H_∞ Filtering with Randomly Occurring Sensor Saturations and Missing Measurements ⋆

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Abstract

In this paper, the H_∞ filtering problem is investigated for a class of nonlinear systems with randomly occurring incomplete information. The considered incomplete information includes both the sensor saturations and the missing measurements. A new phenomenon of sensor saturation, namely, randomly occurring sensor saturation (ROSS), is put forward in order to better reflect the reality in a networked environment such as sensor networks. A novel sensor model is then established to account for both the ROSS and missing measurement in a unified representation by using two sets of Bernoulli distributed white sequences with known conditional probabilities. Based on this sensor model, a regional H_∞ filter with a certain ellipsoid constraint is designed such that the filtering error dynamics is locally mean-square asymptotically stable and the H_∞-norm requirement is satisfied. Note that the regional l_2 gain filtering feature is specifically developed for the random saturation nonlinearity. The characterization of the desired filter gains is derived in terms of the solution to a convex optimization problem that can be easily solved by using the semi-definite programme method. Finally, a simulation example is employed to show the effectiveness of the filtering scheme proposed in this paper.

Key words: Randomly occurring sensor saturations; missing measurements; nonlinear systems; regional H_∞ filters; random incomplete information.

1 Introduction

The past few decades have witnessed an ever increasing research interest in the filtering or state estimation problems that are fundamental to control and signal processing areas. For example, the renowned Kalman filtering theory serves as an essential part of the development of space and military technology [4]. A variety of performance requirements have been proposed in the literature for the filter design, such as the H_∞ specification, the minimum variance requirement, the distributed collaborative behavior and the so-called admissible variance constraint. For example, the extended Kalman filters have been designed in [11] for nonlinear deterministic systems and in [17] for nonlinear stochastic systems. The robust filtering problems have been extensively studied in [19, 22] for systems with norm-bounded uncertainties and in [9, 15] for uncertain systems with integral quadratic constraint. The filters with error variance constraints have been exploited in [19,22] for systems which are subject to the noises with known statistics. The hybrid filtering problems have been investigated in [24] by using Markov chain approaches. The optimal filters have been designed in [1,2] for polynomial systems. Moreover, the H_∞ filtering problems have recently received much research attention by using the linear matrix inequality (LMI) approach, see e.g. [6, 13, 16, 20].

Most filter design approaches available rely on the ideal assumption that there is a *continuous* flow of measurement signals with *unlimited amplitudes*. However, perfect communication is not always possible in many engineering systems especially in a networked environment. For example, due to sensor temporal failure or network transmission delay/loss [6, 18, 19], at certain time points, the system measurement may contain noise only, which means the real signal is missing. Filtering problem with missing measurements has gained considerable research attention and many results have been reported in the literature, see [12, 19]. A common way for handling the missing measurement is to utilize the Bernoulli distributed (binary switching) white sequence specified by a conditional probability distribution in the output equation. Such kind of "binary" description has been employed in many papers such as [7, 12, 19, 25] for filtering problems of linear/nonlinear systems with probabilistic measurement losses. It is worth mentioning that, comparing to large amount of results for missing measurements, the corresponding filter design problem for signals with limited amplitudes or saturation has received much less focus of research despite the fact that sensor saturations occur very often in practical engineering.

In reality, the obstacles in delivering the high performance promises of traditional filter theories are often due to the physical limitations of system components, of which the most commonly encountered one stems from the saturation that occurs in any actuators, sensors, or certain system components. Saturation brings in nonlin-

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ear characteristics that can severely restrict the amount of deployable filter scheme. Such a characteristic not only limits the filtering performance that can otherwise be achieved without saturation, it may also lead to undesirable oscillatory behavior or, even worse, instability. Therefore, the control problem for systems under actuator/sensor saturations have attracted considerable research interest (see e.g. [3, 8, 26]) and the related filtering problem has also gained some scattered research attention [21, 23]. It should be pointed out that, in almost all relevant literature, the saturation is implicitly assumed to occur already. However, in networked environments such as wireless sensor networks, the sensor saturation itself may be subject to random abrupt changes, for example, random sensor failures leading to intermittent saturation, sensor aging resulting in changeable saturation level, repairs of partial components, changes in the interconnections of subsystems, sudden environment changes, modification of the operating point of a linearized model of a nonlinear systems, etc. In other words, the sensor saturations may occur in a probabilistic way and are randomly changeable in terms of their types and/or intensity. Such a phenomenon of sensor saturation, namely, randomly occurring sensor saturation (ROSS), has been largely overlooked in the area. It is, therefore, the main purpose of this paper to bring the issue of ROSS to the readers' attention in order to better reflect the random nature of sensor saturations in large-scale networked systems such as wireless sensor networks.

In this paper, we aim to deal with the H_∞ filtering problem for a class of nonlinear systems with randomly occurring incomplete information. The considered incomplete information includes both the sensor saturations and the missing measurements. A regional H_∞ filter with a certain ellipsoid constraint is designed such that the filtering error dynamics is locally mean-square asymptotically stable and the H_∞-norm requirement is satisfied. Here, the *regional* l_2 gain filtering feature is specifically developed for addressing the random saturation nonlinearity. The characterization of the desired filter gains is derived in terms of the solution to a convex optimization problem that can be easily solved by using the semi-definite programme method. A simulation example is employed to show the effectiveness of the filtering scheme proposed. The main novelty lies in three aspects: 1) the phenomenon of ROSS that typically exists in networked environments is put forward for investigation; 2) a novel sensor model is established to take both the ROSS and missing measurement into account; and 3) a new notion of domain of attraction in the mean square sense is introduced and a certain ellipsoid constraint is imposed on the desired H_∞ filter in the presence of random saturation nonlinearity.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. $\|A\|$ refers to the norm of a matrix A defined by $\|A\| = \sqrt{\text{trace}(A^T A)}$. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of the matrix M. I denotes the identity matrix of compatible dimension. $\text{diag}\{\cdots\}$ stands for a block-diagonal matrix and the notation $\text{diag}_n\{\bullet\}$ is employed to stand for $\text{diag}\{\overbrace{\bullet, \cdots, \bullet}^{n}\}$. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\text{Prob}\{\cdot\}$ means the occurrence probability of the event "$\cdot$". $L_2([0,\infty), \mathbb{R}^n)$ is the space of square summable n-dimensional vector-valued functions. In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

2 Problem Formulation and Preliminaries

Consider a nonlinear discrete-time system

$$\begin{cases} x_{k+1} = f(x_k) + Bw_k \\ z_k = Mx_k \end{cases} \tag{1}$$

and m sensors with both saturation and missing measurements

$$y_k^i = \alpha_k^i \sigma(C_i x_k) + (1 - \alpha_k^i)\beta_k^i C_i x_k + D_i v_k^i, \tag{2}$$

$i = 1, 2, \ldots, m$, where $x_k \in \mathbb{R}^n$ is the state vector, $z_k \in \mathbb{R}^r$ is the output vector to be estimated, $y_k^i \in \mathbb{R}$ is the measurement received by sensor i, $w_k \in \mathbb{R}^p$ and $v_k^i \in \mathbb{R}$ represent, respectively, the process noise belonging to $L_2([0,\infty), \mathbb{R}^p)$ and the measurement noise for sensor i belonging to $L_2([0,\infty), \mathbb{R})$. $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuously vector-valued function. B, M, C_i, and D_i are known matrices with appropriate dimensions.

The saturation function $\sigma: \mathbb{R} \to \mathbb{R}$ is defined as [3,21,23]:

$$\sigma(v) = \text{sign}(v) \min\{1, |v|\} \tag{3}$$

where the notation of "sign" denotes the signum function. Note that, without loss of generality, the saturation level is taken as unity here.

For every i ($1 \leq i \leq m$), $\alpha_k^i \in \mathbb{R}$ and $\beta_k^i \in \mathbb{R}$ are Bernoulli distributed white sequences taking values on 0 and 1 with

$$\begin{cases} \text{Prob}\{\alpha_k^i = 1\} = \mu_i \\ \text{Prob}\{\alpha_k^i = 0\} = 1 - \mu_i \end{cases} \text{and} \begin{cases} \text{Prob}\{\beta_k^i = 1\} = \nu_i \\ \text{Prob}\{\beta_k^i = 0\} = 1 - \nu_i \end{cases},$$

respectively, where $\mu_i, \nu_i \in [0, 1]$ are known constants. Throughout the paper, the stochastic variables α_k^i and β_k^i are independent mutually in all i ($1 \leq i \leq m$).

Remark 1 *The sensor saturation is one of the most important issues in control community that has received an increasing amount of research attention, see e.g. [3, 8, 21, 23, 26]. In practical engineering especially networked control systems, the sensor saturation often occurs in a probabilistic way due to the random abrupt changes. For example, it has been shown in [5] that the battery recovery effect exhibited in wireless sensor networks is subject to the saturation threshold dependent on the random sensing activities, and a Markov chain model has been established to capture the random sensor saturations and further study the effectiveness of duty cycling and buffering. In [10], the problem of distributed average consensus has been investigated for sensor networks with quantized data* *and random link failures leading to random sensor saturations, and the quantizer parameters have then been designed based partially on the desired probability of saturation occurrence. In [14], the transmission of a noisy signal by sensor devices has been analyzed where the sensor devices are linear for small inputs and saturate at large inputs. It has been shown in [14] that large information-carrying signals can be randomly saturated (distorted) in their transmission because of the addition of noise.*

Remark 2 *Another network-induced phenomenon, probabilistic missing measurements, is also inevitable in a networked environment due to the limited bandwidth of the channels for signal transmission. The newly proposed sensor model (2) is capable of accounting for both the phenomena in a unified representation. Specifically, if $\alpha_k^i = 1$, it can be seen that the sensor i is subject to saturation only; if $\alpha_k^i = 0$ and $\beta_k^i = 1$, it means that the sensor i works normally; if $\alpha_k^i = 0$ and $\beta_k^i = 0$, the sensor i receives the noise only, implying that the information transmitted from system (1) to sensor i is missing.*

Assumption 1 *The nonlinear function f satisfies the following sector-bounded conditions:*

$$[f(x) - U_1 x]^T [f(x) - U_2 x] \leq 0, \quad \forall x \in \mathbb{R}^n \tag{4}$$

where U_1, $U_2 \in \mathbb{R}^{n\times n}$ are real matrices of appropriate dimensions and $U = U_1 - U_2$ is a symmetric positive definite matrix.

For notational brevity, we set

$$\begin{aligned}
&\tilde{y}_k = \begin{bmatrix} y_k^1 & y_k^2 & \dots & y_k^m \end{bmatrix}^T, \Lambda_{\alpha_k} = \mathrm{diag}\{\alpha_k^1, \alpha_k^2, \dots, \alpha_k^m\},\\
&\tilde{v}_k = \begin{bmatrix} v_k^1 & v_k^2 & \dots & v_k^m \end{bmatrix}^T, \Lambda_{\beta_k} = \mathrm{diag}\{\beta_k^1, \beta_k^2, \dots, \beta_k^m\},\\
&\bar{\Lambda}_\alpha = \mathrm{diag}\{\mu_1, \mu_2, \dots, \mu_m\}, \bar{\Lambda}_\beta = \mathrm{diag}\{\nu_1, \nu_2, \dots, \nu_m\},\\
&\tilde{C} = \begin{bmatrix} C_1^T & C_2^T & \dots & C_m^T \end{bmatrix}^T, \tilde{D} = \mathrm{diag}\{D_1, D_2, \dots, D_m\}.
\end{aligned}$$

Then, the sensor model (2) can be expressed in the following compact form:

$$\tilde{y}_k = \Lambda_{\alpha_k} \sigma(\tilde{C} x_k) + (I - \Lambda_{\alpha_k}) \Lambda_{\beta_k} \tilde{C} x_k + \tilde{D} \tilde{v}_k$$

where $\sigma(\tilde{C}x_k) := \begin{bmatrix} \sigma(C_1 x_k) & \sigma(C_2 x_k) & \dots & \sigma(C_m x_k) \end{bmatrix}^T$. Here, the notation σ has been slightly abused to denote both the scalar-valued and the vector-valued saturation functions.

In this paper, a full-order filter is adopted that is of the following structure:

$$\begin{cases} \hat{x}_{k+1} = A_f \hat{x}_k + B_f \tilde{y}_k \\ \hat{z}_k = M \hat{x}_k \end{cases} \tag{5}$$

where $\hat{x}_k \in \mathbb{R}^n$ is the state estimate, $\hat{z}_k \in \mathbb{R}^r$ is an estimate of the output z_k, and A_f and B_f are filter parameters to be determined.

By introducing a new vector $\eta_k = \begin{bmatrix} x_k^T & \hat{x}_k^T \end{bmatrix}^T$ and letting filtering error be $\tilde{z}_k = z_k - \hat{z}_k$, an augmented system is obtained as follows:

$$\begin{cases} \eta_{k+1} = \bar{f}(\eta_k) + \bar{A}\sigma(\tilde{C}H\eta_k) + \bar{D}\bar{w}_k \\ \qquad + \displaystyle\sum_{i=1}^m (\alpha_k^i - \mu_i)\bar{B}_i \sigma(\tilde{C}H\eta_k) \\ \qquad + \displaystyle\sum_{i=1}^m \big((1-\alpha_k^i)\beta_k^i - (1-\mu_i)\nu_i\big)\bar{B}_i \tilde{C}H\eta_k \\ \tilde{z}_k = \bar{M}\eta_k \end{cases} \tag{6}$$

where

$$\begin{aligned}
&\bar{f}(\eta_k) = \begin{bmatrix} f(x_k) \\ B_f(I - \bar{\Lambda}_\alpha)\bar{\Lambda}_\beta \tilde{C} x_k + A_f \hat{x}_k \end{bmatrix},\\
&\bar{A} = \begin{bmatrix} 0 \\ B_f \bar{\Lambda}_\alpha \end{bmatrix}, \ \bar{B}_i = \begin{bmatrix} 0 \\ B_f E_i \end{bmatrix}, \ \bar{D} = \begin{bmatrix} B & 0 \\ 0 & B_f \tilde{D} \end{bmatrix},\\
&\bar{w}_k = \begin{bmatrix} w_k \\ \tilde{v}_k \end{bmatrix}, \ H = \begin{bmatrix} I & 0 \end{bmatrix}, \ \bar{M} = \begin{bmatrix} M & -M \end{bmatrix},\\
&E_i = \mathrm{diag}\{\underbrace{0, \dots, 0}_{i-1}, 1, \underbrace{0 \dots, 0}_{m-i}\}.
\end{aligned} \tag{7}$$

Denote by $\eta_{k,\eta_0,\bar{w}}$ the state trajectory of the augmented system (6) starting from the initial value η_0. The notion of "domain of attraction in the mean square sense" is introduced in the following definition.

Definition 1 *The set*

$$\mathscr{D} = \{\eta_0 \in \mathbb{R}^{2n} : \lim_{k\to\infty} \mathbb{E}\|\eta_{k,\eta_0,0}\|^2 = 0\}$$

is said to be the mean-square domain of attraction of the origin of the augmented system (6).

Define an ellipsoid $\Omega(P, \rho)$ as follows:

$$\Omega(P, \rho) = \{\eta \in \mathbb{R}^{2n} : \eta^T P \eta \leq \rho\}$$

where $P \in \mathbb{R}^{2n\times 2n}$ is a positive definite matrix and $\rho \in \mathbb{R}$ is a positive scalar.

The purpose of this paper is to design an H_∞ filter of form (5) for the nonlinear system (1) and the sensors (2) with incomplete information (ROSSs and missing measurements). More specifically, we are interested in looking for the filter parameters A_f and B_f and determining the ellipsoid parameters P and ρ such that the following requirements are met simultaneously:

a) The zero-solution of the augmented system (6) with $\bar{w}_k = 0$ is locally mean-square asymptotically stable, and the ellipsoid $\Omega(P, \rho)$ is contained in its mean-square domain of attraction $\mathscr{D}$.
b) Under the zero-initial condition, if $\eta_k \in \Omega(P, \rho)$ for all $k \in [0, \infty)$, the filtering error $\tilde{z}_k$ satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|\tilde{z}_k\|^2\} \leq \gamma^2 \sum_{k=0}^{\infty} \|\bar{w}_k\|^2 \tag{8}$$

for all nonzero $\bar{w}_k$, where $\gamma > 0$ is a given disturbance attenuation level.

3 Main Results

Let us start with tackling the saturation function σ. According to the definition of the saturation function (3), it is easily known that the nonlinear function σ satisfies $[\sigma(v_i) - a_i v_i][\sigma(v_i) - v_i] \leq 0$ and $|v_i| \leq a_i^{-1}$ where a_i is a positive scalar satisfying $0 < a_i < 1$.
Set $\Lambda = \text{diag}\{a_1, a_2, \dots, a_m\}$ and define

$$\mathscr{L}(\Lambda\tilde{C}H) = \{\eta \in \mathbb{R}^{2n} : |a_i C_i H\eta| \leq 1, i = 1, 2, \dots, m\}.$$

Then, it can be verified that the diagonal matrix Λ satisfies

$$0 < \Lambda < I \tag{9}$$

and the nonlinear function $\sigma(\tilde{C}H\eta)$ satisfies

$$[\sigma(\tilde{C}H\eta) - \Lambda\tilde{C}H\eta]^T[\sigma(\tilde{C}H\eta) - \tilde{C}H\eta] \leq 0, \tag{10}$$

for each $\eta \in \mathscr{L}(\Lambda\tilde{C}H)$.
For the convenience of manipulation, in what follows, the ellipsoid matrix is taken as $P = \text{diag}\{Q_1, Q_2\}$. Then, a sufficient condition is provided in the following theorem which guarantees that the augmented system (6) is locally mean-square asymptotically stable and the ellipsoid $\Omega(P, \rho)$ is contained in its mean-square domain of attraction.

Theorem 1 *Let the filter parameters A_f and B_f be given. If there exist a positive definite matrix $P = diag\{Q_1, Q_2\}$, a diagonal matrix Λ satisfying (9), and positive scalars ρ, ε_1, and ε_2 such that*

$$\Omega(P, \rho) \subset \mathscr{L}(\Lambda\tilde{C}H) \tag{11}$$

and

$$\Phi = \begin{bmatrix} \Upsilon_{11} & -\varepsilon_1\tilde{U}_2 & \tilde{F}^T Q_2 B_f \bar{\Lambda}_\alpha - \varepsilon_2\tilde{V}_2 \\ * & Q_1 - \varepsilon_1 I & 0 \\ * & * & \Upsilon_{33} \end{bmatrix} < 0 \tag{12}$$

where

$$\begin{aligned}
\Upsilon_{11} &= \tilde{F}^T Q_2 \tilde{F} - P - \varepsilon_1\tilde{U}_1 - \varepsilon_2\tilde{V}_1 \\
&\quad + \sum_{i=1}^m (\delta_i + \varsigma_i) H^T\tilde{C}^T E_i B_f^T Q_2 B_f E_i \tilde{C}H, \\
\Upsilon_{33} &= \bar{\Lambda}_\alpha B_f^T Q_2 B_f \bar{\Lambda}_\alpha - \varepsilon_2 I \\
&\quad + \sum_{i=1}^m (\varrho_i + \varsigma_i) E_i B_f^T Q_2 B_f E_i,
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\tilde{U}_1 = H^T(U_1^T U_2 + U_2^T U_1)H/2,\ \tilde{V}_1 = H^T\tilde{C}^T\Lambda\tilde{C}H, \\
&\tilde{U}_2 = -H^T(U_1^T + U_2^T)/2,\ \tilde{V}_2 = -H^T\tilde{C}^T(\Lambda + I)/2, \\
&\tilde{F} = \begin{bmatrix} B_f(I - \bar{\Lambda}_\alpha)\bar{\Lambda}_\beta\tilde{C} & A_f \end{bmatrix}, \quad \varrho_i = \mu_i(1 - \mu_i), \\
&\delta_i = (1 - \mu_i)\nu_i - (1 - \mu_i)^2\nu_i^2,\ \varsigma_i = (1 - \mu_i)\mu_i\nu_i,
\end{aligned}$$

then the zero-solution of the augmented system (6) with $\bar{w}_k = 0$ is locally mean-square asymptotically stable and the ellipsoid $\Omega(P, \rho)$ is contained in the mean-square domain of attraction $\mathscr{D}$.

Proof: Let the Lyapunov function candidate be

$$V(\eta_k) = \eta_k^T P \eta_k$$

and the difference of the Lyapunov function be defined by

$$\Delta V(\eta_k) = \mathbb{E}\{V(\eta_{k+1})|\eta_k\} - V(\eta_k).$$

By noting $P = \text{diag}\{Q_1, Q_2\}$ together with (7), the difference of $V(\eta_k)$ along the system (6) with $\bar{w}_k = 0$ can be calculated as follows:

$$\begin{aligned}
&\mathbb{E}\{\Delta V(\eta_k)\} \\
=&\mathbb{E}\{V(\eta_{k+1}) - V(\eta_k)\} \\
=&\mathbb{E}\Big\{f^T(x_k)Q_1 f(x_k) + \eta_k^T\tilde{F}^T Q_2\tilde{F}\eta_k \\
&+ \sigma^T(\tilde{C}H\eta_k)\bar{\Lambda}_\alpha B_f^T Q_2 B_f \bar{\Lambda}_\alpha \sigma(\tilde{C}H\eta_k) \\
&+ \sum_{i=1}^m \varrho_i \sigma^T(\tilde{C}H\eta_k) E_i B_f^T Q_2 B_f E_i \sigma(\tilde{C}H\eta_k) \\
&+ \sum_{i=1}^m \delta_i \eta_k^T H^T\tilde{C}^T E_i B_f^T Q_2 B_f E_i \tilde{C}H\eta_k \\
&+ 2\eta_k^T\tilde{F}^T Q_2 B_f \bar{\Lambda}_\alpha \sigma(\tilde{C}H\eta_k) - \eta_k^T P\eta_k \\
&- 2\sum_{i=1}^m \varsigma_i \sigma^T(\tilde{C}H\eta_k) E_i B_f^T Q_2 B_f E_i \tilde{C}H\eta_k\Big\}.
\end{aligned}$$

Then, it follows from the inequality

$$\begin{aligned}
&- 2\sigma^T(\tilde{C}H\eta_k)E_i B_f^T Q_2 B_f E_i\tilde{C}H\eta_k \\
\leq&\sigma^T(\tilde{C}H\eta_k)E_i B_f^T Q_2 B_f E_i \sigma(\tilde{C}H\eta_k) \\
&+ \eta_k^T H^T\tilde{C}^T E_i B_f^T Q_2 B_f E_i\tilde{C}H\eta_k
\end{aligned} \tag{14}$$

that $\mathbb{E}\{\Delta V(\eta_k)\} \leq \mathbb{E}\{\zeta_k^T\bar{\Phi}\zeta_k\}$ where

$$\begin{aligned}
&\zeta_k = \begin{bmatrix}\eta_k^T & f^T(x_k) & \sigma^T(\tilde{C}H\eta_k)\end{bmatrix}^T, \\
&\bar{\Phi} = \begin{bmatrix} \bar{\Upsilon}_{11} & 0 & \tilde{F}^T Q_2 B_f\bar{\Lambda}_\alpha \\ * & Q_1 & 0 \\ * & * & \bar{\Upsilon}_{33} \end{bmatrix}, \\
&\bar{\Upsilon}_{11} = \tilde{F}^T Q_2\tilde{F} - P \\
&\qquad + \sum_{i=1}^m (\delta_i + \varsigma_i) H^T\tilde{C}^T E_i B_f^T Q_2 B_f E_i\tilde{C}H, \\
&\bar{\Upsilon}_{33} = \bar{\Lambda}_\alpha B_f^T Q_2 B_f\bar{\Lambda}_\alpha + \sum_{i=1}^m (\varrho_i + \varsigma_i) E_i B_f^T Q_2 B_f E_i.
\end{aligned} \tag{15}$$

For each $\eta_k \in \Omega(P, \rho)$, it can be obtained from (11) that $\eta_k \in \mathscr{L}(\Lambda\tilde{C}H)$. Moreover, it follows from (4) and (10) that

$$
\begin{aligned}
&\mathbb{E}\{\Delta V(\eta_k)\}\\
\leq&\mathbb{E}\big\{\zeta_k^T\bar{\Phi}\zeta_k-\varepsilon_1[f(x_k)-U_1x_k]^T[f(x_k)-U_2x_k]\\
&-\varepsilon_2[\sigma(\tilde{C}H\eta_k)-\Lambda\tilde{C}H\eta_k]^T[\sigma(\tilde{C}H\eta_k)-\tilde{C}H\eta_k]\big\}\\
=&\mathbb{E}\{\zeta_k^T\Phi\zeta_k\}.
\end{aligned}
$$

From (12), we have $\mathbb{E}\{\Delta V(\eta_k)\}<0$ for $\eta_k\neq 0$, which means that $\eta_k\in\mathscr{D}$ (see [8] for details). It follows immediately that $\Omega(P,\rho)\subset\mathscr{D}$, which completes the proof.

Next, we are ready to deal with the regional H_∞ index. In the following theorem, a sufficient condition is given that guarantees the local mean-square asymptotical stability as well as the regional H_∞ performance constraint for the filtering error dynamics.

Theorem 2 *For the given filter parameters A_f and B_f, if there exist a positive definite matrix $P=diag\{Q_1,Q_2\}$, a diagonal matrix Λ satisfying (9), and positive scalars ρ, ε_1, and ε_2 such that*

$$
\Omega(P,\rho)\subset\mathscr{L}(\Lambda\tilde{C}H) \tag{16}
$$

and

$$
\Psi=\begin{bmatrix}\tilde{\Upsilon}_{11} & -\varepsilon_1\tilde{U}_2 & \tilde{F}^TQ_2B_f\bar{\Lambda}_\alpha-\varepsilon_2\tilde{V}_2 & \tilde{F}^TQ_2\tilde{B}_2\\ * & Q_1-\varepsilon_1 I & 0 & Q_1\tilde{B}_1\\ * & * & \Upsilon_{33} & \bar{\Lambda}_\alpha B_f^TQ_2\tilde{B}_2\\ * & * & * & \Upsilon_{44}\end{bmatrix}<0 \tag{17}
$$

where

$$
\begin{aligned}
\tilde{\Upsilon}_{11}=&\tilde{F}^TQ_2\tilde{F}+\sum_{i=1}^m(\delta_i+\varsigma_i)H^T\tilde{C}^TE_iB_f^TQ_2B_fE_i\tilde{C}H\\
&-P+\bar{M}^T\bar{M}-\varepsilon_1\tilde{U}_1-\varepsilon_2\tilde{V}_1,\quad \tilde{B}_1=\begin{bmatrix}B & 0\end{bmatrix},\\
\Upsilon_{44}=&-\gamma^2I+\bar{D}^TP\bar{D},\quad \tilde{B}_2=\begin{bmatrix}0 & B_f\tilde{D}\end{bmatrix},
\end{aligned} \tag{18}
$$

and $\tilde{U}_1$, $\tilde{U}_2$, $\tilde{V}_1$, $\tilde{V}_2$, $\tilde{F}$, and Υ_{33} are defined in (13), then the zero-solution of the augmented system (6) with $\bar{w}_k=0$ is locally mean-square asymptotically stable with the ellipsoid $\Omega(P,\rho)$ contained in the mean-square domain of attraction $\mathscr{D}$, and the filtering error satisfies the regional H_∞ performance requirement (8).

Proof: First, it is easily shown from Theorem 1 that the zero-solution of the system (6) with $\bar{w}_k=0$ is locally asymptotically stable in the mean square, and the ellipsoid $\Omega(P,\rho)$ is contained in the mean-square domain of attraction since the inequality (12) is implied by (17). It remains to show that, under zero-initial condition, the filtering error $\tilde{z}_k$ satisfies the H_∞ performance constraints (8) if $\eta_k\in\Omega(P,\rho)$ for all $k\in[0,\infty)$. Choosing the Lyapunov function similar to one in the proof of Theorem 1 and using the inequality (14), we can calculate that

$$
\begin{aligned}
&\mathbb{E}\{\Delta V(\eta_k)\}+\mathbb{E}\{\|\tilde{z}_k\|^2\}-\gamma^2\|\bar{w}_k\|^2\\
\leq&\mathbb{E}\Big\{f^T(x_k)Q_1f(x_k)+\eta_k^T\tilde{F}^TQ_2\tilde{F}\eta_k+\bar{w}_k^T\bar{D}^TP\bar{D}\bar{w}_k\\
&+\sigma^T(\tilde{C}H\eta_k)\bar{\Lambda}_\alpha B_f^TQ_2B_f\bar{\Lambda}_\alpha\sigma(\tilde{C}H\eta_k)-\eta_k^TP\eta_k\\
&+\sum_{i=1}^m(\varrho_i+\varsigma_i)\sigma^T(\tilde{C}H\eta_k)E_iB_f^TQ_2B_fE_i\sigma(\tilde{C}H\eta_k)\\
&+\sum_{i=1}^m(\delta_i+\varsigma_i)\eta_k^TH^T\tilde{C}^TE_iB_f^TQ_2B_fE_i\tilde{C}H\eta_k\\
&+2f^T(x_k)Q_1\tilde{B}_1\bar{w}_k+2\eta_k^T\tilde{F}^TQ_2B_f\bar{\Lambda}_\alpha\sigma(\tilde{C}H\eta_k)\\
&+2\sigma^T(\tilde{C}H\eta_k)\bar{\Lambda}_\alpha B_f^TQ_2\tilde{B}_2\bar{w}_k+2\eta_k^T\tilde{F}^TQ_2\tilde{B}_2\bar{w}_k\\
&+\eta_k^T\bar{M}^T\bar{M}\eta_k-\gamma^2\|\bar{w}_k\|^2\Big\}\\
=&\mathbb{E}\{\xi_k^T\bar{\Psi}\xi_k\}
\end{aligned}
$$

where

$$
\begin{aligned}
\xi_k=&\begin{bmatrix}\eta_k^T & f^T(x_k) & \sigma^T(\tilde{C}H\eta_k) & \bar{w}_k^T\end{bmatrix}^T,\\
\bar{\Psi}=&\begin{bmatrix}\check{\Upsilon}_{11} & 0 & \tilde{F}^TQ_2B_f\bar{\Lambda}_\alpha & \tilde{F}^TQ_2\tilde{B}_2\\ * & Q_1 & 0 & Q_1\tilde{B}_1\\ * & * & \bar{\Upsilon}_{33} & \bar{\Lambda}_\alpha B_f^TQ_2\tilde{B}_2\\ * & * & * & \Upsilon_{44}\end{bmatrix},\\
\check{\Upsilon}_{11}=&\tilde{F}^TQ_2\tilde{F}-P+\bar{M}^T\bar{M}\\
&+\sum_{i=1}^m(\delta_i+\varsigma_i)H^T\tilde{C}^TE_iB_f^TQ_2B_fE_i\tilde{C}H,
\end{aligned}
$$

and $\bar{\Upsilon}_{33}$ and Υ_{44} are defined in (15) and (18), respectively.

For each $\eta_k\in\Omega(P,\rho)$, it can be easily obtained that $\eta_k\in\mathscr{L}(\Lambda\tilde{C}H)$ by noting the inclusion (11). Along the similar line in the proof of Theorem 1, we have

$$
\begin{aligned}
&\mathbb{E}\{\Delta V(\eta_k)\}+\mathbb{E}\{\|\tilde{z}_k\|^2\}-\gamma^2\|\bar{w}_k\|^2\\
\leq&\mathbb{E}\big\{\xi_k^T\bar{\Psi}\xi_k-\varepsilon_1[f(x_k)-U_1x_k]^T[f(x_k)-U_2x_k]\\
&-\varepsilon_2[\sigma(\tilde{C}H\eta_k)-\Lambda\tilde{C}H\eta_k]^T[\sigma(\tilde{C}H\eta_k)-\tilde{C}H\eta_k]\big\}\\
=&\mathbb{E}\{\xi_k^T\Psi\xi_k\}
\end{aligned}
$$

which, from (17), implies that

$$
\mathbb{E}\{\Delta V(\eta_k)\}+\mathbb{E}\{\|\tilde{z}_k\|^2\}-\gamma^2\|\bar{w}_k\|^2<0
$$

for all nonzero $\bar{w}_k$. By considering the zero-initial value, it follows from the above inequality that (8) holds for each $\eta_k\in\Omega(P,\rho)$ and $k\in[0,\infty)$. The proof of this theorem is now complete.

According to the regional H_∞ performance analysis conducted in Theorem 2, a solution to the regional H_∞ filtering problem with both ROSSs and missing measurements is obtained in the following theorem.

Theorem 3 *For the nonlinear system (1) and sensors (2) with both ROSSs and missing measurements, the addressed regional H_∞ filtering problem is solvable if there exist a positive definite matrix $P = diag\{Q_1, Q_2\}$, a diagonal matrix $Z = diag\{z_1, z_2, \ldots, z_m\}$, matrices X and Y, and positive scalars π, ε_1, and ε_2 such that*

$$0 < Z < \varepsilon_2 I, \tag{19}$$

$$\begin{bmatrix} -P & z_i H^T C_i^T \\ * & -\pi \end{bmatrix} \leq 0, \quad i = 1, 2, \ldots, m, \tag{20}$$

$$\begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 \\ * & -Q_2 & 0 & 0 \\ * & * & -\tilde{Q}_2 & 0 \\ * & * & * & -\tilde{Q}_2 \end{bmatrix} < 0, \tag{21}$$

where

$$\Pi_1 = \begin{bmatrix} \Sigma & -\varepsilon_1 \tilde{U}_2 & H^T \tilde{C}^T (Z + \varepsilon_2 I)/2 & 0 & 0 \\ * & Q_1 - \varepsilon_1 I & 0 & Q_1 B & 0 \\ * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & -\gamma^2 I + B^T Q_1 B & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\Pi_2 = \begin{bmatrix} \vec{F} & 0 & Y\bar{\Lambda}_\alpha & 0 & Y\tilde{D} \end{bmatrix}^T, \quad \tilde{Q}_2 = diag_m\{Q_2\},$$

$$\Pi_3 = \begin{bmatrix} \vec{S}_1 \tilde{C} H & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad \Pi_4 = \begin{bmatrix} 0 & 0 & \vec{S}_2 & 0 & 0 \end{bmatrix}^T,$$

$$\Sigma = -P + \bar{M}^T \bar{M} - \varepsilon_1 \tilde{U}_1 - H^T \tilde{C}^T Z \tilde{C} H,$$

$$\vec{S}_1 = \begin{bmatrix} \sqrt{\delta_1 + \varsigma_1} E_1 Y^T & \ldots & \sqrt{\delta_m + \varsigma_m} E_m Y^T \end{bmatrix}^T,$$

$$\vec{S}_2 = \begin{bmatrix} \sqrt{\varrho_1 + \varsigma_1} E_1 Y^T & \ldots & \sqrt{\varrho_m + \varsigma_m} E_m Y^T \end{bmatrix}^T,$$

$$\vec{F} = \begin{bmatrix} Y(I - \bar{\Lambda}_\alpha)\bar{\Lambda}_\beta \tilde{C} & X \end{bmatrix},$$

and $\tilde{U}_1$ and $\tilde{U}_2$ are defined in (13). Furthermore, if the LMIs (19)-(21) are feasible, the desired filter and ellipsoid parameters are given as

$$\begin{aligned} A_f &= Q_2^{-1} X, & B_f &= Q_2^{-1} Y, \\ P &= diag\{Q_1, Q_2\}, & \rho &= \varepsilon_2^2 \pi^{-1}. \end{aligned} \tag{22}$$

Proof: Setting $Z = \varepsilon_2 \Lambda$, one immediately obtains that $0 < \Lambda < I$ from (19). By using the well-known Schur Complement Lemma and noting the relation of $\rho\pi = \varepsilon_2^2$, the condition (11) is also easily guaranteed by (20). We now consider the inequality (17). Set

$$\tilde{S}_1 = \begin{bmatrix} \sqrt{\delta_1 + \varsigma_1} E_1 B_f^T & \ldots & \sqrt{\delta_m + \varsigma_m} E_m B_f^T \end{bmatrix}^T,$$
$$\tilde{S}_2 = \begin{bmatrix} \sqrt{\varrho_1 + \varsigma_1} E_1 B_f^T & \ldots & \sqrt{\varrho_m + \varsigma_m} E_m B_f^T \end{bmatrix}^T.$$

Then, based on Theorem 2, we only need to show that (17) (i.e. $\Psi < 0$) holds. Ψ can be rewritten as follows:

$$\Psi = \bar{\Pi}_1 + \bar{\Pi}_2 Q_2 \bar{\Pi}_2^T + \bar{\Pi}_3 \tilde{Q}_2 \bar{\Pi}_3^T + \bar{\Pi}_4 \tilde{Q}_2 \bar{\Pi}_4^T$$

where

$$\bar{\Pi}_1 = \begin{bmatrix} -P + \bar{M}^T \bar{M} - \varepsilon_1 \tilde{U}_1 - \varepsilon_2 \tilde{V}_1 & -\varepsilon_1 \tilde{U}_2 & -\varepsilon_2 \tilde{V}_2 & 0 & 0 \\ * & Q_1 - \varepsilon_1 I & 0 & Q_1 B & 0 \\ * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & -\gamma^2 I + B^T Q_1 B & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\bar{\Pi}_2 = \begin{bmatrix} \tilde{F} & 0 & B_f \bar{\Lambda}_\alpha & 0 & B_f \tilde{D} \end{bmatrix}^T, \quad \bar{\Pi}_4 = \begin{bmatrix} 0 & 0 & \tilde{S}_2 & 0 & 0 \end{bmatrix}^T,$$
$$\bar{\Pi}_3 = \begin{bmatrix} \tilde{S}_1 \tilde{C} H & 0 & 0 & 0 & 0 \end{bmatrix}^T.$$

By using the Schur Complement Lemma again, $\Psi < 0$ is equivalent to

$$\begin{bmatrix} \bar{\Pi}_1 & \bar{\Pi}_2 Q_2 & \bar{\Pi}_3 \tilde{Q}_2 & \bar{\Pi}_4 \tilde{Q}_2 \\ * & -Q_2 & 0 & 0 \\ * & * & -\tilde{Q}_2 & 0 \\ * & * & * & -\tilde{Q}_2 \end{bmatrix} < 0. \tag{23}$$

By considering (21) and the relations $Z = \varepsilon_2 \Lambda$, $X = Q_2 A_f$ and $Y = Q_2 B_f$, (23) is true and then the rest of the proof follows from Theorem 2 easily.

Remark 3 *According to Theorem 3, a regional H_∞ filter with an ellipsoid $\Omega(P, \rho)$ can be designed for a class of nonlinear systems subject to both ROSSs and missing measurements in terms of the solution to a set of LMIs. As mentioned in [3], in the presence of saturation, it is difficult to design a controller (or filter) such that the corresponding controlled system (or the filtering error system) is stable and satisfies a desired H_∞ performance requirement in the global sense. A natural yet interesting issue is, therefore, to enlarge the ellipsoid region $\Omega(P, \rho)$ under the premise that the specified H_∞ performance requirement is guaranteed. Such a problem has been well investigated by using the method of introducing a reference set, see [3, 8] for more details.*

4 An Illustrative Example

Consider a nonlinear discrete-time system described by (1) with the matrix parameters

$$B = \begin{bmatrix} 0.5 & 0.1 & 0.1 \end{bmatrix}^T, \quad M = \begin{bmatrix} 0.2 & 0 & 0.15 \\ 0 & 0.1 & 0.2 \end{bmatrix}$$

and the nonlinear function

$$f(x_k) = \begin{bmatrix} -0.7x_{1,k} + 0.05x_{2,k} + 0.05x_{3,k} \\ -0.05x_{1,k} + 0.85x_{2,k} \\ -0.05x_{1,k} - 0.475x_{3,k} + \frac{x_{3,k}\sin x_{1,k}}{\sqrt{x_{1,k}^2 + x_{2,k}^2 + 20}} \end{bmatrix}.$$

It is not difficult to verify that the above nonlinear function f satisfies (4) with

$$U_1 = \begin{bmatrix} -0.5 & 0.1 & 0 \\ 0 & 0.9 & 0 \\ -0.1 & 0 & -0.2 \end{bmatrix}, \quad U_2 = \begin{bmatrix} -0.9 & 0 & 0.1 \\ -0.1 & 0.8 & 0 \\ 0 & 0 & -0.75 \end{bmatrix}.$$

The concerned sensors with both ROSSs and missing measurements are modeled by (2) with the following parameters:

$$C_1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, \; C_2 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \; D_1 = 1, \; D_2 = 1.$$

In this example, the probabilities are taken as $\mu_1 = 0.7$, $\mu_2 = 0.6$, $\nu_1 = 0.7$, and $\nu_2 = 0.75$. The disturbance attenuation level is given as $\gamma = 1.5$. By using the Matlab (with YALMIP 3.0 and SeDuMi 1.1), we solve LMIs (19)-(21) and then, according to (22), the desired filter parameters can be designed as

$$A_f = \begin{bmatrix} 0.2165 & 0.0014 & 0.0081 \\ -0.0004 & 0.1459 & -0.0010 \\ 0.0548 & 0.0060 & 0.1954 \end{bmatrix},$$

$$B_f = \begin{bmatrix} 0.1035 & -0.0096 \\ 0.0000 & 0.0000 \\ -0.0102 & 0.0204 \end{bmatrix},$$

and the ellipsoid parameters are given as

$$P = \begin{bmatrix} 1.0895 & -0.0362 & -0.3883 & 0 & 0 & 0 \\ -0.0362 & 1.1292 & 0.0189 & 0 & 0 & 0 \\ -0.3883 & 0.0189 & 2.7617 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9.1896 & 0.0019 & -0.4682 \\ 0 & 0 & 0 & 0.0019 & 4.5653 & 0.0041 \\ 0 & 0 & 0 & -0.4682 & 0.0041 & 5.1266 \end{bmatrix},$$

$\rho = 0.2228$.

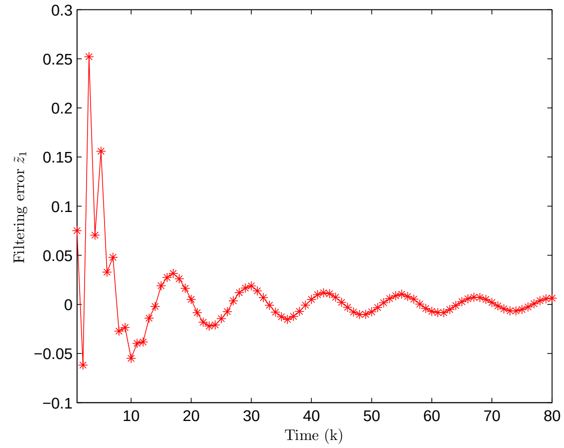

Fig. 1. Filtering error $\tilde{z}_1$

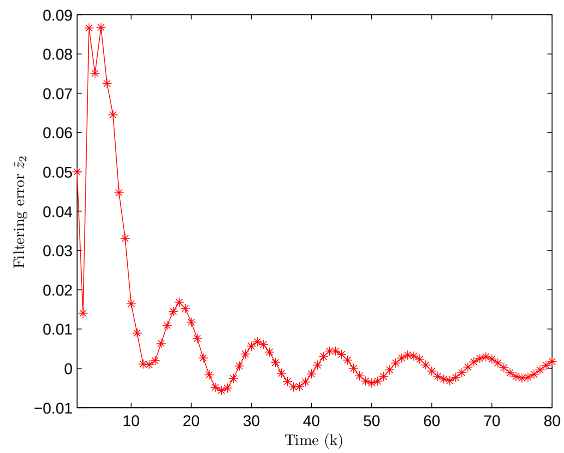

Fig. 2. Filtering error $\tilde{z}_2$

In the simulation, the exogenous disturbance inputs are selected as $w_k = \frac{8\sin(0.5k)}{k+1}$, $v_k^1 = \frac{2\cos(0.3k)}{5(k+1)}$, and $v_k^2 = \frac{2\cos(0.3k)}{5(k+1)}$. The initial values of the state of the system and its estimate are chosen as $x_0 = \begin{bmatrix} 0.3 & 0.3 & 0.1 \end{bmatrix}^T$ and $\hat{x}_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ from the ellipsoid $\Omega(P, \rho)$. Simulation results are shown in Figs. 1-2, where the filtering errors are presented. The simulation results have confirmed that the designed regional H_∞ filter performs very well.

References

[1] M. V. Basin, On optimal filtering for polynomial system states, *Journal of Dynamic Systems Measurement and Control-Transactions of The ASME*, Vol. 125, No. 1, pp. 123-125, Mar. 2003.

[2] M. V. Basin and M. A. A. Garcia, Optimal filtering and control for third degree polynomial system, *Dynamics of Continuous Discrete and Impulsive Systems-Series B-Applications & Algorithms*, Vol. 10, No. 5, pp. 663-680, Oct. 2003.

[3] Y. Cao, Z. Lin and D. Ward, H_∞ antiwindup design for linear systems subject to input saturation, *J. of Guidance Control and Dynamics*, Vol. 25, No. 3, pp. 455-463, May-Jun. 2002.

[4] F. S. Cattivelli and A. H. Sayed, Diffusion strategies for distributed Kalman filtering and smoothing, *IEEE Trans. Automatic Control*, Vol. 55, No. 9, pp. 2069-2084, Sep. 2010
[5] C. K. Chau, F. Qin, S. Sayed, M. H. Wahab and Y. Yang, Harnessing battery recovery effect in wireless sensor networks: experiments and analysis, *IEEE Journal on Selected Areas in Communications*, Vol. 28, No. 7, pp. 1222-1232, Sep. 2010.
[6] H. Gao and T. Chen, H_∞ estimation for uncertain systems with limited communication capacity, *IEEE Trans. Automatic Control*, Vol. 52, No. 11, pp. 2070-2084, Nov. 2007.
[7] F. O. Hounkpevi and E. E. Yaz, Robust minimum variance linear state estimators for multiple sensors with different failure rates, *Automatica*, Vol. 43, No. 7, pp. 1274-1280, 2007.
[8] T. Hu, Z. Lin and B. Chen, Analysis and design for discrete-time linear systems subject to actuator saturation, *Systems & Control Letters*, Vol. 45, No. 2, pp. 97-112, Feb. 2002.
[9] M. R. James and I. R. Petersen, Nonlinear state estimation for uncertain systems with an integral constraint, *IEEE Trans. Signal Processing*, Vol. 46, No. 11, pp. 2926-2937, Nov. 1998.
[10] S. Kar and J. M. F. Moura, Distributed consensus algorithms in sensor networks: quantized data and random link failures, *IEEE Trans. Signal Processing*, Vol. 58, No. 3, pp. 1383-1400, Mar. 2010
[11] A. G. Kallapur, I. R. Petersen and S. G. Anavatti, A discrete-time robust extended Kalman filter for uncertain systems with sum quadratic constraints, *IEEE Trans. Automatic Control*, Vol. 54, No. 4, pp. 850-854, Apr. 2009.
[12] S. Kluge, K. Reif and M. Brokate, Stochastic stability of the extended Kalman filter with intermittent observations, *IEEE Trans. Automatic Control*, Vol. 55, No. 2, pp. 514-518, Feb. 2010.
[13] P. Li, J. Lam and Z. Shu, H_∞ positive filtering for positive linear discrete-time systems: an augmentation approach, *IEEE Trans. Automatic Control*, Vol. 55, No. 10, pp. 2337-2342, 2010.
[14] D. Rousseau, J. Rojas Varela and F. Chapeau-Blondeau, Stochastic resonance for nonlinear sensors with saturation, *Physical Review E*, Vol. 67, No. 2, art. no. 021102, Feb. 2003.
[15] A. V. Savkin and I. R. Petersen, Robust state estimation and model validation for discrete-time uncertain systems with a deterministic description of noise and uncertainty, *Automatica*, Vol. 34, No. 2, pp. 271-274, 1998.
[16] P. Shi, M. Mahmoud, S. K. Nguang and A. Ismail, Robust filtering for jumping systems with mode-dependent delays, *Signal Processing*, Vol. 86, pp. 140-152, 2006.
[17] D. Spinello and D. J. Stilwell, Nonlinear estimation with state-dependent Gaussian observation noise, *IEEE Trans. Automat. Control*, vol. 55, no. 6, pp. 1358-1366, Jun. 2010.
[18] S. Sun, L. Xie, W. Xiao and Y. C. Soh, Optimal linear estimation for systems with multiple packet dropouts, *Automatica*, Vol. 44, pp. 1333-1342, 2008.
[19] Z. Wang, D. W. C. Ho and X. Liu, Variance-constrained filtering for uncertain stochastic systems with missing measurements, *IEEE Trans. Automatic Control*, Vol. 48, No. 7, pp. 1254-1258, Jul. 2003.
[20] L. Wu, J. Lam, W. Paszke, K. Galkowski and E. Rogers, Robust H_∞ filtering for uncertain differential linear repetitive processes, *Int. J. Adaptive Control and Signal Processing*, Vol. 22, No. 3, pp. 243-265, 2008.
[21] Y. Xiao, Y. Cao and Z. Lin, Robust filtering for discrete-time systems with saturation and its application to transmultiplexers, *IEEE Trans. Signal Processing*, Vol. 52, No. 5, pp. 1266-1277, May. 2004.
[22] L. Xie, Y. C. Soh and C. E. de Souza, Robust Kalman filtering for uncertain discrete-time systems, *IEEE Trans. Automatic Control*, Vol. 39, No. 6, pp. 1310-1314, Jun. 1994.
[23] F. Yang and Y. Li, Set-membership filtering for systems with sensor saturation, *Automatica*, Vol. 45, No. 8, pp. 1896-1902, Aug. 2009.
[24] G. Yin and S. Dey, Weak convergence of hybrid filtering problems involving nearly completely decomposable hidden Markov chains, *SIAM Journal on Control and Optimization*, Vol. 41, No. 6, pp. 1820-1842, 2003.
[25] Y. Zhao, J. Lam and H. Gao, Fault detection for fuzzy systems with intermittent measurements, *IEEE Trans. Fuzzy Systems*, Vol. 17, No. 2, pp. 398-410, Apr. 2009.
[26] Z. Zuo, D. W. C. Ho and Y. Wang, Fault tolerant control for singular systems with actuator saturation and nonlinear perturbation, *Automatica*, Vol. 46, No. 3, pp. 569-576, Mar. 2010.

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