UNIVERSITY OF WEST BOHEMIA

Applied and Computational Mechanics 1 (2007) 121 - 126

Applied and Computational Mechanics

Chaos in drive systems

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Received 27 September 2007; received in revised form 4 October 2007

Abstract

The purpose of this article is to provide an elementary introduction to the subject of chaos in the electromechanical drive systems. In this article, we explore chaotic solutions of maps and continuous time systems. These solutions are also bounded like equilibrium, periodic and quasiperiodic solutions.

Keywords: chaos, dynamical system, drive systems

1. Introduction

Dissipative dynamic system can be characterized as systems whose behaviour with increasing time asymptotically approaches steady states if there is no energy added from the outside. Such system description is in many cases possible with relatively simple nonlinear equations of motion. For certain values of parameters of those equations the solution does not converge towards expected values, but chaotically oscillates. Strong dependency on small changes of initial conditions occurs as well. When analyzing such phenomena its mathematical essence can be connected with existence of “strange attractor” in phase plane. Possible creation of chaos can be seen in repeated bifurcation of solution, with so called cumulation point behind which the strange attractor is generated. Phase diagram of system solution then transfers from stable set of trajectories towards new, unstable and chaotic set. Creating the global trajectory diagrams is of essential importance. When succesfull, the asymptotic behavior of systems model is described [4], [1].

2. Drive with DC motor with permanent magnets

Direct current drive with permanent magnets is often used in mechatronic drive systems together with semiconductor transducers. Good properties of such drives are given by the fact that vector of exciting magnetic flux is perpendicular to current direction in armature circuit and motor therefore always produces maximal torque.

Mathematical model of DC motor can be expressed via state variables of current I and angular velocity ω by state equation:

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$$\begin{bmatrix} \frac{di}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{C_\Phi}{L_a} \\ \frac{C_\Phi}{J} & 0 \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} u_a \\ m_z \end{bmatrix} . \tag{1}$$

Where particular symbols represent:

Armature circuit current	I
Motor angular velocity	ω
Motor feeding voltage	u_a
Resistance and inductance of rotor coil	R_a, L_a
Magnetic flux	Φ
Rotor moment of inertia	J

To verify properties and behavior of drive with DC motor in various steady states we used:

a) Theoretical model of DC motor and mathematical model of chaos module
b) Real systems of motor chaos (for input signal generation), amplifier and small DC motor MAXON with revolutions recording and armature current measurement [2].

3. Bifurcation behavior of drive model [3]

For evaluation of DC drive behavior exhibiting possibilities of creation of bifurcation of state variables depending on small change of control parameter we have used chaos module which generated chaotic input signal for DC motor module corresponding to MAXON Re 16. Block diagram of drive model made in Matlab / Simulink is shown on fig. 1.

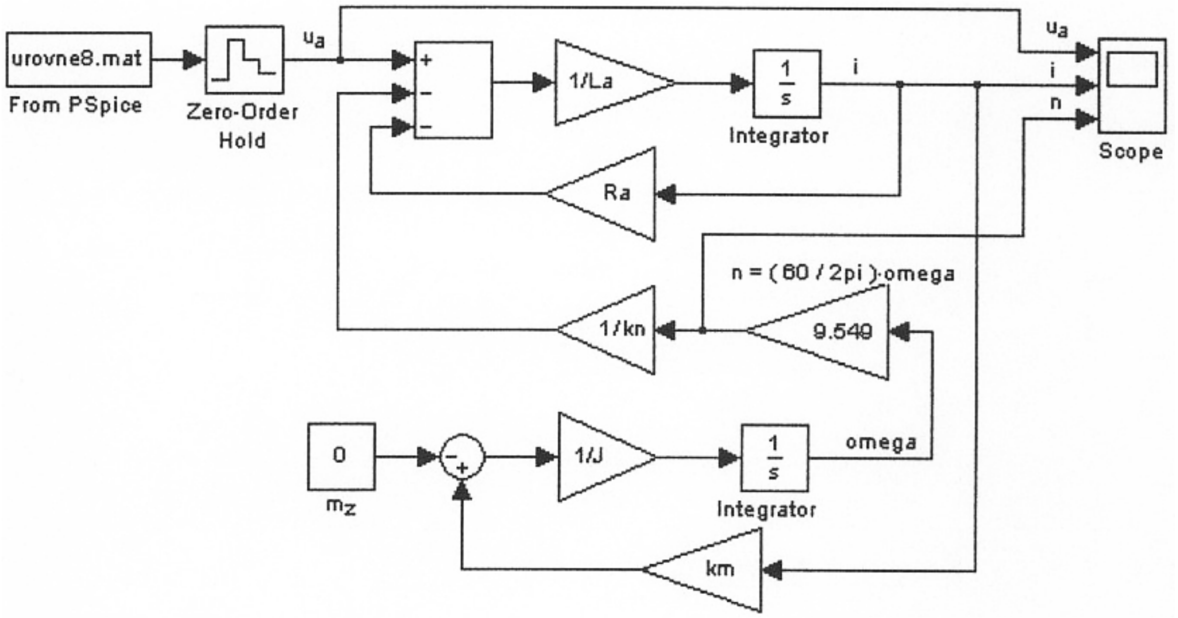

Fig. 1. Block diagram of drive model.

Chaos module is based on NJH 1101 chip, developed in Yamakowa´s Lab & FLSI for modeling and analysis of chaotic states in discrete nonlinear systems. Chaotic signal generated by chaos module in P Spice environment was sampled, saved as value matrix and transferred to Matlab / Simulink environment. We observed the courses of armature voltage $u_a(t)$, angular velocity $\omega(t)$ and armature current $i(t)$, which corresponded to steady states of the system for given values of control current – value of extremely stabilized resistor R_1 in nonlinear circuit, thus the chaos module.

Apart from time dependencies of selected variables we also generated given trajectories (attractors) corresponding to basic stable period, its double, quadruple and following chaotic state when periodicity of the motion can not be ensured.

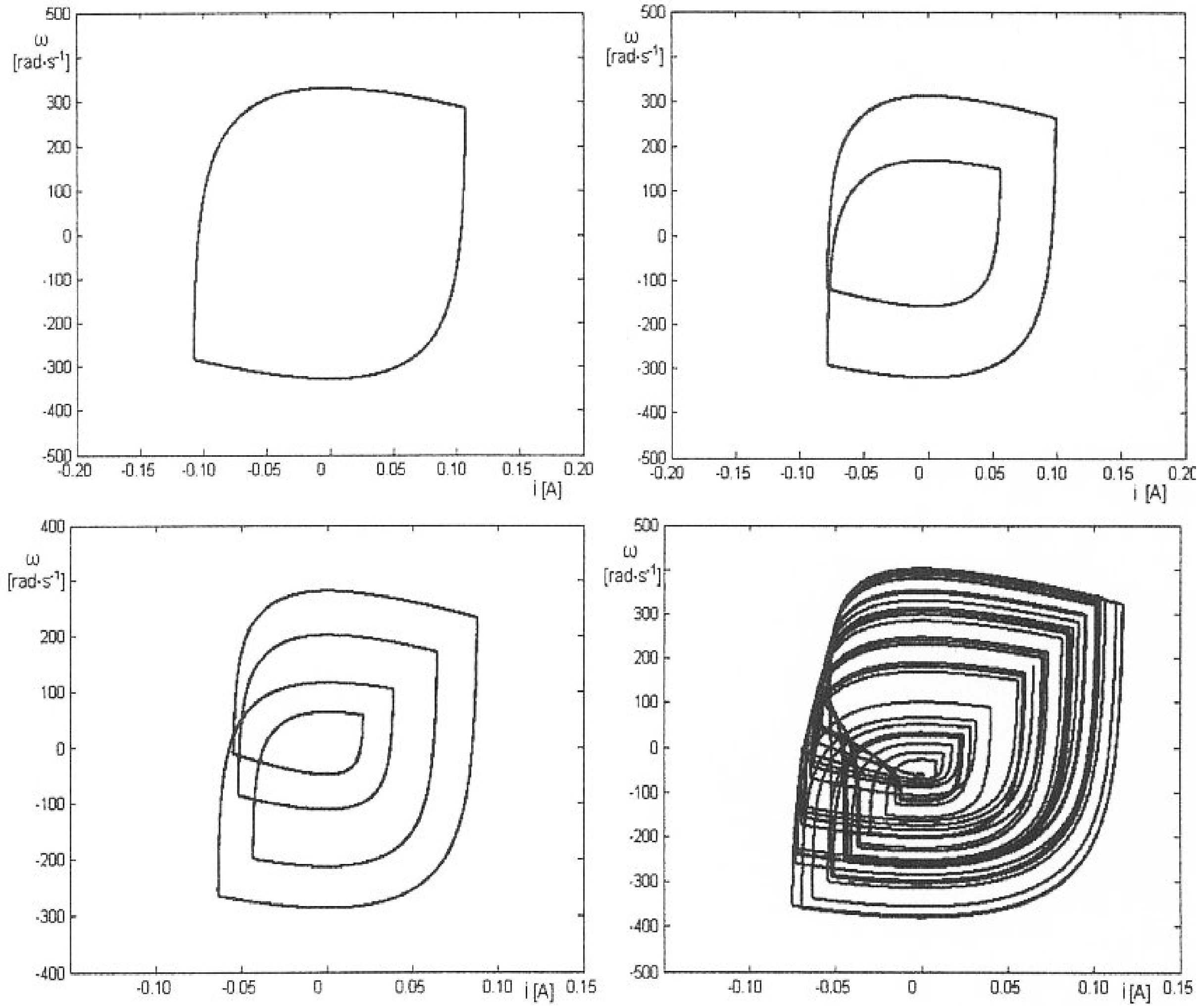

Fig. 2. Attractors of model system.

4. Verification of real system behaviour

Overall layout of the system with MAXON Re-16 DC motor is shown on fig. 3. During the simulation experiments we used oscilloscope to record the courses of feeding voltage $u_a(t)$, induced voltage of tachogenerator $u_i(t)$, corresponding angular velocities of motor $\omega(t)$ and armature current $i(t)$ while changing the control parameter (resistor) R_1 of chaos module.

From given recordings on oscilloscope we can see the existence of limit cycles with primitive period (fig. 4a) and when changing the control parameter with double period (fig. 4b), quadruple period (fig. 4c) and subsequently with chaotic attractor (fig. 4d).

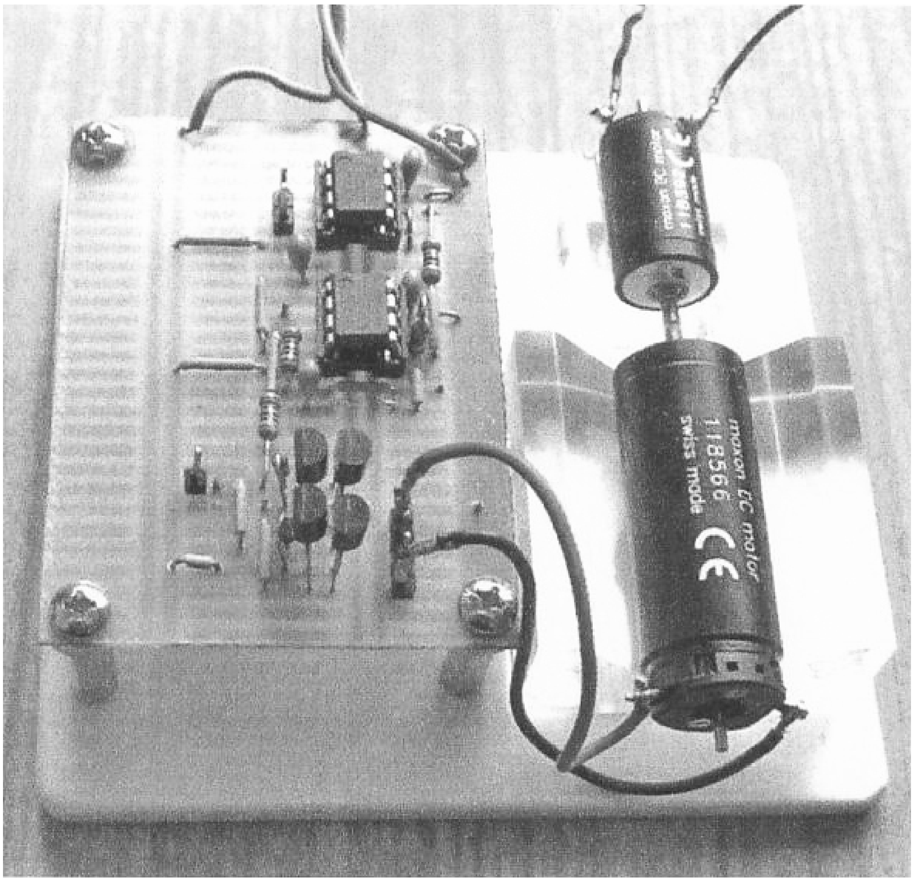

Fig. 3. Layout of DC motor real system.

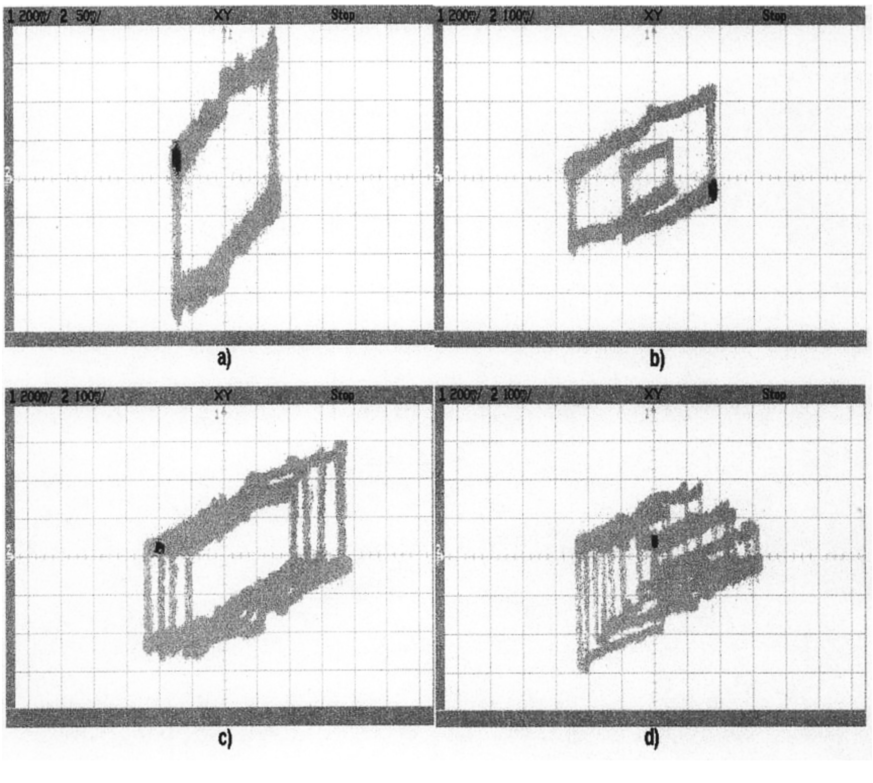

Fig. 4. Attractors of real system: Primitive period (a), double period (b), quadruple period (c) and chaotic attractor (d).

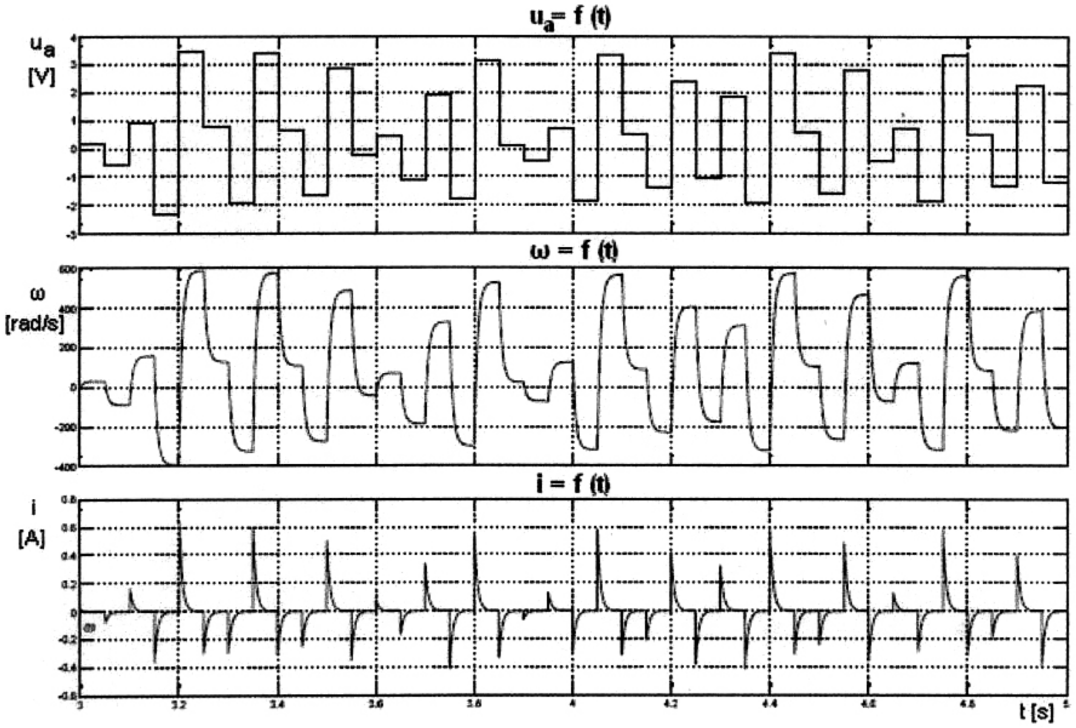

Fig. 5. Time courses of voltage, angular velocity and current – chaos.

Time courses of feeding voltage $u_a(t)$, angular velocity $\omega(t)$ and armature current in chaotic state are recorded on fig. 5. For completeness' sake let us show the “classical” bifurcation diagram as was shown on oscilloscope screen [3]. – fig. 6.

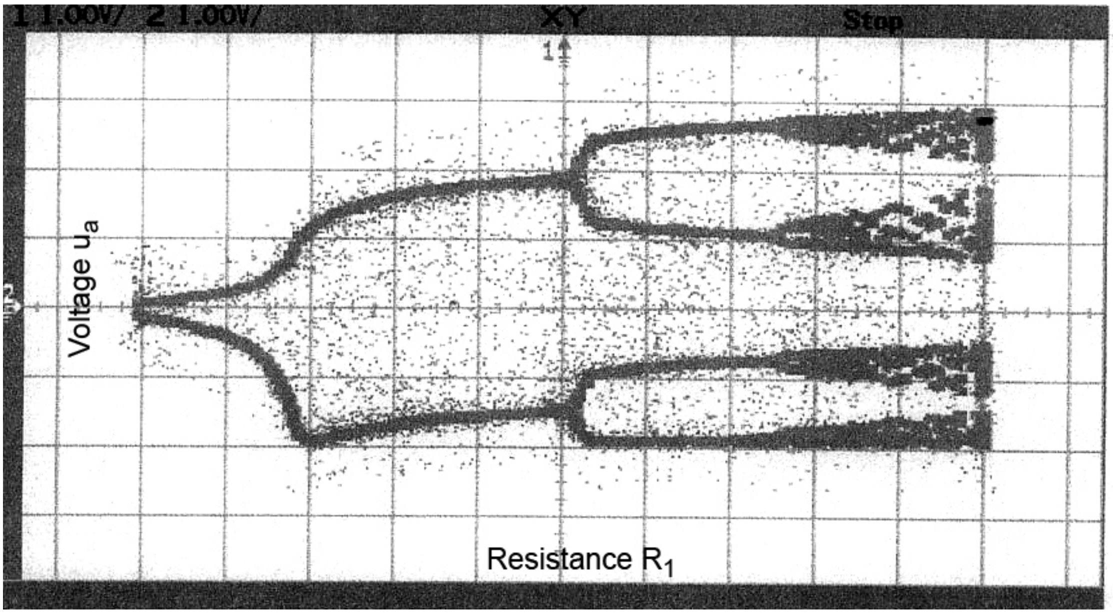

Fig. 6. Bifurcation diagram.

5. Valuation of current results and possibilities of its utilization

If we compare results obtained on model system chaos module + DC motor model and real system chaos module + amplifier + DC motor, we can state that results are very similar.

From performed simulations analysis it is clear that in both cases the existence of steady limit cycles precede the occurrence of chaos. Those cycles can be uniquely determined in phase space using typical tractors (cycles with primitive period, double period and quadruple period). Chaotic behavior in both cases is uniquely described by open state trajectories, which can not be predicted precisely. Those trajectories are situated in certain limiting areas. This has particular technical product:

Both angular velocity of the motor ω and armature current i will not exceed certain limits and observed DC motor is not in direct danger by exceeding the through current even in chaotic state.

Bifurcation analysis can be used as mean for identification of limit values of parameters, or as a mean for drive diagnostic. Bifurcation analysis could extend algorithms used of adaptive controllers design. We can not exclude the use of chaotic signals as control signals for special industrial mechanisms requiring irregular mechanical motion depending on control parameter.

6. Conclusion

Chaos became phenomenon in variety of engineering problems in last years. Therefore we focused on it also in analysis of drive systems. Based on performed analysis we can state following recommendations:

- when evaluating the properties and behavior of dynamic system it is useful to define such parameters of models, which can influence the occurrence of parasitic motion including chaotic one (fluctuation of initial conditions, links gaps, control parameters),
- to observe the evolution of responses in phase planes based on changes of selected parameters and to identify typical chaos effects,
- if such effect occur then evaluate Fourier spectrum of responses. Chaotic motion corresponds to broadband spectra, even when exciting spectra is narrowband.

With respecting given recommendations it is not difficult to identify the areas of possible occurrence of chaos in technical systems using mathematical modelling. However, we do not want to disvalue the analytical approaches with above described alternative approach.

Acknowledgement

Published results were acquired with the support of the Institute of Thermomechanics ASCR – research plan AVO Z20 760514 and Grad Agency CR in the frame of grand project number 101/06/0063.

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