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EXPLORING PRE-SERVICE MATHEMATICS TEACHERS' UNDERSTANDINGS OF COUNTABILITY AND INFINITY IN WEBQUEST BASED LEARNING ENVIRONMENT

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Abstract:
Infinity concept is difficult to understand because of its nature. Even if the concept of infinity doesn't directly take part in mathematics curriculum, it takes place on many topics such as lines, planes, sets, irrational numbers, limit, differentiation, and integration. The present study examined pre-service mathematics teachers' understandings of infinity and countability concepts in WebQuest based learning environment. Twenty-nine pre-service teachers participated at the study. Data were collected through a questionnaire developed by the researchers and semi-structured interviews. Qualitative data were analyzed by using phenomenology research method. The findings of this study revealed that pre-service teachers generally defined infinite sets as the sets whose elements continue infinitely. They inclined to define countable sets as bounded sets, finite sets, and the sets with known elements. Whereas most of the students stated countable finite sets and countable infinite sets were existent, they also expressed uncountable finite sets and uncountable infinite sets were non-existent. They used natural numbers set as an example for the countable infinite sets. This research presented some implications for teacher education programme in the light of obtained findings.

Keywords: mathematics WebQuest, countability, infinity, pre-service teachers

1. Introduction

Characteristics of infinity have confused human minds for many years (Mamolo & Zazkis, 2008). Since infinity does not exist in our world we live in, to imagine and understand this concept is challenging (Fischbein, 2001; Tirosh, 1999). The human mind

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is familiar with finite realities (Narli, 2011). According to Monaghan (2001), infinity is perceived as a very large number and there is an understanding that infinity is unique. In addition, students have conceived the infinity concept to be a potentially inexhaustible process (Fischbein, Tirosh, & Melamed, 1981).

From the perspective of pre-service teachers' own education at undergraduate level, the concept of infinity takes part in many subjects such as irrational number, limits, derivatives, and integrals (Çelik & Akşan, 2013; Özmantar, 2008). In addition, the concept is vital for some courses like Algebra, Topology, and Elementary Number Theory (Narli, 2011). Moreover, at elementary school level, mathematics teachers mention about natural numbers, integers, and real numbers. Whereas natural numbers and integers are countable infinite sets, real numbers are uncountable infinite sets. In addition, any set that is given as $A = \{1, 2, 3, 4, 5\}$ is countable finite set. In-service mathematics teachers use these sets in their courses. They should know the sets' features and the differences between them because it is important to know accurate content knowledge about the subjects they teach in their classes. Some curious students can ask to teachers or some teachers can come across a question that natural numbers and real numbers are infinite but what the differences between these sets are. Teachers should be able to answer such kinds of questions and should have accurate concept images of infinity and countability concept. Various studies (e.g., Dede & Soybaş, 2011; Narli, 2011; Yanik & Flores, 2009; Yanik, 2014) have suggested that in order to make the right decision about students' mathematical understandings, being aware of their understandings is necessary for educators, especially mathematics teachers.

Most of research indicates that pre-service teachers have difficulty in understanding countability and infinity concepts. Furthermore, on examination of the related literature reveals that few studies exist in this bulk of studies especially in the countability concept (e.g., Aztekin, 2013; Ünan & Doğan, 2011). Research is mostly focused on just understanding of infinity concept (e.g., Dreyfus & Tsamir, 2004; Narli, 2011; Aztekin, Arikan, & Sriraman, 2010; Tirosh & Tsamir, 1996; Tsamir & Tirosh, 1999; Tsamir, 2001). It may be worthwhile to explore elementary mathematics pre-service teachers' concept images related to both infinity and countability concepts, additionally; the present study correlates the concept of infinity with the concept of countability. Countability and infinity concepts are closely associated and the concept of infinity that is quite abstract can become apparent by using this association.

Alias, Dewiit, and Sirej (2014) indicated that WebQuests made it easier to understand abstract subjects in physics. Countability and infinity concepts are quite abstract and difficult to understand. WebQuests can be used for teaching these concepts.

The goal of the study was to explore pre-service mathematics teachers' understandings of infinity and countability concepts in WebQuest based learning environment. In order to investigate this "What are the nature of pre-service teachers' understandings of infinity and countability?" research question guided the study.

2. Literature Review

The concept of infinity has been explained in some studies with paradoxes (Dubinsky, Weller, Mcdonald, & Brown, 2005; Mamolo & Zazkis, 2008; Waldegg, 2005). A paradox called "Hilbert's Grand Hotel" is used for introducing infinity by Mamolo and Zazkis (2008). This paradox is as follows:

> *"You are the manager of the Grand Hotel. The hotel has infinitely many rooms and there is no empty room. One person came and demanded a room, how can you accommodate your new client in a personal room?"*

The solution of this paradox is based on the one-to-one correspondence. If each client moves to his/her neighbour's room, the new client can accommodate in the hotel. A is identified as the set of clients' rooms, and B is identified as the set of occupied rooms after the shift. Among the sets $A = \{1, 2, 3, \dots\}$ and $B = \{2, 3, 4, \dots\}$, there is a bijective function, and so the cardinalities of the two sets are the same. Moreover, Hilbert's Grand Hotel itself represents actual infinity; the potentially infinite hotel that creates new rooms corresponds to potential infinity (Mamolo & Zazkis, 2008).

Infinity is classified as classifies as actual and potential infinity by mathematicians (Dubinsky, Weller, Mcdonald, & Brown, 2005; Pehkonen, Hannula, Maijala, & Soro, 2006). According to potential infinity concept accepted by Aristotle (Bagni, 1997), infinity does not exist in reality, it exist potentially and this point of view was accepted for through the years (Tirosh, 1999). Similar difficulties have been also observed in the students' understanding of infinity. For instance, scientists firstly adopted the idea of potential infinity and later formed actual infinity idea. In the same vein, viewed from the educational point, students have primarily formed the potential infinity idea and then actual infinity (Kolar & Cadez, 2012). In the 19th century, Cantor introduced a theory "Cantorian Set Theory". Cantor (1915) suggested the cardinality of an infinite set and defined each element in a set as a "unit". Bolzano's "part-whole" argument is also related to the comparison of two infinite sets that are seemingly different cardinality (Luis, Moreno, & Waldegg, 1991).

The tendency in mathematics education research is to examine students' understanding of infinity through the lens of Cantor's set theory (e.g., Dreyfus & Tsamir, 2004; Narli, 2011). In several studies, some sets are given such as $A = \{1, 2, 3, \dots\}$ and $B = \{2, 4, 6, \dots\}$; students are asked to compare cardinality of these sets. According to research results, while some participants used one-to-one correspondence approach, some participants used comparatively a "part-whole" approach (Mamolo & Zazkis, 2008). In addition, in some studies, students decide whether a set is infinite or not by moving finite sets' characteristics to infinite sets (Fischbein, Tirosh, & Hess, 1979).

A number of studies conducted with several age groups such as elementary school students (Tirosh, 1999; Pehkonen, Hannula, Maijala, & Soro, 2006; Falk, 2010; Narli & Narli, 2012), high school students (Tirosh & Tsamir, 1996; Tsamir & Tirosh, 1999; Tsamir, 2001), undergraduate students and pre-service teachers (Çelik & Akşan, 2013; Mamolo & Zazkis, 2008; Mura & Louce, 1997; Narli, 2011; Tsamir, 2002; Ünan & Doğan, 2011), and also PhD students (Aztekin, Arikan, & Sriraman, 2010) on the students' understanding of infinity have been existent.

Falk (2010) examined children's, whose ages are between 6-15, and adults' understanding of potential and actual infinity. Results revealed that 8-year-old students could comprehend the actual and potential infinity but in the following years, they could not fully understand the actual infinity concept. They did not internalize fully the finite and the infinite space between sets. Furthermore, many adults also had difficulties in understanding this gap. In his study, Tirosh (1999) suggested that responses were same for all age groups and making right classification about infinite or finite sets was not associated with their mathematical background. However, advanced mathematics education students used more systematically logical schemes. These studies have indicated that students' understanding of infinity concept doesn't change with respect to according to their grade levels.

Mamolo and Zazkis (2008) examined approaches to infinity concept of two groups' undergraduate students with different mathematical background. Responses to a paradox used in the study are the same for both groups and having different mathematical background did not affect their responses. Therefore, it can be concluded that students from various disciplines may have similar understanding of infinity concept.

Narli (2011) investigated the effects of Cantor set theory instruction based on constructivist learning environment on secondary mathematics pre-service teachers' understanding retention. Ünan and Doğan (2011) examined secondary school science and mathematics pre-service teachers' finite sets, countable and uncountable infinite sets concept images via mathematical modelling. All these studies results advocate that teaching methods have influenced students' understanding of infinity concept.

On the other hand, according to Narli and Narli (2012) school, namely formal education, did not help students to learn about infinity concept and also teachers; in general, did not have an exact idea of infinity. Eighth-grade elementary school students mostly made infinity definition based on experiences outside of school. They tried to apply finite sets' features to infinite sets and explained the concept of infinity via uncountable set concept. The study by Çelik and Akşan (2013) investigated that elementary mathematics pre-service teachers' responses were intuitional but did not coincide with the concept of actual infinity. These results revealed that students' responses about infinity and other related concepts are based on mostly their daily life experiences.

2.1 The mathematical definitions of infinity and countability concepts

Imagining infinity or countability can be difficult but the formal definitions may help students to construct their own examples.

The formal definitions of countable/uncountable and finite/infinite sets concepts are as follows:

If a function exists, $f: A \to B$ that is bijective, one-to-one correspondence defined between the sets A and B then A is called equivalent to B (Lipschutz, 1965). For example, $\mathrm{N} = \{1, 2, 3, \dots\}$ natural numbers set is equivalent to $\mathrm{E} = \{2, 4, 6, \dots\}$ even numbers set. Because there is a bijective function defined by $f(x) = 2x, f: N \to E$.

If a set is empty or equivalent to $\{1, 2, \dots, n\}$ for some $n \in N$, it is defined to be a *finite* set, or else it is called as *infinite* (Lipschutz, 1965).

If two sets are finite, showing their equi-valence is easy; otherwise, if they are infinite it is hard to prove whether the sets are equivalence or not (Narli, 2011). If two finite sets are equivalent, they contain the same number of elements. Also, an infinite set can be equivalent to a proper subset of itself (Lipschutz, 1965).

Countability concept is one part of Cantorian Set Theory. It is directly related to infinity concept through the countable set and equivalence concepts. The concept of countability is basically based on equivalence of two sets. A set is defined as countable set if it is finite or denumerable (countable set). If a one-to-one function $f: S \to N$ exists from S to the natural numbers $N = \{1, 2, 3, \dots\}$, a set S is countable (Lipschutz, 1965). However, some researchers have not included finite sets to countable sets; they have accepted only countably infinite set as countable sets (Rudin, 1976).

On the other hand, every finite set is a countable set. $\mathrm{A} = \{1, 2, 3, 4, 5\}$ set is an example of the countable finite set; natural numbers, integers, and rational numbers are the example of the countable infinite set; real numbers, complex numbers and real numbers in [0,1] interval are uncountable infinite sets. The sample of an uncountable finite set is non-existent.

2.2 WebQuests

Recently, WebQuests have been extensively used in educational research (Öksüz & Uça, 2010). Webquests were firstly proposed by Dodge and March from San Diego State University in 1995 (Kelly, 2000). A WebQuest composes of 6 stages: introduction, tasks, process, resources, evaluation, and conclusion (Dodge, 1997).

Two types WebQuests are available: the short-term and the long-term WebQuests. While short-term webquests last between 1and 3 courses (minimum 45-minute and maximum 135-minute as class period), the long-term one are completed within 1 week-1 month. Designing of short-term webquests are easy and can be carried out fast (Dodge, 1997; Watson, 1999). Technology-supported learning environments increase the academic success and motivation of the students (Hayes & Billy, 2003; Halat & Peker, 2011); therefore, both types of WebQuests can be applied for different courses. The WebQuest in this research is an example of short-term WebQuests.

In WebQuest activities, students learn successfully with an accurate framework that is given in the tasks and complete them using the Internet (Yang, 2014). Teaching via WebQuests provides students to associate their past training experiences and present learning outputs (Çığrık & Ergül, 2010). WebQuests provide students to develop their creativeness and critical thinking skills (Lim & Hernandez, 2007). Moreover, learners use internet in a funny way and WebQuest learning activities improve their attitude towards the courses positively (Kurtuluş & Kılıç, 2009).

The results of some recent research show that WebQuests are effective tools in pre-service teachers' teaching-learning process. Halat (2008) observed that WebQuests had a positive effect on the geometrical thinking levels of pre-service primary teachers. Halat and Peker (2011) indicated that WebQuests had a positive impact on the motivation of pre-service teachers. Pre-service mathematics teachers who participated in the research of Goktepe-Yildiz and Goktepe-Korpeoğlu (2016) stated that they would use the opportunities of WebQuests in their further courses. Moreover, pre-service teachers favored the technology-aimed activities and the prepared WebQuest provided the students to learn while doing activities and generated examples by using technology (Gülbahar, Madran, & Kalelioğlu, 2010).

The construction of some abstract mathematical concepts is especially vital for pre-service teachers to learn some mathematical concepts better. Therefore, this research developed a WebQuest based teaching activity for mathematics courses and explored the understandings of students about infinity and countability concepts.

3. Methods

Phenomenology is a kind of the qualitative research methods, which was used for the present study. "*Phenomenological analysis gives priority to people's accounts of intentionality and subjective meanings. This is the phenomenological researcher's first and only point of reference.*" (Scott, 2001).

"*The purpose of this phenomenological research is not to aim investigate the phenomenon within the context of an investigation, phenomenology focuses on describing it and seeking to mean in a particular and an actual experience.*" (Hull, 2003). Additionally, in phenomenological studies how individuals construct their meanings of the experience is explored (Vanderstoep & Johnston, 2009).

3.1 Participants

Twenty-nine junior pre-service teachers who were enrolled in the Department of Elementary Mathematics Education at a north-western public university in Turkey were selected randomly. The availability of participants for this study was also used for selection criteria. Purposive sampling that is a feature of qualitative research was employed in the phase of selecting students for semi-structured interviews. Purposive sampling is generally used to access "*knowledgeable people*" (Ball, 1990), and the sample is chosen for a specific purpose (Cohen, Manion, & Morrison, 2007). Thus, seven pre-service mathematics teachers were selected based on their responses to the countability and infinity concept test, definitions, examples and explanations in the written instrument. Five pre-service teachers were female and two of them were male. Selected teacher candidates' average age is 20-22. Two of the pre-service teachers showed high, three of them showed medium, and two of them showed low level according to academic record of three years. All of them voluntarily participated in the study. A difficulty of phenomenological research is to reach a certain number of individuals who have experience the same phenomenon (Vanderstoep & Johnston, 2009). However, in the present study, all participants completed successfully General Mathematics, Abstract Mathematics, and Analysis I-II-III courses including infinity and countability concepts and they faced with these concepts before. Pre-service teachers experienced the correct mathematical definition of the concepts in their education. In the curriculum of Department of Elementary Mathematics Education in the Faculty of Education in Turkey, General Mathematics course contains these topics: set theory, set of natural numbers set of integer, set of rational numbers, and set of real numbers, complex numbers, and their properties. In addition, finite and infinite sets subjects are in Abstract Mathematics course. More detailed information about these topics is given in Analysis I-II-III.

3.2 Developing the mathematics WebQuest learning enviroment

A WebQuest was developed which can be applied for the teaching some topological concepts (e.g., countability, infinity, countable infinity, uncountable infinity). A six-phase WebQuest was prepared for pre-service mathematics teachers. Pre-service teachers were already familiar with infinity and countability concepts; therefore, it was a short-time WebQuest activity that can be completed within two weeks (two 45-minute class period) in an elective mathematics course.

Infinity and countability concepts took part in many subjects such as irrational number, limits, derivatives, and integrals in undergraduate level. The scope of the WebQuest teaching activity was to deepen pre-service teachers' understandings of these concepts. Thus, the WebQuest based learning environment expanded their understandings of some curriculum concepts. Moreover, to stimulate students' high level thinking abilities the researchers asked students to relate these concepts with each other.

The prepared WebQuest composed of 6 sections proposed by Dodge (1997), namely: introduction, task, process, resources, evaluation, and conclusion.

In the introduction section, a mobius strip was given in the introduction page to attract the students' attention with regard to the subject. Klein bottle's properties were introduced to use daily life example. "What we know about countability, infinity, countable infinity and uncountable infinity concepts" question was asked to mention from topological concepts.

In the task section, two main tasks were formed for pre-service teachers. In the first task, the students were asked to search for the definitions of countability, infinity and countable infinity and uncountable infinity concepts by using the web pages in the resources section. In the second task, students can learn the correlation among these concepts. It was explained that they can use the internet and other sources of information resources during this research.

In the process section, students studied individually. Researchers assist students in the phase of reaching relevant mathematical Internet resources. True-false questions which can be answered in the light of knowledge obtained from the task section were available in this section. The students were asked to write reasons for their responses.

Moreover, a crossword puzzle was given. Some topological concepts were placed to crossword puzzle. Pre-service teachers were asked to fill the gaps about these concepts.

In the resources section, carefully-selected web pages related to mathematical countability and infinity concepts were listed. Moreover, it was explained that they could use different sources of their own.

In the evaluation section, the evaluation scale was predesigned and these criteria were given to students in detail. True-false questions and crossword puzzle were evaluated one by one.

In the conclusion section, a summary was given about what they have learned with regard to their WebQuest experience. It was mentioned that they had acquired basic topological concepts (countability and infinity concepts) and could reach more advanced mathematical information.

3.3 Instruments

Two main data resources were used in the study: (1) infinity and countability concept test, (2) semi-structured interviews.

3.3.1 Infinity and countability concept test

Pre-service teachers' understandings related to countability and infinity concepts were tested by means of a written instrument named countability and infinity concept test which was developed by the researchers. After students completed the WebQuest activity, they answered the questions in the written test. The questionnaire consisted of six open-ended questions. The first two questions in the test were concerned with the definition of the concept of countability and infinity. The other four questions were required to give examples associating with these two concepts. Questions were used to see what to extend preservice teachers could recognize countable/uncountable and finite/infinite sets and their features by giving examples and non-examples of them. In three cases (3., 4., and 6. questions) it was possible to generate examples, in one case (5. question) it was not possible. The concept test was examined by two experts, one of them has several pure mathematics research and the other researcher was from Elementary Mathematics Education Department. The content of the questionnaire was found to be valid by experts for this study. The questionnaire lasted 60 minutes during a lesson. Questions in data collection tool (countability and infinity concept test) were provided below:

1. What is countable set? Explain.
2. What is infinite set? Explain.
3. Is a countable finite set available? If it is available, give an example.
4. Is a countable infinite set available? If it is available, give an example.
5. Is an uncountable finite set available? If it is available, give an example.
6. Is an uncountable infinite set available? If it is available, give an example.

3.2.2 Semi-structured interviews

After the written instrument was implemented and analyzed, seven in-depth semi-structured interviews with pre-service mathematics teachers were conducted. In order to determine the points which pre-service teachers had difficulty in understanding of countability and infinity concepts, the second author carried out semi-structured interviews for examining students' concept images deeply. The researcher asked different questions and examples to pre-service teachers from the questionnaire according to their responses in the interview. Interviews lasted in a peaceful atmosphere and the names of the participants were kept hidden. All interviews were audio-typed and transcribed. The interviews were carried out in Turkish and later they were translated into English by the researchers. The interview length varied from 30 minutes to 45 minutes. In designing of the interview, questions were expressed in a flexible way to students by the interviewer to learn their own understanding and personal experiences explicitly. Sometimes leading questions were asked such as "Is there any set whose beginning and end are certain?", "Does the beginning and the end need to be certain so as to be a countable set?" in order to identify the concept images students hold.

3.3 Data Analysis

Qualitative approaches are convenient for exploring the characteristics of complex phenomena (Miles & Huberman, 1994). The data obtained were analysed qualitatively in this study. Data were obtained from three different research methods: a questionnaire, transcripts of the semi-structured interviews.

Firstly, to gain insight about the whole, written data obtained from the infinity and countability test and translation form of the each interview were read again and again. After generating the whole views about the responses of participants, obtained data separated to categories. Students' responses to each statement were examined separately and categorized according to their similarities and differences and presented in percentage and frequency tables. Both of two authors classified separately the data and 90% consistency was found among the categories. During the last phase of the data analysis, excerpts from the semi-structured interviews were presented. Similar type of analysis has been used by different researchers. For example, in their study Dede and Soybaş (2011) used Giorgi's (1985) existential-phenomenological data analysis while analysing pre-service mathematics teachers' concept images of polynomials. Giorgi's (1985) existential-phenomenological data analysis involved similar steps like the present study: initial reading to get a sense about the whole, separating into meaning units, and transformation into mathematics, and synthesis of the structure.

On the other hand, in some statements "correct answer" term was taken place. It means this response from participants is consistent with formal mathematical definition of the concept, "Incorrect answer" means it is inconsistent with formal mathematical definition of the concept.

4. Results

Analyses of participants' answers suggested four categories to probe students' understandings of a countable set and infinite set. For the first question; (a) A set whose elements are countable, (b) Bounded set, (c) A set whose elements are known, (d) Finite set categories; for the second question, (a) A set which has infinite number of elements, (b) A set whose elements are uncountable, (c) Not bounded set, (d) Other categories.

The data obtained from the concept test (third, fourth, fifth, and sixth questions) evaluated by using "existent", "non-existent", and "no answer" categories. The existence of countable finite set, uncountable finite set, countable infinite set, and uncountable infinite set was examined via these questions.

4.1 Pre-service Teachers' Understandings of Countable Set and Infinite Set

Data obtained from the "What is countable set?" and "What is infinite set?" questions in the concept test were shown in Table 1.

Table 1: Pre-service teachers' understandings of countability and infinity

Questions in the form	Categories	f	%
What is countable set?	A set whose elements are countable	12	41
	Bounded set	7	25
	A set whose elements are known	5	17
	Finite set	5	17
What is infinite set?	A set which has infinite number of elements	15	52
	A set whose elements are uncountable	6	21
	Not bounded set	4	14
	Other	4	14

Definitions made for the countable set by pre-service teachers were compounded under four categories. The findings showed that 41% of participants interpreted countable set as a set whose number of elements was countable. A student who held this conception considered that a set was countable if we could count the number of defined set's elements from its first element to infinity. Also, 25% of pre-service teachers' answers were settled in the bounded set category. These students mentioned that countable sets' elements had beginning and end points and the set's elements were in a certain boundary. 17% of them explained countable set as "a set which elements are known", stating that we know the elements of the set. Similarly, 17% of them defined as a finite set. They explained countable set as a set that has a finite number of elements.

The findings also showed that half of the participants (52%) conceived infinite set as a set having an infinite number of elements (see Table 1). For instance, a student defined the infinite set as a set that has infinitely many elements. 21% of them stated infinite set as a set that has an uncountable number of elements. One student explained infinite set as *"a set whose elements are not countable and that have not a certain boundary like natural numbers".* 4 students considered infinite set as not having a boundary. 4 students' responses couldn't be placed in these categories. Two out of three associated infinity with limit conception and one of them stated *"A set that positive and negative limits of goes to infinity"*, the other expressed *"A set whose elements go to infinity"* statements. A student also used correspondence with natural numbers *"A set whose number of elements does not correspond to any natural number set"*.

Countability and infinity concept were asked together in semi-structured interviews and selected pre-service teachers were interviewed to probe their understanding deeply. The following excerpts were from the semi-structured interviews:

Interviewer (I): What are countability, infinity, countable set concepts? Is there any relationship between them?
Student 1 (S1): I think a countable set is a logical expression of the values in the definition range.

I: Then, does the set need to be defined?
SI: Yes, the given expression must be defined. In my opinion, it doesn't have to be finite to be countable. Even if it is finite, values need to be ensured in the definition range.

I: Does the countable sets' the beginning and the end points need to be certain?
SI: I think it needs to be clear.

I: What can you say about the infinite set?
SI: I think an infinite set is the set that at least one or both boundary of is endless.

I: Real numbers within the closed interval [1,2], is this set infinite set? For example, in this set the interval is certain, that is, it doesn't go infinity.
SI: Then, it is an infinite set. We can place an unlimited number of numbers into $[1,2]$. *However, if the integer in this interval was asked, the set wouldn't be infinite.*

I: Then, Can we make a generalization for the infinite set, does the beginning and the end of it need to go necessarily infinity?
S1: What I said before is incorrect. We can comment according to the statements given.

Throughout the interview, Student 1 mentioned that the beginning and the end of a countable set need to be clear. However, initially, he expressed that in order to be an infinite set, one or both of its boundaries need to be infinite. After interviwers' questions, he realized that such a necessity is not required.

Countability and infinity concept were asked to Student 3. Likewise Student 1, Student 3 stated that the beginning and the end of a countable set have to be clear.

I: Which thoughts bring to your mind about the countability and infinity?
S3: A countable set is a set that has a beginning and an end, that is, a set we can specify in any interval. An infinite set has also a beginning and an end but it is a set we cannot know, we cannot express, we cannot limit.

I: Do the beginning and the end of the set need to be certain of being a countable set?
S3: It can be like this, for example, there are numbers from 1 to infinity, perhaps the number of elements is one million, we don't know. It could continue rhythmically and we can write according to the rhythmic continuation.

I: Namely, may it have not an end?
S3: Probably it would be the infinity when it comes to a place we do not know.

Students 3 stated that infinite set is a set that we can not determine the beginning and the end of it. In addition, he considered infinite set as a boundless set by stating: "*An infinite set has also a beginning and an end but it is a set we cannot know, we cannot express, we cannot limit.*"

Another student in the interview, Student 2, used the word mean of countable concept and his first description is *"we can count the elements of countable set."*

I: What are a countable set, an infinite set?
S2: Countable set is a set we can count the elements of.

I: Do the beginning and the end of it need to be certain of being countable?
S2: Yes, it starts with 1 because it must have at least one element and I think it goes to infinity.

I: What is an infinite set? Is a set whose beginning and end points are certain? Does it need to be unclear?
S2: In my opinion, an infinite set should be unclear. So, the beginning and the end of it is not clear. The end of it is not already clear, the beginning of it is not clear as well.

I: For example, let's we think the interval [0,1]. Get real numbers in the closed interval [0,1].
S2: There are an infinite number of real numbers, but…

I: Yes, however, the beginning and the end of it is clear.
S2: Yes, there is something so.

Initially, Student 2 defined the infinite set as a set that has not the beginning and the end points, and then he changed his idea with the example given by interviewer. In addition, he thought that countable sets' first point has to be which is different view from the Student 1 and Student 3.

Similarly, Student 4 used the word mean of countable. He also mentioned the beginning and the end points of this set have to be certain.

I: A countable set and an infinite set, what do they mean, is there any connection between them? Are these things opposite? What do you think about these concepts?
S4: A set is called a countable set if the number of its elements is countable.

I: Does the number of their elements need to be clear, then? Do the beginning and the end need to be certain to be a countable set?
S4: Yes yes, in order to be a countable set it have to be like this.

I: How will we consider the infinite set, then?
S4: The beginning point can be certain for the infinite set but the ending point or both of them can not be certain.

I: Can we count the elements of the infinite set?
S4: Yes.

Student 4 correctly defined infinite set by saying *"The beginning point can be certain for the infinite set but the ending point or both of them can not be certain."* Furthermore, he said that the elements of the infinite set are countable.

S7 explained countable set concept by giving an example: even integers. Among participants only S7 gave an accurate example.

I: What are a countable set, infinite set? Is there any connection between them? What is the relationship, if any?
S7: A countable set, that is, we can count its elements, should I give an example?

I: Yes, you can give an example.
S7: Even integers.

I: An infinite set?
S7: For an infinite set, either the beginning or the end of it must go towards infinity or both of them can go.

I: Then, let's think closed interval [0,1]. How about this set?
S7: A finite set.

I: Let me ask you the real numbers in this range, then?
S7: Infinite.

I: But there is an interval. Actually, the beginning and the end are certain.
S7: Yes, that is true.

I: Do the beginning and the end of it need to be clear, then?
S7: The beginning and the end of an infinite set doesn't need to be certain, in fact, it is important whether the elements of a set is real numbers or integer.

At the beginning of the interview, S7 made correct explanations, but for infinite set definition, he stated *"For an infinite set, either the beginning or the end of it has to go towards infinity or both of them can go."* After interviewers' intervention, he expressed *"The beginning and the end of an infinite set doesn't need to be certain, in fact, it is important whether the elements of a set are real numbers or integers."*

4.2 Pre-service Teachers' Understandings of Countable Finite Set

While 54% of pre-service teachers stated countable finite sets existed, 29% of participants expressed countable finite set were not existent. Also, four students gave no answer. Some students, who acknowledged the existence of countable, presented *"The numbers set"*, *"A = {1, 2, 3, 4, 5}, s(A) = 5"* and *"Two digits numbers set"* examples. A student connected countable finite set concept with daily life and gave *"A set which is consisting of objects that exist in the universe"* response.

The following excerpts showed correct comments about the countable finite set concept. Three students gave directly right examples. While Student 1 presented integer from 5 to 15, for example, Student 3 suggested odd natural numbers set less than 100, and similar to Student 1, Student 6 gave integer samples between 1 to 5.

I: Is there a countable finite set? If any, which example can we give?
Student 1: Let integer be given. When you say the numbers between 5 and 15 it would be considered as a countable finite set.

I: Is there a countable finite set?
Student 3: Yes, possible. A set of odd natural numbers less than 100.

I: Does countable finite set exist?
S6: Yes there is.

I: What can give as an example?
S6: Integer between 1 and 5.

Student 4 reached correct countable finite set example after a little thinking with the researcher's supports.

I: So is there a countable finite set? Can we give an example such a set?
S4: A countable finite set... Is a finite set countable? So I need to think a bit, actually.

I: For example, let's think sets. A set like this {1, 2, 3, 4, 5}. Is this set a finite set?
S4: Yes finite.

I: Is this set countable?
S4: Yes countable.

I: So it is an example of a finite and countable set.
S4: Yes, I misunderstood the question. I understood as a countable infinite set.

Student 5 presented {0, 1, 2} set whose elements were certain as an example but further explanations of him shows that he explained countable infinite set as a set whose first and last element were certain.

I: Let's give a countable finite set example. Is there available such a set?
S5: Many sets might be appropriate.

I: For example?
S5: A set contains 0, 1, and 2 elements or all sets that have a beginning and last points.

Based on interviews conducted with students, it was concluded that all of the students stated that countable finite sets were existent. Similarly, the results obtained from the written instrument indicated that the majority of students thought that countable finite set was available.

4.3 Pre-service Teachers' Understandings of Countable Infinite Set

The half of the students gave correct answers and stated that countable finite sets were existent. About 38% of students reported that countable infinite sets were not existent. The rest of the students (12% of students) could not answer the question. Furthermore, the participants, who accepted the existence of the countable infinite set, gave *"Natural number set"*, *"Prime numbers are countable and infinite"*, and *"Integers set"* examples. Some students', whose opinions are this kind of set did not exist, explanations are such as *"Non-existent. We can not count the infinite sets' elements"*, and *"In my opinion, it is not possible. If we can count the set, it is finite."*

The students' ideas on countable infinite set supported the data obtained from the written instrument-countability and infinity concept test. Excerpts from the students' interviews also illustrated these findings. For instance, S1 gave correct example for a countable infinite set.

I: What is countable infinite set?
S1: Real numbers set from 5 to positive infinity or from -5 to negative infinity are countable infinite sets.

I: Is it right in real numbers set?
S1: Not in real numbers in the integers.

I: What is this set, if the elements of the set are real numbers?
S1: Uncountable infinite set.

Student 1 thought that if a set was infinite, one boundary of it had to be infinite. In addition, he was initially unclear about which of the number sets (integers or real numbers) has to be within the specified set range, then he decided integer has to be. Thus, he gave right example for countable set. Furthermore, in the same dialogue he determined an example of uncountable infinite set. As another example, when asked how he would explain both countable and infinite set, his answer was as follows:

I: How can a set be both countable and infinite?
S2: I don't know but let me think. We say that let it go from 1 to infinity, namely, the elements from 1 to positive infinity. We can count up to a point, but after a while, we can not continue. I do not know this countable infinity.

Similar to Student 1, Student 2 determined a set whose one interval was infinite for infinite set. However, he couldn't give countable infinite set example.

I: Countable infinite set?
S3: Again, even natural numbers, we know how to proceed after a while but we can not write because of not having an end point.

By giving an example of even natural numbers set, Student 3 presented right countable infinite set example.

I: Does a countable infinite set exist?
S6: Yes, it is existent. In statistics course, the teacher defined integers set as an infinite set; because in any case, the elements of them were matching with any numbers.

Since Student 6 learned some information about this concept before, he stated that such a set could be available. However, he couldn't give set example.

I: Can you give an example of countable finite set? Is there such a set? If it is existent - not have to be- how is the set?
S4: Countable finite set is existent but infinite I don't know… That is to say, in my opinion, if it exists, it can not count or I can not think such an example.

I: Is there a countable infinite set example?
S5: According to me, it couldn't be available. Because we knew how many it was and we could count but this time, it is not infinite there is a conflict.

I: So do all of the elements of the set have to be certain?
S5: Yes.

Student 4 and Student 5 expressed countable and infinite set couldn't be available together. Therefore, he couldn't give countable infinite set example. Student 5 also stated that all elements of a countable infinite set had to be certain.

Two out of seven interviewed students gave incorrect comments about countable infinite set; expressing countable infinite set example was not existent. Five pre-service teachers stated that countable infinite set was existent and attempted to give examples. Finding obtained from concept test also indicated that 50% of participants accepted countable infinite set was existent.

4.4 Pre-service Teachers' Understandings of Uncountable Finite Set

Most of the students (62%) gave correct answers and expressed that such a set was not existent. Some of these students stated that this type of set was not existent because the elements of the finite sets were certain and countable. Other students, who also found the correct answer, suggested that uncountable finite set did not exist and they explained their views as: *"If it is uncountable it is considered that it has not the final point"*. Five pre-service teachers also, expressed uncountable finite set was existent. These students, without explaining in detail, presented real numbers from 1 to 10, rational numbers and negative integer examples. In addition, four students gave no answer.

At the end of the interviews, while two pre-service teachers stated that uncountable finite set was existent, five pre-service teachers expressed there was not such a set. For example, responses from S1 and S2 were as follows:

I: Is there an uncountable finite set example?
S1: Integer from 5 to 6, a positive or negative integer from 5 to 6.

I: Uncountable finite set?
S7: The uncountable finite set can be existent. For example, integers from 0 to 1 were uncountable because there was no integer in this interval.

Student 1 and student 7 stated that an uncountable finite set example was existent. Since there were not any elements in the interval, they thought empty set as the uncountable finite set. On the other hand, Student 2 presented incorrect example for uncountable finite set.

I: Does uncountable finite set exist? It will not be counted and it will be finite.
S2: Real numbers in the open interval (2, 3). There was an interval, finite; there was 2 to 3 open interval. We can not count the numbers in it. Because it is too many.

Student 2 gave an uncountable finite set example. He thought real numbers in (2, 3) interval and could not count the elements therefore he gave an infinite set for finite set example.

After interviewers' intervention S3 realised that his example was a countable infinite set example.

I: Uncountable finite set?
S3: It cannot be counted but finite… Real numbers from 0 to 10. An interval of it is certain but expands continuously. We can reduce the real numbers set. However, I don't know it is uncountable finite or countable infinite set.

I: You chose (0, 1) interval, what kind of set real numbers in (0, 1) interval?
S3: Countable infinite.

Student 3 attempted to provide a sample for the uncountable finite set but then he decided that while trying to find examples, the given sample was a countable infinite set. The set was real numbers set but he still thought it was a countable set.

Student 4 thought that if a set was finite, it was countable. From this point of view he decided uncountable finite set did not exist. Student 7 stated directly this kind of set was not possible.

I: Ok. An uncountable finite set? Both it can not be counted and it will be infinite.
S4: If it is finite, I think it is countable.

I: Namely, you say that such a set is not available.
S4: Yes.

I: Is there uncountable finite set?
S7: I think, it is not possible.

Student 4 and student 7 gave the correct answer and stated that uncountable finite set was available.

4.5 Pre-service Teachers' Understandings of Uncountable Infinite Set

Fourteen pre-service teachers answered this question incorrectly. Their view was that uncountable infinite set couldn't be existent. Nine students also stated that such a set could be available. They gave irrational numbers, real numbers, and rational numbers examples. Six students made no comments. In addition, one student presented a different point of view from daily life by stating *"Stars in the universe are infinite and uncountable."*

Unlike the results obtained from the paper-pencil test, in the semi-structured interviews, pre-service teachers expressed uncountable infinite sets were available. Five students gave uncountable infinite set examples. For instance, Student 2 gave $(-\infty, +\infty)$ set for uncountable infinite example and Student 3 gave real numbers example.

I: Uncountable infinite set?
S2: It is already infinite set directly. The interval is from negative infinity to positive infinity.

I: What is uncountable infinite set?
S3: Real numbers. We can both enlarge and reduce the real numbers set.

Student 2 and Student 3 gave correct uncountable infinite set examples by thinking directly infinite sets. Similarly, S4 and S5 stated that an infinite set was uncountable set.

I: Well, uncountable infinite set?
S4: If a set is an infinite set it is already uncountable.

I: Generally, how can you give an example set?
S4: How a set? $(-\infty, +\infty)$ *open interval. We can't count xs in this interval.*

I: Ok, uncountable infinite set?
S5: Since infinite sets are uncountable, they are infinite. In my opinion, it may be for this reason.

I: Can you give an example for uncountable infinite sets?
S5: From negative infinity to positive infinity, because of being not counted.

Student 4 and Student 5 stated infinite sets already can't be countable. Thus, he thought the interval whose boundaries go to infinity was an infinite set. S7 presented real numbers in $(0, +\infty)$ interval as an example set.

I: An uncountable infinite set?
S7: It is available but I can't give such an example now.

I: Numbers won't be able to count and will be infinite. Think infinite sets. But they can't be counted.
S7: Then real numbers from 0 to positive infinity can be.

Student 7 was initially unable to provide any examples for the uncountable infinite set but then by thinking real numbers in (0, ∞) interval, he gave correct example set. Student 1 attempted to give an example for uncountable infinite set, but he couldn't reach correct example.

I: Can you give an uncountable infinite set example?
S1: Integers from negative infinity to positive five were an example of the uncountable infinite set.

The example given by Student 1 for the uncountable infinite set was a sample for the infinite set but elements in the determined set were integers, so this set was countable.

5. Discussion

The purpose of the present study was to examine pre-service mathematics teachers' understandings of countability and infinity concepts in WebQuest Based Learning Environment.

In view of the results, it can be concluded that although pre-service mathematics teachers studied these concepts in General Mathematics and Calculus courses, there were some inconsistencies between the concept definitions and pre-service mathematics teachers' understandings and specific difficulties related to the existence of countable finite, uncountable finite, uncountable finite, and uncountable infinite sets examples. In addition, it can be said that some common examples were existent given by students. For instance, natural numbers set was given as a countable infinite set example.

Although pre-service teachers have already become familiar with the concept of countability and subsequently performed tasks related to this concept in a WebQuest-supported learning environment, almost all of the students could not comprehend the countability concept accurately. The findings revealed that pre-service mathematics teachers participating in the study had no formal understanding regarding countable sets. Neither in written instrument nor during the interviews, no students benefitted from the mathematical definition of countable sets. They tried to make interpretation from the word meaning of countability. In general, they considered countable sets as sets that we could count the elements of it. In addition, countable set was considered as finite, bounded set, and some students considered that its elements were known. The student ideas on countable set were supported from the data obtained during the interviews. Hence, pre-service mathematics teachers considered "countable set" concept as a mathematical concept to be far from the mathematical sense. Some interpretations in this manner also directed them into the wrong way. For instance, the students, who thought countable sets' beginning and end points were existent, didn't accept countable infinite sets. They tended to think infinite sets as uncountable sets. Understandings about countability were limited and their understanding was based on students' daily life experiences. Similar results were reported by Ünan and Doğan (2011) related to infinity concept. There were no students that related the concept of countability with finite set and equivalent concept. Therefore, participants don't have the idea that a set is countable if a set is finite or corresponds with natural numbers set, namely it is equivalent to natural number set.

As in countability concept, pre-service math teachers' responses were based on the meaning of the infinity word such as "a set has infinite elements", and "a set whose elements are countable". In addition, interviews revealed that students thought infinite set as a set whose one or both border goes positive or negative infinity. There were no students who benefitted from mathematical definition of finite set. Students' responses were intuitive about infinity, even though at the university level. Singer and Voica (2003) stated similarly that infinity was felt intuitively in number sets. Moreover, Mamolo and Zazkis (2008) and Tirosh (1999) stated that students with mathematical background gave similar answers about infinity concept. Since students could not observe the concept in real life, they used their common sense perceptions. The findings revealed that students' sentences still depend on infinity concept's daily life meanings. Çelik and Akşan (2013) reported that since students didn't complete a course relating infinity concept directly, their responses did not coincide with the actual infinity.

More than half of participant students (54%) believe that countable finite set were existent. Therefore, the majority of students' responses were consistent with the mathematical definition of it. For instance, "numbers set composed of 0, 1,…, 9 was finite" was an example from a pre-service teacher. Due to the mathematical definition of countable set (Lipschutz, 1965), countable set has to be already finite set; thus countable finite set is existent. Numerous examples can be given. In addition, integers from 5 to 15, or even natural numbers less than 100 were given as examples.

Half of the students stated that countable infinite sets were existent. Although ideas about countability and infinity concepts were conflicting with each other, received responses in written instrument showed that their assessments were correct. Students, who considered that if a set was an infinite set it couldn't be counted, were also existent. Some students attempted to explain infinity concept via uncountability in line with the previous research (Narli & Narli, 2012). Since the students didn't comprehend the mathematical definition of the countability concept, their understanding was like that. One of the students who participated in interviews (S1) concluded that in determining whether a set was countable or not the set members were important. Because two sets whose borders are the same can be countable or uncountable according to their elements. For example, closed interval $[0, 5]$ set is countable set if its elements are integers; this set is uncountable set if its elements are real numbers. Based on the response of S6 about countable infinite set, the student gave an intuitive response since he couldn't remember accurately what it was.

More than half of the students (62%) considered that uncountable finite set were not existent. These responses were correct because being both finite and uncountable set was contrary to the mathematical definition of countability and infinity concepts (Lipschutz, 1965). Set examples given by some students for uncountable finite set were other type of set samples. For instance, an example of a student was real numbers from 1 to 10, but that set was uncountable infinite set thus it was not appropriate for uncountable finite set. Another example given during the interview was integers from 5 to 6, that set was an empty set and student thought that it was an uncountable set; also the boundaries of the set was certain therefore he stated that it was a finite set. However, according to finite set definition empty set was a finite set so that example was not suitable.

In the present study, to probe students' understanding of infinity and countability, semi-structured interviews conducted with selected participants because the questions in the written form were very direct. In order to obtain detailed information about the students' understanding, different questions, sometimes also leading questions, were asked. Some students modified their understandings. In addition, some students mentioned their experiences in the courses. For example, S6 expressed that "*in statistics course, the teacher defined integers set as an infinite set; because in any case, the elements of them were matching with any numbers*".

In the present study, all participants completed successfully General Mathematics, Abstract Mathematics, and Analysis I-II-III courses including infinity and countability concepts. Therefore, they came across the mathematical definition of the concepts before. However, some understandings of them were inconsistent with formal concept definitions. For example, the majority of the participants stated that uncountable infinite sets were non-existent. We may say, in some cases, there is a gap between the concept definition and the concept image.

As mentioned in the introduction part, understandings of mathematics teachers affect their students' understandings, it is important to have accurate understandings of infinity and countability for pre-service mathematics teachers. Yanik and Flores (2009), Yanik (2014), Narli (2011), Dede and Soybaş (2011) have emphasized that in order to make the right decision about students' mathematical understanding, being aware of their understandings is necessary for educators, especially mathematics teachers.

The WebQuest based learning environment enabled students to explore the concepts which are quite abstract more detailed. If the understandings of the students' about these concepts were questioned before the WebQuest based learning environment was carried out, the shallower data could be obtained. Because, despite the WebQuest activity, there are mathematical coherence in the students' understandings.

Data analysis was not made according to students' academic levels, genders, etc. in the present study. However, pre-service teachers' academic records of three years were used as criteria for selection of the interviews.

6. Implications

According to the present research, pre-service mathematics teachers' understandings about the countability and infinity concepts were away from their mathematical meanings. Therefore, prior to entering the teaching profession, it may be useful to have the correct understandings about these concepts for pre-service math teachers. These concepts can be focused on more in general mathematics courses at undergraduate level and the two concepts can be taught by associating with each other. These concepts require students to think at higher levels and it is suitable for undergraduate level students. Students will meet again with these concepts in elementary number theory course when they get to fourth grade. Accurate understanding can be created in the course with accurate reasoning. It may be the research subject for students who want to continue their academic experience with master or PhD. In this study, infinity was associated with countability and the students were asked to give some examples from number sets. In further studies, place of infinity in topics of limit, derivative and integral can be searched and the meaning of these concepts can be investigated. The understanding of limit concept is especially important for undergraduate level. For example, due to lack of accurate knowledge about these concepts, pre-service math teachers attempt to calculate limits, whose values going to positive or negative infinity. In order to comprehend this situation, students have to have accurate understandings of infinity.

Since Webquest-supported activities have influenced the learners' understanding positively, this study examined the understandings of students after the application. However, since this study was not an experimental study, the effectiveness of WebQuest was not evaluated directly. Therefore, experimental studies can be carried out for further research.

These types of technology-aimed activities should be included more frequently so that the learners can control their own knowledge.

Additionally, infinity and countability understandings of students, who are enrolled in the department of mathematics, in science and literature faculty can also be researched and comparisons can be made with the faculty of education students.

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