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The Identity String Field and the Tachyon Vacuum

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Abstract: We show that the triviality of the entire cohomology of the new BRST operator $\mathcal{Q}$ around the tachyon vacuum is equivalent to the $\mathcal{Q}$-exactness of the identity $\mathcal{I}$ of the $\star$-algebra. We use level truncation to show that as the level is increased, the identity becomes more accurately $\mathcal{Q}$-exact. We carry our computations up to level nine, where an accuracy of 3% is attained. Our work supports, under a new light, Sen's conjecture concerning the absence of open string degrees of freedom around the tachyon vacuum. As a by-product, a new and simple expression for $\mathcal{I}$ in terms of Virasoro operators is found.

Keywords: BRST Cohomology, Open String Field Theory, Level Truncation.

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Contents

1. Introduction

Regarding the fate of the tachyon in various systems such as brane-antibrane pairs in Type II theories as well as the D25-brane in the bosonic string theory, Sen proposed his famous three conjectures in [1, 2]. These state that (i) The difference in energy between the perturbative and the tachyon vacuum exactly cancels the tension of the corresponding D-brane system; (ii) After the tachyon condenses, all open string degrees of freedom disappear, leaving us with the closed string vacuum; and (iii) Non-trivial field configurations correspond to lower-dimensional D-branes.

Because tachyon condensation is an off-shell process[2], we must use the formalism of string field theory. Both Witten's cubic open string field theory [4] and his background independent open string field theory [5, 6, 7, 8] seem to be good candidates. Indeed, in the last two years, there has been a host of works aimed to understand Sen's three conjectures by using the above two string field theories as well as the non-linear sigma-model (Born-Infeld action) [9]. Thus far, Sen's first and third conjectures have been shown to be true to a very high level of accuracy ([10] - [27]); they have also been proven analytically in Boundary String Field Theory ([28] - [30]). The second conjecture however, is still puzzling.

Let us clarify the meaning of this conjecture. From a physical point of view, after the tachyon condenses to the vacuum, the corresponding D-brane system disappears and there is no place for open strings to end on. Therefore at least all perturbative conventional open string excitations (of ghost number 1) should decouple from the theory. There has been a lot of work to check this statement, for example ([31]-[42]). In particular, using level truncation, [43] verifies that the scalar excitations at even levels (the Q closed scalar fields) are also Q-exact to very high accuracy.

However, as proposed in [45, 46] there is a little stronger version for the second conjecture. There, Rastelli, Sen and Zwiebach suggest that after a field redefinition, the new BRST operator may be taken[3] to be simply c_0, or more generally a linear combination of operators of the form $(c_n + (-)^n c_{-n})$. For such a new BRST operator, not only should the conventional excitations of ghost number 1 disappear, but more precisely the full cohomology of any ghost number of the new BRST operator around the tachyon vacuum vanishes[4]. Hence these authors propose that Sen's second conjecture should hold in such a stronger level. In fact, Sen's second conjecture suggests also that around the tachyon vacuum, there should be only closed string dynamics. However, we will not touch upon the issue of closed strings in our paper and leave the reader to the references [40, 48, 49, 53].

Considering the standing of the second conjecture, it is the aim of this paper to address to what degree does it hold, i.e., whether the cohomology of Q_{Ψ_0} is trivial only for ghost number 1 fields or

[2]For some early works concerning tachyon condensation please consult [3].

[3]The first String Field Theory action with pure ghost kinetic operator was written down in [47].

[4]An evidence for the triviality of a subset of the discrete ghost number one cohomology was presented recently in [44] which complemented [43].

for fields of any ghost number. We will give evidence which shows that the second conjecture holds in the strong sense, and is hence consistent with the proposal in [45, 46].

Our discussion relies heavily upon the existence of a string field $\mathcal{I}$ of ghost number 0 which is the identity of the $\star$-product. It satisfies

$$\mathcal{I} \star \psi = \psi \star \mathcal{I} = \psi$$

for any state[5] ψ. The state $\mathcal{I}$ was first constructed in the oscillator basis in [55, 56]. Then a recent work [57] gave a recursive way of constructing the identity in the (background independent) total-Virasoro basis which shows its universal property in string field theory. As a by-product of our analysis, we have found a new and elegant analytic expression for $\mathcal{I}$ without recourse to the complicated recursions.

Ignoring anomalies, the fact that Q_{Ψ_0} is a derivation of the $\star$-algebra implies that $\mathcal{I}$ is Q_{Ψ_0} closed and the problem is to determine whether it is also Q_{Ψ_0} exact, i.e., if there exists a ghost number -1 field A, such that $\mathcal{I} = Q_{\Psi_0} A$. If so, then for an arbitrary Q_{Ψ_0} closed state ϕ we would have

$$\begin{aligned} Q_{\Psi_0}(A \star \phi) &= (Q_{\Psi_0} A) \star \phi - A \star (Q_{\Psi_0} \phi) \\ &= \mathcal{I} \star \phi \\ &= \phi, \end{aligned}$$

where in the second step, we used the fact that ϕ is Q_{Ψ_0}-closed, and in the last step, that $\mathcal{I}$ acts as the identity on ϕ. This means that any Q_{Ψ_0}-closed field ϕ is also Q_{Ψ_0}-exact, in other words, the entire cohomology of Q_{Ψ_0} is trivial.

Therefore we have translated the problem of the triviality of the cohomology of Q_{Ψ_0} into the issue of the exactness of the identity $\mathcal{I}$. In this paper, we will use the level truncation method to show that the state A indeed exists for the tachyon vacuum Ψ_0 up to an accuracy of 3.2%.

The paper is structured as follows. In Section 2, we explain the above idea of the exactness of $\mathcal{I}$ in detail. In Section 3, we use two different methods to find the state A: one without gauge fixing and the other, in the Feynman-Siegel gauge. They give the results up to an accuracy of 2.4% and 3.2% respectively. In Section 4, we discuss the behaviour of $\mathcal{I}$ under level truncation and perform

[5] There are some mysteries regarding of the identity. For example, in [57] the authors showed that this identity string field is subject to anomalies, with consequences that $\mathcal{I}$ may be the identity of the $\star$-algebra only on a subspace of the whole Hilbert space. In the following, we will first assume that $\mathcal{I}$ behaves well on the whole Hilbert space, and postpone some discussions thereupon to Section 4.

a few consistency checks on our approximations. Finally, in Section 5 we make some concluding remarks and address some further problems and directions.

A few words on nomenclature before we proceed. By $|0\rangle$ we mean the $SL(2,\mathbf{R})$-invariant vacuum and $|\Omega\rangle := c_1 |0\rangle$. We consider $|\Omega\rangle$ to be level 0 and hence $|0\rangle$ is level 1. Furthermore, in this paper we expand our fields in the universal basis (matter Virasoro and ghost oscillator modes).

2. The Proposal

To reflect the trivial cohomology of the BRST operator at the stable vacuum, Rastelli, Sen and Zwiebach [46] proposed that after a field redefinition, the new BRST operator Q_{new} may be taken to be simply c_0, or more generally a linear combination of operators of the form $(c_n + (-)^n c_{-n})$. For such operators, there is an important fact: there is an operator A such that

$$\{A, Q_{new}\} = I,$$

where I is the identity operator. For example, if $Q_{new} = c_n + (-)^n c_{-n}$, we can choose $A = \frac{1}{2}(b_{-n} + (-)^n b_n)$ because $\{\frac{1}{2}(b_{-n} + (-)^n b_n), Q_{new}\} = \{\frac{1}{2}(b_{-n} + (-)^n b_n), c_n + (-)^n c_{-n}\} = 1$. Therefore, if the state Φ is closed, i.e., $Q_{new}\Phi = 0$, then we have

$$\begin{aligned} \Phi &= \{A, Q_{new}\}\Phi = AQ_{new}\Phi + Q_{new}A\Phi \\ &= Q_{new}(A\Phi) \end{aligned} \tag{2.1}$$

which means that Φ is also exact. Thus the existence of such an A guarantees that the cohomology of Q_{new} is trivial.

In fact the converse is true. Given a Q_{new} which has vanishing cohomology we can always construct an A such that $\{A, Q_{new}\} = I$. Suppose that we denote the string Hilbert space at ghost level g by V_g. Define the subspace V_g^C as the set of all closed elements of V_g. We can then pick a complement, V_g^N, to this subspace[6] satisfying $V_g = V_g^C \oplus V_g^N$. Note that it consist of vectors which are not killed by Q_{new}. This subspace V_g^N, is not gauge invariant but any specific choice will do. The important point is that because we have assumed that Q_{new} has no cohomology, the restriction of Q_{new} to V_g^N given by

$$Q_{new}\Big|_{V_g^N} : V_g^N \to V_{g+1}^C,$$

[6]More precisely, the space V_g could be split as $V_g = V_g^C \oplus (V_g/V_g^C)$ where (V_g/V_g^C) is a vector space of equivalence classes under the addition of exact states. V_g^N should be considered as a space of representative elements in (V_g/V_g^C).

has no kernel and is surjective[7] on V^C_{g+1}. Thus it has an inverse which we denote A

$$A\Big|_{V^C_{g+1}} \equiv Q^{-1}_{new}\Big|_{V^C_{g+1}} : V^C_{g+1} \to V^N_g.$$

This insures that on the space V^C_g, $\{A, Q_{new}\} = I$ holds since if Φ is Q_{new}-closed,

$$\{A, Q_{new}\}\Phi = AQ_{new}\Phi + Q_{new}A\Phi = Q_{new}Q^{-1}_{new}\Phi = \Phi.$$

The above discussion only defines the action of A on V^C_g, what remains is to define its action on the complement V^N_g. Here there is quite a bit of freedom since one can choose any map that takes V^N_g into V^C_{g-1}. Assuming this, we have that for $\Phi \in V^N_g$,

$$\{A, Q_{new}\}\Phi = AQ_{new}\Phi + Q_{new}A\Phi = Q^{-1}_{new}Q_{new}\Phi + Q^2_{new}\chi = \Phi,$$

where by assumption $A\Phi$ is Q_{new}-closed (because it is in V^C_{g-1}) and thus equals $Q_{new}\chi$ for some $\chi \in V^N_{g-2}$. In general one can insist that A satisfies more properties. For example if we set $A|_{V^N_g} = 0$ we get that $A^2 = 0$. We summarize the above discussion as

PROPOSITION 2.1 *The cohomology of Q_{new} is trivial iff there exists an operator A such that $\{A, Q_{new}\} = I$.*

The basic hypothesis of this paper is that not only does such an operator A exist for Q_{Ψ_0}, but also for special choices of A, *the action of A can be expressed as the left multiplication by the ghost number -1 string field* which we denote as $A\star$. Thus we are now interested in satisfying the equation $\{A\star, Q_{new}\} = I$. Writing this out explicitly we have

$$\begin{aligned}
\{A\star, Q_{new}\}\Phi &= A \star (Q_{new}\Phi) + Q_{new}(A \star \Phi) \\
&= A \star Q_{new}(\Phi) + (Q_{new}A) \star \Phi - A \star (Q_{new}\Phi) \\
&= (Q_{new}A) \star \Phi.
\end{aligned}$$

In order for the last line to equal Φ for all Φ we need that

$$Q_{new}A = \mathcal{I}, \tag{2.2}$$

where $\mathcal{I}$ is the identity of the $\star$-algebra.

[7] As remarked in the previous footnote, if we use (V_g/V^C_g) instead of V^N_g, the mapping is an isomorphism of vector spaces.

For the case of interest, we wish to study the physics around the minimum of the tachyon potential. We recall that for a state Φ, the new BRST operator around the solution ψ of the EOM is given by

$$Q_\psi \Phi = Q_B(\Phi) + \psi \star (\Phi) - (-)^\Phi (\Phi) \star \psi. \tag{2.3}$$

Using this expression for the BRST operator we can rewrite the basic equation (2.2) as $Q_\psi A = Q_B(A) + \psi \star (A) + (A) \star \psi = \mathcal{I}$. For general vacua ψ, such a string field A will not exist. For example in the perturbative vacuum, $\psi = 0$, Q_ψ is simply Q_B. It is easy to show here that there is no solution for A because the Q_B action preserves levels while $\mathcal{I}$ has a component at level one (namely $|0\rangle$), but the minimum level of a ghost number -1 state A is 3. Indeed, for a more general solution $\psi \neq 0$ (such as the tachyon vacuum), the star product will not preserve the level and so it may be possible to find A. Our endeavor will be to use the level truncation scheme to find A for the tachyon vacuum Ψ_0, i.e., to find a solution A to the equation

$$Q_{\Psi_0} A = \mathcal{I}. \tag{2.4}$$

Note that this equation is invariant under

$$A \to A + Q_{\Psi_0} B$$

for some B of ghost number -2, thereby giving A a gauge freedom. This is an important property to which we shall turn in the next section.

Having expounded upon the properties of A, our next task is clear. In the following section, we show that for the tachyon vacuum Ψ_0, we can find the state A satisfying (2.4) in the approximation of the level truncation scheme.

3. Finding The State A

Let us now solve (2.4) by level truncation. To do so, let us proceed in two ways. We recall from the previous section that A is well-defined up to the gauge transformation $A \to A + Q_{\Psi_0} B$ where B is a state of ghost number -2. Because in the level truncation scheme, this gauge invariance is broken, we first try to find the best fit results without fixing the gauge of A. The fitting procedure is analogous to that used in [44] and we shall not delve too much into the details. We shall see below that at level 9, the result is accurate to 2.4%. However, when we check the behaviour of the numerical coefficients of A as we increase the accuracy from level 3 to 9, we found that they do not seem to converge. We shall explain this phenomenon as the consequence of the gauge freedom in the definition of A; we shall then redo the fitting in the Feynman-Siegel gauge. With this second method, we shall find that the coefficients do converge and the best fit at level 9 is to 3.2% accuracy. These results support strongly the existence of a state A in (2.4) and hence the statement that the cohomology around the tachyon vacuum is indeed trivial. In the following subsections let us present our methods and results in detail.

3.1 The Fitting without Gauge Fixing A

To solve the condition (2.4), we first need an explicit expression of the identity $\mathcal{I}$. Such an expression has been presented in [55] and [57], differing by a mere normalization factor $-4i$. In this paper, we will follow the conventions of [57] which has[8]

$$|\mathcal{I}\rangle = e^{L_{-2}-\frac{1}{2}L_{-4}+\frac{1}{2}L_{-6}-\frac{7}{12}L_{-8}+\frac{2}{3}L_{-10}+\cdots}|0\rangle \tag{3.1}$$

$$\begin{aligned}
&= |0\rangle + L_{-2}|0\rangle + \frac{1}{2}(L_{-2}^2 - L_{-4})|0\rangle \\
&+ \left(\frac{1}{6}L_{-2}^3 - \frac{1}{4}L_{-2}L_{-4} - \frac{1}{4}L_{-4}L_{-2} + \frac{1}{2}L_{-6}\right)|0\rangle \\
&+ \left(\frac{1}{24}L_{-2}^4 + \frac{1}{4}(L_{-2}L_{-6} + L_{-6}L_{-2}) + \frac{1}{8}L_{-4}^2 - \frac{7}{12}L_{-8}\right. \\
&\quad \left. -\frac{1}{12}(L_{-2}^2L_{-4} + L_{-2}L_{-4}L_{-2} + L_{-4}L_{-2}^2)\right)|0\rangle
\end{aligned} \tag{3.2}$$

where $L_n = L_n^m + L_n^g$, the sum of the ghost (L_n^g) and matter (L_n^m) parts, is the total Virasoro operator. For later usage we have expanded the exponential up to level 9. Furthermore, we split L_n into matter and ghost parts and expand the latter into b_n, c_n operators as $L_m^g := \sum\limits_{n=-\infty}^{\infty}(2m-n) : b_n c_{m-n} : -\delta_{m,0}$. In other words, we write the states in the so-called "Universal Basis" [57].

As a by-product, we have found an elegant expression for $\mathcal{I}$ which avoids the recursions[9] needed to generate the coefficients in the exponent. In fact, one can show that only L_{-m} for m being a power of 2 survive in the final expression, thus significantly reducing the complexity of the computation

[8]With the normalization $\langle c_1, c_1, c_1\rangle = 3$ that we are using, we should scale this expression by a factor of $K^3/3$, where $K = 3\sqrt{3}/4$. However, as the normalization of the identity will not change our analysis, we will use this right normalization only in Section 4, where we are dealing with expressions like $\mathcal{I} \star \Phi$.

[9]Indeed the expression given in [56] has no recursion either, however their oscillator expansion is not normal-ordered due to ghost insertions at the string mid-point.

of level-truncation for $\mathcal{I}$:

$$|\mathcal{I}\rangle = \left(\prod_{n=2}^{\infty} \exp\left\{-\frac{2}{2^n}L_{-2^n}\right\}\right) e^{L_{-2}}\,|0\rangle$$
$$= \ldots \exp(-\frac{2}{2^3}L_{-2^3}) \exp(-\frac{2}{2^2}L_{-2^2}) \exp(L_{-2})\,|0\rangle\,, \tag{3.3}$$

where we emphasize that the Virasoro's of higher index stack to the left *ad infinitum*. We leave the proof of this fact to the Appendix.

It is worth noticing that in the expansion of $\mathcal{I}$ only odd levels have nonzero coefficients. This means that we can constrain the solution A of (2.4), if it exists, to have only odd levels in its expansion. The reason for this is as follows. Equation (2.4) states that $Q_B A + \Psi_0 \star A + A \star \Psi_0 = \mathcal{I}$, moreover we recall that (cf. e.g. Appendix A.4 of [44]) the coefficient $k_{\ell,i}$ in the expansion of the star product $x \star y = \sum\limits_{\ell,i} k_{\ell,i}\psi_{\ell,i}$ is $k_{\ell,i} = \langle\tilde{\psi}_{\ell,i}, x, y\rangle$ for the orthogonal basis $\tilde{\psi}$ to ψ. Now the triple correlator has the symmetry $\langle x, y, z\rangle = (-)^{1+g(x)g(y)+\ell(x)+\ell(y)+\ell(z)}\langle x, z, y\rangle$, where $g(x)$ and $\ell(x)$ are the ghost number and level of the field x respectively. Whence, one can see that the even levels of $\Psi_0 \star A + A \star \Psi_0$ will be zero because the tachyon vacuum Ψ_0 has only even levels and A is constrained to odd levels. Furthermore, $Q_B = \sum\limits_n c_n L^m_{-n} + \frac{1}{2}(m-n) : c_m c_n b_{-m-n} : -c_0$ preserves level. Therefore, in order that both the left and right hand sides of (2.4) have only odd levels, A must also have only odd level fields.

Now the procedure is clear. We expand A into odd levels of ghost number -1 with coefficients as parameters and calculate $Q_{\Psi_0}A$. Indeed as with [44], all the states will be written as Euclidean vectors whose basis is prescribed by the fields at a given level; the components of the vectors are thus the expansion coefficients in each level. Then we compare $Q_{\Psi_0}A$ with $\mathcal{I}$ up to the same level and determine the coefficients of A by minimizing the quantity

$$\epsilon = \frac{|Q_{\Psi_0}A - \mathcal{I}|}{|\mathcal{I}|},$$

which we of course wish to be as close to zero as possible. We refer to this as the *"fitting of the coefficients"*. The norm $|.|$ is the Euclidean norm (for our basis, see the Appendix) . As observed in [43], different normalizations do not significantly change the values from the fitting procedure, so for simplicity we use the Euclidean norm to define the above measure of proximity ϵ. The minimum level of the ghost number -1 field A is 3, so we start our fitting from this level and continue to up to level 9 (higher levels will become computationally prohibitive).

First we list the number of components of odd levels for the fields A and $\mathcal{I}$ up to given levels:

	level 3	level 5	level 7	level 9
Number of Components of A	1	4	14	43
Number of Components of $\mathcal{I}$	4	14	43	118

From this table, we see that at level 3, we have only one parameter to fit 4 components. At level 5, we have 4 parameters to fit 14 components. As the level is increased the number of components to be fitted increases faster that the number of free parameters. Therefore it is not a trivial fitting at all.

3.1.1 A up to level 3

At level 3 the identity is:

$$\begin{aligned}\mathcal{I}_3 &= |0\rangle + L_{-2}\,|0\rangle \\ &= |0\rangle - b_{-3}c_1\,|0\rangle - 2b_{-2}c_0\,|0\rangle + L^m_{-2}\,|0\rangle\end{aligned}$$

and we find the best fit of A (recall that at level 3 we have only 1 degree of freedom) to be

$$A_3 = 1.12237\,b_{-2}\,|0\rangle\,,$$

with an ϵ of 17.1%.

3.1.2 A up to level 5

Continuing to level 5, we have

$$\begin{aligned}\mathcal{I}_5 &= |0\rangle + L_{-2}\,|0\rangle + \tfrac{1}{2}(L^2_{-2} - L_{-4})\,|0\rangle \\ &= |0\rangle - b_{-3}c_1\,|0\rangle - 2b_{-2}c_0\,|0\rangle + L^m_{-2}\,|0\rangle + b_{-5}c_1\,|0\rangle - b_{-2}c_{-2}\,|0\rangle \\ &\quad + b_{-3}c_{-1}\,|0\rangle + 2b_{-3}b_{-2}c_0c_1\,|0\rangle + 2b_{-4}c_0\,|0\rangle - \tfrac{1}{2}L^m_{-4}\,|0\rangle \\ &\quad - b_{-3}c_1L^m_{-2}\,|0\rangle - 2b_{-2}c_0L^m_{-2}\,|0\rangle + \tfrac{1}{2}L^m_{-2}L^m_{-2}\,|0\rangle\,.\end{aligned}$$

To this level we have determined the best-fit A to be

$$A_5 = 1.01893\,b_{-2}\,|0\rangle + 0.50921\,b_{-3}b_{-2}c_1\,|0\rangle - 0.518516\,b_{-4}\,|0\rangle + 0.504193\,b_{-2}L^m_{-2}\,|0\rangle\,,$$

with an ϵ of 11.8%.

The detailed data of the field A to levels 7 and 9 are given in table B.1 of the Appendix. Here we just summarize the results of the best-fit measure ϵ:

	level 3	level 5	level 7	level 9
$\epsilon = \frac{\lvert Q_{\Psi_0} A - \mathcal{I}\rvert}{\lvert\mathcal{I}\rvert}$	0.171484	0.117676	0.0453748	0.0243515

This indicates that up to an accuracy of 2.4% at level 9, there exists an A that satisfies (2.4); moreover the accuracy clearly gets better with increasing levels. This is truly an encouraging result.

3.2 The Stability of Fitting

There is a problem however. Looking carefully at the coefficients of A given in the table B.1, especially the fitting coefficients between levels 7 and 9, we see that these two groups of data have a large difference. Naively it means that our solution for A does not converge as we increase level. How do we solve this puzzle?

We recall that A is well-defined only up to the gauge freedom

$$A \longrightarrow A + Q_{\Psi_0} B.$$

It means that the solutions of (2.4) should consist of a family of gauge equivalent A. However, because $Q_{\Psi_0}^2 \neq 0$ under the level truncation approximation, the family (or the moduli space) is broken into isolated pieces. Similar phenomena were found in [43] where the momentum-dependent closed states were given by points instead of a continuous family. Using this fact, our explanation is that the fitting of levels 7 and 9 are related by $Q_{\Psi_0} B$ for some field B of ghost number -2. To show this, we solve a new $\tilde{A}$ up to level 9 that minimizes

$$\frac{|(\tilde{A})_7 - A_7|}{|A_7|} + \frac{|Q_{\Psi_0} \tilde{A} - \mathcal{I}_9|}{|\mathcal{I}_9|}$$

where A_7 is the known fitting data at level seven, $\mathcal{I}_9$ is the identity up to level nine and $(\tilde{A})_7$ refers to the first 14 components (i.e., the components up to level seven) of the level 9 expansion of $\tilde{A}$. By minimizing this above quantity, we balance the stability of fitting from level 7 to 9. The data is given in the last column of B.1. Though having gained stability, the fitting for level 9 is a little worse, with ϵ increasing from 2.44% to 3.56%.

The next thing is to check whether $\tilde{A} - A_9$ is an exact state $Q_{\Psi_0} B$. We find that this is indeed true and we find a state B such that

$$\frac{|(\tilde{A} - A_9) - Q_{\Psi_0} B|}{|\tilde{A} - A_9|} = 0.28\%.$$

3.3 Fitting A in the Feynman-Siegel Gauge

Alternatively, by gauge-fixing, we can also avoid the instability of the fit. If we require the state A to be in the Feynman-Siegel gauge, A will not have the gauge freedom anymore and the fitting result should converge as we do not have isolated points in the gauge moduli space to jump to. We have done so and do find much greater stability of the coefficients.

Notice that in the Feynman-Siegel gauge, A has the same field bases in levels 3 and 5, so the fitting at these two levels is the same as in Subsections 3.1.1 and 3.1.2. However, in this gauge it has one parameter less at level 7 and 5 less in level 9. Performing the fit with these parameters we have reached an accuracy of $\epsilon = 4.8\%$ at level 7 and $\epsilon = 3.2\%$ at level 9, which is still a good result. The details are presented in Table B.2 in the Appendix.

4. Some Subtleties of the Identity

As pointed out in the Introduction, there are some mysterious and anomalous features of the identity $\mathcal{I}$. For example, $\mathcal{I}$ is not a normalizable state [50], moreover, c_0, contrary to expectation, does not annihilate $\mathcal{I}$ even though it is a derivation [57]. We shall show in the following that with a slight modification of the level truncation scheme, this unnormalizability does not effect the results and furthermore that in our approximation $Q_{\Psi_0}\mathcal{I}$ indeed vanishes as it must for consistency.

Let us first show how problems may arise in a naive attempt at level truncation. Consider the quantity $\mathcal{I}_\ell \star |\Omega\rangle - |\Omega\rangle$, where $\mathcal{I}_\ell$ denotes the identity truncated to level ℓ and $|\Omega\rangle := c_1 |0\rangle$. We of course expect this to approach 0 as we increase ℓ. Using the methods of the previous section, we shall define the measure of proximity

$$\eta \equiv \frac{|\mathcal{I}_\ell \star |\Omega\rangle - |\Omega\rangle\,|}{|\,|\Omega\rangle\,|} = |\mathcal{I}_\ell \star |\Omega\rangle - |\Omega\rangle\,|,$$

where $|.|$ is our usual norm. We list η to levels 3, 5, 7, and 9 in the following Table:

level ℓ	3	5	7	9
$\eta = \|\mathcal{I}_\ell \star \|\Omega\rangle - \|\Omega\rangle\,\|$	2.06852	2.87917	3.56054	3.9452

Our η obviously does not converge to zero, hence star products involving $\mathcal{I}$ do not converge in the usual sense of level truncation. It is however not yet necessary to despair, as weak convergence will come to our rescue[10].

[10]We thank B. Zwiebach for this suggestion.

Indeed, instead of truncating the result to level ℓ, let us use a slightly different scheme. We truncate $\mathcal{I}_\ell \star |\Omega\rangle$ to a fixed level $m < \ell$ and observe how the coefficients of the fields up to level m converge as we increase ℓ. In the following table we list the values of the coefficients coeff(x) of the basis for $m = 2$ (i.e., fields x of level 0, 1 and 2) for the expression $\mathcal{I}_\ell \star |\Omega\rangle$.

$\mathcal{I}_\ell \star \|\Omega\rangle$	coeff$(\|\Omega\rangle)$	coeff$(b_{-1}c_0\,\|\Omega\rangle)$	coeff$(b_{-1}c_{-1}\,\|\Omega\rangle)$	coeff$(b_{-2}c_0\,\|\Omega\rangle)$	coeff$(L^m_{-2}\,\|\Omega\rangle)$
$\ell = 3$	0.6875	0.505181	-0.905093	-0.930556	0.465278
$\ell = 5$	1.16898	-0.278874	0.38846	0.520748	-0.260374
$\ell = 7$	0.911094	0.16252	-0.197833	-0.296607	0.148304
$\ell = 9$	1.05767	-0.0971502	0.0902728	0.163579	-0.0817895

We see that the $|\Omega\rangle$ component converges to 1 while the others converge to 0, as was hoped. We note however that this (oscillating) convergence is rather slow and we thus expect slow weak convergence for other calculations involving the identity.

Having shown that as $\ell \to \infty$ we get a weak convergence $\mathcal{I}_\ell \star |\Omega\rangle \to |\Omega\rangle$, we now consider $Q_{\Psi_0}\mathcal{I}_\ell$ as $\ell \to \infty$, which should tend to zero. Since Q_B preserves level and $Q_B\mathcal{I} = 0$, we have that $Q_B\mathcal{I} = 0$ in the level expansion; thus $Q_{\Psi_0}\mathcal{I} = \Psi_0 \star \mathcal{I} - \mathcal{I} \star \Psi_0$, which should converge to zero.

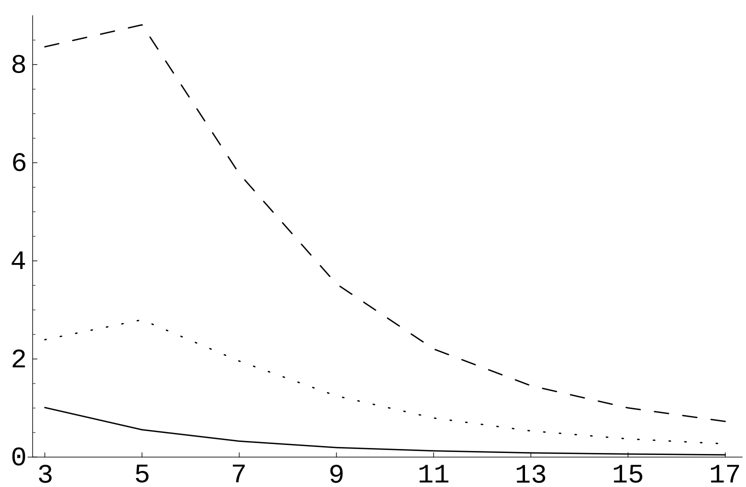

Figure 1: A plot of $q_{0,1}(\ell)$ (solid curve), $q_{2,1}(\ell)$ (dotted curve) and $q_{2,3}(\ell)$ (dashed curve) as functions of the level ℓ of the identity. ℓ goes from 3 to 17.

As the expression $Q_{\Psi_0}\mathcal{I}$ is linear in every component of Ψ_0, that $\mathcal{I}$ is Q_{Ψ_0}-closed will be established if we can show that for each component ϕ in Ψ_0, $\phi \star \mathcal{I} - \mathcal{I} \star \phi \equiv [\phi\star, \mathcal{I}]$ converges to zero as the level of $\mathcal{I}$ is increased. We plot in Fig.1, the absolute values of the coefficient of $c_0 |0\rangle$ in the expressions $[(c_1 |0\rangle)\star, \mathcal{I}_\ell]$, $[(c_{-1} |0\rangle)\star, \mathcal{I}_\ell]$ and $\left[(L^m_{-2} c_1) |0\rangle \star, \mathcal{I}_\ell\right]$, which we denote by $q_{0,1}(\ell)$, $q_{2,1}(\ell)$ and $q_{2,3}(\ell)$ respectively. It seems clear that the coefficients do converge to zero.

The weak convergence we have shown above can be interpreted in a more abstract setting. Let us examine the quantity $|\mathcal{I}_\ell \star \Phi - \Phi|$. It was shown in [51] that the $\star$-algebra of the open bosonic string field theory is a C^*-algebra. A well-known theorem dictates that any C^*-algebra M (with or without unit) has a so-called *approximate identity* which is a set of operators $\{\mathcal{I}_i\}$ in M indexed by i satisfying (i) $\|\mathcal{I}_i\| \leq 1$ for every i and (ii) $\|\mathcal{I}_i x - x\| \to 0$ and $\|x\mathcal{I}_i - x\| \to 0$ for all $x \in M$ with respect to the (Banach) norm $\|.\|$ of M (cf. e.g. [52]).

The level ℓ in our level truncation scheme is suggestive of an index for $\mathcal{I}$. Furthermore the weak convergence we have found in this section is analogous to property (ii) of the theorem (being of course a little cavalier about the distinction of the Banach norm of the C^*-algebra with the Euclidean norm used here). Barring this subtlety, it is highly suggestive that our $\mathcal{I}_\ell$ is an approximate identity of the $\star$-algebra indexed by level ℓ.

5. Conclusion and Discussions

According to a strong version of Sen's Second Conjecture, there should be an absence of any open string states around the perturbatively stable tachyon vacuum Ψ_0. This disappearance of all states, not merely the physical ones of ghost number 1, means that the cohomology of the new BRST operator Q_{Ψ_0} should be completely trivial near the vacuum. It is the key observation of this paper that this statement of triviality is implied by the existence of a ghost number -1 field A satisfying

$$Q_{\Psi_0} A = Q_B A + \Psi_0 \star A + A \star \Psi_0 = \mathcal{I}.$$

That is to say that if the identity of the $\star$-algebra $\mathcal{I}$ is a Q_{Ψ_0} exact state, then the cohomology of Q_{Ψ_0} would be trivial.

The level truncation scheme was subsequently applied to check our proposal. We have found that such a state A exists up to an accuracy of 3.2% at level 9. Although these numerical results give a strong support to the proposal for the existence of A and hence the triviality of Q_{Ψ_0}-cohomology near the vacuum, an analytic expression for A would be most welcome. However, to obtain such an analytic form of A, it seems that we would require the analytic expression for the vacuum Ψ_0, bringing us back to an old problem. It is perhaps possible that by choosing different gauges other than the Feynman-Siegel gauge we may find such a solution.

Our solution A satifies $\{A, Q\} = I$. It would be nice to see whether we can choose A cleverly to make $A \star A = 0$ (our Feynman-Siegel gauge fitting may not satisfy this equation). We are interested with this case because for the proposal of $Q_B = c_n + (-)^n c_{-n}$ made in [45, 46], one could find that $A = \frac{1}{2}(b_{-n} + (-)^n b_n)$ which does satisfy $A^2 = 0$. It would be interesting to mimic this nilpotency within the $\star$-algebra. Furthermore, it would be fascinating to see if we can make a field redefinition to reduce A to a simple operator such as b_0, and at the same time reduce Q_{Ψ_0} to a new BRST operator as suggested in [46], for example, c_0.

Last but not least, an interesting question is about the identity $\mathcal{I}$. In this paper we have given an elegant analytic expression for $\mathcal{I}$ which avoids the usage of complicated recursion relations. Furthermore, we have suggested that though the $\star$-algebra of OSFT may be a non-unital C^*-algebra, $\mathcal{I}$ still may serve as a so-called approximate identity. However, as we discussed before, anomalies related to the identity in the String Field Theory make the calculation in level truncation converge very slowly. It will be useful to understand more about $\mathcal{I}$.

Acknowledgements

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A. The Perturbatively Stable Vacuum Solution at Level $(M, 3M)$

We tabulate the coefficient of the expansion of the stable vacuum solution Ψ_0 at various levels and interaction [13].

$gh = 1$ field basis	level $(2, 6)$	level $(4, 12)$	level $(6, 18)$	level $(8, 24)$
$\|\Omega\rangle$	0.3976548947184288	0.4007200390749924	0.4003790755638671	0.39973608190423154
$b_{-1}c_{-1}\|\Omega\rangle$	−0.1389738152295008	−0.1502869559917484	−0.15477497270540513	−0.15712091953765914
$L^m_{-2}\|\Omega\rangle$	0.0408931493261807	0.04159452148973691	0.04175525359702033	0.041806849347695574
$b_{-1}c_{-3}\|\Omega\rangle$		0.041073385934010505	0.041936906548529496	0.042358626301118626
$b_{-2}c_{-2}\|\Omega\rangle$		0.02419174563180113	0.02489022878379843	0.025301843897808124
$b_{-3}c_{-1}\|\Omega\rangle$		0.013691128644670262	0.013978968849509828	0.014119542100372846
$L^m_{-4}\|\Omega\rangle$		−0.003741923212578628	−0.0037331617302832193	−0.0037279001402683682
$b_{-1}c_{-1}L^m_{-2}\|\Omega\rangle$		0.005013189182427192	0.005410660944694899	0.005620705137023851
$L^m_{-2}L^m_{-2}\|\Omega\rangle$		−0.00043064009114185083	−0.0004545462255696699	−0.0004654022166127481
$b_{-1}c_{-5}\|\Omega\rangle$			−0.02193107815206234	−0.022161386573208323
$b_{-2}c_{-4}\|\Omega\rangle$			−0.013702048066242712	−0.01385275004340868
$b_{-3}c_{-3}\|\Omega\rangle$			−0.00834273227278023	−0.008359650003474304
$b_{-4}c_{-2}\|\Omega\rangle$			−0.0068510240331213544	−0.0069263750217043295
$b_{-5}c_{-1}\|\Omega\rangle$			−0.004386215630412471	−0.0044322773146416965
$b_{-2}b_{-1}c_{-2}c_{-1}\|\Omega\rangle$			−0.005651485281802872	−0.00580655453652034
$L^m_{-6}\|\Omega\rangle$			0.0010658398347450269	0.0010617366766707361
$b_{-1}c_{-1}L^m_{-4}\|\Omega\rangle$			−0.0008498595740547494	−0.0008732233330861659
$b_{-1}c_{-2}L^m_{-3}\|\Omega\rangle$			−0.000046769138331183204	−0.000052284121618944255
$b_{-2}c_{-1}L^m_{-3}\|\Omega\rangle$			−0.000023384569165591568	−0.000026142060809472097
$L^m_{-3}L^m_{-3}\|\Omega\rangle$			$4.479437511126653 \times 10^{-6}$	$5.080488681869039 \times 10^{-6}$
$b_{-1}c_{-3}L^m_{-2}\|\Omega\rangle$			−0.002457790374962076	−0.002528657337188949
$b_{-2}c_{-2}L^m_{-2}\|\Omega\rangle$			−0.0020680241879350277	−0.002125416342663475
$b_{-3}c_{-1}L^m_{-2}\|\Omega\rangle$			−0.0008192634583206926	−0.0008428857790629816
$L^m_{-4}L^m_{-2}\|\Omega\rangle$			0.00022330350231085353	0.00022500193649010967
$b_{-1}c_{-1}L^m_{-2}L^m_{-2}\|\Omega\rangle$			−0.00011131535311028013	−0.00012817322136544294
$L^m_{-2}L^m_{-2}L^m_{-2}\|\Omega\rangle$			$-7.241008154399294 \times 10^{-6}$	$-6.240064701718801 \times 10^{-6}$

continued...

$gh = 1$ field basis	level $(2, 6)$	level $(4, 12)$	level $(6, 18)$	level $(8, 24)$
$b_{-1}c_{-7}\,|\Omega\rangle$				0.014312021693536028
$b_{-2}c_{-6}\,|\Omega\rangle$				0.009158200585940239
$b_{-3}c_{-5}\,|\Omega\rangle$				0.005674268936470511
$b_{-4}c_{-4}\,|\Omega\rangle$				0.004838957768226669
$b_{-5}c_{-3}\,|\Omega\rangle$				0.0034045613618823045
$b_{-6}c_{-2}\,|\Omega\rangle$				0.0030527335286467446
$b_{-2}b_{-1}c_{-3}c_{-2}\,|\Omega\rangle$				-0.0035422558218537676
$b_{-7}c_{-1}\,|\Omega\rangle$				0.0020445745276480116
$b_{-2}b_{-1}c_{-4}c_{-1}\,|\Omega\rangle$				0.0037527555019998804
$b_{-3}b_{-1}c_{-3}c_{-1}\,|\Omega\rangle$				0.0004302428004449616
$b_{-3}b_{-2}c_{-2}c_{-1}\,|\Omega\rangle$				-0.0011807519406179202
$b_{-4}b_{-1}c_{-2}c_{-1}\,|\Omega\rangle$				0.0018763777509999383
$L^m_{-8}\,|\Omega\rangle$				-0.00041801038699211334
$b_{-1}c_{-1}L^m_{-6}\,|\Omega\rangle$				0.00029329813765991303
$b_{-1}c_{-2}L^m_{-5}\,|\Omega\rangle$				$6.281489731737461 \times 10^{-6}$
$b_{-2}c_{-1}L^m_{-5}\,|\Omega\rangle$				$3.140744865868727 \times 10^{-6}$
$b_{-1}c_{-3}L^m_{-4}\,|\Omega\rangle$				0.000500528172313894
$b_{-2}c_{-2}L^m_{-4}\,|\Omega\rangle$				0.00030379159554779373
$b_{-3}c_{-1}L^m_{-4}\,|\Omega\rangle$				0.00016684272410463048
$L^m_{-4}L^m_{-4}\,|\Omega\rangle$				-0.000021999720024591806
$b_{-1}c_{-4}L^m_{-3}\,|\Omega\rangle$				0.00003496149452657495
$b_{-2}c_{-3}L^m_{-3}\,|\Omega\rangle$				$-3.2753561169368668 \times 10^{-6}$
$b_{-3}c_{-2}L^m_{-3}\,|\Omega\rangle$				$-2.1835707446245427 \times 10^{-6}$
$b_{-4}c_{-1}L^m_{-3}\,|\Omega\rangle$				$8.74037363164371 \times 10^{-6}$
$L^m_{-5}L^m_{-3}\,|\Omega\rangle$				$-1.3196771313891132 \times 10^{-6}$
$b_{-1}c_{-1}L^m_{-3}L^m_{-3}\,|\Omega\rangle$				$1.2594432286572633 \times 10^{-6}$
$b_{-1}c_{-5}L^m_{-2}\,|\Omega\rangle$				0.001534533432927412
$b_{-2}c_{-4}L^m_{-2}\,|\Omega\rangle$				0.0013556709245221895
$b_{-3}c_{-3}L^m_{-2}\,|\Omega\rangle$				0.0006166063072874846
$b_{-4}c_{-2}L^m_{-2}\,|\Omega\rangle$				0.0006778354622610939
$b_{-5}c_{-1}L^m_{-2}\,|\Omega\rangle$				0.00030690668658548353
$b_{-2}b_{-1}c_{-2}c_{-1}L^m_{-2}\,|\Omega\rangle$				0.0005782814358972997
$L^m_{-6}L^m_{-2}\,|\Omega\rangle$				-0.00007624602726052426
$b_{-1}c_{-1}L^m_{-4}L^m_{-2}\,|\Omega\rangle$				0.00006375616369006518
$b_{-1}c_{-2}L^m_{-3}L^m_{-2}\,|\Omega\rangle$				$5.9626436110722614 \times 10^{-6}$
$b_{-2}c_{-1}L^m_{-3}L^m_{-2}\,|\Omega\rangle$				$2.9813218055361256 \times 10^{-6}$
$L^m_{-3}L^m_{-3}L^m_{-2}\,|\Omega\rangle$				$-5.422796727699355 \times 10^{-7}$
$b_{-1}c_{-3}L^m_{-2}L^m_{-2}\,|\Omega\rangle$				0.00004728162691342103
$b_{-2}c_{-2}L^m_{-2}L^m_{-2}\,|\Omega\rangle$				0.00010011937816215435
$b_{-3}c_{-1}L^m_{-2}L^m_{-2}\,|\Omega\rangle$				0.000015760542304474034
$L^m_{-4}L^m_{-2}L^m_{-2}\,|\Omega\rangle$				$-4.371565449219928 \times 10^{-6}$
$b_{-1}c_{-1}L^m_{-2}L^m_{-2}L^m_{-2}\,|\Omega\rangle$				$-3.759766768481099 \times 10^{-7}$
$L^m_{-2}L^m_{-2}L^m_{-2}L^m_{-2}\,|\Omega\rangle$				$7.259081254041818 \times 10^{-7}$

(A.1)

B. Fitting of the Parameters of A

B.1 A up to Level 9 without Gauge Fixing

As A is of ghost number -1 and has only odd levels, we here tabulate such field basis at levels 3, 5, 7 and 9. The best-fit numbers are the coefficients of A obtained by best-fit via minimizing $\epsilon = \frac{|Q_{\Psi_0}A-\mathcal{I}|}{|\mathcal{I}|}$. The stable fit at level 9 is constructed so as to control the convergence behaviour of the coefficients.

Field Basis	level 3 fit	level 5 fit	level 7 fit	level 9 fit	stable level 9 fit
$b_{-2}\|0\rangle$	1.12237	1.01893	0.948316	1.25995	0.931864
$b_{-3}b_{-2}c_1\|0\rangle$		0.50921	0.37306	0.660674	0.401547
$b_{-4}\|0\rangle$		−0.518516	−0.753272	−0.25828	−0.753004
$b_{-2}L^m_{-2}\|0\rangle$		0.504193	0.50695	0.400769	0.496562
$b_{-4}b_{-3}c_1\|0\rangle$			0.698601	−0.10683	0.691255
$b_{-5}b_{-2}c_1\|0\rangle$			0.893251	−1.8453	0.888407
$b_{-6}\|0\rangle$			−0.531323	1.40819	−0.541737
$-b_{-3}b_{-2}c_{-1}\|0\rangle$			−1.87167	3.14822	−1.86475
$-b_{-4}b_{-2}c_0\|0\rangle$			−2.54254	3.2966	−2.54625
$b_{-2}L^m_{-4}\|0\rangle$			0.264611	−0.750856	0.255304
$b_{-3}L^m_{-3}\|0\rangle$			0.00193005	−0.0539165	−0.0191971
$b_{-3}b_{-2}c_1L^m_{-2}\|0\rangle$			0.358002	0.301463	0.338645
$b_{-4}L^m_{-2}\|0\rangle$			−0.724095	0.163428	−0.744985
$b_{-2}L^m_{-2}L^m_{-2}\|0\rangle$			0.166002	0.180328	0.169096
$b_{-5}b_{-4}c_1\|0\rangle$				0.0796036	0.273844
$b_{-6}b_{-3}c_1\|0\rangle$				−1.09893	−0.107261
$b_{-7}b_{-2}c_1\|0\rangle$				0.847731	0.195816
$b_{-8}\|0\rangle$				−0.313743	−0.277211
$b_{-3}c_{-3}b_{-2}\|0\rangle$				−19.0376	−4.11409
$-b_{-4}b_{-2}c_{-2}\|0\rangle$				−0.147445	−0.626872
$-b_{-4}b_{-3}c_{-1}\|0\rangle$				1.80597	−0.0745503
$-b_{-5}b_{-2}c_{-1}\|0\rangle$				−0.172462	−0.356920
$b_{-4}b_{-3}b_{-2}c_0c_1\|0\rangle$				1.05994	−0.102556
$-b_{-5}b_{-3}c_0\|0\rangle$				1.48397	−0.319450
$-b_{-6}b_{-2}c_0\|0\rangle$				−0.784562	0.0949989
$b_{-2}L^m_{-6}\|0\rangle$				0.103719	−0.00879977
$b_{-3}L^m_{-5}\|0\rangle$				−0.530976	−0.0537990
$b_{-3}b_{-2}c_1L^m_{-4}\|0\rangle$				0.428303	0.0633010
$b_{-4}L^m_{-4}\|0\rangle$				0.114766	0.111182
$b_{-4}b_{-2}c_1L^m_{-3}\|0\rangle$				0.687831	0.200100
$b_{-5}L^m_{-3}\|0\rangle$				−0.165379	−0.134011
$-b_{-3}b_{-2}c_0L^m_{-3}\|0\rangle$				−2.72288	−0.722198
$b_{-2}L^m_{-3}L^m_{-3}\|0\rangle$				0.3427	0.0910701
$b_{-4}b_{-3}c_1L^m_{-2}\|0\rangle$				−0.01845	0.304266
$b_{-5}b_{-2}c_1L^m_{-2}\|0\rangle$				−0.628564	−0.137309
$b_{-6}L^m_{-2}\|0\rangle$				0.39923	0.195490
$-b_{-3}b_{-2}c_{-1}L^m_{-2}\|0\rangle$				−0.537685	−0.289167
$-b_{-4}b_{-2}c_0L^m_{-2}\|0\rangle$				0.951973	−0.288878
$b_{-2}L^m_{-4}L^m_{-2}\|0\rangle$				−0.237783	−0.0856879
$b_{-3}L^m_{-3}L^m_{-2}\|0\rangle$				−0.332135	−0.0868470
$b_{-3}b_{-2}c_1L^m_{-2}L^m_{-2}\|0\rangle$				0.128844	0.126029
$b_{-4}L^m_{-2}L^m_{-2}\|0\rangle$				−0.00185911	−0.160345
$b_{-2}L^m_{-2}L^m_{-2}L^m_{-2}\|0\rangle$				0.0403381	0.0402361
$\epsilon = \|Q_{\Psi_0}A - \mathcal{I}\|/\|\mathcal{I}\|$	0.171484	0.117676	0.0453748	0.0243515	0.0356226

(B.1)

B.2 Fitting A in the Feynman-Siegel gauge

As A enjoys the gauge freedom $A \to A + Q_{\Psi_0}B$, we can fix it to be in the Feynman-Siegel gauge.

This is another way to control the convergence behaviour of the coefficients.

fields	level 3 fit	level 5 fit	level 7 fit	level 9 fit
$b_{-2}\,\lvert 0\rangle$	1.12237	1.01893	1.12465	1.05322
$b_{-3}b_{-2}c_1\,\lvert 0\rangle$		0.50921	0.467	0.500266
$b_{-4}\,\lvert 0\rangle$		-0.518516	-0.503772	-0.53228
$b_{-2}L^m_{-2}\,\lvert 0\rangle$		0.504193	0.476325	0.504269
$b_{-4}b_{-3}c_1\,\lvert 0\rangle$			0.333428	0.326986
$b_{-5}b_{-2}c_1\,\lvert 0\rangle$			-0.330557	-0.328381
$b_{-6}\,\lvert 0\rangle$			0.346811	0.331188
$-b_{-3}b_{-2}c_{-1}\,\lvert 0\rangle$			0.325862	0.327997
$-b_{-4}b_{-2}c_0\,\lvert 0\rangle$			0	0
$b_{-2}L^m_{-4}\,\lvert 0\rangle$			-0.166799	-0.164306
$b_{-3}L^m_{-3}\,\lvert 0\rangle$			0.00133026	0.000334022
$b_{-3}b_{-2}c_1L^m_{-2}\,\lvert 0\rangle$			0.341592	0.328637
$b_{-4}L^m_{-2}\,\lvert 0\rangle$			-0.332864	-0.327326
$b_{-2}L^m_{-2}L^m_{-2}\,\lvert 0\rangle$			0.1686	0.165931
$b_{-5}b_{-4}c_1\,\lvert 0\rangle$				0.245489
$b_{-6}b_{-3}c_1\,\lvert 0\rangle$				-0.253014
$b_{-7}b_{-2}c_1\,\lvert 0\rangle$				0.250149
$b_{-8}\,\lvert 0\rangle$				-0.257672
$b_{-3}c_{-3}b_{-2}\,\lvert 0\rangle$				0.249999
$-b_{-4}b_{-2}c_{-2}\,\lvert 0\rangle$				-0.256812
$-b_{-4}b_{-3}c_{-1}\,\lvert 0\rangle$				0.246526
$-b_{-5}b_{-2}c_{-1}\,\lvert 0\rangle$				-0.25213
$b_{-4}b_{-3}b_{-2}c_0c_1\,\lvert 0\rangle$				0
$-b_{-5}b_{-3}c_0\,\lvert 0\rangle$				0
$-b_{-6}b_{-2}c_0\,\lvert 0\rangle$				0
$b_{-2}L^m_{-6}\,\lvert 0\rangle$				0.00104113
$b_{-3}L^m_{-5}\,\lvert 0\rangle$				0.0000151443
$b_{-3}b_{-2}c_1L^m_{-4}\,\lvert 0\rangle$				-0.126025
$b_{-4}L^m_{-4}\,\lvert 0\rangle$				0.12448
$b_{-4}b_{-2}c_1L^m_{-3}\,\lvert 0\rangle$				-0.0004548
$b_{-5}L^m_{-3}\,\lvert 0\rangle$				-0.000819122
$-b_{-3}b_{-2}c_0L^m_{-3}\,\lvert 0\rangle$				0
$b_{-2}L^m_{-3}L^m_{-3}\,\lvert 0\rangle$				0.0000905036
$b_{-4}b_{-3}c_1L^m_{-2}\,\lvert 0\rangle$				0.250728
$b_{-5}b_{-2}c_1L^m_{-2}\,\lvert 0\rangle$				-0.251499
$b_{-6}L^m_{-2}\,\lvert 0\rangle$				0.250865
$-b_{-3}b_{-2}c_{-1}L^m_{-2}\,\lvert 0\rangle$				0.249179
$-b_{-4}b_{-2}c_0L^m_{-2}\,\lvert 0\rangle$				0
$b_{-2}L^m_{-4}L^m_{-2}\,\lvert 0\rangle$				-0.123363
$b_{-3}L^m_{-3}L^m_{-2}\,\lvert 0\rangle$				0.000457948
$b_{-3}b_{-2}c_1L^m_{-2}L^m_{-2}\,\lvert 0\rangle$				0.126358
$b_{-4}L^m_{-2}L^m_{-2}\,\lvert 0\rangle$				-0.125248
$b_{-2}L^m_{-2}L^m_{-2}L^m_{-2}\,\lvert 0\rangle$				0.0406385
$\epsilon = \lvert Q_{\Psi_0}A - \mathcal{I}\rvert / \lvert\mathcal{I}\rvert$	0.171484	0.117676	0.0480658	0.0320384

(B.2)

B.3 Expansion of $\mathcal{I}$ up to level 9

Immediately below the field basis at ghost number 0 and levels 1, 3, 5, 7 and 9 is given the coefficient of the expansion of $\mathcal{I}$.

$\lvert 0\rangle$	$b_{-3}c_1 \lvert 0\rangle$	$-b_{-2}c_0 \lvert 0\rangle$	$L^m_{-2} \lvert 0\rangle$	$b_{-5}c_1 \lvert 0\rangle$
1	-1	2	1	1
$-b_{-2}c_{-2} \lvert 0\rangle$	$-b_{-3}c_{-1} \lvert 0\rangle$	$b_{-3}b_{-2}c_0c_1 \lvert 0\rangle$	$-b_{-4}c_0 \lvert 0\rangle$	$L^m_{-4} \lvert 0\rangle$
1	-1	2	-2	$-\frac{1}{2}$
$b_{-2}c_1L^m_{-3} \lvert 0\rangle$	$b_{-3}c_1L^m_{-2} \lvert 0\rangle$	$-b_{-2}c_0L^m_{-2} \lvert 0\rangle$	$L^m_{-2}L^m_{-2} \lvert 0\rangle$	$b_{-7}c_1 \lvert 0\rangle$
0	-1	2	$\frac{1}{2}$	-1
$-b_{-2}c_{-4} \lvert 0\rangle$	$-b_{-3}c_{-3} \lvert 0\rangle$	$b_{-3}b_{-2}c_{-2}c_1 \lvert 0\rangle$	$-b_{-4}c_{-2} \lvert 0\rangle$	$b_{-4}b_{-2}c_{-1}c_1 \lvert 0\rangle$
0	-1	1	0	0
$-b_{-5}c_{-1} \lvert 0\rangle$	$b_{-4}b_{-3}c_0c_1 \lvert 0\rangle$	$b_{-5}b_{-2}c_0c_1 \lvert 0\rangle$	$-b_{-6}c_0 \lvert 0\rangle$	$b_{-3}b_{-2}c_{-1}c_0 \lvert 0\rangle$
1	2	-2	2	2
$L^m_{-6} \lvert 0\rangle$	$b_{-2}c_1L^m_{-5} \lvert 0\rangle$	$b_{-3}c_1L^m_{-4} \lvert 0\rangle$	$-b_{-2}c_0L^m_{-4} \lvert 0\rangle$	$b_{-4}c_1L^m_{-3} \lvert 0\rangle$
0	0	$1/2$	-1	0
$-b_{-2}c_{-1}L^m_{-3} \lvert 0\rangle$	$-b_{-3}c_0L^m_{-3} \lvert 0\rangle$	$L^m_{-3}L^m_{-3} \lvert 0\rangle$	$b_{-5}c_1L^m_{-2} \lvert 0\rangle$	$-b_{-2}c_{-2}L^m_{-2} \lvert 0\rangle$
0	0	0	1	1
$-b_{-3}c_{-1}L^m_{-2} \lvert 0\rangle$	$b_{-3}b_{-2}c_0c_1L^m_{-2} \lvert 0\rangle$	$-b_{-4}c_0L^m_{-2} \lvert 0\rangle$	$L^m_{-4}L^m_{-2} \lvert 0\rangle$	$b_{-2}c_1L^m_{-3}L^m_{-2} \lvert 0\rangle$
-1	2	-2	$-1/2$	0
$b_{-3}c_1L^m_{-2}L^m_{-2} \lvert 0\rangle$	$-b_{-2}c_0L^m_{-2}L^m_{-2} \lvert 0\rangle$	$L^m_{-2}L^m_{-2}L^m_{-2} \lvert 0\rangle$	$b_{-9}c_1 \lvert 0\rangle$	$-b_{-2}c_{-6} \lvert 0\rangle$
$-1/2$	1	$1/6$	1	0
$-b_{-3}c_{-5} \lvert 0\rangle$	$b_{-3}b_{-2}c_{-4}c_1 \lvert 0\rangle$	$-b_{-4}c_{-4} \lvert 0\rangle$	$b_{-4}b_{-2}c_{-3}c_1 \lvert 0\rangle$	$-b_{-5}c_{-3} \lvert 0\rangle$
0	0	1	0	0
$b_{-4}b_{-3}c_{-2}c_1 \lvert 0\rangle$	$b_{-5}b_{-2}c_{-2}c_1 \lvert 0\rangle$	$-b_{-6}c_{-2} \lvert 0\rangle$	$b_{-5}b_{-3}c_{-1}c_1 \lvert 0\rangle$	$b_{-6}b_{-2}c_{-1}c_0 \lvert 0\rangle$
0	-1	0	0	0
$-b_{-7}c_{-1} \lvert 0\rangle$	$b_{-3}b_{-2}c_{-2}c_{-1} \lvert 0\rangle$	$b_{-5}b_{-4}c_0c_1 \lvert 0\rangle$	$b_{-6}b_{-3}c_0c_1 \lvert 0\rangle$	$b_{-7}b_{-2}c_0c_1 \lvert 0\rangle$
-1	-1	2	-2	2
$-b_{-8}c_0 \lvert 0\rangle$	$b_{-3}b_{-2}c_{-3}c_0 \lvert 0\rangle$	$b_{-4}b_{-2}c_{-2}c_0 \lvert 0\rangle$	$b_{-4}b_{-3}c_{-1}c_0 \lvert 0\rangle$	$b_{-5}b_{-2}c_{-1}c_0 \lvert 0\rangle$
-2	2	-2	2	-2
$L^m_{-8} \lvert 0\rangle$	$b_{-2}c_1L^m_{-7} \lvert 0\rangle$	$b_{-3}c_1L^m_{-6} \lvert 0\rangle$	$-b_{-2}c_0L^m_{-6} \lvert 0\rangle$	$b_{-4}c_1L^m_{-5} \lvert 0\rangle$
$-1/4$	0	0	0	0
$-b_{-2}c_{-1}L^m_{-5} \lvert 0\rangle$	$-b_{-3}c_0L^m_{-5} \lvert 0\rangle$	$b_{-5}c_1L^m_{-4} \lvert 0\rangle$	$-b_{-2}c_{-2}L^m_{-4} \lvert 0\rangle$	$-b_{-3}c_{-1}L^m_{-4} \lvert 0\rangle$
0	0	$-1/2$	$-1/2$	$1/2$
$b_{-3}b_{-2}c_0c_1L^m_{-4} \lvert 0\rangle$	$-b_{-4}c_0L^m_{-4} \lvert 0\rangle$	$L^m_{-4}L^m_{-4} \lvert 0\rangle$	$b_{-6}c_1L^m_{-3} \lvert 0\rangle$	$-b_{-2}c_{-3}L^m_{-3} \lvert 0\rangle$
-1	1	$1/8$	0	0
$-b_{-3}c_{-2}L^m_{-3} \lvert 0\rangle$	$b_{-3}b_{-2}c_{-1}c_1L^m_{-3} \lvert 0\rangle$	$-b_{-4}c_{-1}L^m_{-3} \lvert 0\rangle$	$b_{-4}b_{-2}c_0c_1L^m_{-3} \lvert 0\rangle$	$-b_{-5}c_0L^m_{-3} \lvert 0\rangle$
0	0	0	0	0
$L^m_{-5}L^m_{-3} \lvert 0\rangle$	$b_{-2}c_1L^m_{-4}L^m_{-3} \lvert 0\rangle$	$b_{-3}c_1L^m_{-3}L^m_{-3} \lvert 0\rangle$	$-b_{-2}c_0(L^m_{-3})^2 \lvert 0\rangle$	$b_{-7}c_1L^m_{-2} \lvert 0\rangle$
0	0	0	0	-1
$-b_{-2}c_{-4}L^m_{-2} \lvert 0\rangle$	$-b_{-3}c_{-3}L^m_{-2} \lvert 0\rangle$	$b_{-3}b_{-2}c_{-2}c_1L^m_{-2} \lvert 0\rangle$	$-b_{-4}c_{-2}L^m_{-2} \lvert 0\rangle$	$b_{-4}b_{-2}c_{-1}c_1L^m_{-2} \lvert 0\rangle$
0	-1	1	0	0
$-b_{-5}c_{-1}L^m_{-2} \lvert 0\rangle$	$b_{-4}b_{-3}c_0c_1L^m_{-2} \lvert 0\rangle$	$b_{-5}b_{-2}c_0c_1L^m_{-2} \lvert 0\rangle$	$-b_{-6}c_0L^m_{-2} \lvert 0\rangle$	$b_{-3}b_{-2}c_{-1}c_0L^m_{-2} \lvert 0\rangle$
1	2	-2	2	2
$L^m_{-6}L^m_{-2} \lvert 0\rangle$	$b_{-2}c_1L^m_{-5}L^m_{-2} \lvert 0\rangle$	$b_{-3}c_1L^m_{-4}L^m_{-2} \lvert 0\rangle$	$-b_{-2}c_0L^m_{-4}L^m_{-2} \lvert 0\rangle$	$b_{-4}c_1L^m_{-3}L^m_{-2} \lvert 0\rangle$
0	0	$1/2$	-1	0
$-b_{-2}c_{-1}L^m_{-3}L^m_{-2} \lvert 0\rangle$	$-b_{-3}c_0L^m_{-3}L^m_{-2} \lvert 0\rangle$	$(L^m_{-3})^2L^m_{-2} \lvert 0\rangle$	$b_{-5}c_1(L^m_{-2})^2 \lvert 0\rangle$	$-b_{-2}c_{-2}(L^m_{-2})^2 \lvert 0\rangle$
0	0	0	$1/2$	$1/2$
$-b_{-3}c_{-1}(L^m_{-2})^2 \lvert 0\rangle$	$b_{-3}b_{-2}c_0c_1(L^m_{-2})^2 \lvert 0\rangle$	$-b_{-4}c_0(L^m_{-2})^2 \lvert 0\rangle$	$L^m_{-4}(L^m_{-2})^2 \lvert 0\rangle$	$b_{-2}c_1L^m_{-3}(L^m_{-2})^2 \lvert 0\rangle$
$-1/2$	1	-1	$-1/4$	0
$b_{-3}c_1(L^m_{-2})^3 \lvert 0\rangle$	$-b_{-2}c_0(L^m_{-2})^3 \lvert 0\rangle$	$(L^m_{-2})^4 \lvert 0\rangle$		
$-1/6$	$1/3$	$1/24$		

(B.3)

C. The Proof for the Simplified Expression for the Identity

In this section we wish to present the proof for the analytic expression for the identity as given in

(3.3). We remind the reader of the expression:

$$
\begin{aligned}
|\mathcal{I}\rangle &= \left(\prod_{n=2}^{\infty} \exp\left\{-\frac{2}{2^n} L_{-2^n}\right\}\right) e^{L_{-2}}\,|0\rangle \\
&= \ldots \exp(-\frac{2}{2^3} L_{-2^3}) \exp(-\frac{2}{2^2} L_{-2^2}) \exp(L_{-2})\,|0\rangle\,, \qquad \text{(C.1)}
\end{aligned}
$$

or its BPZ conjugate form[11]

$$\langle \mathcal{I}| = \langle 0|\, U_h U_{f_2} U_{f_3} U_{f_4} \ldots, \qquad \text{(C.2)}$$

where $U_{f_n} = e^{-\frac{2}{2^n} L_{2^n}}$ for $n \geq 2$ and $U_h = e^{L_2}$. In [57], the identity is given by $\langle \mathcal{I}| = \langle 0|\, U_{f_\mathcal{I}}$ where $U_{f_\mathcal{I}}$ is the operator corresponding to the function

$$f_\mathcal{I}(z) = \frac{z}{1 - z^2}.$$

Using the composition law $U_{g_1} U_{g_2} = U_{g_1 \circ g_2}$, what we need is to prove

$$U_h U_{f_2} U_{f_3} U_{f_4} \ldots = U_{h \circ f_2 \circ f_3 \circ \ldots} = U_{f_\mathcal{I}}$$

which is equivalent to proving

$$\lim_{k\to\infty} h \circ f_2 \circ \ldots \circ f_k(z) = f_\mathcal{I}(z) = \frac{z}{1 - z^2}. \qquad \text{(C.3)}$$

For the operator $U_f = e^{a L_n}$, the corresponding function f is given by [54]

$$f(z) = \exp\left\{a z^{n+1} \partial_z\right\} z = \frac{z}{(1 - a n z^n)^{1/n}}\,,$$

so we have

$$
\begin{aligned}
h(z) &= \tfrac{z}{(1-2z^2)^{1/2}} \\
f_n(z) &= \tfrac{z}{(1+2z^{2^n})^{1/2^n}}.
\end{aligned}
$$

A useful property of the f_n is that $f_n(z) = (g(z^{2^n}))^{1/2^n}$ where

$$g(z) := \frac{z}{1+2z} = \frac{1}{2 + 1/z}.$$

Before writing down the general form, first let us do an example:

$$f_2 \circ f_3 \circ f_4(z) = f_2 \circ f_3[(g[z^{2^4}])^{1/2^4}]$$

[11]Please notice that, besides the replacement $L_n \to (-)^n L_{-n}$, the orders under BPZ-conjugation are also reversed. This is because we use L_n instead of the oscillators α_m, whose orders do not get reversed under BPZ.

$$
\begin{aligned}
&= f_2[(g[((g[z^{2^4}])^{1/2^4})^{2^3}])^{1/2^3}] \\
&= f_2[(g[g^{1/2}[z^{2^4}]])^{1/2^3}] \\
&= (g[((g[g^{1/2}[z^{2^4}]])^{1/2^3})^{2^2}])^{1/2^2} \\
&= (g[g^{1/2}[g^{1/2}[z^{2^4}]]])^{1/2^2} \\
&= (g^{1/2}[g^{1/2}[g^{1/2}[z^{2^4}]]])^{1/2}.
\end{aligned}
$$

Now it is easy to see that the general form is

$$h \circ f_2 \circ f_3 \circ \ldots \circ f_{k+1}(z) = h \circ (\underbrace{g^{\frac{1}{2}} \circ \ldots \circ g^{\frac{1}{2}}}_{k}(z^{2^{k+1}}))^{\frac{1}{2}}.$$

Thus equation (C.3) is equivalent to showing that

$$\lim_{k\to\infty} \underbrace{g^{\frac{1}{2}} \circ \ldots \circ g^{\frac{1}{2}}}_{k}(z^{2^{k+1}}) = (h^{-1}(f(z)))^2 = \frac{z^2}{1+z^4}.$$

The left hand side can be written as

$$((\underbrace{2 + (2 + \ldots + (2}_{k} + 1/z^{2^{k+1}})^{\frac{1}{2}} \ldots)^{\frac{1}{2}})^{\frac{1}{2}})^{-1} = z^2((2z^{2^2} + (2z^{2^3} + \ldots (2z^{2^{k+1}} + 1)^{\frac{1}{2}} \ldots)^{\frac{1}{2}})^{\frac{1}{2}})^{-1}.$$

Thus (C.3) reduces to the verification of the equation

$$\lim_{k\to\infty} (2z^{2^2} + (2z^{2^3} + \ldots (2z^{2^{k+1}} + 1)^{\frac{1}{2}} \ldots)^{\frac{1}{2}})^{\frac{1}{2}} = 1 + z^{2^2}.$$

This can be done as follows. Consider first squaring both sides of the above equation and canceling $2z^{2^2}$ from the two sides, we get

$$\lim_{k\to\infty} (2z^{2^3} + \ldots (2z^{2^{k+1}} + 1)^{\frac{1}{2}} \ldots)^{\frac{1}{2}})^{\frac{1}{2}} = 1 + z^{2^3}.$$

Repeating the above operation k times, the left hand side gives 1 while the right hand side gives $1 + z^{2^{k+2}}$. Thus as long as $z < 1$, we get that the left and right hand sides do converge to each other as $k \to \infty$.

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