the capability of the charging station. It will be shown later that in this region, no EV can be fully charged.

In Theorem 4.2.1, we provide the optimal solution to the sum of power and the user satisfaction factors for a given feasible schedule at $t \in L_n$.

Theorem 4.2.1 The optimal solution for problem (8) is given as follows:

- *If* $0 \le W_{L_n} < R_1^{L_n}$, *then* $\gamma_i^{*L_n} = 1$ *and* $\sum_{i \in J_n} p_{ij}^{t\,*} = \sum_{i \in J_n} \frac{w_i^{L_n}}{|L_n|}$.
- *If* $R_1^{L_n} \le W_{L_n} \le R_2^{L_n}$, *then* $\gamma_i^{*L_n} = 1 - \frac{2\beta \sum_{i \in J_n} w_i^{L_n} - \alpha |L_n|}{2 w_i^{L_n} (|J_n|\beta + \eta |L_n|)}$

  and $\sum_{i \in J_n} p_{ij}^{t\,*} = \frac{\alpha |J_n| + 2\eta \sum_{i \in J_n} w_i^{L_n}}{2(|J_n|\beta + \eta |L_n|)}$.

- *If* $W_{L_n} > R_2^{L_n}$, *then* $\gamma_i^{*L_n} = 1 - \frac{\sum_{i\in J_n} w_i^{L_n} - |L_n| \min\left(|J_n| p_{safe},\ p_{max}\right)}{|J_n| w_i^{L_n}}$

  and $\sum_{i\in J_n} {p_{ij}^t}^* = \min\left(|J_n| p_{safe},\ p_{max}\right)$.

As aforementioned, this problem can be solved by standard convex optimization techniques. Notice that the the station learns all the information about the EVs currently connected to the station, and therefore its solution can be obtained using the same approach presented in Appendix A for small periods of time.

Remark 4.2.1 We can obtain the following lower and upper bounds of $\gamma_i^{*L_n}$ from Theorem 4.2.1.

- If $0 \le W_{L_n} < R_1^{L_n}$, then $\gamma_i^{*L_n} = 1$.
- If $R_1^{L_n} \le W_{L_n} \le R_2^{L_n}$, then $1 - \frac{2\beta \min(|J_n| p_{\text{safe}},\ p_{max}) - \alpha}{2\eta w_i^{L_n}} \le \gamma_i^{*L_n} \le 1$.
- If $W_{L_n} > R_2^{L_n}$, then $0 \le \gamma_i^{*L_n} < 1 - \frac{2\beta \min(|J_n| p_{\text{safe}},\ p_{max}) - \alpha}{2\eta w_i^{L_n}}$.

From the station owner's point of view, the station is able to compute the expected range of user satisfaction factor guaranteed at a certain time based on the total energy demand.

Remark 4.2.2 The optimal sum power $\sum_{i\in J_n} {p_{ij}^t}^*$ is constant over time for all $t \in L_n$. From the KKT conditions given in the Appendix A, we observe that $\sum_{i\in J_n} {p_{ij}^t}^* = \sum_{i\in J_n} \frac{\gamma_i^{*L_n} w_i^{L_n}}{|L_n|}$, where $\gamma_i^{*L_n} \in [0,1]$. Notice that the right-hand side of the above equation does not depend on t, and therefore the sum power at each time slot is constant over time.

Remark 4.2.3 The optimal power allocation P^{*L_n} may not be unique. The system of equations to solve the power allocation consists of $|J_n| + (\max_{i \in J_n} d_i - \min_{i \in J_n} r_i + 1)$ equations and $|J_n| + \sum_{i \in J_n} (d_i - r_i)$ unknown variables. Since the arrival and departure times satisfy $r_i < d_i$, we have more unknown variables than equations in most cases. This implies that the system of equations is undetermined, and therefore the optimal power allocation P^* may not be unique.

Similar to the offline case, after we obtain the solution to P^{*L_n} and γ^{*L_n}, the algorithm verifies if all EVs satisfy the condition $\gamma_i^{*L_n} \geq \gamma_{\min}$ after the new EV arrives at the station. If not, the new EV is rejected immediately. Here, the first and second stages are not re-executed.

In the next section, we apply the concept of competitive analysis to evaluate the proposed online two-stage charging strategy under non-congested and congested scenarios.

4.3 Competitive Analysis

In this section, we apply the concept of competitive ratio to evaluate the proposed online algorithms against the offline counterparts and derive the closed-form expressions for a special scenario. For general cases, we will illustrate the competitive ratio performance by simulations. The main idea behind competitive analysis is to ensure that an online algorithm could guarantee an acceptable performance compared to the offline algorithm. The concept of competitive ratio is defined below [47].

Definition 1 *An online algorithm is σ-competitive if $\min_{J \in \Upsilon} \frac{F_{on}(J)}{F_{off}(J)} \geq \sigma$, where J is* *an input instance with* N *jobs,* Υ *is the collection of instances,* $F_{on}(J)$ *and* $F_{off}(J)$ *are the unified profit obtained by the online and offline charging algorithms, respectively.*

Here, as an special case, we analyze the competitive ratio when each EV has a different arrival time r_i, with the same departure time d, user satisfaction factor $\gamma = 1$, and energy requirement W. As time goes on, the number of EVs increases, which increases the total demand and the per unit cost of power consumption for the operator.

In the following theorem, we provide lower bounds of the competitive ratio in this special case under non-congested and congested scenarios.

Theorem 4.3.1 Given an arbitrary arrival time, a fixed departure time, the same user satisfaction factor $\gamma = 1$*, and the same energy requirement* W*, the lower bounds of the competitive ratio for non-congested and congested scenarios are given as follows:*

(a) Non-congested (N-C) scenario (i.e. $G_t \leq C$ *for all* $t \in [1, T]$*)*

$$\sigma \geq \frac{\alpha T - 2\beta W N}{\alpha T - \beta W N},$$

(b) Congested (C) scenario (i.e. $G_t > C$ *for any* $t \in [1, T]$*)*

$$\sigma \geq \frac{\alpha W C \left(\frac{|S|-C}{T} + 1\right) - \beta W^2 \left[\frac{2|S|C}{T} + \frac{|S|-C}{C} + \frac{2(|S|-C)^2}{T-2(|S|-C)+1}\right]}{\alpha W |S| - \beta \frac{W^2 |S|^2}{T}},$$

where G_t *is the number of EVs be charged at time* t *and* S *is the set of all EVs scheduled.*

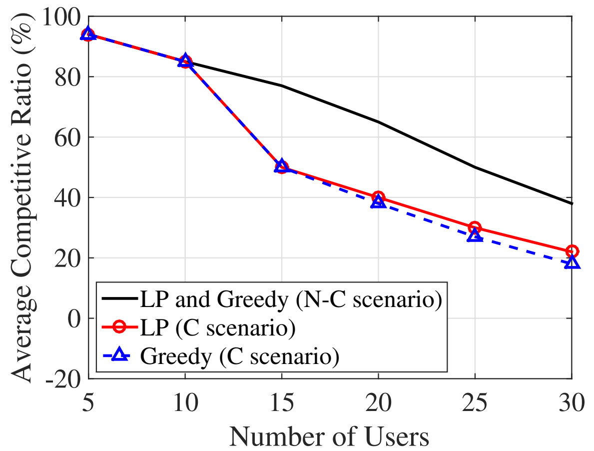

Figure 4.3: Average competitive ratios for the special case under non-congested and congested scenarios

The detailed proof of Theorem 4.3.1 is presented in Appendix B. Figure 4.3 plots the competitive ratios obtained for both LP and greedy algorithms under non-congested and congested scenarios in the mentioned special case, averaged over 100 random realizations. The setup of parameters is given in the next section. The numerical evaluation of competitive ratio for general cases is given in the next section.

4.4 Numerical Analysis

This section presents some simulation results to illustrate the performance of the online two-stage charging algorithms. The numerical analysis was conducted using the MATLAB-based optimization tool CVX [48] on a PC with Intel Core i7-4770, CPU speed 3.40 GHz, and 8 GB RAM.

We consider a public charging station with $C = 12$ charging machines and $T = 24$ time slots. We partition the entire frame $[0, 24]$ into multiple slots, each of which is

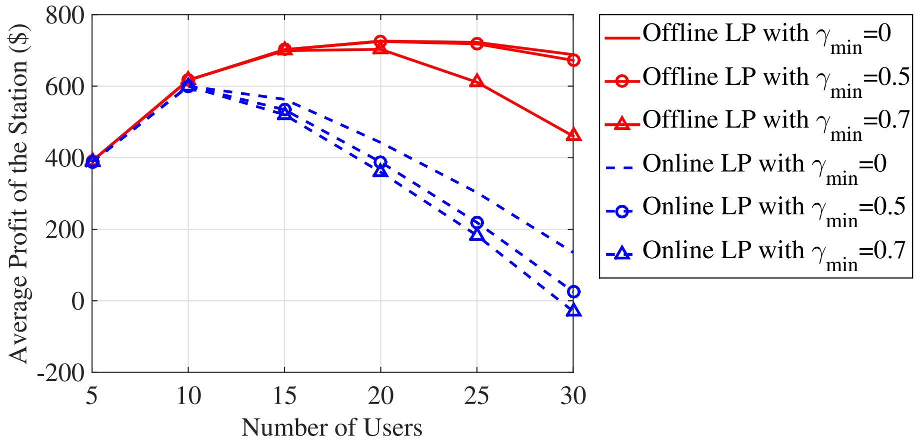

Figure 4.4: Average profit of the online LP two-stage algorithm against its offline counterpart

of length $\Delta = 1$ hour. The total demand of the system and the individual charging speed are limited to $p_{\max} = 1MW$ and $p_{\text{safe}} = 20kW$, respectively. The number of EVs is $N = 30$, and the amount energy (in kWh) that the EV asks is a random real number over [10, 40]. All EV customers have different arrival and departure times. We randomly pick r_i from $[1, 16]$ and d_i from $[r_i + 2, r_i + 8]$. The charging time is restricted to be at least 2 hours since currently available charging stations take around 2-3 hours to charge their EVs [8]. We set $\gamma_{\min}$ to be 0, 0.5 or 0.7, respectively, $\alpha = 4\$/kWh$, $\beta = 0.10\$/(kWh)^2$, and $\eta = 0.20\$/(kWh)^2$.

This section presents some simulation results to illustrate the performance of our online two-stage charging algorithms in terms of the average profit, user satisfaction factor, percentage of EVs serviced, power allocation, load factor, computational time, and competitive ratio, where the average results are taken over 100 random realizations.

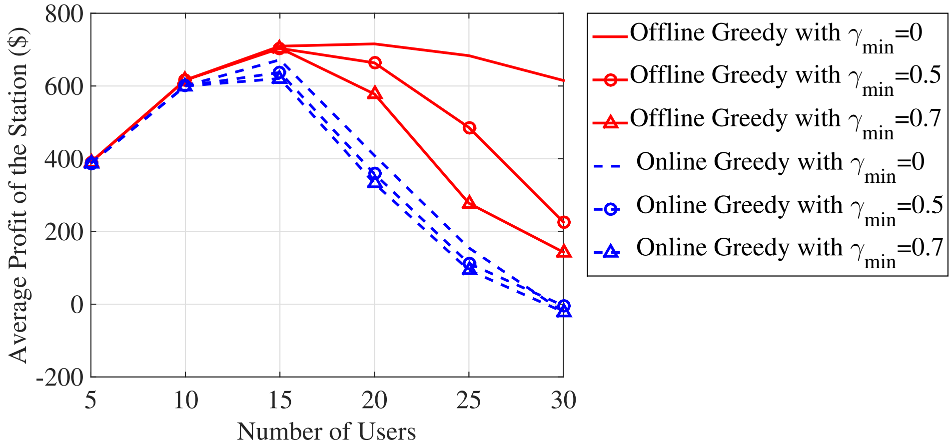

Figure 4.5: Average profit of the online greedy two-stage algorithm against its offline counterpart

In Fig. 4.4 and Fig. 4.5, we show the average profit for both offline and online charging scenarios. Notice that the offline two-stage algorithms provide a better profit as expected due to the knowledge of future charging requests. Also, we observe that the profit obtained by the online two-stage algorithms decreases as $\gamma_{\min}$ increases.

We also show in Fig. 4.6 and Fig. 4.7 the average computational times for both the offline and online charging scenarios. As expected, notice that both online two-stage algorithms consume more overall computational time compared to their offline counterparts. In addition, the computational time of the offline algorithms varies as $\gamma_{\min}$ increases due to the additional iterations needed to find the final solution.

Furthermore, the average user satisfaction factor is shown in Fig. 4.8 and Fig. 4.9, where the average is taken over both random realizations and different customers. The online LP and greedy two-stage algorithms provide respectively about 60% and

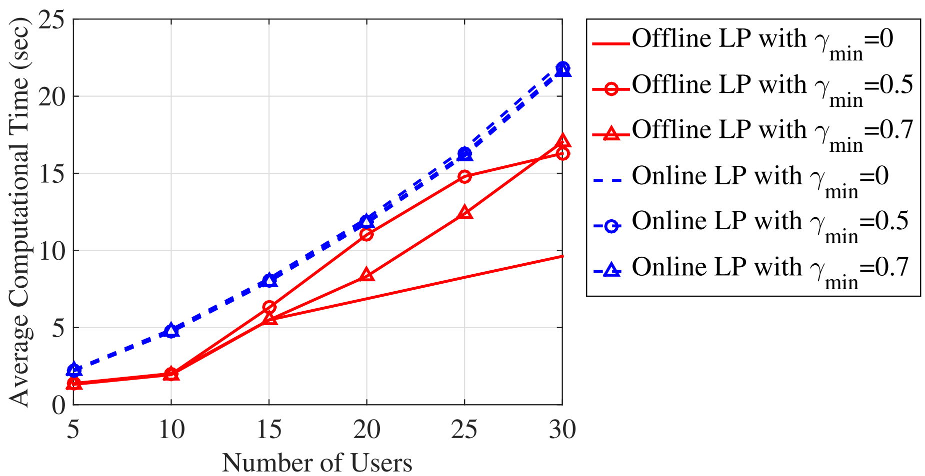

Figure 4.6: Average computational time of the online LP two-stage algorithm against its offline counterpart

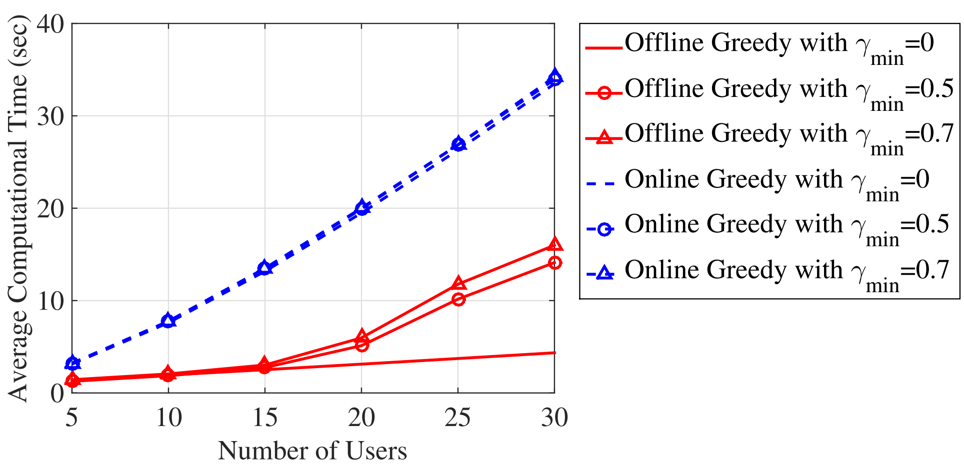

Figure 4.7: Average computational time of the online greedy two-stage algorithm against its offline counterpart

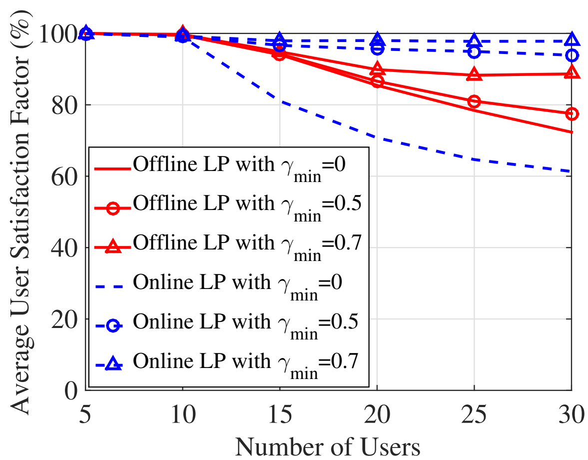

Figure 4.8: Average user satisfaction factor of the online LP two-stage algorithm against its offline counterpart

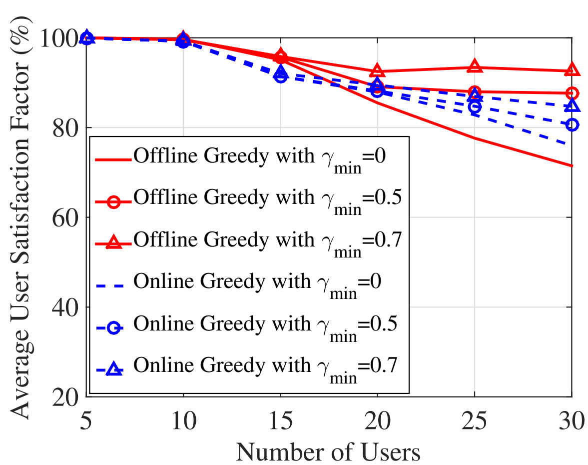

Figure 4.9: Average user satisfaction factor of the online greedy two-stage algorithm against its offline counterpart

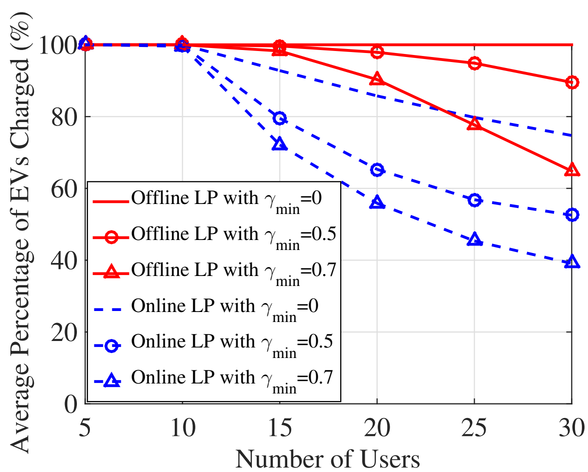

Figure 4.10: Average percentage of vehicles charged of the online LP two-stage algorithm against its offline counterpart

75% (vs. 100% and 85%) of charging when $\gamma_{\min} = 0$ (vs. $\gamma_{\min} = 0.7$). Notice that the user satisfaction factor increases as the value of $\gamma_{\min}$ increases, at the price of rejecting more customers. This information can be utilized to design an online strategy to achieve certain QoS requirement based on the expected number of arriving users.

In Fig. 4.10 and Fig. 4.11, we show the percentages of EVs serviced. The online LP and greedy two-stage algorithms respectively serve 75% and 70% (vs. 40% and 50%) of EVs when $\gamma_{\min} = 0$ (vs. $\gamma_{\min} = 0.7$). This is still a good result in terms of overall customers satisfaction since about half of the expected EVs are successfully scheduled and charged with at least 70% of the desired energy target for both offline and online charging scenarios.

We present the average power consumption in Fig. 4.12 and Fig. 4.13, where the average is taken over both random realizations and time. Notice that both

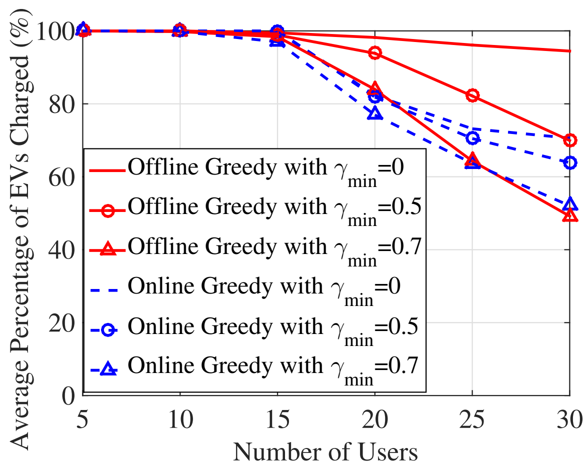

Figure 4.11: Average percentage of vehicles charged of the online greedy two-stage algorithm against its offline counterpart

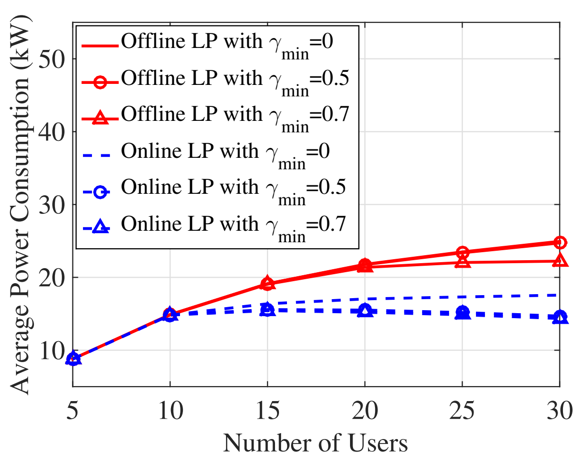

Figure 4.12: Average power consumption of the online LP two-stage algorithm against its offline counterpart

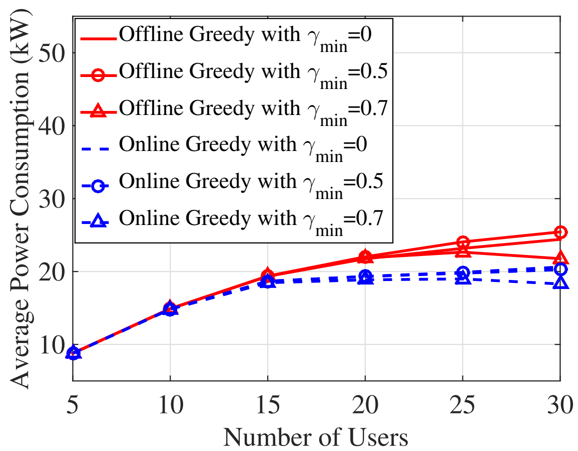

Figure 4.13: Average power consumption of the online greedy two-stage algorithm against its offline counterpart

online algorithms provide a lower power consumption compared with their offline counterparts. This is expected since the offline approach utilize the future charging information to uniformly allocate the total sum of power in order to reduce the power consumption cost.

In Fig. 4.14 and Fig. 4.15, we show the average load factor, where the average is taken over both random realizations and time. We can observe that the online two-stage LP and greedy algorithms respectively achieve about 82% and 78% (vs. 75% and 73%) of load factor when $\gamma_{\min} = 0$ (vs. $\gamma_{\min} = 0.7$). This is an outstanding result since it shows that both offline and online two-stage algorithms provide a very stable power consumption, which reduces power peaks and improves the profit of the operator.

Finally, we illustrate the competitive ratios for general cases. In Fig. 4.16, Fig. 4.17, and Fig. 4.18, we show the competitive ratios achieved under non-congested,

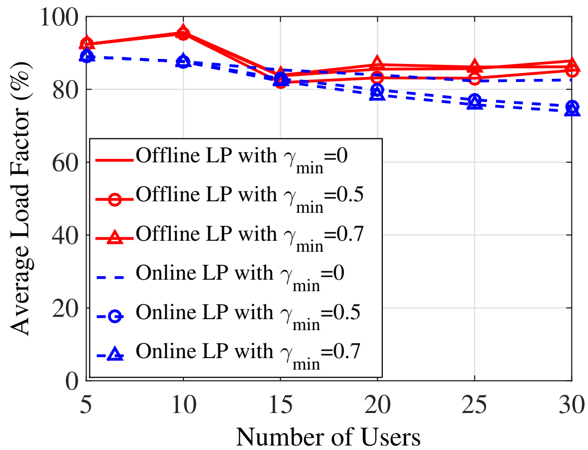

Figure 4.14: Average load factor of the online LP two-stage algorithm against its offline counterpart

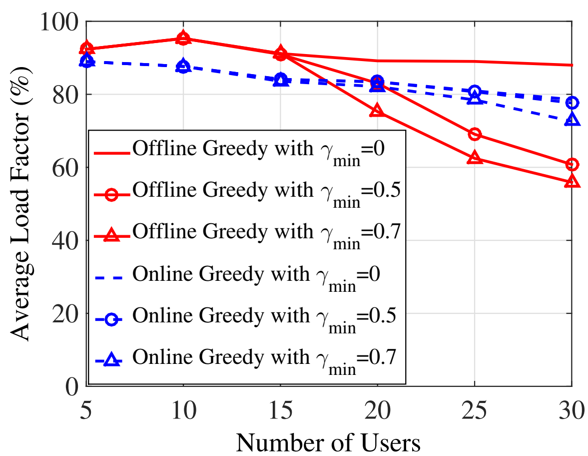

Figure 4.15: Average load factor of the online greedy two-stage algorithm against its offline counterpart

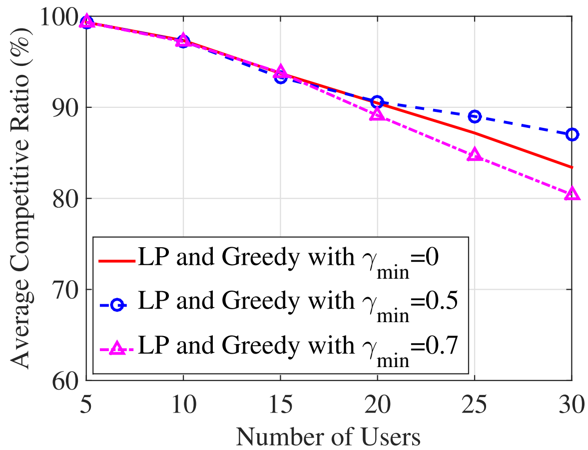

Figure 4.16: Average competitive ratio for general cases under non-congested scenarios.

congested, and very congested environments, respectively. In the non-congested case, the number of customers is small enough such that all EVs can be successfully scheduled and charged on the available machines. Here, both the online LP and greedy algorithms provide a competitive ratio of at least 88% (vs. 80%) when $N = 30$ and $\gamma_{\min}$ is 0 (vs. 0.7). Meanwhile, in the congested case, the number of customers is large but the set of EVs rejected is still small. Here, the online LP and greedy algorithms provide a competitive ratio of at least 55% and 25% (vs. 40% and 65%) when $N = 30$ and $\gamma_{\min}$ is 0 (vs. 0.7), respectively. Finally, in the highly congested case, the number of EVs rejected is large. Here, both online LP and greedy algorithms provide a competitive ratio close to 20% and 18% (vs. 15% and 40%) when $N = 30$ and $\gamma_{\min}$ is 0 (vs. 0.7).

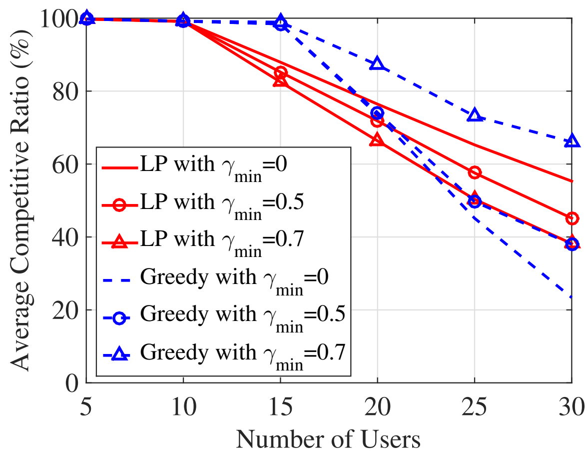

Figure 4.17: Average competitive ratio for general cases under congested scenarios.

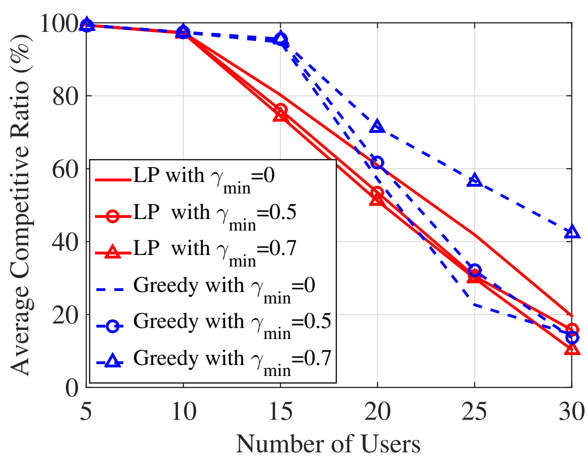

Figure 4.18: Average competitive ratio for general cases under highly congested scenarios.

5. CONCLUSION

This section summarizes the work presented in this dissertation and highlights the results and contribution of this research. We also present some promising future research problems.

5.1 Summary of the Work

In this dissertation, we studied a profit maximization framework for electric vehicle charging under offline and online charging setups. Our algorithms achieve this goal by jointly optimizing EV scheduling, charging power, and user satisfaction factors for multiple EVs, where customers are guaranteed to be charged with at least $\gamma_{\min}$ of the desired energy target.

In Section 2, we described the system model and introduced the overall profit maximization problem for EV charging under both offline and online charging setups. We showed that the profit maximization problem is NP-complete for both cases, and proposed two-stage EV charging strategies to obtain some efficient suboptimal solutions.

In Section 3, we presented an offline two-stage EV charging strategy to obtain suboptimal solutions. In the first stage, the station finds the best EV scheduling that maximizes the unified profit by using either an offline LP relaxation or greedy algorithm with fixed charging power and user satisfaction factors. Then, based on the suboptimal schedule, the second stage optimizes the power allocation and user satisfaction factors to maximize the unified profit of the station operator, where the optimal solutions were derived. Afterwards, the algorithm verifies if every EV is charged with at least $\gamma_{\min}$ percentage of the desired energy target. If not, the EVs with invalid γ's are rejected and the offline two-stage charging algorithm is re-executed until a feasible solution is found.

In Section 4, we introduce an online two-stage EV charging strategy to address the issue when EV arrival information is not available in advance at the station. Similarly, every time a new EV arrives at the station, our online two-stage charging strategy first finds the best EV scheduling that maximizes the instantaneous profit by using either an online LP relaxation or greedy algorithm with fixed charging power and user satisfaction factors. Then, the second stage optimizes the power allocation and user satisfaction factors to maximize the instantaneous profit of the station operator. Here, optimal solutions were derived. Finally, after the solutions to the power allocation and user satisfaction factors are obtained, the station verifies if the previous EVs can still be charged with at least $\gamma_{\min}$ satisfaction factor after the new EV arrives at the station. If not, the new EV is rejected immediately and the previous charging strategy is resumed.

In Sections 3 and 4, simulation results were presented to evaluate our two-stage algorithms by comparing with other charging approaches, and showed that our strategies perform well with respect to the average profit, user satisfaction factor, percentage of EVs serviced, computational time, power consumption, load factor, and competitive ratio.

We showed that for both offline and online cases the profit and the percentage of EVs serviced decrease as the value of $\gamma_{\min}$ increases, while the user satisfaction factor increases as the value of $\gamma_{\min}$ increases, at the price of rejecting more customers. Therefore, we observe that there is a clear tradeoff between the profit obtained by the station owner and the quality of the charging service provided to the customers.

5.2 Summary of the Contributions

The main contributions of this work are summarized as follows:

- A profit maximization framework for charging is proposed, which jointly schedules EVs, allocates power, and adjusts the user satisfaction factor, under peak power and charging facility constraints. It is shown that the profit maximization problem is NP-complete in both offline and online scenarios.

- An efficient two-stage charging strategy is proposed to solve the profit maximization problem for each charging scenario. In the offline case, the first stage finds a suboptimal schedule by using either an offline LP relaxation or greedy scheduling algorithm. Then, given the schedule from the first stage, the second stage optimizes the charging power and user satisfaction factors, where closed-form solutions are derived. After that, the algorithm verifies if each EV is charged with at least the minimum user satisfaction factor. If not, the EVs that violated this condition are rejected and the offline two-stage algorithm is re-executed until a feasible solution is found. In the online case, the first stage also finds the suboptimal schedule that maximizes the instantaneous profit by using either an online LP relaxation or greedy scheduling algorithm. Then, the second stage optimizes the charging power and user satisfaction factors of the EVs currently connected to the station, where closed-form solutions are also derived. Afterwards, the algorithm verifies if all EVs are charged with at least the minimum user satisfaction factor. If not, the new EV is rejected immediately and the previous charging strategy is resumed.

- The computational complexity is analyzed for both offline and online algorithms, where it shows that the greedy scheduling algorithms outperform the LP relaxation scheduling algorithms in terms of computational time by slightly sacrificing the overall profit.

- A competitive analysis for the online two-stage charging algorithm is also considered. The lower bounds of competitive ratio are derived in terms of the unified profit for a special case when all EVs depart at the same time with a high power demand. For this special case, the competitive ratio σ is guaranteed to be at least $\frac{\alpha T-2\beta WN}{\alpha T-\beta WN}$ and $\frac{\alpha WC\left(\frac{|S|-C}{T}+1\right)-\beta W^2\left[\frac{2|S|C}{T}+\frac{|S|-C}{C}+\frac{2(|S|-C)^2}{T-2(|S|-C)+1}\right]}{\alpha W|S|-\beta\frac{W^2|S|^2}{T}}$ for non-congested and congested scenarios, respectively.

- Simulation results show that the proposed offline two-stage LP and greedy strategies respectively provide at least 86% and 76% of the unified profit obtained by the exhaustive search charging strategy when the minimum user satisfaction factor is 0%. Meanwhile, under non-congested scenarios, both online two-stage LP and greedy strategies guarantee at least 88% (vs. 80%) of the unified profit obtained by their offline counterparts when the minimum user satisfaction factor is 0% (vs. 70%). Similarly, under congested scenarios, the online two-stage LP and greedy strategies guarantee respectively at least 55% and 25% (vs. 40% and 65%) of the unified profit obtained by their offline counterparts when the minimum user satisfaction factor is 0% (vs. 70%). Moreover, both offline two-stage LP and greedy algorithms achieve user satisfaction factors of 72% (vs. 88% and 92%) when the minimum user satisfaction factor is 0% (vs. 70%). Meanwhile, the online two-stage LP and greedy algorithms respectively achieve user satisfaction factors of 60% and 75% (vs. 98% and 85%) when the minimum user satisfaction factor is 0% (vs. 70%). Notice that as the value of the minimum user satisfaction factor increases, the profit of the station decreases and the user satisfaction factor achieved at the end of the schedule increases. Therefore, we can say that there is a clear tradeoff between the profit obtained by the operator and the quality of the charging service provided for both offline and online charging strategies. It is worth mentioning that these simulation results are obtained in a congested scenario with a small number of charging machines.

- In addition to the average profit and user satisfaction factor, our offline and online charging strategies provide outstanding results with respect to the average computational time, percentage of EVs serviced, power consumption, competitive ratio, and load factor of the system.

5.3 Future Research Work

Several future research lines related to the EV charging problem presented in this dissertation have been identified. Some of them are summarized as follows:

- Our overall EV charging-scheduling problem was proved to be NP-Complete, and solved using an efficient two-stage algorithm to find suboptimal solutions. An interesting idea would be to consider other approximation algorithms to determine if we can improve the profit of the station and get closer to the optimal solution. An example of such algorithms is the submodular optimization approach.

- In this work, we presented a greedy and deterministic online two-stage EV charging strategy. A possible extension of this work would be to consider certain statistical information to predict the customer arrival in the future. This would help with a better plan for allocation of power and scheduling of charging machines to improve the profit and QoS.

- For both offline and online charging scenarios, we assumed that all EVs have identical battery packs. An attractive idea could be to study the influence on the charging efficiency when considering different battery packs.

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APPENDIX A

OPTIMAL SOLUTION TO THE EV CHARGING PROBLEM

A.1 The proof of Theorem 3.2.1

Since problem (6) is a convex quadratic problem, we can obtain the optimal solution by KKT. The Lagrangian function of problem (6) is given by

$$\begin{aligned} F = &- \left\{ \sum_{t \in D_m} \left[\alpha \sum_{i \in I_m} p_{ij}^t - \beta \left(\sum_{i \in I_m} p_{ij}^t \right)^2 \right] - \eta \sum_{i \in I_m} \left(w_i - \gamma_i w_i \right)^2 \right\} \\ &+ \sum_{t \in D_m} \lambda_t \left(\sum_{i \in I_m} p_{ij}^t - p_{max} \right) + \sum_{i \in I_m} \mu_i \left(\gamma_i w_i - \sum_{t=r_i}^{d_i} p_{ij}^t \right) \\ &+ \sum_{i \in I_m} \upsilon_i \left(\sum_{t=r_i}^{d_i} p_{ij}^t - w_i \right) + \sum_{t \in D_m} \sum_{i \in I_m} \sigma_{ij} \left(p_{ij}^t - p_{safe} \right) + \sum_{i \in I_m} z_i \left(\gamma_i - 1 \right). \end{aligned} \tag{A.1}$$

After taking the derivative with respect to p_{ij}^t and γ_i, respectively, we obtain the following KKT conditions

$$\frac{\partial F}{\partial p_{ij}^t} = -\alpha + 2\beta \sum_{i \in I_m} p_{ij}^{*\,t} + \lambda_t - \mu_i + \upsilon_i + \sigma_{ij} = 0; \tag{A.2}$$

$$\frac{\partial F}{\partial \gamma_i} = 2\eta w_i^2 \left(\gamma_i^* - 1\right) + \mu_i w_i + z_i = 0; \tag{A.3}$$

$$\sum_{i \in I_m} p_{ij}^{*\,t} \leq p_{max}, t \in D_m; \tag{A.4}$$

$$\gamma_i^* w_i \leq \sum_{t=r_i}^{d_i} p_{ij}^{*\,t} \leq w_i, i = 1, 2, \ldots, N; \tag{A.5}$$

$$0 \leq p_{ij}^{*\,t} \leq p_{safe}, i = 1, 2, \ldots, N, r_i \leq t \leq d_i; \tag{A.6}$$

$$0 \leq \gamma_i^* \leq 1, i = 1, 2, \ldots, N; \tag{A.7}$$

$$\lambda_t \geq 0, \mu_i \geq 0, \upsilon_i \geq 0, \sigma_{ij} \geq 0, z_i \geq 0; \tag{A.8}$$

$$\lambda_t \left(\sum_{i \in I_m} p_{ij}^{*\,t} - p_{max} \right) = 0, t \in D_m; \tag{A.9}$$

$$\mu_i \left(\gamma_i^* w_i - \sum_{t \in D_m} p_{ij}^{*\,t} \right) = 0, i = 1, 2, \ldots, N; \tag{A.10}$$

$$\upsilon_i \left(\sum_{t \in D_m} p_{ij}^{*\,t} - w_i \right) = 0, i = 1, 2, \ldots, N; \tag{A.11}$$

$$\sigma_{ij} \left(p_{ij}^{*\,t} - p_{safe} \right) = 0, i = 1, 2, \ldots, N, t \in D_m; \tag{A.12}$$

$$z_i \left(\gamma_i^* - 1 \right) = 0, i = 1, 2, \ldots, N. \tag{A.13}$$

From (A.2) and (A.3), we obtain

$$\sum_{i \in I_m} {p_{ij}^*}^t = \frac{\alpha - \lambda_t + \mu_i - \upsilon_i - \sigma_{ij}}{2\beta}, \tag{A.14}$$

$$\gamma_i^* = \frac{2\eta w_i^2 - \mu_i w_i - z_i}{2\eta w_i^2}. \tag{A.15}$$

After solving for all possible values of λ_t, μ_i, υ_i, σ_{ij}, and z_i, we discuss the optimal solutions in the following cases.

Case 1) $\lambda_t > 0$, $\mu_i > 0$ and $\upsilon_i = 0$, $\sigma_{ij} = 0$, and $z_i = 0$:

According to (A.4), (A.5), (A.9), (A.10), (A.11), (A.12), and (A.13), we have

$$\sum_{i \in I_m} {p_{ij}^*}^t = p_{max},$$

$$\sum_{t \in D_m} {p_{ij}^*}^t = \gamma_i^* w_i,$$

$$\sum_{t \in D_m} {p_{ij}^*}^t < w_i,$$

$${p_{ij}^*}^t < p_{safe},$$

$$\gamma_i < 1.$$

Therefore, the optimal solution to the sum of power is given by

$$\sum_{i \in I_m} {p_{ij}^*}^t = p_{max}. \tag{A.16}$$

Now, let us find the solution to γ_i. We have $\sum_{t\in D_m} p_{ij}^{*\,t} = \gamma_i^* w_i$ for $i \in I_m$ and $0 \leq \gamma_i < 1$. By summing up $\sum_{t\in D_m} p_{ij}^{*\,t}$ over i and switching the summations, we obtain the following

$$\sum_{i\in I_m} p_{ij}^{*\,t} = \sum_{i\in I_m} \frac{\gamma_i^* w_i}{|D_m|}. \tag{A.17}$$

Let us substitute (A.16) into (A.17)

$$\sum_{i\in I_m} \frac{\gamma_i^* w_i}{|D_m|} = p_{max}. \tag{A.18}$$

We solve the equation and find the following optimal solution

$$\begin{aligned} \gamma_i^* &= \frac{|D_m|p_{max} + |I_m|w_i - \sum\limits_{j\in I_m} w(j)}{|I_m|w_i} \\ &= 1 - \frac{\sum_{i\in I_m} w_i - |D_m|p_{max}}{|I_m|w_i}. \end{aligned} \tag{A.19}$$

The solutions (A.16) and (A.19) are valid only when $W_m > \frac{2p_{max}(|I_m|\beta+\eta|D_m|)-\alpha|I_m|}{2\eta}$.

Case 2) $\lambda_t = 0$, $\mu_i > 0$ and $\upsilon_i = 0$, $\sigma_{ij} > 0$, and $z_i = 0$:

According to (A.4), (A.5), (A.9), (A.10), (A.11), (A.12), and (A.13), we have

$$\sum_{i \in I_m} {p_{ij}^*}^t < p_{max},$$

$$\sum_{t \in D_m} {p_{ij}^*}^t = \gamma_i^* w_i,$$

$$\sum_{t \in D_m} {p_{ij}^*}^t < w_i,$$

$${p_{ij}^*}^t = p_{safe},$$

$$\gamma_i < 1.$$

Thus, the optimal solution to the sum of power is given by

$$\sum_{i \in I_m} {p_{ij}^*}^t = \sum_{i \in I_m} p_{safe} = |I_m| p_{safe}. \tag{A.20}$$

Now, let us find the solution to γ_i. We have $\sum_{t \in D_m} {p_{ij}^*}^t = \gamma_i^* w_i$ for $i \in I_m$ and $0 \leq \gamma_i < 1$. Similar to the previous case, we sum up $\sum_{t \in D_m} {p_{ij}^*}^t$ over i and switch the summations to obtain the following

$$\sum_{i \in I_m} {p_{ij}^*}^t = \sum_{i \in I_m} \frac{\gamma_i^* w_i}{|D_m|}. \tag{A.21}$$

Let us substitute (A.20) into (A.21)

$$\sum_{i \in I_m} \frac{\gamma_i^* w_i}{|D_m|} = |I_m| p_{safe}. \tag{A.22}$$

We solve the equation and find the following optimal solution

$$\begin{aligned} \gamma_i^* &= \frac{|D_m||I_m|p_{safe} + |I_m|w_i - \sum\limits_{j \in I_m} w(j)}{|I_m|w_i} \\ &= 1 - \frac{\sum_{i \in I_m} w_i - |D_m||I_m|p_{safe}}{|I_m|w_i}. \end{aligned} \tag{A.23}$$

The solutions (A.20) and (A.23) are also valid only when $W_m > \frac{2|I_m|p_{safe}(|I_m|\beta+\eta|D_m|)-\alpha|I_m|}{2\eta}$.

If we combine the results from the first two cases, we obtain the following:

$$\sum_{i \in I_m} {p_{ij}^*}^t = \min\left(|I_m|p_{safe}, p_{max}\right). \tag{A.24}$$

and

$$\gamma_i^* = 1 - \frac{\sum_{i \in I_m} w_i - |D_m| \min\left(|I_m|p_{safe}, p_{max}\right)}{|I_m|w_i}. \tag{A.25}$$

when $W_m > \frac{2\min\left(|I_m|p_{safe}, p_{max}\right)(|I_m|\beta+\eta|D_m|)-\alpha|I_m|}{2\eta}$.

Case 3) $\lambda_t = 0$, $\mu_i > 0$, $\upsilon_i = 0$, $\sigma_{ij} > 0$, and $z_i = 0$:

According to (A.4), (A.5), (A.9), (A.10), (A.11), (A.12), and (A.13), it can be found that

$$\sum_{i\in I_m} {p_{ij}^*}^t < p_{max},$$

$$\sum_{t\in D_m} {p_{ij}^*}^t = \gamma_i^* w_i,$$

$$\sum_{t\in D_m} {p_{ij}^*}^t < w_i,$$

$${p_{ij}^*}^t = p_{safe},$$

$$\gamma_i^* < 1.$$

So, we can say that $\sum_{t\in D_m} {p_{ij}^*}^t = \gamma_i^* w_i$ for $i \in I_m$ and $0 \le \gamma_i < 1$. Let us sum up this result over i so that

$$\sum_{i\in I_m}\sum_{t\in D_m} {p_{ij}^*}^t = \sum_{i\in I_m} \gamma_i^* w_i.$$

By switching the summations on the left hand side and solving the equation, we obtain that

$$\sum_{i\in I_m} {p_{ij}^*}^t = \sum_{i\in I_m} \frac{\gamma_i^* w_i}{|D_m|}, \tag{A.26}$$

which is the optimal sum of power for this case.

Next, we derive the optimal value of γ_i^*. From (A.15), we know that $\gamma_i^* = \frac{2\eta w_i^2 - \mu_i w_i}{2\eta w_i^2}$, where $\mu_i = 2\eta w_i(1-\gamma_i^*)$. Then, based on the case conditions and (A.14), we obtain

$$\sum_{i\in I_m} {p_{ij}^*}^t = \frac{\alpha+\mu_i}{2\beta} = \frac{\alpha+2\eta w_i(1-\gamma_i^*)}{2\beta}. \tag{A.27}$$

Using the result from (A.26), we have

$$\sum_{i\in I_m} \frac{\gamma_i^* w_i}{|D_m|} = \frac{\alpha+2\eta w_i(1-\gamma_i^*)}{2\beta}. \tag{A.28}$$

After solving this equation, we obtain the following optimal solution

$$\begin{aligned}\gamma_i^* &= \frac{\alpha|D_m| + 2\eta|D_m|w_i + 2\beta\big(|I_m|w_i - \sum_{j\in I_m} w(j)\big)}{2w_i(|I_m|\beta+\eta|D_m|)} \\ &= 1 - \frac{2\beta\sum_{i\in I_m} w_i - \alpha|D_m|}{2w_i(|I_m|\beta+\eta|D_m|)}. \end{aligned} \tag{A.29}$$

Substituting (A.29) into (A.26), we obtain the optimal solution to the sum of power

$$\begin{aligned}\sum_{i\in I_m} {p_{ij}^*}^t &= \sum_{i\in I_m} \frac{\alpha|D_m| + 2\eta|D_m|w_i + 2\beta\big(|I_m|w_i - \sum_{j\in I_m} w(j)\big)}{2|D_m|(|I_m|\beta+\eta|D_m|)} \\ &= \frac{\alpha|I_m| + 2\eta\sum_{i\in I_m} w_i}{2(|I_m|\beta+\eta|D_m|)}. \end{aligned} \tag{A.30}$$

Equations (A.29) and (A.30) are the optimal solutions to the user satisfaction factors and sum of power, respectively. This is valid whenever $\frac{\alpha|D_m|}{2\beta} \leq W_m \leq \frac{2\min\left(|I_m|p_{safe}, p_{max}\right)(|I_m|\beta+\eta|D_m|)-\alpha|I_m|}{2\eta}$.

Case 4) For any other values of λ_t, μ_i, υ_i, σ_{ij}, and $z_i > 0$, we obtain $\gamma_i^* = 1$ and $\sum_{i \in I_m} p_{ij}^{*\,t} = \sum_{i \in I_m} \frac{w_i}{|D_m|}$ whenever $W_m < \frac{\alpha|D_m|}{2\beta}$.

In all, the optimal solutions can be summarized in the form given in Theorem 3.2.1.

APPENDIX B

COMPETITIVE ANALYSIS

B.1 The proof of Theorem 4.3.1

We analyze the competitive ratio for the EV charging problem for non-congested and congested scenarios. Here, we consider a special case where each EV has a different arrival time r_i, with the same departure time d, user satisfaction factor $\gamma = 1$, and energy requirement W.

B.1.1 Non-congested scenario ($G_t \leq C$ for all t)

Let I be the set of EVs be charged simultaneously in the offline case. When $G_t \leq C$ for all $t \in [1, T]$, all EVs are scheduled and charged successfully. The unified profit obtained by the offline algorithm is given by

$$F_{\text{off}} = \sum_{t=1}^{T} \left[\alpha \sum_{j=1}^{C} \sum_{i \in I} p_{ij}^{t} - \beta \left(\sum_{j=1}^{C} \sum_{i \in I} p_{ij}^{t} \right)^{2} \right] - \eta \sum_{i \in I} \left(w_i - \gamma_i w_i \right)^2 , \tag{B.1}$$

$$= \sum_{t=1}^{T} \left[\alpha \sum_{i \in I} \frac{\gamma_i w_i}{T} - \beta \left(\sum_{i \in I} \frac{\gamma_i w_i}{T} \right)^{2} \right] - \eta \sum_{i \in I} \left(w_i - \gamma_i w_i \right)^2 , \tag{B.2}$$

$$= \sum_{t=1}^{T} \left[\alpha \sum_{i \in I} \frac{W}{T} - \beta \left(\sum_{i \in I} \frac{W}{T} \right)^{2} \right] , \tag{B.3}$$

$$= \alpha W N - \beta \frac{W^2 N}{T} - 2\beta \frac{W^2}{T} \sum_{i=1}^{N-1} \left(N - i \right) , \tag{B.4}$$

$$= \alpha W N - \beta \frac{W^2 N}{T} - 2\beta \frac{W^2}{T} \left[N(N-1) - \frac{N(N-1)}{2} \right] , \tag{B.5}$$

$$= \alpha W N - \beta \frac{W^2 N^2}{T} . \tag{B.6}$$

For the online case, let J_n and L_n be the set of scheduled EVs and the length of the charging period after the n^{th} arrival, respectively. Here, the expected charging time of each EV is calculated by $T_i = T + 1 - i$. The profit provided by the online algorithm is given by

$$F_{\text{on}} = \sum_{n=1}^{N} \left\{ \sum_{t \in L_n} \left[\alpha \sum_{j=1}^{C} \sum_{i \in J_n} p_{ij}^t - \beta \left(\sum_{j=1}^{C} \sum_{i \in J_n} p_{ij}^t \right)^2 \right] - \eta \sum_{i \in J_n} \left(w_i^{L_n} - \gamma_i^{L_n} w_i^{L_n} \right)^2 \right\} \tag{B.7}$$

$$= \sum_{n=1}^{N} \left\{ \alpha \sum_{i \in J_n} \frac{\gamma_i^{L_n} w_i^{L_n}}{T_i^{L_n}} - \beta \left(\sum_{i \in J_n} \frac{\gamma_i^{L_n} w_i^{L_n}}{T_i^{L_n}} \right)^2 - \eta \sum_{i \in J_n} \left[w_i^{L_n} - \gamma_i^{L_n} w_i^{L_n} \right]^2 \right\} \tag{B.8}$$

$$= \sum_{n=1}^{N} \left\{ \alpha \sum_{i \in J_n} \frac{W - v_i^{L_n}}{T_i^{L_n}} - \beta \left(\sum_{i \in J_n} \frac{W - v_i^{L_n}}{T_i^{L_n}} \right)^2 \right\} \tag{B.9}$$

$$= \alpha W N - \beta W^2 \sum_{i=1}^{N} \frac{1}{T_i} - 2\beta W^2 \sum_{i=1}^{N-1} \frac{T_i - 1}{T_i} \sum_{k>i}^{N} \frac{1}{T_k} \tag{B.10}$$

$$= \alpha W N - \beta W^2 \sum_{i=1}^{N} \frac{1}{T_i} - 2\beta W^2 \sum_{i=1}^{N-1} \frac{N - i}{T_i} \tag{B.11}$$

$$= \alpha W N - \beta W^2 \sum_{i=1}^{N} \frac{1}{T + 1 - i} - 2\beta W^2 \sum_{i=1}^{N-1} \frac{N - i}{T + 1 - i} \tag{B.12}$$

$$= \alpha W N - \beta W^2 \sum_{i=1}^{N} \frac{1}{T + 1 - i} - 2\beta W^2 \left[N + (N - 1 - T) \sum_{i=1}^{N-1} \frac{1}{T + 1 - i} \right] \tag{B.13}$$

$$= \alpha W N - 2\beta W^2 N + \beta W^2 (2T + 1 - 2N) \sum_{i=1}^{N-1} \frac{1}{T + 1 - i}. \tag{B.14}$$

Notice that

$$\sum_{i=1}^{N} \frac{1}{T + 1 - i} = \frac{1}{T} + \frac{1}{T - 1} + \cdots + \frac{1}{T - (N - 1)} \geq \frac{N}{T}. \tag{B.15}$$

Substituting (B.15) into (B.14), we have

$$F_{\text{on}} \geq \alpha WN - 2\beta W^2 N + \beta W^2 (2T + 1 - 2N)\frac{N}{T} \tag{B.16}$$

$$= \alpha WN + \beta W^2 (1 - 2N)\frac{N}{T} \tag{B.17}$$

$$\geq \alpha WN - \frac{2\beta W^2 N^2}{T} \tag{B.18}$$

Based on the results obtained in (B.6) and (B.18), the lower bound of the competitive ratio under a non-congested scenario is given by

$$\sigma = \frac{F_{\text{on}}}{F_{\text{off}}} \geq \frac{\alpha WN - \frac{2\beta W^2 N^2}{T}}{\alpha WN - \beta \frac{W^2 N^2}{T}} \geq \frac{\alpha T - 2\beta WN}{\alpha T - \beta WN}. \tag{B.19}$$

B.1.2 Congested scenario ($G_t > C$ for any t)

When $G_t > C$ for any $t \in [1, T]$, a set $\{1, 2, \ldots, |S|\}$ of EVs is scheduled and charged, where $|S| < N$. The unified profit obtained by the offline algorithm is determined by

$$F_{\text{off}} = \sum_{t=1}^{T} \left[\alpha \sum_{j=1}^{C} \sum_{i \in S} p_{ij}^{t} - \beta \left(\sum_{j=1}^{C} \sum_{i \in S} p_{ij}^{t} \right)^2 \right] - \eta \sum_{i \in S} (w_i - \gamma_i w_i)^2 \tag{B.20}$$

$$= \sum_{t=1}^{T} \left[\alpha \sum_{i \in S} \frac{\gamma_i w_i}{T} - \beta \left(\sum_{i \in S} \frac{\gamma_i w_i}{T} \right)^2 \right] - \eta \sum_{i \in S} (w_i - \gamma_i w_i)^2 \tag{B.21}$$

$$= \sum_{t=1}^{T} \left[\alpha \sum_{i \in S} \frac{W}{T} - \beta \left(\sum_{i \in S} \frac{W}{T} \right)^2 \right] \tag{B.22}$$

$$= \alpha W |S| - \beta \frac{W^2 |S|}{T} - 2\beta \frac{W^2}{T} \sum_{i=1}^{|S|-1} (|S| - i) \tag{B.23}$$

$$= \alpha W |S| - \beta \frac{W^2 |S|}{T} - 2\beta \frac{W^2}{T} \left(|S|(|S| - 1) - \frac{|S|(|S| - 1)}{2} \right) \tag{B.24}$$

$$= \alpha W |S| - \beta \frac{W^2 |S|^2}{T}. \tag{B.25}$$

For the online case, let I_n and L_n be defined similar to the non-congested scenario. For this specific case, the set of EVs that are partially scheduled is $S_1 = \{1, 2, \ldots, |S| - C\}$, and the set of EVs completely scheduled is $S_2 = \{|S| - C + 1, |S| - C + 2, \ldots, |S|\}$, where $S = S_1 \bigcup S_2$. The profit provided by the online algorithm is given by

$$F_{\text{on}} = \sum_{n=1}^{N} \sum_{t \in L_n} \left\{ \alpha \sum_{j=1}^{C} \sum_{i \in J_n} p_{ij}^{t} - \beta \left(\sum_{j=1}^{C} \sum_{i \in J_n} p_{ij}^{t} \right)^2 - \eta \sum_{i \in J_n} \left[w_i^{L_n} - \gamma_i^{L_n} w_i^{L_n} \right]^2 \right\} \tag{B.26}$$

$$= \sum_{n=1}^{N} \left\{ \alpha \sum_{i \in J_n} \frac{\gamma_i^{L_n} w_i^{L_n}}{T_i^{L_n}} - \beta \left(\sum_{i \in J_n} \frac{\gamma_i^{L_n} w_i^{L_n}}{T_i^{L_n}} \right)^2 - \eta \sum_{i \in J_n} \left[w_i^{L_n} - \gamma_i^{L_n} w_i^{L_n} \right]^2 \right\} \tag{B.27}$$

$$= \sum_{n=1}^{N} \left\{ \alpha \sum_{i \in J_n} \frac{W - v_i^{L_n}}{T_i^{L_n}} - \beta \left(\sum_{i \in J_n} \frac{W - v_i^{L_n}}{T_i^{L_n}} \right)^2 \right\} \tag{B.28}$$

$$\begin{aligned} &= \alpha W C \left(\sum_{i=1}^{|S|-C} \frac{1}{T_i} + 1 \right) - \beta W^2 \left(C \sum_{i=1}^{|S|-C} \left(\frac{1}{T_i} \right)^2 + \sum_{i=|S|-C+1}^{|S|} \frac{1}{T_i} \right) \\ &\quad - 2\beta W^2 \left(\sum_{i=1}^{|S|-C} \frac{1}{T_i} \sum_{k=1}^{|S|-C} \frac{C-k}{T_{i+k}} + \sum_{i=|S|-C+1}^{|S|} \frac{|S|-i}{T_i} \right) \end{aligned} \tag{B.29}$$

$$\begin{aligned} &= \alpha W C \left(\sum_{i=1}^{|S|-C} \frac{1}{T_i} + 1 \right) - \beta W^2 C \sum_{i=1}^{|S|-C} \left(\frac{1}{T_i} \right)^2 \\ &\quad - 2\beta W^2 \left(\sum_{i=1}^{|S|-C} \frac{1}{T_i} \sum_{k=1}^{|S|-C} \frac{C-k}{T_{i+k}} \right) - 2\beta W^2 C \\ &\quad + \beta W^2 \left((2T + 1 - 2|S|) \sum_{i=|S|-C+1}^{|S|} \frac{1}{T_i} \right). \end{aligned} \tag{B.30}$$

Now, let us find the lower bound of F_{on}. Notice that

$$\sum_{i=1}^{|S|-C} \frac{1}{T_i} = \frac{1}{T} + \frac{1}{T-1} + \cdots + \frac{1}{T-(|S|-C-1)} \geq \frac{|S|-C}{T}, \tag{B.31}$$

$$\sum_{i=1}^{|S|-C} \left(\frac{1}{T_i}\right)^2 = \frac{1}{T^2} + \frac{1}{(T-1)^2} + \cdots + \frac{1}{(T-(|S|-C-1))^2} \leq \frac{|S|-C}{C^2}, \tag{B.32}$$

$$\begin{aligned}\sum_{i=|S|-C+1}^{|S|} \frac{1}{T_i} &= \frac{1}{T-(|S|-C)} + \frac{1}{T-(|S|-C+1)} + \cdots + \frac{1}{T-(|S|-1)} \\ &\geq \frac{C}{T},\end{aligned} \tag{B.33}$$

$$\begin{aligned}&\sum_{i=1}^{|S|-C} \frac{1}{T_i} \sum_{k=1}^{|S|-C} \frac{C-k}{T_{i+k}} = \frac{1}{T}\left(\frac{C-1}{T-1} + \cdots + \frac{C-(|S|-C)}{T-(|S|-C)}\right) \\ &+ \frac{1}{T-1}\left(\frac{C-1}{T-2} + \cdots + \frac{C-(|S|-C)}{T-(|S|-C)-1}\right) \\ &+ \cdots + \frac{1}{T-(|S|-C-1)}\left(\frac{C-1}{T-(|S|-C)} + \cdots + \frac{C-(|S|-C)}{T-(|S|-C)-(|S|-C)+1}\right) \\ &\leq \frac{(|S|-C)^2}{T-2(|S|-C)+1}.\end{aligned} \tag{B.34}$$

Substituting (B.31)-(B.34) in (B.30), we have

$$\begin{aligned}F_{\text{on}} &\geq \alpha W C\left(\frac{|S|-C}{T} + 1\right) - \beta W^2 C\left(\frac{|S|-C}{C^2}\right) - 2\beta W^2\left[\frac{(|S|-C)^2}{T-2(|S|-C)+1}\right] \\ &\quad - 2\beta W^2 C + \beta W^2(2T+1-2|S|)\frac{C}{T} \\ &\geq \alpha W C\left(\frac{|S|-C}{T} + 1\right) - \beta W^2\left[\frac{2|S|C}{T} + \frac{|S|-C}{C} + \frac{2(|S|-C)^2}{T-2(|S|-C)+1}\right].\end{aligned} \tag{B.35}$$

Based on the results obtained in (B.25) and (B.35), the lower bound of the competitive ratio under a congested scenario is given by

$$\sigma = \frac{F_{\text{on}}}{F_{\text{off}}} \geq \frac{\alpha WC\left(\frac{|S|-C}{T}+1\right) - \beta W^2\left[\frac{2|S|C}{T} + \frac{|S|-C}{C} + \frac{2(|S|-C)^2}{T-2(|S|-C)+1}\right]}{\alpha W|S| - \beta\frac{W^2|S|^2}{T}}. \tag{B.36}$$

In all, the lower bounds of the competitive ratio for both non-congested and congested scenarios can be summarized in the form given in Theorem 4.3.1.