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How much do Pre-Service Physics Teachers Know about some of the Key Operations in Vector Analysis?

Fizik Öğretmeni Adayları Vektör Analizindeki Bazı Önemli Operatörleri Ne Derece Biliyorlar?

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Abstract

The primary purpose of this study is to find out how accurately pre-service teachers use gradient, divergence and curl, which are key operations in vector analysis, and also how well they know the correct meanings of those operations. The secondary purpose of the research is to determine at what level they use scalar product and vector product, which are key algebraic operations that form a basis for the use of the aforementioned differential operations. The research was conducted with 90 pre-service physics teachers who have all passed the "Mathematical Methods in Physics I-II Courses". Students' understanding and usage level of the operations mentioned above were tested using a paper-and-pencil test (including eight tasks). The analyses of the collected data were based on quantitative and qualitative techniques. Results indicate that pre-service physics teachers have specific and considerable comprehension difficulties with the physical meanings of vector differential operations. In the paper, the conclusions of the study and implications for physical mathematics teaching are discussed.

Keywords: Mathematics education, physics education, differential operations, algebraic operations

Öz

Bu çalışmanın ana amacı, öğretmen adaylarının vektör analizinde anahtar operatörler olan gradyan, diverjans ve rotasyoneli ne derece doğru kullandıklarını ve aynı zamanda bu operatörlerin doğru anlamlarını ne kadar iyi bildiklerini ortaya çıkarmaktır. Çalışmada ayrıca, sözü edilen diferansiyel işlemcilerin kullanımı için bir temel oluşturan ve anahtar cebirsel işlemler olan skaler ve vektörel çarpımları da ne derece kullanabildiklerini belirlemek amaçlanmıştır. Araştırma "Fizikte Matematiksel Yöntemler I-II" derslerinde başarılı olmuş 90 fizik öğretmeni adayı ile gerçekleştirilmiştir. Öğrencilerin söz edilen operatörleri anlayışları ve kullanım düzeyleri Kâğıt-Kalem Testi (sekiz akademik iş) kullanılarak ölçülmüştür. Toplanan verilerin analizi nicel ve nitel tekniklere dayalıdır. Araştırmanın sonuçları, fizik öğretmeni adaylarının vektör diferansiyel operatörlerin fiziksel anlamları ile ilgili dikkate değer ve çeşitli anlama zorluklarına sahip olduklarını göstermektedir. Makalede, çalışmanın sonuçları ve fiziksel matematik öğretimine yönelik uygulamalar tartışılmıştır.

Anahtar Sözcükler: Matematik eğitimi, fizik eğitimi, diferansiyel operatörler, cebirsel operatörler.

Introduction

Mathematics and physics are disciplines that are interlinked. Mathematics serves not only as the "language" of physics, but also often verifies the content and meaning of the concepts and theories themselves. Similarly, concepts, arguments and modes from physics are applied to mathematics. Hence, physics helps the development of the field of mathematics; playing an important role in its creation and development (Tzanakis, 2002). Literature shows several studies that have analyzed the

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relationship between physics and mathematics, and also research that have investigated how well and to what extent students can interrelate what they have learnt in physics and mathematics in theoretical physics courses such as electromagnetism. To exemplify, in a study aimed to demonstrate that physics and mathematics are closely related to Differential Equation Theory, Chaachoua and Sağlam (2006) have examined the relationships between the two disciplines, modelling, the situation of modelling and the role of modelling in students' practices. Arslan and Arslan (2010) collected the views of prospective physics teachers concerning the relationship between physics and mathematics, and their abilities to model a physical phenomenon by using differential equations. Judging by the results of that study, it is clear that prospective physics teachers are informed about the importance of the connection between mathematics and physics because they stated the role of mathematics in physics as being indispensable, necessary, and useful.

Albe et al., (2001) have investigated how students make use of mathematics when studying the physics of electromagnetism. In their study, they have examined the commonly used electromagnetic terms (i.e., magnetic field, magnetic flux) and their mathematical representations and arithmetical tools. The results of the study show that the majority of the students have problems in correlating some of the concepts in electromagnetism to other concepts as well as in formulating them mathematically. Moreover, it was observed that the majority of the students had difficulties regarding the formation of associations between mathematical formalization (vectors, and integral calculus) and physical descriptions of magnetic fields and flux.

De Mul (2004) showed that although those university students have taken courses teaching them mathematical methods before, they have a hard time comprehending the mathematical concepts and skills in a physics course like electromagnetism. Moreover, even if they have understood the mathematical methods and related skills very well, they still have difficulty in applying that knowledge in physics courses.

In the same way, students are also taught everything about vector analysis, integral operations and differential operations in Mathematical Methods in Physics I-II courses before they take electricity, magnetism and electromagnetic theory courses. However, the first author who has been teaching the *Electromagnetic Theory Course* for more than four years has observed that most students have a hard time applying those mathematical operations to a case in physics, and that they cannot explain the results in terms of physics. This observation is of great importance in this current study. Also, the fact that research concerning students' use of vector differential operations is scarce; it motivated the researchers to design this study.

Vector analysis has a major role in the fields of engineering, physical sciences and mathematics. In addition, this type of analysis is frequently used in electromagnetism courses. Scalar function, vector function, vector field, gradient, divergence, curl are key terms in vector analysis. These terms are explained below:

Gradient, Divergence and Curl Operations in Vector Algebra

In a vector field F denoted in region T of space, the function formulated as follows for x, y, z points of T

$$\boldsymbol{F}(x,y,z) = P(x,y,z)\hat{i} + Q(x,y,z)\hat{j} + R(x,y,z)\hat{k} \qquad (1.1.1)$$

is known as a vector-valued function. One can briefly define the vector field $\boldsymbol{F}$by using P, Q, and R component functions as:

$$\boldsymbol{F}(x,y,z) = P\hat{i} + Q\hat{j} + R\hat{k} \qquad (1.1.2) \boldsymbol{F}(x,y,z) = P.\vec{i} + Q.\vec{j} + R.\vec{k}$$

Here, the P, Q, and R components are scalar functions.

Gradient of a scalar field: Let $f = f(x,y,z)$ be a differentiable scalar function in the region of $\Omega \subset \Re^3$. Then, the vector field that forms a gradient vector to the following vector is known as a "gradient vector field".

$$\vec{\nabla}f(x,y,z)=\frac{\partial f}{\partial x}\hat{i}+\frac{\partial f}{\partial y}\hat{j}+\frac{\partial f}{\partial z}\hat{k} \quad (1.1.3)$$

The gradient vector is in the same direction as the maximum derivative of *f* at the points of x, y, z. To illustrate, let $f(x,y,z)$ denote the temperature at x, y, z points in space. Then, to warm up as fast as possible, we should move in the direction of $\vec{\nabla}f(x,y,z)$. The gradient operator is linear, and it enables us to obtain a vector field from a scalar function.

Toexemplify,thegraphandgradientvectorfieldofthescalarfunction $f(x,y,z)=x^2+y^2-z$ is $\vec{\nabla}f=2x\hat{i}+2y\hat{j}-\vec{\hat{k}}$. The graph below shows $f(x,y,z)=x^2+y^2-z=1$ graph of that function, which is one of its level curves, as well as its gradient vector field (see Figure 1).

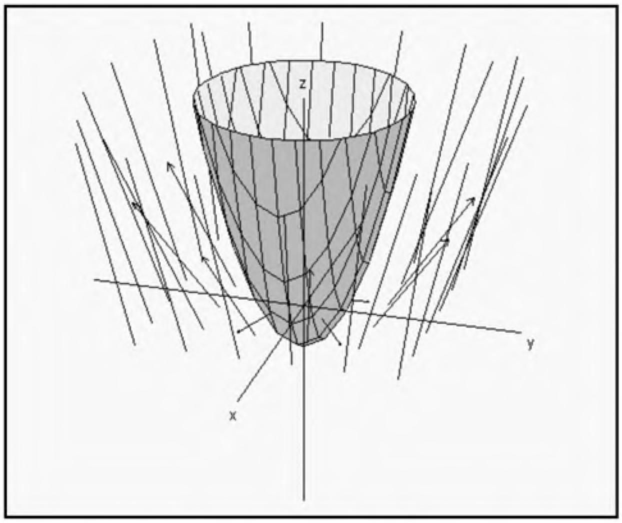


Figure 1. $f(x,y,z)=x^2+y^2-z=1$

Divergence of a vector field: If the vector function, which is as follows, in region $\Omega\subset\Re^3$ $\boldsymbol{F}(x,y,z)=P(x,y,z)\hat{i}+Q(x,y,z)\hat{j}+R(x,y,z)\hat{k}$ $\boldsymbol{F}(x,y,z)=P(x,y,z)\vec{\imath}+Q(x,y,z)\vec{\jmath}+R(x,y,z)\vec{k}$ is continuous and differentiable, then

$$div\vec{F}=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z} \quad (1.1.4)$$

is the divergence of this function. Divergence is used to obtain a scalar function from a vector function. Considering this, the meaning of the symbol for divergence could be explained as follows.

Let's discuss the neighbourhood of the point $(x_0,y_0,z_0)\subset\Re^3$ and e >0. Ω_e denotes a sphere whose exact centre is (x_0,y_0,z_0) and radius is e >0. As usual, we will accept the direction of N exterior normal to the sphere as positive.

If $div\boldsymbol{F}(x_0,y_0,z_0)>0$, then the value of the flux moving from the inside of the surface of the sphere Ω_e to outside is bigger than the value of the flux moving into the sphere.

If $divF(x_0, y_0, z_0)$ <0, then the value of the flux moving from the inside of the surface of the sphere Ω_e to outside is smaller than the value of the flux moving into the sphere.

When $divF(x_0, y_0, z_0)$ =0, then the value of the flux moving from the inside of the surface of the sphere Ω_e to outside is equal to the value of the flux moving into the sphere.

In fluid mechanics, under the circumstance of

$$divF(x_0, y_0, z_0) = 0 \quad (1.1.5)$$

the material is known as "incompressible material" (Halilov *et al.*, 2008).

For instance, the vector function $F(x, y, z) = -y\hat{i} + x\hat{j}$ is a vector field including the tangent vectors of the circles whose centre is the origin with a radius of r.

Curl (Rotational) of a vector field: The vector field of $F(x,y,z) = P(x,y,z)\hat{i} + Q(x,y,z)\hat{j} + R(x,y,z)\hat{k}$ $F(x,y,z) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$ is called the rotation vector or curl, which is described as follows:

$$curlF = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k} \quad (1.1.6)$$

$$curlF$$

The $curl\vec{F}$ vector physically means that if F F is the velocity vector of a liquid flow, the $curl\vec{F}$ vector sets the axis, which passes through x, y, z points, of the liquid rotating or curling as fast as the angular velocity of the rotation itself at the x, y, z points (the point where the vector is different from 0) (Edwards and Penney 2001). For example, by considering the vector field of $F(x,y,z) = -y\hat{i} + x\hat{j}$, the $curlF = 2\hat{k}$ vector at the point of (0,0,0) has been drawn (see Figure 2).

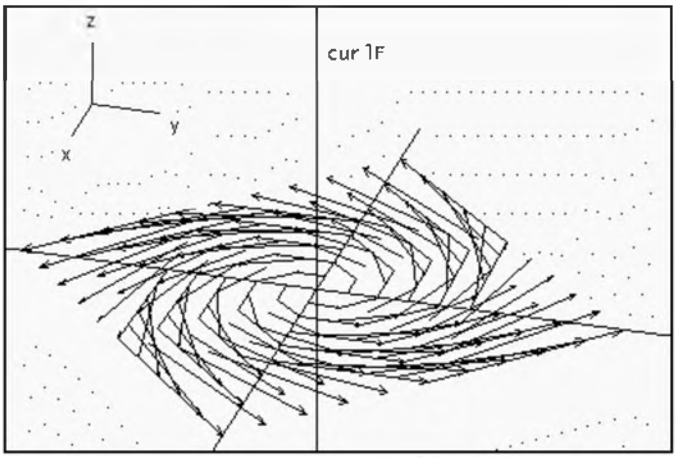


Figure 2. $F(x,y,z) = -y\hat{i} + x\hat{j}$

Present Study

This study aims to reveal how pre-service physics teachers make use of the change in physical quantities in 3D space and the three main vector differential operators (i.e., gradient, divergence, and curl) for a physical quantity (e.g., temperature, electric field, magnetic field); how they interpret the operation performed on the quantity physically. Moreover, this study also aims is to find out at what level the pre-service physics teachers use scalar and vector products, which are two of the major algebraic operations. With these purposes in mind, the researchers attempted to answer the following research questions:

How do pre-service physics teachers interpret gradient, divergence and curl operations on a physical quantity in terms of physics?

At what level do pre-service physics teachers use gradient, divergence and curl in vector analysis?

At what level do pre-service physics teachers use scalar product and vector product, the major algebraic operations in vector analysis?

Method

Both quantitative and qualitative research methods were used in this study. The sample included 90 pre-service physics teachers who were enrolled in the Department of Physics Education at a public university in Turkey. Prior to the study, the participants took Physics I-II (based on mechanics topics), Physics III-IV (based on electrics and magnetism), Modern Physics I-II, Thermodynamics, Mathematical Methods in Physics I-II, Optics, and Vibration and Waves courses at the same university. All of the candidate teachers that participated in the study were the ones who were successful in "Mathematical Methods in Physics I-II." The *data* reported in this study are for the 2008-2009 and 2009-2010 *academic years.*

Data were collected through a paper-and-pencil test (PPT). Students' understanding and usage level of important vector differential calculus and algebraic operations in physics were assessed using this paper-and-pencil methodology. Eight tasks were prepared to solve the sub-problems of this study. These tasks include questions regarding gradient, divergence, curl (rotational), scalar and vector product operations in vector algebra and their applications to some scalar or vector fields. These tasks controlled by three different experts of physics and mathematics for scope validity. The Paper-and-Pencil Test (PPT) was conducted after the instruction of the topics concerning vector algebra. The students completed the PPT under examination conditions during a class lasting 60 minutes.

The data obtained from the test were analyzed quantitatively and qualitatively by the researchers. The data were evaluated and categorized as "complete understanding", "limited/ incomplete understanding" and "no understanding". To determine the degree of agreement between the two evaluations, the Pearson correlation coefficient was calculated. Reliability was found to be 0.90.

Results

In order to find out students' understanding regarding vector differential calculus and algebraic operations, the responses to the PPT have been analyzed in detail and can be seen underneath the related question. The quantitative data have been given in percentage and frequency.

Findings Obtained from the Physical Meaning Questions

Question 1: What is physical meaning of the gradient of a scalar function (or field)? Please explain with example, by drawing the related/necessary figures.

Quantitative and Qualitative Analysis of Responses to Question 1

Clearly, 95% of the students have stated that the expression resulting from the interaction of a del operator ($\vec{\nabla}$) with a scalar function denotes the gradient of that scalar function. The explanations that the pre-service physics teachers made about the physical meaning of a gradient can be divided into two groups. The ones who have explained accurately (30%) stated that the gradient of a scalar function is a vector field which faces the direction of the highest degree of rise in the scalar field, and whose magnitude is the greatest rate of change. On the other hand, the students (65%) who have given a wrong answer claimed that scalar function is a vector quantity that shows the direction of increase and magnitude of the gradient function. Here, their mistake is to think that a scalar function may change only in one direction, and that direction definitely signifies an increase. Excerpts from two students' exam papers illustrate these findings as follows:

Student (S_1): "Gradient reflects the velocity of change of a scalar function in vectors. It is in the direction of increase."

S_{25}: "Gradient can be applied to scalar quantities like temperature. It informs us about the increase and decrease in the temperature. Gradient is a quantity signifying that direction."

Question 2: What is physical meaning of the divergence of a vector function (or field)? Please explain with example, by drawing the related/necessary figures.

Quantitative and Qualitative Analysis of Responses to Question 2

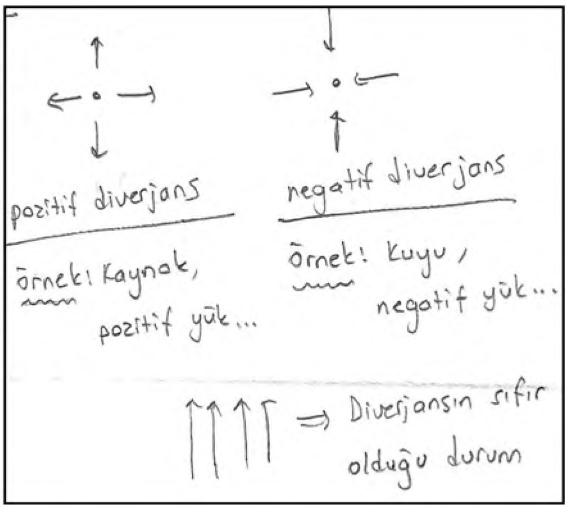
The majority of the students (90%) have expressed that the term divergence means getting farther away from each other, and that it is the scalar product of the del operator with the vector field, $div\vec{F}$, where $\vec{F}$ denotes a vector field. However, it is understood from their exam papers that they have difficulty in explaining the physical meaning of the scalar quantity that they have found. 20% of the students have stated that the divergence of a vector function measures how far the vector lines diffusing from a given point diverge. 70% of them wrote misstatements claiming that the strength of a vector field depends on how far the function is away from the selected point. Excerpts from students' exam papers illustrate these findings as follows:

S_5: "....the divergence of a vector quantity shows us how much the strength will decrease as it gets farther away from the point."

S_{12}: "...divergence means to move away. In other words, divergence tells us how far a function has moved away from a selected point P."

S_9: "Divergence shows diffusion from a given point. A divergence could be positive, negative or zero." (see figure 3)

positive divergence Example: source, positive charge.



negative divergence Example: sink, negative charge

The state where divergence is zero.

Figure 3. Student 9's diagram for divergence of a vector field

Question 3: What is physical meaning of the curl of a vector function (or field)? Please explain with example, by drawing the related/necessary figures.

Quantitative and Qualitative Analysis of Responses to Question 3

Almost all of the students have stated that the vector product of the del operator with the vector field (the vector product of the del operator with the vector field gives the $curl\vec{F}$, where $\vec{F}$ signifies a vector field) denotes the curl ($curl\vec{F}$) of that vector field where F denotes a vector field, and that the quantity found is a vector itself. Yet, it has also been observed that they cannot express the physical quantity found very well. 10% of them mentioned that the $curl\vec{F}$ vector can be calculated by the right-hand rule. Also, that vector is perpendicular to the vector field, and its strength is equal to the velocity of rotation of the vector field. Results show that 90% of the students wrote in their exam papers that the curl of a vector field denotes the circulation of that specific vector around a point. Some of the students (60%), in their drawings, considered the point as a starting or final point, and drew the circulation like a spiral starting from that point. However, the curl is represented by a vector throughout the field, and the traits of this vector (length and direction) signify the rotation at that particular point. Excerpts from students' exam papers illustrate these findings:

S_{19}: "Rotational; in other words, curl is the measure of circulation of a velocity field v. A vector field whose curl is zero is called rotational. For instance, the rotation of electric field is zero."

S_1: "Curl is the measure of circulation of a vector function around a point. To exemplify, if the vector field is the flow velocity of a moving fluid, then sea vortex is different from the situation where the curl is zero. Besides, a magnetic field has a tendency to circulate around a point, too." (see figure 4a and 4b)

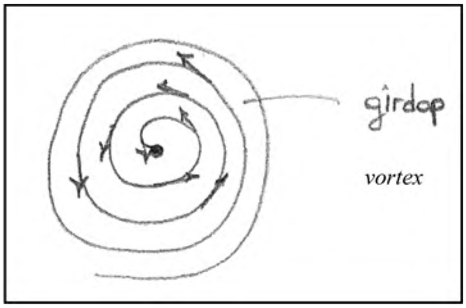


Figure 4(a). Student 1's diagram for the curl of flow velocity of a moving fluid

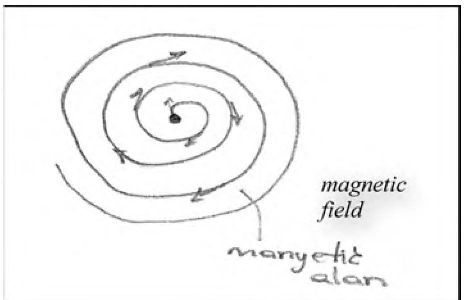


Figure 4(b). Student 1's diagram for the curl of a magnetic field

Here, it is clear that the majority of the students could not grasp the length and direction of the vector $curl\boldsymbol{F}$.

Findings Obtained from the Use of Grad, Div and Curl Operations

Question 4:Find the gradient of $f(x,y,z) = x^2 + y + z^3$ *at the point of (2,1,1). Please comment on your finding.*

Question 5: Find the divergence of $\vec{F}(x,y,z) = 2x\hat{i} + y\hat{j} + z^2\hat{k}$ *at the point of (2,1,1). Please comment on your finding.*

Question 6: Find the curl of $\vec{F}(x,y,z) = 3x^2\hat{i} + 2z\hat{j} - x\hat{k}$ *at the point of (2,1,1). Please comment on your finding.*

Quantitative and Qualitative Analysis of Responses to Questions 4, 5, and 6

When the students' exam papers were assessed, it has been seen that almost all of them could write the equation *gradf* correctly in the 3D Cartesian coordinate system, could do the partial derivation operations on the function f perfectly, and could find the gradient of the function at the given point (i.e., $gradf = 4\hat{i} + \hat{j} + 3\hat{k}$) correctly. Yet, unfortunately, most of the students who could find the correct answer were unable to comment on the result. Moreover, few students were unsuccessful in doing the partial derivation operations.

Among all, 80% of the students could write the equation $div\vec{F}$ correctly in 3D Cartesian coordinate system, and could calculate the divergence of the function given (i.e., $div\vec{F} = 5$) right. Almost half of the students who answered correctly could explain the result correctly. The rest of them calculated incorrectly as they wrote the equation wrong.

Also, 80% of the students could write the equation $curl\vec{F}$ correctly in 3D Cartesian coordinate system, and could calculate the curl of the function given (i.e., $curl\vec{F} = -2\hat{i} + \hat{j}$) correctly. However, almost all of them failed to explain the result. 20% of them, although able to write the equation $curl\vec{F}$ correctly, made some simple calculation errors and were therefore unable to explain the result.

These findings indicate that the students did not have much difficulty in doing the mathematical operations regarding the act of the del operator on a given scalar or vector function in 3D Cartesian coordinate system; however, they could not explain their findings in terms of physics very well.

Findings Obtained from the Questions of Algebraic Operations

The pre-service physics teachers were asked the following questions so as to understand at what level they can use the basic, non-differential algebraic operations in vector algebra. The questions probe the scalar product (multiplication of two vector fields, yielding a scalar field: $\vec{A}.\vec{B}$) and vector product (multiplication of two vector fields, yielding a vector field: $\vec{A} \times \vec{B}$), which are the keystones of basic algebraic operations.

Question 7: Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ be two different vectors. What is the scalar product of these two vectors? Is the quantity found scalar or vector? How can we calculate the angle between these two vectors?

Question 8: Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ be two different vectors. What is the vector product ($\vec{A} \times \vec{B}$) of these two vectors? Is the quantity found scalar or vector? Can you draw the quantity you have found in 3D Cartesian coordinate system by using vectors $\vec{A}$ and $\vec{B}$?

Quantitative and Qualitative Analysis of the Responses to Questions 7 and 8

When the papers of the pre-service teachers were evaluated, it was seen that all the students could write the equation for the scalar product asked in question 7 correctly (i.e.,

$\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|.\text{cosq} = A_x B_x + A_y B_y + A_z B_z$), that the product is scalar, and it can be calculated from the angle between these two vectors (θ) (from the equation of the angle between those two vectors $\text{cosq} = (A_x B_x + A_y B_y + A_z B_z)/|\vec{A}|.|\vec{B}|$). When the researchers examined the students' answers for question 8, it was observed that all the students understood that the quantity obtained as a result of the vector product is a vector itself (e.g., vector $\vec{C}$); almost all of them could write the vector product in the form of a 3x3 determinant; and that few of them made mistakes with plus (+) and minus (-) signs. In their exam papers, 90% of the students both explained and showed by drawing that the vector obtained as a result of the vector product is perpendicular to the plane created by the vectors A and B; and also that its direction can be found by benefiting from the right-hand rule.

Discussion

The primary purpose of this study is to see how accurately pre-service physics teachers use gradient, divergence and curl, which are key operations in vector analysis, and also how well they know the accurate meanings of those operations. The secondary purpose of the research is to determine at what level they use scalar product and vector product, which are key algebraic operations that form a basis for the use of the aforementioned operations. It can be deduced that although pre-service physics teachers know that they can form a vector field from a scalar field, or vice versa by making use of gradient, divergence and curl operations, they still have considerable difficulty in comprehending the various physical meanings of vector algebra operations.

It has been observed that they are comfortable with doing all the mathematical operations concerning the interaction of a del operator with any scalar or vector function in 3D Cartesian coordinate system.

It has been diagnosed that 65% of the students have misunderstood one thing about the physical meaning of gradient of a scalar function. They describe the gradient of a scalar function as a vector quantity indicating the direction and magnitude of increase of a function. Unfortunately, they think that a scalar function can change only in one direction, and that direction absolutely shows the increase.

Moreover, the researchers have realized that the majority of the students fail to understand the physical meaning of the divergence of a vector function. Some students could not explain it very well and stated that divergence measures how far the vector lines diffusing from a given point diverge. Some other students, on the other hand, wrongfully claimed that the strength of a vector field depends on how far the function is away from the selected point.

Among all, 90% of the students wrote in their papers that the curl of a vector field shows the circulation of that vector around a point. That is, they believe that the curl of a vector function is a quantity that signifies the circulation of that vector function around a point; in other words, whether there is circulation or not. Some of the students, in their drawings, considered the point as a starting or final point, and drew the circulation like a spiral starting from that point. However, at every point in the field, the curl is represented by a vector. Besides, they failed to explain the direction and strength of the curl vector clearly.

On the other hand, the pre-service physics teachers have pretty good knowledge of the geometrical consequences of scalar product and vector product of two vectors. The findings prove that they know that a scalar product gives rise to a scalar quantity, and a vector product brings about a new vector. They are also aware that this new vector is perpendicular to the plane created as a result of the product of vectors, and also that its direction can be determined by using the right-hand rule. When all these findings have been evaluated, the researchers have come to the conclusion that pre-service physics teachers have sufficient knowledge of the key algebraic operations fundamental for understanding the use of gradient, divergence and curl operations. In addition, they are comfortable with numeric practices of differential vector operations. Yet, they have considerable difficulty in both understanding and expressing the physical meanings of gradient, divergence and curl.

The findings of the study show parallelism with the findings of a similar research by Saarelainen et al., (2006) to a certain degree. Saarelainen et al., (2006) have revealed that although freshman students generally use vectors in mathematics successfully, they often fail to do well with vectors in electromagnetism courses. They have also observed that freshman students find some of the new concepts such as flux integral and contour integral, which are essential for learning electromagnetism and used in vector algebra, too complex. These findings by Saarelainen et al., (2006) show parallelism with the findings of Dunn and Barbanel (2000), and Nguyen and Meltzer (2003).

It is believed that a likely reason for this problem could be the possibility that the key differential operators used in physics courses such as Electromagnetic Theory and vector algebra are taught with a completely mathematical approach in "Mathematics" and "Mathematical Methods in Physics" courses; that is, without forging a link between mathematics and physics. For this reason, while the pre-service physics teachers are good at numeric use of those operators, they are not very good at understanding and expressing the consequences to result from their implementation with physical quantities.

Conclusion

This study provides some evidence that pre-service physics teachers have great difficulty in comprehending the physical meanings of gradient, divergence, and curl (i.e., the effect of these operators on scalar and vector fields), which are the major operators of vector algebra; in other words.

As a result, it can be said that although the students are able to use these operators and the algebraic operations that are key to their use mathematically, they cannot understand what those operators mean in physics. The required instructional interventions must immediately be made so that the students can realize that these are "operators" operating on a function, and that they reach significance as long as they operate on mathematical structures such as a set, scalar function, and vector function; in other words, they make no sense at all on their own. Namely, the results of an operation must be assigned a meaning after that operator (i.e., addition, extraction, differential, integral) had been performed on main problems in chemistry, physics, economics and engineering. What is more, it is of great importance to put the difference between the operations performed by single variable functions and multivariable functions. While a single variable function can increase only in one direction on a surface; that is, on a function graph with two or more variables, the increase could be in more than one direction. Then, underlining that the increase is in the direction where there is maximum increase, and drawing attention to the functions increasing in more than one direction is of crucial importance.

By modernizing teaching methods, students can be taught how to draw basic graphs in 3D space environment. This will enable them to visualize specific areas of the graphs of the functions they are working on. Therefore, various graphing software could be used for this purpose, and students could be trained about how to use them.

Dunn and Barbanel (2000 present a math/physics course model that can be taught students on the condition that it is integrated with electricity and magnetism related calculus topics. In this model, mathematical subjects such as vector fields, scalar fields, partial derivatives, directional derivatives, surface integrals, gradient, divergence, curl, linear algebra have been integrated with physics subjects such as electric fields, electric flux, Gauss's law, electric potential, magnetic fields topically.

In future studies concerning this issue, (a) students' comprehension difficulties can be analyzed by using deep interview method, (b) new research can be done about other physics topics that are common subjects of both physics and mathematics by benefiting from topical integration method, and the effects of such a model on students' affective behaviours (their attitude towards the course and learning in general, their motivation, etc.) besides their academic success, (c) the effects of teaching with use of math drawing programmes on students perception of the relationships between gradient, divergence and curl operators could be explored.

References

Albe, V., Venturini, P., & Lascours, J. (2001). Electromagnetic concepts in mathematical representation of physics. *Journal of Science and Education Technology, 10,* 197–203.

Arslan, A. S., & Arslan, S. (2010). Mathematical models in physics: A study with prospective physics teacher. *Scientific Research and Essays, 5*(7), 634-640.

Chaachoua, H., & Saglam, A. (2006). Modelling by differential equations. *Teaching Mathematics and its Applications, 25*(1), 15-22.

De Mul, F. F. M., Batlle, C. M., De Bruijn, I., & Rinzema, K. (2004). How to encourage university students to solve physics problems requiring mathematical skills: The 'adventurous problem solving' approach. *European Journal of Physics, 25,* 51–61.

Dunn, W. J., & Barbanel, J. (2000). One model for an integrated math/physics course focusing on electricity and magnetism and related calculus topics. *American Journal of Physics, 68,* 749-757.

Edwards, C. H., & Penney, D. E. (2001). *Fen-Mühendislik Fakülteleri ve Yüksekokul Öğrencileri İçin Matematik Analiz ve Analitik Geometri (Cilt II).* Ankara: Palme.

Halilov, H., Hasanoğlu, A., & Can, M. (2008). *Yüksek Matematik: Tek Değişkenli Fonksiyonlar Analizi (Cilt II).* İstanbul: Literatür.

Nguyen, N-L., & Meltzer, D. E. (2003). Initial understanding of vector concepts among students in introductory physics courses. *American Journal of Physics, 71,* 630–638.

Saarelainen, M., Laaksonen, A., & Hirvonen, P. E. (2006). Students' initial knowledge of electric and magnetic fields—more profound explanations and reasoning models for undesired conceptions. *European Journal of Physics, 28,* 51–60.

Tzanakis, C. (2002). On the relation between mathematics and physics in undergraduate teaching. *Paper presented at the 2nd International Conference on the Teaching of Mathematics July 1-6,* Greece.