

Recursive identification of ship manoeuvring dynamics and hydrodynamics

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Abstract

This article presents new applications of recursive identification methods to estimation of ship manoeuvring dynamics and hydrodynamics. A vessel operated in sea water can be represented by a mathematical model with unknown coefficients called ship manoeuvring dynamic coefficients or hydrodynamic coefficients. Computer simulation and full scale experiments on board a small vessel verify the feasibility of the methods and show that the estimated parameters converge very well. At the Australian Maritime College there are many model vessels without any mathematical models. Hydrodynamic coefficients of these model vessels are determined by experiments to be conducted with hydrodynamics facilities at the College and by the recursive estimation methods we develop.

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1 Introduction

Nowadays computers with large memory, high speed CPU, high performance programming languages and multi-channel data acquisition devices allow engineers to execute very complicated estimation algorithms. The increasing demands for high performance computer controlled systems stimulate researchers to focus on development of estimation algorithms. Recursive estimation methods are among the solutions for dynamic modelling.

Recursive estimation algorithms have theoretically been developed by Ljung et al. [1] since the 1960s. They supposed different versions of the

recursive estimation method. Zhou et al. [5] applied the recursive prediction method in estimation of ship hydrodynamic coefficients based on computer simulations and experiments. The recursive estimation algorithms proved to have very good properties.

I successfully applied two recursive estimation algorithms in modelling of manoeuvring dynamics and design of adaptive control systems for surface vessels [2]. This article systematizes the recent results and proposes new applications.

The article describes the recursive identification algorithms in Section 2, computer simulation and full scale experiments for verification of the applied recursive algorithm in Section 3, discusses applications of recursive estimation algorithms in marine systems in Section 4 and highlights some concluding remarks in Section 5.

2 Recursive identification algorithms

Suitable mathematical models for ship manoeuvring dynamics are selected by a designer. In general, a discrete time model for ship manoeuvring dynamics must be expressed as either a MARX (Multi-variable Auto-Regressive eXogenous) model or a state space model.

A MARX model is of the form

$$\mathbf{A}(z^{-1})\mathbf{y}(t) = \mathbf{B}(z^{-1})\mathbf{u}(t - k) + \varepsilon(t). \quad (1)$$

where k is time delay, $\varepsilon(t)$ is the Gaussian noise with zero mean, $\mathbf{y}(t)$ and $\mathbf{u}(t - k)$ are vectors of measured outputs and inputs, respectively, and $\mathbf{A}(z^{-1})$ and $\mathbf{B}(z^{-1})$ are matrix polynomials of z^{-1} known as the unit backward shift operator, $z^{-i}\mathbf{y}(t) = \mathbf{y}(t - i)$, and defined by

$$\mathbf{A}(z^{-1}) = \mathbf{I} + \mathbf{a}_1z^{-1} + \mathbf{a}_2z^{-2} + \dots + \mathbf{a}_mz^{-m}, \quad (2)$$

$$\mathbf{B}(z^{-1}) = \mathbf{b}_0 + \mathbf{b}_1 z^{-1} + \mathbf{b}_2 z^{-2} + \cdots + \mathbf{b}_n z^{-n}. \quad (3)$$

The system (1) is expressed in the matrix form

$$\mathbf{y}(t) = \boldsymbol{\phi}^T \boldsymbol{\theta} + \varepsilon(t). \quad (4)$$

where $\boldsymbol{\phi} = [-\mathbf{y}^T(t-1), -\mathbf{y}^T(t-2), \dots, -\mathbf{u}^T(t-m), \mathbf{u}^T(t-k), \mathbf{u}^T(t-k-1), \dots, \mathbf{u}^T(t-k-n)]^T$ is the matrix of unknown parameters and $\boldsymbol{\theta} = [\mathbf{A} \ \mathbf{B}] = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_m, \mathbf{b}_0, \mathbf{b}_1, \dots, \mathbf{b}_n]$ is the matrix of input and output measurements.

Alternatively, a state space model is of the form

$$\mathbf{x}(t+1) = \mathbf{F}(\boldsymbol{\theta})\mathbf{x}(t) + \mathbf{G}(\boldsymbol{\theta})\mathbf{u}(t) + \mathbf{K}(\boldsymbol{\theta})\varepsilon(t), \quad (5)$$

$$\mathbf{y}(t) = \mathbf{C}(\boldsymbol{\theta})\mathbf{x}(t) + \varepsilon(t). \quad (6)$$

where $\mathbf{x}(t)$ is the state vector, $\mathbf{u}(t)$ is the input vector, $\mathbf{F}(\boldsymbol{\theta})$ is the dynamic matrix, $\mathbf{G}(\boldsymbol{\theta})$ the input matrix, $\mathbf{C}(\boldsymbol{\theta})$ the output matrix, $\mathbf{K}(\boldsymbol{\theta})$ the Kalman gain matrix, the elements of which will be estimated along with the system parameters $\boldsymbol{\theta}$, and ε is the innovation. The model in equations (5) and (6) is rewritten in the matrix form

$$\mathbf{y}(t) = \boldsymbol{\phi}^T \boldsymbol{\theta} + \varepsilon(t). \quad (7)$$

where $\boldsymbol{\phi}(t)$ is the measured data matrix (including inputs and outputs).

2.1 Recursive prediction error algorithm

The recursive prediction error (RPE) algorithm applied to estimate elements of matrices in (5) and (6) is to minimise the following criterion function

$$V(\boldsymbol{\theta}) = \frac{1}{2} \boldsymbol{\varepsilon}^T(t) \boldsymbol{\Lambda}^{-1}(t) \boldsymbol{\varepsilon}(t). \quad (8)$$

where Λ is a positive definite matrix, and a Gauss–Newton search direction is chosen as

$$f(\mathbf{t}) = \mathbf{H}^{-1}(\mathbf{t})\boldsymbol{\psi}(\mathbf{t}, \boldsymbol{\theta})\Lambda^{-1}(\mathbf{t})\boldsymbol{\varepsilon}(\mathbf{t}). \quad (9)$$

where $\mathbf{H}(\mathbf{t})$ is the Hessian, the second derivative of the criterion function with respect to $\boldsymbol{\theta}$, and $\boldsymbol{\psi}(\mathbf{t}, \boldsymbol{\theta})$ is the gradient of predicted output with respect to $\boldsymbol{\theta}$, and $\boldsymbol{\varepsilon}(\mathbf{t}, \boldsymbol{\theta})$ is the vector of predicted error.

Algorithm 1 summarises the RPE algorithm using the Gauss–Newton search direction to estimate the parameters $\boldsymbol{\theta}$ of (7) [1, 6, 5, 4, 3, 2].

The step size factor $\alpha(\mathbf{t})$ is given by Ljung and Söderström [1], Zhou et al. [5] and Nguyen [3] as

$$\alpha(\mathbf{t}) = \frac{1}{1 + \mathbf{t}}. \quad (16)$$

where \mathbf{t} is the time increasing in value for every sampling time.

Within the practical aspects of implementing the algorithm, the main task is to choose initial settings such as Λ , \mathbf{H} and the sampling time. Calculation of the prediction $\hat{\mathbf{y}}$ and of the gradient of the predicted output $\boldsymbol{\psi}(\mathbf{t})$ depends on the chosen model. The RPE algorithm is applied into both an SISO (single input single output) system model and an MIMO (multi-input multi-output) system (state space) model.

2.2 Recursive least squares algorithm

The recursive least squares (RLS) algorithm is a special case of the RPE algorithm. The RLS algorithm applied to the MARX model is to minimise the cost function

$$J = \sum_{i=1}^{\mathbf{t}} \boldsymbol{\varepsilon}^T(\mathbf{t})\lambda^{t-i}\boldsymbol{\varepsilon}(\mathbf{t}). \quad (17)$$

Algorithm 1: Summary of recursive prediction error algorithm.

1 **repeat**

2 Form the predicted error using

$$\varepsilon(t) = \mathbf{y}(t) - \hat{\mathbf{y}}(t). \quad (10)$$

where $\hat{\mathbf{y}}(t)$ is the predicted output that is often initialised as $\hat{\mathbf{y}}(0) = \boldsymbol{\theta}(0)\boldsymbol{\phi}(0)$

3 Form the weighting matrix by

$$\boldsymbol{\Lambda}(t) = \boldsymbol{\Lambda}(t-1) + \alpha(t)[\varepsilon(t)\varepsilon(t)^T - \boldsymbol{\Lambda}(t-1)]. \quad (11)$$

4 Form the Hessian

$$\mathbf{H}(t) = \mathbf{H}(t-1) + \alpha(t)[\boldsymbol{\psi}(t)\boldsymbol{\Lambda}^{-1}(t)\boldsymbol{\psi}^T(t) - \mathbf{H}(t-1)]. \quad (12)$$

5 Update the estimated parameters

$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}(t-1) + \alpha(t)\mathbf{H}^{-1}(t)\boldsymbol{\psi}(t)\boldsymbol{\Lambda}^{-1}(t)\varepsilon(t). \quad (13)$$

6 Update the predicted output

$$\hat{\mathbf{y}}(t+1) = \boldsymbol{\theta}(t)\boldsymbol{\phi}(t+1). \quad (14)$$

7 Calculate the gradient of predicted output by

$$\boldsymbol{\psi}(t) = \left[\frac{d}{d\boldsymbol{\theta}} \hat{\mathbf{y}}(t, \boldsymbol{\theta}) \right]^T. \quad (15)$$

8 **until** *updated data converges* ;

The forgetting factor (FF) λ is a number from 0.95 to 0.998 and used as a means to progressively reduce the emphasis placed on past information. According to Hang et al. [7] the FF can be either fixed or made variable with time and in practice the FF is often chosen in the above range because it gives the best compromise between the speed of adaptation and sensitivity to noise. Algorithm 2 summarises the RPSA.

Algorithm 2: Recursive least squares algorithm.

1 At time step $\mathbf{t} + 1$:

2 **repeat**

3 Form $\boldsymbol{\theta}(\mathbf{t} + 1)$ using new data.

4 Form the prediction error using

$$\boldsymbol{\varepsilon}(\mathbf{t} + 1) = \mathbf{y}(\mathbf{t} + 1) - \boldsymbol{\phi}^T(\mathbf{t} + 1)\boldsymbol{\theta}(\mathbf{t}). \quad (18)$$

5 Form the covariance matrix using

$$\mathbf{P}(\mathbf{t} + 1) = \lambda^{-1}\mathbf{P}(\mathbf{t}) \left[\mathbf{I}_M - \frac{\boldsymbol{\phi}^T(\mathbf{t})\boldsymbol{\phi}(\mathbf{t})\mathbf{P}(\mathbf{t})}{\lambda + \boldsymbol{\phi}(\mathbf{t})\mathbf{P}(\mathbf{t})\boldsymbol{\phi}^T(\mathbf{t})} \right]. \quad (19)$$

6 Update the parameter matrix by

$$\boldsymbol{\theta}(\mathbf{t} + 1) = \boldsymbol{\theta}(\mathbf{t}) + \mathbf{P}(\mathbf{t} + 1)\boldsymbol{\phi}^T(\mathbf{t} + 1)\boldsymbol{\varepsilon}(\mathbf{t} + 1). \quad (20)$$

7 Wait for the next step to elapse.

8 **until** *finished* ;

The number of unknown parameters is $M = \mathbf{n} + \mathbf{m} + 1$, and the initial covariance matrix is chosen as $\mathbf{P}(0) = \alpha\mathbf{I}_M$ in which α is a large number, for example 10^3 or 10^4 .

In general, any recursive identification algorithm is applied in modelling of dynamic systems and control system design, provided such the recursive

identification algorithm proves its reliability and accuracy to an acceptable degree.

3 Brief description of full scale experiments and simulation study

Full scale experiments and computer simulations have been done based on a non-linear mathematical model of a Japanese small vessel. This section very briefly describes full scale experiments and a simulation study.

Full scale experiments to verify the recursive estimation methods were conducted on board the training vessel belonging to Tokyo University of Marine Science and Technology. The vessel has very good manoeuvrability with a pitch controllable propeller, two side thrusters, advanced computers and instrumentation devices. Simulation programs and control programs for full scale experiments were made in MATLAB codes and Simulink models. full scale experiments required data acquisition devices. REALoop of Xanalog Inc. was used. Nguyen [3] gave detailed information.

Before conducting full scale experiments on board a real ship, many simulation programs in M-files and Simulink models were made to verify the online estimation methods. One typical experiment to estimate ship manoeuvring dynamics is the Kempf's zigzag manoeuvre (also called Z-test). Figure 1 shows an example of time series of yaw angle, rudder angle and yaw rate for a 20-20 Z-test that is used to estimate ship manoeuvring coefficients.

The ship manoeuvring model for this Simulink model is

$$\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{B}\delta(k), \quad (21)$$

where $\mathbf{x}(k)$ is the state vector and $\delta(k)$ is the rudder angle:

$$\mathbf{x} = \begin{bmatrix} \psi \\ r \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \mathbf{b}_{11} \\ \mathbf{b}_{21} \end{bmatrix}.$$

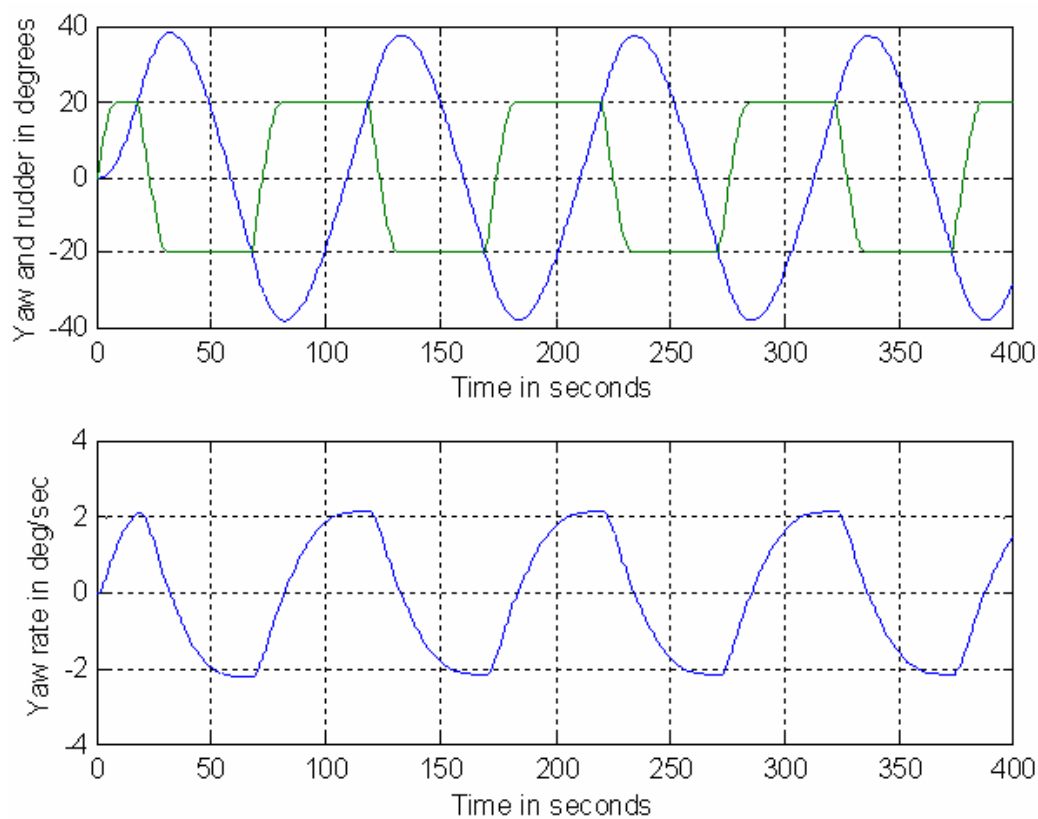


FIGURE 1: Time series of yaw angle, rudder angle and yaw rate for 20-20 Z-test.

Simulated results are shown in Figures 2-3: all estimated parameters show good convergence.

These methods can be used for estimation of manoeuvrability indices (T and K) in Nomoto's first order model

$$T\dot{r} + r = K\delta, \quad (22)$$

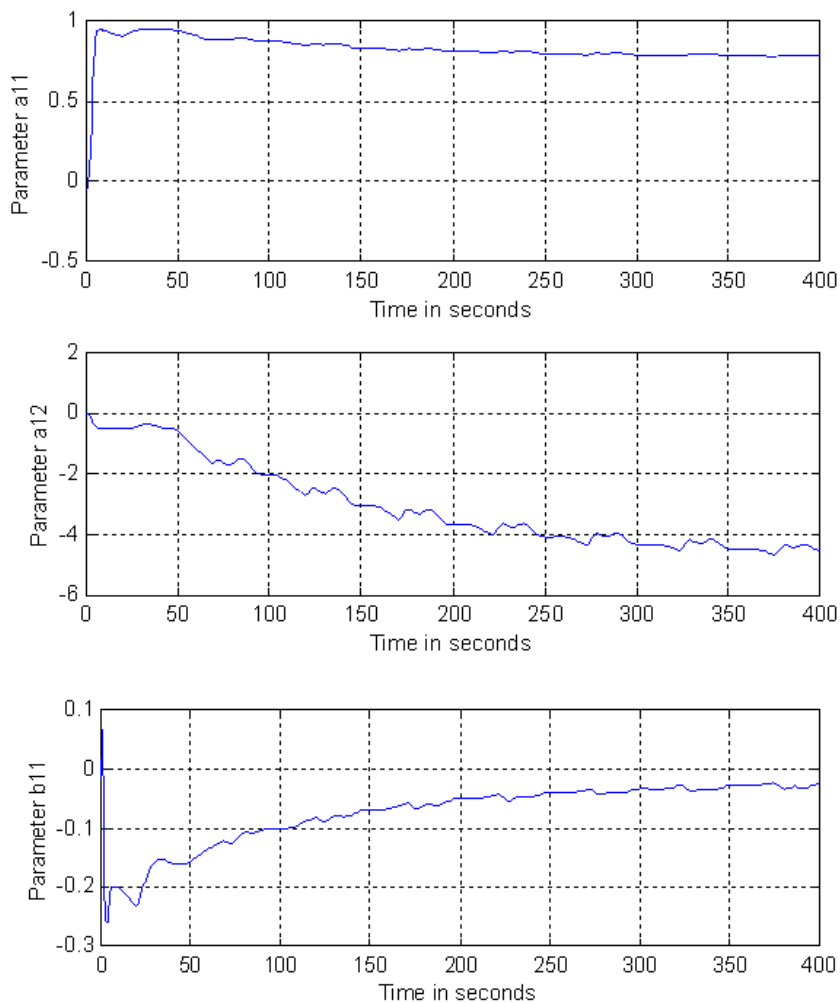
where r and δ are the yaw angle and ruder angle.

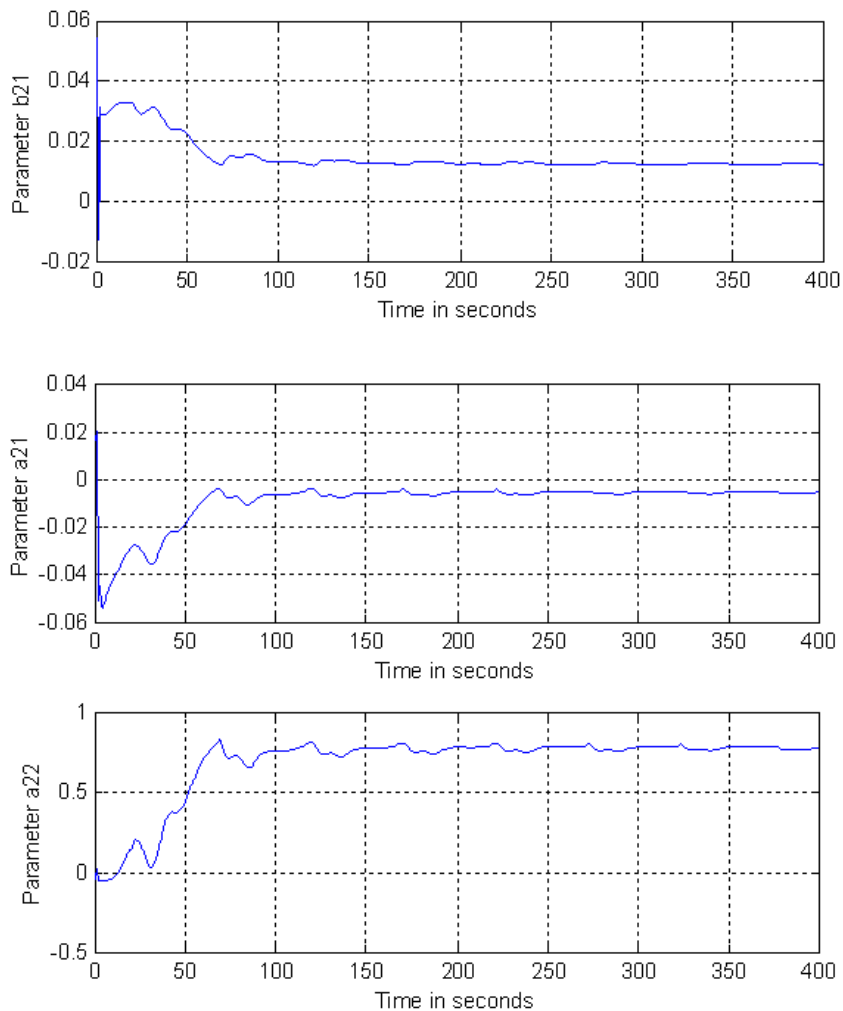
4 Applications of recursive estimation algorithms

The recursive estimation algorithm is applied to estimate manoeuvrability indices, hydrodynamic coefficients, for marine automatic control systems and for fault detection and monitoring systems for ship's structures. In designing an automatic control system, a recursive estimation algorithm is combined with a control law such as a self-tuning pole assignment control or optimal control. This section discusses the recent simulated and experimental results and proposes some possible applications.

4.1 Estimation of ship's manoeuvrability indices and hydrodynamic coefficients

Manoeuvrability (course stability, turning ability and stopping ability) of ocean vessels is important for marine pilots and deck officers and it changes over time because of the ship's changing conditions. It is required that the ship's manoeuvrability should be tested periodically. The recursive estimation methods are suggested to be good methods for periodic manoeuvring

FIGURE 2: Time series of estimated parameters a_{11} , a_{12} and b_{11} .

FIGURE 3: Time series of estimated parameters a_{21} , a_{22} and b_{21} .

trials on board the ship such as Z-test, turning circle test, Dieudonne's spiral manoeuvre, and stopping test. The manoeuvrability testing function is added in the autopilot system on board a ship.

4.2 Autopilot systems

Autopilot systems for marine vessels are designed by the combination of a recursive estimation algorithm with a suitable control law. The following autopilot systems have been verified by computer simulation and full scale experiments on board a real ship. RLSA based self-tuning pole assignment autopilot systems in which the control algorithm is a combination of the recursive least squares method with the self-tuning pole assignment control law. RPEA based self-tuning pole assignment autopilot systems in which the control algorithm is a combination of the recursive prediction error algorithm with the self-tuning pole assignment control law. RLSA-based optimal autopilot systems in which the control algorithm is a combination of the recursive least squares algorithm with the LQG optimal control law. RPEA-based optimal autopilot systems in which the control algorithm is a combination of the recursive prediction error algorithm with the LQG optimal control law. Nguyen [3] gave more information.

4.3 Automatic manoeuvring systems for search and rescue mission

The control algorithm for an automatic manoeuvring system for maritime search and rescue mission was formulated and verified by computer simulation. The control algorithm is based on a recursive estimation method, optimal control and waypoint technique according to Nguyen [2]. Ship positions are obtained by GPS and DGPS/RTK-GPS receivers. The control algorithm should be verified in model test conditions by model experiments or in real

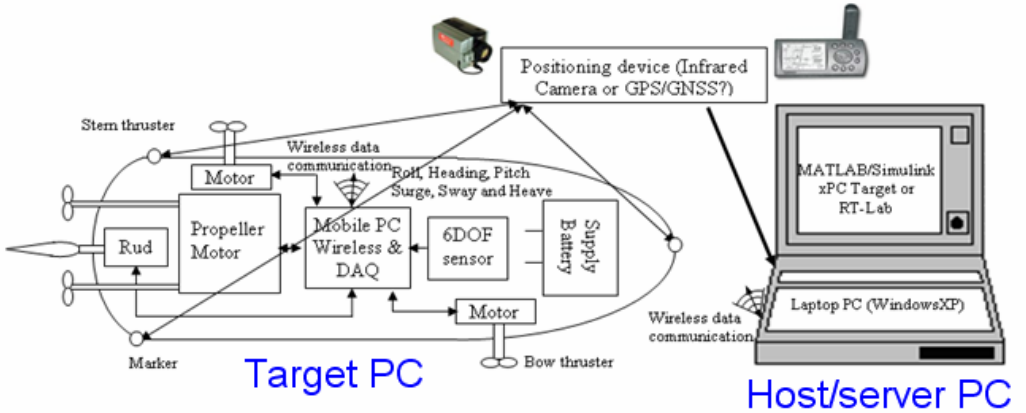


FIGURE 4: Arrangement of experimental facility.

conditions by full scale experiments. The manoeuvring system has been developed into a multitask integrated manoeuvring system with new functions such as manoeuvrability index estimator, rudder-roll stabilizer and ship motion monitor. An arrangement of experimental facility used in the model test basin is shown in Figure 4.

5 Conclusions

Ship manoeuvring dynamics and hydrodynamics are estimated online using a recursive estimation algorithm with experiments, appropriate data acquisition devices and programming languages. In the recursive estimation algorithms, unknown parameters adapt to changes of environment. Control algorithm that is a combination of a recursive estimation algorithm and a control law has been formulated and verified by computer simulation and experiments. New applications of recursive estimation algorithms in marine control systems should be considered and verified in different conditions.

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