General Purpose Technologies and Economic Growth: Electricity Diffusion in the Manufacturing Sector Before WWII

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September 2010

CWPE 1048

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ABSTRACT

This paper evaluates the diffusion of electricity within the context of a GPT perspective. The paper develops a new comparative data set on the usage of electricity in the manufacturing sectors of the US, Britain, France, Germany and Japan and proceeds to evaluate the hypotheses of a *productivity slowdown* and of a *productivity bonus* as postulated by many existing GPT models.

INTRODUCTION

That technological change is an important determinant of modern economic growth is an idea that most economists would agree upon. However the details of this relationship are less clear. One hypothesis that has been extensively discussed is the Schumpeterian perspective of technological clusters. This perspective relating to *major* innovation has been used to model long (Kondratieff) cycles in the rate of economic growth. Studies that have seen modern economic history as displaying a sequence of GPTs have used this as a basis of a theory for long cycles in economic growth of the type that Schumpeter discussed (Freeman and Louçã, 2001). Although the relevance of Kondratieff cycles for understanding modern economic growth remains questionable, since the early 1990s a new theoretical approach has emerged that depicts *long-term* growth unevenness as resulting from the emergence of General Purpose Technologies (GPTs) that create long waves of economic growth (Jovanovic and Rousseau, 2005; Aghion and Howitt, 2009).

To date much of the work on GPTs has been theoretical in nature, informing us of *possible* outcomes but offering limited insights on *actual* historical economic growth. Working with fairly simple prototype models of GPTs a number of macroeconomic growth hypotheses have been accepted in the literature; for example, much of the early literature (Hornstein and Krusell 1996, Greenwood and Yorokoglu 1997), argues that the diffusion of a new GPT will be correlated with a productivity slowdown in the early phase of the diffusion process, and *after some time* this would be followed by productivity acceleration. As the title of a 1998 article by Helpman and Trajtenberg put it there would be "a time to sow and a time to reap".[1] Indeed it is these growth effects that make GPTs inherently different from other technological changes. As Helpman puts it:

> Growth that is driven by general purpose technologies is different from growth driven by incremental innovation. Unlike incremental innovation, GPTs can trigger an uneven growth trajectory, which starts with a prolonged slowdown followed by a fast acceleration.

(Helpman, 2004, p. 51)

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[1] This hypothesis follows David 1990.

However, GPT models are, at present, simple thought experiments and have serious limitations when used to describe and explain historical economic growth. Lipsey et al. (2005, p. 384) review the literature as it developed since the early 1990s and argue that all models to date share the common problem that they deal with a complex historical economic system inappropriately, seeing shifts in the rate of economic growth as the outcome of a single GPT. In reality, at any point in time, change in the rate of economic growth will be an outcome of the stock of GPTs at different stages of their life cycle, and as such the link between a particular GPT and historical economic growth is not uniquely determined. So while episodic effects on economic growth are a reasonable research hypothesis, the expectation that they would generate long-term growth regularities or quasi-cyclical patterns in the shape of a productivity slowdown followed by a productivity surge is far more questionable.

In the last few years the GPT approach has come under growing attack. Joel Mokyr suggested that it can hardly be described as a theory and that its academic currency is on the wane (Mokyr, 2006). Alexander Field (2008) convincingly suggests that GPT theory has little heuristic power. Moreover he shows that the criteria commonly used by the literature to identify GPTs and to separate them from the rest of technical progress (what Helpman describes as incremental innovation) are at best subjective. But they are also flawed in the sense of being too restrictive and to exclude technological transformations likely to produce significant growth effects. Indeed the impossibility of effectively using the proposed criteria for GPT selection has led to the compilation of vastly subjective and heterogeneous lists of historical examples of GPTs. The heterogeneity of choice is such that only three GPTs appear in all the eleven compilations surveyed by Field. They are what Field calls the Big Three namely: (1) IT or ICT (or more specifically semiconductors, or computer, or Internet); (2) electricity; and (3) steam. These are indeed the three GPTs mentioned in the original article by David (1990) from which much of the GPT literature originates.

Field observes that steam has already been dealt a serious blow in its status as a GPT by Crafts and Mills (2004). Moreover, it is very difficult to describe steam as a GPT as its application was restricted to an exceedingly small amount of uses for the first half century after Newcomen's Dudley Castle Machine of 1712. Even once Watt

invented the separate condenser, steam engine take-up remained for many decades slow and limited to a handful of uses (mostly mining). Similarly, the development of high-pressure steam engines in the first two decades of the 19th century did not expand steam use much beyond the realm of mining. It is only with the further advances in high-pressure technology and with the arrival of compounding in the 1840s and 50s, that steam can start to lay claims of generality in its use at least when it comes to textile production, metallurgy, land and, later, sea transport. Yet, even in 1870 two thirds of steam power was concentrated in coal-mining, cotton textiles, and metal manufactures (Crafts and Mills, 2004). In other words it took more than a century and a half for steam to become a GPT (on the relevance of the Corliss engine in the diffusion process of steam see Rosenberg and Trajtenberg, 2004). The development of steam was so slow, its diffusion process so gradual, as to prevent any of the sudden accelerations and decelerations in growth and productivity that theory associates with GPTs.

This leaves GPT literature with only two agreed examples: ICT; and its historical antecedent: electricity. We concentrate on the latter. In particular we look at the diffusion of electricity in the manufacturing sector before World War II. The focus on the manufacturing sector is the result of evidence indicating that the acceleration in US labour productivity growth during the 1920s originated almost entirely in the manufacturing sector (David and Wright, 2003; Field, 2006). This surge in labour productivity in the manufacturing sector was common to all manufacturing industries and accounted for a substantial proportion of the increase in TFP observed in the first part of the interwar years. If there is a productive sector where the effects of electricity as a GPT on productivity should be magnified and easily detectable that is manufacturing.

To evaluate the hypothesis of a link between GPT and productivity growth we have built a new comparative data set of electricity diffusion in the manufacturing sector for the major industrial economies of the 20th Century – covering the US, the UK, France, Germany and Japan. The selection of countries allows us to compare the experience of the US, as the technological leader, with the 'relatively backward' economies of the early 20th Century.

Our research extends the existing literature in a number of significant ways. David and Wright (2003) focus on the considerable acceleration of productivity growth in the US in the 1920s and associate this with a substantial increase in labour productivity and a concurrent fall in capital intensity in the US manufacturing sector. They see the upsurge in labour productivity as a direct result of the diffusion of electricity-based GPTs in production, and of the supply shock to the labour market caused by increasing restrictions on mass-immigration. Although David and Wright also consider the experience of Britain and Japan, they rely on a descriptive analysis of the diffusion of electricity in the manufacturing sectors for these economies[2]. We extend this work by building a comparative data set of electricity consumption per worker in the manufacturing sector for the major industrial countries before WWII as a way of quantifying some of the existing qualitative and theoretical hypotheses.

An important empirical regularity postulated by the GPT literature is the idea of a *productivity slowdown* in the early stages of the arrival of a new GPT. This idea can be found in David (1991), in Bresnahan and Trajtenberg (1995), in Helpman and Trajtenberg (1998a and 1998b), in Aghion and Hewitt (1998)[3] and has been specified as a stylized fact of economic growth in the survey of Jovanovic and Rousseau (2005, p. 1187) who state:

> But overall the evidence clearly supports the view that technological progress is uneven, that it does entail the episodic arrival of GPTs, and that these GPTs bring on turbulence and lower growth early on and higher growth and prosperity later. The bottom line is that with a wider body of data and fifteen more years of it than David (1991) had at his disposal, we confirm his hypothesis that Electrification and IT adoption are manifestations of the same force at work, namely the introduction of a GPT.

From a theoretical point of view the existence of a productivity slowdown is ascribed to a variety of mechanisms: capital obsolescence due to technological uncertainties (Bresnahan and Trajtenberg, 1995; Helpman and Trajtenberg, 1998a and 1998b; Aghion and Howitt 1998; Howitt 1998); to excessive risk-taking in investment

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[2] This leads David and Wright (2003, pp. 147-55) to argue that the use of electricity in manufacturing in Japan and Britain matched that of the US by the 1930s with similar productivity surges, suggesting that follower countries can accelerate the benefits of using the new technology. We show below that the quantitative evidence suggests that the US maintained its leadership position throughout the interwar period.

[3] A similar argument is developed earlier by Phelps-Brown and Handfield-Jones (1952) as an application to the late 19th Century "climacteric" in Britain's economic growth.

decisions in the early phases of a GPT's diffusion process (Aghion and Howitt, 1998); to coordination problems (Bresnahan and Trajtenberg, 1995); to learning processes by firms (Hornstein and Krusell, 1996: Greenwood and Yorokoglu, 1997); the job training of employees (Helpman and Rangel, 1999); to the formation of skill differentials in response to the introduction of a new GPT (Aghion and Howitt 1998); to the required development of complementary technologies (Helpman and Tranjtenberg 1998a); and finally to measurement problems in the national accounts (Howitt 1998).

As for the observation of such growth effects, this has been mostly based on a descriptive evaluation of historical productivity data. For example, David and Wright (2003) use Kendrick's productivity data to show the existence of a low growth phase during 1889-1913, compared to the pre-1889 phase and the 1920s. Jovanovic and Rousseau (2005) use ocular inspection of a Hodrick-Prescott trend of growth rates in output per man-hour to justify evidence of a productivity slowdown. The same technique is then used to detect the subsequent productivity acceleration (the other main stylised fact associated with the arrival of a new GPT)[4]. Our aim is to use time series techniques to describe the trend movements and the episodic long swings of economic growth as a way of avoiding selection problems.

There are two steps to our analysis: first, we present the results of our comparative data set, providing quantitative evidence on electricity usage on a per worker basis for five of the major industrial countries before WWII; we then consider whether this diffusion path is associated with the existence of phases of "productivity slowdown" and "productivity bonus" in the manufacturing sector's productivity growth as a way of evaluating the key empirical macroeconomic hypotheses of the GPT literature.

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[4] They analyse Kendrick's output per man-hour in the non-farm sector.

1. ADOPTION OF ELECTRICITY IN THE MANUFACTURING SECTOR

The traditional indicator of the extent of electrification of production has been the capacity of primary electric motors. These are motors driven by electricity purchased from utilities and, therefore, not produced within the plant. An indicator of the extent of electrification of production in the manufacturing sector that is directly linked to labour productivity is the capacity (in *HP*) of primary electric motors per employee. Data on primary electric capacity is relatively easy to find, but it should be noted that such an indicator has serious drawbacks as a means of thinking about the effects of the electrification of production methods. Electric motors were not necessarily employed at the same rate across countries because of differences at the national level in the number and length of shifts over which industrial machinery is operated in one day. Moreover, the existence of secondary electric motors (those run on electricity produced within the industrial establishment), introduces a further element of confusion in the comparative exercise. A better and more direct way to assess the comparative contribution of the electrification of manufacturing processes to the growth of productivity is to use measures of electricity consumption per worker in the manufacturing sector. The total electricity consumed by the US manufacturing sector in *GWh* is available in the Department of Commerce (1975, series D130). From this and from the data on employment in the manufacturing sector (Kendrick, 1961) we have obtained the observations on the electricity consumed per employee in the US manufacturing sector that appear in Figure 1.

In order to be able to make international comparisons with the experience of the US we have constructed a similar data set for the total electricity consumed per employee in the UK, French, German and Japanese manufacturing sectors. A comparison of the consumption of electricity per employee in the manufacturing sector in these five countries is presented in Figure 2. Figure 2 confirms that the adoption of electrical technologies in production was markedly delayed in the European core countries and Japan as compared to the US. The electricity consumed by each employee in the US manufacturing sector remained three times as high as that consumed by his/her European counterpart for much of the period, and fell below this threshold only in the last part of the 1930s Great Depression.

Two key conclusions emerge from this comparative data set. First, it is clear that the use of electricity in the manufacturing sector was far more extensive in the US than in the other major economies of the inter-war period. The lag between the leader and followers suggests that by the early 1920s the other economies had a comparable usage of electricity in manufacturing to that of the US around 1907. By the mid 1930s the follower countries had a comparable usage to the US in 1920. This result differs from the description of the diffusion of electricity in follower countries during the interwar period by David and Wright (2003) who claim that "...by the end of the 1930s the extent of diffusion of electric power in British manufacturing as a whole essentially matched that in the US."[5]

The quantitative comparative evidence suggests that the US managed to use more electricity per worker earlier than other countries and was able to sustain this lead even when other countries were making significant strides in diffusion in the inter-war period. The reason for the US lead in the use of electricity per worker is related to the rich resource endowment of the US, resulting in the relative cheapness of electricity. This in turn was due to the relative cheapness of thermal generation in the US, owing to the low price of the natural resources (oil and coal) employed. This hypothesis can be evaluated using census data for Britain and the US. In 1935 the British manufacturing sector was reported to have bought 7,100 *GWh* of electric energy at a cost of £ 23,570, 000. This meant that the average cost of a *KWh* for the British manufacturer was £ 0.00332 or 0.80*d* of the time.[6] At the prevailing exchange rate of $ 4.971 per pound (Mitchell 1988, p. 703) in 1935 the price of a *KWh* bought by the average British manufacturer was equal to $ 0.0165. As a term of comparison we observe that in 1929 and 1937 the average price paid by US manufacturers to acquire a *KWh* was $ 0.0127, and $ 0.0102 respectively.[7] This suggests that the price of electricity paid by

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[5] David and Wright, 2003, pp. 149-50.

[6] Our calculation on the basis of the individual industry cost returns of the 1935 UK census of production. The original figures refer to electricity purchased and used, including electricity generated in other works under the same ownership.

[7] Our calculations on data from the 1939 US census of manufactures. Note that the data on the US *KWh* cost in manufacturing given by Broadberry 1997 in Table 7.5 p. 101 is mistakenly expressed in pence, while it should be expressed in US cents (see Melman 1956, p. 206). Also note that the Melman's calculations based on the electricity price paid by large manufacturers are very close to our calculations based on the more comprehensive census returns in the US case, and are substantially lower than for the UK (0.69*d* as opposed to our 0.80*d*).

British manufacturers in 1935 was ca. 50 per cent higher than that paid by their US counterparts. Given the strict correlation between the cost of electricity and the cost of its direct substitutes (coal, oil, etc.), this price differential indicates that sources of power *in general* were considerably cheaper in the US than in the UK. In turn, this should be a strong indication of the *energy intensive* nature of production in the US, provided that one takes the not entirely unreasonable view that other production inputs such as capital and labour were not substantially cheaper in the US than in the UK (Broadberry, 1997, p. 101). Indeed, there is strong evidence to suggest that the cost of labour relative to the cost of electric energy in manufacturing was at least twice as high in the US than in the UK, France, Germany and Japan for the duration of the second quarter of the twentieth century (Broadberry 1997, Table 7.5, p. 101 and Melman 1956, p. 206 and 213). We can only agree with Broadberry's remarks on the likely continuation of the effect of energy endowments on comparative labour productivity in the rest of the century (Broadberry 1997, p. 102).[8]

The Second key feature of the new data set is that the major European economies shared a common experience in the adoption of electricity in the manufacturing sector, despite their differing stages of development and per capita income (Japan shared a similar trend from a lower level of electricity per worker). This common path of electricity adoption by the European countries provides an opportunity to evaluate whether any productivity effects also follow common movements.

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[8] Abramovitz and David (2000, pp. 50-53) stress the fundamental importance of natural resource abundance in shaping the form, rate, and underlying technologies of US growth up to the first quarter of the twentieth century. They also maintain that natural resource abundance continued to play an important role in US growth in the rest of the century. In a similar fashion Wright (1990, p. 651) notes that ' … the single most robust characteristic of American manufacturing exports was intensity in nonreproducible natural resources. In fact, their relative resource intensity was increasing over the half-century prior to the Great Depression.' Resource abundance characterises US industry as a whole in the last two centuries, and it is a necessary condition for many of the distinctively American industrial developments in this period (*ibid*. p. 653, and p. 661).

2. LABOUR PRODUCTIVITY TRENDS IN MANUFACTURING

In this section we evaluate the key empirical hypotheses of the GPT literature - the existence of phases of "productivity slowdown" and "productivity bonus" in the manufacturing sector that can be associated with electricity diffusion. Figure 3 plots the path of US manufacturing sector labour productivity on a logarithmic scale for the period 1889-1950[9]. For most of the 1920s the rate of growth of labour productivity was clearly high, averaging 5 per cent per annum during 1920-29. Yet, given the cyclical nature of American economic growth some care is needed to distinguish the extent, nature and timing of any trend acceleration that may have resulted from electricity usage in this sector.

Economic historians have described the cyclical path of the US economy before World War II as displaying a number of cycles of differing durations. One of the dominant fluctuations noted is the existence of irregular long swings (Abramovitz, 1961, 1968), representing episodes of accelerated and retarded economic growth. The average period of these fluctuations was around the 20-year frequency[10]. To analyse the trends and cycles in the data we use the wavelet methodology to decompose labour productivity data into "approximations" and "details": approximations capture the high-scale, low-frequency (trend) components of the data; and the details are the low-scale, high-frequency (cyclical) components. The major advantage of the wavelet method is that the non-parametric nature of the decomposition is able to capture the irregular nature of the period and amplitude of economic cycles and captures cyclical processes of different durations (an outline of the wavelet method is presented in the methodological Appendix to this paper).

The trend in the US labour productivity is depicted in Figure 4 and the cyclical decompositions in Figure 5. The wavelet decomposition shows the existence of marked long swings of approximately 20 years.[11] US manufacturing sector

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[9] For the period before 1889 we can also use the available benchmark data for the years of 1869, 1879 and 1889.

[10] Jones and Olken (2005) have described this feature of modern growth as "Start-Stop Growth", entailing large swings of growth within twenty-year intervals. Comin and Gertler (2006) have emphasised the importance of "medium-term business cycles". Although both these studies focus on the US experience after World War II, the pre WWII displayed similar growth features.

[11] The long swing pattern of growth observed after 1889 is also seen in the pre-1889 benchmark data, with the rate of growth of labour productivity averaging 0.72% pa over the period 1869-79 and 2.2% pa

productivity growth displays two key features in this period: first, much of the rapid growth of the US in the 1920s can best be seen as cyclical in nature; secondly, the sector also showed a series of accelerations in the 20th Century, particularly over the trough-peak phases c.1909-1924 and c.1933-41.[12]

The analysis of labour productivity trends in the US manufacturing sector raises doubts about the stylised facts postulated in the GPT literature. The idea of a productivity pause in the early stages of the diffusion of the new GPT is not verified by the data. Although there are episodes of slow productivity growth before the 1920s these are generally in the long swing frequency rather than the trended effects that have been postulated to arise from the diffusion process of a GPT. For example, David and Wright put forward the idea that US productivity growth slowed down in the period 1889-1913 compared to the pre-1889 phase. The labour productivity movements for the manufacturing sector conflict with this historical description. The data also suggests that although a productivity bonus is observed in the early 20th Century the trend acceleration begins before the 1920s and most of the gains in the 1920s are cyclical gains. The marked swings in manufacturing sector productivity growth are a striking feature of early 20th Century US economy. The description that arises from this analysis is that a phase of long-term slowdown that can be associated with the diffusion path of a GPT is not a robust stylised fact and the nature of any acceleration that may arise from the US adoption of electricity as a new GPT entails large swings of productivity growth rather than a smooth transition to a high growth trajectory.

Figure 6 plots British manufacturing sector labour productivity during 1869-1938, together with the wavelet estimate of the trend component. The trend displays segmentation in the interwar period with a period of trend acceleration in productivity growth.[13] The data also display irregular long swings in labour productivity growth of durations below 20 years (see Figure 7). Hence, although there is evidence of trend acceleration associated with the late diffusion stage of electricity usage in British manufacturing, there is no evidence of long-run retardation in labour productivity in the early diffusion stage. The nearest we come to observing such a phase is the

over the period 1879-89.
[12] This description is consistent with the description of US productivity growth given by Field (2003).
[13] This feature of the growth process is observed with simple descriptive statistics- over the period 1920-38 the mean growth rate of labour productivity doubled relative to the pre-1913 period.

productivity retardation of the early 20th Century. However, to perceive the retardation of the 1900s as an aspect of GPT growth runs contrary to the evidence of a relatively slow rate of adoption of electricity in British manufacturing in the first decade of the 20th century. The British case provides partial support for the GPT perspective in that there is a major gain in manufacturing sector productivity growth beginning in the 1920s and building momentum in the 1930s that is correlated with our evidence that by the 1930s Britain had a comparable usage of electricity per worker to the US ten years earlier. It seems a strong possibility that by the interwar period British manufacturing was able to make significant productivity gains with the adoption of electricity in manufacturing production processes.

Figure 8 plots labour productivity in the German manufacturing sector on a logarithmic scale using annual data over the period 1885-1938 (there are gaps in the data series over the trans-war period 1914-24). Some benchmarked data for the period 1875-1885 also exist and is included for analysis. The break in the data series in the trans-war period forces us to evaluate a number of empirical hypotheses using the truncated data sets. We use the wavelet decomposition for the pre-1913 period to examine evidence of a productivity pause. The wavelet decomposition suggests that the pre-1913 trend is segmented into long swings of high and low growth (see Figure 9). Table 1 utilises the benchmarked data for the period 1875-1885 and calculates the geometric growth rates for a number of long period comparisons. Labour productivity growth before 1913 is best depicted as following two complete swings of slow and fast rates of growth.

Taking growth measures over complete swings shows that labour productivity growth averaged 1.33 per cent per annum during the complete swing of 1875-1898 and 2.1 per cent during 1898-1913. The German trends are clearly inconsistent with a productivity slowdown in the early diffusion stage of electricity as a new GPT.

Table 1 compares interwar productivity growth with pre-1913 trends. The trans-war shocks were so severe for Germany that the observed labour productivity growth of the inter-war epoch was not able to shift the long-term productivity performance of the German manufacturing sector. Over the period 1913-29 German labour productivity growth averaged 1.3 per cent per annum, comparable to the period before 1900. Although the manufacturing sector's labour productivity growth rate over the inter-war period was high - the growth rate of 1925-38 (2.81% per annum) was higher than that of the high growth episode of 1898-1913 (2.1 per cent per annum) - given

the growth disruptions of the trans-war period this was not sufficient to shift the growth path of manufacturing productivity in Germany onto a path of trend acceleration. The German case fails to find any evidence of productivity slowdown or of a productivity bonus.

Table 1: Labour Productivity Growth in German Manufacturing
(Per cent growth per annum)

	GROWTH RATE	INTER-PERIOD GROWTH CHANGE
1875-1890[14]	0.38	-
1890-1898	2.44	2.06
1898-1902	0.004	-2.44
1902-1913	2.69	2.29
1913-1925[15]	-0.36	-3.05
1925-1929	3.85	4.21
1929-1938	2.34	-1.51

An index for Japanese manufacturing sector labour productivity has been constructed on the basis of the employment data contained in Umemura *et al.* (1988), and of the output of the Japanese manufacturing industry over the period 1896-1940 taken from the Japanese historical national accounts (LTES) [16]. Figure 10 displays Japan's

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[14] The pre-1885 data are based on the benchmark years of 1875, 1882 and 1885.

[15] The calculation is based on the benchmark years of 1913 and 1925.

[16] The data overlap in 1919 and 1920 allowing us to scale the pre-1919 series to be comparable to the interwar series. The employment data were estimated on the basis of annual surveys of establishment with more than nine employees before 1919, of establishments with more than four employees for the period 1919 – 1937, and for all establishments from 1938 onwards. The extrapolation for the smaller establishments before 1938 and the coverage of the State-owned establishments were originally obtained using industrial census information and more comprehensive local surveys when available.

manufacturing sector labour productivity: the growth rate averaged 1.72% per annum over the period 1896-1940 but labour productivity fluctuated significantly about trend. The trend decomposition shows a clear break in the path of Japan's manufacturing productivity in the 1930s. The swings in labour productivity are displayed in Figure 11. The growth rate calculations over the long swing phases are reported in Table 2. Looking at growth movements over complete swings we can see evidence of trend acceleration- the growth rate over the complete swing 1905-1925 averaged 1.17 per cent per annum and the growth rate over the period 1925-37 averaged 1.82 per cent per annum – this is clear in the estimate of the trend component in Figure 10.

Table 2: Labour Productivity Growth in Japan's Manufacturing
(Per cent growth per annum)

	GROWTH RATE	INTER-PERIOD GROWTH CHANGE
1896-1901	6.25	-
1901-1905	-8.11	-14.36
1905-1917	1.70	9.81
1917-1925	0.38	-1.32
1925-1933	3.74	3.36
1933-1937	-2.00	-5.74

In the case of France the availability of data to discuss pre-1914 trends is limited. Observations for Labour input are available only for a few benchmark observations between 1872 and 1911, which limits us to drawing inferences on the underlying trend.[17] The rate of growth of labour productivity growth in industry was

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[17] Benchmark observations exist for 1872, 1876,1881,1886, 1891, 1901, 1906 and 1911.

below 1 per cent per annum during the period 1872-1911. For the interwar period we were able to build an index of manufacturing sector productivity for the period 1921-38.[18] This is presented in Figure 12. It is clear that during the 1920s labour productivity growth in French manufacturing displays high growth rates (representing a doubling of the pre-1911 trend growth rate. However, with the onset of depression in the 1930s, France enters a phase of stagnant productivity growth over the lost decade of the 1930s.

What is emerging from the investigation of the empirical relationship between electricity as a GPT and manufacturing sector productivity growth is a picture that does not fit well with the stylised facts postulated in the GPT literature. In no case do we find evidence of productivity slowdown in the early stages of electricity diffusion in the manufacturing sector. Productivity slowdowns are observed but they are cyclical in nature; such phases of growth are best described by the Kuznets swing literature (of 20-year cycles) and do not appear to be long-term retardations that can be attributed to the diffusion of electricity as a GPT.

Evidence of a productivity bonus is mixed. The US case reveals a productivity bonus but this begins earlier than suggested in the GPT literature spanning the whole period c.1909-1929. Much of the permanent gain is observed during the period 1909-1924, whist the gains of the 1920s are mainly cyclical. Considering the follower countries, despite sharing a common path of electricity usage per worker, they display significant heterogeneity in the time-path of a productivity bonus. Britain sees a productivity bonus mainly in the 1930s, Japan in the 1920s and the 1930s, France in the 1920s and Germany not at all. This clearly shows that we cannot simply relate the productivity trends of a country only to the technological path associated with electricity as a GPT. To explain this heterogeneity of productivity growth would require a much broader framework, discussing, amongst other things, the interactions of technology, the macroeconomic policy framework and the effect of shocks.

The bottom line seems to be that ultimately the key empirical hypotheses of the GPT literature relating to macroeconomic performance cannot be verified from an examination of electricity adoption in the early 20th Century. As a result there is no

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[18] Using the labour input calculations for the manufacturing sector and assuming that the trends in industrial production display significant co-movements with those for the manufacturing sector.

unique cyclical path of slowdown and acceleration that can be predicted using this framework for analysing economic growth. The analysis of the interwar era suggests that there may exist productivity effects. However, these effects can only be analysed in a multivariate framework to capture the heterogeneity of growth experiences observed across countries that shared similar paths in the adoption of electricity.

3. CONCLUDING REMARKS

Our examination of the relationship between electricity and economic growth suggests that the stylized facts of how GPTs affect economic growth need to be re-written.

First, a quasi-cycle of productivity slowdown, followed by a productivity bonus is not a robust empirical observation. Whether we consider the case of the US, as the technological leader of the world economy, or the path of the follower countries we do not observe a long period of productivity slowdown that can be attributed to "a time to sow". Although there is more evidence for a productivity surge that can be associated with the trajectory of electricity usage in the manufacturing sector, some care is needed to interpret the nature of the relationship. A productivity bonus is observed in the case of the US but the lag of this growth effect may be shorter than postulated in the GPT literature –we find evidence of a trend acceleration beginning in the second decade of the 20th Century, reinforced by cyclical growth in the 1920s. The follower countries share the path of electricity diffusion in the manufacturing sector but display heterogeneity in growth effects. For Britain we observe trend acceleration spread out over the interwar period; for France productivity growth accelerates only in 1920s; for Germany there is no trend acceleration; and Japan sees strong trend acceleration in the 1930s. The relationship between electricity diffusion and productivity growth in manufacturing needs to be more nuanced than what is suggested by the stylized facts often found in the GPT literature. The latter should probably not include an initial productivity slowdown.

Secondly, the heterogeneity of growth movements in the major industrial countries of the 20th Century suggests that the role of technology can only be one of a number of important variables determining growth outcomes. The GPT literature has

sought to test evidence for this perspective by looking for evidence of trend acceleration in the time-scale of decades. However, it is clear that within this timescale the determinants of productivity growth are not just technological. The heterogeneity of the productivity paths of industrial countries may be explicable within a technological and a policy perspective. For example, France is able to gain a productivity bonus in the 1920s but is unable to do so in the 1930s under its membership of the Gold Bloc.

Finally, our study of electricity as a GPT can be used in conjunction with other case studies to build a broader picture of GPTs and economic growth. Crafts and Mills (2004) examined the case of steam as a GPT in the 19th Century and found no evidence to support the macroeconomic hypotheses of the GPT framework when trying to explain British 19th Century economic growth. The failure of GPT theory to account for 20th Century growth suggests that caution is needed before we use GPT theory to explain historical economic growth. The bottom line is that the simplicity of GPT theory makes it an appealing theory of episodic growth but its simplicity is also its major weakness as a perspective to investigate historical economic growth. Our analysis thus confirms the growing doubts about the usefulness of the concept of GPT. As Field notices, the GPT theory fails to unequivocally individuate more than three historical examples: steam, electricity, and ICT. Of these at least one (steam) is unlikely to have had much of an effect on growth and certainly shows little in the way of productivity retardation. As for the productivity bonuses they seem to have taken such an inordinate amount of time to materialise (a century and a half after Newcomen's original invention) as to be quantitatively indistinguishable from the growth effects of a multitude of other technological changes of the 19th century. Now we find that even for electricity there is no evidence of productivity retardation and at most we have cases where there might be some long-term productivity bonus. Our evidence on electricity supports the conclusions of Crafts and Mills (2004) that "the newfound enthusiasm for General Purpose Technology models of long run growth processes should not be taken too far". The historical case studies of steam and electricity raise the broader question of whether GPT theory can be a theory of macroeconomic growth – the inability of GPT theory to account for the historical record of growth raises strong doubts.

What are we left with? With a theory that suggest that GPTs are pervasive technological changes (of which there are few agreed examples), that might or might

not produce productivity accelerations. Being pervasive they are more likely than other technological changes to produce effects at the macro level but there are also many examples of less pervasive technological changes that are equally transformational. So we are back to the old truism that big technological changes are going to produce noticeable effects on growth and productivity trends. Yet, the significance for growth of a technological innovation has to be measured on at least three dimensions: *pervasiveness* (of how many productive processes it affects, number of spillovers, etc.); *magnitude* of its economic benefits (the size of social savings, and efficiency gains it affords, and the extent of each individual spillover); and *speed* of the diffusion process as, however pervasive and large the technological changes are, if it requires an exceedingly long time for them to diffuse their effect on growth and productivity would be so diluted as to be indistinguishable from the effects of lesser technological innovations and from those of policy. In summary, we might need to demote GPT theory to a specific case of a more general and mundane relationship between technology and long-term growth according to which a big (not necessarily a pervasive) technological change will have big effects. If it was a small technological change it would be likely to have small effects. This is hardly news and hardly a theory and more in the realm of what the French would describe as a *lapalissade*.[19]

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[19] The term *lapalissade* comes from ironical verses stating the utterly obvious referring to the French General Jacques de la Palice or la Palisse (1470 – 1525).

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APPENDIX: WAVELET TREND-CYCLE DECOMPOSITION[20]

Until recently the windowed Fourier transform (WFT) was the standard method used to detect cyclical components of time series in the frequency domain. The basic idea of the WFT is to break a time series into segments with a selected window function. The Fourier transform is applied to each segment separately.[21] The WFT produces a sequence of 'local' spectrum of the time series $x(t)$ along the time dimension. To assume stationarity and enhance the time information, one must use a short segment (window), which results in a poor frequency resolution. If one uses a long segment (window), frequency resolution improves to the detriment of time resolution, since it is difficult to know what frequencies occur at which time intervals. Clearly a major drawback of the WFT is that it uses a fixed window width to analyze economic time-series that display cycles of low and high frequencies and time-varying features.[22]

The wavelet transform is a new non-parametric method of decomposing nonstationary time series, which solves some of the problems of the windowed Fourier method. The wavelet transform uses a two-parameter family of function: time location (translation) parameter τ and scale parameter λ,

$$WT_x(\tau, \lambda) = \frac{1}{\sqrt{\lambda}} \sum_{t=1}^{N} x(t)\psi^*\left(\frac{t-\tau}{\lambda}\right) \qquad \textbf{(1)}$$

where $\psi^*(.)$ is the complex conjugation of the wavelet function $\psi[(t-\tau)/\lambda]$, the basis function in the wavelet transform. Wavelet algorithms process data at different scales or resolutions. The wavelet is dilated or compressed to extract frequency information from a time-series $x(t)$. The extent of dilation or compression is determined by the scale parameter λ, which is inversely related to the frequency of the wavelet.[23] Thus,

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[20] We would like to thank Weike Wu with helping us with this Appendix on wavelets.

[21] The windowing is accomplished via a weight function that places less emphasis on data near the interval's endpoints.

[22] Clearly this problem is highly relevant to long-run data analysis, when such changes are more likely to occur.

[23] This is similar to sinusoids. For example, the effect of the scale factor a in sine is:

$$f(t) = \sin(t) \quad ; \quad a = 1$$
$$f(t) = \sin(2t) \quad ; \quad a = 1/2$$
$$f(t) = \sin(4t) \quad ; \quad a = 1/4$$

Thus, for a sinusoid $\sin(\omega t)$, the scale factor a is inversely related to the frequency ω.

in the wavelet transform, each frequency component is analyzed with a resolution appropriate for its scale.[24]

To extract time information, the wavelet transform is computed for each value of the scale parameter λ at different time location τ, similar to the WFT, from beginning to end of the signal $x(t)$. In contrast to the WFT, however, the window width of wavelet transform varies with frequency: the wavelet transform uses short windows at high frequencies and long windows at low frequencies. Thus, the difficult problem of determining an optimum window width is avoided.

Whilst the Fourier transform has a single set of basis functions, sines and cosines, the wavelet transforms have an infinite set of possible basis functions. The wavelet $\psi(t)$ is defined to satisfy the following properties: 1) it integrates to 0, which ensures that $\psi(t)$ is wave like, allowing us to extract frequency components around the trend. 2) It decays sufficiently fast and lasts through a finite period of time or space, in contrast to sine and cosine in Fourier transform, allowing us to obtain localisation in time.

Given a wavelet $\psi(t)$, the corresponding scaling function $\phi(t)$ can be defined to satisfy the following properties : 1) it integrates to 1, i.e. the scaling function is an averaging function; 2) it is normalised to have unit norm; 3) it is orthogonal to all corresponding wavelets; 4) it is orthogonal to its discrete translations; 5) at some scale, it can be obtained as a linear combination of itself at the next scale; 6) the wavelet can be obtained as a linear combination of dilates and translates of the scaling function. Thus, a pair of corresponding scaling function ϕ and wavelet function ψ can be used to represent the smooth trend (low-frequency) part and the detailed (high frequency) part of a signal respectively.

In calculating wavelet coefficients one approach is to use the so-called dyadic scales and time locations (translations): $\lambda = 2^j$ and $\tau = 2^j k$, where j and k are integers. Working with this discrete wavelet transform (DWT) gives efficient estimates, without loss of accuracy. The scaling function and wavelet function are given as:

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[24] Because the wavelet is scaled by λ, wavelet analysis is often called a time-scale analysis rather than a time-frequency analysis.

$$\phi_{j,k}(t) = \frac{1}{\sqrt{2^j}} \phi\left(\frac{t - 2^j k}{2^j}\right) \qquad (2)$$

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}} \psi\left(\frac{t - 2^j k}{2^j}\right) \qquad (3)$$

As j (and scale parameter 2^j) increases, the function $\phi_{j,k}$ and $\psi_{j,k}$ get shorter, whereas the translation step (i.e. $2^j k$) gets larger. Thus, time-frequency/scale resolution is not constant but varies with frequency.

There are several types of wavelet functions available. The choice of the wavelet function to be used depends on the application, which requires a trade-off between properties such as smoothness, temporal location, frequency location, symmetry and orthogonality. The Haar, Daubechies, Coiflets and Symlet are four examples of wavelets:

1) The Haar wavelet is the first wavelet, proposed in the 1930s. It is a square wavelet. If a process exhibits discontinuous jumps, the Haar wavelet may be best suited for describing this behaviour. For studies of business cycles, however, the Haar wavelet is not appropriate, given the discontinuous nature of its waveform, which has poor allocation in frequency.

2) Daubechies is the most used family of wavelets. This is nearly symmetric and when the scaling function has the same number of coefficients as the wavelet function, it is orthogonal; otherwise, it is biorthogonal.[25] Daubechies' original paper shows that this wavelet is good for representing polynomial behaviour.

3) Coiflet is more symmetrical than the Daubechies wavelet. Whereas Daubechies wavelet has vanishing moments for the wavelet functions but not for the corresponding scaling functions, Coiflet has vanishing moments for both.

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[25] Classical wavelets require the filters to be orthogonal across both translation and scale, which gives a clean, robust system. In this case, however, the wavelet and scaling functions must have the same length and the length must be even. These restrictions prevent linear phase analysis, which is crucial in image processing. Biorthogonal wavelet is the result of relaxing the restriction of orthogonality.

4) Except for the Haar system, no system of wavelet function and corresponding scaling function can be at the same time compactly supported[26] and symmetric. Nevertheless, for practical purposes (in image processing for example), one can try to be as close as possible. Symmlet is a result of this kind of effort.

Within each family of wavelets (such as the Daubechies family), wavelets are often classified by the number of vanishing moments, a stringent mathematical definition related to the number of nonzero coefficients, e.g. Daubechies 4, Daubechies 6, etc. With more nonzero wavelet coefficients, the functions become smoother, resulting in better frequency location but poorer time location.

Wavelets use a multiresolution decomposition (MRD) method. This means that a given time series $x(t)$, with finite variance, can be decomposed into different approximations at different scales. At the first level, $x(t)$ can be decomposed into two components: S_1 is the (smoothed) approximation of $x(t)$ taking into account the low frequencies of the signal and its resolution is half of $x(t)$, whereas the detail D_1 corresponds to the high frequency details of $x(t)$ that are not in S_1. The decomposition process can be implemented recursively at different scales, i.e. successive approximations being decomposed in turn, so that $x(t)$ is broken down into many lower resolution components (e.g. $x(t)$ split into S_1 and D_1; S_1 split into S_2 and D_2; S_2 split into S_3 and D_3; …). MRD produces a family of hierarchically organized decompositions. In theory, the decomposition process can be iterated until the individual details consist of a single sample or pixel. In practice, the level chosen is based on a desired low-pass cut-off frequency. The detail component D_j is the discrepancy between two successive approximations S_{j-1} and S_j. As j increases, the resolution of D_j becomes poorer, which reflect the lower-frequency parts of the series.

$$x(t) \approx S_J(t) + D_J(t) + D_{J-1}(t) + \ldots + D_1(t) \qquad (4)$$

where J is the number of multi-resolution components or scales, and k ranges from 1 to the number of coefficients in the corresponding component. Of those, S_J denotes cycles with periodicity greater that 2^{J+1} periods (the "trend") and the D_j (j = 1, 2, …, J) captures cycles between 2^j and 2^{j+1}.

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[26] If a wavelet/scaling function vanishes outside of a finite interval, it has compact support, which is useful for local analysis.

Mallat (1985, 1989) verified relationships between quadrature mirror filters, hierarchical algorithms[27] and orthonormal wavelet bases. Following Mallat's scheme, the approximation and details components in MRD are derived as,

$$S_J(t) = \sum_k s_{J,k} \, \phi_{J,k}(t) \tag{5}$$

$$D_j(t) = \sum_k d_{j,k} \, \psi_{J,k}(t) \qquad j = 1, 2, \ldots, J \tag{6}$$

where

- J is the number of multi-resolution components or scales, and k ranges from 1 to the number of coefficients in the corresponding component;

- the functions $\phi_{J,k}$ and $\psi_{j,k}$ (j = 1, 2, …, J) are the approximating scaling functions and wavelet functions respectively

- the $s_{J,k}$ are called the smooth coefficients and the $d_{j,k}$ are called the detail coefficients, which represent the smooth trend of the data at the coarsest scale and the deviations at scale j (j = 1, 2, …, J) respectively. With discrete wavelets, they can be approximated as:

$$s_{J,k} = \frac{1}{\sqrt{2^J}} \sum_{t=1}^{N} \phi\left(\frac{t - 2^J k}{2^J} \right) \tag{7}$$

$$d_{j,k} = \frac{1}{\sqrt{2^j}} \sum_{t=1}^{N} \psi\left(\frac{t - 2^j k}{2^j} \right) \qquad j = 1, 2, \ldots, J \tag{8}$$

The number of coefficients at a given scale j is related to the width of the wavelet function. When the length of the data n is divisible by 2^J, there are $n/2^j$ coefficients $d_{j,k}$ at scale j (j = 1, 2, …, J). Similarly, at the coarsest scale, there are $n/2^J$ $s_{J,k}$ coefficients[28]. A strong output is given where the shape of $x(t)$ is closely matched by the shape of the chosen wavelet.

Thus, the multiresolution decomposition of expression (4) can be re-written as

$$x(t) \approx \sum_k s_{J,k} \, \phi_{J,k}(t) + \sum_k d_{J,k} \, \psi_{J,k}(t) + \sum_k d_{J-1,k} \, \psi_{J-1,k}(t) + \ldots + \sum_k d_{1,k} \, \psi_{1,k}(t) \tag{9}$$

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[27] This is also referred to as a *pyramidal* algorithm.
[28] In total, there are n $(\sum_j 1/2^j + 1/2^J) = $ n coefficients, i.e. $\sum_j n/2^j$ detail coefficients and $1/2^J$ smooth coefficients.
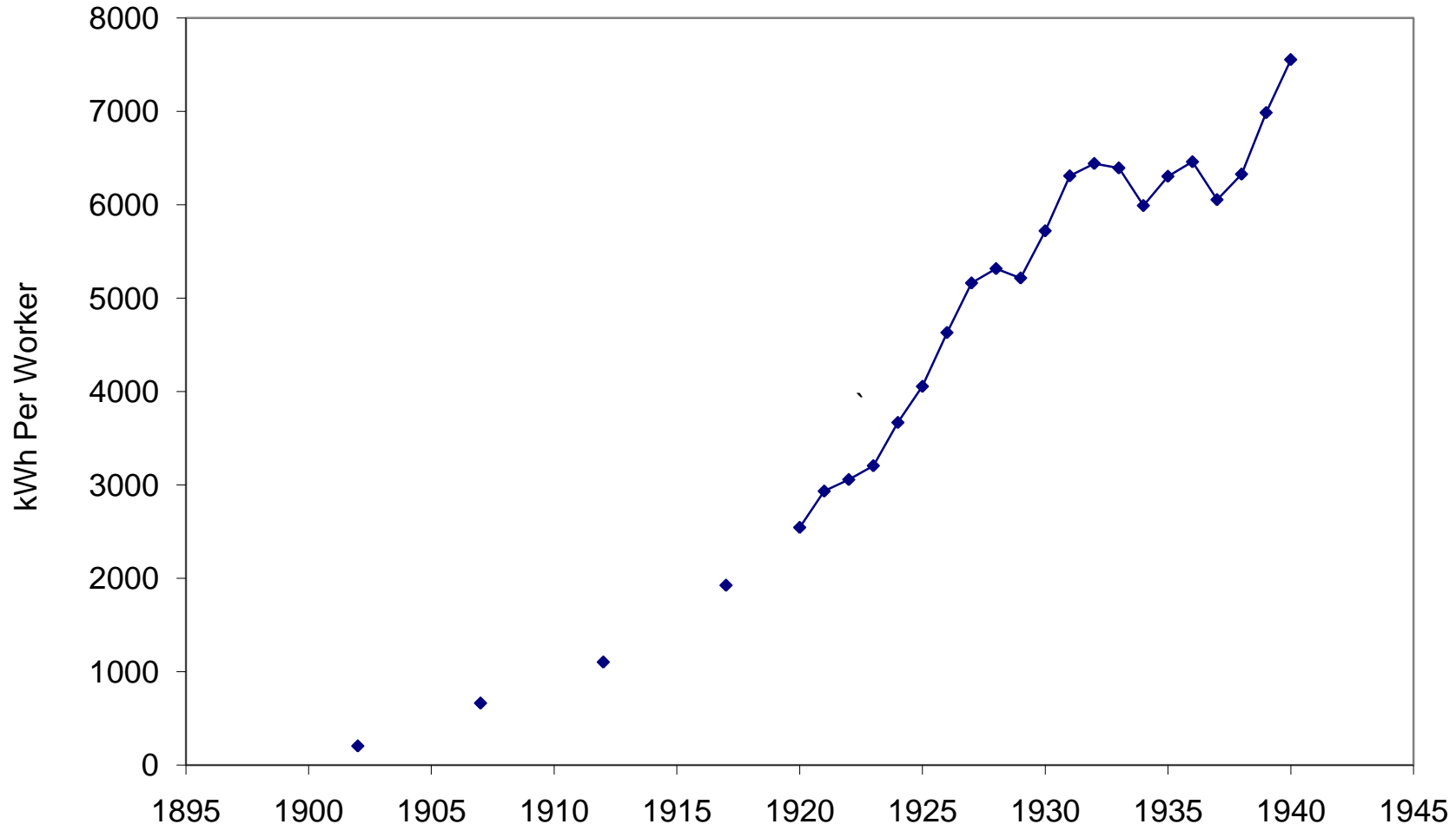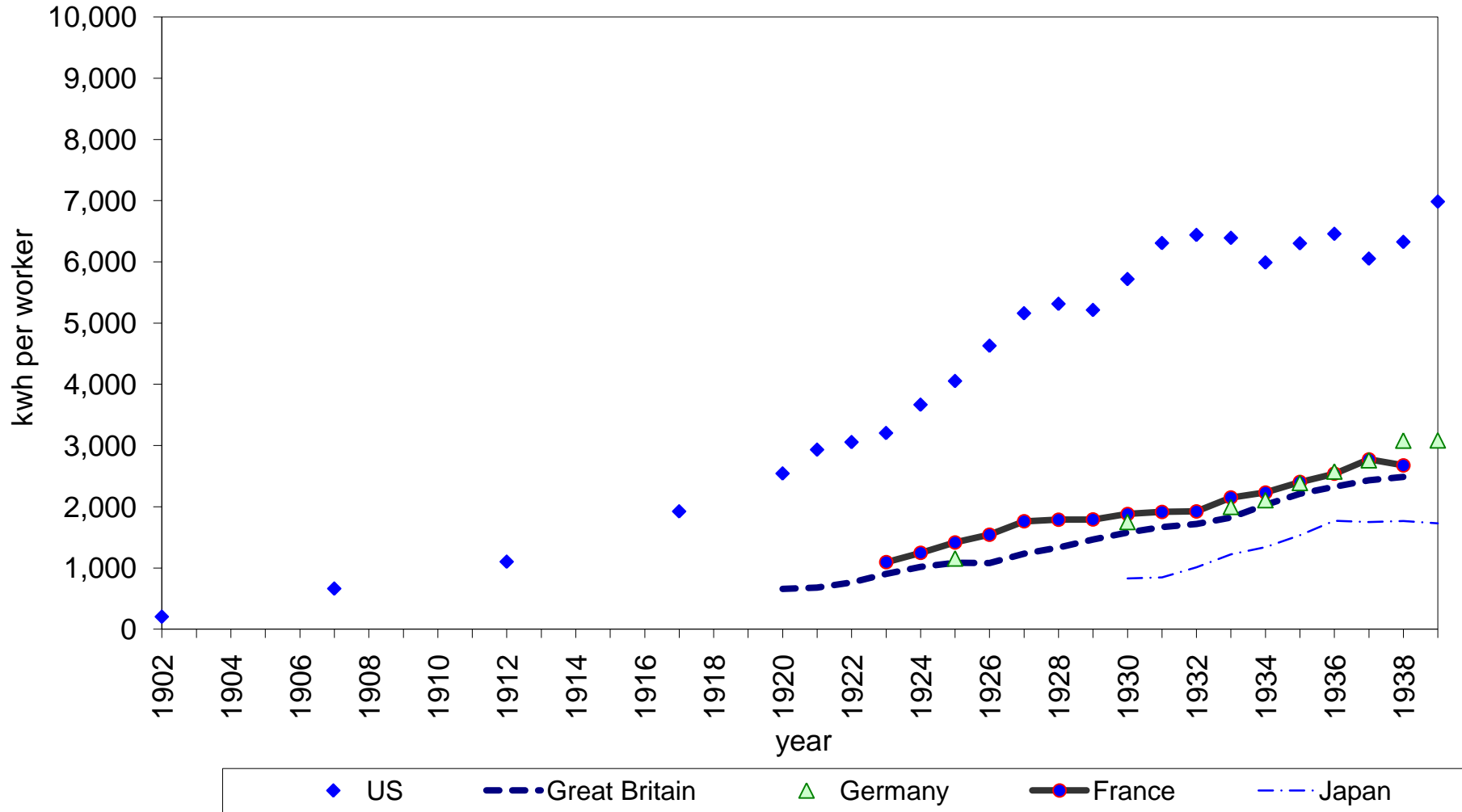
Figure 1: Consumption of Electricity Per Worker in US manufacturing

Figure 2. Consumption of electricity per employee in the manufacturing sector

Figure 3: USA MANUFACTURING SECTOR LABOUR PRODUCTIVITY INDEX 1889-1950 (1929=100)
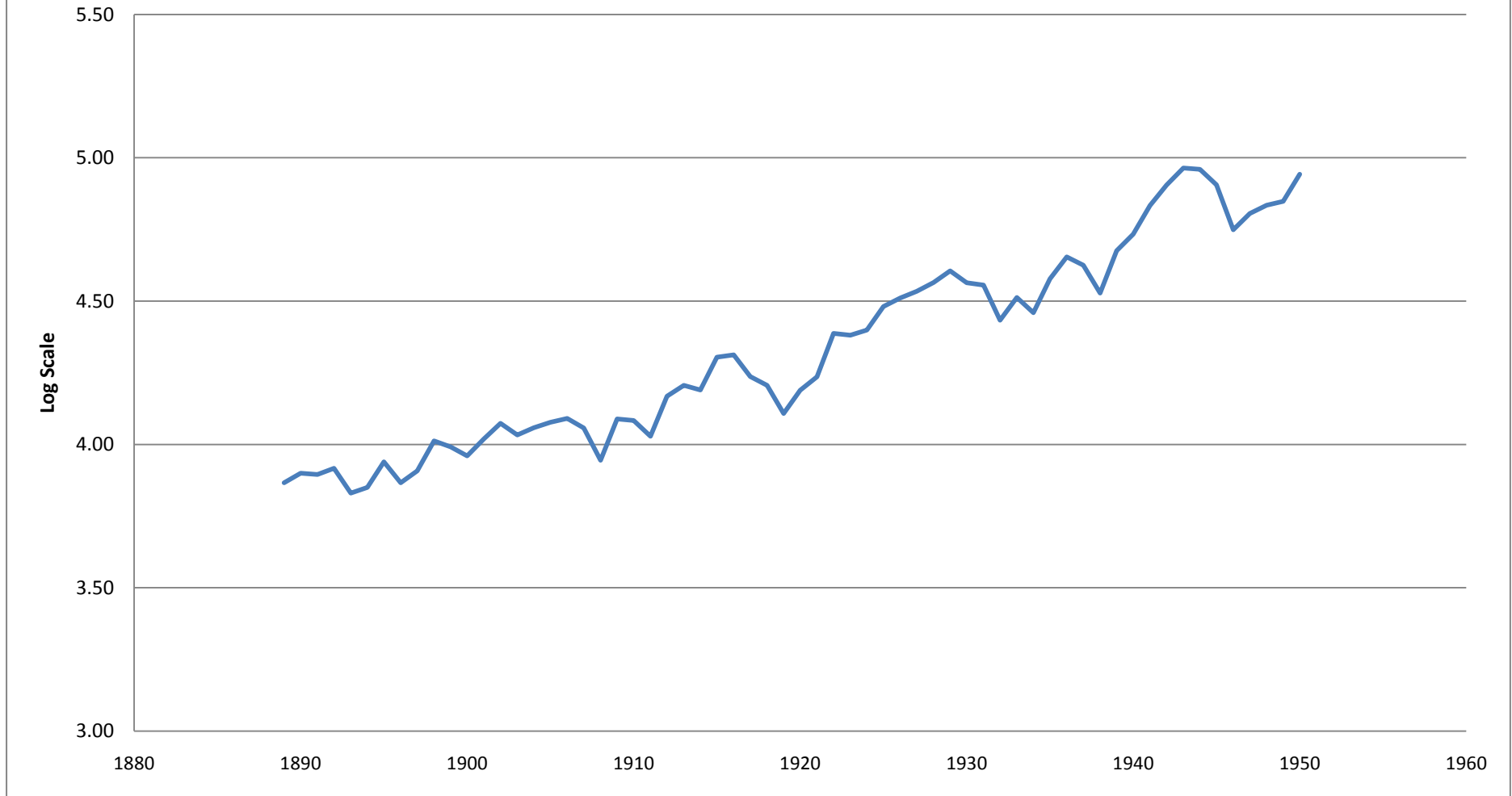
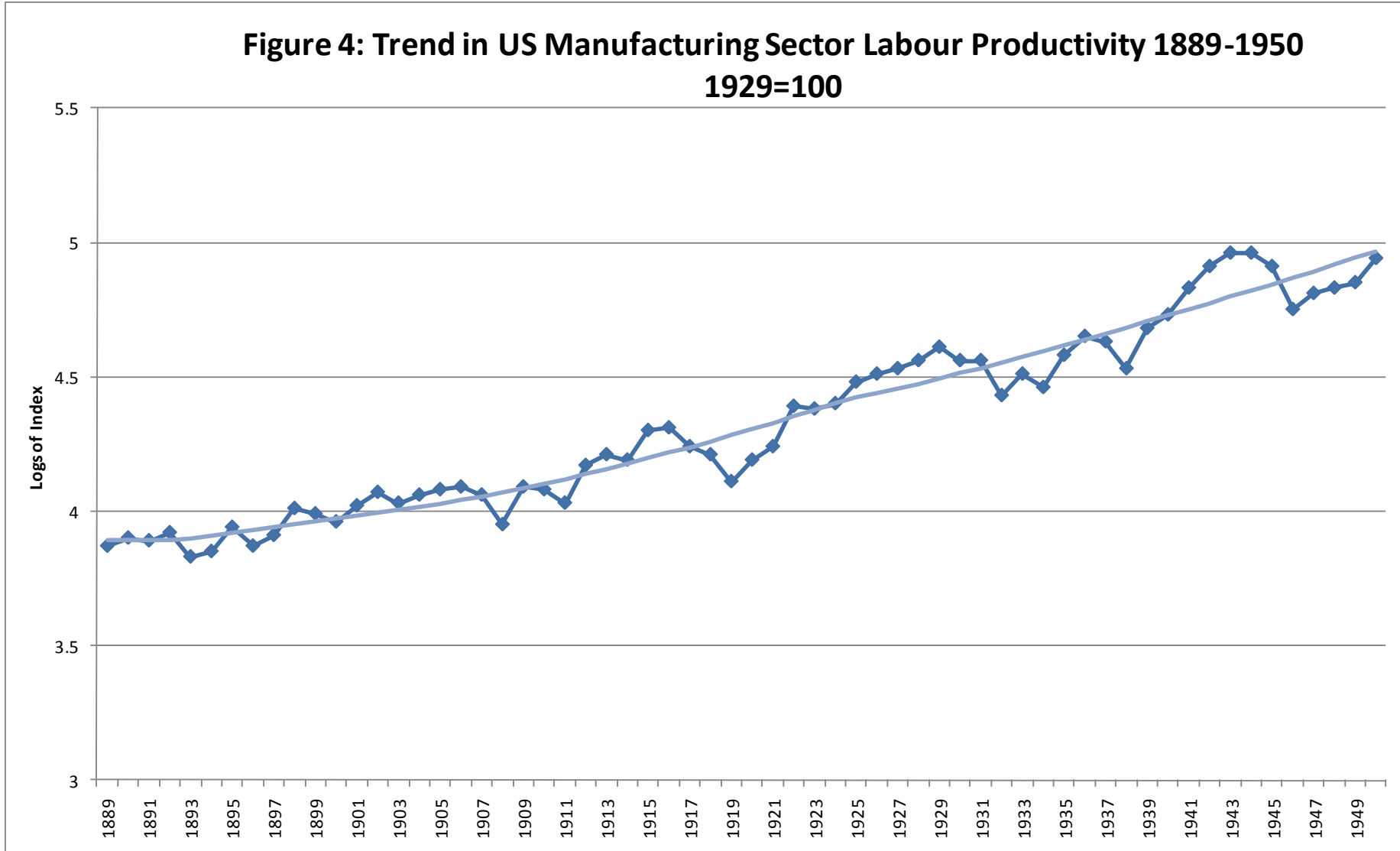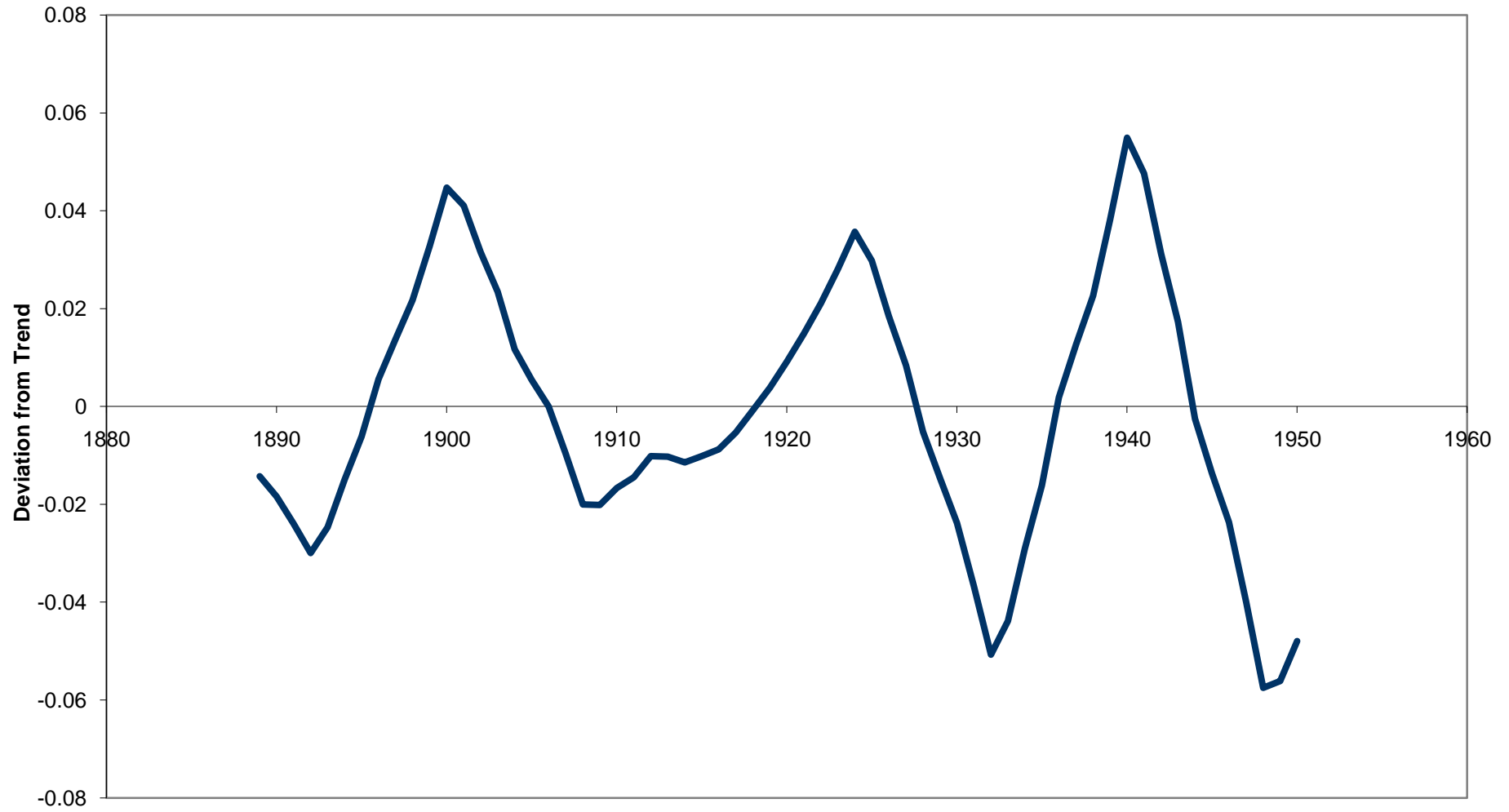Figure 4: Trend in US Manufacturing Sector Labour Productivity 1889-1950
1929=100

Figure 5: Long Swings in US Manufacturing Sector Labour Productivity
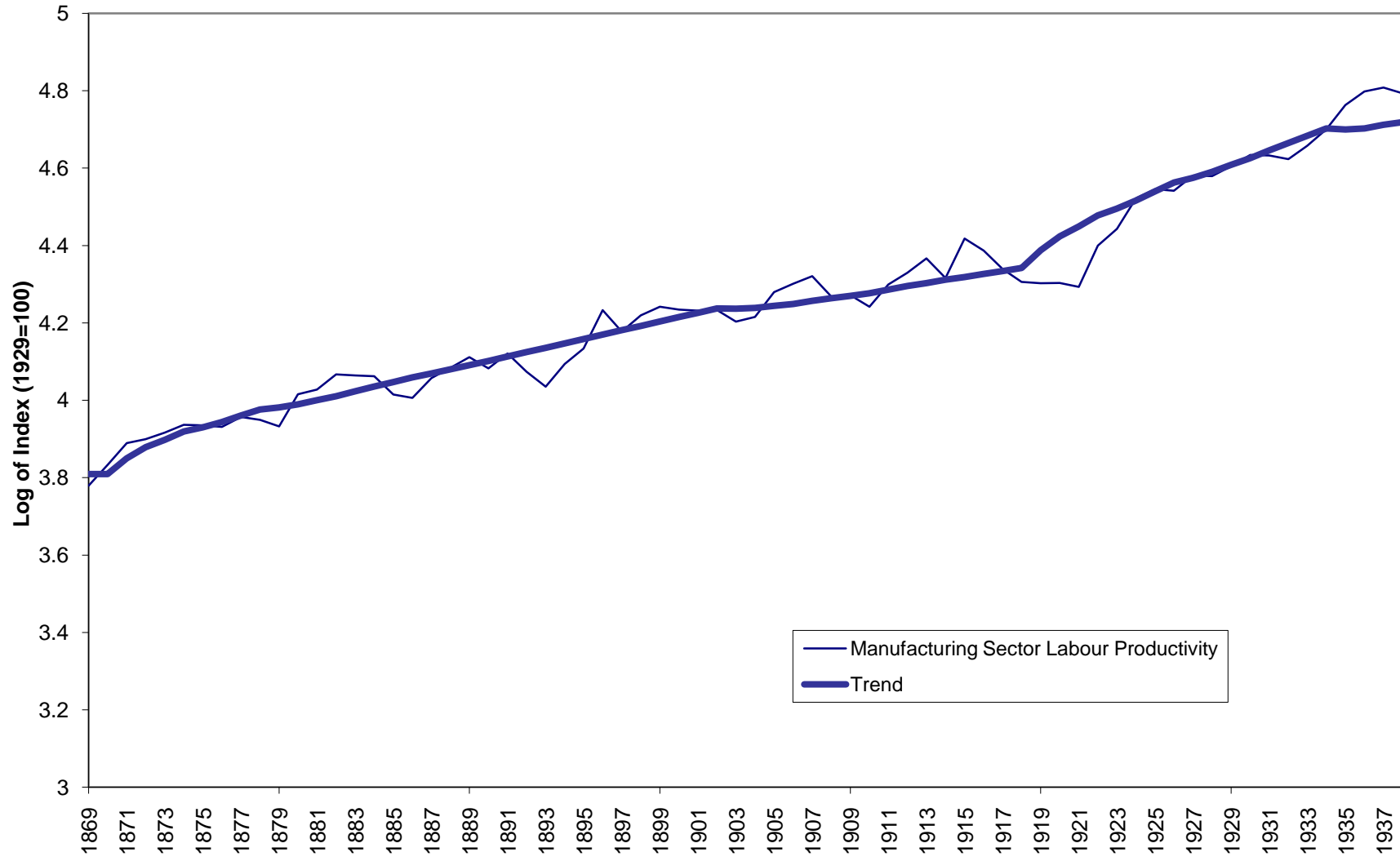
Figure 6: British Manufacturing Labour Productivity 1869-1938

Figure 7: Long Swing Decomposition of UK manufacturing Labour Productivity 1869-1938
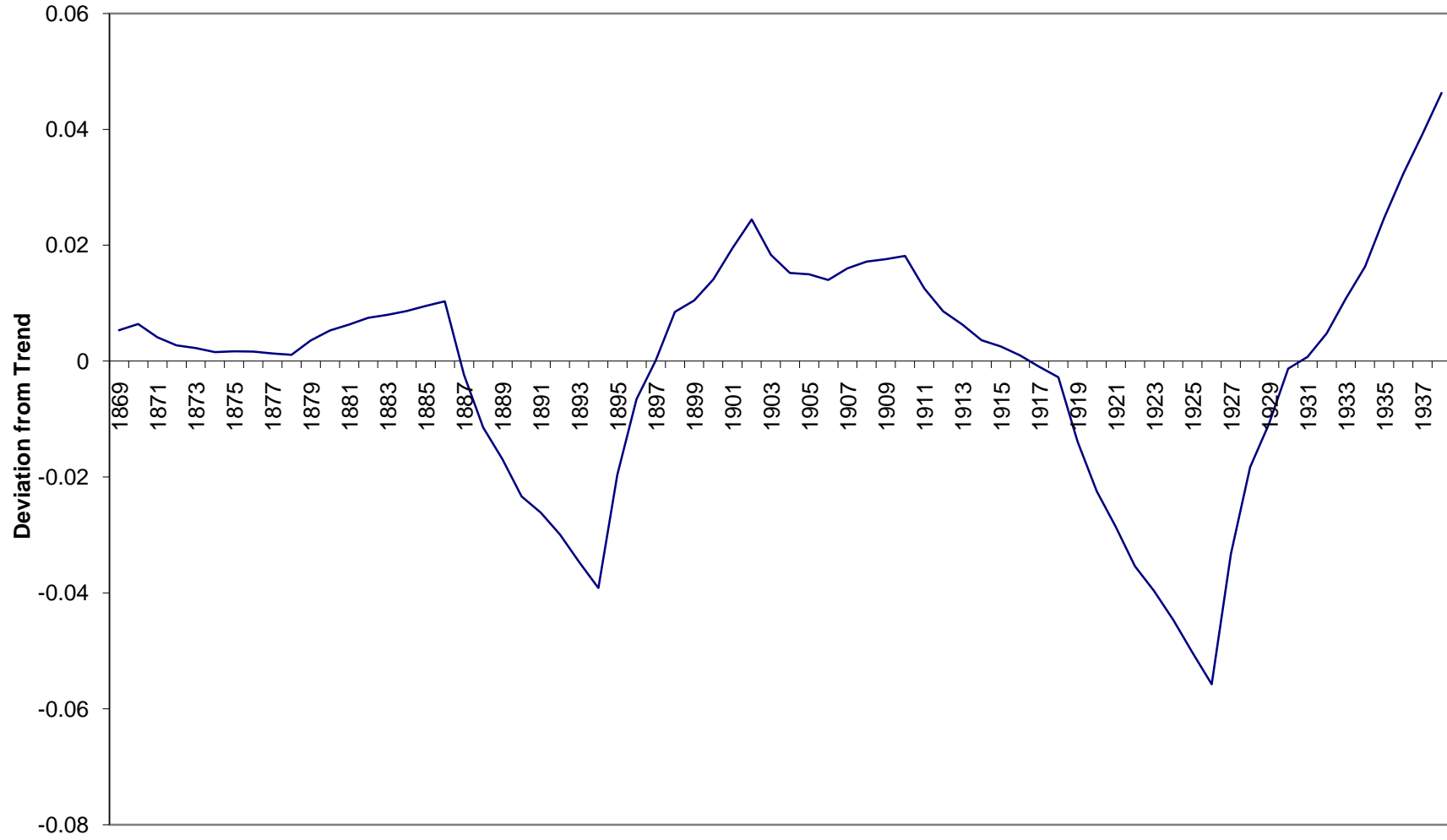
Figure 8 : German Manufacturing Sector Labour Productivity 1875-1938
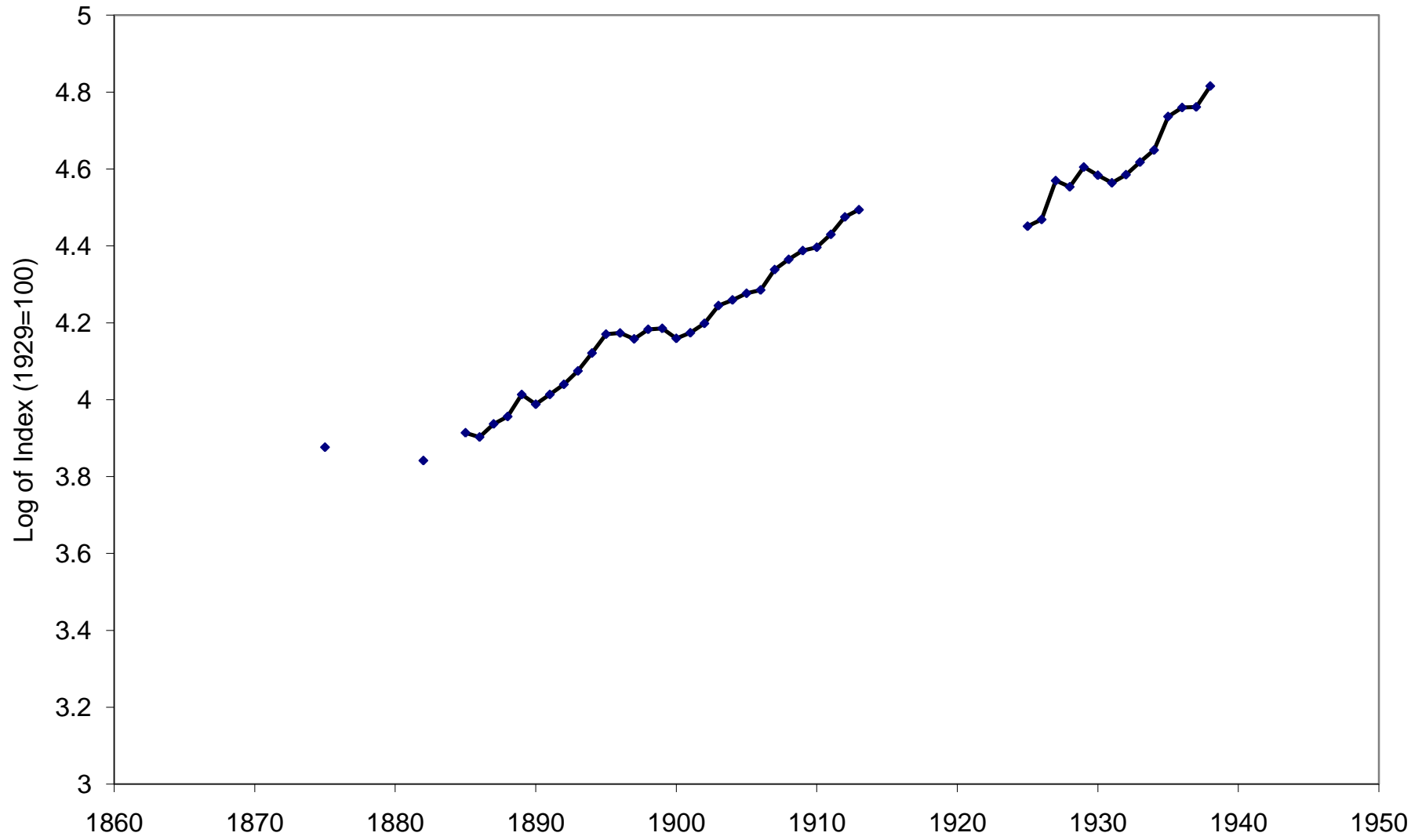
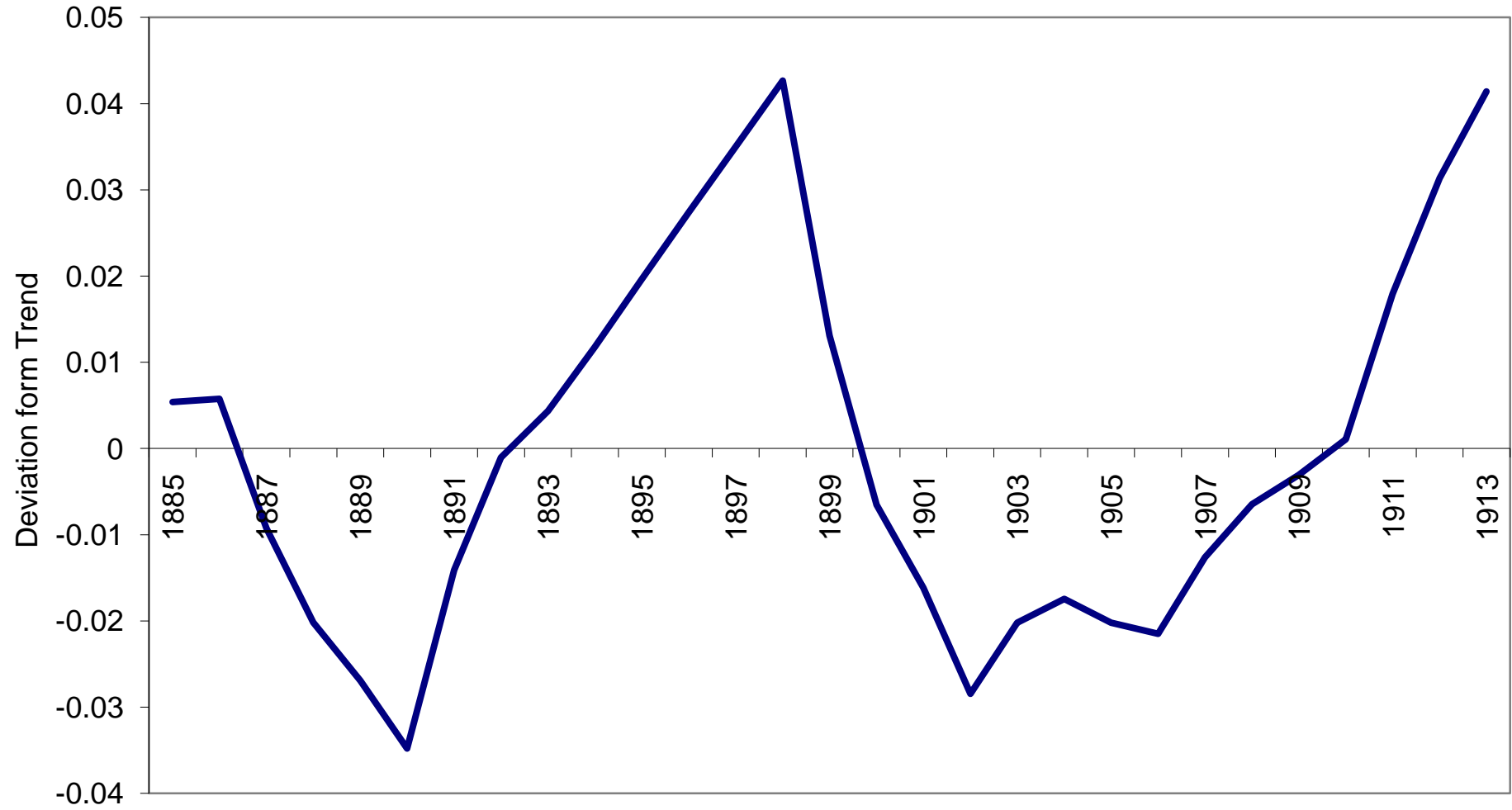Figure 9: Long Swing in German Labour Productivity

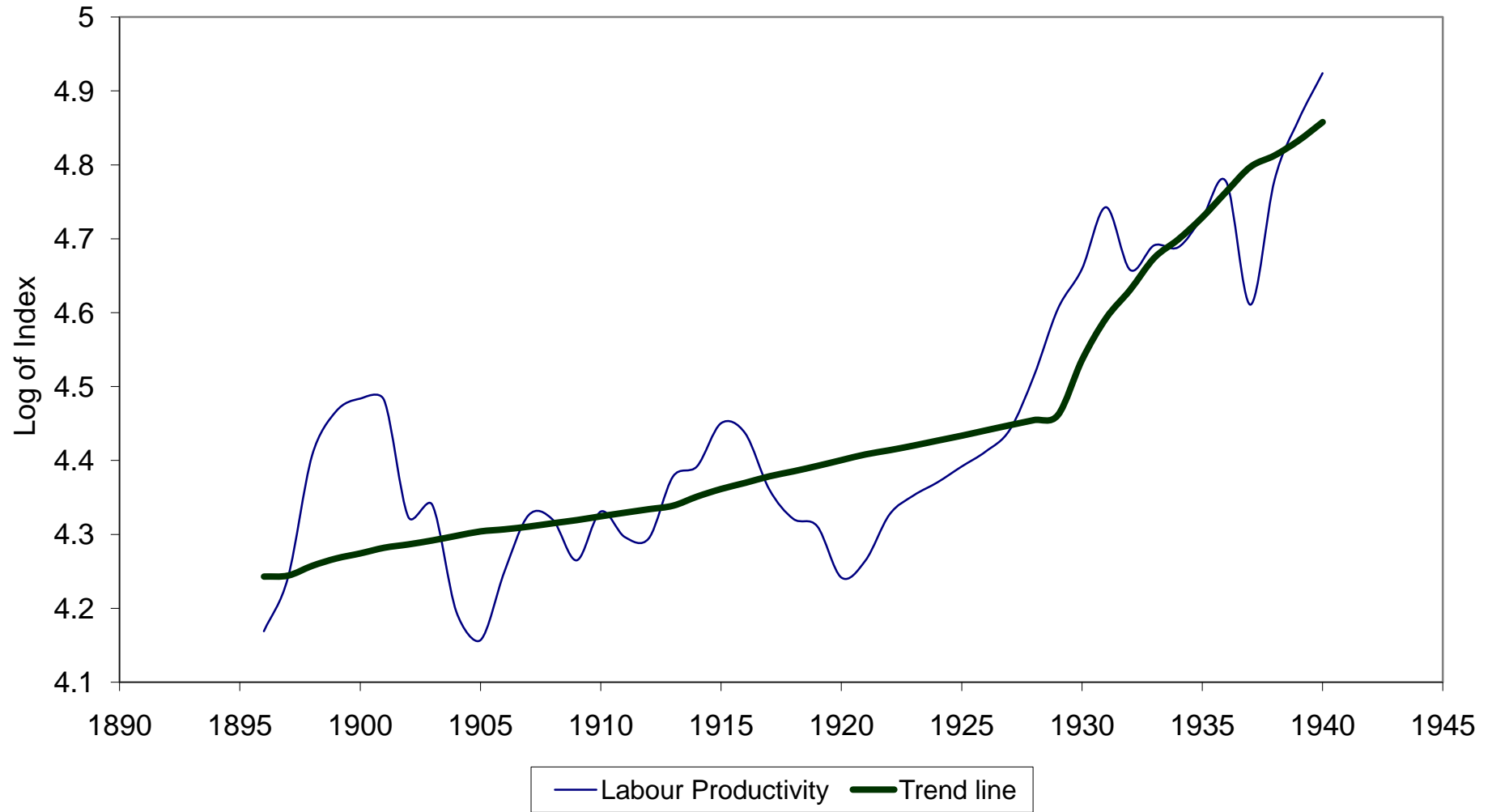Figure 10: Japan's Manufacturing Labour Productivity and Wavelet Trend Line (1929=100)

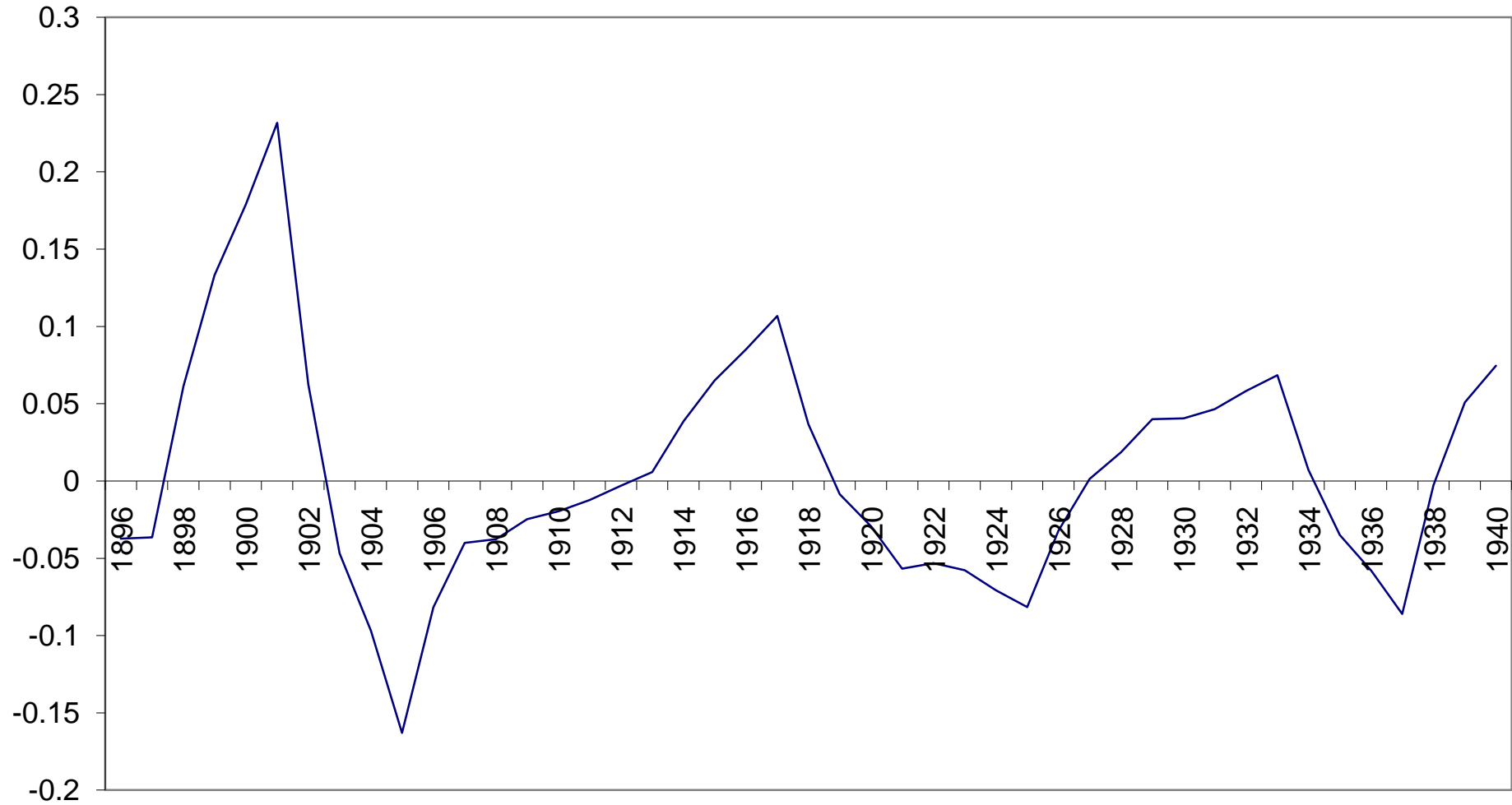Figure 11: Long Swing in Japan's Manufacturing Labour Productivity

Figure 12: French Manufacturing sector Labour Productivity Index 1921-1938