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JORDAN FORMS OF REAL AND COMPLEX MATRICES UNDER RANK ONE PERTURBATIONS

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Abstract. New perturbation results for the behavior of eigenvalues and Jordan forms of real and complex matrices under generic rank one perturbations are discussed. Several results that are available in the complex case are proved as well for the real case and the assumptions on the genericity are weakened. Rank one perturbations that lead to maximal algebraic multiplicities of the "new" eigenvalues are also discussed.

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