

# Channel Impulse Response Equalization based on Genetic Algorithm in Mode Division Multiplexing

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**Abstract**—This paper proposed to use genetic algorithm (GA) as an adaptive algorithm for mode division multiplexing (MDM) equalization in order to minimize the mean square error as well as to maximize the similarity between the ideal signal and the MDM distorted signal. A significant result has been obtained of implementing GA on MDM equalization compared to other conventional algorithm such as least mean square (LMS) which is used dominantly in current equalizations.

**Index Terms**—Mode Division Multiplexing; Equalization; Genetic Algorithm; Mode Coupling.

## I. INTRODUCTION

The exponential rate of increase in Internet-driven demand in recent years is leading to the nonlinear Shannon limit in single mode fibers (SMFs) being approached [1]. Therefore, the need for new technologies and subsystems is necessary to cost-effectively increase capacity in a single fiber as well as in multimode fiber.

Recently, an additional multiplexing technique that is actively being investigated to overcome the capacity limit is mode division multiplexing (MDM). In MDM, multiple spatial modes over multimode fiber (MMF) has been used [1-6], using spatial modes to transfer the data as independent channels accordingly, a larger transmission capacity with respect to conventional MMFs can be achieved.

The imperfection in MMF manufacturing such as microbending lead to the coupling between different modes where modes tend to interchange the power and causes a delay in receiving modes and disparity of power distribution between modes these undesirable phenomena lead to ISI [7]. For that, this undesirable effect of ISI causes neighboring symbols to interfere with each other at the receiver [8-16]. In addition, the received signal is wrongly decoded as the receiver cannot predict the correct form of the wave guides. As a result, this will cause higher bit error rate and reduce data rate of MMF. Recently, multiple-input-multiple-output (MIMO) digital signal processing (DSP) has been successfully applied to enable MDM in transmission experiments.

This paper proposed a new equalization scheme for MDM system based on GA as an adaptive algorithm to mitigate mode coupling. GA based equalization implementation to mitigate the ISI effects on the MDM signals. The current algorithms which are used in MDM equalization are basically based on the conventional algorithms such as LMS and RLS these algorithms suffer from some problems; the main drawback of the LMS and RLS algorithms is that it is

sensitive to the scaling of its input. This makes it very hard (if not impossible) to choose a learning rate that guarantees stability of the algorithm as well as the disability to solve the nonlinear problems [17], for these reasons GA based equalization is proposed to solve the ISI problem and overcome the conventional algorithm limitations.

This paper organized as follows, the MDM is presented in Section II. Section III presents the analytical model. Section IV presents the MDM channel equalization. Section V presents the results and discussion. Section VI presents the performance evaluation of GA against other algorithms and the paper conclusion is presented in Section VII.

## II. MODE DIVISION MULTIPLEXING MODEL

MDM has been modeled to identify the ISI problem practically in order to collect the distorted signal from it; MDM is modeled in OptiSystem 7.0, as shown in Figure 1. Main elements for the model starting from the transmitter, Spatial optical transmitter is used which contain the PRBS, the coder which responsible for the signal generator for each channel, spatial VCSEL. Three channels transmitted over the MMF the transmitted modes are two LG01 and one LG00 over 1550 nm wavelength and 40Gbits/s data rates. Spatial connector is used to connect the MMF spatially, this component connects signals with transverse mode profiles. Modes can be translated and rotated. The output modes from the first MMF will be as an input for the second MMF and the something with the third MMF until reaching the receiver, spatial optical receiver is used this component is an optical receiver subsystem built using the spatial Aperture and the optical receiver components such as a PIN or APD photodetector, a Bessel filter and a 3R regenerator. The received modes are 5 channels which increased because of the mode coupling phenomena.

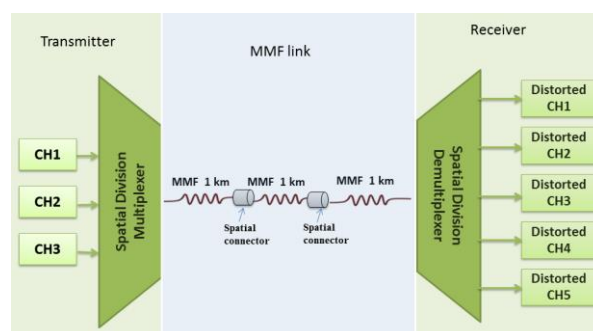


Figure 1: MDM model

### III. ANALYTICAL MODELING FOR MDM CHANNEL EQUALIZATION

In MDM system, the detrimental effects of ISI may be mitigated by equalization. It is one of the earliest techniques to alleviate ISI, since the channel is time shifting. an equalization evens out the signal in a manner in which the equalizer adjusts itself depends on the changing channel the adaptive equalization will be used for this purpose [6].

Many equalization schemes have been investigated for MDM equalization. Equalization scheme based on least mean square algorithm (LMS) and Recursive least square algorithm (RLS) are used to solve the linear distortion of the transmitted signals. The structure of the adaptive channel equalizer based on LMS algorithm is shown in Figure 2. As illustrated in Figure, the received signal  $y(n)$  is different from the original signal  $x(n)$  because it was distorted by the overall channel transfer function  $C(z)$ , which includes the transmit filter, the transmission medium, and the receive filter. To recover the original signal  $x(n)$ , we need to process  $y(n)$  using the equalizer  $W(z)$ , which is the inverse of the channel's transfer function  $C(z)$  in order to compensate for the channel distortion. That is, we have to design the equalizer

$$w(z) = \frac{1}{c(z)} \tag{1}$$

such that  $x^{(n)} = x(n)$ . An adaptive filter requires the desired signal  $d(n)$  for computing the error signal  $e(n)$  for the LMS adaptive algorithm. During the training stage, the adaptive equalizer coefficients are adjusted by transmitting a short training sequence. This known transmitted sequence is also generated in the receiver and is used as the desired signal  $d(n)$  for the LMS algorithm. After the short training period, the transmitter begins to transmit the data sequence. In the data mode, the output of the equalizer.  $x^{(n)}$  is used by a decision device to produce binary data. Assuming that the output of the decision device is correct, the binary sequence can be used as the desired signal  $d(n)$  to generate the error signal  $e(n)$  for the LMS algorithm. The signal samples at the equalizer input are of the form:

$$Y(n) = \sum_{j=0}^{n-1} h(j)x(n-j) + v(n) \tag{2}$$

where  $x(n)$  denotes the data sample at time index  $n$ ,  $v(n)$  is the additive noise with the variance  $2\sigma$ , and  $h(j)$  is the channel impulse response. The data samples take on values of  $x(n) = \pm 1$ , and the noise is assumed to be independent.

The equalizer output is:

$$\hat{x}(n) = \mathbf{w}^T(n)\mathbf{x}(n) \tag{3}$$

where  $x(n) = [x(n), x(n-1), x(n-2), x(n-N+1)]^T$  is the vector of data sample at the equalizer input, and  $w(n) = [w(n), w(n-1), w(n-2), w(n-N+1)]^T$  is the vector of weighting coefficients of the adaptive filter.

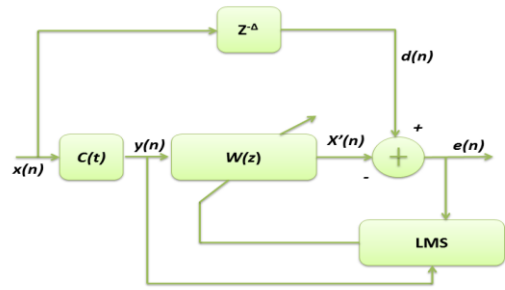


Figure 2: LMS channel equalizer

The output  $\hat{x}(n)$  is used in estimating the transmitted data symbol  $x(n-k)$ , with  $k$  denoting the delay. The  $n$ -th output error sample is:

$$e(n) = \hat{x}(n) - x(n-K) \tag{4}$$

The weighting coefficients in the LMS algorithm are updated according to the following expression:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e^H(n)\mathbf{x}(n) \tag{5}$$

where  $\mu$  is the step size which controls the rate of convergence of the LMS algorithm. The output means square error (MSE) is:

$$\begin{aligned} \varepsilon(n) &= E[e^2(n)] = \mathbf{w}^T(n)\mathbf{R}(n)\mathbf{w}(n) \\ &+ E[x^2(n)] - 2\mathbf{w}^T(n)E[\hat{x}(n)x(n-K)] \end{aligned} \tag{6}$$

where  $R = \frac{1}{N} \sum_{n=1}^N x(n)x(n)$ . The average output MSE after  $n$ -th iteration can be expressed as:

$$\varepsilon_{avg}(n) = \varepsilon(n) + E[V^T(n)RV(n)] \tag{7}$$

where  $\varepsilon(n)$  is the minimum MSE as given by (6) for optimal weighting coefficients vector  $w_{opt}(n)$ . In contrast to the LMS algorithm, the RLS algorithm uses information from all past input samples (and not only from the current tap-input samples) to estimate the (inverse of the) autocorrelation matrix of the input vector. To decrease the impudence of input samples from the far past, a weighting factor for the impudence of each sample is used. The limitation of using LMS and RLS equalization scheme are the disability of solving the nonlinear distortion as well as the instability where by using these algorithms the filter weights do not reach to their optimum values due to the mean square error (MSE) being trapped to local minimum, for that purpose GA based equalization has been proposed to solve the ISI as shown in the following sections.

### IV. GA BASED MDM EQUALIZATION

The GA implements a multi-objective optimization approach, the general process of GA can be illustrated in Figure 3. In this work, there are two discrete signals: a reference signal which is Gaussian signal and a distorted signal which we get it from the MDM simulation, and the main target of using GA is to rearrange and to change values

of the distorted signal to get a MSE value between the reference signal and distorted signal less than a predefined limit with minimum change in values of the distorted signal. In order to achieve this target, the work has been split into two stages each stage will achieve one objective: rearranging the distorted signal and then changing values of the distorted signal.

*A. The first stage*

In order to rearrange the distorted signal, careful analysis of the proposed problem has been done. Let’s assume that the length of the signals is n, so the search space must be n. The bigger the length is, the bigger the search space is. Therefore, a search algorithm must be used because it is impossible to check the whole search space in numerous cases. GA is a great heuristic search algorithm, and it has been used in this research. In order to use GA, crossover, mutation, objective function, and structure of the chromosome must be defined. The chromosome is an array of two elements representing two indices from the distorted signal (two elements to swap). The value of each one of these two elements must be integer value in the range [1: n]. The objective function of the first stage is the MSE value between the Gaussian signal and the distorted signal after swap operation. GA has been used in repeated manner to determine the best swap operations minimizing the MSE value as low as possible. This stage finishes when the MSE value converges into a constant value, and the output of this stage is the “new arranged distorted signal”.

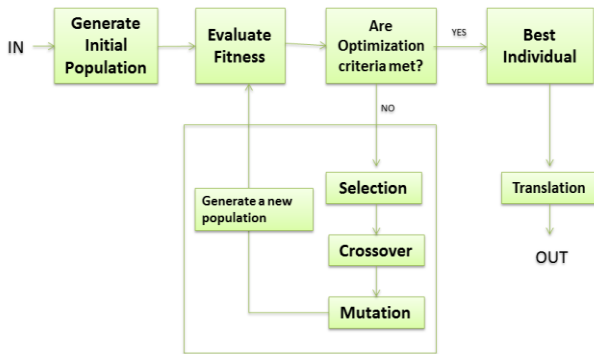


Figure 3: GA main processes

*B. The second GA stage*

After minimizing the MSE value as low as possible by rearranging the signal elements, the second stage starts. The output of the last stage constitutes with the Gaussian signal the input to this stage.

The objective function of the second stage is to minimize the MSE by changing some of the value of elements of arranged distorted signal produces a new signal with the name “Equalized signal”, and must guarantee that the MSE value between the changed distorted signal and the Gaussian signal is less than the allowed limit, and the MSE value between the arranged distorted signal and the changed distorted signal is as small as possible (i.e. minimizing the distortion in the arranged distorted signal). The chromosome is an array of n elements representing the values which must be added to or subtracted from the arranged distorted signal to minimize the MSE. To determine the range of these elements, refer to the following equations:

$$Difference = Gaussian\ signal - Arranged\ distorted\ signal \quad (8)$$

For each element, I in the chromosome:

If the difference (I) is equal to zero then Lower bound = 0 and higher bound = 0, If the difference (i) is bigger than zero then:

$$Lower\ bound = difference\ L(i) * (1 - exp(-Const * Max\_limit)) \quad (9)$$

and higher bound = difference (I) , If the difference (I) is smaller than zero then Lower bound = difference (I) and:

$$Higherbound = difference(i) * (1 - exp(-Const * Max\_limit)) \quad (10)$$

where Lower bound and higher bound constitute together the required range, exp is the exponential function, cost is a constant chosen as needed, maxLimit is the maximum allowed limit of the MSE value and the final equalized signal is computed from the following equation:

$$Equalized\ signal = Arranged\ distorted\ signal\ Chromosome \quad (11)$$

Optimization problem must satisfy the following constraints: MSE between the Gaussian signal and the equalized signal < maxLimit (maxLimit is predefined). Objective function is minimizing the MSE value between the Gaussian signal and the arranged distorted signal if the previous constraint is satisfied or the fitness value will be infinity. When the maxLimit variable is chosen to be very small, the range of value of chromosome elements must also be small in order to guarantee the convergence of GA into suitable solution (i.e. searching between more effective solutions making the distorted signal more similar to the reference signal), and that is the main reason for using the exponential function and constant in lower bound and higher bound equation. If decrease the maxLimit, increase the constant until you get a suitable solution.

V. RESULT AND DISCUSSION

The distorted signals which collected from the MDM system are compared with the Gaussian signals to compute the MSE. The SSIM for the distorted signal before the equalization is shown in Figure. 4. The MSE is high and the pulse shape for the distorted signal is broadening. While the LMS and RLS results show a slight improvement in both MSE and in the pulse shape similarity the LMS main parameters and results can be summarized below in Table 1 and Table 2.

Table 1  
RLS parameters and results

| CH  | FIR weight length | RLS forget factor | RLS initialization parameter | MSE    | SSIM  | CPU time in sec |
|-----|-------------------|-------------------|------------------------------|--------|-------|-----------------|
| CH1 | 18                | 0.54              | 0.999                        | 0.0327 | 0.303 | 0.4836          |
| CH2 | 22                | 1                 | 0.55                         | 0.0607 | 0.280 | 0.4368          |
| CH3 | 55                | 0.99              | 0.912                        | 0.0441 | 0.435 | 0.4680          |
| CH4 | 18                | 0.9               | 0.12                         | 0.0325 | 0.557 | 0.4872          |
| CH5 | 18                | 0.8               | 0.912                        | 0.0590 | 0.561 | 0.4684          |

Table 2  
LMS parameters and results

| CH  | FIR weight length | LMS step size | LMS leakage factor | MSE      | SSIM   | CPU time |
|-----|-------------------|---------------|--------------------|----------|--------|----------|
| CH1 | 18                | 1             | 0.03               | 0.03086  | 0.5157 | 0.405    |
| CH2 | 18                | 0.8           | 0.03               | 0.014014 | 0.5926 | 0.374    |
| CH3 | 18                | 0.2           | 0.03               | 0.0576   | 0.5527 | 0.405    |
| CH4 | 18                | 0.39          | 0.03               | 0.0465   | 0.4852 | 0.390    |
| CH5 | 18                | 0.9           | 0.03               | 0.014987 | 0.6942 | 0.421    |

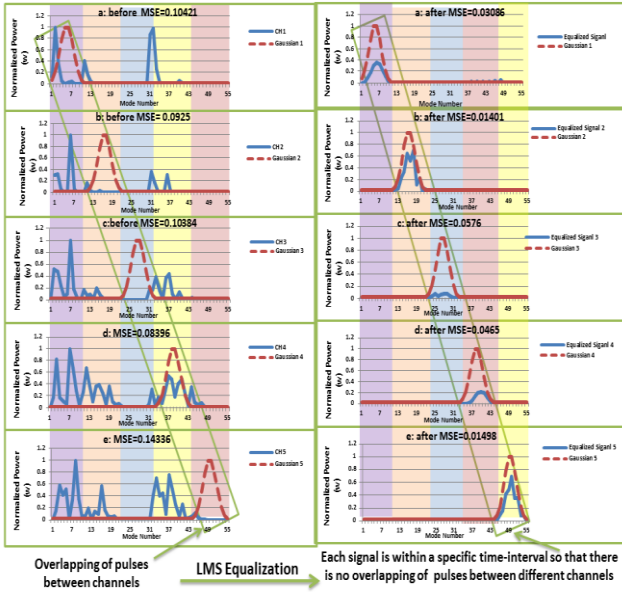


Figure 4: Distorted signal compared with Gaussian signal

From the Figure 5 and 6 it can be seen that LMS successfully solve the ISI problem while RLS fail where there is still overlapping between the channels and in terms of pulse shape both LMS and RLS couldn't recover the signal to be as optimal as Gaussian signal.

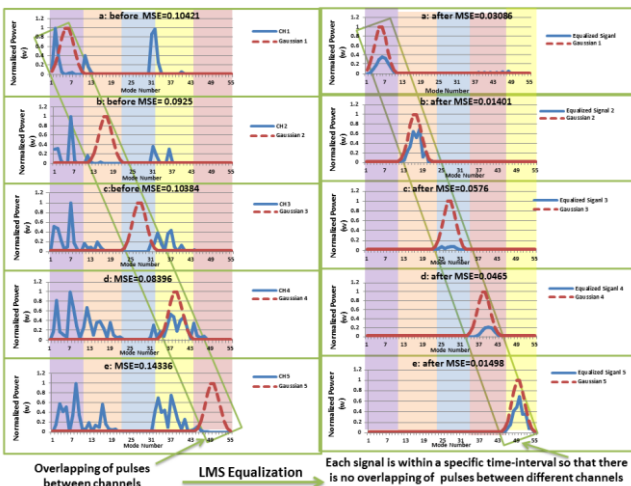


Figure 5: LMS running results

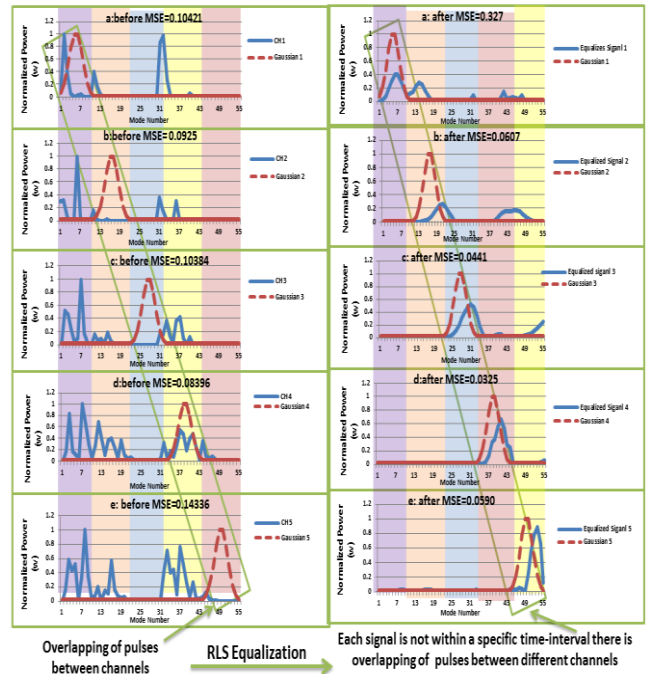


Figure 6: RLS running results

## VI. EVALUATION PERFORMANCE

In this section, the performance of GA based equalization will be compared with the performance of LMS equalizer based on the following evaluation measurements:

### A. Mean Square Error (MSE)

Table 3 shows the comparison of MSE for 5 channels between LMS and GA. from the Figure it is obviously can be seen that the best MSE is obtained from GA based equalization which successfully minimizes the MSE to be almost 0 for the five channels while LMS also minimize the MSE but very slight which make GA exceed the LMS in term of minimizing MSE.

Table 3  
MSE comparison for LMS, RLS and GA

| Channel | MSE before equalization | MSE after LMS equalization | MSE after RLS equalization | MSE after GA equalization |
|---------|-------------------------|----------------------------|----------------------------|---------------------------|
| CH 1    | 0.10421                 | 0.03086                    | 0.0327                     | 0.00007452                |
| CH 2    | 0.0925                  | 0.01401                    | 0.0607                     | 0.00007592                |
| CH 3    | 0.10384                 | 0.05761                    | 0.0441                     | 0.00006215                |
| CH 4    | 0.08396                 | 0.04658                    | 0.0325                     | 0.00006974                |
| CH 5    | 0.14336                 | 0.01502                    | 0.0590                     | 0.00006972                |

Figure 7 shows the comparison between the performance of LMS and the performance of GA in terms of minimizing the MSE.

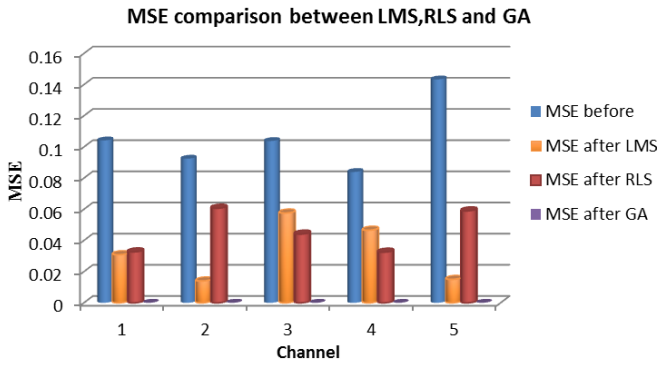


Figure 7: MSE comparison for LMS, RLS and GA

**B. SSIM**

The implementation of LMS and GA gave 5 equalized channels each channel are compared with Gaussian channel to evaluate the similarity index between them from Table 4 which shows the SSIM comparisons between LMS and GA and from the Figure it can easily see that the GA successfully shape the output signal to be almost the same as the Gaussian the SSIM index improved from around 0.1 to be almost the same where the GA record almost 0.9 similarity between the two signals for the all channels while the LMS reduce the difference gap between the distorted signal and the Gaussian signal but very slightly , generally the GA is the best which can be seen clearly in Figure 8.

Table 4  
SSIM comparison between LMS, RLS and GA

| Channel | SSIM before equalization | SSIM after LMS equalization | SSIM after RLS equalization | SSIM after GA equalization |
|---------|--------------------------|-----------------------------|-----------------------------|----------------------------|
| CH 1    | 0.1271                   | 0.5157                      | 0.3033                      | 0.9407                     |
| CH 2    | 0.0817                   | 0.5926                      | 0.2805                      | 0.9450                     |
| CH 3    | 0.1001                   | 0.5527                      | 0.4354                      | 0.9529                     |
| CH 4    | 0.1850                   | 0.4852                      | 0.5571                      | 0.9525                     |
| CH 5    | 0.0664                   | 0.6942                      | 0.5611                      | 0.9498                     |

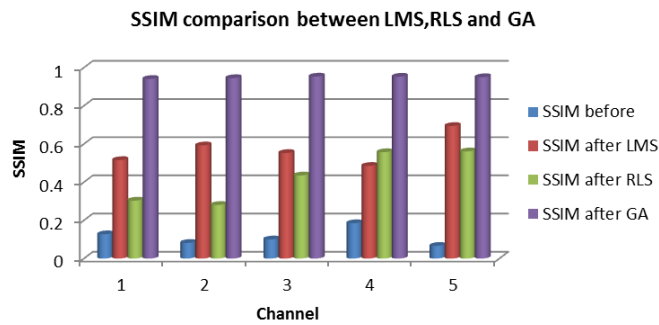


Figure 8: SSIM comparison for GA, LMS and RLS

**C. CPU time**

The CPU consumption is compared between the three algorithms where the GA consider the most time-consuming comparing with LMS which consume around 0.5 sec to get the result per channel while the GA exceed the 2 minutes to produce the result per channel, the GA is the worse in terms of CPU time consuming, Table 5 and Figure 9 shows the comparison between LMS and GA in terms of CPU time consumption.

Table 5  
CPU time comparison for GA and LMS

| Channel | CPU time for LMS equalization | CPU time for RLS equalization | CPU time for GA equalization in sec |
|---------|-------------------------------|-------------------------------|-------------------------------------|
| CH 1    | 0.4056                        | 0.4836                        | 120.2456                            |
| CH 2    | 0.3744                        | 0.4368                        | 124.5356                            |
| CH 3    | 0.4056                        | 0.4680                        | 124.0542                            |
| CH 4    | 0.3900                        | 0.4872                        | 142.8969                            |
| CH 5    | 0.4122                        | 0.4684                        | 129.7441                            |

\* CPU times are measured under similar conditions

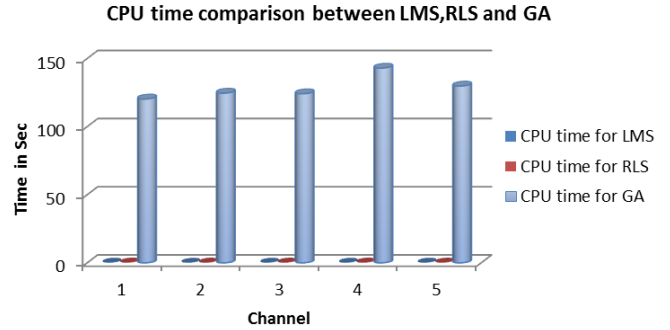


Figure 9: CPU time comparison for GA and LMS

**VII. CONCLUSION**

This paper proposed an equalization scheme for MDM system based on using GA the results compared with the LMS and RLS results based on performance measurement MSE, SSIM and CPU time, it is proven that GA is the best on solving ISI problems comparing with RLS and LMS even if it is slower than them but the improvement in the MSE and SSIM is very good which makes using GA effective compared with the traditional algorithms such as LMS and RLS.

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