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# The Impact of Immigrant Dynasties on Wage Inequality

Michael Ben-Gad  
University of Haifa\*

June 2005

## Abstract

I construct a set of dynamic macroeconomic models to analyze the effect of unskilled immigration on wage inequality. The immigrants or their descendants do not remain unskilled—over time they may approach or exceed the general level of educational attainment. In the baseline model, the economy's capital supply is determined endogenously by the savings behavior of infinite-lived dynasties, and I also consider models in which the supply of capital is perfectly elastic, or exogenously determined. I derive a simple formula that determines the time discounted value of the skill premium enjoyed by college-educated workers following a change in the rate of immigration for unskilled workers, or a change in the degree or rate at which unskilled immigrants become skilled. I compare the calculations of the skill premiums to data from the U.S. Current Population Survey to determine the long-run effect of different immigrant groups on wage inequality in the United States.

*JEL Classification Numbers:* J61, O41.

*Key Words:* Immigration, Educational Attainment, Wage Inequality.

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\*Department of Economics, Mt. Carmel, Haifa 31905, Israel, [mbengad@econ.haifa.ac.il](mailto:mbengad@econ.haifa.ac.il). I wish to thank Hillel Rapoport and two anonymous referees for their comments and suggestions.

# 1 Introduction

In this paper I construct a set of macroeconomic models to analyze how increases in the number of unskilled immigrants may affect wage inequality over time. In these models, a change in the number of such immigrants does not necessarily alter the composition of the workforce permanently. Rather than remaining unskilled forever, a portion of the additional immigrants, or their descendants, join the ranks of skilled workers. Indeed, as is the case for some immigrant groups in the United States, the native-born children or grandchildren of immigrants with low levels of education may not merely assimilate by matching the general level of educational attainment, but exceed it.

For the baseline model, I adopt the Weil (1989) overlapping dynasties optimal growth framework. Changes in immigration policy not only affect wages directly by altering the size and composition of the labor force, but also alter the rate of return to capital, inducing changes in savings behavior that gradually affect the size of the capital stock. These changes to the size of the capital stock indirectly affect wages as well. I derive a simple reduced form that encapsulates all these different effects on one measure of wage inequality—the ratio between the discounted values of skilled and unskilled wages.

The effect of a change in immigration policy on wages is neither constant nor immediate. A change in immigration policy generates changes in the size and composition of the population that accumulate over time. The effect of these changes on factor returns may or may not be permanent, depending on whether the labor supplied by the immigrants or their descendants perfectly substitutes for the pre-existing labor supply. Therefore by examining the ratio between the discounted value of the two different wages, I can determine to what degree, in the long run, high educational attainment by the descendants of unskilled immigrants either ameliorates or reverses their short-run impact on wage inequality.

In my baseline model, capital is endogenously determined but adjusts slowly. Borjas (1999) analyzes the impact of immigration in static models under two alternative assumptions—capital supply is either completely elastic, or fixed. To better understand the sensitivity of my measure of wage inequality to different assumptions about the supply of capital, I compare the behavior of my model to one in which the stock of capital adjusts immediately to policy changes, but where the rate of return is exogenously determined. In addition, I also consider the case where the size of the capital stock grows at a fixed exogenous rate.

In Section 2, I briefly review recent U.S. immigration policy. I present data from the U.S. Census Current Population Survey that demonstrates the vast differences in educational attainment among different immigrant groups—differences that span at least two generations. These data also highlight the higher degree of intergenerational mobility between the immigrant generation and the second generation, when compared to the analogous native populations.

I present in Section 3 the dynamic optimal growth model with overlapping dynasties, developed by Weil (1989). Ben-Gad (2004) used the model to examine the behavior of an economy that is absorbing immigrant dynasties over time. In this paper, I distinguish between two types of households: households with skilled workers (college-educated adults), with unskilled workers (adults without college degrees).

Section 4 describes the dynamic system and the general perturbations method I use to simulate its behavior. I also derive the formula used to calculate the discounted skill premium (the percentage gap between the present value of wages for college and non-college educated workers) in the baseline model. I also present the explicit reduced form for the special case where the elasticities of substitution between the factors of production are all identical.

Section 5 briefly describes two alternative assumptions about the elasticity of the capital supply. I demonstrate that for the special case in which all the elasticities of substitution between the inputs are identical, the ratio between the two wages at any moment in time is identical, regardless of what mechanism governs the dynamic behavior of the capital stock. Nonetheless, even in this case, the discounted premium to education is sensitive to the model we choose.

In Section 6, I present my procedure for modelling the effect of immigration policy on the composition of the labor force over time. There is first, a direct effect as the ratio of skilled to unskilled workers among the extra new arrivals seldom matches the veteran population. There is a second effect because I permit the descendants of these immigrants to switch between the two categories during the periods after they arrive. One serious limitation to this approach is that membership in either category is determined exogenously—I am modeling the impact of observed changes in educational attainment but make no attempt to explain them.

Section 7 explains the choice of parameters I use to calibrate the model. In Section 8, I consider the impact of a twenty-year surge in the immigration of unskilled workers on wages and wage inequality, within the context of three different assumptions for the elasticity of capital supply, and three different specifications of the production function. I consider three different scenarios. First, what if the immigrants and their descendants remain permanently unskilled? No one ever attends college, and their arrival permanently lowers the share of unskilled workers within the economy. This scenario is a crude approximation of perhaps the most pessimistic outcome for immigration: the creation of a permanent unskilled under class. In the second scenario, the immigrants are initially unskilled, but over time, they or their descendants gradually attain the levels of college education prevalent in the general population. This process of immigrant ‘assimilation’ ultimately restores the distribution of college-educated and non-college educated people in the workforce to its initial level. In the third scenario all the immigrants or their descendants eventually attend college. This last case is perhaps the least likely, yet paradoxically the most informative. Finally, I compare the results to those obtained for an

identically sized influx of college-educated immigrants.

Finally, in Section 9, I consider more realistic examples, where not all immigrants are skilled or unskilled and neither are all their descendants. I relate the results for the discounted skill premium to the data on college attainment for the different immigrant groups in the Current Population Survey.

In this paper I do not presume to explain the decisions made by households to immigrate. Because legal migration from the developing world to the developed world is regulated by the rationing of visas, and illegal migration by the resources invested in interdiction, or the harshness of penalties imposed on those violating immigration laws, I believe it is possible to treat modest changes in rates of immigration for unskilled workers as exogenous policy decisions.<sup>1</sup> More importantly, this paper ignores the decisions made by immigrants or their descendants to acquire education. Instead, I focus on the long-run implications of differential educational attainment among immigrants and their descendants for wage inequality.

## **2 U.S. Immigration Policy and Educational Attainment Among Immigrant Groups**

### **2.1 The Rate of Net International Migration**

The share of foreign-born within the population of the United States declined steadily between 1910 to 1970, from 14.7% to 4.7%. Since then it has climbed swiftly, reaching 11.7% by the end of 2003. What has generated such a dramatic rise in just over three decades?

The official rate of immigration presented in Figure 1, Panel a), features the data tabulated by the U.S. Bureau of Citizenship and Immigration Services. These numbers show immigration rising steadily from a rate of 1.5 per thousand in 1960 to 2.6 per thousand in 1988, then rising much more steeply, reaching 7.1 per thousand in 1991, and then declining to 2.3 per thousand in 1999.

The rate at which people arrived in the United States did indeed rise between 1960 and 1991, but not by nearly so much, nor was the rate nearly so volatile. The official rate of immigration in Panel a) of Figure 1 does not show the date at which foreigners arrive in the United States or join its workforce, but merely captures the number who attain the official status of immigrant. Hence there was no massive influx of immigrants in 1991, but rather a large number of people, many living and working illegally in the United States for a decade or more, who took advantage of the amnesty provisions in the Immigration Reform and Control Act of 1986 (IRCA), to register as legal immigrants.

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<sup>1</sup>See Galor (1986), Djajic (1989), Borjas (1994), and Zak et. al. (2002) for models with endogenously determined levels of immigration.

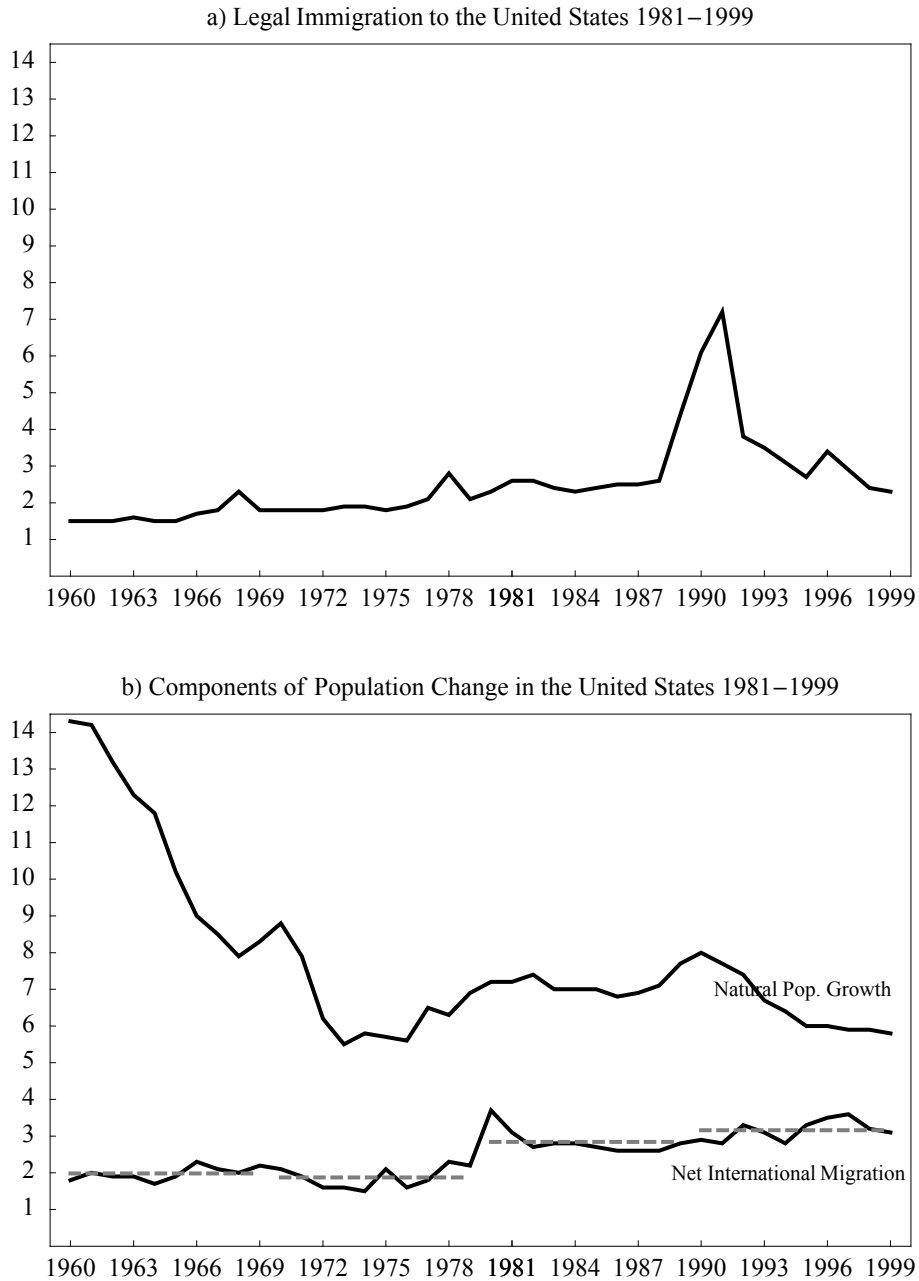


Figure 1: Annual rates per thousand, of legal immigration and components of population growth in the United States, 1960 to 1999. Natural population growth is number of births, less deaths. Net international migration (NIM) from 1960 to 1984 includes migration by U.S. civilians, but excludes military personnel, NIM from 1985 to 1994 excludes both U.S. civilians and military personnel, and 1995 to 1999 NIM excludes both U.S. civilians and military personnel. The dashed gray lines correspond to decade averages for NIM. Sources: *U.S. Census Bureau, Population Division and Housing and Household Economic Statistics Division and U.S. Census Bureau, Statistical Abstract of the United States*, various years.

Net international migration (NIM) in Figure 1 Panel b), measures the physical movement of people between the United States and the rest of the world. The rise in the NIM was far less dramatic than either the changes in the official rate of immigration or the steep decline in the rate of natural population growth in Panel b) of Figure 1.<sup>2</sup> Between 1960 and 1999 the birth rate in the United States dropped from 23.8 to 14.4 per thousand, causing the steep decline in the rate of natural population growth. The sharp decline in the birth rate combined with the gradual increase in net migration between 1970 and 2000 to generate the large increase in the share of the foreign born within the U.S. population over the same period.

## 2.2 U.S. Immigration Policy

Since passage of the Immigrant and Nationality Act of 1965, most legal immigrants have arrived in the United States through some form of family sponsorship. Immediate relatives of United States citizens may enter without limit; during the 1990's about a quarter of a million arrived each year. Other relatives of U.S. citizens are admitted as family-sponsored preference immigrants—the Immigration Act of 1990 set the limit for all family sponsored immigrants as either 226,000, or 480,000 minus the number of people admitted under the category of immediate relatives during the previous year, whichever is larger. The United States also allocates 140,000 employment-based preference visas for workers with special skills or training (as well as investors), and an additional 55,000 visas are allocated by lottery under the diversity program. Finally, the United States admits refugees and asylum seekers (refugees are admitted from abroad on the basis of a yearly quota set annually by the president). After a year, refugees and asylees are eligible for permanent residence—between 1991 and 2000, just over one million gained admission.

In addition to immigration visas, in 1992 the United States began granting 65,000 H-1B visas to temporary workers with special skills—nearly all recipients have college or advanced degrees.<sup>3</sup> To ameliorate a perceived shortage of qualified workers in the information technology sector, Congress passed the American Competitiveness in the Workforce Act of 1998, temporarily increasing the number of H-1B visas to 115,000 per year in 1999 and 2000, and 107,500 in 2001. The American Competitiveness in the Twenty-First Century Act of 2000 (AC21) added an extra 347,500 visas by raising the cap to 195,000 for each of the years 2001, 2002, and 2003, for a total of 585,000 H-1B visas over three years. The cap for 2004 and beyond is once again 65,000.

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<sup>2</sup>Data for the rate of net international migration are available for calendar years up until 1999, and for years 2001 and beyond. Data for 2000 are available for only part of the year. Therefore I compare the decades 1960-1969, 1970-1979, 1980-1989, 1990-1999.

<sup>3</sup>H-1B visas are granted for a maximum of two consecutive three-year stays. However, workers are no longer required to demonstrate an intention to return to their home countries and most recipients are soon eligible to apply for permanent residency. In the past at least half of those admitted under the program changed status and ultimately became permanent residents (see Lowell (2001)).

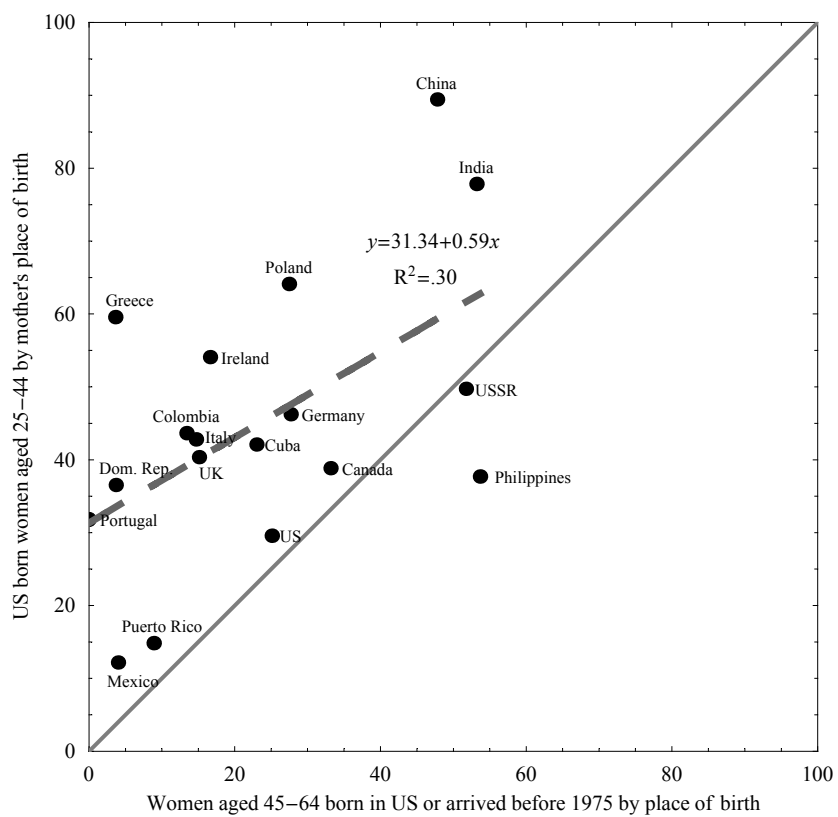
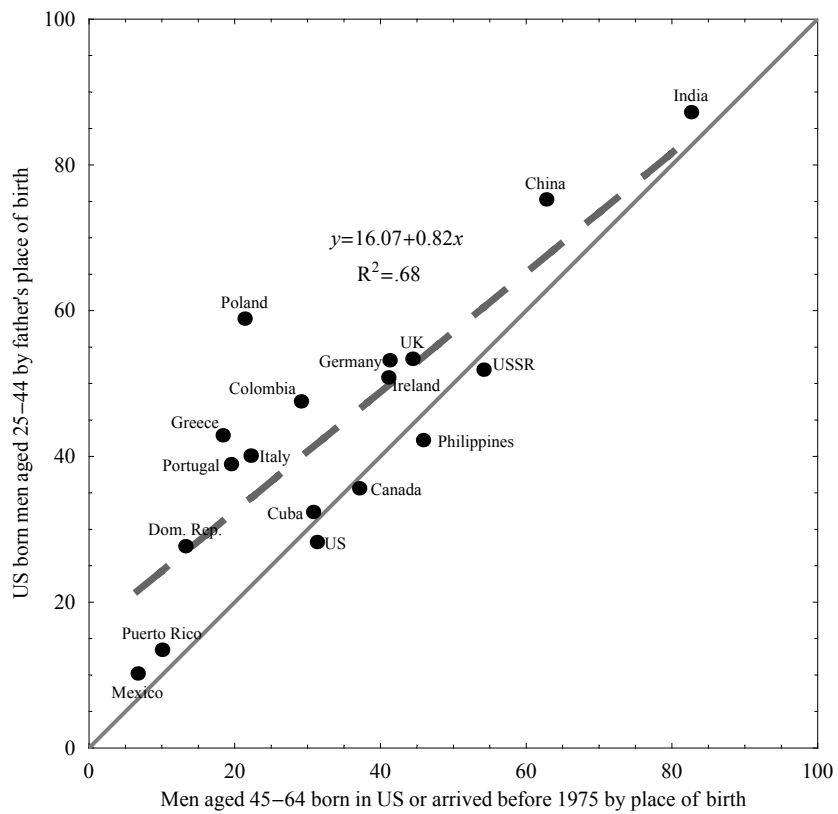


Figure 2: Percentage of the U.S. population with four-year college degrees by age, sex, birthplace, and parent's birthplace. Data for the USSR includes all respondents from any of the former republics in the sample, the data for the UK includes respondents from Northern Ireland, and data for Portugal includes respondents from the Azores. Pooled data for 2001, 2002, and 2003 from the U.S. Census, Current Population Survey. Source: Miriam King, Steven Ruggles, and Matthew Sobek. Integrated Public Use Microdata Series, Current Population Survey: Preliminary Version 0.1. Minneapolis: Minnesota Population Center, University of Minnesota, 2003.



Finally, the gross inflow of illegal immigrants is about 350,000 per year.<sup>4</sup> The net increment to the population from this source is smaller—eighty percent of those who leave the United States are foreign born, and a substantial fraction of these are illegal aliens returning home. In the year 2000 there were approximately seven million people living in the United States illegally, of whom 1.5 million arrived between 1991 and 2000—a net inflow of 150,000 per year.<sup>5</sup>

### 2.3 College Attainment and Immigrant Groups

Clearly, any government considering a serious change in its immigration policy should be concerned not only with the skills immigrants bring to their new country, but also with the levels of education attained by their children—the members of the second generation. In the two graphs in Figure 2, I pool data from the Current Population Survey of the U.S. Census for the years 2001, 2002, and 2003. The horizontal axes correspond to the share of people aged 45-64 with four-year Bachelor's degrees by place of birth. I sample only those immigrants who arrived prior to 1975. So as to focus on people who have immigrated to the United States near the beginning of their working lives, and at about the time they are establishing a household. By restricting the sample I exclude older people whose immigration was perhaps sponsored by their adult children—themselves immigrants to the United States.

Examining immigrants by country of origin reveals an enormous degree of heterogeneity in the shares of people with college degrees. By this measure, male immigrants from India are the best educated—nearly 83% have college degrees, followed by male immigrants from China with 63%. By contrast, the average share of college educated U.S. born males within the same age group is only 31%.

Among all the males in this sample, the least educated are Mexican immigrants, of whom just under 7% have completed college, followed by Puerto Ricans (I treat respondents who report Puerto Rico as their birthplace as immigrants rather than natives even though they are U.S. citizens by birth). Although formal education does not completely encompass all labor market skills or perfectly predict labor market outcomes, it is not hard to imagine the direct impact of these immigrants from Mexico or Puerto Rico on the wages of unskilled workers in the United States. Indeed, Borjas (2003) estimates the overall wage elasticity within skill groups to be -0.4 (there are four levels of educational attainment in his model, and he also controls for labor force experience).

By contrast the arrival of highly educated immigrants from India or China is likely to depress the wages of skilled workers. Abstracting from the overall level of wages, further immigration

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<sup>4</sup>U.S. Department of Homeland Security, Office of Immigration Statistics, *2002 Yearbook of Immigration Statistics*.

<sup>5</sup>U.S. Immigration and Naturalization Service, Office of Policy and Planning, *Estimates of the Unauthorized Immigration Population Residing in the United States: 1990 to 2000*.

from India or China is likely to lower the wage premium enjoyed by male college graduates, whereas immigration from Mexico and Puerto Rico is likely to enhance it. Furthermore, the impact of each particular group of immigrants on the composition of the labor force and wage inequality is likely to last long after the immigrants themselves retire.

To understand what this means, consider the vertical axes of the graphs in Figure 2. These measure the rate of college completion among U.S. born individuals aged 25-44, according to the birthplace of their fathers (for men) or mothers (for women). These are members of a second generation, counterparts to the immigrant generation whose rates of college education are measured on the horizontal axis.

Notice that in the upper panel of Figure 2, the points representing Mexico, Puerto Rico and India are above, but fairly close to the solid grey 45° line through the origin, and the point representing China is not too far from it either. If we treat the younger people in our sample, represented on the vertical axes as surrogates for the children of the immigrant cohort on the horizontal axes, we can conclude that the impact of each immigrant group on the share of college educated within the population is fairly constant over the course of at least two generations. This does not mean that their quantitative impact on wages is constant—the overall level of wages, and under some circumstances the ratio between wages for skilled and unskilled workers, is very dependent on the rate of adjustment of the capital stock to any surge in immigration. Nonetheless, qualitatively, we can confidently predict the impact on wage inequality generated by each of these four immigrant groups across at least two generations and probably more.<sup>6</sup>

This confidence quickly dissipates when we consider some of the other groups represented in Figure 2, or worse still, when we attempt to compare between them. Among male immigrants from Poland in our sample, only 21% have college degrees—well below the average for the population as a whole. Yet the levels of college graduation among young American-born sons of Polish-born fathers are immediately below the high levels attained by their Indian and Chinese counterparts—59% in the sample completed college. How do we compare the impact of male immigrants from places like Poland, with their high levels of intergenerational upward mobility, with the impact of better educated immigrants from Colombia, Cuba, Germany, Ireland, Italy or the United Kingdom? How much does it matter that the children of these more-educated immigrants seem to experience relatively little upward mobility, and graduate from college at lower rates than men whose fathers are from Poland? Indeed, what about immigrants from Canada, the Philippines or the former USSR? There we see a slight drop across the two generations in the share who report completing college.

Chiswick (1978) found that controlling for various factors, including age, and schooling, immigrants to the United States earn more than their native counterparts provided they have

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<sup>6</sup>Considering recent studies on intergenerational mobility (see the survey by Solon (1999)) or the model of ethnic capital estimated by Borjas (1992), these effects are likely to be felt in the third generation and beyond.

worked in the U.S. for a long enough period of time. If we interpret this finding as a measure of motivation, it would seem that some of this motivation spills over to the next generation or is expressed in a greater effort by immigrant parents to provide a college education for their children. Among males, the Polish immigrant group presents the most obvious example of this phenomenon. However, nearly all the points in Figure 2 cluster along a regression line ( $R^2 = .68$ ) above the  $45^\circ$  line.<sup>7</sup>

The intergenerational outcomes for the women among these same immigrant groups in the lower panel of Figure 2 are far less predictable. Consider female immigrants from Greece. In terms of college completion, they are as a group the second least educated people in our sample. Less than four percent of this group have college degrees, while among the American-born women between the ages of 45 and 64 in our sample, the rate of college completion is 25%. Indeed, only women born in Portugal have lower rates of college completion (.5%). Yet the point in the lower panel of Figure 2 corresponding to Greece is well above the  $45^\circ$  line. This is because just under 60% of second generation American born women aged 25 to 44 who report having Greek-born mothers completed college, twice the rate of daughters of American-born women in the same age group, and behind only women with mothers from China, India, and Poland. What does the arrival of immigrants like these women from Greece mean for wage inequality in the U.S. over time? Which dominates, the low levels of education in the first, or the high levels of education in the second generation and perhaps beyond?

### 3 Immigration in a model with endogenous capital accumulation

Suppose there are only two types of workers in the economy, either skilled or unskilled, and each supplies a distinct labor input. These workers are members of infinite-lived households that grow in size at a constant rate, and the number of these households is constantly augmented by immigration.

To model an economy with both natural population growth and immigration, we treat each resident as a member of an infinite-lived immigrant dynasty. In the absence of uncertainty, the behavior of each new immigrant of type  $i$ , and all of his or her descendants, can be characterized as the maximization of the dynasties' infinite horizon discounted utility function beginning at time  $s$ :

$$\max_{c_i} \int_s^\infty e^{(\rho-n)(s-t)} \ln c_i(s, t) dt, \quad i \in \{U, S\}, \quad (1)$$

subject to a time  $t$  budget constraint:

$$\dot{k}_i(s, t) = w_i(t)l_i + (r(t) - n)k_i(s, t) - c_i(s, t), \quad \forall s, t, \quad i \in \{U, S\}, \quad (2)$$

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<sup>7</sup>However we cannot reject the null hypothesis that its slope is equal to one.

where  $c_i(s, t)$ , and  $k_i(s, t)$  represent the time  $t$  consumption and holdings of capital of the members of a type  $i$  dynasty with arrival date  $s$ ;  $w_i(t)$  and  $r(t)$  represent their time  $t$  wages and the rate of return of capital;  $\rho$  is the subjective discount rate; and  $n$  is the rate of natural population increase—the rate of growth of the dynasties themselves.

The consumption rule for dynasty  $s$  at time  $t$  is:

$$c_i(s, t) = (\rho - n) [\omega_i(t) + k_i(s, t)], \quad \forall s, t, \quad i \in \{U, S\}, \quad (3)$$

where  $\omega_i(t) = \int_t^\infty e^{-\int_t^u r(v)dv} w_i(u) l_i du$  is the present discounted value of all future income from labor of type  $i$  from time  $t$  forward. Immigrant households of type  $i$  enter the economy at time  $t$  at a rate of  $m_i(t)$ , and we assume that all immigrants arrive in their new homeland. Aggregate consumption and capital evolve according to:

$$\dot{C}_i(t) = (\rho - n) [r(t) (\Omega_i(t) + K_i(t)) - C_i(t) + P_i(t) m_i(t) \omega_i(t)], \quad i \in \{U, S\}, \quad (4)$$

$$\dot{K}_i(t) = w_i(t) L_i(t) + r(t) K_i(t) - C_i(t) \quad (5)$$

where  $C_i(t)$ ,  $K_i(t)$ , and  $\Omega_i(t)$  are, respectively, the time  $t$  consumption, physical capital holdings, and the present value of future earnings aggregated over all the households with skill-level  $i$ ;  $M_i(s)$  is the number of households with skill-level  $i$ , that have accumulated by time  $s$ ; and  $P_i(s) = e^{n(t-b)} M_i(s)$  represents the overall size of each portion of the population.<sup>8</sup> The total labor input supplied by a household of type  $i \in \{U, S\}$  at time  $t$  is  $l_i$ , and the total supply of each type is  $L_i(t) = P_i(t) l_i$ ,  $i \in \{U, S\}$ .

The production function  $F : \mathbb{R}^3 \rightarrow \mathbb{R}$  has constant returns to scale in both types of labor and aggregate capital. Factors receive their marginal products:

$$r(t) = F_K(k_U(t) + \eta(t) k_S(t), l_U, \eta(t) l_S) - \delta, \quad (6)$$

$$w_i(t) = F_{H_i}(k_U(t) + \eta(t) k_S(t), l_U, \eta(t) l_S), \quad (7)$$

where  $\eta(t) = P_S(t) / P_U(t)$  is the ratio of households with skilled workers to unskilled workers in the economy at time  $t$ , and  $\delta$  is the rate of depreciation for physical capital.

The behavior of the economy is determined by four laws of motion for per-capita consumption  $c_i(t) = \frac{C_i(t)}{P_i(t)}$  and capital  $k_i(t) = \frac{K_i(t)}{P_i(t)}$ :

$$\dot{c}_i(t) = (r(t) - \rho) c_i(t) - (\rho - n) k_i(t) m_i(t) \kappa_i(t) \quad i \in \{U, S\}, \quad (8)$$

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<sup>8</sup>Define  $t = b$  as a date in the arbitrarily distant past  $b < 0$ , when the economy was founded by an initial cohort of size  $M_U(b) + M_S(b)$ . Then  $C_i(t)$ ,  $K_i(t)$ , and  $\Omega_i(t)$  are the consumption, capital and the future earnings for the initial type  $i$  population at time  $b$ , and all the additional cohorts accumulated at rate  $m_i(s)$  since  $b$ , all growing at the rate of  $n$ . Hence  $C_i(t) = e^{n(t-b)} \int_b^t M_i(s) m_i(s) c_i(s, t) ds + e^{n(t-b)} M_i(b) c_i(b, t)$ ,  $K_i(t) = e^{n(t-b)} \int_b^t M_i(s) m_i(s) k_i(s, t) ds + e^{n(t-b)} M_i(b) k_i(b, t)$ ,  $\Omega_i(t) = e^{n(t-b)} \left( \int_b^t M_i(s) m_i(s) ds + M_i(b) \right) \omega_i(t)$ , and  $M_i(s) = e^{\int_b^s m_i(v) dv}$ .

$$\dot{k}_i(t) = w_i(t)l_i + (r(t) - n - m_i(t)\kappa_i(t))k_i(t) - c_i(t) \quad i \in \{U, S\}, \quad (9)$$

where  $\kappa_i(t) = (k_i(t) - k_i(t, t)) / k_i(t)$  is the fractional difference between per-capita capital holdings and the capital immigrants bring with them.

In our simulations we analyze the model using a family of production functions whose most general expression is the nested constant elasticity of substitution (nested CES) aggregate production function with constant returns to scale developed by Sato (1967):

$$F(K(t), L_U(t), L_S(t)) = \left[ (1 - \alpha) L_U(t)^\vartheta + \alpha (\beta K(t)^\nu + (1 - \beta) L_S(t)^\nu)^{\frac{\vartheta}{\nu}} \right]^{\frac{1}{\vartheta}}, \quad (10)$$

where  $K(t) = K_U(t) + K_S(t)$  is the total stock of capital.<sup>9</sup>

## 4 The Dynamic System, and the Discounted Skill Premium

The sets of equations (8) and (9) for each skill-type are very similar, as the savings and consumption decisions of each type of household in the economy are not very different from each other. Finding a sufficiently precise approximation of the saddle path that corresponds to this dynamic system is very difficult because the condition number of the Jacobian matrix of the linearized system is very high. To overcome this problem, we define the variables  $a_U(t) = \ln \tilde{c}_U(t)$  and  $\chi(t) = \frac{w_S(t)}{w_U(t)}$ , which equals  $\frac{c_S(t) - (\rho - n)k_S(t)}{c_U(t) - (\rho - n)k_U(t)}$ , and replace the two laws of motion for consumption (8) with:

$$\dot{a}_U(t) = r(t) - \rho - (\rho - n) e^{-a_U(t)} k_U(t) m_U(t) \kappa_U(t) \quad (11)$$

$$\dot{\chi}(t) = (\rho - n) \frac{\chi(t) w_U(t) - w_S(t)}{e^{a_U(t)} - (\rho - n) k_U(t)} \quad (12)$$

$$\dot{k}_U(t) = w_U(t) + (r(t) - n - m_U(t) \kappa_U(t)) k_U(t) - e^{a_U(t)} \quad (13)$$

$$\dot{k}_S(t) = w_S(t) + (r(t) - \rho - m_S(t) \kappa_S(t)) k_S(t) + \left( (\rho - n) k_U(t) - e^{a_U(t)} \right) \chi(t) \quad (14)$$

Redefining the variables of the system has an additional benefit. The variable  $\chi(t)$  directly expresses the ratio between the discounted values of all the future skilled and unskilled wages.

In steady state, rates of immigration for skilled and unskilled must be equal—we employ perturbation methods (see Judd (1998)) to study the dynamic behavior of the model following temporary changes in the flow of skilled or unskilled immigrants.<sup>10</sup> Define  $m$  as the initial

<sup>9</sup>Also known as the two stage CES production function. The first stage combines skilled labor and raw capital to develop and maintain production capital:  $K^* = (\lambda K^\nu + (1 - \lambda) (H_S)^\nu)^{\frac{1}{\nu}}$ .  $K^*$  is used by unskilled labor in the second stage to manufacture final goods:  $Y = \left[ \mu (H_U)^\vartheta + (1 - \mu) (K^*)^\vartheta \right]^{\frac{1}{\vartheta}}$ . See Goldin and Katz (1998).

<sup>10</sup>The general theory of perturbations was first developed by Euler, Laplace, and most importantly Lagrange in the late 18th century to study celestial mechanics. The movement of a planet around the sun was ‘perturbed’ from its elliptical orbit by the gravitational pull of other planets which varied over time (Ekeland (1988)). Judd (1982), (1985) introduced perturbations to economics to study fiscal policy where the perturbations are changes in tax rates. Here changes in immigration policy perturbs the economy from its balanced growth path.

steady state rate of immigration, and replace  $m_i(t)$  in (11)-(14) with  $m + \epsilon\pi_i(t)$ , where  $\pi_i(t)$  is a bounded dynamic perturbation to the rate of migration by type- $i$  workers, and  $\epsilon$  is a small positive number that regulates its magnitude. Similarly we define  $\eta$  as the steady-state ratio of skilled to unskilled workers and replace the the terms  $\sigma(t)$  with  $\eta + \epsilon\xi(t)$ , where  $\xi(t)$  is a bounded dynamic perturbation to the skill ratio.

Defining  $\pi(t) = \{\pi_S(t), \pi_U(t)\}_{t=0}^{\infty}$ , consumption and capital for each skill-type are all functions of  $\pi$ ,  $\xi$  and  $\epsilon$ .<sup>11</sup> We differentiate (8), (9) with respect to  $\epsilon$  at the point  $\epsilon = 0$ :

$$\begin{bmatrix} \frac{\partial}{\partial \epsilon} \dot{a}_U(t, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} \dot{\chi}(t, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} \dot{k}_U(t, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} \dot{k}_S(t, \epsilon, \pi, \xi) \end{bmatrix} = \mathbf{J} \begin{bmatrix} \frac{\partial}{\partial \epsilon} a_U(t, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} \chi(t, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} k_U(t, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} k_S(t, \epsilon, \pi, \xi) \end{bmatrix} - \begin{bmatrix} (\rho - n) e^{-a_U} k_U \kappa_U \pi_U(t) \\ 0 \\ k_U \kappa_U \pi_U(t) \\ k_S \kappa_S \pi_S(t) \end{bmatrix} - \begin{bmatrix} \Omega_K \\ \frac{(\rho - n)(\chi l_U \Omega_U - l_S \Omega_S)}{e^{a_U} - (\rho - n)k_U} \\ k_U \Omega_K + l_U \Omega_U \\ k_S \Omega_K + l_S \Omega_S \end{bmatrix} \xi(t), \quad (15)$$

where  $\mathbf{J}$  is a  $4 \times 4$  Jacobian matrix;  $a_U$ ,  $\chi$ ,  $k_U$ , and  $k_S$  are the initial steady state values of log consumption, the ratio between the present values of skilled and unskilled wages, and capital; and  $\Omega_K = l_S \frac{\partial^2 F}{\partial L_S \partial K} + k_S \frac{\partial^2 F}{\partial K \partial K}$ ,  $\Omega_U = l_S \frac{\partial^2 F}{\partial L_S \partial L_U} + k_S \frac{\partial^2 F}{\partial K \partial L_U}$ ,  $\Omega_S = l_S \frac{\partial^2 F}{\partial L_S \partial L_S} + k_S \frac{\partial^2 F}{\partial K \partial L_S}$ .

To better understand how immigration affects the behavior of the model, I divide the shocks in (15) between two separate vectors that operate autonomously. The first contains the terms  $\kappa_U$  and  $\kappa_S$ , and if positive [negative] reflects the effects of capital dilution [enhancement] generated by the arrival of capital-poor [rich] immigrants, as described by Borjas (1995) in his static model and Ben-Gad (2004) in a dynamic setting. The terms  $\Omega_K$ ,  $\Omega_U$ , and  $\Omega_S$  in the second shock vector, capture those changes in the returns to capital, unskilled wages, and skilled wages respectively, that result from the change in the composition of the labor force, i.e.  $\xi(t)$ .

The portion of a policy change that operates through the channel represented by the first vector are completely transitory if the shocks have bounded support. Even permanent changes in the rate of immigration produce few changes in factor returns, or in the welfare of the native population. By contrast, the second vector can generate permanent changes in the economy even if the policy changes it represents are transitory. Small differences in the rate of immigration between the two types of immigrants accumulate over time, and permanently affect the composition of the labor force. Only if the value of all the perturbations operating through the second vector is zero in the limit—as will be the case if immigrant dynasties gradually assimilate until they replicate the overall skill distribution—do all the variables in the economy return to their original steady-state values.

<sup>11</sup>To guarantee convergence to an interior balanced growth path we also impose the restriction on  $\pi_S(t)$  and  $\pi_U(t)$  that they must satisfy  $\left| \lim_{T \rightarrow \infty} \int_0^T (\pi_S(t) - \pi_U(t)) dt \right| < \infty$

We solve (15) using Laplace Transforms:

$$\begin{aligned} & \begin{bmatrix} \mathcal{L}_v \left[ \frac{\partial}{\partial \epsilon} a_U(t, \epsilon, \pi, \xi) \right] \\ \mathcal{L}_v \left[ \frac{\partial}{\partial \epsilon} \chi(t, \epsilon, \pi, \xi) \right] \\ \mathcal{L}_v \left[ \frac{\partial}{\partial \epsilon} k_U(t, \epsilon, \pi, \xi) \right] \\ \mathcal{L}_v \left[ \frac{\partial}{\partial \epsilon} k_S(t, \epsilon, \pi, \xi) \right] \end{bmatrix} \\ &= (v\mathbf{I} - \mathbf{J})^{-1} \left( \begin{bmatrix} \frac{\partial}{\partial \epsilon} a_U(0, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} \chi(0, \epsilon, \pi, \xi) \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} (\rho - n) e^{-a_U} k_U \kappa_U \mathcal{L}_v [\pi_U] \\ 0 \\ \kappa_U k_U \mathcal{L}_v [\pi_U] \\ \kappa_S k_S \mathcal{L}_v [\pi_S] \end{bmatrix} - \begin{bmatrix} \Omega_K \\ \frac{(\rho - n)(\chi l_U \Omega_U - l_S \Omega_S)}{e^{a_U} - (\rho - n)k_U} \\ k_U \Omega_K + l_U \Omega_U \\ k_S \Omega_K + l_S \Omega_S \end{bmatrix} \mathcal{L}_v [\xi] \right) \end{aligned} \quad (16)$$

where  $\frac{\partial}{\partial \epsilon} a_U(0, \epsilon, \pi, \xi)$  and  $\frac{\partial}{\partial \epsilon} \chi(0, \epsilon, \pi, \xi)$  are the initial changes in the values of the two control variables<sup>12</sup>. The matrix  $\mathbf{J}$  has four eigenvalues, two negative and two positive. Define the two positive eigenvalues as  $\mu_1$  and  $\mu_2$ . Each element of the left-hand vector must be bounded for any positive value  $v$ , including the eigenvalues  $\mu_1$  and  $\mu_2$ , and yet the determinants  $|\mu_h \mathbf{I} - \mathbf{J}|$ ,  $h \in \{1, 2\}$  are zero by definition. The only way for (16) to be bounded when  $v = \mu_h$ ,  $h \in \{1, 2\}$  is for the numerator, the adjoint of  $\mu_h \mathbf{I} - \mathbf{J}$ ,  $h \in \{1, 2\}$  multiplied by the term in parentheses, to be equal to zero:

$$\begin{aligned} \text{adj} [\mu_h \mathbf{I} - \mathbf{J}] & \left( \begin{bmatrix} \frac{\partial}{\partial \epsilon} a_U(0, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} \chi(0, \epsilon, \pi, \xi) \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} (\rho - n) e^{-a_U} k_U \kappa_U \mathcal{L}_{\mu_h} [\pi_U] \\ 0 \\ \kappa_U k_U \mathcal{L}_{\mu_h} [\pi_U] \\ \kappa_S k_S \mathcal{L}_{\mu_h} [\pi_S] \end{bmatrix} - \begin{bmatrix} \Omega_K \\ \frac{\rho - n(\chi l_U \Omega_U - l_S \Omega_S)}{e^{a_U} - (\rho - n)k_U} \\ k_U \Omega_K + l_U \Omega_U \\ k_S \Omega_K + l_S \Omega_S \end{bmatrix} \mathcal{L}_{\mu_h} [\xi] \right) \\ &= 0, h \in \{1, 2\} \end{aligned} \quad (17)$$

both of which can then be solved for the values of  $\frac{\partial}{\partial \epsilon} a_U(0, \epsilon, \pi, \xi)$  and  $\frac{\partial}{\partial \epsilon} \chi(0, \epsilon, \pi, \xi)$ .

Whereas the evolution of  $\chi(t)$ , the ratio between the present discounted value of skilled and unskilled wages, is not of particular interest, (19)—the change in its value at time  $t = 0$ —gives us all the relevant information for determining how a change in immigration affects discounted wage inequality over time—without explicitly calculating the impulse responses for wages or the rate of return to capital following a policy announcement. The discounted skill (or college) premium  $S^P$  is the percentage difference between the present discounted value of skilled and unskilled wages:

$$S^P = \left( \chi + \epsilon \frac{\partial}{\partial \epsilon} \chi(0, \epsilon, \pi, \xi) - 1 \right) \times 100 \quad (18)$$

where  $\chi$  is once again the initial steady-state value of  $\chi(t)$ .

The first element in the second row of matrix  $\mathbf{J}$  is zero. The third and fourth are zero as well, if the elasticities of substitution between the various inputs in the production function are equal—if  $v = \vartheta$  in (10), only the diagonal element  $\mathbf{J}_{22}$  of the second row of matrix  $\mathbf{J}$  is non-zero, which is one of the two positive eigenvalues. Hence, the value of  $\frac{\partial}{\partial \epsilon} \chi(0, \epsilon, \pi, \xi)$  is the second

<sup>12</sup>The Laplace transform of a function  $f(t)$  and a positive number  $v$  is  $\mathcal{L}_v[f] = \int_0^\infty f(t) e^{-vt} dt$ .

element in the third vector in (17), where the eigenvalue is  $\mathbf{J}_{22} = \rho - n$ :

$$\frac{\partial}{\partial \epsilon} \chi(0, \epsilon, \pi, \xi) = -\frac{(\rho - n)(\chi l_U \Omega_U - l_S \Omega_S)}{e^{a_U} - (\rho - n)k_U} \int_0^\infty e^{-(\rho - n)t} \xi(t) dt \quad (19)$$

**Result 1** *If the elasticities of substitution between all the inputs are equal, the discounted skill premium is not a function of capital dilution  $\kappa_U$  and  $\kappa_S$ .*

Result 1 tells us that even though the rate of return to capital is affected by changes in immigration policy, for the special case of Cobb-Douglas or CES production, capital dilution itself has no effect on the discounted wage gap. A surge in immigration of type  $i$  certainly lowers the wages of all the workers of type  $i$  in the economy as long as the values of  $\kappa_U$  and  $\kappa_S$  are not negative. The wages of type  $j \neq i$  may either rise or fall depending on whether the relative scarcity of this type of labor has a stronger effect than the decline in per-capita capital. Either way, as long as the elasticity of substitution between the inputs is constant, we can separate the analysis of the discounted skill premium from the effects on the economy generated by the dilution of the capital stock.

If  $F$  is CRS and all the inputs are complementary in production,  $F_{ij} > 0$  for all  $i \neq j$  then  $\Omega_U > 0$  and  $\Omega_S < 0$ . Furthermore  $\frac{(\rho - n)}{e^{a_U} - (\rho - n)k_U} = 1/\omega_U(t)$ , the inverse value of the per-capita present value of unskilled wages.

**Result 2** *If the elasticities of substitution between all the inputs are equal, the production function is CRS and all the inputs are complementary in production, the college premium increases [decreases] if the present value of the shock  $\xi(t)$ , discounted by  $\rho - n$ , is negative [positive].*

From Result 2 we know how skilled and unskilled workers fare relative to each other, but we do not know what happens to the overall level of wages in the economy. To calculate the dynamic behavior of the wage levels we need to know the evolution of capital over time. We apply the inverse Laplace transforms to (16):

$$\begin{aligned} \begin{bmatrix} \frac{\partial}{\partial \epsilon} a_U(t, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} \chi(t, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} k_U(t, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} k_U(t, \epsilon, \pi, \xi) \end{bmatrix} &= e^{\mathbf{J}t} \begin{bmatrix} \frac{\partial}{\partial \epsilon} a_U(0, \epsilon, \pi, \xi) \\ \frac{\partial}{\partial \epsilon} \chi(0, \epsilon, \pi, \xi) \\ 0 \\ 0 \end{bmatrix} \quad (20) \\ - \int_0^t e^{\mathbf{J}(t-q)} \begin{bmatrix} (\rho - n) e^{-a_U} k_U \kappa_U \pi_U(q) \\ 0 \\ k_U \kappa_U \pi_U(q) \\ k_S \kappa_S \pi_S(q) \end{bmatrix} dq &- \int_0^t e^{\mathbf{J}(t-q)} \begin{bmatrix} \Omega_K \xi(q) \\ \frac{(\rho - n)(\chi l_U \Omega_U - l_S \Omega_S)}{e^{a_U} - (\rho - n)k_U} \xi(q) \\ (k_U \Omega_K + l_U \Omega_U) \xi(q) \\ (k_S \Omega_K + l_S \Omega_S) \xi(q) \end{bmatrix} dq. \end{aligned}$$

The time path of capital is approximated by  $k_i(t, \epsilon, \pi, \xi) \approx k_i + \epsilon \frac{\partial}{\partial \epsilon} k_i(t, \epsilon, \pi, \xi)$ ,  $i \in \{U, S\}$ , and to determine the time path of each wage in isolation, as well as the rate of return to capital, we insert  $k_i(t, \epsilon, \pi, \xi)$  together with the time path of  $\eta(t)$  into (6)-(7).



## 5 Inelastic and Perfectly Elastic Capital Supply

In the literature on immigration, capital supply is usually treated in one of two ways (see Borjas (1999)). One approach is to assume that changes in the supply of labor do not induce changes in the stock of capital—i.e., the capital stock is fixed. The other approach is to assume that capital flows freely between countries, and the capital stock adjusts immediately to accommodate the arrival of new immigrants. If capital is fixed, or in the case of this model, constrained to grow at the constant baseline rate of population growth  $m + n$ , the wage responses to changes in immigration will generally be very strong, and changes in the rate of return to capital will be large and permanent. By contrast, if capital adjustment is instantaneous, the rate of return to capital is fixed exogenously, and if labor is completely homogenous, immigration does not affect wages either. Of course, in models with heterogenous labor, a surge in immigration that upsets the composition of the labor force will alter wages.

The model developed in Sections 3 and 4 stakes out a middle position between these two extremes. Capital is neither fixed, nor does it adjust immediately. Instead it accumulates gradually through the savings decisions of the agents in the economy. In the long run, the rate of return to capital is fixed as in the open economy—a function of time preference. In the short run, the rate of return to capital does respond to changes in the flow of immigration, and these changes induce changes in savings behavior and the accumulation of capital.

To better understand the different implications of what we assume about the capital supply, set  $v = \vartheta$  in (10). The result is a constant elasticity of substitution production function, where  $\alpha$  and  $\beta$  control the relative shares of the three inputs, and the elasticity of substitution between each pair of inputs is  $\sigma_{US} = \sigma_{UK} = \sigma_{SK} = \frac{1}{1-\vartheta}$ . From (7), factors receive their marginal products and the wages under CES production reduce to:

$$w'_S(t) = \vartheta\alpha(1 - \beta)L_S(t)^{\vartheta-1}/Y(t) \quad (21)$$

$$w'_U(t) = \vartheta(1 - \alpha)L_U(t)^{\vartheta-1}/Y(t) \quad (22)$$

where  $Y(t)$  is total output. Dividing (21) by (22), the ratio of the two wages, the instantaneous college premium, is identical regardless of what we assume about the supply of capital.

**Result 3** *If the production function is CES, shocks  $\xi(t)$  and  $\pi(t)$  induce the same changes in the instantaneous college premium if capital is fixed, perfectly elastic, or endogenously supplied.*

Does this mean that the endogeneity of the capital supply is only relevant for determining the wage level or the discounted college premium  $S^P$  if the elasticities of substitution between the inputs are not equal? No. First, the total levels of output  $Y(t)$  in (21) and (22) will differ depending on what we assume about the nature of the capital supply. Hence, even if wage ratios are identical, wage levels are not. Second, the rate at which we discount the evolution of

each wage over time, the rate of return to capital less the natural rate of population growth, is sensitive to what we assume about the supply elasticity of capital.

What we assume about capital supply does not affect the ratio between the two wages, but does affect the ratio between their discounted values. Nonetheless, the ratio between the present values of each stream of wages is likely to be close to each other when the elasticity of substitution is constant, even if neither the levels of these wages nor the rate at which each is discounted is the same. Indeed the lower the elasticity of substitution, the smaller the gap between the rate of return to capital under free capital flows and endogenous capital supply, and the smaller the difference in the discounted college premium.

## 6 Modelling the Impact of Immigration and Educational Attainment Over Time

Most of the empirical work that measures the performance of immigrants and their families focuses on labor market outcomes—either the wages they command when they work, or their rates of employment. Work by Chiswick (1978), (1986), Borjas (1985), (1987), (1992), and Card, DiNardo, and Estes (2000) document the earnings of immigrants to the United States and work by Altonji and Card (1991), Borjas *et. al.* (1997) and Johannsson *et. al.* (2003) focus on employment. My focus here is on educational attainment as a proxy for labor market skills (see Jasso, Rosensweig, and Smith (2000) for a discussion). Wages in the model derive directly from that. In addition, I am interested in the educational attainment not only of the immigrants themselves, but of their descendants. Recent empirical work on assimilation, and more particularly on educational attainment in the second generation, includes Gang and Zimmerman (2000), Riphahn (2003) (both study Germany), and van Ours and Veenman (2003) (who study the Netherlands).

My focus here is on unskilled immigrants and their children. Immigrants with baccalaureate degrees remain educated to the end of their lives, while only a small number of uneducated people who arrive in North America, Australia or Western Europe as adults, subsequently complete college. From Figure 2 we see that most of the points are above the 45° line, suggesting that at least some of the children of unskilled immigrants do attend college. Indeed comparing the educational outcomes of the different immigrant groups to the most convenient reference point—the older generation of U.S. natives and the children of U.S. natives—there seems to be far more upward mobility among the immigrants. Bauer and Riphahn (2004) observe a similar pattern when comparing educational attainment among the native Swiss population with that of second generation immigrants to Switzerland, using micro-level data that includes direct measurement of parental education.

The immigration flows of each type and the relative size of the two types of the population,

are clearly related, and yet we distinguish throughout between the two. In an economy without assimilation, a surge of immigration, even if temporary, can permanently alter the distribution between the two skill-types. By treating changes in the rates of immigration and changes in the shares of the two skill types as distinct and separate shocks in (15), I distinguish between the immediate impact of an immigrant group's arrival, and its long-run affect on the economy as the group's members and their families either assimilate, or exceed the general population's rate of college completion.

To simplify the analysis, I will assume that changes in the rates of immigration begin at time zero, last  $T$  years and are constant over the entire period. The change in the overall rate of immigration is defined as  $\epsilon$ , and the increase in the overall population that results from the new policy is  $e^{\epsilon T} - 1$ . The share of skilled workers within the immigration surge is  $\Pi_S(T)$ , and the share of unskilled workers in the immigration surge is  $\Pi_U(T) = 1 - \Pi_S(T)$ . Beginning at time  $Q$ , some of these workers, or their descendants, shift between the two categories. By time  $V$  the share of skilled workers within this population stabilizes at  $\Pi_S(V)$ , and the share of unskilled workers is  $\Pi_U(V) = 1 - \Pi_S(V)$ . Define a set of dynamic perturbations:

$$\pi_i(x, j) = \frac{1}{\epsilon T} \ln \left[ 1 + \frac{(e^{\epsilon T} - 1) \Pi_i(j)}{P_i(0)} \right] \mathcal{U}(T - x), \quad i \in \{U, S\}, \quad j \in \{T, V\}. \quad (23)$$

where the unit step indicator function  $\mathcal{U} : \mathbb{R} \rightarrow \{0, 1\}$  returns the value of one for all numbers greater than, or equal to zero, and the value zero for all numbers less than zero. The perturbations directly affect the ratio of skilled to unskilled workers in the economy:

$$\lambda(t, j) = \frac{\eta(0)}{\epsilon} \left( e^{\epsilon \int_0^t (\pi_S(x, j) - \pi_U(x, j)) dx} - 1 \right), \quad j \in \{T, V\}. \quad (24)$$

The perturbation to the ratio of skilled to unskilled workers  $\eta(t)$  is the sum of two components:

$$\xi(t) = \lambda(t, T) + \left[ \frac{(t - Q) \mathcal{U}(t - Q) - (t - V) \mathcal{U}(t - V)}{Q - V} \right] (\lambda(V, V) - \lambda(T, T)) \quad (25)$$

We replace the perturbations  $\pi_i(t)$ ,  $i \in \{U, S\}$  in (15)–(20) with  $\pi_i(t, T)$ ,  $i \in \{U, S\}$ . These represent the actual perturbations to the rates of immigration, whereas the terms  $\pi_i(x, V)$   $i \in \{U, S\}$  correspond to a counterfactual policy under which the shares of skilled and unskilled households within the population of additional immigrants are  $\Pi_S(V)$  and  $\Pi_U(V)$ , in the short run and not merely in the long run. The first component in (25),  $\lambda(t, T)$ , expresses the direct cumulative effect of the actual changes in immigration flows, and  $\lambda(V, V)$  expresses the long-run effect of the additional immigration, after they have completed their shift from the initial rate of college attainment  $\Pi_i(T)$  to the final rate  $\Pi_i(V)$ ,  $i \in \{U, S\}$ .

The first component of  $\xi(t)$ ,  $\lambda(t, T)$ , expresses the initial, direct effect of the immigration surge on the composition of the work force from the moment the new immigration policy is announced till time  $T$ , when the immigration surge has concluded. If  $\lambda(T, T) > 0$ , then the share of skilled workers among the additional immigrants is higher than in the local population,

and the immigrants initially raise the value of  $\eta(t)$ . If  $\lambda(T, T) < 0$ , the share of skilled workers is lower, and the immigrants initially lower the value of  $\eta(t)$ . The second component in (25) expresses movements of the immigrants between the two skill categories and their effect on the overall labor force composition between the periods  $Q$  and  $V$ .

A few examples of possible time paths for  $\eta(t)$  will help illustrate the behavior of the model. The left-hand panels of Figure 3 illustrate the evolution of  $\eta(t)$  following a surge in unskilled immigration only, and the right-hand panels of Figure 3 illustrate the evolution of  $\eta(t)$  following a surge in skilled immigration. The black curves represent the behavior of  $\eta(t)$  if there is no assimilation, the dark grey curves represent the behavior of  $\eta(t)$  if all the immigrants assimilate, and the light gray curves correspond to instances where the shares of skilled and unskilled workers within the immigrant population reverse over time.

Suppose the skill composition of the workers in the immigration surge does not match the prevailing composition of the host country, but neither the immigrants nor their descendants switch between the two categories after they arrive. In this case  $\Pi_i(V) = \Pi_i(T)$ ,  $i \in \{U, S\}$ , which implies  $\lambda(V, V) = \lambda(T, T)$ . The second term in (25) is zero, the behavior of  $\xi(t)$  is determined by  $\lambda(t, T)$ , and  $\eta(t)$  either declines or increases until period  $T$ , and then remains fixed at its new steady state value. Each set of black curves in Figure 3 corresponds to the extreme sub-cases in which an immigration surge is uniformly composed of either unskilled (left-hand side of Figure 3) or skilled (right-hand side of Figure 3) workers. Note that in each column the black curves are identical for different values of  $Q$  and  $V$ , and serve as points of reference.

Suppose the surge of additional immigrants initially upsets the balance between skilled and unskilled workers in the labor force, but gradually, over the time period between  $Q$  and  $V$ , these immigrants assimilate until the shares of skilled and unskilled workers exactly mimics that of the general population. Under this scenario  $\Pi_i(V) = \Pi_i(T) = P_i(0)$ ,  $i \in \{U, S\}$  and therefore  $\lambda(V, V) = 0$ . Consider a surge in unskilled immigration. If the process of assimilation begins immediately and ends soon after the last of the additional immigrants arrive (Panel a) of Figure 3), the dark grey curve barely declines below its initial value. In Panel c) assimilation begins immediately, but the process lasts longer and the decline in  $\eta(t)$  is steeper.

Immigrants, or more likely their descendants, may not merely assimilate. As we see in Figure 2, few of the women among Greek immigrants to the United States have college degrees, but a disproportionate fraction of second-generation Greek-American women do. Similarly, the value of  $\eta(t)$  may first decline because of a surge of unskilled immigration, but rise above its initial value by time  $V$ . Similarly it is possible (though a good deal less likely) that a surge in immigration may initially raise, but ultimately lower the value of  $\eta(t)$ . If the value of  $Q$  is set above  $T$ , then the value of  $\eta(t)$  first behaves according to  $\lambda(t, T)$ , before beginning its ascent (the light gray curves in Panel g) of Figure 3) or descent (the light gray curves in Panel h) of

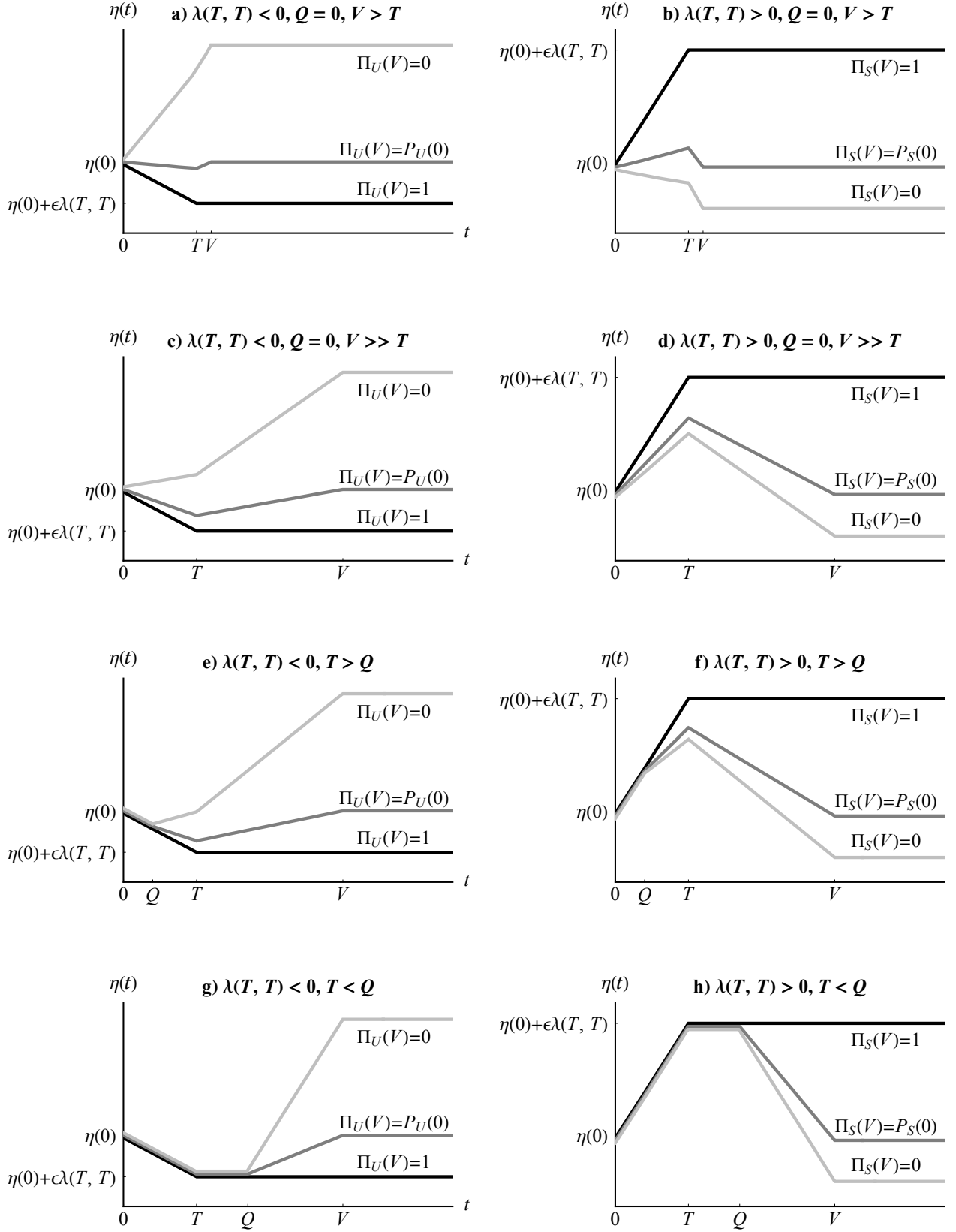


Figure 3: The time paths of  $\eta(t)$ , the ratio of skilled to unskilled workers, for different degrees of assimilation following different influxes of skilled and unskilled immigrants.

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PREFERENCES, TECHNOLOGY AND FACTOR SHARES:

$\rho = .0495$	Matches average rate of return on capital of 5%.
$\delta = .061$	Average rate of depreciation on capital: 1991-2000. <sup>†</sup>
$\alpha = .283$	Average U.S. capital's share of national income: 1991-2000. <sup>†</sup>
$\beta = .482$	Matches ratio of initial earnings. <sup>†‡§</sup>

POPULATION:

$n = .0067$	Average U.S. natural rate of population growth: 1991-2000. <sup>‡</sup>
$m_S = m_U = .0034$	Average U.S. rate of net migration: 1991-2000. <sup>‡</sup>
$\kappa_S = \kappa_U = 1$	Immigrants arrive without physical capital.
$P_S(0) = .256$	Population with college degrees. <sup>‡</sup>
$d = 2.7$	Ratio of initial earnings and wealth. for households with/without college degrees. <sup>‡§</sup>

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Table 1: Parameterization of Baseline Model. <sup>†</sup>Bureau of Economic Analysis. <sup>‡</sup>U.S. Census Bureau. <sup>§</sup>1998 Survey of Consumer Finances.

Figure 3). However if the value of  $V$  is no longer zero, but below  $T$ , then the reversal in the direction of  $\eta(t)$  begins at time  $Q$  (the light gray curves in Panel e) or Panel f) of Figure 3). If  $Q=0$ , then as in Panels a), b), and c) the direction of  $\eta(t)$  may be completely determined by the value of  $\Pi_i(V)$ .

## 7 Parameterizing the Model

Between 1990 and 1999 the net rate of migration to the United States was just under 3.2 per thousand. Although a much larger fraction of immigrants have less than nine years of schooling, the percentage of the foreign-born with baccalaureate degrees closely matches that of the general population—25.8% of foreign-born people in the United States over the age of 25 have college degrees, as compared to 25.6% of the total U.S. population. For the initial stock of skilled and unskilled workers we set  $P_S(0) = .256$  and  $P_U(0) = .744$ , and set the steady state rates of immigration for both skill types to  $m_S = m_U = .0032$ .<sup>13</sup> If the rates of legal and illegal immigration to the United States during the decade of the 1990's carry forward, and the rate of out migration continues to hold steady at one per thousand, foreign migration will augment the U.S. population with close to ten million additional people over the course of this decade.

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<sup>13</sup>At the high end, graduate education declines slightly with the degree of nativity: 9.7% of the foreign born have graduate degrees, as do 8.9% of natives with foreign-born parents, but only 8.2% of natives with native-born parents. Grade school education rises more steeply with nativity—22.2% of the foreign-born and 10.1% of the natives with foreign-born parents have less than nine grades of schooling (7.2% of the foreign-born have less than five), against only 4.5% with less than nine grades among the native-born population with native parents (See U.S. Department of Commerce, Bureau of Census, *Profile of the Foreign-Born Population in the United States: 2000*, December 2001).

There is no readily available data on the financial assets or physical capital that new immigrants to the United States bring. Given the large gap in income between sending countries and the United States, and the relative youth of most immigrants when they arrive, it is unlikely that capital holdings for the typical immigrant, skilled or unskilled, approaches U.S. per-capita capital holdings for either skill type. I set  $\kappa_S = \kappa_U = 1$ , which implies that after financing their move to the United States and setting up a household, immigrants have exhausted their savings. I also assume that both types of workers supply the same amounts of labor and set  $l_U = l_S = 1$ .

The ratios of mean earnings and income for households, as well as individuals, with bachelor's degrees to those without, range from 2.13 to 2.71, as measured by the U.S. Census. The 1998 Survey of Consumer Finances reports on net wealth as well as income and earnings. The ratio of mean earnings is 2.35, that of income is 2.3 while net wealth is 3.3. The gap between median earnings and wealth is smaller—2.4 versus 3.06. In steady state, the ratio of capital held by skilled and unskilled agents must be equal to the ratio of their wages. I choose an intermediate number 2.7, and combine this with the 1991 to 2000 average share of capital in national income, 28.3%, to set the values of the parameters in the production function for different elasticities of substitution.

Both the cross-country estimations of the nested CES production function (10) by Fallon and Layard (1975) and Duffy *et. al.* (2004) and the time-series estimations using U.S. data by Krussel *et. al.* (2000) and Swedish data by Lindquist (2003), find that the difference between the values of the parameters  $\vartheta$  and  $v$  in (10) is statistically significant—implying the existence of the capital skill complementarities first postulated by Griliches (1969).

I simulate the baseline model setting  $\vartheta = .401$  and  $v = -.495$  in (10) to match the estimates by Krussel *et. al.* (2000) (their distinction between skilled and unskilled workers based on college education matches my own). These parameter values correspond to elasticities of substitution between capital and unskilled labor  $\sigma_{UK}$ , and between skilled and unskilled labor  $\sigma_{US}$ , that are equal to  $\frac{1}{1-\vartheta} = 1.67$ , and an elasticity of substitution between capital and skilled labor  $\sigma_{SK}$  equal to  $\frac{1}{1-v} = .67$ .<sup>14</sup> When  $\vartheta$  is set equal to  $v$ , the production function (10) becomes the standard CES function with three inputs, and when both approach the value of one in the limit we have the Cobb-Douglas function.

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<sup>14</sup>The Allen Hicks partial elasticity of substitution between capital and unskilled labor, and between skilled and unskilled labor are also equal to .67, but the Allen Hicks partial elasticity of substitution between capital and skilled labor is  $\frac{1}{1-\vartheta} + \frac{1}{\phi_{SK}} \left( \frac{1}{1-v} - \frac{1}{1-\vartheta} \right) = .36$ , where  $\phi_{SK}$  is the combined share of skilled labor and capital.

## 8 A Twenty Year Surge in Immigration

### 8.1 Wage Levels

Comparing the period 1980-1989, and the period 1990-1999 in Figure 1, Panel b) the rate of net international migration rose by just over .3 per thousand. In the next two sections I consider the implications of an additional rise of a similar magnitude, that lasts for two decades. To begin with, let us suppose that all the additional immigrants are people without college degrees. The overall rate of immigration rises from 3.2 to 3.5, per thousand and the rate of immigration by the unskilled rises to just over 3.6 per thousand.

How many additional people does such a rise in immigration imply? If present trends continue, the United States will absorb about twenty million legal and illegal immigrants over the course of a two decades, and fifteen million of them will not have Baccalaureate degrees. The change considered here need not entail an increase in the number of legal immigrants alone. A slight curtailment in enforcement efforts along the border could easily cause the number of unskilled immigrants to rise by the additional seventy five thousand people per year (one million-and-three-quarters over the course of twenty years) that we are considering here.

Consider first the effect of the policy change on each type of wages when assimilation does not occur. Setting  $\Pi_S(T) = \Pi_S(V) = 0$ , and the elasticities of substitution in the production function (10)  $\sigma_{SK} = .67$ , and  $\sigma_{UK} = 1.67$ , the surge of unskilled immigration produces a permanent change in the skill composition of the work force, and generates the changes in the wages of unskilled workers shown in the upper left-hand corner of Figure 4. The dotted lines represent the impulse response in an economy with an inelastic supply of capital. Here, because of the permanent dilution of the capital stock, the long-run response of unskilled wages, a drop of just above .35% for the Nested CES production function, is twenty percent higher than the long-run decline in wages if capital is either completely elastic or endogenously determined. By contrast, the rise in skilled wages of over three-tenths of a percent if capital is elastic, in the upper left-hand corner of Figure 5, is well over twice the rise in skilled wages if capital is fixed.

The impulse responses in Figures 4 and 5 are all relatively modest because they were calculated for an economy in which the elasticity of substitution between the two types of labor is high. If the elasticity of substitution between all the inputs is lower, the effect of immigration on both skilled and unskilled wages is larger. Setting all the elasticities of substitution to two-thirds, the long-run drop in unskilled wages is close to nearly nine-tenths of a percent if capital is fixed, and nearly sixth-tenths of a percent if capital is elastic. The long-run rise in skilled wages is one-third of a percent if capital is fixed, and nearly twice that amount if capital is elastic (see Figures 9 and 10 in the Appendix).

Whether or not there is free movement of capital, or if capital is elastically supplied but only from internal savings, the long-run effect on wages is always the same. The difference in



the responses of wages is only a short-term phenomenon, and for the case of unskilled immigration, this difference is very small. In general, the response of wages if capital is endogenously determined falls between two extremes, i.e., between the case where the capital supply grows at an exogenous rate, and that where capital is perfectly elastic—but is much closer to the latter than the former. Of course it must be emphasized that in the overlapping dynasties model there is no representative consumer. Ensuring aggregability requires a logarithmic utility function, and hence a relatively high degree of intertemporal elasticity of substitution in consumption. If the elasticity of substitution were lower, the accumulation of capital would be slower, and the short-term response of wages in the case of endogenously supplied capital would not be quite so close to the responses generated by the model with perfectly elastic capital.

The upper right-hand graphs in Figures 4 and 5 illustrate the response of wages if  $\Pi_S(T) = 0$  but  $\Pi_S(V) = P_S(0)$  and  $P_S(0) = .256$ . In the twenty-fifth year ( $Q=25$ ), five years after the immigration surge has ended, the descendants of these additional unskilled immigrants enter the labor force. Of these new workers 25.6% are skilled, exactly mirroring the proportion of skilled workers within the larger population. By year forty-five ( $V=25$ ) this additional population has completely assimilated. The arrival of these additional workers dilutes capital and causes both skilled and unskilled wages to drop in the short run, but will not affect long-run wages unless capital is inelastic. If capital supply is inelastic, unskilled wages drop by between a third of a percent by year twenty (as in Figure 4), or just under nine-tenths of a percent (as in the Appendix), depending on the elasticity of substitution. The drop is caused by the combined effect of an increase in the relative share of unskilled workers in the labor force, and the overall rise in the size of the labor force itself. In the long run, as the descendants of immigrants assimilate, only the latter of these two effects remain, and the unskilled wage recovers approximately half its short-term loss.

Wages for skilled workers initially climb, as the additional immigrants upset the balance between skilled and unskilled workers. In each case, this change in the composition of the labor force initially dominates the effects of capital dilution. If capital supply is completely elastic or endogenous, the skilled wage also returns to its initial level, once the immigrants or their descendants completely assimilate.

If capital is in fixed supply, the skilled wage initially rises but ultimately declines. If  $\sigma_{SK} = .67$ , and  $\sigma_{UK} = 1.67$ , the wage is one-tenth of a percent higher by year twenty, but then begins to gradually decline, until it is half a percent below where it was before the new policy was initiated. If both elasticities of substitution are two-thirds, the wage initially rises by one third but is ultimately four-tenths of a percent lower, and if both elasticities are one the wage first rises by two-tenths of a percent and then by half a percent, until it is three-tenths of a percent below its initial level (see Figures 9 and 10 in the Appendix). The greater the complementarity between skill and capital, the more capital dilution mitigates the initial rise in skilled wages

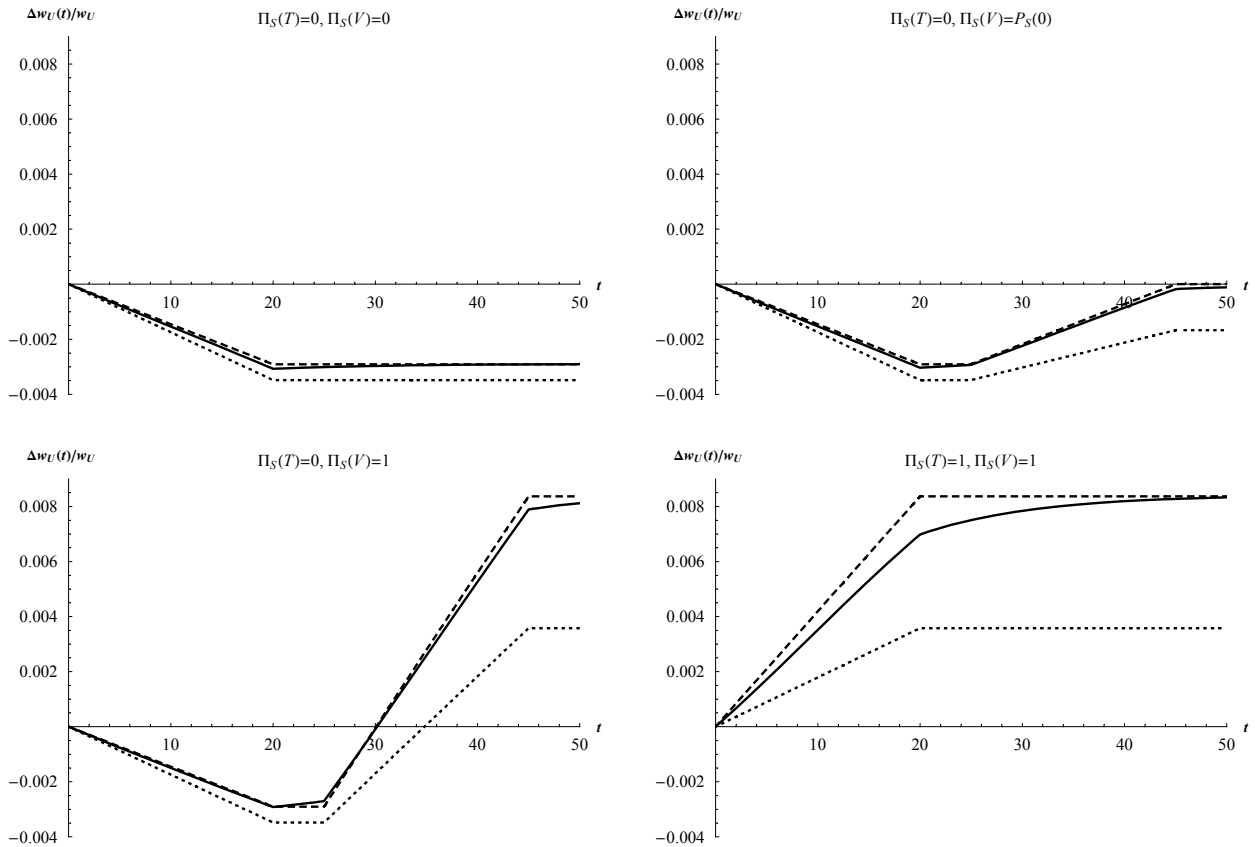


Figure 4: Nested CES production function  $\sigma_{SK} = .67$ , and  $\sigma_{US} = \sigma_{UK} = 1.67$ . Impulse response for unskilled wages following a twenty year surge in the rate of immigration from 3.2 to 3.5 per thousand,  $Q=25$  and  $V=45$ . The solid, dashed, and dotted curves represent respectively, the impulse responses generated by the model with capital supply that is elastic, completely elastic, and inelastic.

generated by the change in the composition of the labor force. In every case the permanent dilution of the capital stock guarantees a lower skilled wage in the long run.

The lower left-hand graphs in Figures 4 and 5 are perhaps the most interesting—they show the behavior of wages when  $\Pi_S(T) = 0$  and  $\Pi_S(V) = 1$ . A surge of immigrants, all unskilled, enter the country over the course of twenty years. Starting in year twenty-five they or their descendants begin a remarkable transformation. Rather than merely assimilating as in the previous experiment, this group crosses in its entirety from the skilled to the unskilled category during the course of twenty years. Wages for unskilled workers initially decline, and wages for skilled workers rise as these immigrants arrive and join the workforce.

Starting in year twenty, if capital is endogenous, or in year twenty-five if capital is either inelastic or completely elastic, unskilled wages begin to rise and skilled wages to decline. If the capital supply is completely elastic, both sets of wages pass through their original levels before continuing to rise or decline, at precisely the same time—year thirty. If the supply of capital is endogenous, the remaining effects of capital dilution will cause unskilled wages to begin to

Nested CES Production Function,  $\sigma_{UK}=\sigma_{US}=1.67$ ,  $\sigma_{SK}=.67$

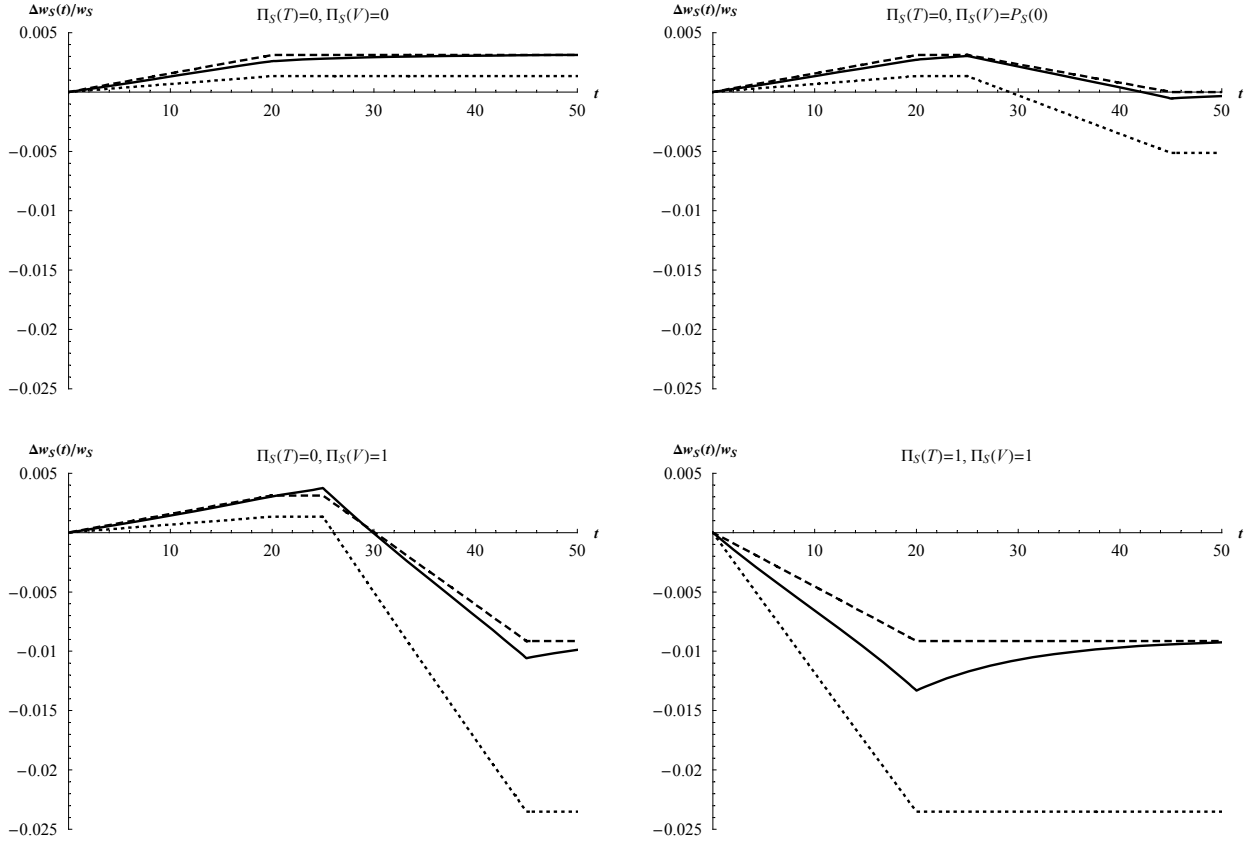


Figure 5: Nested CES production function  $\sigma_{SK} = .67$ , and  $\sigma_{US} = \sigma_{UK} = 1.67$ . Impulse response for skilled wages following a twenty year surge in the rate of immigration from 3.2 to 3.5 per thousand,  $Q=25$  and  $V=45$ . The solid, dashed, and dotted curves represent respectively, the impulse responses generated by the model with capital supply that is elastic, completely elastic, and inelastic.

decline below their original level during the second half of the twenty ninth year—approximately half a year before skilled wages have completely recovered. If capital supply is exogenous, the gap between the point when skilled wages reach their initial level and the moment when unskilled wages have completely recovered will be much larger—the former occurs between year twenty-six or twenty-seven (depending on the elasticities of substitution), and the latter at year thirty-five. This gap of a few years, when both wages are below their initial levels, directly results from capital dilution.

In the long run, wages in the lower left-hand sides of Figures 4 and 5 are identical to the long-run wages in the lower right-hand sides. The latter represent the behavior of wages when all the immigrants arrive as skilled workers. The long-run changes in wages are the same for a completely elastic and an endogenously determined capital supply—capital dilution affects the former not at all, and the latter only in the short-run. Indeed the long-run drops in skilled wages and increases in unskilled wages are nearly symmetric in this particular example, ranging from nine-tenths of a percent when the elasticity of substitution between the two types of labor

is at its highest value, to approximately 1.8% when the elasticity of substitution is lowest.

If the supply of capital is fixed or grows at a fixed rate, capital dilution is permanent. As long as all the elasticities of substitution are identical (as in Figures 9 and 10 in the Appendix), capital dilution's effect on skilled and unskilled wages is the same: each drops by one half or three-quarters of a percent, depending on whether the elasticity of substitution is one or .67. If the elasticity of substitution between capital and unskilled labor is raised to 1.67, capital dilution generates a much larger drop in both the short and long-run wages of skilled workers, as their labor complements the capital that is now relatively more scarce. Hence the permanent rise in wages for unskilled workers is only four-tenths of a percent, while the drop in the wages of skilled workers is 2.3%.

## 8.2 The Discounted College Premium

As we have seen, a change in the rate of immigration does not change the wage structure overnight. A shift in policy means a change in the flow of immigrants that gradually alters the size and composition of the workforce. In addition, the economy does not necessarily adjust immediately to these changes. Unless the amount of capital adjusts immediately or is permanently fixed, a surge of immigration will generate an initial shock to the savings rate, followed by the gradual convergence of the size of the capital stock to its new steady-state level. Finally, immigrants themselves, or at least their descendants, may not permanently remain within their initial skill categories, generating further disturbances to wages long after the surge in immigration has subsided.

The impact of immigration on wage inequality must reflect the behavior of wages over time. Encapsulating this behavior into a number requires us to discount by the rate of return available to these workers, and this rate of return is just as sensitive to immigration policy as the wages themselves, unless we assume from the outset that the capital supply is perfectly elastic and that its rate of return is determined exogenously.

The need to take into account the endogeneity of the discount rate is apparent if we compare the time paths of wages in Figures 4 and 5 to the behavior of the discounted college premium in Table 2. The time paths of wages when capital supply is endogenously determined, fall between the two extreme cases of completely elastic or inelastic capital supply, and closer to the former than the latter. By contrast, the effect of immigration on the discounted college premium does not necessarily follow this pattern.

The first column of Table 2 illustrates the effect of the immigration surge on the discounted college premium, if all the additional immigrants are and remain unskilled workers. The new policy implies that the premium will rise from an initial value of 170% to anywhere from 170.87% to 172.17%, depending on what we assume about the elasticity of capital supply and, more importantly, about the elasticity of substitution between the factors of production. Changes in

inequality of this magnitude might appear small, but considering the magnitude of the change in policy—a rise in the immigration rate of a mere three per ten thousand during twenty years—even the smallest of these changes are quite impressive.

Even more impressive is the effect of the immigration surge when it is comprised wholly of skilled workers, as in the last column of Table 2. The small rise in the number of skilled immigrants over the course of twenty years has the potential to lower the value of  $S^P$  from 170% to 166.52% if capital is supplied endogenously, and capital and skill are relative complements. The drop in the discounted skill premium is even larger under nearly all the other scenarios in Table 2.

It is important to keep in mind that the number of additional skilled immigrants who arrive under the policy considered here is approximately seventy-five thousand per year. This number is similar to the sixty-five thousand visas available under the H1-B visa program through most of the years of its existence, and gives us some idea of what that program’s impact on wage inequality might be. How robust are these results? The impact of a surge of skilled immigration of this magnitude might cause a much smaller drop in the value of  $S^P$ —perhaps from 170% to 169%—but only if the elasticities of substitution between the different inputs are very high (in this model close to five).

In the middle column of Table 2— $\Pi_S(T) = \Pi_S(V) = P_S(0)$ —the distribution of skills among the immigrants exactly replicates that of the general population. As long as the cross elasticities of substitution are equal across the different inputs, such an influx of immigration has no effect on the college premium. Even if the supply of capital is not perfectly elastic, capital dilution causes both types of wages to drop by the same proportion. Only if the elasticities are not equal and capital supply is not completely elastic can a surge in immigration of this type lead to a change in the discounted skill premium.

The third to last row of Table 2, in the middle column, combines the nested CES production function and fixed capital supply for a surge of immigration whose rates of college attainment match those of the general public. This entry isolates the effect of capital dilution on the discounted college premium—at most a drop from 170% to 169.38%. A rise in immigration lowers wages for everyone because everyone’s labor complements capital. However the higher degree of complementarity between skilled labor and the factor of production that is being diluted ensures that their wages decline more, and the wage premium declines. If capital is endogenously determined, capital dilution is transitory and the effect of the immigration surge is smaller—the discounted wage premium is 169.89%.

### 8.3 Assessing the Impact of Skill Acquisition

The results in Table 2 reveal that a relatively small influx of unskilled immigrants substantially raises the discounted college premium if these immigrants and their descendants remain perma-

	$\Pi_S(T) = \Pi_S(V) = 0$	$\Pi_S(T) = \Pi_S(V) = P_S(0)$	$\Pi_S(T) = \Pi_S(V) = 1$
COBB DOUGLAS: $\sigma_{UK} = \sigma_{US} = \sigma_{SK} = 1$			
<i>Fixed Capital</i>	171.45	170.00	165.89
<i>Completely Elastic Capital</i>	171.45	170.00	165.86
<i>Endogenous Capital</i>	171.45	170.00	165.76
CES: $\sigma_{UK} = \sigma_{US} = \sigma_{SK} = .67$			
<i>Fixed Capital</i>	172.16	170.00	163.91
<i>Completely Elastic Capital</i>	172.17	170.00	163.85
<i>Endogenous Capital</i>	172.16	170.00	163.67
NESTED CES: $\sigma_{UK} = \sigma_{US} = 1.67, \sigma_{SK} = .67$			
<i>Fixed Capital</i>	170.87	169.38	165.20
<i>Completely Elastic Capital</i>	171.08	170.00	166.88
<i>Endogenous Capital</i>	171.05	169.89	166.52

Table 2: The values of the discounted skill premium  $S^P$ , following a rise in the rate of immigration from 3.2 to 3.5 per thousand during the course of two decades. The baseline value of  $S^P$  is 170%.

nently unskilled. A similar sized influx of skilled immigrants produces an opposite and much larger change. How might these results change if the immigrants, or at least their descendants, switch between the two skill categories? More specifically, how much can college attainment by immigrants subsequent to their arrival in the United States as unskilled workers mitigate or reverse their initial impact on wage inequality? How much does future college attainment among the members of the second generation affect the discounted college premium?

Suppose once again that the additional immigrants arrive initially as unskilled workers,  $\Pi_S(T) = 0$ , but over time a fraction of these immigrants or their children join the ranks of the skilled, until ultimately the distribution between skilled and unskilled within this additional population matches that of the general population— $\Pi_S(V) = P_S(0)$ . As in the second columns of Figures 4 and 5, we are again modeling a process of assimilation, at least in terms of educational attainment. Table 3 illustrates the implications of this process on the discounted skill premium for the nested CES specification of the production function,  $\sigma_{UK} = \sigma_{US} = 1.67, \sigma_{SK} = .67$ . The same calculations, with all the elasticities of substitution set equal to each other, are presented in Tables 5 and 6 in the Appendix.

As should be expected, the values in Table 3 fall somewhere between the corresponding entries in the first two columns of Table 2 (last three rows). The lower the values of  $Q$  and  $V$ —the earlier the date at which the process of assimilation begins, and the earlier the date at which it is completed—the closer the values of the discounted college premium are to the

values in the second column of Table 2. Higher values of  $Q$  and  $V$  correspond to delayed and extended periods of assimilation, and are closer to the higher values in the first column. As in the second column of graphs in Figure 4 and 5, unskilled wages rise and skilled wages decline starting in period  $Q$ . Even if  $Q$  is large, implying that only members of the second generation attend college, assimilation mitigates in the long run the initial rise in inequality.

Compare the discounted college premium in Table 2 for the values  $\Pi_S(T) = 0$  and  $\Pi_S(V) = 0$ , and the nested CES production function with their corresponding values in Table 3. If  $Q=20$  and  $V=40$ , the discounted college premium in Table 3 is 170.26% if the supply of capital is exogenously determined, and 170.64% if capital supply is perfectly elastic. The corresponding values in Table 2 are 170.87% and 171.08%. The less elastic the supply of capital, the higher the rate at which changes in wages that take place after time  $Q$  are discounted, and yet when capital supply is inelastic it seems that the relative impact on the discounted college premium is larger. Hence we conclude that the disproportional downward pressure on skill wages induced by capital dilution dominates the higher discount rate.

The arrival of a small number of unskilled immigrants generates a significant rise in inequality in Table 2, regardless of how the production function is specified, or the nature of the capital supply. By contrast, an immigration surge that replicates the existing distribution of skills in the general population, and the underlying immigration flow, slightly lowers the discounted college premium if capital and skill are relative complements and the supply of capital is not completely elastic. Hence a surge of immigration need not raise inequality, even if the immigrants and their descendants merely match the skill distribution decades in the future, and in the meantime increase the relative size of the unskilled workforce. Of course, only if as in the top part of Table 3, the supply of capital is inelastic, and the process of assimilation begins soon after these immigrants begin to arrive. Otherwise assimilation mitigates the effects of immigration on the value of  $S^P$ , but cannot reverse it.

The values of the discounted college premium in Table 3 are far more sensitive to the value of  $Q$  than to the value of  $V$ . The point at which the share of college educated people has returned to its initial value of  $P_S(0)$  is not as important as how quickly the first immigrants begin their transformation into skilled workers. This difference becomes even more obvious if we consider a more radical degree of transformation on the part of the new immigrants.

What if unskilled immigrants, or their children, do not merely match the general level of educational attainment but far surpass it? Again, consider the most extreme scenario: the immigrants arrive initially as unskilled workers,  $\Pi_S(T) = 0$ , but over time they or their children all join the ranks of the skilled,  $\Pi_S(V) = 1$ . In nearly every case the values of the discounted skill premium in Table 4 (calculated for nested CES specification of the production function,  $\sigma_{UK} = \sigma_{US} = 1.67$ ,  $\sigma_{SK} = .67$ ) fall well below 170%. The same is true if the elasticities of substitution are identical, as in Tables 7 and 8.

	$Q = 0$	$Q = 10$	$Q = 20$	$Q = 30$	$Q = 40$	$Q = 50$
<i>Inelastic Capital</i>						
$V = 20$	169.38	169.71	*	*	*	*
$V = 30$	169.63	169.92	170.13	*	*	*
$V = 40$	169.82	170.08	170.26	170.40	*	*
$V = 50$	169.97	170.20	170.37	170.49	170.57	*
$V = 60$	170.09	170.30	170.45	170.55	170.63	170.69
<i>Completely Elastic Capital</i>						
$V = 20$	170.00	170.24	*	*	*	*
$V = 30$	170.18	170.39	170.54	*	*	*
$V = 40$	170.32	170.50	170.64	170.74	*	*
$V = 50$	170.43	170.59	170.71	170.80	170.87	*
$V = 60$	170.52	170.67	170.77	170.85	170.91	170.95
<i>Endogenous Capital</i>						
$V = 20$	169.93	170.18	*	*	*	*
$V = 30$	170.11	170.34	170.49	*	*	*
$V = 40$	170.26	170.45	170.59	170.69	*	*
$V = 50$	170.37	170.54	170.67	170.75	170.82	*
$V = 60$	170.46	170.61	170.72	170.80	170.86	170.90

Table 3: Nested CES specification of the production function:  $\sigma_{UK}=\sigma_{US}=1.67$ ,  $\sigma_{SK}=.67$ . The values of the discounted skill premium  $S^P$ , following a rise in the rate of immigration from 3.2 to 3.5 per thousand during the course of two decades, for different values of  $V$  and  $Q$ . The immigrants are initially unskilled  $\Pi_S(T)=0$ , but beginning at time  $Q$ , some of these immigrants, or their descendants, become skilled workers. By time  $V$  the share of skilled workers within this population stabilizes at  $\Pi_S(V)=P_S(0)$  where  $P_S(0)=.256$  is the prevailing share of skilled workers within the general population. The baseline value of  $S^P$  is 170%.



	$Q = 0$	$Q = 10$	$Q = 20$	$Q = 30$	$Q = 40$	$Q = 50$
<i>Inelastic Capital</i>						
$V = 20$	165.20	166.46	*	*	*	*
$V = 30$	166.14	167.26	168.08	*	*	*
$V = 40$	166.87	167.87	168.59	169.12	*	*
$V = 50$	167.44	168.34	168.98	169.45	169.79	*
$V = 60$	167.90	168.70	169.28	169.69	169.99	170.21
<i>Completely Elastic Capital</i>						
$V = 20$	166.88	167.79	*	*	*	*
$V = 30$	167.57	168.37	168.97	*	*	*
$V = 40$	168.11	168.83	169.35	169.74	*	*
$V = 50$	168.53	169.18	169.65	169.99	170.24	*
$V = 60$	168.87	169.45	169.88	170.18	170.40	170.57
<i>Endogenous Capital</i>						
$V = 20$	166.64	167.63	*	*	*	*
$V = 30$	167.38	168.24	168.86	*	*	*
$V = 40$	167.94	168.71	169.25	169.63	*	*
$V = 50$	168.38	169.06	169.54	169.88	170.13	*
$V = 60$	168.73	169.34	169.77	170.07	170.29	170.46

Table 4: Nested CES specification of the production function:  $\sigma_{UK}=\sigma_{US}=1.67$ ,  $\sigma_{SK}=.67$ . The values of the discounted skill premium  $S^P$ , following a rise in the rate of immigration from 3.2 to 3.5 per thousand during the course of two decades, for different values of  $V$  and  $Q$ . The immigrants are initially unskilled  $\Pi_S(T)=0$ , but beginning at time  $Q$ , all these immigrants, or their descendants, become skilled workers. By time  $V$  the share of skilled workers within this population stabilizes at  $\Pi_S(V)=1$ . The baseline value of  $S^P$  is 170%.

Consider the following scenario: a man and woman arrive in the United States at age twenty-five, at the very beginning of the immigration surge, meet and start a family. Neither are college educated and both spend the next forty years employed as unskilled workers. These parents place a high value on education and all their American-born children complete college. The first child graduates and joins the labor force perhaps twenty five years after the parents' arrival. Such a high level of college attainment among the second generation does not merely mitigate the negative effect on wage inequality of their immigrant parents' decades of participation in the labor force, but completely reverses it. Hence, admitting these two people into the United States raises the college premium for thirty years, but seen from the perspective of several generations, discounted by the economy's prevailing rate of return, the arrival of this family has lowered overall wage inequality.

## 9 Immigrant Groups and Wage Inequality

The simple examples of immigration surges in Section 8 are completely hypothetical and hardly match the heterogeneity of the bi-generational patterns of educational attainment for the different U.S. immigrant groups in Figure 2. Compare for example the shares of college-educated male immigrants to the United States from Poland and the Philippines, and their corresponding second generations. A relatively small share of male immigrants from Poland aged 45 to 64 have college degrees—21.3% versus 25.6% for the overall U.S. population. However, a very large fraction of American born sons aged 25 to 44 with Polish-born fathers have college degrees—64.1%. Admitting Polish immigrants prior to 1975 raised the gap between the wages of skilled and unskilled workers, but a generation later, the high levels of educational attainment achieved by the second generation have had the opposite effect on the skill premium.

Indeed, compare the experience of Polish immigrants over the course of two generations with immigrants from the Philippines. The share of male immigrants from the Philippines aged 45 to 64 with college degrees is much higher—45.8%—but among second generation Americans aged 25 to 44 with Philippine-born fathers, the share completing college is significantly lower than for their Polish-American counterparts—only 42.3% (indeed, there is even some reversion to the mean, as members of the second generation are less educated than members of the first). How do we compare the impact of the entire experience of Philippine immigration, including subsequent generations, with immigration from Poland?

Once again  $P_S(0)$  is the initial steady-state share of college-educated people in the economy prior to the change in policy. The initial share of college educated among the additional immigrants is  $\Pi_S(T)$ , and  $\Pi_S(V)$  is the share of college educated among these same immigrants or their descendants at time  $V$ . Starting at time  $Q$ , the share of skilled workers among these additional dynasties begins changing (linearly) until the share of skilled workers among these

same additional immigrants or their descendants is  $\Pi_S(V)$ .

I focus on the case of endogenously determined capital supply, and once again set  $\epsilon = .0003$ , the value of  $T$  equal to twenty, the value of  $Q$  to twenty-five, and the value of  $V$  to forty-five, in (23). I insert the resulting values of the perturbations into (24), and use these to calculate the value of  $\xi$  in (25). Setting (19) to different values of  $S^P$  yields an implicit function in the values  $\Pi_S(T)$  and  $\Pi_S(V)$ . These implicit functions are represented as iso-curves in Figures 6—7, corresponding to the Nested CES specification, with the initial steady state discount rate set at .05 and .03 respectively (the Cobb-Douglas and CES production functions appear respectively in Figures 11 and 12 in the Appendix). These curves are superimposed on the points representing the intergenerational rates of college completion for different immigrant groups in Figure 2.

What do we learn from Figures 6—7? First, though the function  $S^P$  is a complicated non-linear function, it appears nearly linear on the  $\Pi_S(T)$  and  $\Pi_S(V)$  plane. Second, the distances between the iso-curves appear constant—changing the values of either  $\Pi_S(T)$  or  $\Pi_S(V)$  generates nearly linear changes in the value of the discounted college premium.

Each iso-curve that correspond to  $S^P=170\%$  bisects a plane between two regions—immigrant groups with values of  $\Pi_S(T)$  and  $\Pi_S(V)$  that fall to the right of the curve lower the discounted college premium, while those with values to the left of the curve raise it. In Figures 6—7 the curve corresponding to  $S^P=170\%$  passes to the left of the point (25.6,25.6). Therefore even if both the immigrant and second generation fall below the general levels of educational attainment, the discounted college premium may still decline because when capital and skill are relative complements, capital dilution harms the wages of skilled workers more than it harms the wages of the unskilled.<sup>15</sup>

A group of immigrants with a low value of  $\Pi_S(T)$  but a high value of  $\Pi_S(V)$  raise the ratio between skilled and unskilled wages in the short run, but cause it to drop in the long-run. The short-run effect of  $\Pi_S(T)$  is temporary, whereas the long-run effect of  $\Pi_S(V)$  in this model lasts forever. On the other the long-run is discounted at a rate of return that varies (in response to immigration policy) around five percent. The discounting dominates and the angles measured between the iso-curves and the horizontal axis in Figure 6 (and Figures 11 and 12)— are above  $45^\circ$ .

Consider the effect of Greek immigrants on the values of  $S^P$  in Figure 6. Among members of the second generation, the rate of college completion is 59.60% for women against 42.91% for men. Nonetheless, the very low levels of education among the women of the immigrant

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<sup>15</sup>In Figures 11 and 12 (see Appendix), the elasticities of substitution between all the inputs are equal, and the iso-curve passes through the point (25.6,25.6), the value of  $P_S(0)$ . Again, a surge of immigration that merely replicates the existing skill distribution has no effect on the discounted discount premium, unless the elasticities of substitution between the inputs are not equal. For example, if production is Cobb Douglas or CES the impact of Italian women on the value of  $S^P$  is slightly negative, rather than slightly positive, as in Figure 6.

generation, 3.62%, means that the combined effect of the mother-daughter pair on the value of  $S^P$  is positive—they raise inequality, if only slightly. The male immigrant from Greece is more likely to have a college degree (18.34%), though less likely than members of the population at large. Combined with a higher-than-average rate of college attainment in the second generation, the father-son pairs fall to the right of the  $S^P = 170$  iso-curve. Hence males as a group lower inequality, and the combined effect of all Greek immigrants, male and female, on the value of the discounted college premium is close to neutral.

Indeed women (and their daughters) from Colombia, the Dominican Republic, Greece, and Portugal, together with men from the Dominican Republic, all raise the value of  $S^P$  even though members of the second generation are better educated than the general population. Female immigrants from Cuba and Ireland, and male immigrants from Greece, Italy, Poland, and Portugal have lower rates of college completion than the general population, but the high rates of college attainment for corresponding members of the second generation mean their overall effect on  $S^P$  is negative.

The best educated group in our sample are Indian men. For the immigrant generation 82.62% have college degrees, and for members of the second generation this rises to 87.26%. An additional 87,000 visas per year to members of this group over the course of twenty years profoundly lowers the value of  $S^P$ , to 167.24%. The next group, Chinese men, lowers the value of  $S^P$  to 168.03%.

Among the people in the sample born in the United States, the least educated are those whose parents were born in Mexico or Puerto Rico. Only 6.68% of immigrant men from Mexico have college degrees, and for American-born sons of Mexican-born fathers, the share with college degrees is only 10.23%. These values generate a value of 170.7% for  $S^P$ . The corresponding figures for women, 3.98% and 12.21%, produce an  $S^P$  of 170.8%. Overall, the values of  $S^P$  for men rise in the following order: India, China, U.S.S.R., U.K., Germany, Philippines, Ireland, Canada, Colombia, Poland, Cuba, U.S., Italy, Greece, Portugal, Dominican Republic, Puerto Rico, and Mexico. For women the order is China, India, U.S.S.R., Philippines, Poland, Canada, Germany, Cuba, Ireland, U.S., Italy, U.K., Colombia, Greece, Dominican Republic, Puerto Rico, Portugal, and Mexico.

If the production function is Cobb-Douglas or CES all the iso-curves shift outward, including the curve corresponding to 170%. Women from the U.K, and Italy now fall to the left of the curve; they now lower inequality in an economy without capital-skill complementarities (Figures 11 and 12 in the Appendix). More generally, the specification of the production function employed in Figure 6 implies a high rate of substitution between the two types of labor. The lower the elasticity of substitution the closer together are the iso-curves and the more sensitive is the value of  $S^P$  to different immigration surges. Consider setting all the elasticities of substitution to .67 (Figure 12 in the Appendix). Raising the number of male immigrants by

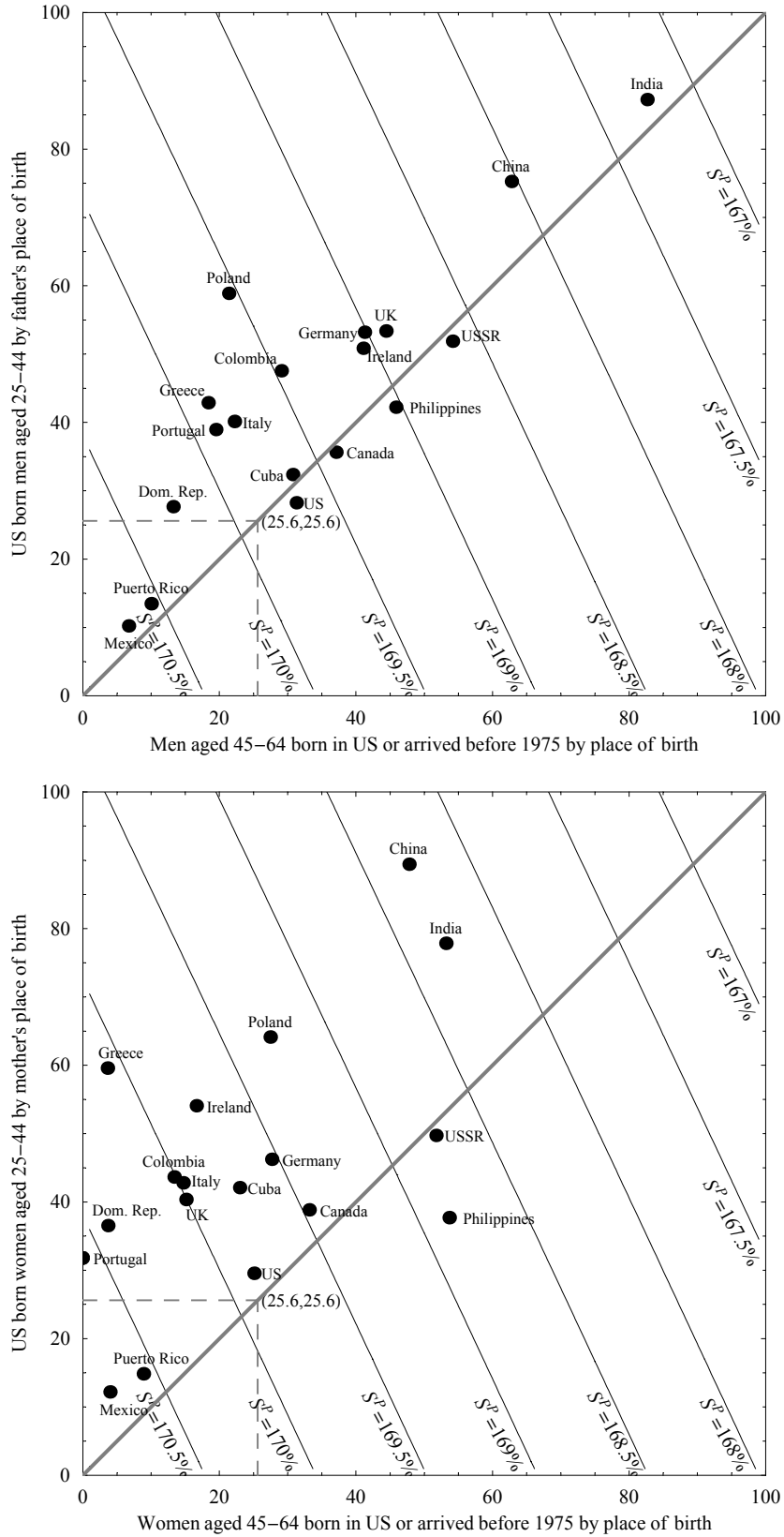


Figure 6: Nested CES production function:  $\sigma_{UK} = \sigma_{US} = 1.67$ ,  $\sigma_{SK} = .67$ . Iso-curves for the values of  $S^p$  following a twenty year twenty-five percent rise in the rate of immigration. The baseline rate of return of capital is set to .05, and  $Q=25$  and  $V=45$ . Points represent the percentage of the U.S. population with four-year college degrees by age, sex, birthplace, and parent's birthplace from Figure 2.

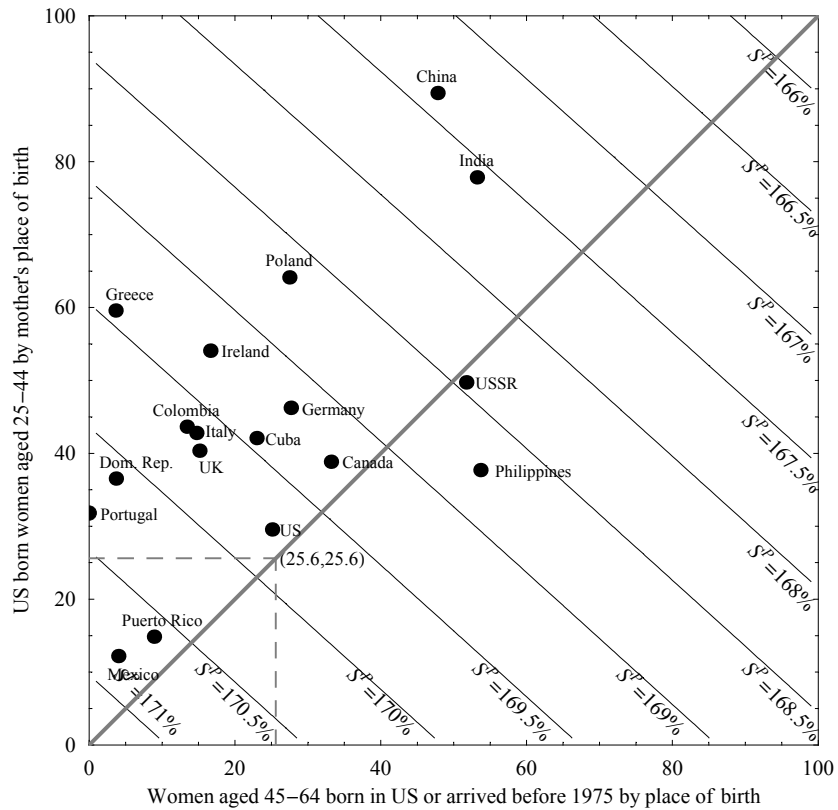
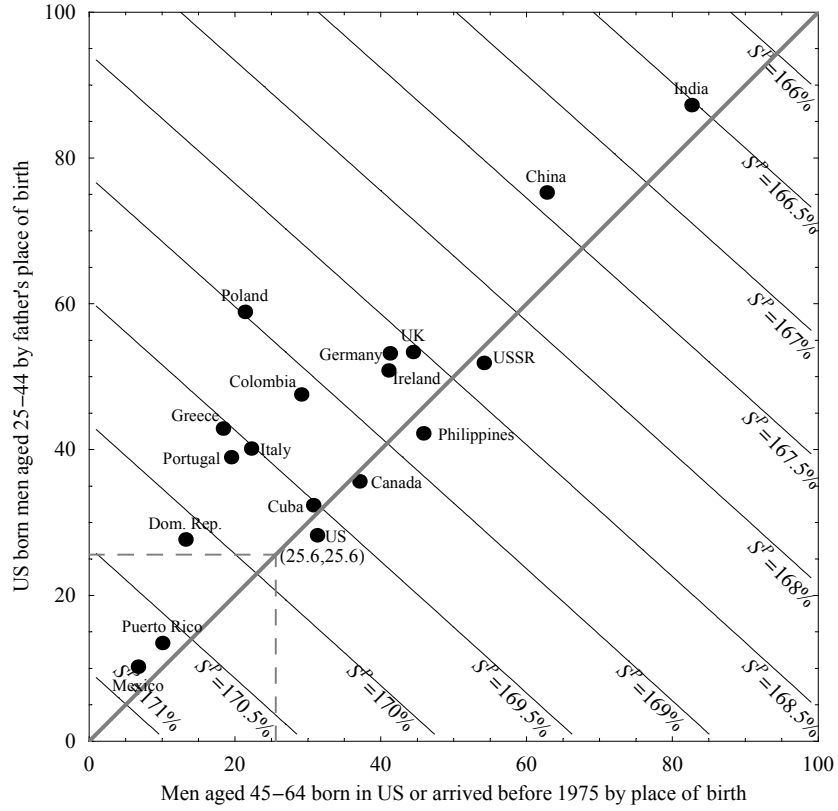


Figure 7: Nested CES production function:  $\sigma_{UK} = \sigma_{US} = 1.67$ ,  $\sigma_{SK} = .67$ . Iso-curves for the values of  $S^p$  following a twenty year twenty-five percent rise in the rate of immigration. The baseline rate of return of capital is set to .03, and  $Q=25$  and  $V=45$ . Points represent the percentage of the U.S. population with four-year college degrees by age, sex, birthplace, and parent's birthplace from Figure 2.

seventy-five thousand per year for two decades lowers the value of  $S^P$  to 165.02%. At the other end of the ordinal ranking, the arrival of the same number of Mexican women to the United States raises the value of  $S^P$  to 171.50%.

Changing the initial discount rate to .03 changes the shapes and not merely the placement of the iso-curves in Figure 7. The angles measured between the iso-curves and the horizontal axis drops below  $45^\circ$ , because the rate of educational attainment in the future designated by  $\Pi_S(V)$ , is no longer as heavily discounted. This creates a new ranking for some of the immigrant groups. The values of  $S^P$  for men rise starting with India, and moving through China, U.S.S.R., U.K., Germany, Ireland, Philippines, Poland, Colombia, Canada, Italy, Cuba, Greece, Portugal, U.S., Dominican Republic, Puerto Rico, and Mexico. For women the order is: China, India, U.S.S.R., Poland, Philippines, Germany, Ireland, Canada, Greece, Cuba, Italy, Colombia, U.K., U.S., Dominican Republic, Portugal, Puerto Rico, and Mexico.

Comparing these two lists to those obtained under the higher discount rate for men, the order of the first five, India, China, U.S.S.R., U.K., and Germany, does not change; nor does the order of the last three, Dominican Republic, Puerto Rico, and Mexico. Ireland and Philippines switch places at positions six and seven—the rates of college attainment for second generation members of the Irish community displace the higher rates of college attainment of Philippine immigrants. Colombia remains in ninth place, but eighth-placed Canada switches with tenth-placed Poland. Of the remaining countries, Italy jumps two places to number eleven, and Greece and Portugal each rise by one place, to thirteen and fourteen respectively. Cuba drops by one place to number twelve, and the U.S. drops three places to fifteen.

The shifts between the different countries are more pronounced for the women in the sample—only six, India, China, U.S.S.R., at the beginning, Italy and Dominican Republic in the middle, and Mexico at the end of the list maintain their positions. The lowering of the discount rate has a particularly large impact on the position of Greek women—they rise from fourteenth to ninth place. Furthermore, they no longer raise the value of  $S^P$  even slightly; with the lower discount rate these women lower the value of  $S^P$ . In summary, those immigrant groups with stable intergenerational rates of college attainment (those closest to the  $45^\circ$ ) have the same effect on the discounted college premium, regardless of which discount rate is employed in the calibration of the model. However, for the immigrant groups that experience large changes between the immigrant and second generations, the discount rate is an important factor in determining their overall impact.

Four important caveats. First, in this model there are only two skill types. Although educational attainment is by necessity a discrete variable in empirical studies, ordinarily there are more than just two categories. Second, in order to keep the model tractable, I do not treat educational attainment as a choice made by either the immigrants or the other individuals in the economy. Third, I abstract from the important distinction Chiswick (1978) found for the effect

of additional schooling prior to immigration and after immigration for immigrant men's earnings in the United States. Here a college degree from China is equivalent to a college degree attained in the United States. Finally, I assume that once immigrant groups have achieved a level of college education at time  $V$ , this remains their permanent level from there on in. This means not only that a disproportionate number of immigrants from India and China arrive with college degrees, and that the rates of college graduation among members of the second generations are even higher, but that subsequent generations do not 'assimilate' by lowering their educational performance.

## 10 Conclusion

Countries throughout the developed world find themselves grappling with the question of immigration. The combination of declining birth rates and increased life-span is already raising the average age of citizens throughout the west, and increasing substantially the share of pensioners within the population. The ability of governments to maintain generous systems of old-age pensions is now in serious doubt. Some see more liberal immigration policies as a possible alternative to raising minimum retirement ages, slashing benefits, or raising the tax burden on the dwindling population of native younger workers (see Auerbach and Oreopoulos (1999), Storesletten (2000), and Fehr *et. al.* (2004)). This paper abstracts from the motivations for higher rates of immigration, instead focusing on its consequences for wage inequality, both in the short and long term.

The traditional immigration-absorbing countries, the United States, Canada, and Australia, presently implement programs that grant residence to carefully selected foreigners with at least a Baccalaureate degree and a marketable skill. Increasingly other countries, including Germany and the United Kingdom, are also considering programs that target highly-skilled engineers and scientists. At the same time, as traditional source countries, particularly in Asia, develop and opportunities there grow, perhaps fewer engineers and scientists will choose to permanently leave their homelands. Although there is still a vast pool of well educated people willing to move to the West, competition between Western countries to attract them may intensify in the future.

By contrast, the supply of unskilled immigrants—from the most impoverished and unstable countries in the world, and even from those portions of the world experiencing rapid growth—is enormous and may even grow during the next few decades. First, the cost of long-distance travel is likely to continue to decline. Second, the wages paid to unskilled workers in the poorest countries are not likely to move anywhere near the wages available in the developed world for the foreseeable future. Furthermore, the spread of mass communication in recent years has greatly raised awareness among inhabitants of poor countries about standards of living in the



West. Finally, whereas once international migration entailed the near complete severing of ties to family and culture, the telecommunications revolution of the past two decades now allows migrants to retain links to their homeland by telephone, internet, and satellite television. Given these factors, Western countries need only relax their present interdiction efforts among illegal immigrants and they can receive nearly any influx of low-skilled workers they want. The only question is how many do they want?

A surge in the number of unskilled immigrants will at least initially exacerbate wage inequality. In countries with a strong commitment to income equality, governments may find themselves spending relatively less on old-age pensions, but more on other types of transfer payments. Can assimilation or enhanced educational attainment by members of the second generation ameliorate these effects? Since few unskilled adult immigrants attend school after arriving in their new home, their absorption typically entails a rise in the share of unskilled workers in the labor force until the end of their working lives. Nonetheless a rise in unskilled immigration today need not imply the creation of a self-perpetuating community of unskilled workers for generations to come and a permanent rise in the wage gap.

As I have demonstrated, the disproportionate share of unskilled workers among certain immigrant groups that arrived in the United States a generation ago did not necessarily cause wage inequality, if we compare the ratio of discounted wages over time. The effect of high levels of educational attainment among the members of the second generation can easily overwhelm the low levels of education that often characterize the immigrant generation.

What is the likely impact of immigration to the United States today? Consider the results in Figure 8. The horizontal axis is the same as the horizontal axis in Figure 2, but I replace the vertical axis with U.S.-born and young immigrants that arrived after 1975. The large changes are mostly towards greater shares of college education. The most noticeable improvements are among Canadian and Italian men, women from Ireland, and both men and women from Greece. Overall the men are reasonably close to the 45°. We cannot predict the rates of educational attainment among members of the second generation, twenty-five years hence, but perhaps the patterns in Figure 2 offer some clue. Similarly, the methods developed here for calculating the discounted skill premium provide some guidance for determining how these immigrants and their families will affect wage inequality in the future.

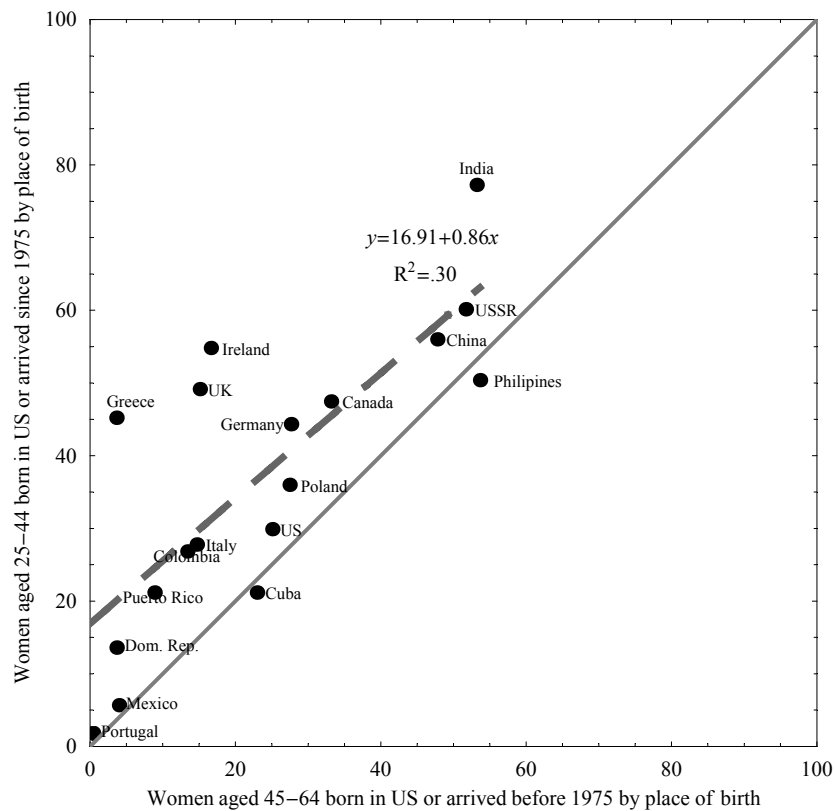
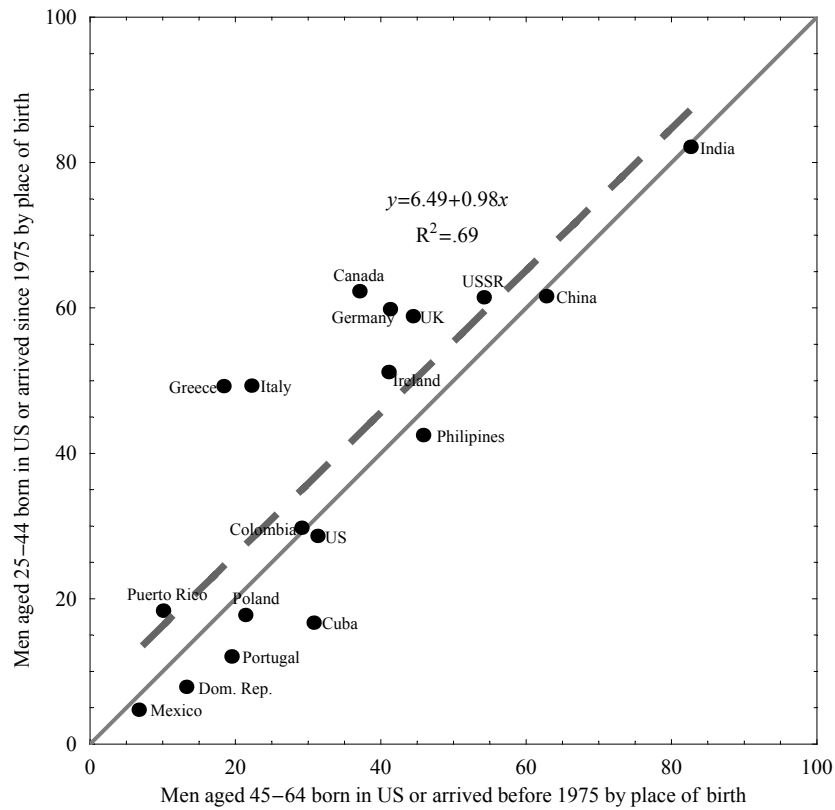
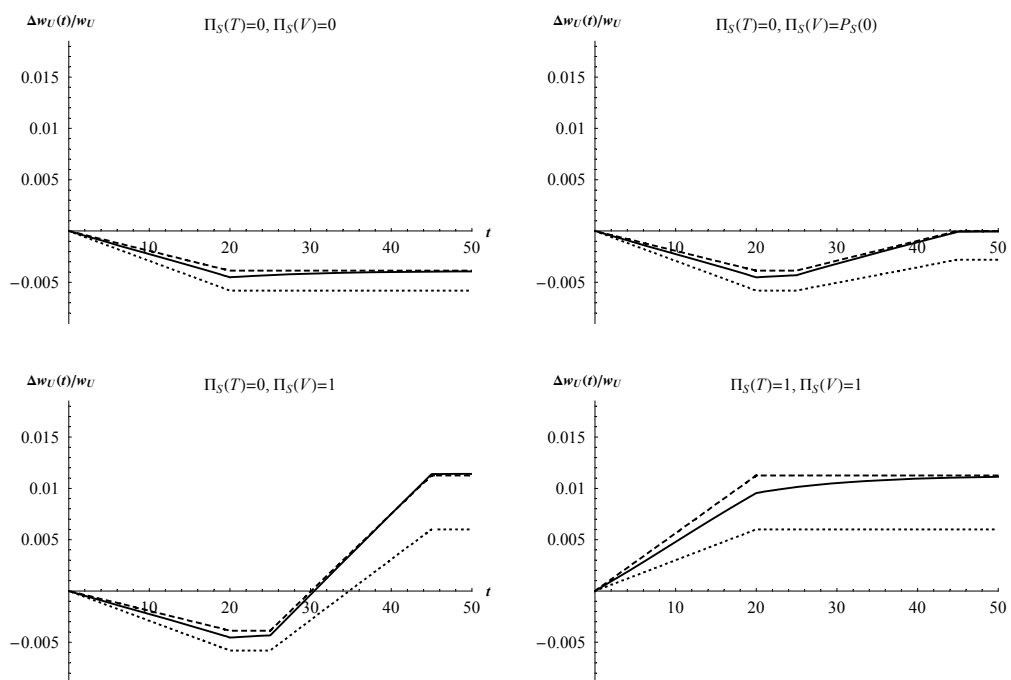


Figure 8: Percentage of the U.S. population with four-year college degrees by age, sex, and birthplace. Data for the USSR includes all respondents from any of the former republics in the sample, the data for the UK includes respondents from Northern Ireland, and data for Portugal includes respondents from the Azores. Pooled data for 2001, 2002, and 2003 from the U.S. Census, Current Population Survey. Source: Miriam King, Steven Ruggles, and Matthew Sobek. Integrated Public Use Microdata Series, Current Population Survey: Preliminary Version 0.1. Minneapolis: Minnesota Population Center, University of Minnesota, 2003.

# 11 Appendix—Cobb Douglas and CES Production

a) Cobb Douglas Production Function,  $\sigma_{UK}=\sigma_{US}=\sigma_{SK}=1$



b) CES Production Function,  $\sigma_{UK}=\sigma_{US}=\sigma_{SK}=.67$

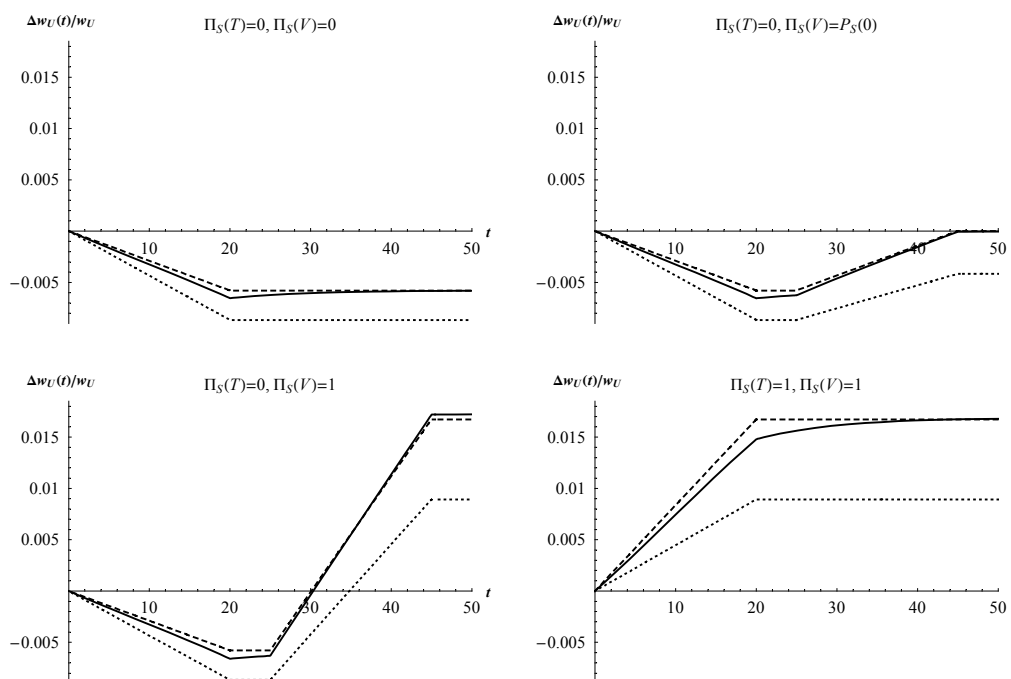
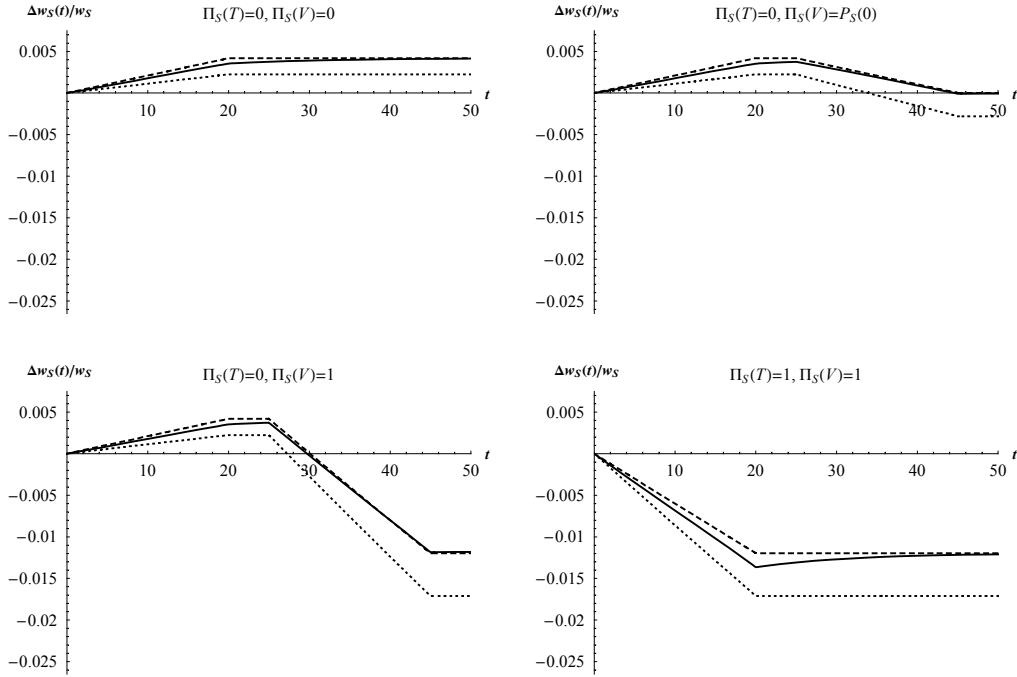


Figure 9: Cobb-Douglas and CES production functions. Impulse response for unskilled wages following a twenty year surge in the rate of immigration from 3.2 to 3.5 per thousand,  $Q=25$  and  $V=45$ . The solid, dashed, and dotted curves represent respectively, the impulse responses generated by the model with capital supply that is elastic, completely elastic, and inelastic.

**a) Cobb Douglas Production Function,  $\sigma_{UK}=\sigma_{US}=\sigma_{SK}=1$**



**b) CES Production Function,  $\sigma_{UK}=\sigma_{US}=\sigma_{SK}=.67$**

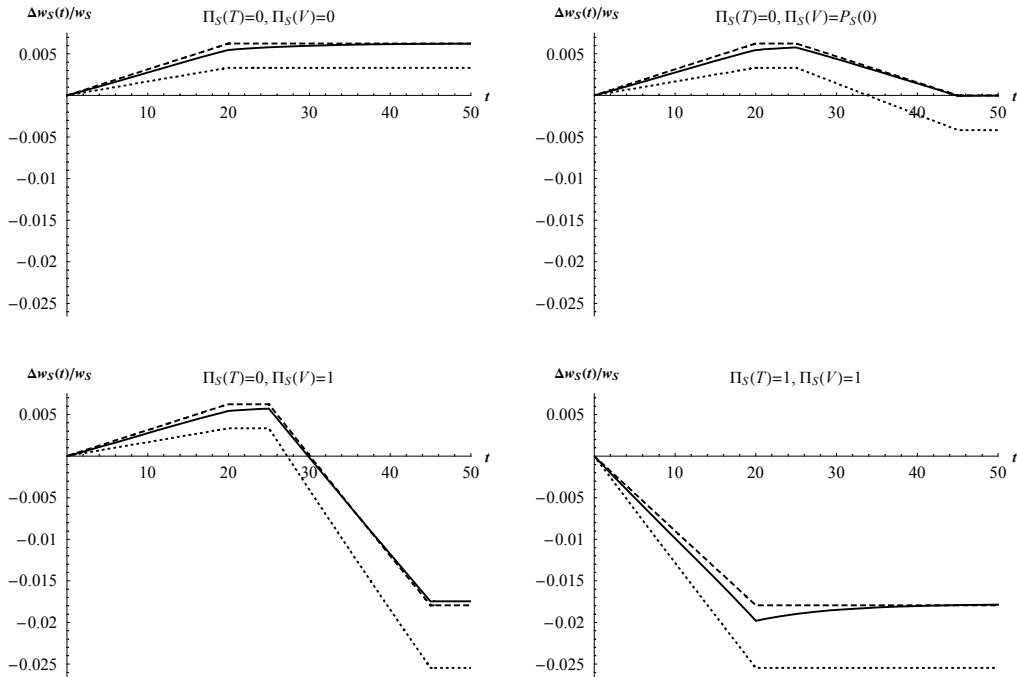


Figure 10: Cobb-Douglas and CES production functions. Impulse response for skilled wages following a twenty year surge in the rate of immigration from 3.2 to 3.5 per thousand,  $Q=25$  and  $V=45$ . The solid, dashed, and dotted curves represent respectively, the impulse responses generated by the model with capital supply that is elastic, completely elastic, and inelastic.

	$Q = 0$	$Q = 10$	$Q = 20$	$Q = 30$	$Q = 40$	$Q = 50$
<i>Inelastic Capital</i>						
$V = 20$	170.00	170.32	*	*	*	*
$V = 30$	170.24	170.52	170.73	*	*	*
$V = 40$	170.43	170.68	170.86	170.99	*	*
$V = 50$	170.57	170.80	170.96	171.08	171.16	*
$V = 60$	170.69	170.89	171.04	171.14	171.22	171.27
<i>Completely Elastic Capital</i>						
$V = 20$	170.00	170.31	*	*	*	*
$V = 30$	170.24	170.52	170.72	*	*	*
$V = 40$	170.43	170.67	170.86	170.99	*	*
$V = 50$	170.57	170.80	170.96	171.07	171.16	*
$V = 60$	170.69	170.89	171.04	171.14	171.22	171.27
<i>Endogenous Capital</i>						
$V = 20$	170.00	170.31	*	*	*	*
$V = 30$	170.24	170.51	170.71	*	*	*
$V = 40$	170.42	170.66	170.84	170.97	*	*
$V = 50$	170.56	170.78	170.94	171.05	171.14	*
$V = 60$	170.68	170.88	171.02	171.12	171.19	171.25

Table 5: Cobb-Douglas Production Function:  $\sigma_{UK}=\sigma_{US}=\sigma_{SK}=1$ . The values of the discounted skill premium  $S^P$ , following a rise in the rate of immigration from 3.2 to 3.5 per thousand during the course of two decades, for different values of  $V$  and  $Q$ . The immigrants are initially unskilled  $\Pi_S(T)=0$ , but beginning at time  $Q$ , some of these immigrants, or their descendants, become skilled workers. By time  $V$  the share of skilled workers within this population stabilizes at  $\Pi_S(V)=P_S(0)$  where  $P_S(0)=.256$  is the prevailing share of skilled workers within the general population. The baseline value of  $S^P$  is 170%.

	$Q = 0$	$Q = 10$	$Q = 20$	$Q = 30$	$Q = 40$	$Q = 50$
<i>Inelastic Capital</i>						
$V = 20$	170.00	170.47	*	*	*	*
$V = 30$	170.36	170.78	171.09	*	*	*
$V = 40$	170.64	171.01	171.28	171.48	*	*
$V = 50$	170.86	171.19	171.43	171.61	171.74	*
$V = 60$	171.03	171.33	171.55	171.70	171.82	171.90
<i>Completely Elastic Capital</i>						
$V = 20$	170.00	170.47	*	*	*	*
$V = 30$	170.36	170.77	171.08	*	*	*
$V = 40$	170.64	171.01	171.28	171.48	*	*
$V = 50$	170.86	171.19	171.43	171.61	171.73	*
$V = 60$	171.03	171.33	171.55	171.70	171.82	171.90
<i>Endogenous Capital</i>						
$V = 20$	170.00	170.46	*	*	*	*
$V = 30$	170.35	170.76	171.06	*	*	*
$V = 40$	170.63	170.99	171.25	171.45	*	*
$V = 50$	170.84	171.17	171.40	171.57	171.70	*
$V = 60$	171.01	171.31	171.52	171.67	171.78	171.86

Table 6: CES Production Function:  $\sigma_{UK}=\sigma_{US}=\sigma_{SK}=.67$ . The values of the discounted skill premium  $S^P$ , following a rise in the rate of immigration from 3.2 to 3.5 per thousand during the course of two decades, for different values of  $V$  and  $Q$ . The immigrants are initially unskilled  $\Pi_S(T)=0$ , but beginning at time  $Q$ , some of these immigrants, or their descendants, become skilled workers. By time  $V$  the share of skilled workers within this population stabilizes at  $\Pi_S(V)=P_S(0)$  where  $P_S(0)=.256$  is the prevailing share of skilled workers within the general population. The baseline value of  $S^P$  is 170%.

	$Q = 0$	$Q = 10$	$Q = 20$	$Q = 30$	$Q = 40$	$Q = 50$
<i>Inelastic Capital</i>						
$V = 20$	165.88	167.11	*	*	*	*
$V = 30$	166.81	167.89	168.68	*	*	*
$V = 40$	167.52	168.49	169.19	169.70	*	*
$V = 50$	168.08	168.95	169.57	170.03	170.36	*
$V = 60$	168.52	169.31	169.87	170.28	170.57	170.78
<i>Completely Elastic Capital</i>						
$V = 20$	165.86	167.06	*	*	*	*
$V = 30$	166.77	167.85	168.64	*	*	*
$V = 40$	167.49	168.45	169.15	169.66	*	*
$V = 50$	168.05	168.91	169.54	169.99	170.33	*
$V = 60$	168.50	169.28	169.84	170.25	170.54	170.76
<i>Endogenous Capital</i>						
$V = 20$	165.75	166.97	*	*	*	*
$V = 30$	166.68	167.76	168.54	*	*	*
$V = 40$	167.40	168.36	169.05	169.57	*	*
$V = 50$	167.97	168.83	169.45	169.90	170.23	*
$V = 60$	168.42	169.19	169.75	170.15	170.44	170.66

Table 7: Cobb-Douglas Production Function:  $\sigma_{UK}=\sigma_{US}=\sigma_{SK}=1$ . The values of the discounted skill premium  $S^P$ , following a rise in the rate of immigration from 3.2 to 3.5 per thousand during the course of two decades, for different values of  $V$  and  $Q$ . The immigrants are initially unskilled  $\Pi_S(T)=0$ , but beginning at time  $Q$ , all these immigrants, or their descendants, become skilled workers. By time  $V$  the share of skilled workers within this population stabilizes at  $\Pi_S(V)=1$ . The baseline value of  $S^P$  is 170%.

	$Q = 0$	$Q = 10$	$Q = 20$	$Q = 30$	$Q = 40$	$Q = 50$
<i>Inelastic Capital</i>						
$V = 20$	163.90	165.73	*	*	*	*
$V = 30$	165.27	166.89	168.08	*	*	*
$V = 40$	166.33	167.77	168.82	169.59	*	*
$V = 50$	167.16	168.46	169.39	170.07	170.56	*
$V = 60$	167.82	168.99	169.83	170.43	170.87	171.19
<i>Completely Elastic Capital</i>						
$V = 20$	163.84	165.63	*	*	*	*
$V = 30$	165.20	166.79	167.97	*	*	*
$V = 40$	166.26	167.68	168.73	169.50	*	*
$V = 50$	167.09	168.38	169.31	169.99	170.49	*
$V = 60$	167.76	168.92	169.76	170.37	170.81	171.14
<i>Endogenous Capital</i>						
$V = 20$	163.66	165.48	*	*	*	*
$V = 30$	165.05	166.65	167.83	*	*	*
$V = 40$	166.13	167.55	168.59	169.35	*	*
$V = 50$	166.97	168.25	169.17	169.85	170.34	*
$V = 60$	167.64	168.79	169.62	170.22	170.66	170.98

Table 8: CES Production Function:  $\sigma_{UK}=\sigma_{US}=\sigma_{SK}=.67$ . The values of the discounted skill premium  $S^P$ , following a rise in the rate of immigration from 3.2 to 3.5 per thousand during the course of two decades, for different values of  $V$  and  $Q$ . The immigrants are initially unskilled  $\Pi_S(T)=0$ , but beginning at time  $Q$ , all these immigrants, or their descendants, become skilled workers. By time  $V$  the share of skilled workers within this population stabilizes at  $\Pi_S(V)=1$ . The baseline value of  $S^P$  is 170%.



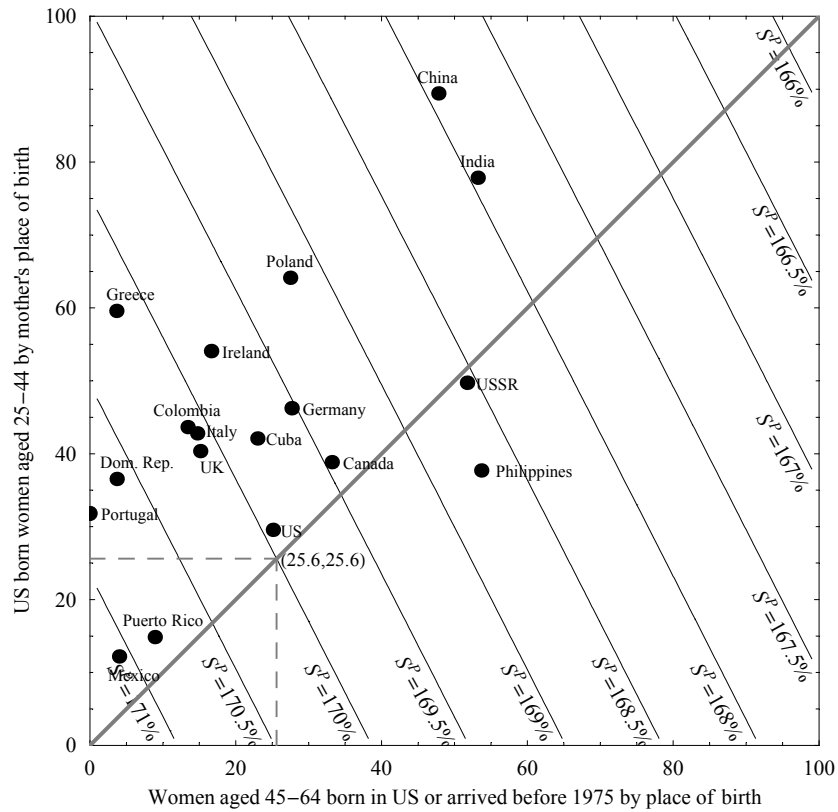
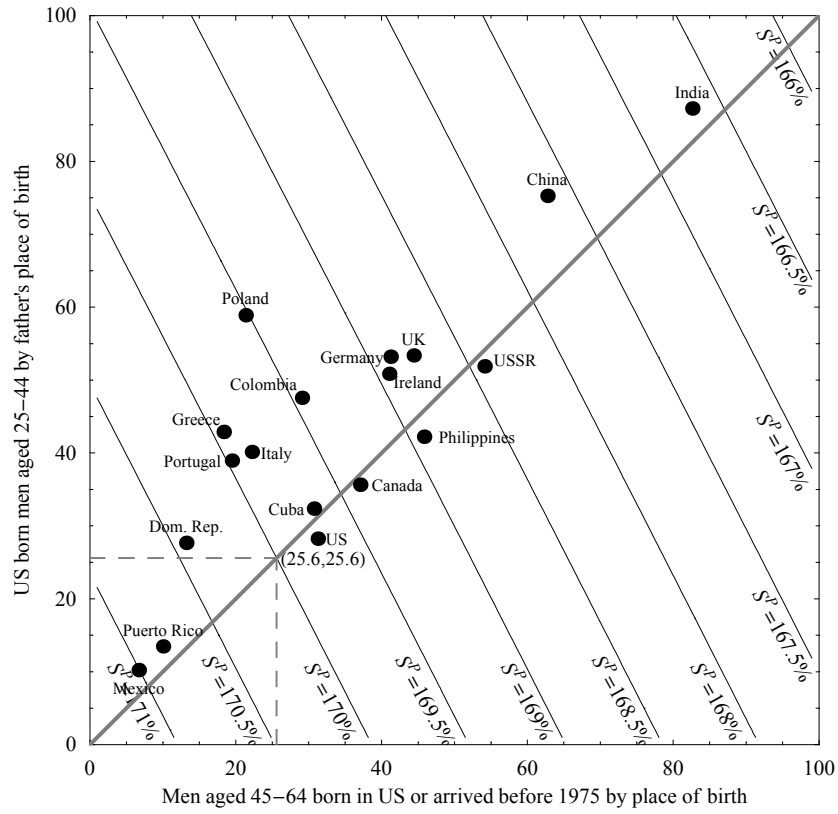


Figure 11: Cobb-Douglas production function:  $\sigma_{UK} = \sigma_{US} = \sigma_{SK} = 1$ . Iso-curves for the values of  $S^p$  following a twenty year twenty-five percent rise in the rate of immigration. The baseline rate of return of capital is set to .05, and  $Q=25$  and  $V=45$ . Points represent the percentage of the U.S. population with four-year college degrees by age, sex, birthplace, and parent's birthplace from Figure 2.

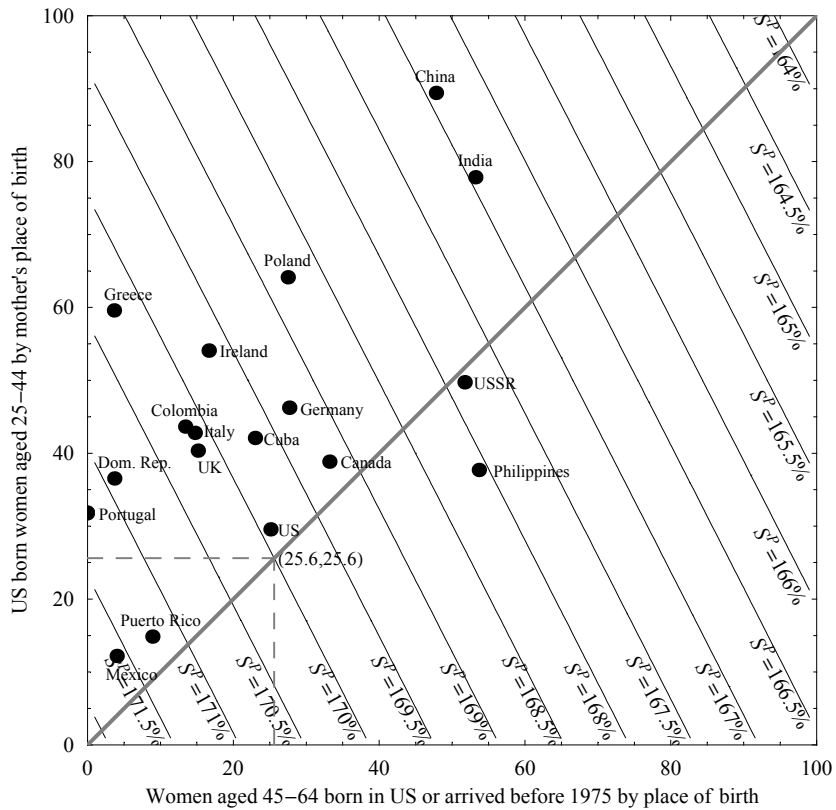
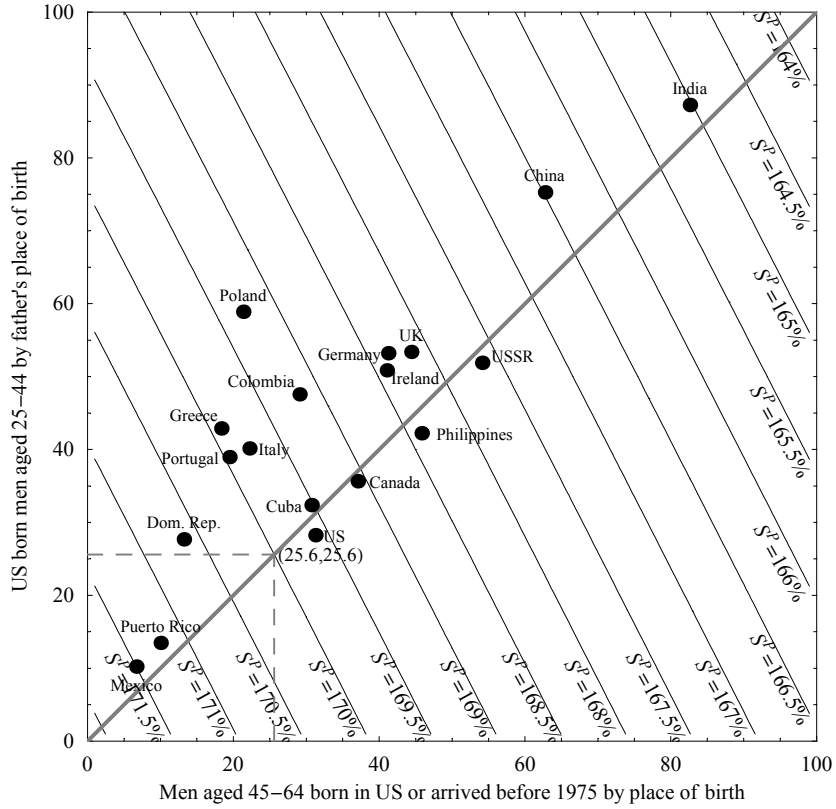


Figure 12: CES production function:  $\sigma_{UK}=\sigma_{US}=\sigma_{SK}=.67$ . Iso-curves for the values of  $S^p$  following a twenty year twenty-five percent rise in the rate of immigration. The baseline rate of return of capital is set to .05, and  $Q=25$  and  $V=45$ . Points represent the percentage of the U.S. population with four-year college degrees by age, sex, birthplace, and parent's birthplace from Figure 2.

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