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D-branes, Orientifolds and K-theory

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Abstract: The complete D-brane spectrum in $\mathbb{Z}_2$ orientifolds is computed. Stable non-BPS D-branes with both integral and torsion charges are found. The relation to K-theory is discussed and a new K-theory relevant to orientifolds is suggested.

1 Introduction

D-branes [2] play an important role in string theory. They classify a large number of string vacua, provide a canvas for gauge theories and are a key ingredient of many dualities. Understanding the spectrum of D-branes in a given on-shell closed string background is an important and often difficult subject. For closed string backgrounds with a conformal field theory description, D-branes are most succinctly represented as boundary states [8] - coherent states constructed out of closed string oscillators.

Another tool used in the study of D-brane spectra is K-theory [9, 5]. Since D-branes couple to R-R potentials it was initially thought that they may be classified by (possibly rational) cohomology. With the discovery of brane annihilation through tachyon condensation [3] though the K-theory description of D-branes seems to be the more natural description. The two descriptions differ in the classification of torsion-charged branes. Hence, torsion charged D-branes provide a good test of the K-theory description.

In this note we review the boundary conformal field theory (BCFT) construction of D-branes in the $\Omega \times \mathcal{I}_4$ orientifolds [10, 11, 12] of type IIB theory [1]. We shall focus on D-branes in the GP model and refer the reader to [1] for details of the BZDP model. The GP model has an O9-plane and sixteen O5-planes, and in order to cancel the resulting tadpoles one introduces 16 D9-branes and 16 D5-branes. Placing one D5-brane at each of the O5-planes one obtains a locally charge canceling configuration. The D5-branes can only move of the O5-planes in pairs, much as fractional branes in orbifolds. Unlike fractional branes, they do not couple to twisted sectors and have thus been dubbed stuck [1].

When classifying D-branes, we will find it convenient to follow the notation of [7]: we will refer to a Dp-brane as a D(r, s)-brane with $r + s = p$ and the brane having $r + 1$ Neumann direction in the non-compact six-dimensional spacetime and s Neumann directions on T^4. For example the tadpole canceling D5- and D9-branes will be labeled as D$(5, 0)$- and D$(5, 4)$-branes, respectively.

The rest of this note is organised as follows. In section 2 we discuss integrally charged non-BPS D-branes and their decay channels, while in section 3 we present some examples of torsion charged non-BPS D-branes. In section 4 we compare the BCFT construction to K-theory predictions; we show that the BZDP model is described by orthogonal equivariant K-theory. In particular there is good agreement between K-theory and BCFT for torsion charged branes. We also suggest a new twisted version of the equivariant orthogonal K-theory which should describe D-branes in the GP orientifold.

2   Integrally charged non-BPS D-branes

Integrally charged non-BPS D-branes carry twisted R-R charges but are neutral under the untwisted R-R charge. In the GP model two types of such truncated branes were encountered. Firstly Ω-invariant truncated branes from the $\mathcal{I}_4$ orbifold [7] are present for $r = -1, 3$ and $s = 1, 3$. Secondly, D$\bar{\text{D}}$pairs, in which Ω maps the D-brane to the $\bar{\text{D}}$-brane, occur for $r = -1, 3$ and $s = 0, 4$. In the orbifold the latter are bound states of two fundamental objects; in the orientifold however, they form one object.

The decay channels and stability regions of the Ω-invariant truncated branes are modified by the Ω projection. In the orbifold these are stable for[1]

$$R_\perp \; \geq \; \frac{1}{\sqrt{2}}\,, \tag{1}$$

$$R_\| \; \leq \; \sqrt{2} \tag{2}$$

where $R_\perp$, $R_\|$ are radii of the compact directions inverted by $\mathcal{I}_4$ which are perpendicular and parallel to the truncated brane. For small $R_\perp$ the $\hat{\text{D}}(r, s)$-brane decays into a D$\bar{\text{D}}$pair of superimposed $(r, s+1)$-branes, while for large $R_\|$ it decays into a D$\bar{\text{D}}$pair of $(r, s-1)$-branes at opposite fixed points. In the orientifold the $s = 1$ ($s = 3$) $\hat{\text{D}}$-branes decay along $x^\perp$ ($x^\|$) as in the orbifold.[2] On the other hand the orientifold $s = 1$ ($s = 3$) $\hat{\text{D}}$-branes are stable for all values of $R_\|$ ($R_\perp$) as the Ω projection removes the open-string momentum (winding) states.[3]

3   Torsion charged non-BPS D-branes

In the GP model two kinds of torsion charged D-branes were encountered. The first type, present for $s = 2$, couples to both the untwisted and twisted NS-NS sector

$$|h_\pm, (r, s = 2)\rangle = |NSNS\rangle \pm |NSNS, T\rangle\,, \tag{3}$$

and in the non-compact theory carry $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ charge. The second kind of torsion-charged branes couple only to the untwisted NS-NS sector

$$|g, (r, s)\rangle = |NSNS\rangle\,. \tag{4}$$

The latter branes are very similar to the torsion branes encountered in $\mathcal{I}_n\Omega$ orientifolds of type II theories [3, 5, 6]. We focus here on $r = 5$ branes and refer the reader to [1] for other cases.

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[1]We take $\alpha' = 1$ throughout.

[2]This is consistent since there are $r = -1, 3$, $s = 2$ BPS fractional branes in the GP model.

[3]This is fortunate as in the GP orientifold there are no BPS fractional $r = -1, 3$ $s = 0, 4$ branes into which the truncated branes could decay.

In the uncompactified theory there are two (5,2)-branes with untwisted and twisted NS-NS couplings: h_+ and h_-, both of which are stable for[4]

$$\frac{1}{R_{||}^2} + R_\perp^2 \geq \frac{1}{2} \, . \tag{5}$$

Further, it was shown in [1] that

$$|h_+, (5,2)\rangle + |h_-, (5,2)\rangle = |g(5,2)\rangle \, . \tag{6}$$

g can be thought of as a Z_2 D7-brane of type I and its image under $\mathcal{I}_4$ and so satisfies

$$|g\rangle + |g\rangle = |0\rangle \, , \tag{7}$$

with $|0\rangle$ representing the closed string vacuum. Since h_+ and h_- are distinct, the (5,2)-branes in the non-compact theory carry $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ charge. In the compact theory the h-branes will couple to four twisted NS-NS sectors, and by turning on suitable Wilson lines and fluxes in the compact directions one finds many more h-branes [1].

The above torsion charged D7-branes are in fact unstable since the open string stretching between them and tadpole-canceling D-branes has a tachyonic groundstate. The D7-branes will as a result decay; the above discussion does however indicates the presence of a torsion charge in the theory. In the BZDP orientifold we found [1] torsion-charged D-branes (for example a D-particle) which have no such tachyonic instabilities. It is interesting to note that such a D-particle is *stable* for all values of compactification radii.

The g D7-brane of equation (4), has no tachyonic states from open strings with both endpoints on its worldvolume for radii satisfying equations (1) and (2). For radii outside these regions the D7-brane becomes unstable and decays. The decay channels are most easily found by considering the brane as an Ω-invariant superposition of fractional (5,2)-branes in the $\mathcal{I}_4$ orbifold as depicted in figure 1. The decays in the orbifold are known and turn out to be Ω-invariant. For example for $R_{||} \geq \sqrt{2}$ the g(5,2)-brane decays into a g(5,1)-brane (also constructed in [1]) which carries $\mathbb{Z}_2$ charge. This D6-brane for $R_{||} \geq \sqrt{2}$ decays into a stuck D(5,0)-brane at one fixed point and stuck $\bar{\text{D}}$(5,0)-brane at another. The $\bar{\text{D}}$(5,0)-brane can annihilate with a tadpole canceling D(5,0)-brane, should the latter be present at that fixed point. In this case the configuration ends up being BPS and further changes of radii will not affect it. On the other hand if there is no tadpole canceling D(5,0)-brane at the fixed point the configuration remains non-BPS.

Unlike the original D7-brane, both final configurations have no tachyons coming from open strings with one endpoint on the torsion-charged configurations and one on a tadpole canceling D-brane; they describe the endpoint of condensation of these tachyons. We note that the positions of D5-branes are T-dual to Wilson lines on D9-branes and so the g D7-branes can also be thought to decay into gauge configurations on the worldvolumes of the D9-branes.

4   Orientifolds and K-theory

D-branes on orbifolds are described by equivariant K-theory. On the other hand, D-branes in Type II theories projected by $\mathcal{I}_n\Omega$ are classified by Real K-theory. It is natural

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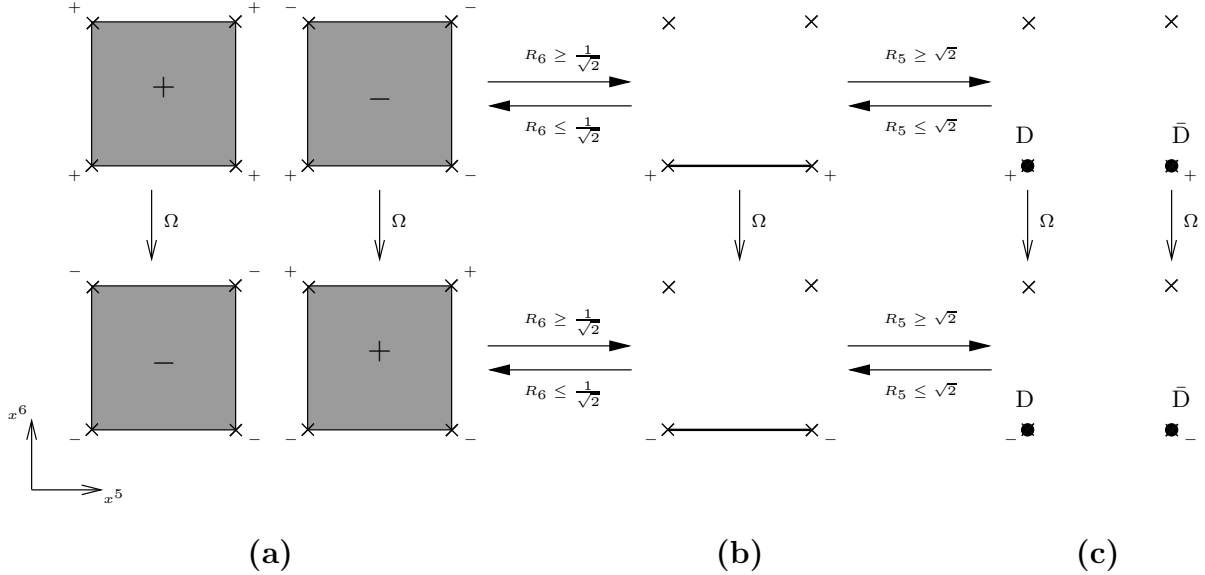[4]In particular they are stable in the decompactified theory.

Figure 1: The decay channels of $\mathbb{Z}_2$-charged branes are most easily seen as an Ω invariant process in the $\mathcal{I}_4$ orbifold. The first line in the figure shows the standard decent of an $s = 2$ D$\bar{\text{D}}$pair (a), via an $s = 1$ $\hat{\text{D}}$-brane (b), into an $s = 0$ D$\bar{\text{D}}$pair. The second line is the Ω-image of this decay. Together, the diagrams show the decays between $\mathbb{Z}_2$-charged $s = 2, 1, 0$-branes in the GP orientifold. **(a)** A $\mathbb{Z}_2$-charged $(5,2)$-brane (called h in the text) is an Ω invariant configuration of four fractional $(5, 2)$-branes in the orbifold. The twisted R-R charges of each of the fractional branes are shown as $\pm$ next to the fixed points denoted by crosses. The untwisted R-R charge is ± 1 and shown in the middle of each brane. **(b)** A $\mathbb{Z}_2$ charged $(5,1)$-brane is an Ω invariant configuration of two truncated $(5, 1)$-branes in the orbifold. The twisted R-R charges of each of the $\hat{\text{D}}$-branes are shown as $\pm$ next to the fixed points denoted by crosses. **(c)** In the orbifold cover a stuck $(5,0)$-(anti-)brane is an Ω-invariant pair of fractional $(5, 0)$-(anti-)branes with opposite twisted R-R charges. Here we show a stuck brane at one fixed point with a stuck anti-brane at the other.

then to expect that D-branes in orientifolds of the form $\Omega\mathcal{I}_n \times G$ be classified by Real G-equivariant K-theory. It is in fact straightforward to compute[5]

$$KR_{\mathbb{Z}_2 \times \mathbb{Z}_2}(pt) = \mathbb{Z} \oplus \mathbb{Z}. \tag{8}$$

As a result D9-branes classified by $KR_{\mathbb{Z}_2 \times \mathbb{Z}_2}$ should carry two integral charges - untwisted and twisted R-R charges. The D(5,4)-branes of GP only carry untwisted R-R charge[6] and hence $KR_{\mathbb{Z}_2 \times \mathbb{Z}_2}$ *cannot* describe this model. Nonetheless the D(5,4)-branes of BZDP do carry both untwisted and twisted R-R charges[7]. Hence, $KR_{\mathbb{Z}_2 \times \mathbb{Z}_2}$ can only describe D-branes in the BZDP model. As further evidence of this using long exact sequences much like those in [7] one can for example show that

$$KR_{Z_2 \times \mathbb{Z}_2}(R^{0,1}) = \mathbb{Z}_2 \oplus \mathbb{Z}_2, \tag{9}$$
$$KR_{Z_2 \times \mathbb{Z}_2}(R^{1,0}) = \mathbb{Z}, \tag{10}$$

indicating that there should be $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ charged $(4, 4)$-branes and $\mathbb{Z}$ charged $(5, 3)$-branes in the BZDP model. Such D-branes have been found to be consistent using BCFT techniques [1].

D-branes in the GP orientifold are not described by $KR_{\mathbb{Z}_2 \times \mathbb{Z}_2}$, and at first sight there is no obvious alternate candidate for it. A similar problem occurs in $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

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[5]By $KR_{\mathbb{Z}_2 \times \mathbb{Z}_2}$ we mean K-theory with an anti-linear involution τ and a linear order two map g which together form the $\mathbb{Z}_2 \times \mathbb{Z}_2$ group.

[6]This is immediate since there is no twisted sector tadpole in this model.

[7]In the BZDP model the twisted tadpole has to be canceled.

of Type II theories. There are in fact two such orbifolds [13] which differ by the action of g_i, on the g_j-th twisted sector ($i \neq j$). Since the closed string spectrum differs in the two theories, the stable D-branes are also distinct [14]. In fact D-branes in the theory with no torsion are described by the equivariant K-theory $K_{\mathbb{Z}_2 \times \mathbb{Z}_2}$ while those in the theory with torsion by $K^{[c]}_{\mathbb{Z}_2 \times \mathbb{Z}_2}$, a twisted equivariant K-theory.[8] Bundles classified by $K^{[c]}_{\mathbb{Z}_2 \times \mathbb{Z}_2}$ differ from those of $K_{\mathbb{Z}_2 \times \mathbb{Z}_2}$ in that the two generators of $\mathbb{Z}_2 \times \mathbb{Z}_2$ anticommute as linear maps on the fibres in the twisted case. The GP and BZDP models differ by the action of Ω on the $\mathcal{I}_4$-twisted sector, and so D-branes in the GP model should be described by $KR^{[c]}_{\mathbb{Z}_2 \times \mathbb{Z}_2}$: a twisted version of $KR_{\mathbb{Z}_2 \times \mathbb{Z}_2}$. Bundles classified by $KR^{[c]}_{\mathbb{Z}_2 \times \mathbb{Z}_2}$ will have anti-commuting complex conjugation and $\mathbb{Z}_2$ action on the fibres. Such a K-group has not been previously studied and it is not clear whether it forms part of some generalised cohomology theory. For a detailed discussion of these issues see [15].

5 Conclusions

We have discussed the full spectrum of D-branes in $\mathcal{I}_4 \times \Omega$ orientifolds. In particular non-BPS D-branes with torsion and integer charges were identified, and their stability regions and decay channels were discussed. The BCFT constructions were compared to K-theory; we saw in particular good agreement between BCFT and K-theory for torsion-charged D-branes, supporting the K-theory conjecture. We have also found that Orthogonal (or Real) equivariant K-theory does not always describe the spectrum of D-branes in orientifolds. Rather, there are twisted versions of such K-groups which are also relevant.

It would be interesting to find if the twisted K-groups form a generalised cohomology. We hope to comment on this in the near future [15]. The torsion charges we have identified should have an interpretation in terms of dual heterotic and F- theories. Understanding these charges in the dual theories would provide a very non-trivial, non-BPS test of the conjectured dualities.

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