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Numerical Study of the Casson Non-Newtonian Fluid Flow over a Nonlinear Stretching Sheet

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Abstract

In this paper, numerical study of Casson non-Newtonian fluid over a nonlinear stretching sheet using shooting method will be presented. The governing nonlinear partial differential equation is converted to an ordinary differential equation by using similarity transformations. This ordinary differential equation is handled numerically with the use of well-known shooting technique aided by Runge-Kutta method. For comparison of the results, MATLAB in-built solver BVP4C is used. The discussion of the results is provided in the forms of tables as well as graphically.

Keywords: Casson fluid; stretched sheet; porous medium; shooting method; porous medium; Runge-Kutta method.

1. Introduction

In the past few years, different spectral techniques have been suggested for solving problems on unbounded sections [1-2]. A brief study on some of the recent progresses in the spectral techniques for unbounded regions is shown in [3]. The resent study is about to present the numerical solutions of the Casson non-Newtonian fluid of an incompressible viscous flow over a nonlinear stretching sheet using a porous medium. Many researchers have studied the flow of this type.

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Rashidi [4] used the modified differential transform technique and Hayat and his colleagues [5] employed the modified Adomian decomposition technique. There are many techniques that also been used to solve these types of problems. Some of them are homotopy perturbation technique [6], homotopy analysis technique [7], spectral homotopy analysis technique [8], perturbation homotopy transformation technique [9], rational Chebyshev collocation technique [10] and optimal homotopy asymptotic technique [11].

2. Problem Formulation

In this section, the detailed discussion of the two-dimensional Casson nanofluid flow with MHD effects over a porous medium has been investigated. $B(x)$ is magnetic field functional normally to the flow direction. The induced magnetic field is neglected. The governing continuity and momentum equations [12-15] are given as:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(1+\frac{1}{\alpha}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u-\frac{\nu}{k}u. \quad (2)$$

Here $B(x)=B_0 x^{\frac{n-1}{2}}$, and B_0 is magnetic field strength. The boundary conditions of the nonlinear stretching sheet are

$$\begin{aligned} u(x,0)&=dx^n, \qquad v(x,0)=0, \\ u(x,0)&\to 0 \quad as \quad y\to\infty. \end{aligned} \quad (3)$$

By using the similarity transformations

$$\begin{aligned} u&=dx^n f'(z), \qquad z=\sqrt{\frac{d(n+1)}{2\nu}}x^{\frac{n-1}{2}}y, \\ v&=-\sqrt{\frac{d\nu(n+1)}{2}}\left(f(z)+\frac{n-1}{n+1}zf'(z)\right)x^{\frac{n-1}{2}}. \end{aligned} \quad (4)$$

Using the above similarity transformations (4), the non-dimensional form of Eqs. (1)- (3) are transformed as

$$\left(1+\frac{1}{\alpha}\right)f'''(z)+f(z)f''(z)-\beta f'^2(z)-(M+\lambda)f'(z)=0. \quad (5)$$

With boundary conditions

$$f(0)=0,\ f'(0)=1,\ f'(z)=0 \text{ as } z\to\infty. \quad (6)$$

Different dimensionless parameters are given as

$$\beta = \frac{2n}{n+1}, \quad M = \frac{2\sigma B_0^2}{\rho d(n+1)}, \quad \lambda = \frac{2\nu}{dk(n+1)}. \tag{7}$$

Where *u* and *v* are velocity components, α is the Casson fluid factor, σ electrical conductivity parameter, ρ is the fluid density, B_0 is the magnetic field strength, *k* is the non-uniform permeability, β is the stretching parameter, *M* is the magnetic parameter, λ is the porosity parameter and *d* and *n* are used as constants.

3. Solution Methodology

In this section, an overview is narrated for solving the nonlinear ordinary differential equations (ODEs) along with the boundary conditions. The numerical solution of the above nonlinear ODEs has been provided by using a famous method named as shooting method. The efficiency and performance of this scheme is to use for the purpose of comparison to the other numerical techniques. The obtained numerical values are compared with the bvp4c which is in-built MATLAB solver and the domain is taken as [0, z_{max}]. The representation *f* by y_1 has been implemented to obtain the IVPs from BVPs.

3.1. Shooting Method Modelling

$$\begin{cases} y_1' = y_2, & y_1(0) = 0 \\ y_2' = y_3, & y_2(0) = 1 \\ y_3' = \dfrac{\alpha}{\alpha+1}\left(-y_1 y_3 + \beta y_2^2 + M y_2 + \lambda y_2\right), & y_3(0) = i_1 \end{cases} \tag{8}$$

The numerical solution of the above converted IVPs from BVPs is presented with the use of shooting technique enhanced by Runge-Kutta scheme. The missing initial condition is represented as $y_3(0) = i_1$. The conventional Newton's scheme is operational for the improvement of missing initial conditions. The shooting method is used for the missing value i_1 until it cannot meet the tolerance, which is given below as:

$$\max\left\{\left|y_3(z_{\max})\right|\right\} < \xi \tag{9}$$

Where $\xi > 0$ is very small number and taken as $\xi = 10^{-07}$.

Three comparison tables based on the present results of $f''(0)$ with the literature results and BVP4C against the Magnetic number *M* is provided in Tables 1, 2 and 3. The comparison of these numerical results in Table. 1 shows the correctness of the scheme. The values of $\alpha = 10$ and $\lambda = 0$.

Table 1: Comparison of the numerical results for $\beta = 1$

M	Literature	Present	BVP4C
0	-1.0000	-1.0000	-1.0000
1	-1.4142	-1.4072	-1.4072
5	-2.4494	-2.4373	-2.4373
10	-3.3166	-3.3002	-3.3002
50	-7.1414	-7.1060	-7.1060
100	-10.0498	-10.0000	-10.0000
500	-22.3830	-22.2719	-22.2719
1000	-31.6385	-31.4816	-31.4816

Table 2: Comparison of the numerical results for $\beta = 1.5$

M	Literature	Present	BVP4C
0	-1.1486	-1.1430	-1.1430
1	-1.5252	-1.5177	-1.5177
5	-2.5161	-2.5037	-2.5037
10	-3.3663	-3.3496	-3.3496
50	-7.1647	-7.1292	-7.1292
100	-10.0664	-10.0165	-10.0165
500	-22.3904	-22.2794	-22.2794
1000	-31.6438	-31.4868	-31.4868

Table 3: Comparison of the numerical results for $\beta = 5$

M	Literature	Present	BVP4C
0	-1.9025	-1.8931	-1.8931
1	-2.1528	-2.1422	-2.1422
5	-2.9414	-2.9268	-2.9268
10	-3.6956	-3.6773	-3.6773
50	-7.3256	-7.2893	-7.2893
100	-10.1816	-10.1816	-10.1816
500	-22.4425	-22.3311	-22.3311
1000	-31.6806	-31.5235	-31.5235

The comparison of the literature results with the shooting and BVP4C in Tables 1,2 and 3 indicate the impressively substantial arrangements of the results, that motivate the authors to solve this problem based on Casson non-Newtonian fluid flow over a nonlinear stretching sheet. For more understanding of the designed model, the effects of different parameter on dimensionless velocity mathematically written as $f'(z)$ are given.

The plots of different parameters are given in Figs. (01-03). Fig. 1 shows the effects of magnetic parameter M on velocity profile. By increasing the values of M the velocity profile decrease and it happens because of Lorentz forces that are also called drag forces, that oppose the fluid and as a result decrements is seen in the velocity profile. Some useful findings of stretching parameter are plotted in Fig. 2. The result is by increasing the stretching parameter the velocity profile decreases and same effects have been seen for the porosity parameter on the velocity profile.

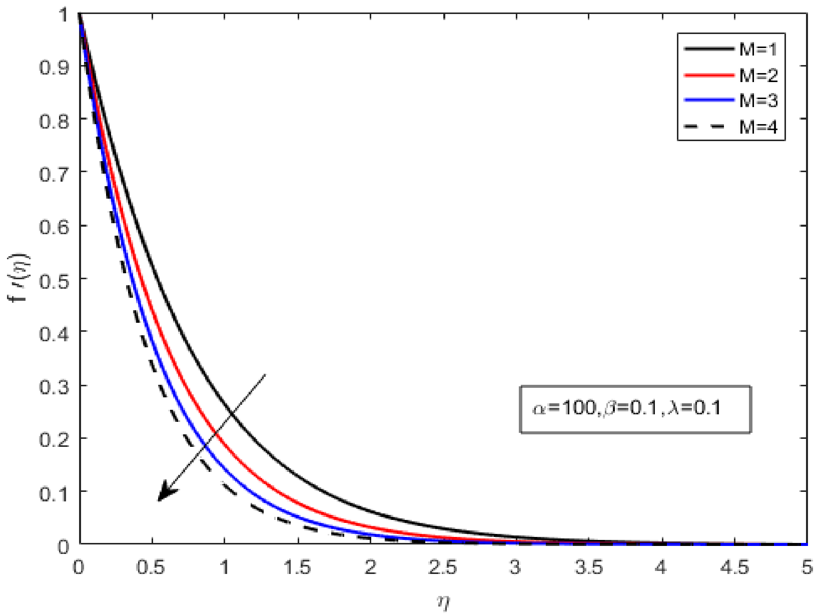

Figure 1: Variation of M on f'

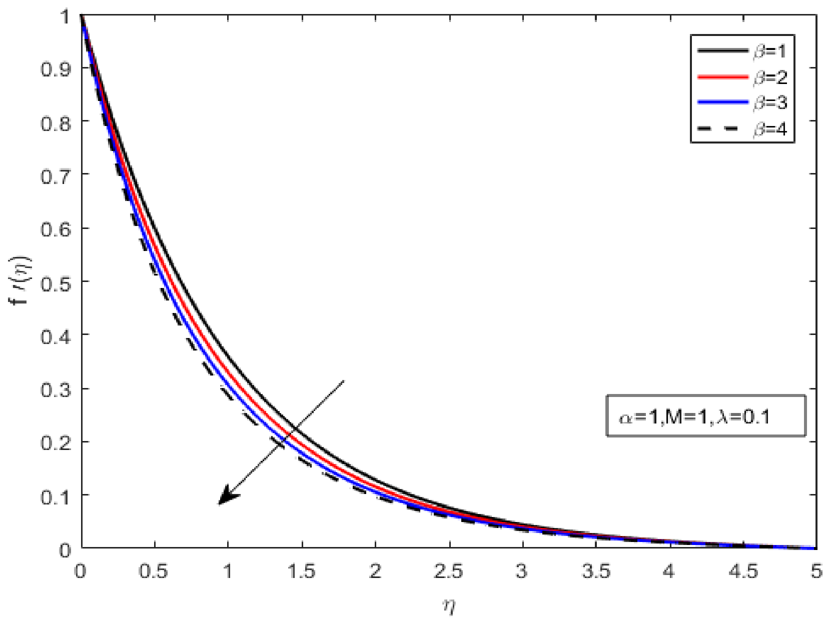

Figure 1: Variation of β on f'

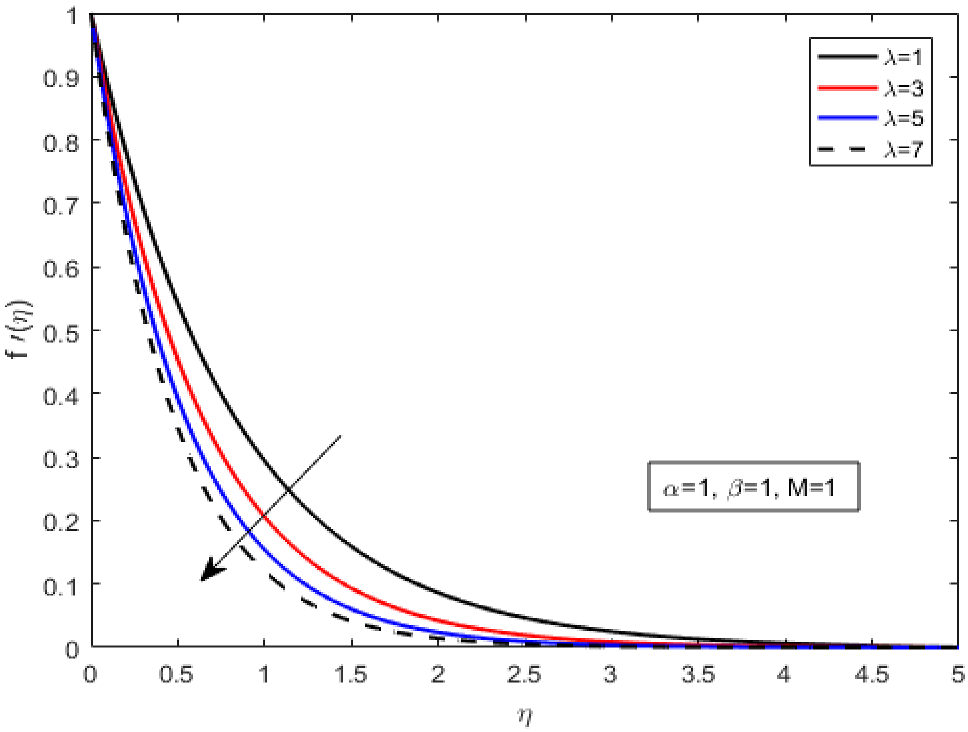

Figure 1: Variation of λ on f'

4. Conclusion

The motivation behind the present study is to solve the designed model by using shooting technique. The comparison of the present results with the literature results and Matlab built-in solver BVP4C shows the worth of the shooting method. The plots of different parameter have been drawn on the velocity profile.

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