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A REVIEW OF BALL LIGHTNING MODELS

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ABSTRACT

Ball lightning is a natural phenomenon that occurs in the atmosphere. However due to its brevity and rarity, its occurrence is not well understood. Three models based on electromagnetic properties are discussed in this paper to explain the rare phenomenon of ball lightning. The first model incorporates the idea of electron bunching, electrons moving with different velocities. This creates a plasma bubble by recombining electrons with ionized gas to form plasma that is stabilized by a standing microwave. The second model explains the idea of streamers being tangled and linked in a magnetic field while stabilized by the conservation of helicity. The third model is a lab created skyrmion that when evaluated exhibits qualities similar to ball lightning.

Keywords: ball lightning, electron bunching, streamers, skyrmion

INTRODUCTION

Ball lightning is a rare natural phenomenon where a ball of light about the size of a grapefruit appears momentarily before fading away or exploding. Because of the brevity and rarity of ball lightning, it has rarely been recorded, and most information comes from eyewitness accounts. Despite the fact that almost all descriptions of ball lightning are from eyewitnesses, sources that can be unreliable, witness accounts are surprisingly consistent (Jerauld et al. 2008). Witnesses say that ball lightning appears suddenly, ranging in size from that of a golf ball to a beach ball. It is reported to be white, blue, or orange in color, with glowing tendrils (Keul and Diendorfer 2018; Lee et al. 2018; Meir et al. 2013). It moves erratically, sometimes followed by smoke trails. It is not directly harmful to humans, but a resulting explosion can be. Additionally, ball lightning shows several unusual properties including the ability to travel through solid objects such as windows and walls; it has even appeared inside airplanes. It has also been seen originating or terminating at electrical devices such as radios, electrical sockets, and power transformers (Keul and Diendorfer 2018). The aforementioned are some of the sparse observations of ball lightning properties. However, the cause of ball lightning is even more elusive. It is believed to be associated with cloud-to-ground lightning and, in addition, the luminosity and affinity for electrical hardware suggests that electromagnetic fields play a role (Nikitin et al. 2018). Here, we present background information on the electromagnetic environment and conditions that could potentially lead to ball lightning, as well as several models that may explain this phenomenon.

Atmospheric Conditions

It is generally agreed upon that ball lighting is associated with cloud-to-ground lightning (Meir et al. 2013). Therefore, the first step in understanding ball lightning is to understand its precursor cloud-to-ground lightning. In the form of lightning studied here, the most general definition is an extreme case of static discharge occurring between the cloud and the ground. Inside the cloud, charges separate from each other via friction between water molecules. Positive charges separate towards the anvil of the cloud, while negative charges separate towards the base of the cloud. At first, air acts as a natural insulator between the cloud and the ground. However, when opposite charges reach a peak, the insulating factor of air is nullified. A channel of negative charge called the "stepped leader" descends to the ground, by an arbitrary path of least resistance. Upward streamers, of a positive charge in this instance, reach out with their own channels. The electrical transfer between the stepped leader and the upward streamers is the result formally known as the return stroke. The charged regions become temporarily equalized, until opposite charges build back up and overcome the insulating nature of air once more (Peer and Kendl 2010). Normally, the ground has a slight negative charge, which is an inherent property of the Earth. However, because the base of thunderstorms is negatively charged, a natural repulsion occurs whenever a thunderstorm forms over ground. This leaves a positive charge on the ground whenever thunderstorms are present. Typically, there is a steady current of electrons flowing upwards from the Earth. Thunderstorms reverse the charge of this flow which can be modeled by the following equation:

$$I_{up} = N_{storm}\ x\ I_{storm} \tag{1}$$

where I_{up} is the total upward current, N_{storm} is the number of global thunderstorms, and I_{storm} is the upward positive current to the ionosphere that a single thunderstorm is estimated to produce. This equation demonstrates how many amperes of positive charge are transferred upwards from Earth via thunderstorms assuming this is negatively charged lightning (Peer and Kendl 2010). In the case of positively charged lightning, which occurs between 5% to 10% of the time, the separation of charges due to the friction of water molecules still follows the same process (Rañada et al. 1998). However, a positively charged stepped leader descends to the ground instead. This stepped leader occurs in the anvil instead of the base, as with negatively charged lightning. Because positive lightning occurs in the anvil, it must be more intense than negative lightning, as it has to travel through more air. Most cases of positive lightning only have one return stroke, while negative lightning typically consists of multiple return strokes. On average, a negative bolt of lightning has 500 MJ of energy, transfers 15 C of electrical charge, and delivers 30,000 A of current. The peak current of a positive bolt of lightning can be 300,000 A to 400,000 A, transfers "several hundreds" of coulombs of electrical charge, and has a voltage of a billion volts (Rañada et al. 1998; Turner 2003). Because positive lightning happens more rarely, robust data on positive lightning is not readily available. These two separate charges are important for the distinction of cloud-to-ground lightning; however, eyewitness reports seem to indicate that the electric charge of the lightning involved is inconsequential to the formation of ball lightning. In an assessment of ball lightning cases done by correlating the eyewitness accounts of ball lightning in Europe, roughly 55% of those cases correlated with positive cloud-to-ground lightning (Meir et al. 2013). While the occurrence of ball lightning does seem to happen more frequently with positively charged lightning, the recurrence at which positively charged lightning occur is not significant enough to point to the charges being the sole cause of ball lightning's formation. To understand the relationship between cloud-to-ground lightning and ball lightning, it might prove more beneficial to look at the magnetic and electric properties of both cases.

The majority of information for the electric and magnetic properties of lightning focused on in the remainder of this section comes from the International Center for Lightning Research and Testing, now closed, which was located in Gainesville, Florida. The center gathered data from lightning strikes that occurred in its vicinity. Parameters studied include the electric field, magnetic field, electric field derivative, and magnetic field derivative. All lightning strikes that the International Center for Lightning Research and Testing measured occurred within 1 km^2 of the facility. The center included six electrical field stations, two magnetic field stations, four magnetic field derivative stations, four electrical field derivative stations, and two optical stations. It should be noted that there were stations that have more than one function, and there were only six physical stations available in this project. Data was gathered on 18 negative cloud-to-ground lightning strikes, one positive cloud-to-ground lightning strike, one subsequent positive cloud-to-ground lightning strike, and several subsequent negative cloud-to-ground lightning strikes.

Based on the center's 18 negative first strokes of cloud-to-ground lightning, it appears that the stepped leader's electric field increases at very close distances, and remains constant at further distances until $t = 0$ μs, the instant of the return stroke. Afterwards the electric field increases greatly during the return stroke phase before leveling off. Increasing or decreasing the time of the data varies the averages slightly, but at 100 μs the average can range anywhere from 39.5 to 18.9 kVm^{-1}, with the higher voltage being at the 100–200 m range and the lower voltage being at the 900–1100 m range with the other range of voltages occurring in between those distances (Jerauld et al. 2008). It should be noted that the leveling off of the voltage should be expected, as lightning acts as an equalizer between two areas of differing charges. Once the exchange of charges reaches equilibrium, so does the electrical field. The magnetic field has a similar data trend, with subtle differences. The magnetic field the stepped leader produces is constant at first until a signature pulse occurs, where there is a small jump in the magnetic field. This small spike in the magnetic field occurs with a small lag before the streamers connect to the stepped leader (i.e. $t = 0$). A peak is reached shortly afterwards during the return stroke phase, and typically decreases afterwards, although there may be subsequent peaks during the overall decrease. The average median of data for the magnetic field for all lightning strikes in this survey is not available, but between six lightning strikes the maximum peak after $t = 0$ ranged from 30.3 μT to 14.0 μT (Uman and Krider 1989). It should be noted the peak occurs slightly after $t = 0$, which is due to the current of lightning strengthening the magnetic field during the stepped leader and upward streamer exchange of charge. As the current of the lightning strike gets closer to equilibrium, the magnetic field starts to decrease.

MODELS

Electromagnetic Radiation by Relativistic Electron Bunch

Because of the unpredictable and variable characteristics of ball lightning, it is hard to find one model that can explain all observations. In this section we focus on one theory of ball lighting involving a concept called an electron bunch. We chose to use these ideas in order to answer many of the variabilities of ball lightning all at once. A proposed theory by H.C. Wu invokes the formation of a plasma bubble created from an electron bunch at the tip of a lightning strike colliding with the ground. This electron bunch then causes the ionized air in the area to be electrically bound and creates a spherical plasma bubble or plasma shell (Wu 2016). The bubble is held together by radiation pressure in the microwave range of the electromagnetic spectrum. We further explain these concepts that set up the initial conditions for ball lightning to occur.

Electron Bunch

An electron bunch is said to occur right after a lightning strike. At the tip of the stepped leader of a lightning strike, a bunch of electrons are accelerated by x-ray bursts which cause the electrons to accelerate. These bursts have enough energy to accelerate approximately 10^{11} electrons (Wu 2016). This causes the group of electrons to avalanche down. Electrons moving at relativistic speeds can have transition radiation through media that account for the energies of the x-ray bursts (Wu 2016). The striking of the electron bunch to the ground excites an intense microwave radiation. The theory of H. C. Wu proposes a standing microwave model that keeps the plasma bubble formed after the strike. This idea is imposed to explain the seamless passing of ball lightning through glass plates as well as other characteristics. The size of the plasma bubble is about the size of the electron bunch with and an additional scale height of the plasma shell. This implies that the bunch is randomly created and not dependent on there being a certain number of electrons. There may be a threshold in which there is a maximum number of electrons and a minimum needed to form a bunch, but there is not a certain number of electrons needed. The bunch moves uniformly together; however, each individual electron at a given time will have a different velocity than at another time. It is as if the electrons are electromagnetically knotted together, constantly repelled by each other in all directions, effectively leading to a continuous movement of the ball lightning itself.

Microwave Generation

There is no evidence that there are strong microwaves generated from a lightning strike itself. There is, however, evidence that transition radiation can be generated through medium surfaces when an electron passes in or out of the medium (Wu et al. 2016). The radiation from the electron bunch can be coherent, in which the waves have a constant phase shift relative to each other while also having about the same frequency. As the electron bunch approaches relativistic energies, the electric fields of each individual electron are approximately equal to the magnetic field created multiplied by the speed of light,

$$E \simeq cB. \tag{2}$$

In other words, the fields are mostly transverse, which makes them similar to the fields in an electromagnetic wave. For this case, the coherent transition radiation can be viewed as the reflected wave on the electron bunch from the medium surface. The radiation energy can be written as

$$W_{TR} = R_{\epsilon}\ x\ W_{bf} \tag{3}$$

where $R = |(\sqrt{\epsilon} - 1)/(\sqrt{\epsilon} + 1)|^2$ is the Fresnel reflection formula, $W_{bf}$is the total energy of the bunch field, and ϵ is the permittivity of the medium. The radiation is strongest for a perfect conductor where the permittivity approaches infinity. A Boltzmann distribution of the electron bunch can produce almost the same transition radiation pulse as a monoenergetic one (Wu et al. 2016).

Electromagnetic Knot

As previously stated, the phenomenon of ball lighting is a debated topic due to its rare tendencies and its unpredictable nature. Scientists continue to struggle with the explanation of its occurrence. To continue the theory of ball lightning, a new model of its formation is expressed by Rañada et al. in the article *A Model of Ball Lightning as a Magnetic Knot with Linked Streamers*. This article provides a new basis and explanation of ball lightning. This model explains an electromagnetic knot with linked streamers and magnetic field lines tangled together, being possible through the conservation of helicity. This new model is able to incorporate experimentally tested results of ball lightning characteristics, as well as the varying results of different ball lightning observations such as lack of radiation from the ball, explosion, and severe burns from extremely high temperatures. There tends to be inconsistencies in the observations and this model of an electromagnetic knot seeks to explain the phenomenon. Initial conditions begin with a lightning strike with high enough electric potential difference to create a closed looped streamer. Experiments conducted by Alexeff and Rader (1992) concluded that closed loop streamers can be created in the presence of a high voltage of around 10 MV. This amount of electric potential difference can be produced by a lightning strike in certain cases. The formation of a streamer occurs when a nonlinear ionization wave propagates into a previously unionized region, creating a nonequilibrium plasma behind it (Abrahamson and Dinniss 2000). So, as stepped leaders approach the ground the electric field produced increases in magnitude and once it reaches a high enough value the charges in the ground react by letting off streamers of highly conducting plasma. These streamers are assumed to have ionized gas trapped inside of a thin tube of highly conductive plasma. Due to the high conductivity of the current inside the streamer the temperature inside the system is believed to be ranging from 16,000 K and 19,000 K with little radiating temperature outside of the streamer (Wu 2016). In this model the streamers are assumed to have a diameter of 50–100 μm with a large current (infinite) produced on the inside and zero current on the outside (Wu 2016). If these streamers are produced within a voltage of roughly 10 MV they can short circuit and close off. When a lightning strike occurs, multiple streamers are created and as a result the formation of the closed streamers link together acting as a highly conductive coil with a given magnetic and electric field. The magnetic field of the system is described below.

$$B = -\left[\frac{\sqrt{a}sin^2(\pi R)}{\pi L^2 R^2}\right]\left[(ncos\psi\ x\ e_r) - n\pi Rcot(\pi R)sin\psi\ x\ e_{\psi}\right) + (\pi Rsin\psi\ x\ e_{\psi})] \tag{4}$$

For explaining conservation of helicity in this model, the electric field is set equal to zero to allow for simpler mathematical calculations. To create an electromagnetic knot for this model there must be both an electric and magnetic field produced to link and tangle the system together. In this case a lightning strike cannot create a magnetic field, but the continuous distribution of current in the closed looped streamers act as a highly conducting coil to create a magnetic and electric field needed for the system. The magnetic field of the streamers ends up looped and linked together through helicity. Helicity is the self-linkage of magnetic field lines and can be explained through the simple example of twisting a shoe string in two opposite directions. After multiple twists we feel a force of compression where the string tends to collapse inwards. To keep twisting the string you bring your hands closer together and this action forces the string to loop together. Twisting the string tight enough will end up compressing it into a tight spherical clump of loops. This is the idea proposed for creating the electromagnetic knot. As the closed looped streamers create current with comparable conductivity near infinite. The magnetic field becomes twisted and increases in energy. The current in the streamers coupled with the magnetic field can be expressed as

$$j = \frac{\nabla x B}{\mu_o}. \qquad (5)$$

The energy of the system can then be expressed as

$$E = \left(\frac{B_o^2 L_o^3}{\mu_o}\right) x \left(\frac{T}{T_o}\right)^2. \qquad (6)$$

Magnetic field lines, by nature, tend to be in a state with the lowest energy the system; so, for magnetized plasma the lowest energy state is helicity defined as $h = \int AB\, d^3x = na = constant$, which creates the stability between the magnetic field lines and streamers. The streamers and magnetic field looped together are coupled along the lines of current which act as filament-linked tubes. The stability of the system is then determined by the number of times each of these fields link (the number of electric field lines linked; the number of magnetic field lines linked).

Analyzing the decay of the model leads us to see that as time passes the radius of the knot expands and, due to this expansion, the magnetic field decreases. This small expansion is measured to be 1% to 6% depending on the initial temperature of the streamers, magnetic field intensity, and form of the magnetic knot. The ball of lighting last as long as the system is looped and tangled together. The system loses energy over time based on the power emitted by the knot expressed as

$$\frac{dE}{dt} = -P(T)\, x\, V, \text{ where } V = \frac{4\pi L_o^2 x^2}{3}. \qquad (7)$$

As the system loses its conductivity, resistivity increases and the conservation of magnetic helicity is lost and due to this the system begins to untangle and dissipate. Observably this model can be related to a light bulb where the streamers in the ball lightning model act similarly to the filament inside a light bulb. When further away from the glowing light bulb the less observable the filament is. Similarly, this model proposes that the linked streamers act as the filament and are conducting little to no radiation, but still letting off photons creating the glow observed. This explanation provides a reason where in some observed cases people are burnt by the ball's touch and others feel no heat coming from the object. This also explains why there seems to be loose streams coming off the ball of lighting when, in fact, they are streamers being untangled from the system due to loss of conservation of magnetic helicity.

Quantum Knot

The electromagnetic phenomena that occur in the topological model of ball lightning described previously are also described by a synthetic electromagnetic knot in a three-dimensional skyrmion. Synthetic electromagnetism causes the atomic wave function of neutral ultracold atoms to undergo changes as if they were charged particles acted upon by gauge potential. Studying this model could help reveal some properties of the knots that occur in real ball lightning. The skyrmion is a finite sized object bounded by uniformly oriented triads. The triads within the bounds are represented by a spin texture described by measuring the spin axis and rotation angle of each triad. The spin axes in the center are fully inverted and there are no discontinuities or singularities within the triads. The synthetic electromagnetic fields that arise from this spin texture and the presence of the knot can be described starting with a mean-field description of a Bose-Einstein condensate defined by

$$\Psi(r,t) = \psi(r,t)\zeta(r,t) = \sqrt{n(r,t)}e^{i\phi(r,t)\zeta(r,t)} \tag{8}$$

where Ψ is the scalar order parameter, n is the atomic density, ϕ is the scalar phase and $\zeta = (\zeta_{-1}, \zeta_o, \zeta_{+1})^T_{z_0}$ is the spinor quantized along the z-axis with $\zeta^\dagger\zeta = 1$.

A quantum test charge that is acted upon by the synthetic scalar and vector potentials Φ^* and A^* is defined by

$$\Phi^* = -i\frac{h}{q_e^*}\zeta^\dagger\frac{\delta}{\delta t}\zeta \text{ and } \mathrm{A}^* = -i\frac{h}{q_e^*}\zeta^\dagger\nabla\zeta. \tag{9}$$

These potentials lead directly to the following definitions of synthetic electric and magnetic fields

$$E^* = -\nabla\Phi^* - \frac{\delta A^*}{\delta t} \text{ and } B^* = \nabla \mathrm{x} \mathrm{A}^* \tag{10}$$

both of which follow from Faraday's law and Gauss's law. After one spin rotation around the condensate radius R, the synthetic magnetic field is

$$B^*(r',\theta',\psi') = -\left(\frac{4\pi^2 h}{q_e^* R^2}\right)\frac{sin^2(\pi\rho')}{(\pi\rho')^2}x\left[cos\theta'\hat{r}' - \pi\rho' sin\theta' cot(\pi\rho')\hat{\theta}' - \pi\rho' sin\theta'\hat{\psi}'\right]. \tag{11}$$

And the corresponding electric field is

$$E^*(r',\theta',\psi') = \left(\frac{2g_r\mu_B b_q}{q_e^*}\right)sin^2(\pi\rho')\ x\left[\hat{\theta}' - cot(\pi\rho')\hat{\psi}'\right]sin\theta' \tag{12}$$

where both are given in terms of primed spherical coordinates, the reduced coordinate $\rho = \frac{r'}{R}$, and Lande g-factor $g_f$and the Bohr magneton μ_B.

These equations are very similar to the equations shown in the previous section from the topological description of ball lightning, despite being from a different system. It is possible that the electromagnetic knot can be described in quantum systems as well as classical ones.

Proposed Model

Instead of thinking of the plasma bubble as constrained by a standing microwave, it may be insightful to view the system of a plasma bubble like a system of a star in hydrostatic equilibrium. In hydrostatic equilibrium, a star is supported by the balance of the inward gravity of the star itself and the outward radiation pressure from fusion occurring within the star. Since many atoms fuse in short times within a star, the size of the star is extremely large. We can model a system for ball lightning in hydrostatic equilibrium in balance by the inward electromagnetic and gravitational collapse of the plasma and the outward radiation pressure. Then one can think of the proposed standing wave within the bubble as many generations of microwaves caused by the constant changing of velocities in the electron bunch. It is also important to apply chemical studies more about ball lightning because they lead to answering many questions of ball lightnings characteristics. Burning is a common characteristic of ball lightning and may be explained by the recombination of oxygen molecules with air molecules and atoms or molecules from the ground. For example, quartz is one of the most abundant compounds in the ground, especially in sandy soil areas. Quartz has a bond energy of 408 kJ/mol. At temperatures of slightly above 30,000 K there is enough average kinetic energy to overcome this potential and therefore the bonds can break. The average temperatures from severe lightning strikes are about 30,000 K. This is enough energy to break many molecules' bonds with oxygen. A single microwave, which is considered to have a wavelength of about 1 mm to 1 m, does not have enough energy to make oxygen recombine with its partners in the air and ground. This is where the imposed continuous generation of microwaves comes in. If an oxygen molecule is hit with enough microwave photons, it will have enough energy to recombine with silicon and other air molecules to cause burning in the plasma shell. There are also probably neutral air molecules within the borders of the shell as well. Due to the sheer number of electrons, it is very likely that many will combine with recently ionized molecules from the lightning strike creating neutral molecules.

CONCLUSIONS

While ball lightning is still a poorly understood phenomenon, the understanding and comparison of models and information will remain of utmost importance. From the initial conditions that are favorable for ball lightning to the details of their composition and dissipation, each model has important information to give about how ball lightning really works. The electron bunching and microwave theory describes many qualities that ball lightning possesses. The electromagnetic knot topology is a well-conceived model but the high variance in ball lightning descriptions and lack of recorded ball lightning events makes it difficult to say if it is the correct one. The skyrmion acts as a great model to help understand what could be happening inside of a ball lightning. While it is clear that a skyrmion is not created during real ball lightning events, understanding how the electromagnetic fields knot together can provide insight into how real ball lightnings stay stable. In order to truly understand what ball lightning is and how it works, there must be a scientifically recorded event, or a method to reliably reproduce ball lightning in a laboratory and, until that day, comparing current models is the best way to describe ball lightning.

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